Mathematical modeling of polyurethane foam elasticity measuring process

4 authors:


Amna Bajtarevic-Jelec
University of Zenica


Sabahudin Ekinovic
University of Zenica


Fuad Hadžikadunić
University of Zenica


Amra Talić – Čikmiš
University of Zenica

ISSN 1847-7917

Mathematical modeling of polyurethane foam elasticity measuring process

Amna BAJTAREVIĆ[1)]

Sabahudin EKINOVIĆ[1)]

Fuad HADŽIKADUNIĆ[1)]

Amra TALIĆ-ČIKMIŠ[1)]

1) Faculty of Mechanical Engineering
University of Zenica, Fakultetska 1,
Bosnia and Herzegovina

amna.bajtarevic@mf.unze.ba
sekinovic@mf.unze.ba
hfuad@mf.unze.ba
acikmis@mf.unze.ba

Keywords

polyurethane foam
elasticity
elastic rebound
regression analysis

Original scientific paper

Abstract: Nowadays, production of upholstered furniture and similar products could not be possible without material which is called polyurethane foam. According to main application of this material, elasticity is one of the most important characteristic. The focus of this paper is based on polyurethane foam elasticity measuring by testing of steel ball elastic rebound. In order to determining steel ball elastic rebound dependency from change of foam hardness and ball diameter, regression analysis is performed.

1. Introduction

Polymer materials are the most used materials for production of furniture, besides wood, glass and metal. Specific polymer material which is called polyurethane foam (hereafter: PU foam), is made by chemical reaction of two main substances polyol and isocyanate with additives [1]. Production of upholstered furniture and sleep products is not possible without this material. According to the main use, important characteristics of PU foam are: density, hardness, elongation, plasticity and elasticity. The last above mentioned characteristic is considered as the most important because it determines quality and competitiveness of final product. Very common method of PU foam elasticity analysis is measurement of steel ball elastic rebound that is defined by standard ISO 8307. Method is based on free fall of ball from defined high and rebound high measure.

In the domain of stochastic modeling, method of mathematical modeling of polynomial is used for determining of functional dependence between dependent magnitude (y) and independent magnitudes (x_i). Accordingly, dependence is defined by regression model in the form of $y=f(x_i)$ [2]. In this paper, method of mathematical modeling of polynomial is conduced to defining dependence of elastic ball rebound (y) from ball diameter (x_1) and foam hardness (x_2). Mathematical principle of this dependency is not known in advance.

2. The main characteristics of measurement subject and process

2.1. Term of polyurethane foam

Two main raw materials for production of PU foam are petroleum products: polyol and toluene diisocyanate, e.g. "TDI". Additional substances are activators: Sn-octoate, Sn-isooctoate, stabilizers and others [3]. In short, process of PU foam production is started with dosage main raw materials via dispensing pumps with additives to the "head for mixing" of machine that working on principle of mixer. After process of mixing, homogenous mixed raw materials flow to the special container where chemical reactions are happened. From that container material is started to foam and flow through canal, so in place of special bath, block of PU foam is formed. Produced blocks are stored in storage shed for ripening process that lasts 48 hours. Matured blocks can be stored or processed on machines for PU foam cutting.

The main characteristics of PU foam are defined by proportion of main raw materials and additives. Types of PU foam are distinguished by characteristics of density and hardness. Four main parameters of PU foam such as type of foam, level of hardness, value of density and value of hardness are shown in standard mark of this material, as it is shown in figure 1.

