Simulation of stress and strain distribution using finite element method

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1. Explanation of aim

The aim of this exercise was to create model of physical phenomenon - Stress and strain distribution using finite element method. Implementation of computer simulation helps to understand the mechanisms describing the phenomenon and it theoretical concept. Another objective is to compare results of simulation and the theory and model some more complex geometries.

2. Used tools

Simulation was created using Matlab. Libraries and their use:

- *pdetool* - creating model of stress and strain distribution and visualizing result of calculations.

3. Results

a. Parameters

For model shown in diagram 3.1 used parameters are:

- Application: Structural Mechanics, Plane Stress;
- Boundary:
  - left beam boundary condition type: Dirichlet;
  - bottom beam boundary condition type: Neumann;
  - top beam boundary condition type: Neumann;
  - left beam boundary condition type: Neumann with g2 = -10000;
- PDE specification:
  - Type of PDE: Eliptic;
  - Young's modulus (E): 2e11;
  - Poisson ratio (nu): 0.3;

    - Volume force, x-direction (Kx): 0.0;
    - Volume force, y-direction (Ky): 0.0;
    - Density (rho): 8000.0;
- Solve parameters: Adaptive mode.

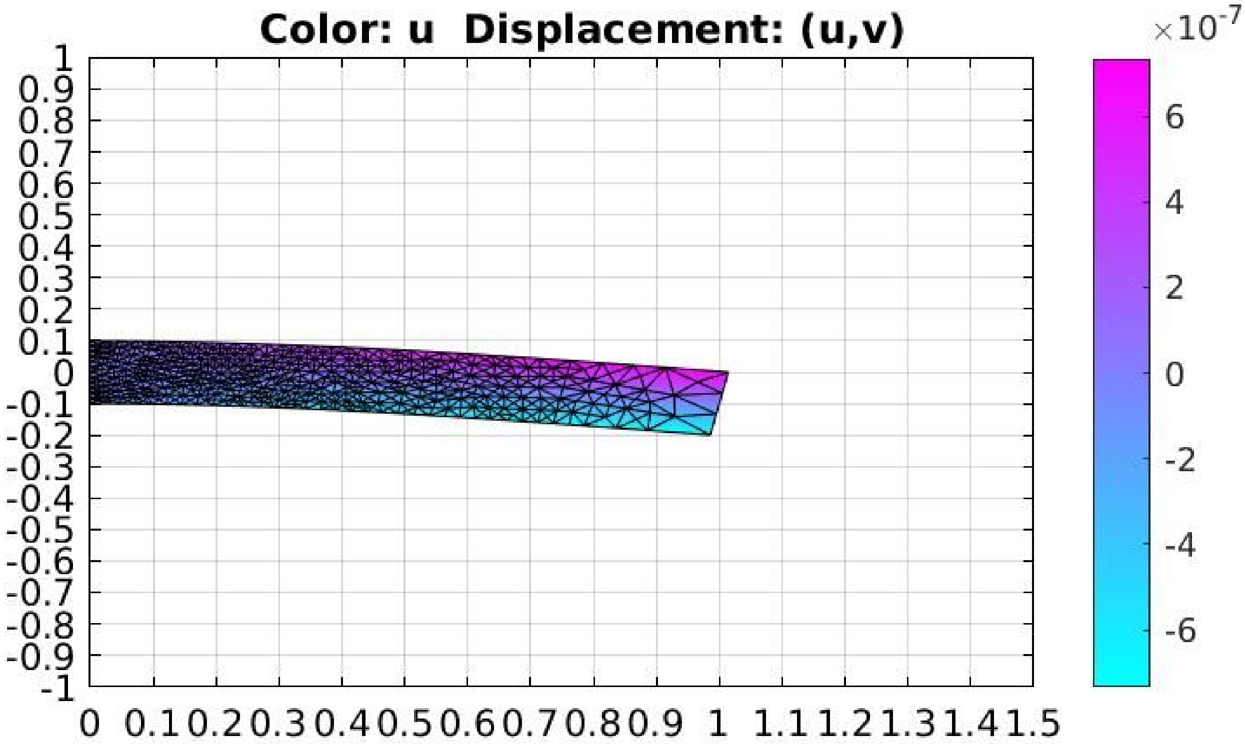

Diagram 3.1. Visualization of deformed beam.

b. Comparison of the calculated beam deformation with the theory

Table 3.1. shows the comparison of the calculated beam deformation with the theory where value:

- *F* - loading force;
- *L* - beam length;
- *E* - is Young's modulus;
- *g* - beam thickness;
- *d* - beam width;
- *theory* - *beam deform value* computed using theoretical formula;
- *simulation* - value computed by simulating model in pdetool;
- *error* - error between calculated beam deformation and the theory.

