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Aggregating Opinions Expressed by Bipolar Argumentation Debates

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Abstract

The process of exchanging opinions can be seen as argumentation process. An interesting step is on how to draw the conclusion based on those opinions, i.e., aggregating them. Various aggregation methods have been published; however, many challenges still remain to be solved. It is especially true for aggregating bipolar argumentation frameworks. The dynamic caused by the presence of both attacks and supports relations among the arguments make the preservation of the framework properties more complicated. The inconsistencies in the definition of supports are one of the major reasons that directly aggregating bipolar argumentation frameworks is a difficult task. One of the current methods to tackle this challenge is to use quantitative argumentation framework to help draw the conclusion.

In this project, another method to solve the problems on aggregating bipolar argumentation frameworks is studied. Bipolar Assumption-based Argumentation Framework is used to remove the inconsistencies of the supports. Along with Social Choice Theory, especially judgement aggregation, it is possible to produce preservation results for the frameworks' properties. There are some positive results and negative results as well, and to evaluate the aggregation rules, an e-polling scenario is used. To complete the e-polling procedures, analyses of quota rules and oligarchic rules are provided to give comparisons of the strengths and weaknesses of each rule.

Keywords: *argumentation, bipolar, aggregation, Social Choice Theory, judgement aggregation, Bipolar Assumption-based Argumentation Framework, e-polling*

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Chapter 1

Introduction

1.1 Motivation

It is well known that problem statements, opinions, or answers can be represented as the process of arguments exchange or debates. Indeed, with the emergence of Artificial Intelligence, argumentation is an active research domain that tries to represent those problems and opinions in order to reach the answers. One initiating work was done by Dung [11], in which he proposed an *Abstract Argumentation Framework* to represent problems or opinions. In that particular work, Dung introduced a set of arguments as the statements on a particular problem, i.e., opinions, and attack relationships between the arguments as the only relationship necessary to formulate the answer.

Then, it is natural to think about what happens if several opinions are to be combined. The matter of aggregating several opinions is also an emerging research area in the argumentation domain. It is concerned with how to get a collective opinion from a set of opinions and ensure that the agents' opinions are well portrayed in the collective opinion. Chen and Endriss [7, 8] studied the aggregation process of Abstract Argumentation frameworks with the help of Social Choice Theory [14], especially judgement aggregation [13], and presented the preservation results of the Abstract Argumentation framework' properties. The preservation of a property means that if all agents accept the property on their own frameworks, then the property is preserved if and only if the aggregated framework accepts it as well.

However, in a debate context, there also exists another type of relationship between the arguments, which is called the support relationship. To deliberate more, given any problem, it is common to have pros and cons opinions, or yes and no opinions. *Bipolar Argumentation Framework* [6, 5] is an extension of the Abstract Argumentation framework that introduces the existence of both attack and support relationship in a framework.

Although some efforts had been made to aggregate the Bipolar Argumentation framework, for example by quantifying the frameworks beforehand as *Quantitative Argumentation Debate Framework* [1, 18], but unfortunately it is very challenging to aggregate Bipolar Argumentation frameworks directly. The reason is that there are many different definitions of the support relationship, such as support as necessary and support as deductive, which have different semantics and cause inconsistencies on the aggregation [6, 5].

Hence, taking the problems and challenges of aggregating Bipolar Argumentation frameworks as the motivation, in this project a study on the aggregation of bipolarity in the frameworks is conducted. The inconsistencies of Bipolar Argumentation framework are resolved in a structured argumentation framework called *Bipolar Assumption-based Argumentation Framework* [9]. This framework is a special type of the more general framework, *Assumption-based Argumentation Framework* [10, 20] in the sense that it is more restricted as there are no rules with empty body. With the Bipolar Assumption-based Argumentation framework, the definitions of support in the Bipolar Argumentation framework can be safely translated and have the same semantics.

Getting the preservation results of Bipolar Assumption-based Argumentation framework's properties are important. However, to evaluate the aggregation rules, an e-polling scenario is used [17]. After the agents vote on many opinions or statements in e-polling, the problem that lies ahead is on the aggregation method. To simply take every vote and opinion of the agents might not provide a desirable result. Hence, analyses and evaluation of aggregation rules used in the preservation studies hold an important part to give a sense of a better method. This phase concludes the project as the aggregation rules are presented and evaluated to get the aggregation result.

1.2 Problem Formulation

As mentioned in the previous section, there are many ways for the agents to form their own opinion of the same problem, especially with the bipolarity in the frameworks. Thus, to be able to formulate the answer, there has to be a way to combine all the opinions to get a single collective opinion. In this project, there are several problems that will be studied and attempted to answer:

1. How to represent bipolarity in the argumentation?

   Adding support relationships along with the attack relationships introduces inconsistencies mostly because of several meanings of the support relationships. Thus, it is necessary to understand different ways to represent bipolarity, e.g. Bipolar Argumentation Framework, Quantitative Argumentation Debate Framework, and Bipolar Assumption-based Argumentation Framework.

2. How to aggregate the bipolar opinions into a collective opinion?

   If support relationships exist in the frameworks, the aggregation process is not straightforward. After converting the bipolarity into the Bipolar Assumption-based Argumentation frameworks, the questions that remain are which element needs to be aggregated and what aggregation rules are able to aggregate them.

3. What are the preservation results of the properties of the collective opinion?

   On aggregating the frameworks, the preservation of the properties is crucial as it is desirable to be able to keep the agents' opinions to reflect the collective opinion. Hence, if a property is true in all the agents' frameworks, then it is preserved if and only if it is true in the aggregated framework. Some important properties include the semantics, such as conflict-freeness and closure; and other properties such as acyclicity and coherence.

4. What are the limitations in the aggregation method and results?

   Limitations sometimes are needed to preserve some properties. Due to the bipolarity of the frameworks, some limitations emerge as the effect of introducing support relationships. Thus, not every aggregation rule or method is able to give a satisfying result.

5. How to put the aggregation method into use in real application?

   Using the aggregation method in the real application is another way of evaluating the aggregation rules. Various applications may utilise the aggregation of Bipolar Assumption-based Argumentation frameworks. In this project, the procedure of e-polling is used to reflect the aggregation process of several opinions.

1.3 Objectives

The objectives of this project are set to address the problems formulated in Section 1.2.

1. To study different types of frameworks and their properties.

   Although the focus is on Bipolar Assumption-based Argumentation frameworks, other frameworks are also important to understand the basis of argumentation. Moreover, most of the properties are inheritable from other types of frameworks.

2. To propose aggregation procedure to combine Bipolar Assumption-based Argumentation frameworks.

   With the help of quota rules and oligarchic rules from judgement aggregation and social choice theory, the procedure of aggregating Bipolar Assumption-based Argumentation frameworks might involve different components of the frameworks.

3. To study the preservation properties of the aggregated framework.

   It is important to preserve the opinion of each agent such that it is still contributing into the collective opinion. The way to do it is to preserve the properties of the agents' frameworks.

4. To study the limitations of the aggregation method.

   Each aggregation rule may have some caveats in preserving the properties of Bipolar Assumption-based Argumentation frameworks. However, the limitations are put into place in order to preserve the properties. Thus, it is necessary to find the connection between the limitations and the preservation.

5. To put the aggregation method into an application.

   In the e-polling scenario, the agents voted for the statements. These statements can be represented in the form of arguments or assumptions. The aggregation process takes place after the voting. To complete the e-polling procedure, the preferable aggregation rule is needed.

1.4 Scope Limitation

Due to time and resource constraints, several limitations are put to keep this project more focused and yield better results. Overcoming these limitations is one possible future work of this project.

1. The aggregation is performed only on Bipolar Assumption-based Argumentation frameworks, which are the special type of non-flat Assumption-based Argumentation frameworks.

   The unique characteristic of Bipolar Assumption-based Argumentation frameworks, which is the head of the rules that never empty, makes the preservation study more systematically. Moreover, the aggregation process uses only the rules as the component to be combined.

2. The aggregation rules considered in this project are only quota rules and oligarchic rules.

   Aggregation rules are basically any mapping functions that take several frameworks for input and give a single collective framework as the output. However, quota rules and oligarchic rules are the most-known aggregation rules with special characteristics that are well used in a lot of applications.

3. All agents in the aggregation process are assumed to be rational.

   Irrational agents may give wrong or invalid opinions, such as being self-attacking, self-supporting, or both attacking and supporting the same argument; and affect the preservation study and the computation of semantics. However, this is very likely to happen, especially in the real application such as e-polling. Thus, [17] proposes a procedure to remove the irrationality.

4. E-polling is chosen to represent the application of the aggregation of Bipolar Assumption-based Argumentation frameworks.

   There are many applications of aggregation such as decision maker and recommender systems. However, e-polling has a unique property which make it interesting to be studied further. Moreover, the aim is to complete the e-polling procedures initially studied in [17].

1.5 Methodologies Overview

The background study in Chapter 2 is carried out by choosing several papers aligned with the project's topic. Most of the papers are within argumentation and aggregation domain, such as Bipolar ABA frameworks and Social Choice Theory. The work from Individual Study Option [19] (ISO) contributes as the literature survey as well. However, the direction of this project is completely different from the ISO as in this project, bipolarity is the main focus of the aggregation.

The study of preservation results in Chapter 3 is conducted by finding the proofs, counter examples, and limitation when bipolarity comes into play. Different from [8], which only considers the presence of attacks, with the existence of supports in the framework, there are some limitations on the preservation of the properties of Bipolar ABA frameworks. More detailed results are given by presenting corner cases scenario.

The application of the aggregation on e-polling in Chapter 4 uses a Brexit scenario. To complete the process of e-polling, appropriate aggregation rules are needed. Several analyses based on each quota rule and oligarchic rule are given by evaluating the utility rate of the rules. It compares every rule on the properties it preserves, the number of agents or opinions it considers, and the number of assumptions in the scenario or frameworks.

1.6 Outcomes

The aggregation of Bipolar Assumption-based Argumentation frameworks combines the rules of each agent into a collective set of rules. Hence, it is assumed that the agents share the language, the assumptions, and the contrary of the framework. The preservation results show both positive and negative results. For the preservation of conflict-freeness and closure, every quota rule and oligarchic rule are able to preserve them. For other results, some are still positive although they are more restricted. For example, the admissibility and set-stable semantics are preserved by nomination rule. For other properties, the results are more negative in terms of the use of oligarchic rules, dictatorship, and veto powers. For instance, the aggregation rule must be dictatorial to preserve the acceptability of an assumption, the preferred, complete, well-founded, ideal semantics, and coherence. In addition, at least one agent must have veto powers in order to preserve the non-emptiness of well-founded extension and acyclicity. Some corner cases show more positive results as the complexity of the problems is reduced. Compared with the preservation results from [8], many of them can be directly extended such as the conflict-freeness, admissibility, and set-stable; and they show that the presence of supports does not affect the preservation. On the other hand, some preservation results face more restrictions because of the support relationships between the assumption, for example the acceptability of an assumption, the preferred, well-founded, and coherence. However, the restrictions are more natural such that it does not greatly affect the final results. Finally, closure and ideal are new properties that are not introduced in [8].

To evaluate the results, analyses on each quota rule and oligarchic rule are conducted in e-polling scenario. Nomination rule is able to preserve one of the stronger semantic, which is set-stable. It also performs well in terms of keeping every agent's opinion into the aggregated opinion. However, one weakness is the possibility of keeping unnecessary or even wrong opinions due to the agent's error. Majority rule and unanimity rule have lower performance ability as both of them can only preserve conflict-freeness and closure, which are most often not enough to get the final answer. The aggregation rule that is dictatorship is able to preserve more properties, such as the acceptability of an assumption, preferred, complete, well-founded, ideal semantics, the non-emptiness of well-founded extension, acyclicity, and coherence. However, it means to ignore most of the agents' opinions. This might lead to a worse answer if the agent with veto powers makes mistakes.

1.7 Report Structure

This report is structured as follows. Chapter 1 gives motivation, aims, and overall outcomes of the project. Chapter 2 provides necessary background study and related work to the project, including the Individual Study Option [19]. Ethics and professional consideration are also included in Chapter 2. Chapter 3 produces the design and implementation of the aggregation of Bipolar ABA frameworks in the form of preservation results of the Bipolar ABA frameworks' properties. Chapter 4 tries to put the preservation results into e-polling application and proposes analyses on the suitable aggregation rules. Chapter 5 concludes the project and shows possible future works of this project.

Chapter 2

Background

2.1 Background Scenario

The scenario is taken from [17] for a more intuitive example used in this chapter. It depicts several arguments on Brexit, some arguments agree with the Brexit and some other arguments disagree with it.

Example 1. There are several opinions of the public on whether the UK should leave the EU or not. Each opinion is given a label for easier representation.

A: The UK should leave the EU.

B: The UK staying in the EU is good for its economy.

C: The EU's immigration policies are bad for the UK's economy.

D: EU membership fees are too high.

E: The UK staying in the EU is good for world peace.

It can be seen that some arguments *attack* or *support* other arguments. For instance, argument B and E attack argument A because while A prefer the UK to leave the EU, B and E argue that staying in the EU is good for the economy and world peace. Furthermore, B is attacked by argument C and D such that staying in the EU does not improve the UK's economy because the immigration policies and the membership fees are too high. In a sense, C and D can be seen as supporting A.

2.2 Abstract Argumentation Framework

Abstract Argumentation Framework (AAF, for short) is commonly used to translate questions and problems into computable and logical form. An abstract argumentation framework consists of a set of arguments and the attack relations [11]. It is written as $\langle Args, attacks \rangle$ and can be represented as directed graph with $argument \in Args$ as the nodes and $att \in attacks$ as the edges.

Definition 2.2.1. (AAF). An *Abstract Argumentation Framework* / AAF is a tuple $\langle Args, attacks \rangle$ where $Args$ is a finite and non-empty set of arguments and *attacks* is binary relations of $Args$.