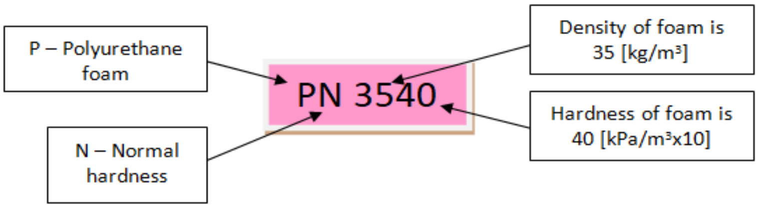


Figure 1. Marking of PU foams

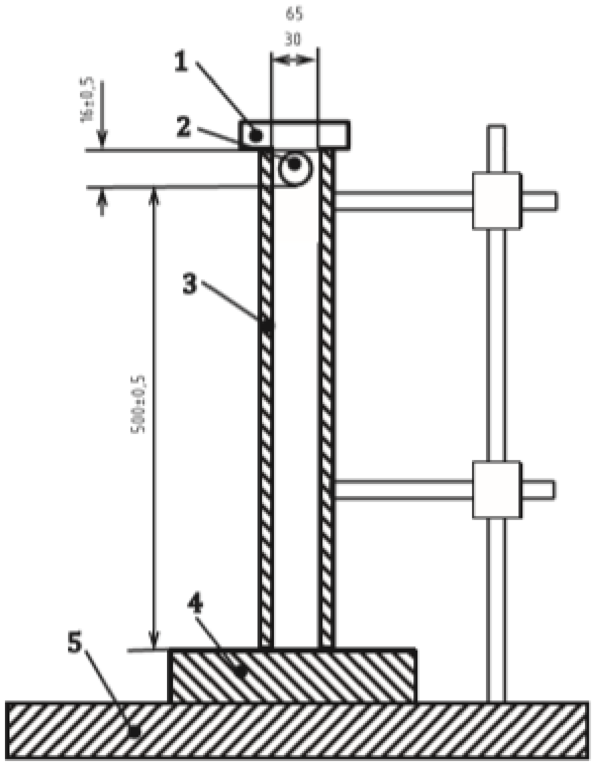
2.2. Analysis of PU foam elasticity

Polyurethane foams are high-elastic materials therefore their main application is material for production of upholstered furniture and systems for sleep (mattresses, toppers and etc.). Elasticity testing methods for foams are very similar such as methods for steel. Specimen of foam is exposed tension force until fracture. Length of elongation before fracture is measured magnitude. However, foam is very nonplastic material, so after fracture, parts of specimen are returned to the original form and dimension [3]. On the other hand, for above mentioned application of foams, it is very important to analyze characteristic of elastic rebound for PU foams.

Clearly, this property is crucial for quality of final product in which main material is PU foam.

Elasticity analysis of polymer materials by measuring of elastic rebound is defined by standard ISO 8307 [4]. Equipment for this measuring consists of vertical, transparent tube with defined scale and diameter from 30 till 65 mm, steel ball and other devices for automatic reading of rebound. Diameter of steel ball should be 16+/-5 mm while mass should be in tolerance of 16.8+/-1.5 g. Measuring is performed by releasing of steel ball from 500+/-5mm height. Fault of measuring could be happened if steel ball make contact with internal surface of tube. In that case, measuring would be invalid.

In figure 2, sketch of equipment for elastic rebound measuring is shown.



Key

1 magnet or other suitable device
2 steel ball
3 transparent tube
4 test piece
5 rigid baseplate

Figure 2. Equipment for elasticity analysis of polymer materials defined by ISO 8307

Dimensions of PU foam specimen should be 100x100x100 [mm]. Tested specimen is mounted at position 4, as it is presented in figure 2.

3. Conducting an experiment

3.1. Defining of problem

Essence of this paper is based on procedure of conduction an experiment with processing of experimental results. Thus, mathematical model is established that shows results dependency of measuring from main factors. In fact, it should be defined dependency of elastic rebound for PU foam from steel ball diameter and hardness of foam.

3.2. Preparing of model

During experiment, simple device for testing PU foam elasticity with manual reading is used. Research is conducted in laboratory for stress and strain analysis at ambient temperature. Steel ball is released from height of 500 mm. Experiment requirement is to prepare three steel balls and three specimens of PU foam. It is important to emphasize that specimens are not subjected to any loads before experiment. Three types of PU foam are used, as follows:

- PE2520 – elastic PU foam, hardness: 2.0 kPa/m^2
- PN3540 – PU foam with normal hardness, hardness: 4.0 kPa/m^2,
- PT4060 – PU foam with increased hardness, hardness: 6.0 kPa/m^2.

Experiment is conducted by achieving of ball free fall. Measure of elastic rebound carried out according to scale of device. Simulation of experiment conducting is shown in the figure 3.