Table 3.1. Comparison of the calculated beam deformation with the theory.

F	L	E	g	d	theory	simulation	error
-1000	1	2e11	1	0.2	-2.5e-06	-2.0184e-06	4.8156e-07
-3000	1	2e11	1	0.2	-7.5e-06	-6.0553e-06	1.4447e-06
-5000	1	2e11	1	0.2	-1.25e-05	-1.0092e-05	2.4078e-06
-7000	1	2e11	1	0.2	-1.75e-05	-1.4129e-05	3.3709e-06
-10000	1	2e11	1	0.2	-2.5e-05	-2.0184e-05	4.8156e-06

c. Simulation of more complicated geometry

On diagram 3.2 We can see the geometry of second model.

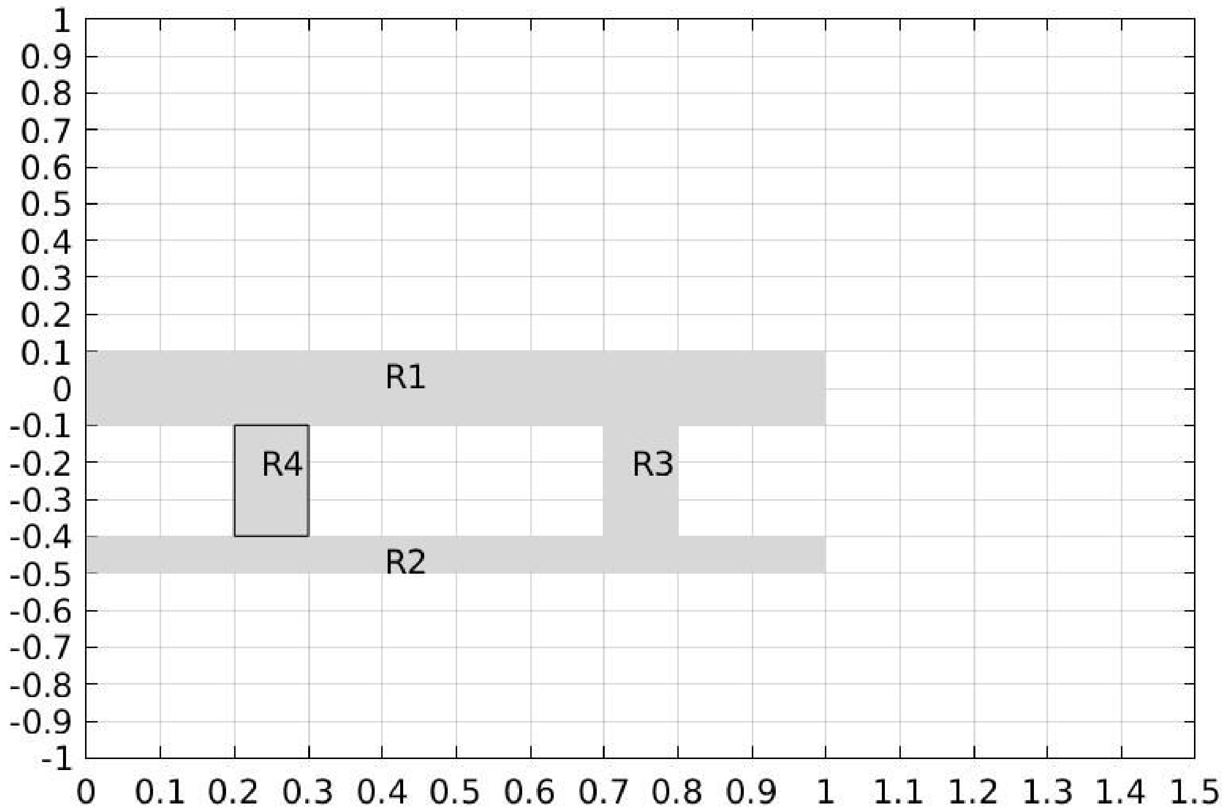

Diagram 3.2. Geometry of model 02.

