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Consensus problems of first-order dynamic multi-agent systems with multiple time delays*

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Consensus problems of first-order multi-agent systems with multiple time delays are investigated in this paper. We discuss three cases: 1) continuous, 2) discrete, and 3) a continuous system with a proportional plus derivative controller. In each case, the system contains simultaneous communication and input time delays. Supposing a dynamic multi-agent system with directed topology that contains a globally reachable node, the sufficient convergence condition of the system is discussed with respect to each of the three cases based on the generalized Nyquist criterion and the frequency-domain analysis approach, yielding conclusions that are either less conservative than or agree with previously published results. We know that the convergence condition of the system depends mainly on each agent's input time delay and the adjacent weights but is independent of the communication delay between agents, whether the system is continuous or discrete. Finally, simulation examples are given to verify the theoretical analysis.

Keywords: multi-agent, time delays, consensus, first-order, convergence

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1. Introduction

The distributed coordination of dynamic multi-agent systems has received more and more attention from many research communities due to its broad applications in many areas including formation control,[1] congestion control,[2] distributed sensor networks,[3] and so on. Recently, as one type of critical problem in the coordinated control of multiple agents, consensus problems have attracted increasing attention. The common property of consensus problems is that each individual agent lacks a global knowledge of the whole system and can interact only with its neighbors to achieve certain global behavior. In the past few years, research about consensus problems has been developing very fast.[4–12]

In multi-agent systems, time-delays may arise naturally. Therefore, it is very important to study the effect of time-delays on the convergence of the consensus protocol. Basically, there are two kinds of delays in multi-agent systems. There is, first of all, a communication delay, which is related to communication from one agent to another. The other is an input delay which is related to processing and connecting time for the packets arriving at each agent. The effect of time delays on agents' consensus behavior has been analyzed in some related references. For example, in Ref. [7], the average-consensus problem is discussed for a system with communication time-delay when the communication topology is the simplest case, i.e. fixed, undirected, and connected. In Ref. [8], the communication topology discussion is also undirected. An investigation of the average consensus problem in directed networks of autonomous agents with both switching topology and a time delay is reported in Ref. [9]. In Ref. [10], the authors provide a set-valued consensus condition for general linear undirected MAS with heterogeneous delays. In Ref. [11], by dividing the topology into a combination of directed trees, some necessary and sufficient conditions are derived for all agents to asymptotically reach a single consensus and multiple consensuses. As a comparison, reference [12] applies a set-valued Lyapunov approach to consider discrete-time consensus algorithms with unidirectional time-dependent communication links. In Ref. [13], a system with changing communication topologies and bounded time-varying communication delays is also studied. In Ref. [14], the authors also investigate a discrete-time system with fixed and undirected topology. In Ref. [15], consensus problems of first-order and second-order multi-agent systems with communication and input delays are proposed. The topology of the system is directed. In addition, reference [16] discusses two kinds of cases, i.e., undirected and directed topology with diverse input and communication delays, and some consensus conditions are obtained. Much of the existing research is based on a network with undirected or strongly connected topology, which is too special to study the general problems.

Motivated by the related work, the objective of our work

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reported in this paper was to investigate the consensus problem of a first-order multi-agent system with multiple time delays. Supposing that the topology of the dynamic system is a fixed digraph which owns a globally reachable node, we analyze and get the sufficient conditions for system convergence in all three cases based on the generalized Nyquist criterion and the frequency-domain analysis approach. The three cases are 1) continuous, 2) discrete, and 3) a continuous system with a proportional plus derivative (PD) controller. The continuous system depends only on each agent's input time delays, the adjacent weight of its neighbors, and the control gain of the system. Compared with results in Refs. [16] and [17], this new result is relatively more conservative. Moreover, inspired by related research, we discuss another protocol introducing a PD controller, and get a convergence condition for the system that is less conservative than the result in Ref. [18]. Finally, we also study a discrete system with diverse delays and obtain a convergence condition that coincides with the existing results in some references; but note that the protocol we discuss is different from theirs.