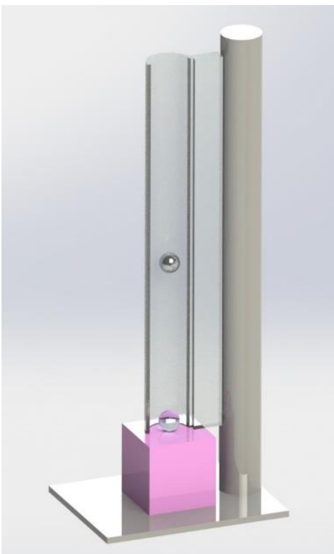


Figure 3. Simulation of experiment

Parameters of used factors are presented in table 1. As it is shown, three values of each factor are marked over three levels: upper, basic and lower. Distribution of levels is performed towards method of planning an experiment [5].

Table 1. Levels of used factors

Factor	Diameter of ball – factor X_1 [mm]	Hardness of PU foam – factor X_2 [kPa/m^2]
Upper level (+1)	22	6.0
Basic lever (0)	16.5	4.0
Lower level (-1)	11	2.0

4. Plan of experiment

Former research is conducted in order to defining if requested dependency can be express by mathematical model of first order with two levels of factors. Analysis is conducted by calculating of linear model in which is defined $N=r^k=2^2=4$. However, model is marked as inadequate by implementing of F-criteria.

Thus, this calculation is left out and analysis of model of second order is conducted.

In the theory of regression models, one of the most used is composition plan, especially, symmetrical composition plan that is applied in this paper too. Consequently, variables: x_0, x_1, x_2,$x_1x_,x_1^2$,x_2^2 and results for three repetition in each point of plan are defined by experimental plan. Number of experiments is defined as:

$$N = 2^k + 2 \cdot k + n_0 = 2^2 + 2 \cdot 2 + 1 = 9 \tag{1}$$

Values from equitation (1) are defined as:

- N – total number of experiments,
- k=2 – number of factors,
- n_0 – number of experiments in central point.

Figure 4 represents scheme of composition orthogonal plan of second order.

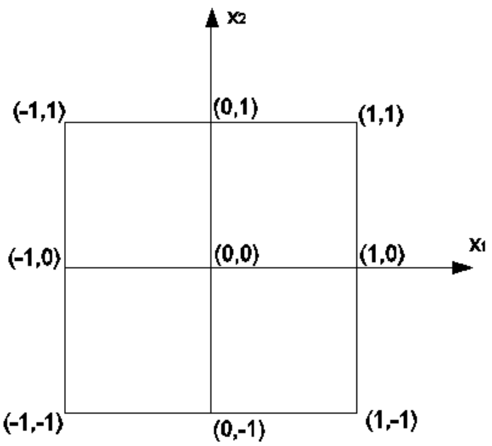


Figure 4. Scheme of orthogonal plan

In order to code main factors, equations of transformation are used, as it is shown:

$$x_i = \frac{X_i - X_{0i}}{w_i} \tag{2}$$

Values from equitation (2) are defined as:

- x_i – coded value of factor,
- X_i – natural value of factor,
- X_{0i} – basic level of factor.

Accordingly, basic levels of factor are defined as:

$$X_{01} = \frac{X_{g1} + X_{d1}}{2} = \frac{22+11}{2} = 16.5 \tag{3}$$

$$X_{02} = \frac{X_{g2} + X_{d2}}{2} = \frac{6+2}{2} = 4 \tag{4}$$

Intervals of variation are calculated in next equations:

$$w_2 = \frac{X_{g2} - X_{d2}}{2} = \frac{6-2}{2} = 2 \tag{5}$$

$$w_1 = \frac{X_{g1} - X_{d1}}{2} = \frac{22-11}{2} = 5.5 \tag{6}$$

Coded values of factors are presented by next formulas:

$$x_{g1} = \frac{X_{g1} - X_{01}}{w_1} = \frac{22-16.5}{5.5} = 1 \tag{7}$$

$$x_{d1} = \frac{X_{d1} - X_{01}}{w_1} = \frac{11-16.5}{5.5} = -1 \tag{8}$$

$$x_{g2} = \frac{X_{g2} - X_{02}}{w_1} = \frac{6-4}{2} = 1 \tag{9}$$

$$x_{d2} = \frac{X_{d2} - X_{02}}{w_1} = \frac{2-4}{2} = -1 \tag{10}$$

Condition of orthogonal is shown as:

