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A new framework of consensus protocol design for complex multi-agent systems

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ABSTRACT

This paper aims to find a simple but efficient method for consensus protocol design. This paper presents two consensus protocols to solve the consensus problem of complex multi-agent systems that consist of inhomogeneous subsystems. The limitations of current studies are analyzed, and a novel model based on transfer functions is presented. This model can be used to describe both homogeneous and inhomogeneous multi-agent systems in a unified framework. Based on this model, two sufficient and necessary conditions for the consensus of complex multi-agent systems have been obtained. One is for the systems without any external input, and the other is for the systems with the same external input. Then, two corresponding distributed consensus protocols are presented. Considering that the complex multi-agent systems may require different outputs sometimes, the relationship between inputs and outputs is analyzed. Finally, some simulations are given to demonstrate the performance and effectiveness of the proposed approaches.

1. Introduction

A multi-agent system consists of some intelligent agents which have the capability of reacting to the variety of environment automatically, such as robots, automatic vehicles, sensors, controllers, and so on. Multi-agent systems are always distributed, autonomous, and cooperative. Due to these advantages, multi-agent systems are more and more widely applied in all kinds of fields, such as micro-satellite clusters [1,2], unmanned aerial vehicles (UAVs) [3,4], air traffic management (ATM) [5,6], cooperative communication [7], and so on. One basic problem of these applications is the consensus of multi-agent systems. Recently, more and more researchers are paying attention to the consensus problem.

The most commonly used models to study the multi-agent systems are single integrator and double integrator models as shown in Eqs. (1) and (2). For example, Tumerdem [8] analyzed the constraints that the forces as well as the positions should be transmitted between all the robots in haptic teleoperation with multiple operators and multiple robots, and relaxed these constraints on the control system by using information graphs and consensus algorithms. His work realized robust teleoperation with multiple telerobots under changing communication topologies based on the single integrator model. Sadeghzadeh [9] solved the consensus problem in homogenous sensor networks in the presence of the communication time delay based on the single integrator model. Olfati-Saber [10] studied the flocking and obstacle avoidance of multi-agent systems based on the double integrator model. Yang et al. [11] applied the decentralized simultaneous estimation to solve the problem of formation control based on the double integrator model. However, many complex systems cannot be described by these two models. To solve the consensus problem for all kinds of complex multi-agent systems in a uniform framework, we need to find a new way to design consensus protocols based on a more common model.

$$\dot{x}_i = u_i \tag{1}$$

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i. \end{cases} \tag{2}$$

Recently, some methods have been presented to study the consensus problem of the complex multi-agent systems based on the state-space models. For example, Xie [12] studied the consensus in high-dimensional multi-agent systems by a linear state feedback control protocol, while the similar conclusions are achieved by a distributed observer-type consensus protocol in [13]. Scardovi [14] investigated the synchronization of a network of identical linear time-invariant state-space models under a possibly time-varying and directed interconnection structure. However, since each agent in these multi-agent systems is described by the same model, the proposed consensus protocols still failed to solve the consensus problem of complex multi-agent systems in a uniform framework, i.e. the multi-agent systems consisting of inhomogeneous agents.

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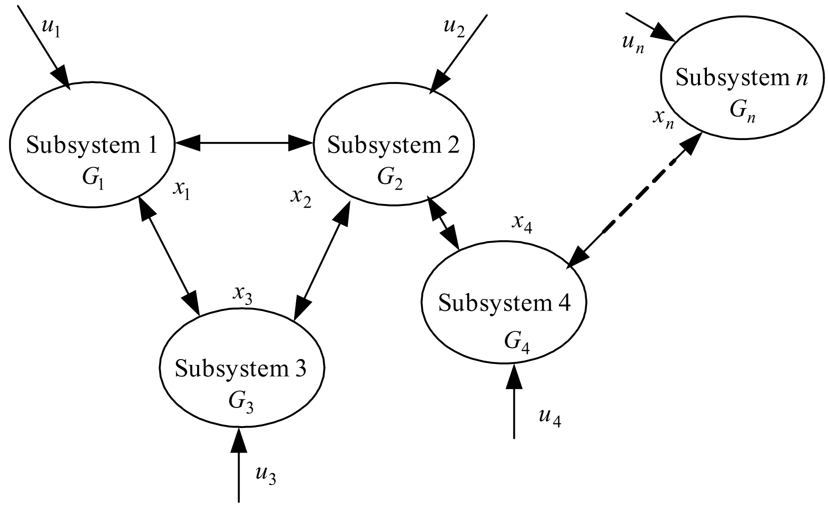

Fig. 1. Topology structure of a complex multi-agent system.

In this paper, a more general model is considered as shown in Eq. (3). Obviously, the homogenous multi-agent systems are the special cases of this model. Based on this model, we present a method for consensus protocol design. This paper mainly has the following contributions: First, the model of each agent, which is also called the subsystem in this paper, is created based on transfer functions. Second, according to the configurations of the multi-agent systems, a feedback system is designed for each subsystem. Two sufficient and necessary conditions are presented to make the complex multi-agent systems be consensus. Third, the relationship between inputs and outputs of the complex multi-agent systems is analyzed. Furthermore, it is observed from the simulation results that the advantages of the proposed method can be described as follows:

(1) Using the proposed model, it is easier to design a consensus protocol for inhomogeneous multi-agent systems;

(2) The performances of subsystems can be adjusted through setting the appropriate parameters in the designed controllers; and

(3) The proposed consensus protocols are distributed. Increasing or decreasing the number of subsystems will not influence the system consensus.