Definition 2.2.2. (Attacks). An attack relation $X\ attacks\ Y$ is defined as a relation between two arguments X and Y, where $\{X, Y\} \in Args$, such that $(X, Y) \in attacks$ or $(X \rightarrow Y)$.

Taking Example 1 from the background scenario in Section 2.1, there are multiple ways to translate it into AAF. Depending on the opinion of the agent, the attack relations can be different. Figure 2.1 depicts one possible way to interpret the scenario. It can be depicted as the UK should leave the EU (argument A) even though staying in the EU is good for its economy (argument B) and for world peace (argument E), because the immigration policies are bad for the UK (argument C) and the membership fees are too high (argument D).

To put it in a more formal notation, then the abstract argumentation framework in Figure 2.1 is $\langle Args, attacks \rangle$ with:

$$Args = A, B, C, D, E$$

$$attacks = \{(B, A), (E, A), (D, B), (C, B), (C, E), (E, C)\}$$

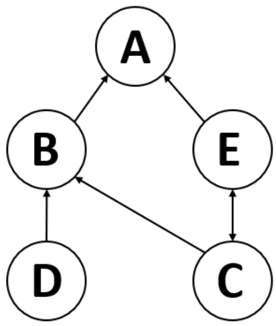

Figure 2.1: Abstract Argumentation Framework Example

The attack relations can be seen as an effort of the attacking argument to defeat the attacked argument. Thus, a set of *winning* or *accepted* arguments can be retrieved from a framework. The procedure to find these arguments follow common logic, for example in Figure 2.1 it is only logical to accept arguments A, C, and D. Argument B and E are both attacked and undefended in this case. For the more formal procedure, the definition of which arguments should be accepted is given by semantics and the set of accepted arguments under specific semantic is called an extension Δ of that particular semantic. To avoid confusion, the terms semantic and extension are used interchangeably throughout this report.

Definition 2.2.3. (Conflict-free). A set of arguments is *conflict-free* if and only if there does not exist any attack relation between arguments in the set, i.e., it does not attack itself.

From the abstract argumentation framework in Figure 2.1, the conflict-free extensions are $\Delta = \{\}, \{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{A, C\}, \{A, D\}, \{B, E\}, \{C, D\}, \{D, E\}$, $\{A, C, D\}$. Extension $\Delta = \{A\}$ and other extensions with a single argument are conflict-free in this example because there does not exist an attack relation $\{\alpha \rightarrow \alpha,\ \alpha \in Args\}$ and thus, $\{(\alpha, \alpha) \notin attacks,\ \alpha \in Args\}$. The extension $\Delta = \{A, C\}$ is also conflict-free as $\{(A, C) \notin attacks\}$. However, extension $\Delta = \{A, B\}$ is not conflict-free because argument B attacks argument A or $\{(B, A) \in attacks\}$.

Remark. An empty set $\{\}$ of any abstract argumentation framework is conflict-free as it does not include any arguments and thus, does not attack itself.

Definition 2.2.4. (Self-defending). A set of arguments is *self-defending* if and only if some arguments in the set attack all arguments that attack the other arguments in the set.

Definition 2.2.5. (Admissible). A set of arguments is *admissible* if and only if it is conflict-free and self-defending, or if each attacked argument has to be defended by attacking the attackers.

The admissible extensions of the framework in Figure 2.1 are $\Delta = \{\}, \{C\}, \{D\}$, $\{E\}, \{A, C\}, \{C, D\}, \{E, D\}, \{A, C, D\}$. The empty extension $\Delta = \{\}$ is admissible because it is conflict-free and as it does not contain any arguments, it does not need to defend anything. The extension $\Delta = \{D\}$ is admissible as it only has one conflict-free argument and it is not attacked by other arguments, therefore, it does not need to defend from any attackers. The extension $\Delta = \{A, C, D\}$ is conflict-free as well because although argument A is attacked by both argument C and D, it is defended by both argument C and D. An example of not admissible extension is $\Delta = \{B, D\}$ because argument D attacks argument B, making the extension not conflict-free in the first place. $\Delta = \{A\}$ is not admissible as well as it is not defended when it is attacked by argument B and E.

Definition 2.2.6. (Preferred). A set of arguments is *preferred* if and only if it is maximally admissible with respect to the subset of arguments.

The only preferred extension of the framework in Figure 2.1 is $\Delta = \{A, C, D\}$. It is quite easy to see the reason as the extension has the maximum number of arguments among the other admissible extensions.

Definition 2.2.7. (Reinstating). A set of arguments is *reinstating* if and only if each argument in the set is defended by other argument.

Definition 2.2.8. (Complete). A set of arguments is *complete* if and only if it is self-defending and reinstating, or if it is admissible and all arguments it defends are included in it.

From the framework in Figure 2.1, the complete extensions are $\Delta = \{D\}, \{D, E\}$, $\{A, C, D\}$. The extension $\Delta = \{A, C, D\}$ is complete because it is admissible and argument A, which is defended by argument C and D, is included in the extension. Therefore, the extension $\Delta = \{C, D\}$ is not complete because it does not include argument A, which it defends.

Definition 2.2.9. (Stable). A set of arguments is *stable* if and only if it is conflict-free and for all arguments it does not include, it attacks each of them.

The stable extensions of the framework in Figure 2.1 are $\Delta = \{D, E\}, \{A, C, D\}$. The extension $\Delta = \{A, C, D\}$ is stable because it is conflict-free and attacks both argument B and E, which are not included in the extension. The extension $\Delta = \{A, C\}$ is not stable because even though it is conflict-free, it does not attack argument D.

Definition 2.2.10. (Grounded). A set of arguments is *grounded* if and only if it is minimally complete with respect to the subset of arguments.

From the framework in Figure 2.1, the only grounded extension is $\Delta = \{D\}$ because it is the minimal complete extension.

Remark. Every argumentation framework is always guaranteed to have only one grounded extension with an empty set as the smallest grounded extension.

Definition 2.2.11. (Ideal). A set of arguments is *ideal* if and only if it is maximally admissible with respect to the subset of arguments and it is contained in all preferred extensions.

As there is only one preferred extension in the framework in Figure 2.1, then the only ideal extension is $\Delta = \{A, C, D\}$ because it is admissible and it is maximal as it is the only preferred extension.

From the definition of each semantic, it is clear that some semantics provide several choices of extensions and the other semantics only provide one possible extension. These semantics are classified as either credulous or sceptical.

Definition 2.2.12. (Credulous semantics). A semantic is *credulous* if there are several possible alternatives of accepted extensions. Semantics that are categorised as credulous are conflict-free, admissible, preferred, complete, and stable.

Definition 2.2.13. (Sceptical semantics). A semantic is *sceptical* if there is only one single accepted extension. Semantics that are categorised as sceptical are grounded and ideal.

2.3 Assumption-Based Argumentation Framework

Assumption-based argumentation framework (ABA, for short) is a form of structured argumentation and can be seen as an extension of Abstract Argumentation framework. As a result, the notion of attacks in ABA is different from the Abstract Argumentation framework. In ABA, attacks are denoted by rules, assumptions, and contraries in a deductive manner. [10, 20]

Definition 2.3.1. (ABA). An *Assumption-based Argumentation Framework* / ABA is a tuple $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ where $\langle \mathcal{L}, \mathcal{R} \rangle$ is a deductive system, with $\mathcal{L}$ the language or sentences, and $\mathcal{R}$ a set of rules in the form $\sigma_0 \leftarrow \sigma_1, \cdots, \sigma_m (m \geq 0)$ with $\sigma_i \in \mathcal{L}(i = 0, \cdots, m)$. As a convention, σ_0 is referred to as the head and $\sigma_1, \cdots, \sigma_m$ as the body of the rule. $\mathcal{A} \subseteq \mathcal{L}$ is a non-empty set, referred to as assumptions. $\overline{}$ is a total mapping from $\mathcal{A}$ to $\mathcal{L}$; $\overline{\alpha}$ is referred to as the contrary of α.

Referring to the background scenario in Section 2.1, Example 2 shows a possible depicted assumption-based argumentation framework.

Example 2. The scenario can be transformed into several sentences, where p is 'leave the EU', q is 'stay in the EU', a is 'bad immigration policies', b is 'good for economy', c is 'good for world peace', and r is 'high membership fees'. Then, the depicted ABA framework is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ where

- Language $\mathcal{L} = \{a, b, c, p, q, r\}$
- Rules $\mathcal{R} = \{p \leftarrow a, r, \quad r \leftarrow, \quad q \leftarrow b, c\}$
- Assumptions $\mathcal{A} = \{a, b, c\}$
- Contraries: $\overline{p} = q,\ \overline{a} = c,\ \overline{b} = r$

The assumptions are actions or observations, such as 'bad immigration policies', 'good for economy', and 'good for world peace'. The non-assumptions ($\mathcal{L} \setminus \mathcal{A}$) are goals to be achieved or avoided, for example the goal of Example 2 is to 'leave the EU' and avoid 'stay in the EU'. The contraries of the assumptions are ways to defeat them, for example 'high membership fees' is the contrary of 'good for economy'. Note that contraries are different from negations, which may occur in the language.

Rules in ABA can be chained to derive conclusions. These conclusions are *deductions*, which are important to establish relations in the framework.

Definition 2.3.2. (Deductions in ABA). A *deduction* for $\sigma \in \mathcal{L}$ supported by $S \subseteq \mathcal{L}$ and $R \subseteq \mathcal{R}$, denoted $S \vdash^{R} \sigma$, is a (finite) tree with

- nodes are labelled by sentences in $\mathcal{L}$ or by τ, where $\tau \notin \mathcal{L}$ denotes *true* value or empty body of rules
- the root is labelled by σ
- leaves are either τ or sentences in S
- non-leaves σ' are with, as children, the elements of the body of some rule in $\mathcal{R}$ with head σ' (τ if this body is empty), and R the set of all such rules

From Example 2, several deductions that can be made are $\{a, r\} \vdash^{R_1} p$ for $R_1 = \{p \leftarrow a, r\}$, $\{\} \vdash^{R_2} r$ for $R_2 = \{r \leftarrow\}$, $\{a\} \vdash^{R_3} p$ for $R_3 = R_1 \cup R_2$, and $\{b, c\} \vdash^{R_4} q$ for $R_4 = \{q \leftarrow b, c\}$. They are all can be represented as trees shown in Figure 2.2. However, $\{a, r, b\} \vdash^{R1} p$ is not a deduction, due to the presence of b in the support as it is irrelevant to p, given $R_1 = \{p \leftarrow a, r\}$.

Finding semantics in ABA means finding the accepted set of assumptions. However, it needs to be *closed* under deduction.

Definition 2.3.3. (Closure). The *closure* of a set of sentences $S \subseteq \mathcal{L}$ is $Cl(S) = \{\sigma \in \mathcal{A} \mid \exists S' \vdash^{R} \sigma,\ S' \subseteq S,\ R \subseteq \mathcal{R}\}$. Then, a set of assumptions $A \subseteq \mathcal{A}$ is *closed* if and only if it consists of all the assumptions deducible from it. In other words, it is closed if and only if $A = Cl(A)$.

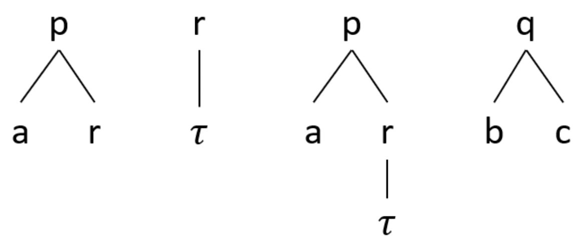

Figure 2.2: Deductions in ABA

Definition 2.3.4. (Flat ABA). An ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ is *flat* if and only if A is closed for every $A \subseteq \mathcal{A}$.

The ABA framework on Example 2 is *flat*. It is a restricted type of ABA framework and it is easy to see that as it has no assumption at the head of the rules, it is guaranteed to be closed and flat.

Similar to the abstract argumentation framework, extensions are affected by relations between arguments. In ABA, specifically, the relations are between the assumptions. However, it is more intuitive to start with arguments. Arguments in ABA are deductions of claims using rules and supported by sets of assumptions.

Definition 2.3.5. (Arguments). An *argument* for a claim $\sigma \in \mathcal{L}$ supported by $A \subseteq \mathcal{A}$ ($A \vdash \sigma$) is a deduction for σ supported by A and some $R \subseteq \mathcal{R}$.

Remark. Support for a sentence from a set of assumptions in ABA arguments is not equal to support for an argument in bipolar argumentation.

From Example 2, the arguments are $\{\} \vdash r, \{a\} \vdash p, \{b, c\} \vdash q, \{a\} \vdash a, \{b\} \vdash b, \{c\} \vdash c$. There are no other arguments exist, for example $\{a, b\} \vdash p$ is not an argument as it is not a deduction and $\{a, r\} \vdash p$ is also not an argument as $\{a, r\} \not\subseteq \mathcal{A}$.

Then, attacks between arguments in ABA are directed at the assumptions in the support of arguments.

Definition 2.3.6. (Attacks in Argument Level). An argument $A_1 \vdash \sigma_1$ *attacks* an argument $A_2 \vdash \sigma_2$ if and only if σ_1 is the contrary of one of the assumptions in A_2.

From Example 2, $\{\} \vdash r$ attacks both $\{b\} \vdash b$ and $\{b, c\} \vdash q$. In addition, $\{a\} \vdash a$ and $\{c\} \vdash c$ attack each other. Arguments may be attacked by the same arguments if they share assumptions in their support.

Definition 2.3.7. (Attacks in Assumption Level). A set of assumptions $A \subseteq \mathcal{A}$ attacks a set of assumptions $A' \subseteq \mathcal{A}$ if and only if an argument supported by a subset of A attacks an argument supported by a subset of A'.