The rest of the paper is organized as follows. In Section 2, some preliminaries on graph theory and model formulation are given. Analysis about the consensus of continuous time and discrete time with multiple time delays are discussed in Section 3 and Section 4, respectively. In Section 5, numerical examples are simulated to verify the theoretical analysis. Conclusions are finally drawn in Section 6.

2. Preliminaries

In this section, some basic concepts and results about algebraic graph theory and model formulation are introduced.

Let $G=\{V,E,A\}$ be a weighted digraph consisting of a set of vertices $V=\{v_1,\ldots,v_n\}$, a set of edges $E\subseteq V\times V$ and a weighted adjacency matrix $A=\{a_{ij}\}\in R^{n\times n}$. The node indexes belong to a finite index set $N=\{1,2,\ldots,n\}$. We assume that the adjacency elements associated with the edges of the digraph are positive, i.e., $a_{ij}>0\Leftrightarrow e_{ij}\in E$, which means that node v_j can receive information from node v_i. Moreover, we assume $a_{ii}=0$ for all $i\in N$. The set of the neighbors of v_i is denoted by $N_i=\{v_j\in V:(v_i,v_j)\in E\}$. For the digraph G, the out-degree of node i is defined as $d_i=\sum\limits_{v_j\in N_i}a_{ij}$. Let $\boldsymbol{D}$ be the degree matrix of G, which is defined as a diagonal matrix with the out-degree of each node along its diagonal. The Laplacian matrix of the weighted digraph is defined as $\boldsymbol{L}=\boldsymbol{D}-\boldsymbol{A}$.

A node v_i in the digraph G is said to be reachable from another node v_j if there is a path in G from v_j to v_i. A node is said to be globally reachable if it can be reached from every other node in the digraph. Digraph G is said to be strongly connected if and only if there exists at least one path between any pair of vertices. Obviously, in contrast with the strongly connected digraph, the weakest requirement is that digraph G is said to be connected is that G owns at least one globally reachable node.

In a dynamic multi-agent system, each agent can be considered as a node in a digraph, and the information flow between two agents can be regarded as a directed path. So the interconnected topology in a multi-agent system can be described as a digraph $G=\{V,E,A\}$. Moreover, each agent updates its current state based upon the information received from its neighbors.

The following two systems have been widely studied, solving consensus problems in a network of continuous-time (CT) integrator agents with dynamics or agents with a discrete-time (DT) model:

$$\dot{x}_i(t)=u_i(t),\ i\in N, \tag{1}$$

$$x_i(k+1)=x_i(k)+u_i(k),\ i\in N, \tag{2}$$

where $x_i\in R^m$ and $u_i\in R^m$ denote the state and the control input of the i-th agent, respectively. For simplicity, we assume $m=1$ in this paper.

In the past few years, consensus protocol (3) has been extensively studied.

$$u_i(t)=\sum_{V_j\in N_i}a_{ij}(x_j(t)-x_i(t)). \tag{3}$$

Taking into account the influence of delays, the following two protocols also have been widely investigated by many researchers

$$u_i(t)=\sum_{V_j\in N_i}a_{ij}(x_j(t-T_{ij})-x_i(t)), \tag{4}$$

$$u_i(t)=\sum_{V_j\in N_i}a_{ij}(x_j(t-T_{ij}-T_i)-x_i(t-T_i)), \tag{5}$$

where T_{ij} and T_i represent the communication and the input delay, respectively.

Lemma 1[8] The Laplacian matrix $\boldsymbol{L}$ has a simple eigenvalue 0 and all the other eigenvalues have positive real parts if and only if the directed network has a globally reachable node.

Lemma 2[19] Let

$$E_i(\mathrm{j}w)=\frac{\pi}{2T}\times\frac{\mathrm{e}^{-\mathrm{j}wT}}{\mathrm{j}w};$$

then $(-1,\mathrm{j}0)\notin\gamma Co(0\cup\{E_i(\mathrm{j}w),i\in N\})$ holds for all $T\geq 0$, $\gamma\in[0,1)$ and $w\in[-\pi,\pi]$.