$$
\begin{aligned}
& a_{i,n}\frac{\mathrm{d}^n(c_i)}{\mathrm{d}t^n}+a_{i,n-1}\frac{\mathrm{d}^{n-1}(c_i)}{\mathrm{d}t^{n-1}}+\cdots+a_{i,1}\frac{\mathrm{d}(c_i)}{\mathrm{d}t}+a_{i,0}c_i \\
&\quad = b_{i,m}\frac{\mathrm{d}^m(r_i)}{\mathrm{d}t^m}+b_{i,n}\frac{\mathrm{d}^{m-1}(r_i)}{\mathrm{d}t^m}+\cdots+b_{i,1}\frac{\mathrm{d}(r_i)}{\mathrm{d}t}+b_{i,0}r_i, \\
&\quad i=1,\ldots,N.
\end{aligned} \tag{3}
$$

The remainder of this paper is organized as follows. In Section 2 some basic concepts are introduced. In Section 3 the model of the complex multi-agent system is proposed. Two consensus protocols are presented in Section 4. Simulation results are given in Section 5. Section 6 concludes the paper.

2. Preliminaries

In this section, some basic concepts about the complex multi-agent system are introduced. The concepts related to graph theory refer to [15,16].

2.1. Complex multi-agent systems

In this paper we consider a connected complex multi-agent system which consists of multiple inhomogeneous subsystems. For convenience, the multi-agent system can be represented by a graph $G(V, E)$ as shown in Fig. 1, where, the vertex $i \in V$ in the graph denotes the subsystem i, and the edge $(i, j) \in E$ denotes the interaction between any two subsystems, meaning that the output x_i of one system is part of the input of another subsystem. Taking Eq. (1) as an example, researchers usually let the output x_i as part of the input of subsystem j, where subsystem j is connected to the subsystem i. u_i is the external input of subsystem i. In Fig. 1, G_i denotes the transfer function of subsystem i.

Remark 1. By custom, we use u_i to denote the external input of subsystem i. Note that the symbol u_i in this paper is different from the same symbol in Eqs. (1) and (2) which is the consensus protocol.

If two subsystems are connected, we say they are neighbors. The neighbor set of subsystem i is defined as $N_i = j \in V, (i, j) \in E$. Let $A = [a_{ij}] \in R^{N\times N}$ denote the adjacency matrix of the complex multi-agent system, where $a_{ij} = 1$, if subsystem i and subsystem j are adjacent, otherwise, $a_{ij} = 0$.

2.2. Laplacian matrix

The degree matrix of a graph G is a diagonal matrix ΔA with diagonal elements $\Sigma_{j=1}^{N} a_{ij}$ that are the row-sum of adjacency matrix A. Then the Laplacian matrix $L \in R^{N\times N}$ is defined as

$$L = \Delta A - A = DD^\top \tag{4}$$

where D is the incidence matrix of the graph. For a connected graph, the rank of L is $N-1$. An important property of the Laplacian matrix is that

$$LX = 0 \tag{5}$$

where $X = c(1, 1, \ldots, 1)^\top$ and c is any constant.

3. Model of complex multi-agent systems

In this section, we present a new model of the complex multi-agent systems based on transfer functions.

Considering subsystem i of the complex multi-agent system as shown in Fig. 1, we can use a block diagram to describe the relation between inputs and outputs as shown in Fig. 2, where $X_i(s) = \mathscr{L}(x_i)$, and $\mathscr{L}$ denotes the Laplace transform. $X_j(s), X_k(s), \ldots, x_l(s)$ are the outputs of the neighbors of subsystem i. From Fig. 2, it is easy to obtain the model of the subsystems as follows:

$$X_i(s) = \left(U_i(s) + \sum_{j\in N_i} X_j(s)\right) G_i(s). \tag{6}$$

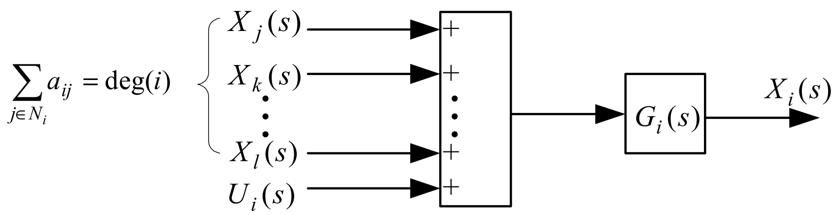

Fig. 2. Block diagram of subsystem i.

Remark 2. The model of Eq. (6) overcomes the limitations discussed in the section of introduction. When $G_i(s)$, $(1 = 1, \ldots, n)$ are the same, it describes the homogeneous system, otherwise, the described system is inhomogeneous. Single integrator, double integrator and state-space function models analyzed in the introduction can all be described by this model.

4. Consensus protocols

In this section, two consensus protocols of the complex multi-agent systems are presented. One is the protocol for the systems without external input, and the other is for the systems with the same external input. Our major work aims to make all the outputs of the complex multi-agent systems the same as shown in Eq. (7).

$$\lim_{s\to 0} sX_1(s) = \lim_{s\to 0} sX_2(s) = \cdots = \lim_{s\to 0} sX_N(s) \quad \text{or}$$
$$\lim_{t\to\infty} x_1(t) = \lim_{t\to\infty} x_2(t) = \cdots = \lim_{t\to\infty} x_N(s). \tag{7}$$

4.1. Consensus protocol for complex multi-agent systems without external input

A subsystem of the complex multi-agent system without external input is shown in Fig. 3. Here, we mainly use the classical feedback control theory to design the consensus protocol. Take the subsystem i into consideration just as shown in the block diagram in Fig. 3. The transfer function of this subsystem is shown as follows:

$$\frac{X_i(s)}{\sum\limits_{j\in N_i} a_{ij}X_j(s)} = \frac{G_{ci}(s)G_i(s)}{1+\sum\limits_{j\in N_i} a_{ij}G_{ci}(s)G_i(s)} \tag{8}$$

where, $G_{ci}(s)$ is the designed controller that is used to control the subsystem $G_i(s)$, and $\deg(i) = \sum_{j\in N_i} a_{ij}$ is the feedback gain.