From Example 2, $\{a\} \subseteq A$ attacks $\{c\} \subseteq A'$. It corresponds to the argument-level attacks from $\{a\} \vdash a$ to $\{c\} \vdash c$. The opposite holds as well, i.e., $\{c\} \subseteq A'$ attacks $\{a\} \subseteq A$. The empty set of assumptions $\{\}$ also attacks $\{b\}$ and $\{b, c\}$.

Having the attack relations between the closed set of assumptions, an extension of assumptions $A \subseteq \mathcal{A}$ can be computed according to different semantics shown in Table 2.1. Most of the semantics' names are the same with the semantics in Abstract Argumentation framework. However, some semantics in ABA have different conditions, especially because of the closure.

Semantics	Conditions
Conflict-free	There does not exist any attack relation between assumptions in the set, i.e., it does not attack itself.
Admissible	It is closed, conflict-free and for every $B \subseteq \mathcal{A}$, if B is closed and attacks A, then A attacks B.
Preferred	It is maximally admissible with respect to the subset of assumptions.
Complete	It is admissible and all assumptions it defends are included in it.
Stable	It is closed, conflict-free and for all assumptions it does not include, it attacks each of them.
Well-founded/Grounded	It is the intersection of all complete extensions.
Ideal	It is maximally admissible with respect to the subset of assumptions and it is contained in all preferred extensions.

Table 2.1: ABA Semantics

From Example 2, $\{\}, \{a\}$, and $\{c\}$ are closed, conflict-free, admissible, and complete. The set $\{a\}$ and $\{c\}$ are also preferred. The empty set $\{\}$ is grounded in this example. Finally, there are no stable and ideal extensions.

There are some properties on the connection between the semantics on the general ABA framework, which should hold for AA framework as well. Take a set of assumptions $A \subseteq \mathcal{A}$,

1. It is preferred, if and only if A is stable
2. If A is admissible, then there is some $B \subseteq \mathcal{A}$ such that B is preferred and $A \subseteq B$
3. It is complete, if and only if A is stable
4. $A \subseteq B$ if and only if A is ideal and $B \subseteq \mathcal{A}$ is the intersection of all preferred extensions
5. It is ideal, if and only if A is the intersection of all preferred extensions and admissible
6. If A is ideal, then for each set of assumptions B attacking A there exists no admissible set of assumptions $B' \subseteq \mathcal{A}$ such that $B' \supseteq B$

Moreover, there are some existential properties of the semantics on general ABA framework. As a preferred extension depends on the existence of admissible extension, it is easy to see that if there is an admissible extension, then there is at least one preferred extension. The preferred extension is also guaranteed in the ABA framework if the empty set of assumptions is closed as it is also admissible. The closure of the empty set of assumptions also leads to the existence of ideal extension as the admissible and preferred extension are both guaranteed in this case.

Flat ABA frameworks also have their own unique properties on relations of some semantics. Take a set of assumptions $A \subseteq \mathcal{A}$ from a flat ABA framework,

1. It is complete, if and only if A is preferred
2. It is minimally complete with respect to the subset of assumptions, if and only if A is grounded
3. If A is grounded, then for every $B \subseteq \mathcal{A}$, if B is preferred, then $A \subseteq B$
4. It is complete, if and only if A is ideal
5. $A \supseteq B$ if and only if A is ideal and $B \subseteq \mathcal{A}$ is grounded
6. It is preferred, if and only if A is maximally complete with respect to the subset of assumptions
7. If A is admissible, then it is ideal if and only if each set of assumptions B attacking A there exists no admissible set of assumptions $B' \subseteq \mathcal{A}$ such that $B' \supseteq B$

There are also some additional existential properties unique only for flat ABA frameworks. The preferred and ideal extensions are guaranteed as an empty set of assumptions is always closed in flat ABA and therefore, it is admissible as well. From the unique property of flat ABA frameworks, if a set of assumptions is preferred, then it is complete. As the preferred extension is guaranteed to exist, then there is at least one complete extension in any flat ABA framework. There is also a unique grounded extension with minimum number of defends of other assumptions not in the extension.

As an extension of Abstract Argumentation framework, there is a method to translate ABA framework to AA framework. Then, the Abstract Argumentation framework's semantics can be used to compute the accepted arguments in the ABA framework.

Definition 2.3.8. (From ABA to AA). Each ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ gives a corresponding AA framework $\langle Args, attacks \rangle$ where $Args$ is the set of all arguments in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$; and $(\alpha, \beta) \in Attacks$ if and only if α attacks β in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$.

For example. the AA framework corresponding to the ABA framework from Example 2 is shown in Figure 2.3.

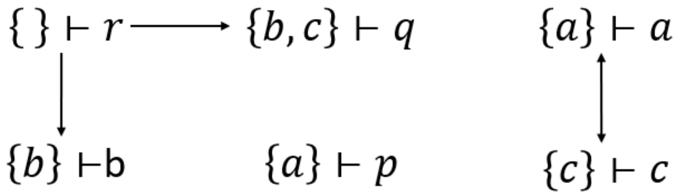

Figure 2.3: ABA to AA

2.4 Bipolar Argumentation Framework

Abstract Argumentation Framework in Section 2.2 introduces attack relations among the arguments. However, in debates there is another form of relation, which is called support relations. The framework with the existence of both relations is known as Bipolar Argumentation Framework (BAF, for short), which is also an extension of the Abstract Argumentation Framework. It is written as $\langle Args, attacks, supports\rangle$ and can also be represented as directed graph with $arg \in Args$ as the nodes, att $(\alpha \rightarrow \beta) \in attacks$ and $supp$ $(\alpha \Rightarrow \beta) \in supports$ as the edges. [6]

Definition 2.4.1. (BAF). A *Bipolar Argumentation Framework* / BAF is a tuple $\langle Args, attacks, supports\rangle$, where $Args$ is a finite and non-empty set of arguments, $attacks$ is binary relations of $Args$, and $supports$ is another binary relations of $Args$.

As the agents are assumed to be rational, then there cannot exist a self-attacking argument, self-supporting argument, or one argument attacks and supports another argument. Using Example 1 in the background scenario in Section 2.1, there are multiple ways to translate it into BAF as well. Depending on the opinion of the agent, the attack relations and support relations can be different. Figure 2.4 depicts one possible way to interpret the scenario. It can be seen as the statement staying in the EU is good for its economy (argument B) and the statement staying in the EU is good for world peace (argument E) attack the statement the UK should leave the EU (argument A). On the other hand, the statement the immigration policies are bad for the UK (argument D) and the statement the membership fees are too high (argument C) support the statement the UK should leave the EU (argument A).

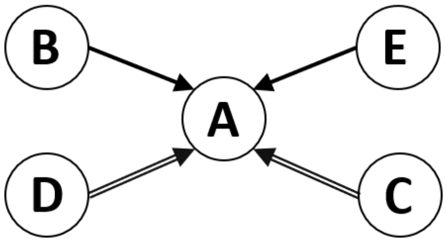

Figure 2.4: One Possible Bipolar Argumentation Framework Interpretation

Definition 2.4.2. (Safety). A set of arguments $S \subseteq Args$ is *safe* if and only if there are no arguments $\alpha, \beta \in S$, and $\gamma \in Args$ such that β $supports$ γ or $\gamma \in S$ and there is an attack from α to γ.

Definition 2.4.3. (Closure). A set of arguments $S \subseteq Args$ is *closed* under $supports$ if and only if $\forall\alpha \in S,\ \forall\beta \in Args$, if $\alpha \Rightarrow \beta$ then $\beta \in S$.

From the bipolar argumentation framework in Figure 2.4, the set $\{A, D\}$ is both safe and closed. It is safe because argument A and D do not attack other arguments. It is closed because argument A is included in the set while argument D supports it. On the contrary, the set $\{B, D\}$ is neither safe nor closed. It is not safe because argument A is supported by argument D and argument B attacks argument A. It is also not closed because it does not contain argument A, which is supported by argument D.

The definition of the attack relations is the same as Definition 2.2.2. However, due to the presence of support relations, another type of attack relation, called supported attack relation, exists.

Definition 2.4.4. (Supported attacks). Let $\alpha, \beta \in Args$. The *supported* attack from α to β, denoted by $\alpha\ attacks_{sup}\ \beta$, if and only if there is $\exists\gamma \in Args$ such that $\alpha\ supports$ some arguments $\{\gamma\}$ attacking β. In other words, α is able to attack everything its supported arguments can attack.

In the BAF in Figure 2.4, there does not exist any supported attack relations. However, if an argument F is added and $F\ supports\ B$, then $F\ attacks_{sup}\ A$.

Unfortunately, there is no universal definition of the support relations and as a result, direct aggregation is a difficult task in BAF. Some works refer support relations as necessary relation, others refer them as deductive relation. Moreover, many other works try to come up with different meanings of support relation [5]. However, in this project the focuses are on deductive support relations and necessary support relations.

2.4.1 Deductive Supports

The goal of a deductive support (*d-support*, for short) is to apply a constraint: If α *d-support* β then to accept α implies to accept β [4]. In the definition of support relations as deductive, there is a new attack relation, called super-mediated attack.

Definition 2.4.5. (Super-mediated attacks). Let $\alpha, \beta \in Args$. The *super-mediated* attack from α to β, denoted by $\alpha\ attacks_{s-med}\ \beta$, if and only if there is $\exists\gamma \in Args$ such that $\beta\ supports\ \gamma$ and either $\alpha\ attacks\ \gamma$ or $\alpha\ attacks_{sup}\ \gamma$. In other words, α is able to attack every supported arguments that α can $attack$ or $attack_{sup}$.

In the framework in Figure 2.4, the super-mediated attacks are $\{B\ attacks_{s-med}\ D$, $B\ attacks_{s-med}\ C$, $E\ attacks_{s-med}\ D$, $E\ attacks_{s-med}\ C\}$. Moreover, any arguments that support either argument B or E also $attack_{s-med}$ argument C, argument D, and any arguments that support either argument C or D.

To compute semantics from BAF with deductive supports, firstly the framework needs to be translated into AAF. The associated abstract argumentation framework for the deductive support is $\langle Args, attacks' \rangle$, where $attacks' = attacks \cup attacks_{sup} \cup$

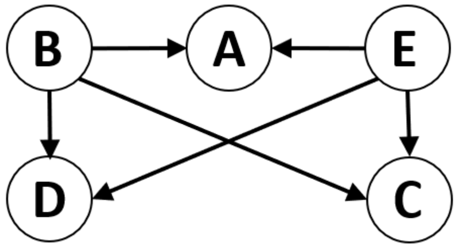

Figure 2.5: Translated AAF from BAF with deductive supports

$attacks_{s-med}$. Thus, the translated AAF of the bipolar argumentation framework in Figure 2.4 with deductive supports is shown in Figure 2.5.

The semantics can then be calculated using the translated abstract argumentation framework. In other words, a set of arguments $S \subseteq Args$ is a *d-admissible / d-preferred / d-stable* extension of $\langle Args, attacks, supports \rangle$ if and only if S is an admissible / preferred / stable extension of $\langle Args, attacks' \rangle$. The purpose of putting 'd' in front of the semantics is to differentiate between semantics for deductive support and the other semantics. As an example, the d-stable extension of BAF in Figure 2.4 can be computed by finding the stable extension of the translated AAF in Figure 2.5, which yields $\Delta = \{B, E\}$.

2.4.2 Necessary Supports

The goal of a necessary support (*n-support*, for short) is to apply constraint: If α *n-support* β then to accept α, it is necessary to accept β [16, 15]. In the definition of support relation as necessary, there is a new attack relation as well, called extended attack.

Definition 2.4.6. (Extended attacks). Let $\alpha, \beta \in Args$. The *extended* attack from α to β, denoted by α $attacks_{ext}$ β, if and only if either α $attacks$ β, or α $attacks$ γ and γ $supports$ β for $\gamma \in Args$, or γ $attacks$ β and γ $supports$ α for $\gamma \in Args$.

In the BAF in Figure 2.4, the extended attacks are $\{B\ attacks_{ext}\ A, E\ attacks_{ext}\ A\}$. Moreover, every argument that attacks either argument C or D $attacks_{ext}$ argument A. Every argument that is supported by either argument B or E $attacks_{ext}$ argument A as well.

Similar to the deductive supports, to compute semantics from bipolar argumentation framework with necessary supports, the AAF representation is needed. It is helpful to understand that d-supports and n-supports correspond to dual interpretations such that α *d-supports* β is equivalent to β *n-supports* α. Therefore, the BAF with necessary supports $\langle Args, attacks, supports \rangle$ is equivalent to BAF with deductive supports $\langle Args, attacks, supports^{-1} \rangle$, with $supports^{-1}$ denotes symmetric relation of $supports$ or $supports^{-1} = \{(\beta, \alpha) | (\alpha, \beta) \in supports\}$. Thus, the translated AAF of the bipolar argumentation framework in Figure 2.4 with necessary supports is the same as the translated AAF of the bipolar argumentation framework with deductive supports using $supports^{-1}$ instead. Figure 2.6 shows the scenario with $supports^{-1}$ and Figure 2.7 shows the translated abstract argumentation framework.

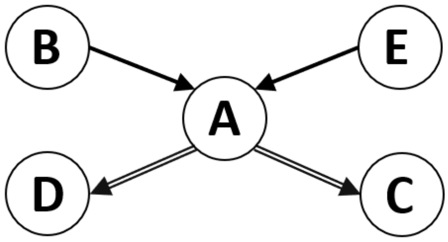

Figure 2.6: BAF with $supports^{-1}$

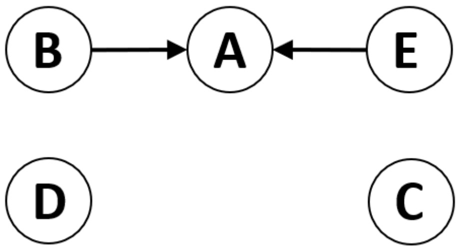

Figure 2.7: Translated AAF from BAF with necessary supports

The semantics can then be computed using the translated abstract argumentation framework. A set of arguments $S \subseteq Args$ is a n-$admissible$ / n-$preferred$ / n-$stable$ extension of $\langle Args, attacks, supports \rangle$ if and only if S is an admissible / preferred / stable extension of $\langle Args, attacks' \rangle$. As an example, the stable extension of BAF in Figure 2.4 can be computed by finding the stable extension of the translated AAF in Figure 2.7, which yields $\Delta = \{B, C, D, E\}$.