$$\alpha^2 = \frac{1}{2} \cdot \left[\sqrt{\left(2^k + 2 \cdot k + n_0\right)} - 2^k\right] \tag{11}$$

$$\alpha^2 = \frac{1}{2} \cdot \left[\sqrt{\left(2^2 + 2 \cdot 2 + 1\right)} - 2^2\right] = 1 \tag{12}$$

$$\alpha = \pm 1 \tag{13}$$

New variable that makes orthogonal of plan matrix is:

$$\lambda_2 = N^{-1} \cdot \sum_{u=1}^{n} X_{iu}^2 = \frac{1}{9}\left(1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2\right) = \frac{2}{3} \tag{14}$$

Accordingly, new variables would be defined:

$$X_i^{'} = X_i^2 - \lambda_2 \tag{15}$$

$$X_1^{'} = X_1^2 - \frac{2}{3} \tag{16}$$

$$X_2^{'} = X_2^2 - \frac{2}{3} \tag{17}$$

According to all calculations, plan experiment matrix could be defined, such as it is shown in table 2.

Table 2. Plan matrix of experiment

Experiment No	Factors								Results of measuring				
	x_0	x_1	x_2	x_1^2	x_2^2	x_1x_2	x_1'	x_2'	y_1	y_2	y_3	$\bar{y}$	$\hat{y}$
1	+1	-1	-1	+1	+1	+1	1/3	1/3	67.5	68	67	67.50	67.22
2	+1	+1	-1	+1	+1	-1	1/3	1/3	61	61.5	60	60.83	60.75
3	+1	-1	+1	+1	+1	-1	1/3	1/3	58	59	58	58.33	58.13

4	+1	+1	+1	+1	+1	+1	1/3	1/3	51	50	51.5	50.83	50.83
5	+1	-1	0	+1	0	0	1/3	-2/3	65	64	66	65.00	65.48
6	+1	+1	0	+1	0	0	1/3	-2/3	58.5	59	58	58.50	58.59
7	+1	0	-1	0	+1	0	-2/3	1/3	64.5	64.5	64	64.33	64.70
8	+1	0	+1	0	+1	0	-2/3	1/3	55.5	55	54.5	55.00	55.20
9	+1	0	0	0	0	0	-2/3	-2/3	63.5	63	63.5	63.33	62.76

5. Regression analysis

Regression model analyzed process of measuring could be shown by formula (18):

$$\hat{y}=\left(b_0+\lambda_2\cdot\sum_{i=1}^{k}b_{ii}\right)+$$
$$+\sum_{i=1}^{k}b_i\cdot x_i+\sum_{i\triangleleft j}^{k}b_{ij}\cdot x_i\cdot x_j+\sum_{i=1}^{k}b_{ii}\cdot\left(x_i^2-\lambda_2\right) \quad (18)$$

If new basic factor can be defined as:

$$b_0^{'}=b_0+\lambda_2\cdot\sum_{i=1}^{k}b_{ii} \quad (19)$$

Then, regression model is shown as:

$$\hat{y}=b_0^{'}+\sum_{i=1}^{k}b_i\cdot x_i+\sum_{j=1}^{k}b_{ij}\cdot x_i\cdot x_j+\sum_{i=1}^{k}b_{ii}\cdot x_i^{'} \quad (20)$$

Values of regression coefficient are calculated by following calculations:

$$b_0^{'}=\frac{\sum_{i=1}^{N}n_i\cdot\overline{y_i}}{\sum_{i=1}^{N}n_i}=\frac{\sum_{i=1}^{9}3\cdot\overline{y_i}}{3\cdot(1+1+1+1+1+1+1+1+1)} \quad (21)$$

$$b_0^{'}=\frac{1}{9}(67.5+60{,}83+58.33+65+ \quad (22)$$
$$+58.5+65.33+55+63.33)=60.41$$

Regression coefficients b_1 and b_2 are determined by formula (23):

$$b_j=\frac{\sum_{i=1}^{N}n_i\cdot x_{ji}\cdot\overline{y_i}}{\sum_{i=1}^{N}n_i\cdot x_{ji}^{2}} \quad (23)$$

