A3/ $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu} = -\frac{\partial \Gamma^{\lambda}_{\mu\lambda}}{\partial x^{\lambda}} + \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} - \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\nu\eta} + \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\eta\lambda}$

$\Gamma^{t}_{ii} = -\frac{\dot{a}}{a} g_{ii}$

$T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} - p g^{\mu\nu}$, $u^{\mu} = \gamma \begin{pmatrix} 1 \\ \vec{v} \end{pmatrix} \overset{\vec{v}\to\vec{0}}{=} 1 \cdot \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix}$

$T^{00} = \rho + p - p = \rho$

$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & & \\ & \frac{-a^2}{1-kr^2} & & 0 \\ & & -a^2 r^2 & \\ 0 & & & -a^2 r^2 \sin^2\theta \end{pmatrix}$

$T^{0i} = T^{i0} = 0$

$T^{ij} = -p g^{ij} \Rightarrow T^{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & -p g^{ii} \end{pmatrix}$

$T_{\mu\nu} = g_{\mu\sigma} g_{\nu\rho} T^{\sigma\rho} = \begin{pmatrix} \rho & 0 \\ 0 & -p g_{ii} \end{pmatrix}$

$R_{00} = \frac{\partial \Gamma^{\lambda}_{0\lambda}}{\partial x^{0}} + \frac{\partial \Gamma^{\lambda}_{00}}{\partial x^{\lambda}} - \Gamma^{\eta}_{0\lambda}\Gamma^{\lambda}_{0\eta} + \Gamma^{\eta}_{00}\Gamma^{\lambda}_{\eta\lambda}$

$\Gamma^{i}_{ti} = \frac{\dot{a}}{a} \Rightarrow R_{00} = \frac{\partial (3\frac{\dot{a}}{a})}{\partial t} - 3\left(\frac{\dot{a}}{a}\right)^2$

$\Gamma^{\lambda}_{0\lambda} = \Gamma^{0}_{00} + \Gamma^{i}_{0i} + \dots$

$= 3\frac{\ddot{a}a - \dot{a}\dot{a}}{a^2} - 3\left(\frac{\dot{a}}{a}\right)^2$

$= -3\frac{\ddot{a}}{a}$

$R_{22} = \frac{\partial \Gamma^{\lambda}_{22}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\lambda}_{2\lambda}}{\partial x^{2}} + \Gamma^{\eta}_{22}\Gamma^{\lambda}_{\eta\lambda} - \Gamma^{\eta}_{2\lambda}\Gamma^{\lambda}_{2\eta}$ $\Big\}\ 2 \to \vartheta$

$= \frac{\partial \Gamma^{t}_{\vartheta\vartheta}}{\partial t} + \frac{\partial \Gamma^{r}_{\vartheta\vartheta}}{\partial r} - \frac{\Gamma^{\varphi}_{\vartheta\varphi}}{\partial \vartheta} + \Gamma^{t}_{\vartheta\vartheta}\left(\Gamma^{i}_{ti}\right) - \Gamma^{t}_{\vartheta\vartheta}\Gamma^{\vartheta}_{\vartheta t}$

$+ \Gamma^{r}_{\vartheta\vartheta}\Gamma^{i}_{ri} - \Gamma^{r}_{\vartheta\vartheta}\Gamma^{\vartheta}_{\vartheta r} - \Gamma^{\vartheta}_{\vartheta t}\Gamma^{t}_{\vartheta\vartheta} - \Gamma^{\vartheta}_{\vartheta r}\Gamma^{r}_{\vartheta\vartheta} - \Gamma^{\varphi}_{\vartheta\varphi}\Gamma^{\varphi}_{\vartheta\varphi}$

$= -\frac{\ddot{a}a - \dot{a}^2}{a^2} g_{\vartheta\vartheta} - (1 - kr^2 - 2kr^2) - \frac{-\sin^2(\vartheta) - \cos^2(\vartheta)}{\sin^2(\vartheta)} - \frac{\dot{a}}{a} g_{\vartheta\vartheta}\left(3\frac{\dot{a}}{a}\right)$

$+ 2\left(\frac{\dot{a}}{a} g_{\vartheta\vartheta} \frac{\dot{a}}{a}\right) - 3(1 - kr^2) + 2r\left(\frac{1 - kr^2}{r}\right) - \cot^2(\vartheta)$

$$= -\frac{\ddot{a}a - 2\dot{a}^2}{a^2} g_{rr} - 1 + 4kr^2$$ (irgendwie falsch, vermutlich Γ's vergessen!)

eigentlich

$$\overset{!}{=} -\frac{\ddot{a}a + 2\dot{a}^2 + 2k}{a^2} g_{rr} \Rightarrow k = -\frac{\ddot{a}a + 2\dot{a}^2 + 2k}{a^2}$$

$$R = R_{\mu\nu\rho}{}^{\mu\nu} = R_{00} + 3k = -3\frac{\ddot{a}a}{a^2} - 3\frac{\ddot{a}a + 2\dot{a}^2 + 2k}{a^2}$$

$$= -6\left(\frac{\ddot{a}a + \dot{a}^2 + k}{a^2}\right)$$

d) $8\pi G\, T_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$

0-Komp.: $8\pi G\, \rho = -3\frac{\ddot{a}}{a} + 3\frac{\ddot{a}a + \dot{a}^2 + k}{a^2} = 3\frac{\dot{a}^2 + k}{a^2}$

ii-Komp.: $-8\pi G p g_{ii} = -\frac{\ddot{a}a + 2\dot{a}^2 + 2k}{a^2} g_{ii} + 3\frac{\ddot{a}a + \dot{a}^2 + k}{a^2} g_{ii}$

$$= \frac{2\ddot{a}a + \dot{a}^2 + k}{a^2} g_{ii}$$

e) $T^{\mu\nu}_{;\mu} = \partial_\mu T^{\mu\nu} + \Gamma^{\mu}_{\mu\alpha} T^{\alpha\nu} + \Gamma^{\nu}_{\mu\alpha} T^{\mu\alpha}$

$$T^{\mu 0}_{;\mu} = \partial_\mu T^{\mu 0} + \Gamma^{\mu}_{\mu\alpha} T^{\alpha 0} + \Gamma^{0}_{\mu\alpha} T^{\mu\alpha} = 0$$

$$= \partial_t \rho + 3\frac{\dot{a}}{a}\rho + \left(-\frac{\dot{a}}{a}\right) \underbrace{g_{ii} T^{ii}}_{-3p} = 0$$

$$\Rightarrow \dot{\rho} + 3\frac{\dot{a}}{a}\rho = -3\frac{\dot{a}}{a} p \;\Big|\cdot a^3$$

$$\left[\frac{da^3}{dt} = 3\dot{a}a^2\right.$$

$$\dot{\rho}a^3 + 3\dot{a}a^2\rho = -3\dot{a}a^2 p$$

$$\frac{d\rho}{dt}a^3 + \frac{da^3}{dt}\rho = -\frac{da^3}{dt} p$$

$$\Leftrightarrow \underbrace{d\rho a^3}_{d(\rho a^3) \hat{=} dE} + \rho \underbrace{da^3}_{dV} = -\underbrace{da^3}_{dV} p$$

Toll! Fein! WoW!