Diagram 3.3 shows the deformed model 02.

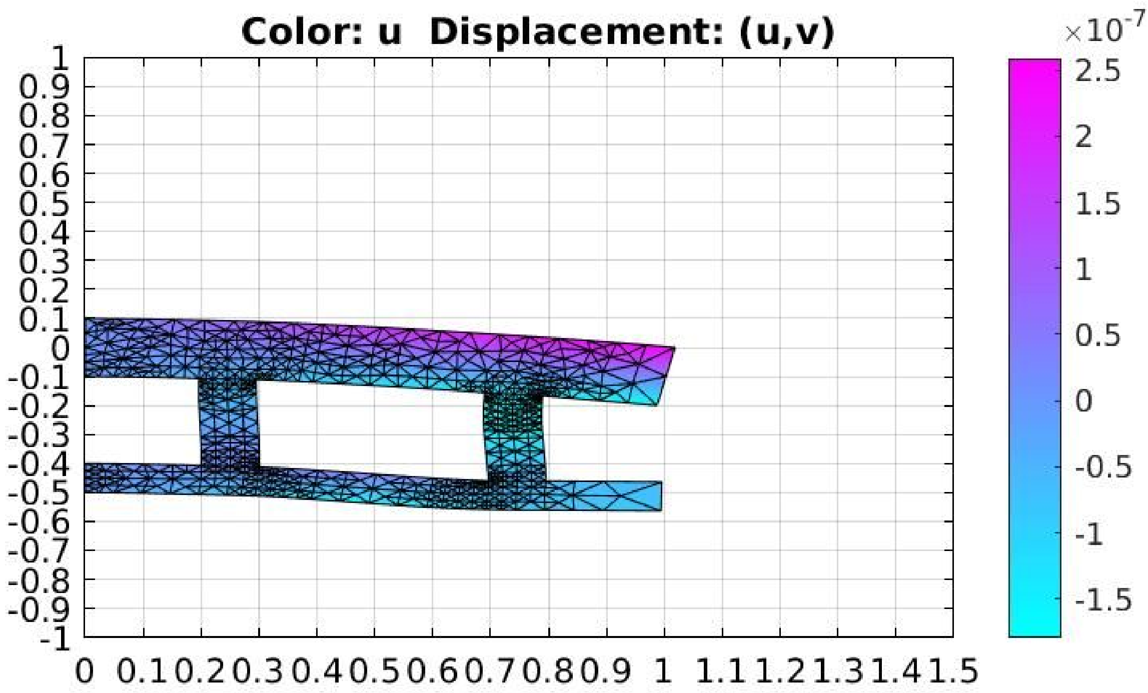

Diagram 3.3. Visualization of deformed beam model 02.

4. Conclusions

Created simulation of stress and strain distribution using finite element method show us different deformation for various models and parameters of simulation. Moreover We compared calculated by simulation beam deformation to the theory and it shows that the result are similar to each other.

5. Code listening

a. Values calculation script

```
pdetool
sim_result = min(v)
points = 4
sim_result = sim_result * points

F = -1000; L = 1; E = 2e11; g = 1; d = 0.2;

h = (F * L^3) / (3 * E * (g * d^3 / 12))

error = abs(sim_result - h)
```