Lemma 3[20] For all $w\in[-\pi,\pi]$, $\bigcup\limits_{i\in I}G_i\subseteq Co(0\cup\{W_i(\mathrm{j}w),i\in N\})$, where G_i denotes the set of disks, and $W_i(\mathrm{j}w)=\gamma_iE_i(\mathrm{j}w)$, $\gamma_i\in[0,1)$.

Lemma 4[16] The following inequality

$$\frac{\sin\big((2D+1)w/2\big)}{\sin(w/2)}\leq 2D+1$$

holds for all nonnegative integers D and all $w\in[-\pi,\pi]$.

Definition 1 Consensus in a first-order multi-agent system (1) or (2) is said to be achieved asymptotically if and only if for any initial conditions, the states of agents satisfy $\lim\limits_{t\to\infty}\|x_i(t)-x_j(t)\|=0$ or $\lim\limits_{k\to\infty}\|x_i(k)-x_j(k)\|=0$, $\forall i,j=$

$1,2,\ldots,N$.

3. Consensus of a continuous system with multiple delays

In this section we consider a novel consensus protocol of a multi-agent system with different communication and input delays at the same time

$$u_i(t)=k_i\sum_{V_j\in N_i}a_{ij}(x_j(t-T_{ij})-x_i(t-T_i)),\quad i\in N,\tag{6}$$

where T_{ij} and T_i denote the communication delay and the input delay, respectively; $k_i(k_i>0)$ is the control gain to the i-th agent.

With the protocol (6), the closed-loop form of the system (1) is

$$\dot{x}_i(t)=k_i\sum_{V_j\in N_i}a_{ij}(x_j(t-T_{ij})-x_i(t-T_i)).\tag{7}$$

Theorem 1 Consider a dynamic multi-agent system of integrators with fixed topology $G=\{V,E,A\}$ that is a digraph containing a globally reachable node. Then, the system (7) can achieve consensus asymptotically if

$$\max\{k_id_iT_i,i\in N\}\prec\frac{\pi}{4},\tag{8}$$

where $d_i=\sum_{v_j\in N_i}a_{ij}$.

Proof Taking the Laplace transformation of the system (7), we can easily get the characteristic equation $\det(sI+KL(s))=0$, where $K=\mathrm{diag}\{k_i,i\in N\}$ and

$$\boldsymbol{L}(s)=(l_{ij}(s))=\begin{cases}-a_{ij}\,\mathrm{e}^{-sT_{ij}}, & v_j\in N_i,\\ \sum\limits_{V_j\in N_i}a_{ij}\,\mathrm{e}^{-sT_i}, & j=i,\\ 0, & \text{otherwise}.\end{cases}$$

Obviously, $\boldsymbol{L}(s)$ is the Laplacian matrix when $s=0$.

Let

$$\boldsymbol{F}(s)=\det(s\boldsymbol{I}+K\boldsymbol{L}(s)).\tag{9}$$

Now, we will prove that all the zeros of $\boldsymbol{F}(s)$ have a modulus less than unity except for a zero at $s=0$.

If $s=0$, then $\boldsymbol{F}(0)=\det(\boldsymbol{L})$. Since the system's topology is a digraph containing a globally reachable node, then by Lemma 1, we know that $\det(\boldsymbol{L})=0$; thus $\boldsymbol{F}(s)$ indeed has a simple zero at $s=0$.

If $s\neq0$, let $\boldsymbol{P}(s)=\det(\boldsymbol{I}+\boldsymbol{G}(s))$, where $\boldsymbol{G}(s)=K\boldsymbol{L}(s)/s$.

By the general Nyquist stability criterion, this is the case if the loci of the eigenvalue of $\boldsymbol{G}(\mathrm{j}w)=K\boldsymbol{L}(\mathrm{j}w)/(\mathrm{j}w)$ does not enclose the point $(-1,\mathrm{j}0)$ for all $w\in[-\pi,\pi]$.