Thus, values of coefficient are:

$$b_1=\frac{\sum_{i=1}^{9}3\cdot x_{1i}\cdot\overline{y_i}}{\sum_{i=1}^{9}3\cdot x_{1i}^{2}} \quad (24)$$

$$b_1=\frac{3\cdot(-67.5+60.83-58.33+50.83-65+58.5)}{3\cdot(1+1+1+1+1+1)}$$
(25)

$$b_1=-3.44 \quad (26)$$

$$b_2=\frac{\sum_{i=1}^{9}3\cdot x_{2i}\cdot\overline{y_i}}{\sum_{i=1}^{9}3\cdot x_{2i}^{2}} \quad (27)$$

$$b_2=\frac{3\cdot(-67.5-60.83+58.33+50.83-64.33+55)}{3\cdot(1+1+1+1+1+1)} \quad (28)$$

$$b_2=-4.75 \quad (29)$$

Coefficients with combined index are determined by:

$$b_{ij}=\frac{\sum_{u=1}^{N}n_u\cdot x_{iu}\cdot x_{ju}\cdot\overline{y_u}}{\sum_{i=1}^{N}n_u\cdot x_{iu}^{2}\cdot x_{ju}^{2}} \quad (30)$$

$$b_{12}=\frac{\sum_{u=1}^{9}n_u\cdot x_{1u}\cdot x_{2u}\cdot\overline{y_u}}{\sum_{i=1}^{9}n_u\cdot x_{1u}^{2}\cdot x_{2u}^{2}} \quad (31)$$

$$b_{12}=\frac{3\cdot(67.5-60.83-58.33+50.83)}{3\cdot(1+1+1+1)}=-0.21 \quad (32)$$

Coefficients of regression with similar index are determined by following calculation:

$$b_{ii}=\frac{\sum_{u=1}^{N}n_u\cdot x_{iu}^{'}\cdot\overline{y_u}}{\sum_{i=1}^{N}n_u\cdot x_{iu}^{'}} \quad (33)$$

$$b_{11}=\frac{3\left[\frac{1}{3}(67.5+68.83+58.33+50.83+65+58.5)\right]}{3\cdot\left(6\cdot\frac{1}{9}+3\cdot\frac{4}{9}\right)}-$$
$$\frac{3\left[\frac{2}{3}(64.33+55+63.33)\right]}{3\cdot\left(6\cdot\frac{1}{9}+3\cdot\frac{4}{9}\right)}=-0.72 \quad (34)$$

$$b_{22}=\frac{3\left[\frac{1}{3}(67.5+68.83+58.33+50.83+64.33+55)\right]}{3\cdot\left(6\cdot\frac{1}{9}+3\cdot\frac{4}{9}\right)}-$$

$$b_{22}=\frac{3\left[\frac{2}{3}(65+58.5+63.33)\right]}{3\cdot\left(6\cdot\frac{1}{9}+3\cdot\frac{4}{9}\right)}=-2.81 \tag{35}$$

If equitation (19) is used, value of factor b_0 is determined:

$$b_0=b_0^{'}-\lambda_2\cdot\sum_{i=1}^{k}b_{ii}=b_0^{'}-\lambda_2\cdot(b_{11}+b_{22}) \tag{36}$$

$$b_0=60.41-\frac{2}{3}\cdot(-0.72-2.81)=62.76 \tag{37}$$

Therefore, expression for digression equitation in coded form is defined as:

$$\hat{y}=b_0^{'}+b_1x_1+b_2x_2+b_{12}x_1x_2+b_{11}x_1^{'}+b_{22}x_2^{'} \tag{38}$$

Regression equitation in coded (39) and natural (40) form is determined by following expressions:

$$\hat{y}=62.76-3.44x_1-4.75x_2-0.21x_1x_2-0.72x_1^2-2.81x_2^2 \tag{39}$$

$$\hat{y}=23.1+3.78x_1+9.22x_2-0.019x_1x_2-0.131x_1^2-1.41x_2^2 \tag{40}$$

6. Dispersion analysis

Dispersion analysis is conducted in order to defining of factors significance. Conducting of dispersion analysis is very important for mathematical modeling. Thus, if one significant factor is missing, value of experiment fault decreases.