Firstly, we present a theorem to show the sufficient and necessary condition of the consensus of the complex multi-agent system without external input as follows:

Theorem 1. *Suppose the complex multi-agent system has no external input. Each subsystem is designed as the block diagram in Fig. 3, and it is stable. Then, all the outputs of this complex multi-agent system will be consensus if and only if $G_{ci}(s)G_i(s)$ contains the factor $1/s$.*

Proof (*Sufficiency*)**.** Assume that contains the factor $1/s$. Rearrange Eq. (8), one can obtain

$$\frac{X_i(s)}{G_{ci}(s)G_i(s)} = \sum_{j\in N_i} a_{ij}(X_j(s) - X_i(s)). \tag{9}$$

Let $X(s) = (X_1(s), X_2(s), \ldots, X_N(s))$, then the complex multi-agent system can be described in the form

$$\Lambda X(s) = -LX(s) \tag{10}$$

where L is the Laplacian matrix defined in Eq. (4), and Λ is a diagonal matrix as follows:

$$\Lambda = \begin{pmatrix} \frac{1}{G_{c1}G_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{G_{c2}G_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{G_{cN}G_N} \end{pmatrix}. \tag{11}$$

Apparently, Eq. (10) is equivalent to the following equation:

$$(\Lambda + L)X(s) = 0. \tag{12}$$

When $s \to 0$, $\lim_{s\to 0}\left(\frac{1}{G_{ci}(s)G_i(s)}\right) = 0$, therefore $\lim_{s\to 0}\Lambda = 0$. Then one obtains that:

$$\lim_{s\to 0}(\Lambda + L)sX(s) = L\lim_{s\to 0} sX(s) = 0. \tag{13}$$

For the stable subsystems, according to the Laplacian Final Value Theorem, one obtains that $\lim_{s\to 0} sX(s) = \lim_{t\to\infty} x(t)$. Since the rank of L is $\mathrm{Rank}(L) = N - 1$, has only one basic solution set $1_N^\top = (1, 1, \ldots, 1)^\top$. Therefore, the solution of $\lim_{s\to 0} sX(s)$ is $\lim_{s\to 0} sX(s) = c(1, 1, \ldots, 1)^\top$, where c is a constant. It is easy to deduce that:

$$\lim_{s\to 0} sX_1(s) = \lim_{s\to 0} sX_2(s) = \cdots = \lim_{s\to 0} sX_N(s).$$

In other words, the complex multi-agent system is consensus when $t \to \infty$.

(Necessity) All the outputs of the complex multi-agent system converge to the same value. Assume that $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = c \cdot 1_N$, where c is a constant. Applying $\lim_{s\to 0} sX(s)$ into Eq. (10), it is easy to find that $L\lim_{s\to 0} sX(s) = 0$. Then, the following equation is obtained:

$$c\lim_{s\to 0}\Lambda \cdot 1_N = 0. \tag{14}$$

It then follows from Eq. (14) that

$$\lim_{s\to 0}\frac{1}{G_{ci}(s)G_i(s)} = 0, \quad i = 1, 2, \ldots, N. \tag{15}$$

Apparently, $G_{ci}(s)G_i(s)$ contains the factor $1/s$. □

Theorem 1 provides the sufficient and necessary condition to achieve consensus for a complex multi-agent system without considering the initial values. Actually, since the multi-agent systems are linear, the initial values don't influence the consensus, but different initial values will lead to different consensus values which will be illustrated by the simulations in Section 5.

From Theorem 1, the consensus protocol of complex multi-agent systems without external input can be obtained as follows:

Consensus protocol 1: Suppose the complex multi-agent system has no external input, and each subsystem is designed as the

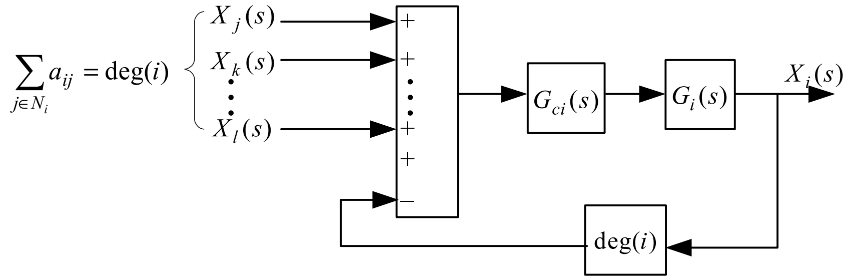

Fig. 3. Block diagram of subsystem *i* for consensus protocol design without external input.

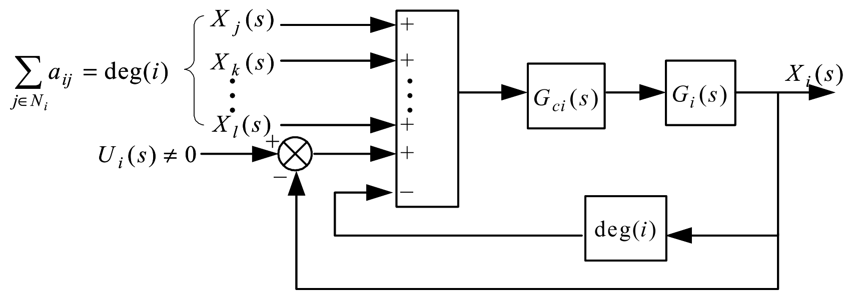

Fig. 4. Block diagram of subsystem *i* for consensus protocol design with external input.

block diagram in Fig. 3. Then, the complex multi-agent system will achieve consensus if Eq. (8) satisfies the following two conditions:

(1) All roots of the characteristic equation Eq. (20) have a negative real part, which means that the control systems as shown in Fig. 3 are stable;

$$1 + \sum_{j\in N_i} a_{ij} G_{ci}(s) G_i(s) = 0, \quad i = 1, 2, \ldots, n. \tag{16}$$

(2) $G_{ci}(s)G_i(s)$ contains the factor $1/s$.

