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Gruppe ②

Aufgabe 4) Zum Einstieg

a) 1. $\eta_{\mu\nu}\eta^{\nu\lambda} = \sum_{\nu=0}^{3} \eta_{\mu\nu}\eta^{\nu\lambda}$

$= (\eta_{\mu 0}\eta^{0\lambda} + \eta_{\mu 1}\eta^{1\lambda} + \eta_{\mu 2}\eta^{2\lambda} + \eta_{\mu 3}\eta^{3\lambda})$

$= \delta_{\mu}^{\lambda} = \begin{cases} 1 \;, & \mu = \lambda \\ 0 \;, & \mu \neq \lambda \end{cases}$ ✓

2. $\eta_{\mu\nu}\eta^{\mu\nu} = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \eta_{\mu\nu}\eta^{\mu\nu}$

$= \sum_{\mu=0}^{3} (\eta_{\mu 0}\eta^{\mu 0} + \eta_{\mu 1}\eta^{\mu 1} + \eta_{\mu 2}\eta^{\mu 2} + \eta_{\mu 3}\eta^{\mu 3})$

$= (\eta_{00}\eta^{00} + \eta_{01}\eta^{01} + \eta_{02}\eta^{02} + \eta_{03}\eta^{03})$
$+ (\eta_{10}\eta^{10} + \eta_{11}\eta^{11} + \eta_{12}\eta^{12} + \eta_{13}\eta^{13})$
$+ (\eta_{20}\eta^{20} + \eta_{21}\eta^{21} + \eta_{22}\eta^{22} + \eta_{23}\eta^{23})$
$+ (\eta_{30}\eta^{30} + \eta_{31}\eta^{31} + \eta_{32}\eta^{32} + \eta_{33}\eta^{33})$

$= 4$ ✓

3. $x_\mu x_\nu \eta^{\mu\nu} = x_\mu x^\mu$ ✓

4. $\eta^{\mu}_{\nu} x_\mu \eta^{\nu}_{\lambda} x^\lambda = \delta^{\mu}_{\nu} x_\mu \delta^{\nu}_{\lambda} x^\lambda = x_\nu x^\nu$ ✓

b) $x'^\mu x'_\mu = \eta_{\mu\nu} x'^\mu x'^\nu$

$= \eta_{\mu\nu} \Lambda^{\mu}_{\rho} x^\rho \Lambda^{\nu}_{\sigma} x^\sigma$

$= \eta_{\rho\sigma} x^\rho x^\sigma$

$= x'_\sigma x^\sigma$ ✓

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