On the other hand, if all factors are included (also insignificant factors), research becomes very prolonged and expensive. In the theory of experimental researches, set of included factors affect at results of research by degree of its importance. To conclude, insignificant factors have to be found and excluded.

6.1. Dispersion of experiment

Dispersion of experimental results is defined by following expressions:

$$s^2(y)=\frac{1}{\sum_{v=1}^{n}f_v}\cdot\sum_{v=1}^{N}f_v\cdot s_v^2 \tag{41}$$

$$s^2(y)=s_E^2=\frac{S_E}{f_E}=\frac{1}{N\cdot(n-1)}\cdot\sum_{u=1}^{N}\sum_{i=1}^{n}\left(y_{ui}-\bar{y}_u\right)^2 \tag{42}$$

$$S_E=\sum_{u=1}^{N}\sum_{i=1}^{n}\left(y_{ui}-\bar{y}_u\right)^2=\left(y_{11}-\bar{y}_1\right)^2+\ldots+\left(y_{93}-\bar{y}_9\right)^2 \tag{43}$$

$$S_E=(67.5-67.5)^2+(68-67.5)^2+(67-67.5)^2+\ldots+$$
$$\ldots+(63.5-63.33)^2=6.83 \tag{44}$$

Thus, dispersion of experiment is determined as:

$$s^2(y)=\frac{6.83}{9\cdot(3-1)}=0.379 \tag{45}$$

Values from equitation (45) are defined as:

- N=9 – number of different points.
- n=3 – number of experiment repetition in each point.

6.2. Dispersion of regression coefficients

Calculation of regression coefficients dispersion is shown by following expressions:

$$s^2(b_0^{'})=\frac{s^2(y)}{\sum_{u=1}^{N}n_u}=\frac{0.379}{3\cdot 9}=0.01403 \tag{46}$$

$$s(b_0^{'})=0.1185 \tag{47}$$

$$s^2(b_i)=\frac{s^2(y)}{\sum_{u=1}^{N}n_u\cdot x_{iu}^{2}} \tag{48}$$

$$s^2(b_1)=\frac{s^2(y)}{\sum_{u=1}^{9}n_u\cdot x_{1u}^{2}}=\frac{0.379}{3\cdot(1+1+1+1+1+1)}=0.02105 \tag{49}$$

$$s^2(b_2)=\frac{s^2(y)}{\sum_{u=1}^{9}n_u\cdot x_{2u}^{2}}=\frac{0.379}{3\cdot(1+1+1+1+1+1)}=0.02105 \tag{50}$$

$$s(b_1)=s(b_2)=0.1452 \tag{51}$$

$$s^2(b_{ii})=\frac{s^2(y)}{\sum_{u=1}^{N}n_u\cdot x_{iu}^{'2}} \tag{52}$$

$$s^2(b_{11})=\frac{s^2(y)}{\sum_{u=1}^{9}n_u\cdot x_{1u}^{'2}}=\frac{0.379}{3\cdot\left(6\cdot\frac{1}{9}+3\cdot\frac{4}{9}\right)}=0.06317 \tag{53}$$

$$s^2(b_{22})=\frac{s^2(y)}{\sum_{u=1}^{9}n_u\cdot x_{2u}^{'2}}=\frac{0.379}{3\cdot\left(6\cdot\frac{1}{9}+3\cdot\frac{4}{9}\right)}=0.06317 \tag{54}$$

$$s(b_{11})=s(b_{22})=0.2513 \tag{55}$$

$$s^2(b_{ij})=\frac{s^2(y)}{\sum_{u=1}^{N}n_u\cdot x_{iu}^{2}\cdot x_{ju}^{2}} \tag{56}$$

$$s^2(b_{12})=\frac{s^2(y)}{\sum_{u=1}^{9}n_u\cdot x_{1u}^{2}\cdot x_{2u}^{2}}=\frac{0.379}{3\cdot(1+1+1+1)}=0.0316 \tag{57}$$

$$s(b_{12})=0.1777 \tag{58}$$

$$s^2(b_0)=s^2(b_0^{'})+\lambda_2^2\cdot\sum_{i=1}^{k}s^2(b_{ii}) \tag{59}$$

$$s^2(b_0) = 0.01403 + \frac{4}{9}\left(0.06317 + 0.06317\right) = 0.0703 \quad (60)$$

