

## INTRODUCTION TO PROBABILITY

### HOMEWORK 2

#### Problem 29

- a) The experiment is randomly selecting two jurors from a group of two women and four men. A sample point is selecting one of the men and one of the women.
- b) Given that we have  $W_1, W_2, M_1, M_2, M_3, M_4$  the sample space is:
- $$\begin{aligned} E_1 &= \{W_1, W_2\}, & E_6 &= \{W_2, M_1\}, & E_{10} &= \{M_1, M_2\}, & E_{13} &= \{M_2, M_3\}, & E_{15} &= \{M_3, M_4\}. \\ E_2 &= \{W_1, M_1\}, & E_7 &= \{W_2, M_2\}, & E_{11} &= \{M_1, M_3\}, & E_{14} &= \{M_2, M_4\}, \\ E_3 &= \{W_1, M_2\}, & E_8 &= \{W_2, M_3\}, & E_{12} &= \{M_1, M_4\}, \\ E_4 &= \{W_1, M_3\}, & E_9 &= \{W_2, M_4\}, \\ E_5 &= \{W_1, M_4\}, \end{aligned}$$
- c) The probability that both of the jurors are women is:  $P(E_1) = 1/15$

#### Problem 37

- a) Since order does matter, the business women can travel in  $6 * 5 * 4 * 3 * 2 = 720$  ways.
- b) The woman either travels to Denver first or San Francisco first. Since we only have two outcomes the probability is .5.

#### Problem 55

- a) Since the order the nurses getting picked does not matter, we can use combinations:

$$\binom{90}{10} = \frac{90!}{10! 80!} = 2.07 \times 10^{18}$$

- b) To get the number of times that 4 male nurses and 6 female nurses are seen in the sample points, we first need to choose 4 male nurses out of 20. Then choose 6 female nurses out of the remaining 70 nurses. The total number of combinations of nurses is given by:

$$P(A) = \frac{n_a}{N} = \frac{\binom{20}{4} \binom{70}{6}}{\binom{90}{10}}$$

#### Problem 59

- a) There are  $\binom{52}{5}$  sample points. Since there are 4 of each kind of card in the deck there is  $4^5$  ways to draw the straight we want. Therefore the probability of drawing the straight we want is:

$$P(A) = \frac{n_a}{N} = \frac{1024}{\binom{52}{5}} = 3.94 \times 10^{-4}$$

- b) Using the result of part a, with the knowledge that there are 10 other ways to rearrange the straight, we get:

$$P(A) = 3.94 \times 10^{-3}$$

## Problem 71

Given that,  $P(A) = .5$ ,  $P(B) = .3$ ,  $P(A \cap B) = .1$

a)  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.3} = \frac{1}{3}$

b)  $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{.1}{.5} = \frac{1}{5}$

c)

$$\begin{aligned} P(A|A \cup B) &= \frac{P(A \cap (A \cup B))}{P(A \cup B)} \\ &= \frac{P(A \cup (A \cap B))}{P(A) + P(B) - P(A \cap B)} \\ &= \frac{P(A) + P(A \cap B) - P(A \cap (A \cap B))}{P(A) + P(B) - P(A \cap B)} \\ &= \frac{.5}{.5 + .3 - .1} \\ &= \frac{5}{7} \end{aligned}$$

d)  $P(A|A \cap B) = \frac{P(A \cap (A \cap B))}{P(A \cap B)} = 1$

e)

$$\begin{aligned} P(A \cap B|A \cup B) &= \frac{P((A \cap B) \cap (A \cup B))}{.7} \\ &= \frac{P(A \cap B)}{.7} \\ &= \frac{1}{7} \end{aligned}$$

## Problem 74

To figure out which events are independent we must first divide each Yes/No row by its percent of the total. This will allow us to compare the Yes and No rows.

- a) **A and D:** Since  $\frac{.20}{.30} \neq \frac{.41}{.70}$  we see that it does matter if we selected a parent with a child in college so these events are **dependent**.
- b) **B and D:** Since  $\frac{.09}{.30} = \frac{.21}{.70}$  we can see that a child in college does not influence the *About Right* group. Therefore, B and D are **independent**.
- c) **C and D:** Since  $\frac{.01}{.30} \neq \frac{.08}{.70}$  we see (from the same argument as a) that C and D are **dependent**.