Based on Gerschgorin disk criterion, we obtain $\lambda(G(\mathrm{j}w))\in\bigcup_{i\in N}G_i$, where

$$G_i=\left\{\zeta:\zeta\in C,\left|\zeta-k_i\sum_{v_j\in N_i}a_{ij}\frac{\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{j}w}\right|\le\left|k_i\sum_{v_j\in N_i}a_{ij}\frac{\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{j}w}\right|\right\}.\tag{10}$$

Defining $d_i=\sum_{v_j\in N_i}a_{ij}$, from Eq. (10) we can easily get the center of the disk

$$G_{i0}(\mathrm{j}w)=k_id_i\frac{\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{j}w}.\tag{11}$$

Suppose the origin point of the complex plane is O, and the intersection point is W_i which is made by the boundary of the disk and the line connecting point O and point G_{i0}, Then the track of point W_i is

$$W_i(\mathrm{j}w)=2k_id_i\frac{\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{j}w}.\tag{12}$$

Letting $W_i(\mathrm{j}w)=\gamma_iE_i(\mathrm{j}w)$, we can get $\gamma_i=4k_id_iT_i/\pi$. As $\gamma_i<1$, it is easy to know that $k_id_iT_i<\pi/4$.

Now defining $\gamma=\max\{\gamma_i,i\in N\}$, obviously, when $\gamma<1$, the following equation holds for all $i\in N$:

$$\begin{aligned}\gamma Co(0\cup\{E_i(\mathrm{j}w),i\in N\})&\supseteq\gamma_iCo(0\cup\{E_i(\mathrm{j}w),i\in N\})\\&=Co(0\cup\{W_i(\mathrm{j}w),i\in N\}).\end{aligned}\tag{13}$$

By Lemma 2, we know that

$$(-1,\mathrm{j}0)\notin\gamma Co(0\cup\{E_i(\mathrm{j}w),i\in N\}),$$

so $(-1,\mathrm{j}0)\notin Co(0\cup\{W_i(\mathrm{j}w),i\in N\})$ holds. Based on Lemma 3, as $Co(0\cup\{W_i(\mathrm{j}w),i\in N\})\supseteq\bigcup_{i\in N}G_i$, we can get $(-1,j0)\notin\bigcup_{i\in N}G_i$.

Hence, we can conclude that the loci of the eigenvalue of $G(\mathrm{j}w)=KL(\mathrm{j}w)/(\mathrm{j}w)$ does not enclose the point $(-1,\mathrm{j}0)$ for all $w\in[-\pi,\pi]$, which implies that all the zeros of $F(s)$ have moduli less than unity except for a zero at $s=0$. So the proof of Theorem 1 is accomplished.

Remark 1 From Theorem 1, we can see that the convergence condition of the system is related only to the input delay, control gain and the adjacent weights between each pair of agents but is independent of communication delays.

Remark 2 When $T_i=T$, i.e., all the agents have the same input delay, it is obvious that the protocol (6) contains the protocol studied in Ref. [17]. Furthermore, the convergence condition in it is more conservative than expression (8) when $k_i=1$.

Remark 3 In Refs. [7] and [16], the authors report obtaining some conditions that are rather similar to the condition given by Theorem 1, but they are all based on a special situation, that is, the topology of the system is undirected or strongly connected. In contrast, Theorem 1 is more general.

Inspired by Ref. [18], in order to improve the dynamic performance of the system, on the foundation of protocol (6), we introduce a PD controller, the new protocol is as follows:

$$u_i(t)=\sum_{V_j\in N_i}a_{ij}(x_j(t-T_{ij})-x_i(t-T_i)+\eta\dot{x}_i(t-T_i)),\ i\in N,\tag{14}$$

where $\eta\,(\eta\succ0,\eta\in R)$ is the feedback strength and T_{ij} and T_i

denotes the communication delay and the input delay, respectively.