4.2. Consensus protocol for complex multi-agent systems with external input

The external input in this paper is defined as the state of target which is always seen as the state of a leader in other work such as in the studies of flocking and formation control. Usually, the target may be only known by some of the subsystems, in other words, some subsystems have external input, while others have none. In this part we assume that the complex multi-agent system consists of M subsystems with external input and N_M subsystems without external input. The controller design to those subsystems without external input is the same as in Part 4.1 just as shown in Fig. 3. But there are some differences with the controller design of subsystems that have external input as shown in Fig. 4. In this case, there is a negative unit feedback to the external input $U_i(s)$.

The transfer function of this subsystem can be described as follows:

$$\frac{X_i(s)}{U_i(s) + \sum_{j\in N_i} a_{ij} X_j(s)} = \frac{G_{ci}(s)G_i(s)}{1 + \left(1 + \sum_{j\in N_i} a_{ij}\right) G_{ci}(s) G_i(s)}. \tag{17}$$

Before presenting the second consensus protocol of the complex multi-agent systems with the same input, we firstly present the following results:

Theorem 2. *Assume that some subsystems of the complex multi-agent system have the same external input, and they are designed as the block diagram in Fig. 4, while others are designed as the block diagram in Fig. 3. Also assume that these subsystems are stable. Then, the complex multi-agent system will asymptotically converge to the external input if and only if $G_{ci}(s)G_i(s)$ contains the factor $1/s$.*

Proof (*Sufficiency*)**.** Assume that $G_{ci}(s)G_i(s)$ contains the factor $1/s$. Rearrange Eq. (17), one obtains the following equation:

$$\frac{X_i(s)}{G_{ci}(s)G_i(s)} = U_i(s) - X_i(s) + \sum_{j\in N_i} a_{ij}(X_j(s) - X_i(s)). \tag{18}$$

Let $X^1(s) = (X_1(s), X_2(s), \ldots, X_M(s))^\top$ denote the output vector of the subsystems with the same external input, $X^2(s) = (X_{M+1}(s), X_{M+2}(s), \ldots, X_N(s))^\top$ denote the output vector of the subsystems without external input and $U^1(s) = (U_1(s), U_2(s), \ldots, U_M(s))^\top$ denote the input vector of the subsystems with external input. Note that $U_1(s) = U_2(s) = \cdots = U_M(s)$. Then, the complex multi-agent system can be described as follows:

$$\begin{aligned} \Lambda \begin{bmatrix} X^1(s) \\ X^2(s) \end{bmatrix} &= \begin{bmatrix} U^1(s) \\ 0 \end{bmatrix} - \begin{bmatrix} X^1(s) \\ 0 \end{bmatrix} + \left(-L \begin{bmatrix} X^1(s) \\ X^2(s) \end{bmatrix}\right) \\ &= \begin{bmatrix} U^1(s) \\ 0 \end{bmatrix} - \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X^1(s) \\ X^2(s) \end{bmatrix} - L \begin{bmatrix} X^1(s) \\ X^2(s) \end{bmatrix} \end{aligned} \tag{19}$$

where I is the identity matrix. Since $\lim_{s\to 0} \frac{1}{G_{ci}(s)G_i(s)} = 0$, one obtains that $\lim_{s\to 0} \Lambda = 0$, and

$$\left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + L\right) \lim_{s\to 0} \begin{bmatrix} sX^1(s) \\ sX^2(s) \end{bmatrix} = \lim_{s\to 0} \begin{bmatrix} sU^1(s) \\ 0 \end{bmatrix}. \tag{20}$$

Consider the following equation:

$$\left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + L\right) Z = 0 \tag{21}$$

where $Z \in R^{N\times 1}$. Multiply $Z^\top$ to the left of Eq. (21), one obtains that

$$Z^\top \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} Z + Z^\top D D^\top Z = 0.$$

Since $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ and $DD^\top$ are both positive semidefinite, $Z^\top \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} Z = 0$ and its solution is $Z = [0, \ldots, 0, z_{M+1}, \ldots, z_N]^\top$. Applying Z to Eq. (21), one obtains that $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} Z = 0$ and $LZ = 0$. Since $\operatorname{rank}(L) = N - 1$, $Z = c \cdot 1_N$ is the only solution. Combining

$Z = [0, \dots, 0, z_{M+1}, \dots, z_N]^\top$, it is easy to find that $Z = 0$ is the only solution of Eq. (21), therefore rank $\left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + L\right) = N$. So Eq. (20) has only one solution. Assume that the final values of external inputs are $\lim_{s\to 0} sU_1(s) = \lim_{s\to 0} sU_2(s) = \cdots = \lim_{s\to\infty} sU_M(s) = c$, $\lim_{s\to 0} sX(s) = \lim_{s\to 0} \begin{bmatrix} sX^1(s) \\ sX^2(s) \end{bmatrix} = c \cdot 1_N$ will be the unique solution. In other words, the outputs of this complex multi-agent system asymptotically converge to the same value when $t \to \infty$.