$$s(b_0) = 0.265 \quad (61)$$

6.3. Defining of significance of regression coefficients

For estimation regression coefficient significance, Student criterion could be used. Criterion is conducted by determining of relation between absolute value of regression coefficient and dispersion of coefficient value. Calculated relation is compared with literature value [2]. If calculated relation is higher than literature value, factor is considered as significant. Thus, following condition is defined by criterion:

$$t_{ri} = \frac{|b_i|}{s(b_i)} \geq t_{t(f_0,\alpha)} \quad (62)$$

Literature value is defined by degree of freedom (f_0) and limit of significance (α=1-P). In this case, for degree of freedom f_0=27-9=18 and defined limit of significance α=5, according to table II [2], literature value **t_t=1.73**.

Calculation of relation between absolute value of regression coefficient and dispersion of coefficient value for each factor is determined as:

$$t_{r0} = \frac{|b_0|}{s(b_0)} = \frac{62.76}{0.265} = 236.83 > 1.73 \quad (63)$$

$$t_{r1} = \frac{|b_1|}{s(b_1)} = \frac{3.44}{0.1425} = 24.14 > 1.73 \quad (64)$$

$$t_{r2} = \frac{|b_2|}{s(b_2)} = \frac{4.75}{0.1425} = 32.71 > 1.73 \quad (65)$$

$$t_{r12} = \frac{|b_{12}|}{s(b_{12})} = \frac{0.21}{0.1777} = 1.18 < 1.73 \quad (66)$$

$$t_{r11} = \frac{|b_{11}|}{s(b_{11})} = \frac{0.72}{0.2513} = 2.86 > 1.73 \quad (67)$$

$$t_{r22} = \frac{|b_{22}|}{s(b_{22})} = \frac{2.81}{0.2513} = 11.18 > 1.73 \quad (68)$$

To conclude, factor b_{12} is insignificant, so it could be excluded from regression equitation. Therefore, final form of regression equitation is determined as:

$$\hat{y} = 23.1 + 3.78x_1 + 9.22x_2 - 0.131x_1^2 - 1.41x_2^2$$
(69)

7. Determining of model adequacy

Condition of model adequacy is defined by Fisher-method, as follows:

$$F_r = \frac{\frac{S_{LF}}{f_{LF}}}{\frac{S_E}{f_E}} \leq F_t\left(f_1, f_2\right)$$
(70)

Literature value of F-distribution is determined by following parameters:

$$f_{LF} = N - m = N - (d+1) = 9 - (5+1) = 3 \quad (71)$$

$$f_E = N_0 - N = 27 - 9 = 18 \quad (72)$$

Used magnitudes are defined as:

- N=9 – number of different points,
- m=5+1 – number of regression coefficients,
- N_0=27 – total number of conducted measurements,
- f_{LF} – number of degree of freedom, factor of distribution f_1,
- f_E – degree of freedom, factor of distribution f_2.

According to above mentioned calculation, literature value [2] of F-criterion for defined f_1 and f_2 and limit of significance α=5% is defined as **F_t=3.16**.

Dispersion for plan adequacy is determined by expression:

$$S_{LF} = n \cdot \sum_{u=1}^{N}\left(\bar{y}_u - \hat{y}_u\right)^2 = 2 \cdot \left[(67.5 - 67.21)^2 + \ldots + \ldots\right] = 2.627 \quad (72)$$

Therefore, by means of formula (70), adequacy of model is defined, as follows:

$$F_r = \frac{\frac{2.627}{3}}{\frac{6.83}{18}} = 2.31 < 3.16 \quad (73)$$

Concerning that calculated value is lower than value from literature, it can be concluded that hypothesis of model adequacy can be accepted. Researched process is adequately described by defined regression model.

