Aufgabe 2) Geodäten auf einer Kugel

a) $$\frac{d^2u^m}{d\tau^2} + \Gamma^m_{ij}\frac{du^i}{d\tau}\frac{du^j}{d\tau}$$

$$= \frac{d^2u^m}{d\tau^2} + \Gamma^m_{rr}\frac{du^r}{d\tau}\frac{du^r}{d\tau} + \Gamma^m_{r\vartheta}\frac{du^r}{d\tau}\frac{du^\vartheta}{d\tau} + \Gamma^m_{r\varphi}\frac{du^r}{d\tau}\frac{du^\varphi}{d\tau}$$
$$+ \Gamma^m_{\vartheta r}\frac{du^\vartheta}{d\tau}\frac{du^r}{d\tau} + \Gamma^m_{\vartheta\vartheta}\frac{du^\vartheta}{d\tau}\frac{du^\vartheta}{d\tau} + \Gamma^m_{\vartheta\varphi}\frac{du^\vartheta}{d\tau}\frac{du^\varphi}{d\tau}$$
$$+ \Gamma^m_{\varphi r}\frac{du^\varphi}{d\tau}\frac{du^r}{d\tau} + \Gamma^m_{\varphi\vartheta}\frac{du^\varphi}{d\tau}\frac{du^\vartheta}{d\tau} + \Gamma^m_{\varphi\varphi}\frac{du^\varphi}{d\tau}\frac{du^\varphi}{d\tau} = 0$$

$\underline{m = r:}$

$$\ddot{r} + \Gamma^r_{\vartheta\vartheta}\dot{\vartheta}^2 + \Gamma^r_{\vartheta\varphi}\dot{\vartheta}\dot{\varphi} + \Gamma^r_{\varphi\vartheta}\dot{\varphi}\dot{\vartheta} + \Gamma^r_{\varphi\varphi}\dot{\varphi}^2 = 0$$
$$\Leftrightarrow -R\dot{\vartheta}^2 - R\dot{\varphi}^2\sin^2\vartheta = 0$$
$$\Leftrightarrow \dot{\vartheta}^2 + \dot{\varphi}^2\sin^2\vartheta = 0 \quad (1)$$

r-Gleichung nicht sinnvoll, da $r = R = \text{const.}$

$\underline{m = \vartheta:}$

$$\ddot{\vartheta} + \Gamma^\vartheta_{\vartheta\vartheta}\dot{\vartheta}^2 + \Gamma^\vartheta_{\vartheta\varphi}\dot{\vartheta}\dot{\varphi} + \Gamma^\vartheta_{\varphi\vartheta}\dot{\varphi}\dot{\vartheta} + \Gamma^\vartheta_{\varphi\varphi}\dot{\varphi}^2 = 0$$
$$\Leftrightarrow \ddot{\vartheta} - \dot{\varphi}^2\sin\vartheta\cos\vartheta = 0 \quad (2)\ \checkmark$$

$\underline{m = \varphi:}$

$$\ddot{\varphi} + \Gamma^\varphi_{\vartheta\vartheta}\dot{\vartheta}^2 + \Gamma^\varphi_{\vartheta\varphi}\dot{\vartheta}\dot{\varphi} + \Gamma^\varphi_{\varphi\vartheta}\dot{\varphi}\dot{\vartheta} + \Gamma^\varphi_{\varphi\varphi}\dot{\varphi}^2 = 0$$
$$\Leftrightarrow \ddot{\varphi} + 2\dot{\vartheta}\dot{\varphi}\cot(\vartheta) = 0 \quad (3)\ \checkmark$$

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