SMD Blatt 4

A11:

Pop 0: $\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}1,5\\2\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix}\begin{pmatrix}3\\3\end{pmatrix}$

$\vec{\mu}_0 = \begin{pmatrix}23/12\\2\end{pmatrix}$ $\quad S_w = \sum_j S_j$ und $S_j = \sum_i (\vec{x}_i - \vec{\mu}_j)(\vec{x}_i - \vec{\mu}_j)^T$

$$S_0 = \sum_i (\vec{x}_i - \vec{\mu}_0)(\vec{x}_i - \vec{\mu}_0)^T$$

$$= \begin{pmatrix}-11/12\\-1\end{pmatrix}(-11/12, -1) + \begin{pmatrix}1/12\\-1\end{pmatrix}(1/12, -1) + \begin{pmatrix}-5/12\\0\end{pmatrix}(-5/12, 0)$$

$$+ \begin{pmatrix}1/12\\0\end{pmatrix}(1/12, 0) + \begin{pmatrix}1/12\\1\end{pmatrix}(1/12, 1) + \begin{pmatrix}13/12\\1\end{pmatrix}(13/12, 1)$$

$$= \begin{pmatrix}121/144 & 11/12\\11/12 & 1\end{pmatrix} + \begin{pmatrix}1/144 & -1/12\\-1/12 & 1\end{pmatrix} + \begin{pmatrix}25/144 & 0\\0 & 0\end{pmatrix}$$

$$+ \begin{pmatrix}1/144 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}1/144 & 1/12\\1/12 & 1\end{pmatrix} + \begin{pmatrix}169/144 & 13/12\\13/12 & 1\end{pmatrix}$$

$$= \begin{pmatrix}53/24 & 2\\2 & 4\end{pmatrix}$$

Pop 1: $\begin{pmatrix}1,5\\1\end{pmatrix}\begin{pmatrix}2,5\\1\end{pmatrix}\begin{pmatrix}3,5\\1\end{pmatrix}\begin{pmatrix}2,5\\2\end{pmatrix}\begin{pmatrix}3,5\\2\end{pmatrix}\begin{pmatrix}4,5\\2\end{pmatrix}$

$$\vec{\mu}_1 = \begin{pmatrix}3\\3/2\end{pmatrix}$$

$$S_1 = \begin{pmatrix}-3/2\\-1/2\end{pmatrix}(-3/2, -1/2) + \begin{pmatrix}-1/2\\-1/2\end{pmatrix}(-1/2, -1/2) + \begin{pmatrix}1/2\\-1/2\end{pmatrix}(1/2, -1/2)$$

$$+ \begin{pmatrix}-1/2\\1/2\end{pmatrix}(-1/2, 1/2) + \begin{pmatrix}1/2\\1/2\end{pmatrix}(1/2, 1/2) + \begin{pmatrix}3/2\\1/2\end{pmatrix}(3/2, 1/2)$$

$$= \begin{pmatrix}9/4 & 3/4\\3/4 & 1/4\end{pmatrix} + \begin{pmatrix}1/4 & 1/4\\1/4 & 1/4\end{pmatrix} + \begin{pmatrix}1/4 & -1/4\\-1/4 & 1/4\end{pmatrix}$$

$$+ \begin{pmatrix}1/4 & -1/4\\-1/4 & 1/4\end{pmatrix} + \begin{pmatrix}1/4 & 1/4\\1/4 & 1/4\end{pmatrix} + \begin{pmatrix}9/4 & 3/4\\3/4 & 1/4\end{pmatrix}$$

$$= \begin{pmatrix}22/4 & 6/4\\6/4 & 6/4\end{pmatrix} = \begin{pmatrix}11/2 & 3/2\\3/2 & 3/2\end{pmatrix}$$

$$\Rightarrow S_w = S_0 + S_1 = \begin{pmatrix}185/24 & 7/2\\7/2 & 11/2\end{pmatrix}$$

$$S_B = (\vec{\mu}_1 - \vec{\mu}_2)(\vec{\mu}_1 - \vec{\mu}_2)^T$$

$$= \begin{pmatrix} -13/12 \\ 1/2 \end{pmatrix} \left(-13/12 \;,\; 1/2 \right) = \begin{pmatrix} 169/144 & -13/24 \\ -13/24 & 1/4 \end{pmatrix}$$