With protocol (14), the closed-loop form of the system (1) is

$$\dot{x}_i(t) = \sum_{V_j \in N_i} a_{ij}(x_j(t-T_{ij}) - x_i(t-T_i) + \eta \dot{x}_i(t-T_i)), \ i \in N. \quad (15)$$

Theorem 2 Consider a dynamic multi-agent system of integrators with fixed topology $G = \{V, E, A\}$ that is a digraph which contains a globally reachable node. Then, the system (15) can achieve consensus asymptotically if

$$\max\left\{2d_i\left(\frac{\sin wT_i}{w} - \eta \cos wT_i\right), i \in N\right\} < 1,$$

where

$$d_i = \sum_{v_j \in N_i} a_{ij}, \ w \in [-\pi, \pi].$$

Proof Taking the Laplace transformation of the system (15), we can easily get the characteristic equation $\det(s\boldsymbol{I} + \boldsymbol{L}(s)) = 0$, where

$$\boldsymbol{L}(s) = (l_{ij}(s)) = \begin{cases} -a_{ij}\,\mathrm{e}^{-sT_{ij}}, & v_j \in N_i, \\ \sum\limits_{V_j \in N_i} a_{ij}\,\mathrm{e}^{-sT_i}(1 - \eta s), & j = i, \\ 0, & \text{otherwise.} \end{cases}$$

If $s = 0$, $\boldsymbol{L}(s)$ is the Laplacian matrix. By Lemma 1, we can easily find that it is the simple zero point at $s = 0$.

If $s \neq 0$, let $\boldsymbol{P}(s) = \det(\boldsymbol{I} + \boldsymbol{G}(s))$, where $\boldsymbol{G}(s) = \boldsymbol{L}(s)/s$. Based on the Gerschgorin disk criterion, we obtain $\lambda(G(\mathrm{j}w)) \in \bigcup\limits_{i \in N} G_i$, where

$$\begin{aligned} G_i = &\left\{\zeta : \zeta \in C, \left| \zeta - \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}(1 - \mathrm{j}w\eta)}{\mathrm{j}w} \right| \right. \\ &\left. \leq \left| \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}(1 - \mathrm{j}w\eta)}{\mathrm{j}w} \right| \right\}. \end{aligned} \quad (16)$$

By the general Nyquist criterion, we know that if the point $(-a, \mathrm{j}0)$ is not in the disk G_i for all $w \in [-\pi, \pi]$ and $a \geq 1$, then the zeros of the characteristic equation have moduli less than unity. From Eq. (16), the following inequality can be held

$$\left| -a + \mathrm{j}0 - \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}(1 - \mathrm{j}w\eta)}{\mathrm{j}w} \right| > \left| \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}(1 - \mathrm{j}w\eta)}{\mathrm{j}w} \right|. \quad (17)$$

From inequality (17), by simple calculations, we can easily get the inequality,

$$a\left[a - 2d_i\left(\frac{\sin wT_i}{w} - \eta \cos wT_i\right)\right] > 0. \quad (18)$$

Since $a \geq 1$, if condition (19) holds, inequality (18) can obviously be held

$$2d_i\left(\frac{\sin wT_i}{w} - \eta \cos wT_i\right) < 1. \quad (19)$$

The proof of Theorem 2 is accomplished.

Remark 4 When $\eta = 0$, the result of Theorem 2 coincides with the result of Theorem 1. And compared with the result in Ref. [18], it is not difficult to find that the convergence condition we have obtained is less conservative.

4. Consensus of a discrete system with multiple delays

Although systems of discrete-time with time delays have been extensively studied, unlike the aforementioned studies, now we will discuss the following protocol with multiple delays

$$u_i(k) = \sum_{V_j \in N_i} a_{ij}(x_j(k - T_{ij}) - x_i(k - T_i)). \quad (20)$$

Then the system of Eq. (2) with Eq. (20) is

$$x_i(k+1) = x_i(k) + \sum_{V_j \in N_i} a_{ij}(x_j(k - T_{ij}) - x_i(k - T_i)), \ i \in N. \quad (21)$$

Theorem 3 Consider a multi-agent system (21) with a fixed topology $G = \{V, E, A\}$ that is a digraph containing a globally reachable node. Then the system achieves a consensus asymptotically if

$$\max\{d_i(2T_i + 1)\} \prec 1, \ i \in N, \quad (22)$$

where

$$d_i = \sum_{v_j \in N_i} a_{ij}.$$

Proof Taking the z-transformation of the system (21) and getting the characteristic equation is $\det\{(z-1)\boldsymbol{I} + \boldsymbol{L}(z)\} = 0$, where $\boldsymbol{L}(z) = \{l_{ij}(z)\}$ is defined as follows:

$$\boldsymbol{L}(z) = (l_{ij}(z)) = \begin{cases} -a_{ij}z^{-T_{ij}}, & v_j \in N_i, \\ \sum\limits_{V_j \in N_i} a_{ij}z^{-T_i}, & j = i, \\ 0, & \text{otherwise,} \end{cases}$$

and $\boldsymbol{L}(1) = \boldsymbol{L}$, which is the Laplacian matrix.