(Necessity) All the outputs of the complex multi-agent system converge to the external input when $t \to \infty$. Assume that $\lim_{s\to 0} sX(s) = c \cdot 1_N \neq 0$ and $\lim_{s\to 0} sU^1(s) = c \cdot 1_M$, where c is a constant. Applying these to Eq. (19), one obtains the equation as follows:

$$\lim_{s\to 0} \Lambda sX(s) = c \lim_{s\to 0} \Lambda \cdot 1_N = 0. \tag{22}$$

It then follows from Eq. (22) that

$$\lim_{s\to 0} \frac{1}{G_{ci}(s)G_i(s)} = 0, \quad i = 1, 2, \dots, N. \tag{23}$$

Apparently, $G_{ci}(s)G_i(s)$ contains the factor $1/s$. □

From Theorem 2, the second consensus protocol of the complex multi-agent systems with the same external input can be obtained as follows:

Consensus protocol 2: Assume that some subsystems of the complex multi-agent system have the same external input, and they are designed as the block diagram in Fig. 4, while others are designed as the block diagram in Fig. 3. Then, the complex multi-agent system will asymptotically converge to the external input if Eq. (21) satisfies the following conditions:

(1) All roots of the characteristic equations Eqs. (20) and (21) have a negative real part, which means all subsystems are stable

$$1 + \left(1 + \sum_{j\in N_i} a_{ij}\right) G_{ci}(s)G_i(s) = 0, \quad i = 1, 2, \dots, M \tag{24}$$

$$1 + \sum_{j\in N_i} a_{ij} G_{ci}(s)G_i(s) = 0, \quad i = M+1,\ M+2, \dots, N \tag{25}$$

where M is the number of subsystems with external input.

(2) $G_{ci}(s)G_i(s)$ contains the factor $1/s$.

It can be observed that these two protocols are similar in the form and can be designed easily. Their advantages include (i) the forms of subsystems are arbitrary; (ii) the conditions are easy to satisfy; and (iii) both protocols are distributed.

Although we normally consider one target in the consensus problem, and design the protocols to make the outputs converge to the same target, the following situation may exist. That is we want the outputs of a connected multi-agent system to be different when it needs to do some different tasks at the same time. Then, how to decide the inputs? The next theorem shows the relationship between inputs and outputs.

Theorem 3. *The subsystems of the complex multi-agent system have different inputs, and the design of the controllers satisfies the conditions of consensus protocol 2. When $s \to 0$, the inputs and outputs of the subsystems satisfy the following condition:*

$$\lim_{t\to\infty} \begin{bmatrix} u_1(t) \\ 0 \end{bmatrix} = \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + L\right) \lim_{t\to\infty} x(t). \tag{26}$$

Proof. The proof can refer to the derivation of Eq. (20). □

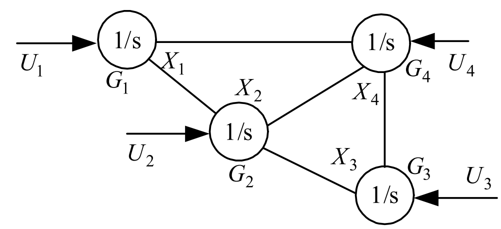

Fig. 5. Multi-agent system constituted by single integrator subsystems.

5. Simulations

In this section, we provide some simulations for two complex multi-agent systems. One system consists of four connected subsystems whose models can be described by Eq. (1), and the other system consists of four inhomogeneous subsystems. During these simulations, we provide the distributed controllers and analyze the relationship between the parameters of the controllers and the performance of the system. The influences of initial values to the consensus value are also considered. To illustrate the relationship between the inputs and outputs of the complex multi-agent systems, the corresponding simulations are carried out as shown in Part 5.3.

5.1. Homogeneous multi-agent system with single integrator subsystems

Many studies have investigated the consensus protocols based on the single integrator system. Here, we also give the example of a multi-agent system which consists of 4 single integrator subsystems. The topology structure of this multi-agent system is shown in Fig. 5.

First, we consider the situation that there is no external input. The distributed controller of each subsystem is constructed according to Fig. 3. To satisfy the condition of consensus protocol 1, the controllers can be designed as follows:

$$G_{ci}(s) = \frac{b}{s+a}, \quad i = 1, 2, \dots, N \tag{27}$$

where a and b are positive constants. Since the subsystems are homogeneous, the controllers have the same form. Then applying Eq. (27) to Eq. (9), one obtains the following characteristic equation:

$$s^2 + as + b\sum_{j\in N_i} a_{ij} = 0, \quad i = 1, 2, \dots, N. \tag{28}$$

Two roots are $s_{1,2} = \frac{-a\pm\sqrt{a^2-4b\sum_{j\in N_i} a_{ij}}}{2}$. The damped coefficient is $\zeta = \frac{a}{2\sqrt{b\sum_{j\in N_i} a_{ij}}}$, and the free frequency is $\omega_n = \sqrt{b\sum_{j\in N_i} a_{ij}}$. Therefore, we can assign the poles of each subsystem arbitrarily by adjusting the parameter a and b. Some simulation results with different parameters are shown in the following:

(1) Let $a = b = 1$ for all $G_{ci}(s)$, $(i = 1, 2, \dots, 4)$, and the values of $s_1, s_2, \zeta, \omega_n$ and of every subsystem are shown in Table 1. The initial values are set as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10, 14, 19, 25)$. The simulation results are shown in Fig. 6(a), and the consensus value is 17.00.

(2) Keep the same initial values. To obtain better performance, a, b are chosen differently. The relationship of the parameters $a, b, s_{1,2}, \zeta$, and ω_n are shown in Table 2. The simulation results are shown in Fig. 6(b), and the consensus value is 17.25.