b) $\vec{\lambda}^* = S_w^{-1} (\vec{\mu}_1 - \vec{\mu}_2)$

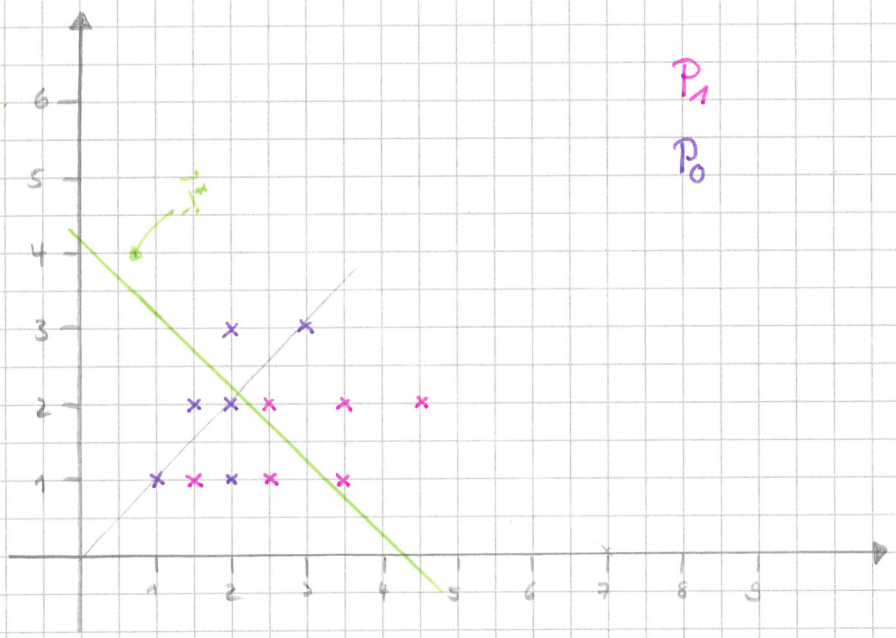
$$S_w^{-1} = \frac{1}{\det(S_w)} \begin{pmatrix} 11/2 & -7/2 \\ -7/2 & 185/24 \end{pmatrix}$$

$$\det(S_w) = 1447/48$$

$$S_w^{-1} = \begin{pmatrix} 264/1447 & -168/1447 \\ -168/1447 & 370/1447 \end{pmatrix}$$

$$\vec{\lambda}^* = S_w^{-1} (\vec{\mu}_1 - \vec{\mu}_2)$$

$$= S_w^{-1} \begin{pmatrix} -13/12 \\ 1/2 \end{pmatrix} = \begin{pmatrix} -370/1447 \\ 367/1447 \end{pmatrix}$$



$$|S_w^{-1} S_B - D| = 0 \Rightarrow EW = 0,\ 0,403824$$

$$\vec{v}_1 = \begin{pmatrix} 0,70557 \\ 0,419058 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} -0,704263 \\ 0,907358 \end{pmatrix}$$

$(\vec{\lambda}^*)^2 \approx 0{,}064853 \approx 0{,}065$

Pop 0: $\vec{\lambda}^* \vec{x}_1 = (-370/1447 \cdot 1 + 367/1447 \cdot 1)$

$= -3/1447 \approx -0{,}002 \rightarrow p_1 \approx -0{,}03$

$\vec{\lambda}^* \vec{x}_2 = -373/1447 \approx -0{,}258 \rightarrow p_2 \approx -3{,}97$

$\vec{\lambda}^* x_3 = 179/1447 \approx 0{,}124 \rightarrow p_3 \approx +1{,}91$

$\vec{\lambda}^* x_4 = -6/1447 \approx -0{,}004 \rightarrow p_4 \approx -0{,}06$

$\vec{\lambda}^* x_5 = 361/1447 \approx 0{,}249 \rightarrow p_5 \approx +3{,}83$

$\vec{\lambda}^* x_6 = -9/1447 \approx -0{,}006 \rightarrow p_6 \approx -0{,}09$

Pop 1:

$\vec{\lambda}^* \vec{x}_1 = -183/1447 \approx -0{,}13 \rightarrow p_1 \approx -2$

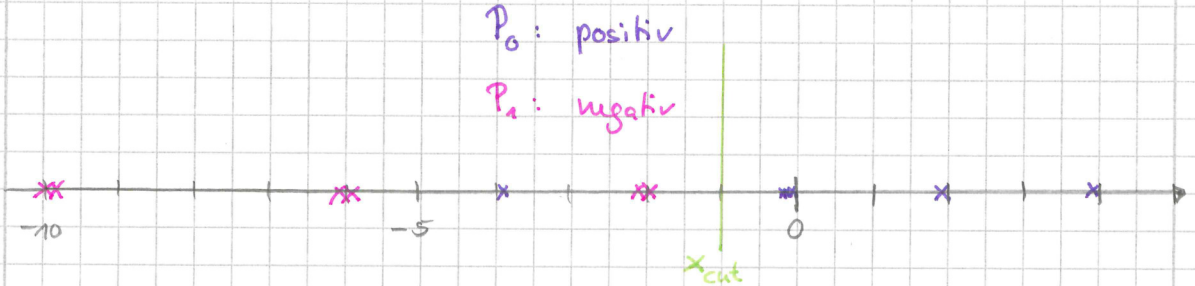
$\vec{\lambda}^* \vec{x}_2 = -558/1447 \approx -0{,}386 \rightarrow p_2 \approx -5{,}94$

$\vec{\lambda}^* \vec{x}_3 = -928/1447 \approx -0{,}641 \rightarrow p_3 \approx -9{,}861$

$\vec{\lambda}^* \vec{x}_4 = -191/1447 \approx -0{,}132 \rightarrow p_4 \approx -2{,}03$

$\vec{\lambda}^* \vec{x}_5 = -561/1447 \approx -0{,}388 \rightarrow p_5 \approx -5{,}97$

$\vec{\lambda}^* \vec{x}_6 = -931/1447 \approx -0{,}643 \rightarrow p_6 \approx -9{,}89$



Reinheit: $\frac{t_p}{t_p + f_p}$ Effizienz = $\frac{t_p}{t_p + f_n}$

→ minimiere f_p und f_n → mit dem x_{cut} $f_p = 0$, $f_n = 1$

→ Reinheit = $\frac{5}{5+0} = 1$ Effizienz = $\frac{5}{5+1} = \frac{5}{6} \approx 0{,}8\overline{3}$