b. Pdetool model 01 (Simple beam)

```
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB
script
%    to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See
%    http://www.mathworks.com/help/pde/examples/index.html for examples
%    of this approach.
function pdemodel
[pde_fig,ax]=pdeinit;
pdetool('appl_cb',3);
set(ax,'DataAspectRatio',[1 2 1]);
set(ax,'PlotBoxAspectRatio',[1.5 1 2]);
set(ax,'XLim',[0 1.5]);
set(ax,'YLim',[-1 1]);
set(ax,'XTick',[ -1.5,...
 -1.3999999999999999,...
 -1.3,...
 -1.2,...
 -1.1000000000000001,...
 -1,...
 -0.89999999999999991,...
 -0.79999999999999993,...
 -0.69999999999999996,...
 -0.59999999999999998,...
 -0.5,...
 -0.39999999999999991,...
 -0.29999999999999982,...
 -0.19999999999999996,...
 -0.099999999999999867,...
 0,...
 0.099999999999999867,...
 0.19999999999999996,...
 0.29999999999999982,...
 0.39999999999999991,...
 0.5,...
 0.59999999999999998,...
 0.69999999999999996,...
 0.79999999999999993,...
 0.89999999999999991,...
 1,...
 1.1000000000000001,...
 1.2,...
```

```
 1.3,...
 1.3999999999999999,...
 1.5,...
]);
set(ax,'YTick',[ -1,...
 -0.90000000000000002,...
 -0.80000000000000004,...
 -0.69999999999999996,...
 -0.59999999999999998,...
 -0.5,...
 -0.39999999999999991,...
 -0.29999999999999993,...
 -0.19999999999999996,...
 -0.099999999999999978,...
 0,...
 0.099999999999999978,...
 0.19999999999999996,...
 0.29999999999999993,...
 0.39999999999999991,...
 0.5,...
 0.59999999999999998,...
 0.69999999999999996,...
 0.80000000000000004,...
 0.90000000000000002,...
 1,...
]);
pdetool('gridon','on');

% Geometry description:
pderect([0 1 0.10000000000000001 -0.10000000000000001],'R1');
set(findobj(get(pde_fig,'Children'),'Tag','PDEEval'),'String','R1')

% Boundary conditions:
pdetool('changemode',0)
pdesetbd(4,...
'dir',...
2,...
char('1','0','0','1'),...
char('0','0'))
pdesetbd(3,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(2,...
```

```
'neu',...
2,...
char('0','0','0','0'),...
char('0','-10000'))
pdesetbd(1,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))

% Mesh generation:
setappdata(pde_fig,'Hgrad',1.3);
setappdata(pde_fig,'refinemethod','regular');
setappdata(pde_fig,'jiggle',char('on','mean',''));
setappdata(pde_fig,'MesherVersion','preR2013a');
pdetool('initmesh')

% PDE coefficients:
pdeseteq(1,...
char('2*((2E11)./(2*(1+(0.3))))+(2*((2E11)./(2*(1+(0.3)))).*(0.3)./(1-(0
.3)))','0','(2E11)./(2*(1+(0.3)))','0','(2E11)./(2*(1+(0.3)))','2*((2E11
)./(2*(1+(0.3)))).*(0.3)./(1-(0.3))','0','(2E11)./(2*(1+(0.3)))','0','2*
((2E11)./(2*(1+(0.3))))+(2*((2E11)./(2*(1+(0.3)))).*(0.3)./(1-(0.3)))'),
...
char('0.0','0.0','0.0','0.0'),...
char('0.0','0.0'),...
char('8000.0','0','0','8000.0'),...
'0:10',...
'0.0',...
'0.0',...
'[0 100]')
setappdata(pde_fig,'currparam',...
['2E11  ';...
'0.3   ';...
'0.0   ';...
'0.0   ';...
'8000.0'])

% Solve parameters:
setappdata(pde_fig,'solveparam',...
char('1','1520','10','pdeadworst',...
'0.5','longest','0','1e-4','','fixed','inf'))

% Plotflags and user data strings:
setappdata(pde_fig,'plotflags',[1 1 1 1 1 1 1 1 0 1 0 1 1 0 0 0 1 1]);
```

```
setappdata(pde_fig,'colstring','');
setappdata(pde_fig,'arrowstring','');
setappdata(pde_fig,'deformstring','');
setappdata(pde_fig,'heightstring','');

% Solve PDE:
pdetool('solve')
```