(3) Keep the same parameters $a, b, s_{1,2}, \zeta$, and ω_n as in the case (2). Let the initial values be $(x_1(0), x_2(0), x_3(0), x_4(0)) = (15, 18, 21, 28)$. The simulation results are shown in Fig. 6(c), and the consensus value is 20.75.

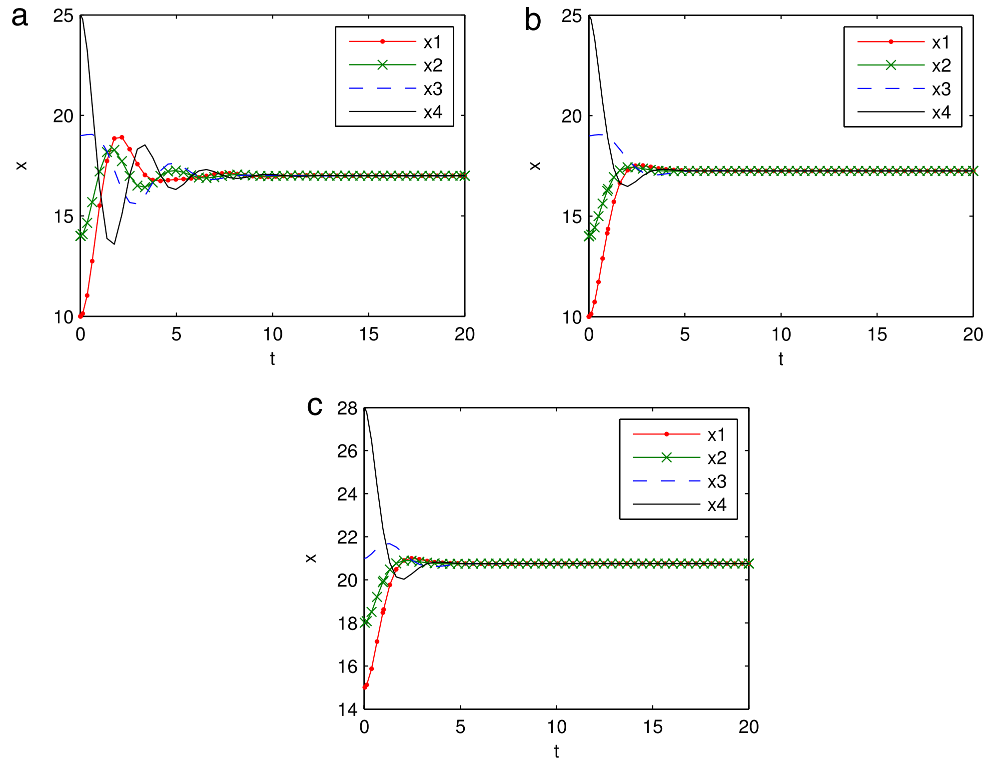

Fig. 6. Consensus of the homogeneous multi-agent system without external input.

Table 1
$s_{1,2}$, ζ, ω_n of multi-agent system without external input.

	a	b	$s_{1,2}$	ζ	ω_n
Subsystem 1	1	1	$\frac{-1\pm j\sqrt{7}}{2}$	$\frac{1}{2\sqrt{2}}$	$\sqrt{2}$
Subsystem 2	1	1	$\frac{-1\pm j\sqrt{11}}{2}$	$\frac{1}{2\sqrt{3}}$	$\sqrt{3}$
Subsystem 3	1	1	$\frac{-1\pm j\sqrt{7}}{2}$	$\frac{1}{2\sqrt{2}}$	$\sqrt{2}$
Subsystem 4	1	1	$\frac{-1\pm j\sqrt{11}}{2}$	$\frac{1}{2\sqrt{3}}$	$\sqrt{3}$

Table 2
Relationships of a, b, $s_{1,2}$, ζ, and ω_n of multi-agent system without external input.

	a	b	$s_{1,2}$	ζ	ω_n
Subsystem 1	2	1	$-1\pm j$	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$
Subsystem 2	$\sqrt{6}$	1	$\frac{-\sqrt{6}\pm j\sqrt{6}}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}$
Subsystem 3	2	1	$-1\pm j$	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$
Subsystem 4	$\sqrt{6}$	1	$\frac{-\sqrt{6}\pm j\sqrt{6}}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}$

Comparing Fig. 6(b) with Fig. 6(a), one can find that Fig. 6(b) has a smaller overshoot, which indicates that adjusting the parameters of controllers appropriately will be able improve the performance of the multi-agent system.

It can be observed from these three simulations in Fig. 6 that the consensus value of the multi-agent system is related to the initial values of subsystems and the parameters of controllers.

Next, we consider the multi-agent system with the same external input. Assume that subsystem 1 and subsystem 2 have the same external input $u_1(t) = u_2(t) = 20$. The controllers are set as Eq. (27). Applying Eqs. (28) and (25), the characteristic equations of this multi-agent system are shown as follows:

$$s^2 + as + (1 + b\Sigma_{j\in N_i} a_{ij}) = 0, \quad i = 1, 2 \tag{29}$$

$$s^2 + as + b\Sigma_{j\in N_i} a_{ij} = 0, \quad i = 3, 4. \tag{30}$$

One can obtain

$$s_{1,2} = \frac{-a \pm \sqrt{a^2 - 4(1+b)\sum_{j\in N_1} a_{ij}}}{2}, \qquad \zeta = \frac{a}{2\sqrt{1 + b\sum_{j\in N_i} a_{ij}}},$$

$$\omega_n = \sqrt{1 + b\sum_{j\in N_i}}$$

for Eq. (29), and

$$s_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b\sum_{j\in N_i} a_{ij}}}{2}, \qquad \zeta = \frac{a}{2\sqrt{b\sum_{j\in N_i} a_{ij}}},$$

$$\omega_n = \sqrt{b\sum_{j\in N_i} a_{ij}}$$

for Eq. (30). Some simulation results with different parameters are shown as follows:

(1) Let $a = b = 1$ for all $G_{ci}(s)$, $(i = 1, 2, \ldots, 4)$, and the initial values are set as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10, 14, 19, 25)$, that is the same as simulation (1) of the system without external input. Then the values of s_1, s_2, ζ, and ω_n of every subsystem are shown in Table 3. The simulation results are shown in Fig. 7(a). One can find that all the outputs converge to the same value.

