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Math 340

Sections:2.7-2.10
Problems: 77a,c,h, 95, 129, 133, 137
9/16/13

Introduction to Probability
Homework 3

Problem 77a

$P(A) = \mathbf{.40}$

Problem 77c

$P(A \cap B) = \mathbf{.10}$

Problem 77h

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.10}{.37} = \mathbf{\frac{10}{37}}$

Problem 95

Two events A and B are such that $P(A) = .2$, $P(B) = .3$, and $P(A \cup B) = .4$

a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .2 + .3 - .4 = \mathbf{.1}$

b) $P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = \mathbf{.9}$

c) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = \mathbf{.6}$

d) $P(\overline{A}|B) = \frac{P(\overline{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{2}{3}$

Problem 129

Given: $\begin{array}{lll} P(+|F) = .7 & P(+|M) = .4 & P(M) = .25 \\ P(-|F) = .3 & P(-|M) = .6 & P(F) = .75 \end{array}$

Using Bayes Theorem,

$$\begin{aligned} P(M|-) &= \frac{P(-|M)P(M)}{P(-|M)P(M) + P(-|F)P(F)} \\ &= \frac{.6 \times .25}{.6 \times .25 + .3 \times .75} \\ &= .4 \end{aligned}$$

Problem 133

Let K = Knows the answer to the question, R = Gets the right answer, G = Guesses the answer, W = Gets the wrong answer

Then $\begin{array}{lll} P(K) = .8 & P(R|G) = .25 & P(R|K) = 1 \\ P(G) = .2 & P(W|G) = .75 & P(W|K) = 0 \end{array}$

The probability that the student really knew the answer given that they got it correct is:

$$\begin{aligned} P(K|R) &= \frac{P(R|K)P(K)}{P(R|K)P(K) + P(R|G)P(G)} \\ &= \frac{.8}{.8 + .25 \times .2} \\ &\approx .9412 \end{aligned}$$

Problem 137

Calculating the probability of getting two white balls for any given bowl:

Bowl	1	2	3	4	5
White	1	2	3	4	5
Black	4	3	2	1	0
P(2 White)	0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{5}$	1

a) The probability that both balls are white is:

$$\sum_{i=1}^{5} \frac{P(\text{2 White in bowl i})}{5} = \frac{2}{5}$$

b) Given that both balls selected are white, the probability that bowl 3 was selected:
Using Bayes' Theorem,

$$\begin{aligned} P(\text{bowl 3}|\text{both white}) &= \frac{P(\text{both white}|\text{bowl 3})P(\text{bowl 3})}{\sum_{i=1}^{5} P(\text{both white}|\text{bowl i})P(\text{bowl i})} \\ &= \frac{\frac{3}{10}}{\frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1} \\ &= \frac{3}{20} \end{aligned}$$