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Introduction to Probability
Homework 6

Problem 103

A warehouse contains ten printing machines, four of which are defective. A company selects five of the machines at random, thinking all are in working condition. What is the probability that all five of the machines are non defective.

Answer.

$$\frac{\binom{6}{5}}{\binom{10}{5}} = \frac{6}{252} = \frac{1}{42}$$

Problem 107

A group of six software packages available to solve a linear programming problem has been ranked from 1 to 6 (best to worst). An engineering firm, unaware of the rankings, randomly selected and then purchased two of the packages. Let Y denote the number of packages purchased by the firm that are ranked 3, 4, 5, or 6. Give the probability distribution for Y.

Answer. We have a hypergeometric distribution with parameters $N = 6$, $n = 2$, and $r = 4$.

$$y = 0 : \frac{\binom{2}{2}}{\binom{6}{2}} = \frac{1}{15} \qquad y = 1 : \frac{\binom{2}{1}\binom{4}{1}}{\binom{6}{2}} = \frac{8}{15} \qquad y = 2 : \frac{\binom{4}{2}}{\binom{6}{2}} = \frac{6}{15}$$

Problem 115

Suppose that a radio contains six transistors, two of which are defective. Three transistors are selected at random, removed from the radio, and inspected. Let Y equal the number of defectives observed, where $Y = 0, 1$, or 2. Find the probability distribution of Y. Express your results graphically as a probability histogram.

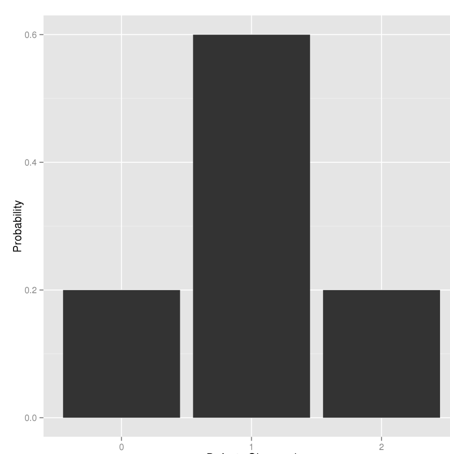
```
library(ggplot2)

y <- 0:2

prob <- dhyper(0:2, 2, 4, 3)

Y <- data.frame(y, prob)

ggplot(Y, aes(x = factor(y), y = prob)) +
    geom_bar(stat = "identity") +
    xlab("Defects Observed") +
    ylab("Probability")
```



Problem 119

Cards are dealt at random and without replacement from a standard 52 card deck. What is the probability that the second king is dealt on the fifth card.

Answer. Let A be the even that the first four cards contain one king and three non-kings. Let B be the even that the fifth card is the second king. The probability of A is a straight hypergeometric probability of drawing one of the four kings and three of the remaining non-kings.

$$P(A) = \frac{\binom{4}{1}\binom{48}{3}}{\binom{52}{4}}$$

Now given that the first four cards have been drawn and the one is a king and the other three are non-kings, the porbability tat the fifth card is a king is given below:

$$P(B|A) = \frac{3}{48}$$

since there are three kings left in the eck of 48 cards. Now we have:

$$P(B) = P(B|A)P(A) = \frac{3}{48}\frac{\binom{4}{1}\binom{48}{3}}{\binom{52}{4}} = 0.0159719$$