Then, it is easy to see the reason why it is difficult to aggregate Bipolar Argumentation frameworks directly. Both definition of supports have their own unique attack relationship that changes the translation into Abstract Argumentation framework. It leads into a different calculation of the semantics and causes inconsistencies. Hence, in the next section, another framework with bipolarity is introduced to handle the inconsistencies in Bipolar Argumentation framework.

2.5 Bipolar Assumption-Based Argumentation Framework

Bipolar Assumption-Based Argumentation Framework (Bipolar ABA, for short) is a special type of ABA framework, which is a more restricted version of non-flat ABA framework. The unique property of Bipolar ABA is that there are no rules with empty body. As indicated in the name, Bipolar ABA framework also has both attack and support relationships in the rules of the framework. [9]

Definition 2.5.1. (Bipolar ABA). A *Bipolar Assumption-based Argumentation Framework* / Bipolar ABA is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with $\mathcal{L}$ is the language, $\mathcal{R}$ are the rules, $\mathcal{A}$ are the assumptions, and $\overline{}$ are the contraries. It is non-flat as every rule in $\mathcal{R}$ has the form of $\beta \leftarrow \alpha$ where $\forall \alpha \in \mathcal{A}$, and β is either an assumption $\beta \in \mathcal{A}$ or a contrary $\beta \in \overline{}$.

Another unique property of Bipolar ABA is the existence of both admissible and preferred extensions. While in the general non-flat ABA frameworks those extensions are not guaranteed, but because in Bipolar ABA there is no rule with empty body so that the empty set of assumption is closed, and therefore, admissible. The preferred extension is guaranteed to exist as there is at least one admissible extension.

It is also easy to see that if a closed set of assumptions $Cl(\{\alpha\})$ attacks a closed set of assumptions $Cl(\{\beta\})$, then the set of assumption $\{\alpha\}$ also attacks the closed set of assumption $Cl(\{\beta\})$. Therefore, if a set of assumptions A is admissible and defends $Cl(\{\alpha\})$ and $Cl(\{\alpha\})$, then $A \cup Cl(\{\alpha\})$ is admissible as well and defends $Cl(\{\beta\})$.

While most of the semantics are the same as the semantics in Assumption-based Argumentation framework in Section 2.3, there is a new set-stable semantic that is important for Bipolar ABA framework, although it might also hold for the more general ABA frameworks.

Definition 2.5.2. (Set-stable). A set of assumptions $A \subseteq \mathcal{A}$ is *set-stable* if and only if A is closed, conflict-free, and A attacks every closed assumption not in the set.

The set-stable semantic is different from the general stable semantic in a way that set-stable does not need to attack each assumptions not in the extension, but only need to attack the closure of the assumptions. It is not guaranteed to exist in any Bipolar ABA frameworks, however, if a set of assumptions is stable, then it is guaranteed to be set-stable as well. For non-flat ABA frameworks, the opposite does not hold, but for flat ABA framework, it a set of assumptions is set-stable, then it is also stable. Moreover, if the set of assumptions is set-stable, then it is preferred.

Similar to Bipolar Argumentation Framework in Section 2.4, the support relations in Bipolar ABA are categorised as either deductive or necessary. Each definition of support in BAF results into different translation method from BAF to Bipolar ABA. However, while different types of supports in BAF have different semantics; both deductive and necessary supports in Bipolar ABA framework have the same semantics.

2.5.1 Deductive Support in Bipolar ABA

The translation from bipolar argumentation framework with deductive support to Bipolar ABA with deductive support (d-ABA, for short) uses the notation α^c with c denotes the contrary and the set of attacks and supports are translated into rules. The translation preserves the direction of support such that if α supports β, then $\beta \leftarrow \alpha$.

Definition 2.5.3. (d-ABA). A bipolar ABA with *deductive support* or d-ABA corresponding to a BAF $\langle Args,\ attacks,\ supports \rangle$ is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with

- $\mathcal{L} = Args \cup \{\alpha^c \mid \alpha \in Args\}$
- $\mathcal{R} = \{\beta^c \leftarrow \alpha \mid (\alpha, \beta) \in attacks\} \cup \{\beta \leftarrow \alpha \mid (\alpha, \beta) \in supports\}$

- $\mathcal{A} = Args$
- $\overline{\alpha} = \alpha^c, \forall \alpha \in \mathcal{A}$

For example, the BAF in Figure 2.4 is translated into d-ABA $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$, where

- $\mathcal{L} = \{A, B, C, D, E, A^c, B^c, C^c, D^c, E^c\}$
- $\mathcal{R} = \{A^c \leftarrow B, \quad A^c \leftarrow E, \quad A \leftarrow C, \quad A \leftarrow D\}$
- $\mathcal{A} = \{A, B, C, D, E\}$
- $\overline{A} = A^c,\ \overline{B} = B^c,\ \overline{C} = C^c,\ \overline{D} = D^c,\ \overline{E} = E^c$

As the BAF can be translated into Bipolar ABA, it is natural that there exist connections between the $attacks$ of both frameworks. For any two arguments $\alpha, \beta \in Args$, if $\{\alpha\}$ attacks $\{\beta\}$ in d-ABA, then α attacks β or α attacks$_{sup}$ β in BAF. In addition, if $\{\alpha\}$ attacks $Cl(\{\beta\})$ in d-ABA, then α attacks$_{s-med}$ β in BAF.

Not only the attacks relations that coincide, several semantics between the two frameworks have connection as well. If a set of assumptions is admissible in d-ABA, then it is closed and d-admissible in BAF under deductive support. In BAF with deductive support, a d-preferred extension is always closed and therefore, it is preferred in Bipolar ABA. The connection also exists between set-stable and d-stable with a set of assumptions is d-stable if and only if it is set-stable in d-ABA.

Unless mentioned otherwise, from this point all Bipolar ABA frameworks are with deductive support. Therefore, depicting the frameworks into a graphical representation retains the direction of supports. An example of graphical representation of Bipolar ABA frameworks with deductive support above is given in Figure 2.8. This representation is useful for understanding complex frameworks more clearly.

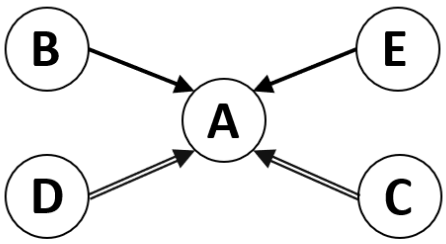

Figure 2.8: Graphical Representation of Bipolar ABA frameworks

2.5.2 Necessary Support in Bipolar ABA

Using a similar notation with the translation into Bipolar ABA from bipolar argumentation framework with deductive support, the difference of the translation from BAF with necessary support is the direction of supports. In Bipolar ABA with necessary support (n-ABA, for short), the direction is reversed such that if α supports β, then $\alpha \leftarrow \beta$.

Definition 2.5.4. (n-ABA). A bipolar ABA with *necessary support* or n-ABA corresponding to a BAF $\langle Args,\ attacks,\ supports\rangle$ is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{\ \ }\rangle$ with

- $\mathcal{L} = Args \cup \{\alpha^c : \alpha \in Args\}$
- $\mathcal{R} = \{\beta^c \leftarrow \alpha : (\alpha, \beta) \in attacks\} \cup \{\alpha \leftarrow \beta : (\alpha, \beta) \in supports\}$
- $\mathcal{A} = Args$
- $\overline{\alpha} = \alpha^c, \forall \alpha \in \mathcal{A}$

Then, the BAF in 2.4 is translated into n-ABA $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{\ \ }\rangle$, where

- $\mathcal{L} = \{A, B, C, D, E, A^c, B^c, C^c, D^c, E^c\}$
- $\mathcal{R} = \{A^c \leftarrow B, \quad A^c \leftarrow E, \quad C \leftarrow A, \quad D \leftarrow A\}$
- $\mathcal{A} = \{A, B, C, D, E\}$
- $\overline{A} = A^c,\ \overline{B} = B^c,\ \overline{C} = C^c,\ \overline{D} = D^c,\ \overline{E} = E^c$

For a closed set of assumptions in n-ABA, it is also coherent in bipolar argumentation framework with necessary support, and it is strongly coherent if the set is conflict-free as well. Therefore, the connection between attacks in n-ABA and extended attacks in BAF with necessary support is that for any two sets of arguments $\alpha, \beta \subseteq Args$, if α attacks β in n-ABA, then α attacks$_{ext}$ β in BAF with necessary support. Thus, it is easy to see the connections between some semantics in BAF with necessary support and n-ABA. If a set of assumptions is admissible / preferred / set-stable in n-ABA, then it is also n-admissible / n-preferred / n-stable respectively in the bipolar argumentation framework.

To sum up, even with both definitions of supports, the translation from BAF into Bipolar ABA framework have exactly the same procedures. It handles the inconsistencies introduced in BAF as the calculation for the semantics are the same. Thus, aggregating Bipolar ABA framework instead of BAF is more preferable.

2.6 Quantitative Argumentation Debate Framework

Quantitative Argumentation Debate Framework (QuAD, for short) is a special type of Bipolar Argumentation framework for computing the *strength* of the arguments, usually used to answer a question [1, 18]. The question is usually represented as the root of a tree with other attacking (con) and supporting (pro) arguments as the leaf. QuAD is introduced in this project in order to understand the e-polling scenario used in Chapter 4.

Definition 2.6.1. (QuAD). A *Quantitative Argumentation Debate Framework* / QuAD is a tuple $\langle \mathcal{A}, \mathcal{C}, \mathcal{P}, \mathcal{R}, \mathcal{BS}\rangle$, where $\mathcal{A}$ is a finite set of *answer* arguments, $\mathcal{C}$ is a finite set of *con* arguments, $\mathcal{P}$ is a finite set of *pro* arguments, $\mathcal{R}$ is an acyclic binary relation, and $\mathcal{BS}$ is a total function of *base score*. The sets $\mathcal{A}, \mathcal{C}, \mathcal{P}$ are pairwise disjoint and the base score $\mathcal{BS} \in [0, 1]$.

Definition 2.6.2. (Attacks and supports). For $\alpha \in \mathcal{A} \cup \mathcal{C} \cup \mathcal{P}$, the set of attackers is $\mathcal{R}^-(\alpha) = \{\beta \in \mathcal{C} | (\beta, \alpha) \in \mathcal{R}\}$. The set of supporters is $\mathcal{R}^+(\alpha) = \{\beta \in \mathcal{P} | (\beta, \alpha) \in \mathcal{R}\}$.

The QuAD representation of the background scenario in Section 2.1 is a tuple $\langle \mathcal{A}, \mathcal{C}, \mathcal{P}, \mathcal{R}, \mathcal{BS} \rangle$, where $\mathcal{A} = \{A\}$, $\mathcal{C} = \{B, E\}$, $\mathcal{P} = \{C, D\}$, and $\mathcal{R} = \{(B, A), (C, A), (D, A), (E, A)\}$. The base score $\mathcal{BS}$ is used to give strength on each argument. The final strength of each argument is then calculated with *Discontinuity Free Quantiative Argumentation Debate* (DF-QuAD, for short) algorithm, which is based on strength aggregation function [18]. The algorithm has four functions, which are the base function, strength aggregation function, combination function, and strength function.

Definition 2.6.3. (Base function). For $\mathcal{I} \in [0, 1]$, the *base function* $f : \mathcal{I} \times \mathcal{I} \to \mathcal{I}$ for $v_1, v_2 \in \mathcal{I}$ is defined as $f(v_1, v_2) = v_1 + (1 - v_1) \cdot v_2 = v_1 + v_2 - v_1 \cdot v_2$.

Definition 2.6.4. (Strength aggregation function). The *strength aggregation function* $\mathcal{F} : \mathcal{I}^* \to \mathcal{I}$, where for $S = (v_1, \cdots, v_n) \in \mathcal{I}^*$, is defined as:

$$\begin{aligned}
&\text{if } n = 0 : \mathcal{F}(S) = 0 \\
&\text{if } n = 1 : \mathcal{F}(S) = v_1 \\
&\text{if } n = 2 : \mathcal{F}(S) = f(v_1, v_2) \\
&\text{if } n > 2 : \mathcal{F}(S) = f(\mathcal{F}(v_1, \cdots, v_{n-1}), v_n)
\end{aligned}$$

Definition 2.6.5. (Combination function). The *combination function* $c : \mathcal{I} \times \mathcal{I} \times \mathcal{I} \to \mathcal{I}$ for $v_0, v_a, v_s \in \mathcal{I}$, where v_0 is the base score, v_a is the attackers' strength, and v_s is the supporters' strength, is defined as:

$$\begin{aligned}
c(v_0, v_a, v_s) &= v_0 - v_0 \cdot |v_s - v_a| & if v_a \geq v_s \\
c(v_0, v_a, v_s) &= v_0 + (1 - v_0) \cdot |v_s - v_a| & if v_a < v_s
\end{aligned}$$

Definition 2.6.6. (Strength function). The *strength function* $\mathcal{S} : \mathcal{A} \cup \mathcal{C} \cup \mathcal{P} \to \mathcal{I}$ for any $\alpha \in \mathcal{A} \cup \mathcal{C} \cup \mathcal{P}$ is defined as $\mathcal{S}(\alpha) = c(\mathcal{BS}(\alpha), \mathcal{F}(\mathcal{R}^-(\alpha)), \mathcal{F}(\mathcal{R}^+(\alpha)))$.

Intuitively, the strength function is used to compute the strength of an argument α. It is based on the strength of the attackers and supporters of α, which are computed using the strength aggregation function and base function. The combination function then calculates the effect of the attackers' strength and supporters' strength to the base score of α.