When $z = 1$, by Lemma 1, we can easily find that it is the simply zero point at $z = 1$.

When $z \neq 1$, similar to the proof of Theorem 2, based on the Gerschgorin disk criterion, we can get the disk set:

$$G_i = \left\{\zeta : \zeta \in C, \left| \zeta - \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{e}^{\mathrm{j}w} - 1} \right| \leq \left| \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{e}^{\mathrm{j}w} - 1} \right| \right\}. \quad (23)$$

By the general Nyquist criterion, we know that if the point $(-a, \mathrm{j}0)$ with $a \geq 1$ is not in the disc G_i for all $w \in [-\pi, \pi]$, then the zeros of $\det\{\boldsymbol{I} + \boldsymbol{L}(z)/(z-1)\}$ have a modulus less than unity. So from Eq. (23), we have

$$\left| -a + j0 - \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{e}^{\mathrm{j}w} - 1} \right| \succ \left| \frac{d_i\,\mathrm{e}^{-\mathrm{j}wT_i}}{\mathrm{e}^{\mathrm{j}w} - 1} \right|. \quad (24)$$

Based on Lemma 4 and by some simple calculations, we can show that if expression (22) holds, then expression (24) can be satisfied. The proof of Theorem 3 is accomplished.

Remark 5 From expression (22), we know the convergence condition of the system, like in Theorem 1, is independent of communication delays, but is related to input delays and the adjacent weights among agents. Meanwhile, it is not difficult to find that the condition we have obtained coincides with the conclusions in some references, such as Ref. [16] and so on.

5. Simulation results

In this section, some simulation experiments are presented to verify the theoretical analysis.

Suppose the multi-agent system's topology is shown in Fig. 1 which includes ten agents. Node 1 is the globally reachable node.

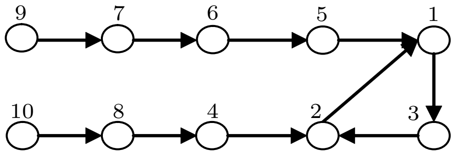

Fig. 1. (color online) Topology of the multi-agent system.

To Fig. 1, without loss of generality, we consider the corresponding adjacent weights between agents to be $a_{13} = 0.1$, $a_{21} = 0.1$, $a_{32} = 0.15$, $a_{42} = 0.25$, $a_{51} = 0.2$, $a_{65} = 0.1$, $a_{76} = 0.15$, $a_{84} = 0.15$, $a_{97} = 0.5$, $a_{108} = 0.2$, and all the others are 0. Suppose the communication time delays among the ten agents are $T_{13} = 1.0$ s, $T_{21} = 0.75$ s, $T_{32} = 1.8$ s, $T_{42} = 2.0$ s, $T_{51} = 0.8$ s, $T_{65} = 0.8$ s, $T_{76} = 1.5$ s, $T_{84} = 1.0$ s, $T_{97} = 1.2$ s, $T_{108} = 0.8$ s. And assume the initial state of the agents are $x(0) = \{2, 3.5, 2.5, 3, 5, 1.5, 6, 5.5, 4, 4.5\}^{\mathrm{T}}$.

Experiment 1

To Theorem 1, without loss of generality, we can assume $k_i = 1$ for all $i \in N$. Based on the adjacent weights given above, from Theorem 1, we can show that the input time delay $T_i \in [0, \pi/2)$. However, under the same conditions, from Ref. [17], it is easy to find that the input time delay $T \in [0, 1)$. For simplicity, in the experiment, let $T_i = 1.5$ s for all $i \in N$, i.e., all the agents have the same input time delays and the value is 1.5 s. If the results of this experiment show that the system (7) can reach consensus asymptotically under these conditions, we can easily conclude that the condition given by Ref. [17] is more conservative than that given by Theorem 1. The state trajectories of all the agents are shown in Fig. 2. Obviously, a consensus is reached.

