Aufgabe 3) Affiner Zusammenhang

$$\Gamma'^{\mu}_{\nu\sigma} = \frac{\partial x'^{\mu}}{\partial \xi^{\varrho}} \frac{\partial^2 \xi^{\varrho}}{\partial x'^{\nu} \partial x'^{\sigma}} \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial \xi^{\varrho}} \frac{\partial^2 \xi^{\varrho}}{\partial x'^{\nu} \partial x'^{\sigma}} \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial \xi^{\varrho}} \frac{\partial}{\partial x'^{\nu}} \left(\frac{\partial \xi^{\varrho}}{\partial x'^{\sigma}} \right) \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial \xi^{\varrho}} \frac{\partial}{\partial x'^{\nu}} \left(\frac{\partial \xi^{\varrho}}{\partial x^{\beta}} \frac{\partial x^{\beta}}{\partial x'^{\sigma}} \right) \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial \xi^{\varrho}} \left(\frac{\partial^2 \xi^{\varrho}}{\partial x'^{\nu} \partial x^{\beta}} \frac{\partial x^{\beta}}{\partial x'^{\sigma}} + \frac{\partial \xi^{\varrho}}{\partial x^{\beta}} \frac{\partial^2 x^{\beta}}{\partial x'^{\nu} \partial x'^{\sigma}} \right) \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial \xi^{\varrho}} \left(\frac{\partial^2 \xi^{\varrho}}{\partial x^{\gamma} \partial x^{\beta}} \frac{\partial x^{\gamma}}{\partial x'^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\sigma}} + \frac{\partial \xi^{\varrho}}{\partial x^{\beta}} \frac{\partial^2 x^{\beta}}{\partial x'^{\nu} \partial x'^{\sigma}} \right) \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\gamma}}{\partial x'^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\sigma}} \Gamma^{\alpha}_{\beta\gamma} + \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial \xi^{\varrho}} \frac{\partial \xi^{\varrho}}{\partial x^{\beta}} \frac{\partial^2 x^{\beta}}{\partial x'^{\nu} \partial x'^{\sigma}} \checkmark$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\gamma}}{\partial x'^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\sigma}} \Gamma^{\alpha}_{\beta\gamma} + \frac{\partial x'^{\mu}}{\partial x^{\beta}} \frac{\partial^2 x^{\beta}}{\partial x'^{\nu} \partial x'^{\sigma}} \checkmark$$

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Gruppe 2

Allgemeine Relativitätstheorie
5. Übungsblatt

Aufgabe 1) Symmetrien des Krümmungstensors

a) $$R_{\mu\nu\varsigma\lambda} = \frac{1}{2}\left(\frac{\partial^2 g_{\mu\varsigma}}{\partial x^\nu \partial x^\lambda} + \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\varsigma} - \frac{\partial^2 g_{\nu\varsigma}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\varsigma}\right)$$
$$+ g_{\eta\sigma}\left(\Gamma^\eta_{\varsigma\mu}\Gamma^\sigma_{\nu\lambda} - \Gamma^\eta_{\lambda\mu}\Gamma^\sigma_{\nu\varsigma}\right) \checkmark$$
$$= -\frac{1}{2}\left(\frac{\partial^2 g_{\nu\varsigma}}{\partial x^\mu \partial x^\lambda} + \frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\varsigma} - \frac{\partial^2 g_{\mu\varsigma}}{\partial x^\nu \partial x^\lambda} - \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\varsigma}\right)$$
$$- g_{\eta\sigma}\left(\Gamma^\eta_{\varsigma\nu}\Gamma^\sigma_{\mu\lambda} - \Gamma^\eta_{\lambda\nu}\Gamma^\sigma_{\mu\varsigma}\right) \checkmark \checkmark$$
$$= -R_{\nu\mu\varsigma\lambda}, \text{ denn}$$

$$\Gamma^\eta_{\lambda\nu}\Gamma^\sigma_{\mu\varsigma} = \frac{\partial x^\eta}{\partial \xi^\alpha}\frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\nu}\frac{\partial x^\sigma}{\partial \xi^\alpha}\frac{\partial^2 \xi^\alpha}{\partial x^\mu \partial x^\varsigma} = \Gamma^\sigma_{\lambda\nu}\Gamma^\eta_{\mu\varsigma} \checkmark$$