c. Pdetool model 02

```
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB
script
%    to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See
%    http://www.mathworks.com/help/pde/examples/index.html for examples
%    of this approach.
function pdemodel
[pde_fig,ax]=pdeinit;
pdetool('appl_cb',3);
set(ax,'DataAspectRatio',[1 2 1]);
set(ax,'PlotBoxAspectRatio',[1.5 1 1]);
set(ax,'XLim',[0 1.5]);
set(ax,'YLim',[-1 1]);
set(ax,'XTick',[ -1.5,...
 -1.3999999999999999,...
 -1.3,...
 -1.2,...
 -1.1000000000000001,...
 -1,...
 -0.89999999999999991,...
 -0.79999999999999993,...
 -0.69999999999999996,...
 -0.59999999999999998,...
 -0.5,...
 -0.39999999999999991,...
 -0.29999999999999982,...
 -0.19999999999999996,...
 -0.099999999999999867,...
 0,...
 0.099999999999999867,...
 0.19999999999999996,...
 0.29999999999999982,...
```

```
 0.39999999999999991,...
 0.5,...
 0.59999999999999998,...
 0.69999999999999996,...
 0.79999999999999993,...
 0.89999999999999991,...
 1,...
 1.1000000000000001,...
 1.2,...
 1.3,...
 1.3999999999999999,...
 1.5,...
]);
set(ax,'YTick',[ -1,...
 -0.90000000000000002,...
 -0.80000000000000004,...
 -0.69999999999999996,...
 -0.59999999999999998,...
 -0.5,...
 -0.39999999999999991,...
 -0.29999999999999993,...
 -0.19999999999999996,...
 -0.099999999999999978,...
 0,...
 0.099999999999999978,...
 0.19999999999999996,...
 0.29999999999999993,...
 0.39999999999999991,...
 0.5,...
 0.59999999999999998,...
 0.69999999999999996,...
 0.80000000000000004,...
 0.90000000000000002,...
 1,...
]);
pdetool('gridon','on');

% Geometry description:
pderect([0 1 0.10000000000000001 -0.10000000000000001],'R1');
pderect([0 1 -0.40000000000000002 -0.5],'R2');
pderect([0.69999999999999996 0.79999999999999993 -0.10000000000000003
-0.40000000000000002],'R3');
pderect([0.20000000000000001 0.30000000000000004 -0.10000000000000003
-0.40000000000000002],'R4');
set(findobj(get(pde_fig,'Children'),'Tag','PDEEval'),'String','R1+R2+R3+
```

```
R4')

% Boundary conditions:
pdetool('changemode',0)
pdesetbd(20,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(18,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(16,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(15,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(13,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(11,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(10,...
'dir',...
2,...
char('1','0','0','1'),...
char('0','0'))
pdesetbd(9,...
'dir',...
2,...
char('1','0','0','1'),...
char('0','0'))
pdesetbd(8,...
```

```
'neu',...
2,...
char('0','0','0','0'),...
char('0','-10000'))
pdesetbd(7,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(6,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(5,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(4,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(3,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(2,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))
pdesetbd(1,...
'neu',...
2,...
char('0','0','0','0'),...
char('0','0'))

% Mesh generation:
setappdata(pde_fig,'Hgrad',1.3);
setappdata(pde_fig,'refinemethod','regular');
setappdata(pde_fig,'jiggle',char('on','mean',''));
setappdata(pde_fig,'MesherVersion','preR2013a');
```

```
pdetool('initmesh')

% PDE coefficients:
pdeseteq(1,...
char('2*((2E11)./(2*(1+(0.3))))+(2*((2E11)./(2*(1+(0.3)))).*(0.3)./(1-(0
.3)))','0','(2E11)./(2*(1+(0.3)))','0','(2E11)./(2*(1+(0.3)))','2*((2E11
)./(2*(1+(0.3)))).*(0.3)./(1-(0.3))','0','(2E11)./(2*(1+(0.3)))','0','2*
((2E11)./(2*(1+(0.3))))+(2*((2E11)./(2*(1+(0.3)))).*(0.3)./(1-(0.3)))'),
...
char('0.0','0.0','0.0','0.0'),...
char('0.0','0.0'),...
char('8000.0','0','0','8000.0'),...
'0:10',...
'0.0',...
'0.0',...
'[0 100]')
setappdata(pde_fig,'currparam',...
['2E11  ';...
'0.3   ';...
'0.0   ';...
'0.0   ';...
'8000.0'])

% Solve parameters:
setappdata(pde_fig,'solveparam',...
char('1','1841','10','pdeadworst',...
'0.5','longest','0','1e-4','','fixed','inf'))

% Plotflags and user data strings:
setappdata(pde_fig,'plotflags',[1 1 1 1 1 1 1 1 0 1 0 1 1 0 0 0 1 1]);
setappdata(pde_fig,'colstring','');
setappdata(pde_fig,'arrowstring','');
setappdata(pde_fig,'deformstring','');
setappdata(pde_fig,'heightstring','');

% Solve PDE:
pdetool('solve')
```