Continuing the example taken from the background scenario and with the assumption that each argument base score $\mathcal{BS} = 0.5$, then the first step is to use the strength aggregation function to find the aggregated strength of attackers $\{(B, A), (E, A)\}$ and supporters $\{(C, A), (D, A)\}$, i.e., in a bottom-up manner. There are two attackers and two supporters with the same base score, as a result both aggregated strength will be the same.

$$\mathcal{F}(S) = f(0.5, 0.5) = 0.5 + 0.5 - 0.5 \cdot 0.5 = 0.75$$

Then, to get the final strength of argument A, which is the question or the root of the framework, the strength function and combination function are used.

$$\mathcal{S}(A) = c(0.5, 0.75, 0.75) = 0.5 - 0.5 \cdot |0.75 - 0.75| = 0.5$$

The final strength of argument A in this example is 0.5, which is the same as the base score. This can happen because of the same number of attackers and supporters (symmetric).

2.6.1 QuAD-V

The QuAD for Voting Framework (QuAD-V, for short) is an extension of QuAD framework, which focus on the voting mechanism as opinion polling to represent deliberative democracy [17]. It is able to facilitate reasoning behind the agents' vote and thus, solidifying their opinions and expanding the opinion polling process. This is important as many agents are either being irrational or not expressing their opinions fully.

Definition 2.6.7. (QuAD-V). A *QuAD for Voting framework* / QuAD-V is a tuple $\langle \mathcal{A}, \mathcal{C}, \mathcal{P}, \mathcal{R}, \mathcal{U}, \mathcal{V} \rangle$, where $\mathcal{A}$ is a finite set of *answer* arguments, $\mathcal{C}$ is a finite set of *con* arguments, $\mathcal{P}$ is a finite set of *pro* arguments, $\mathcal{R}$ is an acyclic binary relation, $\mathcal{U}$ is a finite set of users or agents, and $\mathcal{V}$ is a total function with $\mathcal{V}(u, a)$ is the vote of user $u \in \mathcal{U}$ on argument $a \in \mathcal{A} \cup \mathcal{C} \cup \mathcal{P}$. The sets $\mathcal{A}, \mathcal{C}, \mathcal{P}$ are pairwise disjoint and the vote $\mathcal{V} \in \{-, ?, +\}$, where - is con, ? is neutral, and + is pro.

Definition 2.6.8. (Attacks and supports). For $\alpha \in \mathcal{A} \cup \mathcal{C} \cup \mathcal{P}$, the set of attackers or the set of users voting *against* α is $\mathcal{V}^-(\alpha) = \{u \in \mathcal{U} : \mathcal{V}(u, \alpha) = -\}$. The set of supporters of the set of users voting *for* α is $\mathcal{V}^+(\alpha) = \{u \in \mathcal{U} : \mathcal{V}(u, \alpha) = +\}$.

Definition 2.6.9. (Vote count). For any argument $\alpha \in \mathcal{A} \cup \mathcal{C} \cup \mathcal{P}$, the positive vote count is $\mathcal{N}^+(\alpha) = |\mathcal{V}^+(\alpha)|$, i.e., the number of users voting for argument α. The negative vote count is $\mathcal{N}^-(\alpha) = |\mathcal{V}^-(\alpha)|$, i.e., the number of users voting against argument α.

In QuAD-V, the agents can be neutral as well. The neutrality is denoted by a base score of 0.5. Then, the other arguments' base score is affected by the positive vote count and the negative vote count. Note that the vote counts affect the base score in a different way from the attacks and supports relations.

Definition 2.6.10. (Vote base score). For any argument $\alpha \in \mathcal{A} \cup \mathcal{C} \cup \mathcal{P}$, the *vote base score* is defined as $\mathcal{BS}_v : \mathcal{A} \cup \mathcal{C} \cup \mathcal{P} \to \mathcal{I}$, where

$$\mathcal{BS}_v(\alpha) = 0.5 \qquad if|\mathcal{U}| = 0$$

$$\mathcal{BS}_v(\alpha) = 0.5 + (0.5 \times \frac{\mathcal{N}^+(\alpha) - \mathcal{N}^-(\alpha)}{|\mathcal{U}|}) \qquad if|\mathcal{U}| \neq 0$$

The vote base score starts from 0.5 and increases the higher the positive vote count is, and decreases the higher the negative count is. For example taken from the background scenario depicted in Figure 2.4, if $\mathcal{N}^+(A) = 5$ and $\mathcal{N}^-(A) = 5$ as well, and there are 20 users, then the vote base score $\mathcal{BS}_v(\alpha)$ is 0.5. However, if $\mathcal{N}^+(A) = 5$, but $\mathcal{N}^-(A) = 2$, then $\mathcal{BS}_v(\alpha)$ is 0.65. After that, using the algorithm of DF-QuAD and the vote base score $\mathcal{BS}_v$ as the base score $\mathcal{BS}$, the final aggregated strength of each argument can be computed. The arguments with strength 1 are accepted, 0 are denied, and 0.5 are neutral.

By this point, six types of frameworks are already introduced. First, the Abstract Argumentation framework is the most basic framework that gives foundation to the other frameworks. The Assumption-based Argumentation framework is a structured version of Abstract Argumentation framework and serves as the basis of structured argumentation. Then, the Bipolar Argumentation framework is introduced as the first framework to have support relationships. It has different notions of supports that make it difficult to aggregate due to the inconsistencies of the semantics. To solve this problem, Bipolar Assumption-based Argumentation framework is given. Although there are also two types of supports in Bipolar ABA framework, but both of them have the same set of semantics, which make the aggregation simpler. Thus, in this project, the aggregation is conducted on the Bipolar ABA frameworks. After the aggregation, then it is easy to translate it back to the Bipolar Argumentation framework. Next, QuAD and QuAD-V frameworks play a role in the application of e-polling. The procedure before and after the aggregation involve the use of various formulas in both frameworks. In the next section, Social Choice Theory, to be exact the judgement aggregation, is introduced as one way to aggregate the frameworks.

2.7 Social Choice Theory

Social choice theory is a study of how to combine people's interests or preferences and output a single aggregated or collective decision. One of the many applications of social choice theory is election voting process, in which people differ in opinions and interests. The goal is to aggregate the opinions into a final conclusion, in this case is the winner of the election. In addition, social choice theory covers broader topics as well, such as economic and social welfare. The practitioners in social choice theory often ask questions such as 'what is the process to choose the best outcome from a given set of options by a number of agents?' and 'what procedures that number of agents need to take to arrive at the outcome, given that each of them has personal interest?'. There are mainly two types of aggregation, which are preference aggregation and judgement aggregation. [14]

Preference aggregation is the earliest type of aggregation in social choice theory which aggregates preference rankings of several arguments from a group of agents into a collective preference ranking. Each agents proposes a ranking for the arguments such that if an argument α is more preferable than another argument β, then α ranks higher than β. Collecting all the preference rankings of each agents result in a *profile* $< R_1, ..., R_n >$. Then, a preference aggregation rule F combines all preference ranking into a collective preference ranking $R = F(R_1, ..., R_n)$. [2, 14]

Judgement aggregation uses mathematical formulations to aggregate opinions into a single collective opinion. Hence, it is necessary to study methods to aggregate opinions to resolve the difference in opinions. These methods have been popular research topics in economics or political sciences. However, in the emergence of artificial intelligence and computer science, judgement aggregation has a leap of improvement in solving various problems. In this project, judgement aggregation is the main focus of social choice theory method. [13]

An example of judgement aggregation problem is a presidential election voting. With several candidates, the agents' opinions must be divided as each agent has his own reason on why he chooses the candidate. Using social choice theory and judgement aggregation, it is possible to predict the election outcome by getting the overall positive sentiment of the agents.

The aggregation rules, which output an aggregated framework, can be seen as extensions of graph aggregation [12] and social choice theory [14], especially the judgement aggregation [13]. The judgement aggregation rules are categorised as either quota rules or oligarchic rules.

Definition 2.7.1. (Aggregation Rule). An *aggregation rule* is a mapping function $F : \mathcal{AF}_1 \times \mathcal{AF}_n \rightarrow \mathcal{AF}$ from several argumentation frameworks into a single framework or $F : \Delta_1 \times \Delta_n \rightarrow \Delta$ from several extensions into a single extension.

In this section, the Abstract Argumentation frameworks are used to demonstrate the aggregation rules as based on [7, 8] for more intuitive understanding. In later chapters, the rules will be redefined to follow the project direction more closely. Therefore, Figure 2.9 shows some opinions of the background scenario in Section 2.1 in the form of abstract argumentation framework.

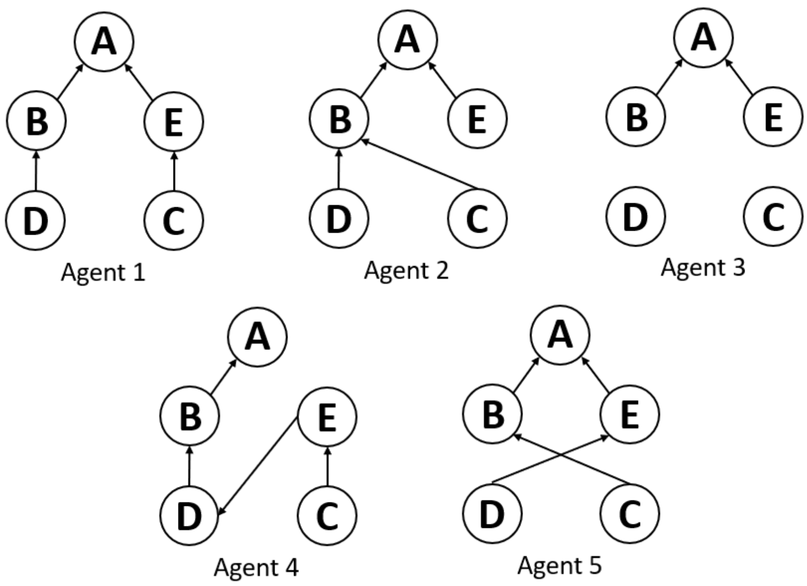

Figure 2.9: Different Opinions of Agents on the Scenario

The different frameworks of the agents show their own opinion on the same scenario. Depending on the agent's point of view, he can depict different attack relations between the arguments. For example, agent 1 thinks that the EU's immigration policies are bad (argument C) so that staying in the EU is not good for world peace (argument E). On the other hand, agent 2 thinks that the EU's immigration policies are bad (argument C) so that the UK's economy will decline (argument B).

2.7.1 Quota Rules

The quota rules use the number of quota $q \in \{1, ..., n\}$ as a threshold to accept any arguments or attack relations, i.e., there should be at least *n* agents that accept selected attack relationships $\{att \in attacks\}$ or arguments from extensions $\{Arg \in \Delta\}$.

Definition 2.7.2. (Quota Rules on attack relationships). Take n abstract argumentation frameworks $\langle$*Args, attacks*$\rangle$, a set of agents *n*, and quota $q \in \{1, ..., n\}$. Then quota rules F_q accept attack relations that are supported by at least *q* agents.

$$F_q(attacks) = \{att \in attacks \mid \#n_{att} \geq q\}$$

Definition 2.7.3. (Quota Rules on arguments). Take an abstract argumentation framework $\langle$*Args, attacks*$\rangle$, a set of agents *n*, quota $q \in \{1, ..., n\}$, and chosen extension from each agent $\{\Delta_1, ..., \Delta_n\}$. Then quota rules F_q accept arguments that are supported by at least *q* agents.

$$F_q(Args) = \{Arg \in Args \mid \#\{Arg \in \Delta_{1...n}\} \geq q\}$$

Although the quota q can be any number less than or equal to the number of agents, but there are several numbers of quota that are considered special. First, the weak majority rule accepts attack relations or arguments that are supported by at least half the number of agents, rounded down. In the scenario depicted on Figure 2.9, there are 5 agents so that the weak majority quota is 2. To get the aggregated set of accepted arguments, each agent needs to provide a set of accepted argument from his own framework. For example, the accepted set of arguments is respectively $\Delta_1, ..., \Delta_5 = \{A, C, D\}$, $\{C, D, E\}$, $\{B, C, D, E\}$, $\{C\}$, $\{C, D\}$; all of which are admissible, then the aggregated set of accepted arguments is $\{C, D, E\}$, which is also admissible in this specific case.

Definition 2.7.4. (Weak Majority Rule). Take a set of agents *n*. *Weak majority rule* accepts arguments or attack relations that are supported by at least $q = \lfloor \frac{n}{2} \rfloor$ agents.

Figure 2.10 shows the aggregated framework using weak majority rule. It accepts all attack relations which are supported by at least 2 agents.

Second, the strict majority rule is a quota rule which accepts attack relations or arguments that are supported by at least half the number of agents, rounded up. For the argument aggregation, using the same example as in the weak majority, i.e., taking accepted admissible sets of arguments from each agent as $\Delta_1, ..., \Delta_5 = \{A, C, D\}$,

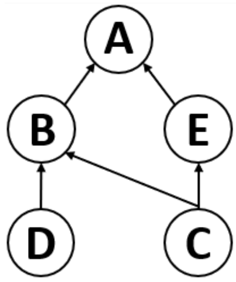

Figure 2.10: Weak Majority Rule

$\{C, D, E\}$, $\{B, C, D, E\}$, $\{C\}$, $\{C, D\}$ respectively, the aggregated accepted set of arguments is $\{C, D\}$, which is also admissible in this specific case.

Definition 2.7.5. (Strict Majority Rule). Take a set of agents *n*. *Strict majority rule* accepts arguments or attack relations that are supported by at least $q = \lceil \frac{n}{2} \rceil$ agents.

Figure 2.11 shows the aggregated framework using strict majority rule, which accepts every attack relation supported by at least 3 agents.

