SMD A26

a) $\bar{x} = \frac{1}{n} \sum_{i=1}^{N} x_i$

$$E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(x_i) = \frac{1}{n} \cdot n\mu = \mu$$

$\rightarrow$ Erwartungsgetreue Schätzfunktion für μ

b) $$V(\bar{x}) = V\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2} V\left(\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} V(x_i)$$

$$= \frac{1}{n^2} \cdot n\sigma^2 = \frac{\sigma^2}{n} \quad \square$$

c) $S_0^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2$

$$E(S_0^2) = \frac{1}{n} E\left(\sum_{i=1}^{n} (x_i^2 - 2x_i\mu + \mu^2)\right)$$

$$= \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - 2\mu \sum_{i=1}^{n} x_i + n\mu^2\right)$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n} E(x_i^2) - 2\mu E\left(\sum_{i=1}^{n} x_i\right) + n\mu^2\right)$$

$$= \frac{1}{n}\left(n(\sigma^2 + \mu^2) - 2n\mu^2 + n\mu^2\right)$$

$$= \sigma^2$$

$\rightarrow$ Erwartungsgetreuer Schätzer für σ^2

d) $$S_1'^2 = \frac{1}{n} E\left(\sum_{i=1}^{N} (x_i - \bar{x})^2\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} [x_i^2 - 2x_i\bar{x} + \bar{x}^2]\right)$$

$$= \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2\right)$$

$$= \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(x_i^2) - nE(\bar{x}^2)$$

$$= \frac{1}{n}(\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)$$

$$= \frac{1-n}{n}\sigma^2 \quad \Rightarrow \text{nicht Erwartungsgetreu}$$

$\rightarrow$ mit Korrektur: $S_1^2 = \frac{1}{1-n}\sum_{i=1}^{n} (x_i - \bar{x})^2$

da $E\left(\frac{n}{1-n} S_1'^2\right) = \sigma^2$