8. Optimization of regression analysis factors

As it is calculated, defined regression model adequately (69) describes dependency of steel ball elastic rebound from ball diameter and hardness of PU foam. In order to get values of factors for the best case, e.g. the highest elastic rebound, mathematical calculation is conducted. Thus, factors x_1 and x_2 are optimized in order to determine its values for the highest value of ŷ.

$$23.1 + 3.78x_1 + 9.22x_2 - 0.131x_1^2 - 1.41x_2^2 =$$
$$= -(0.131x_1^2 - 3.78x_1) - (1.41x_2^2 - 9.22x_2) + 23.1 =$$

$$=-\left(\sqrt{0.131}x_1-\frac{1.89}{\sqrt{0.131}}\right)^2-\left(\sqrt{1.41}x_2-4.61\right)^2+$$

$$+23.1+\left(\frac{1.89}{\sqrt{0.131}}\right)^2+\left(\frac{4.61}{\sqrt{1.41}}\right)^2=$$

$$\approx 23.1+27.28+15.07-\left(\sqrt{0.131}x_1-\frac{.89}{\sqrt{0.131}}\right)^2-$$

$$-\left(\sqrt{1.41}x_2-\frac{4.61}{\sqrt{1.41}}\right)^2\approx$$

$$\approx 65.45-\left(\sqrt{0.131}x_1-\frac{1.89}{\sqrt{0.131}}\right)^2-\left(\sqrt{1.41}x_2-\frac{4.61}{\sqrt{1.41}}\right)^2 \quad (74)$$

Thus, the highest value of (74) will be achieved if

$$\sqrt{0.131}x_1-\frac{1.89}{\sqrt{0.131}}=0 \quad (75)$$

and:

$$\sqrt{1.41}x_2-\frac{4.61}{\sqrt{1.41}}=0. \quad (76)$$

If above mentioned equations are solved, requested values of x_1 and x_2 are:

$$x_1=14.42 \quad (77)$$

$$x_2=3.27$$

(78)

Accordingly, the highest elastic rebound, apropos the highest value of PU foam elasticity could be achieved in case of steel ball with diameter 14.42 mm (factor x_1) and PU foam with 3.27 kPa/m^2 of hardness (factor x_2).

9. Conclusion

Quality of polyurethane foam as main structural material in furniture industry is defined by main mechanical properties such as: density, hardness, elongation, permeability and elasticity. One of the basic methods for analysis of PU foam elasticity is method of measuring steel ball elastic rebound that is defined by standard ISO 8307. Above mentioned standard clearly describes instruments, equipment, conditions, preparation of specimens and procedure of conducting an experiment.

According to basic configuration of ISO defined procedure [4], procedure for conducting of experimental analysis in order to performing of regression analysis is defined. Thus, model that shows dependency of PU foam elasticity from steel ball diameter and PU foam hardness, as independently variables is set.

First conducted experiment presents analysis that is based on model of first order. It is proven that this model is not adequate for describing of process. Finally, regression model of second order is carried out by setting of symmetrical composition plan with two independent factors.

Checking of significance of individual regression coefficients is achieved by Student method. It is proven that one of six coefficients is insignificant. so it is excluded from model. Considering that factor b_{12} is shown as insignificant. it is concluded that mutually action of both factors does not have significant level of effect to process.

Fisher's method is used to define if set model adequately describes process PU foam elasticity measuring. It is shown that model of second order, made by two-factor plan with three iteration in each point of plan is adequate.

On the end, optimization is conducted in order to determine values of factor for the best case – case of the highest elasticity of PU foam. It can be concluded that if steel ball with diameter 14.42 mm and PU foam with hardness 3.27 kPa/m^2 are used, the height of rebound will be the highest, so elasticity of foam will be the most desirable.

10. References

REFERENCES

[1] N.V. Gama, A. Ferreira, A. Barros-Timmons (2018), *Polyurethane foams: Past, Present and Future*, Materials 2018, MDPI, Basel, Switzerland

[2] Jurković Milan (1999), *Matematičko modeliranje inženjerskih procesa i sistema*, Mašinski fakultet u Bihaću, Bihać

[3] Rok Brinc (2007), *Poročilo nostranjega izobreževanja, Vlivanje pene*, Podgrad, Slovenija

[4] Stanić Joko (1981), *Metod inženjerskih mjerenja, Mašinski fakultet*, Beograd

[5] Richard A. Jonson (2011), *Probability and Statistics for Engineers*, Pearson, Eighth Edition

[6] Douglas C. Montgomery, *Design and Analysis, International Student Version*, John Viley and sons, Eighth Edition