(2) Keep the same initial values. To obtain better performance, a, b are chosen differently. The relationship of the parameters a, b, $s_{1,2}$, ζ, and ω_n are shown in Table 4. The simulation results are shown in Fig. 7(b).

(3) Keep the same parameters a, b, $s_{1,2}$, ζ, and ω_n as in simulation (2). Let the initial values be $(x_1(0), x_2(0), x_3(0), x_4(0)) = (15, 18, 21, 28)$. The simulation results are shown in Fig. 7(c).

It can be observed from these three figures in Fig. 7 that all the outputs track the inputs in these three situations. Fig. 7(b) also

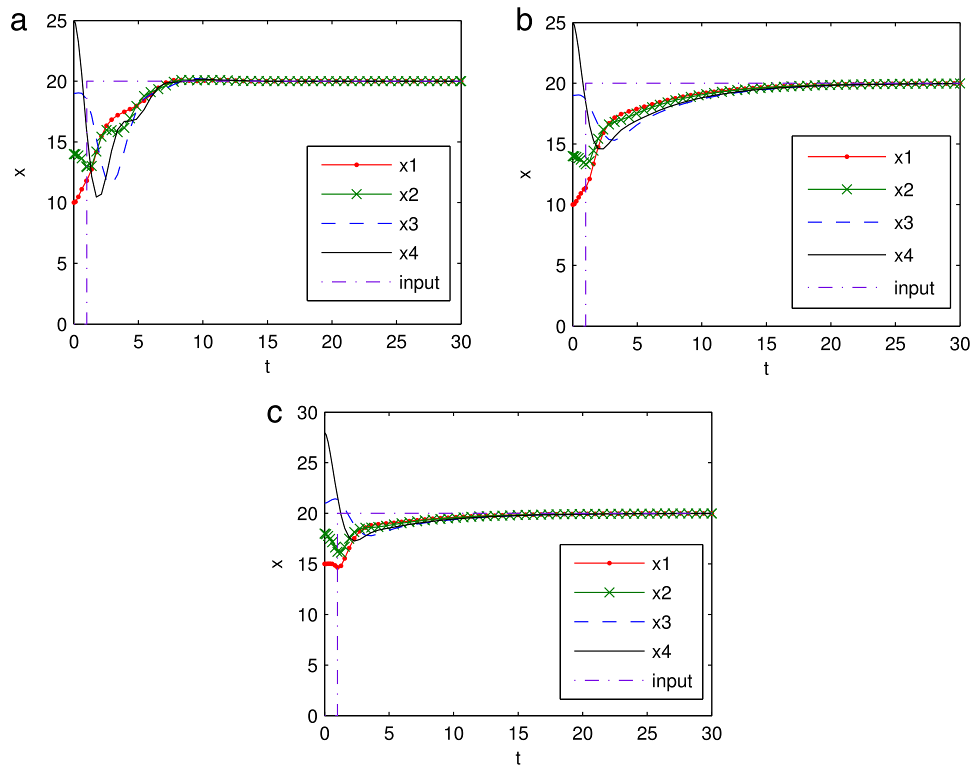

Fig. 7. Consensus of the homogeneous multi-agent system with external input.

Table 3
$s_{1,2}$, ζ, ω_n of the multi-agent system with external input.

	a	b	$s_{1,2}$	ζ	ω_n
Subsystem 1	1	1	$\frac{-1\pm j\sqrt{11}}{2}$	$\frac{1}{2\sqrt{3}}$	$\sqrt{3}$
Subsystem 2	1	1	$\frac{-1\pm j\sqrt{15}}{2}$	$\frac{1}{4}$	2
Subsystem 3	1	1	$\frac{-1\pm j\sqrt{7}}{2}$	$\frac{1}{2\sqrt{2}}$	$\sqrt{2}$
Subsystem 4	1	1	$\frac{-1\pm j\sqrt{11}}{2}$	$\frac{1}{2\sqrt{3}}$	$\sqrt{3}$

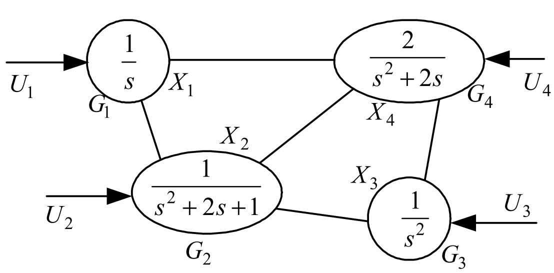

Fig. 8. Multi-agent system constituted by inhomogeneous subsystems.

Table 4
Relation of a, b, $s_{1,2}$, ζ, and ω_n.

	a	b	$s_{1,2}$	ζ	ω_n
Subsystem 1	$\sqrt{6}$	1	$\frac{-\sqrt{6}\pm j\sqrt{6}}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}$
Subsystem 2	$2\sqrt{2}$	1	$-\sqrt{2}\pm j\sqrt{2}$	$\frac{1}{\sqrt{2}}$	2
Subsystem 3	2	1	$-1\pm j$	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$
Subsystem 4	$\sqrt{6}$	1	$\frac{-\sqrt{6}\pm j\sqrt{6}}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}$

shows that setting appropriate controller parameters can decrease overshoot.