b) $$R_{\mu\nu\varsigma\lambda} = \frac{1}{2}\left(\frac{\partial^2 g_{\mu\varsigma}}{\partial x^\nu \partial x^\lambda} + \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\varsigma} - \frac{\partial^2 g_{\nu\varsigma}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\varsigma}\right)$$
$$+ g_{\eta\sigma}\left(\Gamma^\eta_{\varsigma\mu}\Gamma^\sigma_{\nu\lambda} - \Gamma^\eta_{\lambda\mu}\Gamma^\sigma_{\nu\varsigma}\right) \checkmark$$
$$= -\frac{1}{2}\left(\frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\varsigma} + \frac{\partial^2 g_{\nu\varsigma}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\varsigma} - \frac{\partial^2 g_{\mu\varsigma}}{\partial x^\nu \partial x^\lambda}\right)$$
$$- g_{\eta\sigma}\left(\Gamma^\eta_{\lambda\mu}\Gamma^\sigma_{\nu\varsigma} - \Gamma^\eta_{\varsigma\mu}\Gamma^\sigma_{\nu\lambda}\right) \checkmark$$
$$= -R_{\mu\nu\lambda\varsigma} \checkmark$$

c) $$R_{\mu\nu\varsigma\lambda} = \frac{1}{2}\left(\frac{\partial^2 g_{\mu\varsigma}}{\partial x^\nu \partial x^\lambda} + \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\varsigma} - \frac{\partial^2 g_{\nu\varsigma}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\varsigma}\right)$$
$$+ g_{\eta\sigma}\left(\Gamma^\eta_{\varsigma\mu}\Gamma^\sigma_{\nu\lambda} - \Gamma^\eta_{\lambda\mu}\Gamma^\sigma_{\nu\varsigma}\right) \checkmark$$
$$= R_{\varsigma\lambda\mu\nu}, \text{ denn} \checkmark$$

$\Gamma^\gamma_{\alpha\beta} = \Gamma^\gamma_{\beta\alpha}$ und da der Satz von Schwarz gilt.

d) $R_{\mu\nu\rho\lambda} + R_{\mu\lambda\nu\rho} + R_{\mu\rho\lambda\nu}$

$$= \frac{1}{2}\left(\frac{\partial^2 g_{\mu\rho}}{\partial x^\nu \partial x^\lambda} + \frac{\partial^2 g_{\nu\lambda}}{\partial x^\mu \partial x^\rho} - \frac{\partial^2 g_{\nu\rho}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\rho}\right)$$

$$+ g_{\eta\sigma}\left(\Gamma^\eta_{\rho\mu}\Gamma^\sigma_{\nu\lambda} - \Gamma^\eta_{\lambda\mu}\Gamma^\sigma_{\nu\rho}\right)$$

$$+ \frac{1}{2}\left(\frac{\partial^2 g_{\mu\nu}}{\partial x^\lambda \partial x^\rho} + \frac{\partial^2 g_{\lambda\rho}}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 g_{\lambda\nu}}{\partial x^\mu \partial x^\rho} - \frac{\partial^2 g_{\mu\rho}}{\partial x^\lambda \partial x^\nu}\right)$$

$$+ g_{\eta\sigma}\left(\Gamma^\eta_{\nu\mu}\Gamma^\sigma_{\lambda\rho} - \Gamma^\eta_{\rho\mu}\Gamma^\sigma_{\lambda\nu}\right)$$

$$+ \frac{1}{2}\left(\frac{\partial^2 g_{\mu\lambda}}{\partial x^\rho \partial x^\nu} + \frac{\partial^2 g_{\rho\nu}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\rho\lambda}}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 g_{\mu\nu}}{\partial x^\rho \partial x^\lambda}\right)$$

$$+ g_{\eta\sigma}\left(\Gamma^\eta_{\lambda\mu}\Gamma^\sigma_{\rho\nu} - \Gamma^\eta_{\mu\nu}\Gamma^\sigma_{\rho\lambda}\right)$$

$$= 0 \quad \checkmark$$

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