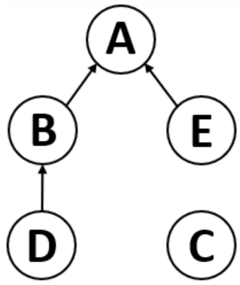

Figure 2.11: Strict Majority Rule

The unanimity rule requires all agents to agree with attack relations or arguments before accepting them, i.e., they are unanimously accepted. For arguments aggregation, referring to the same example as before, the final set of accepted argument is $\{C\}$ according to the unanimity rule.

Definition 2.7.6. (Unanimity Rule). Take a set of agents *n*. *Unanimity rule* accepts arguments or attack relations that are supported by $q = n$ agents.

The result for the attack relations aggregation is given in Figure 2.12. It only has one attack relation from B to A as it is the only attack that is accepted by each agent's framework.

In contrast with the unanimity rule, nomination rule accepts all arguments or attack relations as long as there is at least one agent supporting them. With nomination rule and the same example from before, the aggregated set of arguments is $\{A, B, C, D, E\}$. It has all the arguments in the set because each argument has at least one agent who supports it.

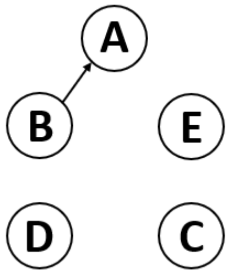

Figure 2.12: Unanimity Rule

Definition 2.7.7. (Nomination Rule). Take a set of agents *n*. *Nomination rule* accepts arguments or attack relations that are supported by at least $q = 1$ agents.

The resulting aggregation of attack relations can be seen in Figure 2.13 with all attacks from every framework included in it.

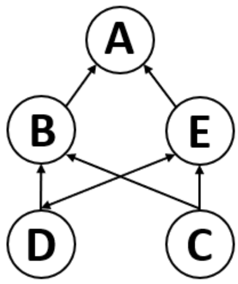

Figure 2.13: Nomination Rule

2.7.2 Oligarchic Rules

Oligarchic rules use dictatorship to give agents the power to veto the accepted attack relations or arguments. It is intuitive to say that oligarchic rules are not fair as the opinions of other agents without veto power are disregarded. However, in some situations it is necessary to use oligarchic rules to avoid conflicts among the agents.

Definition 2.7.8. (Oligarchic Rules). Take a set of agents *n*, and some agents with veto power $n_v \subseteq n$. Then the accepted attack relations or arguments are those supported by all agents with veto power.

$$Relations : F_O(attacks) = \{att \in attacks \mid att \in n_v\}$$
$$Arguments : F_O(Args) = \{Arg \in Args \mid Arg \in \Delta_{n_v}\}$$

Figure 2.14 shows the aggregation of attack relations of the scenario when agent 1 and 4 have veto power. It accepts all attacks that are supported by only agent 1 and agent 4. For the aggregation of arguments, the accepted admissible sets of arguments in the example scenario used before are $\Delta_1, ..., \Delta_5 = \{A, C, D\}$, $\{C, D, E\}$, $\{B, C, D, E\}$, $\{C\}$, $\{D\}$ respectively. Therefore, the aggregated argument is $\{C\}$ for

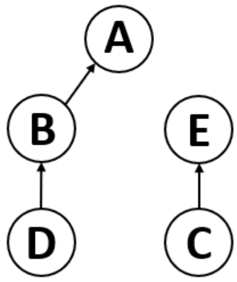

Figure 2.14: Oligarchic Rule

oligarchic rule with agent 1 and 4 because argument C is the only argument in the intersection of the sets $\{A, C, D\}$ and $\{C\}$.

In some sense, oligarchic rules behave like the unanimity rule. If all agents are given veto powers, then it is exactly the same as the unanimity rule. If only some agents have veto power, those agents still need to unanimously agree on the accepted arguments.

2.8 Related Works

Chen and Endriss [8] study the aggregation of Abstract Argumentation frameworks. Assuming that all the agents agree with the set of arguments, the aggregation takes the attacks from each of the agents' frameworks and combines them into a collective set of attacks. The aggregation rules to combine the attacks are quota rules and oligarchic rules from judgement aggregation, although the paper also mentions a more general aggregation rule. The question after the aggregation is whether the opinions of all the agents are represented in the final collective framework. To deliberate more, the preservation of the Abstract Argumentation framework's properties is studied as well. The preservation results aim to give the conditions needed to preserve the properties. Some of the conditions might be related to the aggregation rules, the number of arguments, and the number of agents. The final results show some positive results, such as conflict-freeness, which is preserved by every aggregation rule; and some negative results, in which oligarchic rules and dictatorship come into play. In the following chapter, the preservation results in this work are extended to include the bipolarity.

Rago and Toni [17] take the scenario of e-polling and try to solve some inconsistencies and irrationality of the voting procedure with argumentation process. During the voting process, some agents might agree or disagree with the arguments. However, it is possible that the agents' vote does not have any basis, i.e., irrational. Thus, the paper proposes a novel framework called Quantitative Argument Debate for Voting (QuAD-V), which is an extension of the classic QuAD framework. In the framework, there is a phase to remove the irrationality of the agents' opinions by giving some dynamic questions. This iterative process adds or removes the arguments, and change the vote of the agents. However, in this paper there is not any mention on the aggregation process of the rational agents' opinions. It is proposed as the future work.

Endriss and Grandi [12] work in detail on a more general graph aggregation. The aggregation has a goal of producing a single graph from several graphs as the input. Graph aggregation also has many applications in real life, for example to solve a dispute, to conduct a fair debate, and to find the middle path of several groups of data. Social choice theory plays a role even in the general graph aggregation. Similar to other aggregation, the study focuses on finding methods to preserve the properties of the individual graphs. The results show various conditions and scenarios of graph aggregation with both possibility and impossibility results. An important part of the work is that the results are easily transferable into another type of argumentation frameworks, which basically can still be seen as graphs.

The other related work is the Individual Study Option (ISO) that was completed earlier this year as a survey of the aggregation process. It is described in more detail in the next section with explicit explanation on the difference between the ISO and this project.

2.9 Relation to Individual Study Option (ISO)

The individual study option was conducted for about three months and was limited in the survey of the research topic [19]. It was closely related to this project in terms of the focus, which is an aggregation. The motivation to study opinion aggregation was to resolve conflicts between some agents. The scenario was to find an agent that should be responsible for an accident. All agents in this scenario spoke their own opinion by defending themselves and attacking the other agents as no one wanted to be held responsible. Thus, this process was a form of argument exchange in a debate.

There are several goals for the study. First, to survey various methods to aggregate opinions and extend the survey with latest state of the art methods. Next, to study Social Choice Theory and the role it has on opinion aggregation. Last, along with the limitations on the methods, some directions for further work are listed.

Before dealing with the aggregation methods, some basic knowledge about argumentation are needed. In the ISO, the focus of the aggregation is only with the Abstract Argumentation framework as in Section 2.2. The additional part is the labelling method, which is one way to get the set of the accepted arguments by giving the arguments with label IN, OUT, or UNDEC [11]. Opinions aggregation is also a part of social choice study. Social Choice Theory, especially judgement aggregation with the quota rules and oligarchic rules are all studied in the ISO as in the Section 2.7 of this project.

The survey on aggregation methods had been done in [3]. However, it is limited to the methods published until the year of 2017. Thus, in the ISO there are several new papers that are studied and used to extend the survey. The works on opinions aggregation are categorised into four major focus. The first focus is on the standard aggregation methods. Moreover, the directions of the aggregation methods are by aggregating the frameworks or aggregating the extensions, called framework level approach and argument level approach respectively.

The framework level approach of aggregating opinions is to take the frameworks of all the agents, which have the same set of arguments, and aggregate the attacks relationships into a collective set of attacks. The challenge to this approach is to preserve the properties of the agents' individual frameworks in the aggregated framework. On the other hand, the argument level approach deals with only one framework, which the agents propose one extension each. The aggregation combines the arguments in the extension based on various aggregation rules. In other words, if the property P holds in the individual extensions, then the study tried to preserve the property in the aggregated extension.

The next category is rationality properties, which tries to use several different concepts to further improve the performance of the aggregation. Social choice theory, judgement aggregation, and game theory belong to the rationality properties. Dynamic argumentation is another focus of aggregation study. The process of arguments exchange depends heavily on the behaviour of the agents. Hence, given many trials of the aggregation, the results may not be the same. Therefore, it is an important direction to study algorithms and procedures to deal with the dynamics in order to smoothen the aggregation process. Finally, as social study is closely related to both argumentation and aggregation, some studies focused on social argument assessment. When agents agree or disagree with some arguments, there are levels that differentiate how much they agree or disagree. The assessment is done mostly by quantifying the argumentation process by giving weight, strength, or distance.

In the ISO, the main direction was to study into more detail about the framework level and argument level approaches aggregation. There are two main works by the same author [7, 8] that studies both topics in detail. In the framework level aggregation, an important point is that the agents have exactly the same knowledge about the arguments, but may differ only on the attack relationships.

Definition 2.9.1. (Framework level aggregation). Take a number of agents $N \in \{1, \cdots, k\}$ and a number of Abstract Argumentation frameworks $\{\langle Args, attacks_1\rangle, \cdots, \langle Args, attacks_k\rangle\}$. Framework level aggregation combines $attacks_i$ for all $i \in N$ and produces a collective framework $\langle Args, attacks_{agg}\rangle$.

The preservation study concerned about some important properties of the Abstract Argumentation framework, which are the conflict-freeness, admissibility, preferred, complete, stable, grounded, the acceptability of an argument, acyclicity, coherence, and the non-emptiness of the grounded extension. The preservation results showed some positive results, however, most of the properties can only be preserved if the aggregation rule is dictatorship or at least one agent must have veto power. The usage of oligarchic rules is generally not desirable as they limit the opinions of the agents.

The argument level aggregation, or extensions level aggregation, puts the fact that within one framework, there might be some properties with many extensions. The agents then form their opinion by choosing one of the extensions. Thus, some important points of extensions level aggregation are one framework for all the agents and some proposed extensions with the same property.

Definition 2.9.2. (Extensions level aggregation). Take a number of agents $N \in \{1, \cdots, k\}$ and an Abstract Argumentation framework $\langle Args, attacks \rangle$. Extensions level aggregation combine the extensions Δ_i for all $i \in N$ and produces a collective extension Δ_{agg}.

In other words, if a property P holds in all the agents' extension, then to preserve them means that P should also hold in the aggregated extension. This is a very different approach than the framework level aggregation. To complement the study, there are two new terms introduced in the paper. *MaxAtt* is the highest number of attackers on any argument in the framework and *MaxDef* is the highest number of arguments that defend the other arguments.

Another unique mechanism in extension level approach is to encode various semantics into propositional logic equivalent. For example, a conflict-free extension Δ can be encoded into $\bigwedge_{\substack{A,B \in Arg \\ A \rightharpoonup B}} (\neg A \vee \neg B)$. This encoding is important to determine the preservation of the semantics with a formula $q \cdot (k_2 - k_1) > n \cdot (k_2 - 1) - k_1$, where q is the quota, k_1 is the number of positive literals, and k_2 is the number of negative literals. Both k_1 and k_2 are retrieved from the propositional logic of the semantic given a framework.

The preservation results showed that the grounded semantic is the only property preserved by every quota rule and oligarchic rule as it is always exist in every framework. Some new properties were also introduced in the study. The self-defending property concerns with an argument that attacks each of its attacker and the reinstating property concerns with the ability of the framework to have all arguments defended. Both of them created impossibility results for the admissible and complete semantics. The conflict-freeness is preserved if the aggregation rule has a quota higher than half the number of the agents. Preferred and stable semantics can be joined into I-maximal group of properties, which means both semantics try to get the maximum set of arguments in the extension.

The ISO is completed with a deeper study on the framework level and extensions level aggregation. Although both the ISO and this project have the opinion aggregation as the focus, but there are major differences. First of all, due to the nature of ISO, it was limited to only the survey of the aggregation methods. In other words, there were no original contributions in the ISO.

This project also puts the focus on the bipolarity of the frameworks. Hence, it is not only concern with the Abstract Argumentation framework as in the ISO, but also the other type of frameworks such as Bipolar Argumentation framework, Quantitative Argumentation Debate framework, Assumption-based Argumentation framework, and Bipolar Assumption-based Argumentation framework. The number of works on the aggregation of bipolar frameworks are not as many as the works on the aggregation of the Abstract Argumentation frameworks. Thus, this study proposes the aggregation procedure for Bipolar Assumption-based Argumentation framework.

The properties that are studied in this project overlap with the work of framework level approach aggregation [8]. However, several new semantics are also introduced, including closure, set-stable, well-founded, and ideal semantics. The proof of every preservation result is more complex because of the presence of supports relationships. Moreover, this project covers more scenarios of the preservation study, i.e., the corner cases, which are not mentioned in the paper.

Finally, this project also proposes a useful application of opinions aggregation in the form of e-polling scenario. Furthermore, it completes the e-polling procedure mentioned in [17]. Some careful and detailed analyses on each aggregation rules are given to fit different cases of e-polling. In summary, this project and the ISO relate to the opinion aggregation as the focus. However, this project especially takes a non-trivial direction for the research, which is bipolarity.

2.10 Ethical and Professional Considerations

When considering ethical and professional considerations, there are several issues that must be explained. First, the danger the project might cause due to some misused applications. Argumentation in terms of Artificial Intelligence itself dates from decades ago. It is still an important research area in the hope to help solve daily problems. Humans themselves actually have a better capability in logic and argumentation. Hence, opinion aggregation is a common topic and practically used in debates, decisions making, and disputes solving; all of which do not pose dangers in any kind.

Next, it is necessary to consider the people involved in this project. The people involved in this project are the author and the supervisors. There is not any third party or other person's involvement in this project. The author and the supervisors are all aware about this project and hence, give their consent naturally.