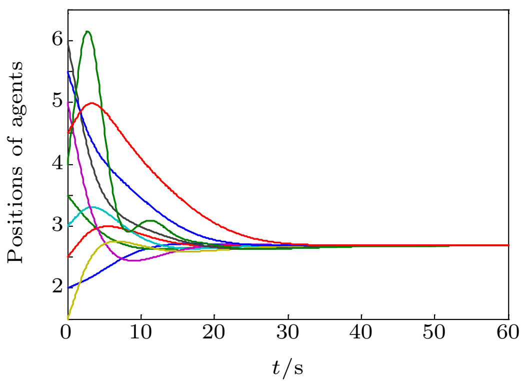

Fig. 2. (color online) State trajectories of the agents, $T_i = 1.5$ s.

Experiment 2

For Theorem 2, in the simulation, we chose a feedback strength of $\eta = 0.5$, and based on the result given in Ref. [18], we know that the input time delay $T < 1.5$ s. In order to make a comparison easily, we let $T_i = 1.5$ s for all $i \in N$ in this experiment, i.e., all the agents have the same input time delay and the value is 1.5 s. The state trajectories of all the agents are shown in Fig. 3. Obviously, all agents reach a consistent value asymptotically, so it is verified that the condition we have obtained is less conservative than the result in Ref. [18]. Meanwhile, from the results in Fig. 2 and Fig. 3, we also see that the PD controller can indeed improve the dynamic performance of the system.

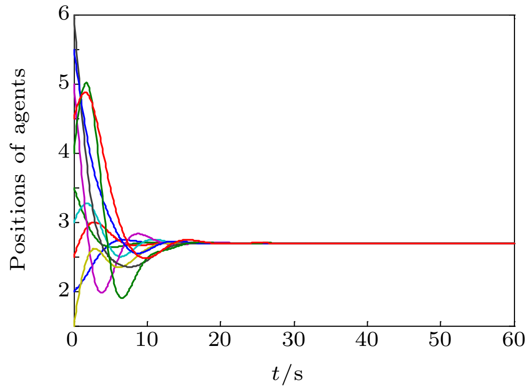

Fig. 3. (color online) State trajectories of the agents, $\eta = 0.5$, $T_i = 1.5$ s.

Experiment 3

To Theorem 3, in the experiment, in order to simplify the problems, we assume all the input time delays are the same, thus let $T_i = 0.3$ s. Obviously, the condition (22) can be held. The state trajectories of all the agents are shown in Fig. 4. Certainly, consensus of the multi-agent system is achieved asymptotically.

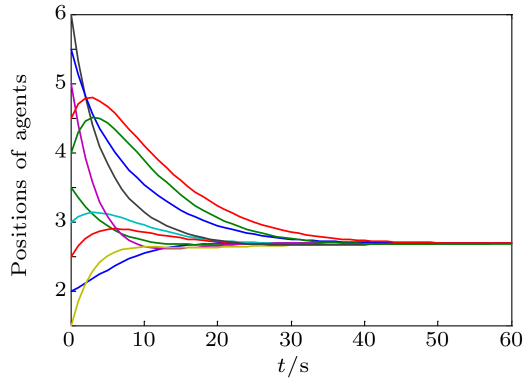

Fig. 4. (color online) State trajectories of the agents of the discrete system, $T_i = 0.3$ s.

6. Conclusion

In this paper, we report our investigation of consensus problems in first-order multi-agent dynamic systems with communication and input time delays. Moreover, some convergence results are obtained and each of them is less conservative or coincides with the conditions given in the related literature. Furthermore, it is found that the convergence condition of a system with directed topology that contains a globally reachable node depends mainly on each agent's input time delay, and the weights between adjacent agents. However, the communication time delays between the agents cannot influence the convergence property of the system, whether it is continuous or discrete. Through considering a PD controller, we know that it can indeed improve the dynamic performance of the system.

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