The simulations above demonstrate that these two proposed consensus protocols are efficient, and the performances can be improved easily through adjusting the parameters of controllers.

5.2. Inhomogeneous multi-agent systems

Distinct from other studies, we provide the simulations of an inhomogeneous multi-agent system. The topology of this complex multi-agent system is shown as Fig. 8.

First, we consider the system without external input. To satisfy the condition of consensus protocol 1, the controllers can be designed as follows:

$$G_{c1}(s) = \frac{1}{s+1}, \qquad G_{c2}(s) = \frac{s^2+2s+1}{s^2+s},$$
$$G_{c3}(s) = \frac{s}{s+1}, \qquad G_{c4} = \frac{s+2}{s+1}. \tag{31}$$

Of course, the controllers also can be designed to be more complex. The initial values of the subsystems are set as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (15, 18, 21, 28)$. The simulation results are shown in Fig. 9.

From Fig. 9, one can find that the inhomogeneous multi-agent system asymptotically converges to the same value, which demonstrates that the consensus protocol 1 is efficient.

Then add an external input $u(t) = 20$ to subsystem 1 and subsystem 3, and keep the same initial values. The simulation results are shown in Fig. 10.

From Fig. 10, one can find that the outputs of the inhomogeneous multi-agent system track the input, which demonstrates that the consensus protocol 2 is also efficient.

5.3. Relationship between inputs and outputs

To validate Theorem 3, the complex multi-agent system as shown in Fig. 8 is added by four different external inputs. Assume

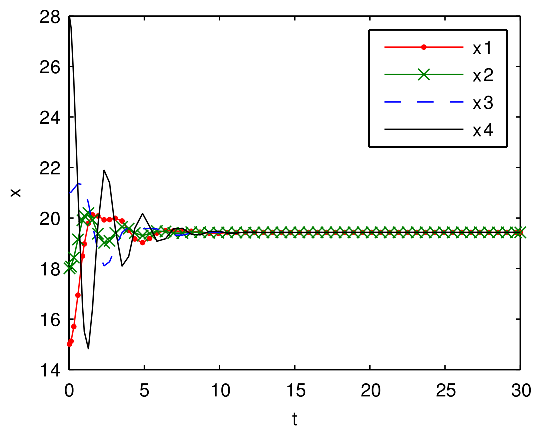

Fig. 9. Consensus of the inhomogeneous multi-agent system without external input.

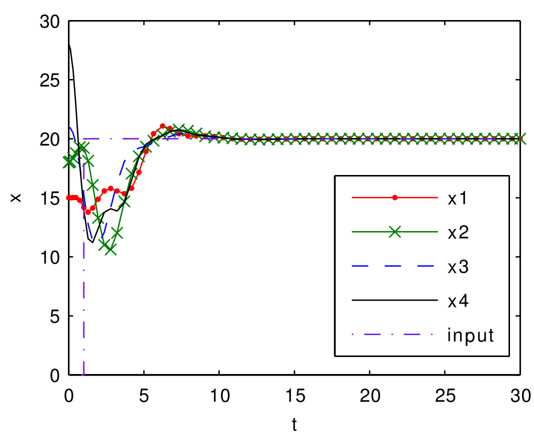

Fig. 10. Consensus of the inhomogeneous multi-agent system with external input.

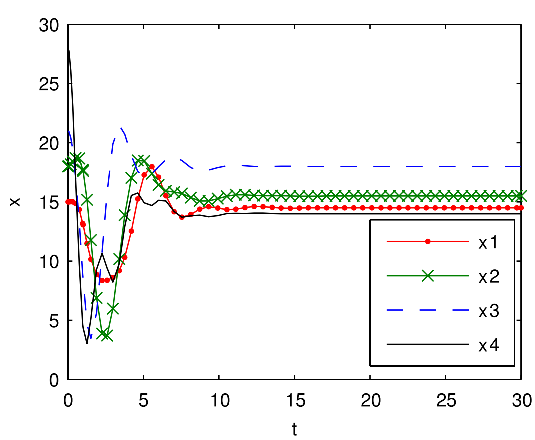

Fig. 11. Outputs of the complex multi-agent system with different inputs.

that the four external inputs are $u(t) = (u_1(t), u_2(t), u_3(t), u_4(t))$. The controllers and their parameters are the same as in Eq. (31). The initial values of the subsystems are $x(0) = (x_1(0), x_2(0), x_3(0), x_4(0)) = (15, 18, 21, 28)$. Then the outputs of the subsystems are $\lim_{t\to\infty} x(t) = \lim_{t\to\infty}(x_1(t), x_2(t), x_3(t), x_4(t)) = (14.50, 15.50, 18.00, 14.00)$ that are shown in Fig. 11.

$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$ can be calculated by Eq. (4). Then, applying L, $\lim_{t\to\infty} u(t)$, and $\lim_{t\to\infty} x(t)$ to Eq. (29), one will find the relationship between inputs and outputs satisfies the conclusion of Theorem 3.

6. Conclusion

This paper presents a new framework to design consensus protocols for complex multi-agent systems. A new model of multi-agent systems is proposed based on transfer functions, and two consensus protocols are presented. To make the outputs be consensus, each subsystem of the complex multi-agent system is designed to be a feedback control system. In this way, the performance of the multi-agent system can be adjusted easily to meet the desired requirement. Two simulations for the homogeneous and inhomogeneous multi-agent systems are provided to demonstrate the performance of the proposed protocols. The relationship between the inputs and outputs of the multi-agent system is also investigated. One possible future work could be the consensus protocols of MIMO multi-agent systems and their applications for formation control.

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