Finally, any information used in this project needs to be considered as well. The knowledge of argumentation, aggregation, and Social Choice Theory are all published without some access restriction. In this case, the information used are some well-known knowledge. Thus, there does not exist any copyright issue. All graphs and pictures attached to this project are all common graphs that can be easily interpreted. Lastly, the Brexit scenario used in Chapter 4 is a scenario that is based on past events. Therefore, any use of the scenario on this project does not affect the event itself.

The full ethics checklist can be found in Appendix A.

Chapter 3

Preservation Results

In this chapter, the preservation results of Bipolar ABA frameworks aggregation are studied. These covers the preservation of the semantics, the acceptability of an assumption, acyclicity, and coherence; all of which are properties of Bipolar ABA framework. Some of the results are new, while others are the extension of [8] by introducing the presence of support and applying it to the Bipolar ABA framework in contrast to the Abstract Argumentation framework.

Definition 3.0.1. (Preservation). Let P be a property of Bipolar ABA framework and take a number of agents $N \in \{1, \cdots, k\}$. If P holds for each agents' framework $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$, then P is preserved if and only if P holds in the aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$.

To design a preservation study for the properties, several things to consider are the aggregation rules, the number of assumptions, the number of agents, and the presence of supports. The aggregation rules considered in the study are quota rules and oligarchic rules. Dictatorship is a special type of oligarchic rule, which gives only one agent with veto power. The number of assumptions affects the study as the more complex a framework is, the more restrictions it has. On the contrary, corner cases for a small number of assumptions might be easier to preserve P. The number of agents determines the number of rules that need to be considered. Finally, the presence of supports in a Bipolar ABA framework forms closures of assumptions that affect the calculation of the property P. Then, as the dynamic of the aggregation happens in the rules of the frameworks, consequently the aggregation takes place on the rules $\mathcal{R}_i$ for all $i \in N$.

Remark. It is important to notice that all agents' frameworks need to satisfy P. For example, if a set of assumptions Δ is P, then Δ is acceptable in all agents' framework. Furthermore, all agents agree on the $\mathcal{L}, \mathcal{A}$, and $\overline{}$.

Definition 3.0.2. (Bipolar ABA frameworks aggregation). Take a number of agents $N \in \{1, \cdots, k\}$ and a number of Bipolar ABA frameworks $\{\langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, \overline{} \rangle, \cdots, \langle \mathcal{L}, \mathcal{R}_k, \mathcal{A}, \overline{} \rangle\}$. Aggregating the frameworks means to combine the rules $\{\mathcal{R}_1, \cdots, \mathcal{R}_k\}$ into a collective rule $\mathcal{R}_{agg}$ resulting in a single aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$.

3.1 Conflict-freeness

A set of assumptions is conflict-free in Bipolar ABA framework if there does not exist any attacks among the assumptions in the set. The preservation of conflict-freeness aims to keep the property of not having attacks in the set of the aggregated rules. The result below shows that conflict-freeness is preserved under all quota rules and oligarchic rules. It is also an extension of the result in [**Theorem 2**, 8].

Theorem 1. *Every quota rules and oligarchic rules preserve conflict-freeness in Bipolar ABA frameworks.*

Proof. Let a Bipolar ABA framework property P be the conflict-freeness. Take a number of agents N and assume that an extension Δ is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. To prove by contradiction, assume that Δ is not conflict-free in the aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, i.e., P does not hold. In other words, there exist two assumptions $\alpha, \beta \in \Delta$ such that α attacks β, i.e., $\beta^c \leftarrow \alpha \in \mathcal{R}_{agg}$. Then, due to the attribute of both quota rules and oligarchic rules, there has to be at least one agent $i \in N$ (in case of nomination rule or oligarchic rule with single veto power) or more (in case of other quota and oligarchic rules) such that α attacks β in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$, i.e., $\beta^c \leftarrow \alpha \in \mathcal{R}_i$ such that Δ is not conflict-free in the agent's framework either. Thus, it contradicts the initial assumption that Δ is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. $\square$

From the proof of Theorem 1, it is clear that the presence of supports does not affect the preservation of conflict-freeness as long as the agents get into an agreement of the conflict-free extensions. As an example, take several Bipolar ABA frameworks with rules $\mathcal{R}_1 = \{D^c \leftarrow A, \quad C^c \leftarrow A, \quad B \leftarrow A\}$, $\mathcal{R}_2 = \{D^c \leftarrow A, \quad C^c \leftarrow B, D^c \leftarrow C\}$, and $\mathcal{R}_3 = \{D^c \leftarrow C, \quad C^c \leftarrow D\}$; illustrated in Figure 3.1. The set of assumptions $\{A, B\}$ is conflict-free in each framework. Then, using unanimity rule yields to $\mathcal{R}_{agg} = \{\}$ such that $\{A, B\}$ is conflict-free as well. Using majority rule, $\mathcal{R}_{agg} = \{D^c \leftarrow A, \quad D^c \leftarrow C\}$ also makes $\{A, B\}$ conflict-free. It is also conflict-free using nomination rule with $\mathcal{R}_{agg} = \{D^c \leftarrow A, \quad C^c \leftarrow A, \quad C^c \leftarrow B, \quad D^c \leftarrow C, \quad C^c \leftarrow D, \quad B \leftarrow A\}$. Finally, using oligarchic rules with veto power given to the first and second frameworks ($\mathcal{R}_1$ and $\mathcal{R}_2$) produce an aggregated set of rules $\mathcal{R}_{agg} = \{D^c \leftarrow A\}$ and $\{A, B\}$ is also conflict-free. Thus, the presence of supports does not affect the preservation of conflict-freeness.

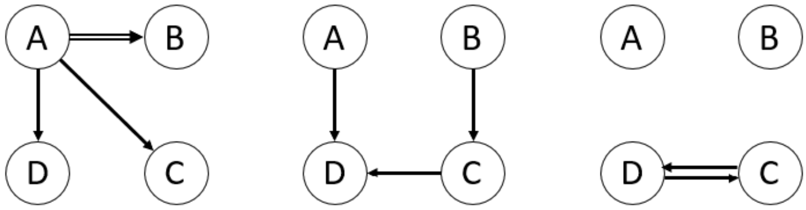

Figure 3.1: Example of Preservation of Conflict-freeness

3.2 Closure

Closure is an important property in the Bipolar ABA framework. It is a consequence of supports between the assumptions. A set of assumptions is closed if all assumptions deducible from the set are included in it. The preservation of the closedness property of a set of assumptions tries to keep that set of assumptions closed in the aggregated framework as well. This is a positive result as the preservation of closure property is preserved under all quota rules and oligarchic rules. It is a new result as closure does not exist in Abstract Argumentation framework.

Theorem 2. *Every quota rules and oligarchic rules preserve closedness of a set of assumptions in Bipolar ABA frameworks.*

Proof. Let a Bipolar ABA framework property P be the closure of assumptions. Take a number of agents N and assume an extension Δ is closed in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. To prove by contradiction, assume that Δ is not closed in the aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, i.e., P does not hold. In other words, there is some assumptions deducible from Δ but not included in it. Using supports notation, it can be said that there exist an assumption $\alpha \in \Delta$ and α supports β, i.e., $\beta \leftarrow \alpha \in \mathcal{R}_{agg}$, for $\beta \in \mathcal{A} \setminus \Delta$ in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$ but $\beta \notin \Delta$. Then, due to the attribute of both quota rules and oligarchic rules, there has to be at least one agent $i \in N$ (in case of nomination rule or oligarchic rule with single veto power) or more (in case of other quota and oligarchic rules) such that $\beta \leftarrow \alpha \in \mathcal{R}_i$ for $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$, i.e., Δ is not closed in the agent's framework either. Thus, it contradicts the initial assumption that Δ is closed in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. □

As an example for the preservation of closure, take several Bipolar ABA frameworks illustrated in Figure 3.2. They are frameworks with rules $\mathcal{R}_1 = \{D^c \leftarrow A, \quad B \leftarrow A, \quad C \leftarrow B\}$, $\mathcal{R}_2 = \{A^c \leftarrow C, \quad A \leftarrow B\}$, and $\mathcal{R}_3 = \{C \leftarrow B, \quad C \leftarrow D\}$. The set of assumptions $\{A, B, C\}$ is closed in each framework. Then, aggregating with unanimity rule makes the aggregated rule $\mathcal{R}_{agg} = \{\}$ and thus, $\{A, B, C\}$ is closed as well. Using majority rule, $\mathcal{R}_{agg} = \{C \leftarrow B\}$ also has the closed set of assumptions $\{A, B, C\}$. With nomination rule, the aggregated rule is $\mathcal{R}_{agg} = \{D^c \leftarrow A, \quad B \leftarrow A, \quad C \leftarrow B, \quad A^c \leftarrow C, \quad A \leftarrow B, \quad C \leftarrow D\}$ and $\{A, B, C\}$ is also closed in this case. Finally, using oligarchic rules with veto power given to the frameworks with rules $\mathcal{R}_1$ and $\mathcal{R}_3$ yields the same result as majority rule, which accepts $\{A, B, C\}$ as closed.

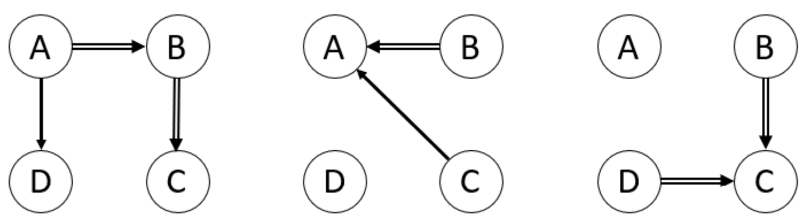

Figure 3.2: Example of Preservation of Closure

3.3 Admissibility

There are three properties for a set of assumptions to be admissible. It needs to be closed, conflict-free, and attack all closed assumptions that attack the set. The preservation results of admissibility depend on the number of assumptions in the frameworks. Thus, Theorem 3 is an extension of [**Theorem 3**, 8] for the number of assumptions $|\mathcal{A}| \geq 4$, while Theorem 4 is corner cases and a new result in the preservation of admissibility.

Theorem 3. *For $|\mathcal{A}| \geq 4$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, nomination rule is the only quota rule that preserves admissibility.*

Proof. Let a Bipolar ABA framework property P be the admissibility. Take a number of agents N and assume that an extension Δ is admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. To prove by contradiction, assume that Δ is not admissible in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$. In other words, there exists an assumption $\alpha \in \Delta$ that is attacked by another assumption $\beta \in \mathcal{A} \setminus \Delta$, i.e., $\alpha^c \leftarrow \beta \in \mathcal{R}_{agg}$, and there does not exist a rule $\beta^c \leftarrow \gamma \notin \mathcal{R}_{agg}$ denoting an attack from $\gamma \in \Delta$ to β. From the characteristic of nomination rule, the rule $\alpha^c \leftarrow \beta$ must exist in some agents' frameworks $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for $i \in N$. Moreover, as the rule $\beta^c \leftarrow \gamma$ does not exist in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, it means that by nomination rule there also does not exist in any of the agents' framework. Thus, Δ is not admissible in the agents' frameworks $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ as well. It contradicts the initial assumption that Δ is admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$.

To complete the proof, it needs to be showed that for $|\mathcal{A}| \geq 4$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, other quota rules except for nomination rule do not preserve admissibility. It can be proved by giving a counter example showing the case. Take a number of agents N and a number of Bipolar ABA frameworks illustrated in Figure 3.3. If $N - q$ agents choose rule $\mathcal{R} = \{\}$, $q - 1$ agents choose rule $\mathcal{R} = \{D^c \leftarrow B, \quad C^c \leftarrow D\}$, and 1 agent chooses rule $\mathcal{R} = \{D^c \leftarrow A, \quad C^c \leftarrow D, \quad A \leftarrow B\}$; the extension $\Delta = \{A, B, C\}$ is admissible in all frameworks. Using nomination rule with $q = 1$, Δ is still admissible with aggregated rules $\mathcal{R}_{agg} = \{D^c \leftarrow A, \quad C^c \leftarrow D, \quad A \leftarrow B\}$. However, using quota rules with $q > 1$, i.e., majority rule and unanimity rule, the aggregated rule is $\mathcal{R}_{agg} = \{C^c \leftarrow D\}$ and Δ is not admissible anymore as assumption C is attacked by D and it is undefended by some other assumptions in Δ. $\square$

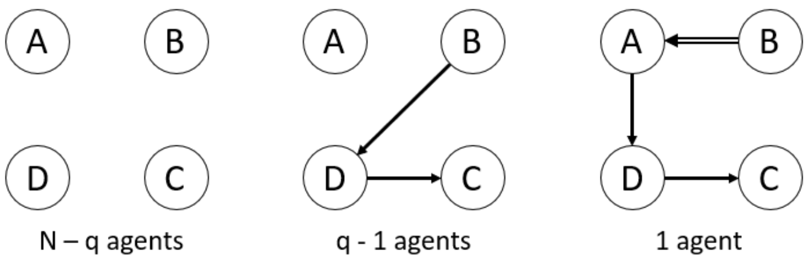

Figure 3.3: Counter Example of the Preservation of Admissibility

The presence of support in the Bipolar ABA framework does not affect the preservation of admissibility. The supports create closure of assumptions, such that if an assumption α supports another assumption β, then $\{\alpha\}$ is not closed but $\{\alpha, \beta\}$ is closed. Assume that $\Delta \supseteq \{\alpha, \beta\}$ is also admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{\ \ } \rangle$ for all $i \in N$, and thus, based on the reasoning above, using the nomination rule Δ is also admissible in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{\ \ } \rangle$ as well. It means there cannot exist an attack between assumptions α and β in any of the agent's framework and no assumptions defend it. Otherwise, Δ is closed but not admissible, which contradicts the initial assumption.

The corner cases for the preservation result of admissibility is taken for the number of assumptions less than or equal to three. Corner cases are not provided in [8]. Therefore, it is a new result on both Bipolar ABA framework in Theorem 4 and Abstract Argumentation framework in Corollary 1.

Theorem 4. *For $|\mathcal{A}| \leq 3$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, every quota rules and oligarchic rules preserve admissibility.*

Proof. Let a Bipolar ABA framework property P is admissibility. If $|\mathcal{A}| = 1$, the result holds vacuously as there does not exist any attack or support. If $|\mathcal{A}| = 2$, assume that $\mathcal{A} = \{\alpha, \beta\}$, then the only possible rules are $\alpha^c \leftarrow \beta$, $\beta^c, \leftarrow \alpha$, $\alpha \leftarrow \beta$, and $\beta \leftarrow \alpha$. If assume that $\{\alpha\}$ is admissible, then $\alpha^c \leftarrow \beta \notin \mathcal{R}_i$ for all $i \in N$. In this case, every quota rules and oligarchic rules yields $\{\alpha\}$ as an admissible extension in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{\ \ } \rangle$.

If $|\mathcal{A}| = 3$, assume that $\mathcal{A} = \{\alpha, \beta, \gamma\}$ and $\Delta = \{\alpha, \beta\}$ is admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{\ \ } \rangle$ for all $i \in N$. To prove by contradiction, assume that Δ is not admissible in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{\ \ } \rangle$. In other words, there is a rule $\alpha^c \leftarrow \beta$ or $\beta^c \leftarrow \alpha$ that violates the conflict-freeness requirement, or $\alpha^c \leftarrow \gamma$ or $\beta^c \leftarrow \gamma$ and there does not exist rule $\gamma^c \leftarrow \alpha$ and $\gamma^c \leftarrow \beta$ in $\mathcal{R}_{agg}$. By quota rules and oligarchic rules, there must be such conditions in some of the agents' frameworks $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{\ \ } \rangle$ for some $i \in N$, i.e, Δ is not admissible in those individual frameworks as well. It contradict the initial assumption that Δ is admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{\ \ } \rangle$ for all $i \in N$. □

Naturally, Theorem 4 can be generalised into the Abstract Argumentation framework. The reason is that the presence of supports does not change the admissibility when the number of assumptions are less or equal than 3. There is no support when $|\mathcal{A}| = 1$. If $|\mathcal{A}| = 2$, if one assumption supports the other, then it forms a closure that is also admissible. If $|\mathcal{A}| = 3$, taking the proof of Theorem 4 into account, assume that $\Delta = \{\alpha, \beta\}$ is admissible in all agents' frameworks and the aggregated framework, and there is a rule $\beta \leftarrow \alpha \in \mathcal{R}_{agg}$. By quota rules and oligarchic rules, then the rule $\beta \leftarrow \alpha$ must exist in some of the agents frameworks $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{\ \ } \rangle$ for some $i \in N$ and Δ is still admissible due to the assumption. Thus, supports do not affect the preservation of admissibility.

Corollary 1. *For $|Arg| \leq 3$, where Arg is the arguments in Abstract Argumentation framework, every quota rules and oligarchic rules preserve admissibility.*

3.4 Set-stable

A set of assumptions is set-stable in Bipolar ABA framework if it is closed, conflict-free, and it attacks all other closed assumptions not in the set. The result shows that the only quota rule that is able to preserve set-stable semantic is the nomination rule. It is an extension of [**Proposition 5**, 8]. Although set-stable semantic in Bipolar ABA framework and stable semantic in Abstract Argumentation framework have different definitions, but the presence of supports in Bipolar ABA does not affect the result and thus, can be extended.

Theorem 5. *The nomination rule is the only quota rule that preserves set-stable extension in Bipolar ABA frameworks.*

Proof. Let a Bipolar ABA framework property P be the set-stable. Take a number of agents N and assume an extension Δ is set-stable in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. Referring to both Theorem 1 and Theorem 2, it can be said that the nomination rule preserves closure and conflict-freeness properties. Therefore, Δ is both closed and conflict-free in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$. To be set-stable, Δ has to attack every closed assumptions not in the set, i.e., $\beta^c \leftarrow \alpha \in \mathcal{R}_{agg}$ for $\alpha \in \Delta$ and for all $\beta \in \mathcal{A} \setminus \Delta$. It is automatically true if $\Delta = \mathcal{A}$ as there does not exist any unattacked assumptions. For other cases, all closed assumptions $\beta \in \mathcal{A} \setminus \Delta$ need to be attacked by some assumptions $\alpha \in \Delta$. As Δ is set-stable in all agents' frameworks, then using nomination rule, it is true that if $\beta \in \mathcal{A} \setminus \Delta$ is attacked by some assumptions $\alpha \in \Delta$, i.e., $\beta^c \leftarrow \alpha \in \mathcal{R}_{agg}$, then there must exist the same rule in some of the agents' framework $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for some $i \in N$. Therefore, Δ is also set-stable in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$.

Other quota rules do not preserve the set-stable because although they preserve the conflict-freeness as in Theorem 1, because they do not guarantee for every closed set of assumptions not in the set-stable extension to be attacked. The counter example is depicted in Figure 3.4. Assume three Bipolar ABA frameworks with rules $\mathcal{R}_1 = \{D^c \leftarrow B, \quad B \leftarrow A\}$, $\mathcal{R}_2 = \{D^c \leftarrow C\}$, and $\mathcal{R}_3 = \{D^c \leftarrow A, \quad C^c \leftarrow D, \quad A \leftarrow B\}$. In each framework, the set of assumptions $\{A, B, C\}$ is set-stable. Using nomination rule, it preserves the set-stable $\{A, B, C\}$. However, using other quota rules with $q > 1$, the aggregated rule is $\mathcal{R}_{agg} = \{\}$, and $\{A, B, C\}$ is not set-stable anymore as the assumption D is not included in it and not attacked either. □

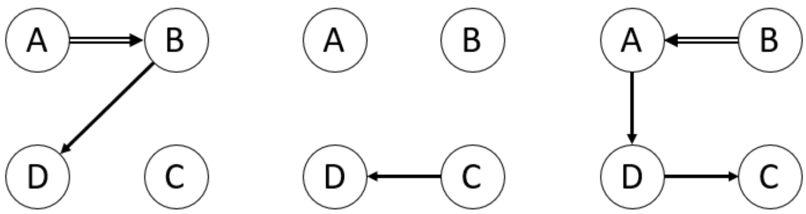

Figure 3.4: Counter Example of the Preservation of Set-stable

It is clear to see that the presence of supports does not affect the preservation of set-stable semantic because the closed set of assumptions as a form of support is always either included in the set or attacked by the set.

3.5 The acceptability of an assumption

The acceptability of an assumption concerns with the preferred, complete, set-stable, well-founded, or ideal semantics, which are *strong* semantics to get the final extension. If an assumption is acceptable in one of those semantics in each of the agents' framework, then the preservation study aims to find whether it is still acceptable under the same semantics in the aggregated Bipolar ABA framework. The presence of supports generally limits the preservation capabilities as the closed sets of assumptions might affect the calculation for the semantics.

Definition 3.5.1. (Acceptability). An assumption α is deemed acceptable under preferred, complete, set-stable, well-founded, or ideal semantics if and only if $\alpha \in \Delta$, where Δ is a preferred, complete, set-stable, well-founded, or ideal extension respectively on each agents' framework.

The proof for this preservation result and a few others use the adaptation of graph theorem from [12] on *implicative* and *disjunctive* properties to represent impossibility results using dictatorship. It basically says that it is impossible to use straightforward aggregation rules to be able to preserve the property P (in this specific case, it is the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics); unless some restrictions are put into place, in which dictatorship comes into play.

Remark. Proving with the concept of implicative and disjunctive shows an impossibility result as there does not exist a simple aggregation rule that preserves the property P except that aggregation rule must be dictatorship.

Definition 3.5.2. (Implicative). A Bipolar ABA property P is *implicative* in a Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$, if there exist three rules $R_1, R_2, R_3 \notin \mathcal{R}$, such that P holds in the aggregated framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$ for $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$, for all $\mathcal{S} \subseteq \{R_1, R_2, R_3\}$, except for $\mathcal{S} = \{R_1, R_2\}$; i.e., P holds if $\mathcal{S} = \{\}$, $\mathcal{S} = \{R_1\}$, $\mathcal{S} = \{R_2\}$, $\mathcal{S} = \{R_3\}$, $\mathcal{S} = \{R_1, R_3\}$, $\mathcal{S} = \{R_2, R_3\}$, and $\mathcal{S} = \{R_1, R_2, R_3\}$.

Remark. Thus, if the framework adopts R_1 and R_2 as the additional rules $\mathcal{S}$, then it should adopt R_3 as well to preserve the property P.

As an example, let a Bipolar ABA property P be the acceptability of an assumption under preferred semantic. Let $\mathcal{R} = \{C^c \leftarrow A, \quad D \leftarrow A\}$, $R_1 = \{B^c \leftarrow C\}$, $R_2 = \{A^c \leftarrow B\}$, and $R_3 = \{C^c \leftarrow D\}$. The assumption B is included in the preferred extension for any combinations of R_1, R_2 and R_3, except for $\mathcal{S} = \{R_1, R_2\}$. If $\mathcal{S} = \{R_1, R_2\}$, then the preferred extension is $\Delta = \{D\}$. Therefore, as there exist three rules $R_1, R_2, R_3 \notin \mathcal{R}$, such that P holds for all $\mathcal{S} \subseteq \{R_1, R_2, R_3\}$, except for $\mathcal{S} = \{R_1, R_2\}$, then P is implicative.

Definition 3.5.3. (Disjunctive). A Bipolar ABA property P is *disjunctive* in a Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$, if there exist two rules $R_1, R_2 \notin \mathcal{R}$, such that P holds in the aggregated framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$ for $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$, for all $\mathcal{S} \subseteq \{R_1, R_2\}$, except for $\mathcal{S} = \{\}$; i.e., P holds if $\mathcal{S} = \{R_1\}$, $\mathcal{S} = \{R_2\}$, and $\mathcal{S} = \{R_1, R_2\}$.

Remark. In other words, the framework has to adopt at least either R_1 or R_2 to preserve the property P.

As an example, let a Bipolar ABA property P be the acceptability of an assumption under preferred semantic. Let $\mathcal{R} = \{B^c \leftarrow A, \quad D \leftarrow C\}$, $R_1 = \{A^c \leftarrow C\}$, and $R_2 = \{A^c \leftarrow D\}$. The assumption B is included in the preferred extension for any combinations of R_1 and R_2, except for $\mathcal{S} = \{\}$. If $\mathcal{S} = \{\}$, then the preferred extension is $\Delta = \{A, C, D\}$. Therefore, as there exist two rules $R_1, R_2 \notin \mathcal{R}$, such that P holds for all $\mathcal{S} \subseteq \{R_1, R_2\}$, except for $\mathcal{S} = \{\}$, then P is disjunctive.

Lemma 1. *Let a Bipolar ABA framework property P be implicative. Then, unanimity rule preserves P.*

Proof. Take a number of agents $N \in \{1, \cdots, k\}$ proposing a number of Bipolar ABA frameworks $\langle \mathcal{L}, \mathcal{R}_1 \cup \mathcal{S}_1, \mathcal{A}, \overline{} \rangle, \cdots, \langle \mathcal{L}, \mathcal{R}_k \cup \mathcal{S}_k, \mathcal{A}, \overline{} \rangle$ and $\mathcal{S}_n \subseteq \{R_1, R_2, R_3\}$ for all $n \in N$. Let $\mathcal{S}_i \neq \{R_1, R_2\}$ for all $i \in N$, i.e., P holds in all agents' frameworks. Then, unanimity rule preserves P because it is impossible to get $\mathcal{S}_{agg} = \{R_1, R_2\}$. □

If P is implicative, nomination rule and majority rule do not preserve P as it is possible to get $\mathcal{S}_{agg} = \{R_1, R_2\}$. For example, let $\mathcal{S}_1 = \{R_1\}$, and $\mathcal{S}_2 = \{R_2\}$. Using nomination rule and majority rule, the aggregated rule is $\mathcal{S}_{agg} = \{R_1, R_2\}$; hence, P is not preserved.

Lemma 2. *Let a Bipolar ABA framework property P be implicative and disjunctive. Then, the aggregation rule that preserves P must be dictatorial.*

Proof. The proof for the implicativeness can be found in Lemma 1. For the disjunctiveness, take a number of agents $N \in \{1, \cdots, k\}$ proposing a number of Bipolar ABA frameworks $\langle \mathcal{L}, \mathcal{R}_1 \cup \mathcal{S}_1, \mathcal{A}, \overline{} \rangle, \cdots, \langle \mathcal{L}, \mathcal{R}_k \cup \mathcal{S}_k, \mathcal{A}, \overline{} \rangle$ and $\mathcal{S}_n \subseteq \{R_1, R_2\}$ for all $n \in N$. Let $\mathcal{S}_i \neq \{\}$ for all $i \in N$, i.e., P holds in all agents' frameworks. Then, nomination rule or majority rule preserves P because it is impossible to get $\mathcal{S}_{agg} = \{\}$.

As P is implicative and disjunctive, to preserve P the aggregation rule must be dictatorial instead. From the implicativeness, unanimity rule preserves P and from the disjuntiveness, nomination rule or majority rule preserves P. However, as P is implicative and disjunctive, then none of the quota rules preserve P because using nomination or majority rule, it is possible to get $\mathcal{S}_{agg} = \{R_1, R_2\}$ and violating the implicativeness; and using unanimity rule, it is possible to get $\mathcal{S}_{agg} = \{\}$ and violating the disjunctiveness. Thus, to preserve P, the aggregation rule must be dictatorship to avoid such cases with $\mathcal{S}_{agg} = \{\}$ or $\mathcal{S}_{agg} = \{R_1, R_2\}$. □

If P is disjunctive, unanimity rule does not preserve P as it is possible to get $\mathcal{S}_{agg} = \{\}$. For example, let $\mathcal{S}_1 = \{R_1\}$ and $\mathcal{S}_2 = \{R_2\}$. Using unanimity rule, the aggregated rule is $\mathcal{S}_{agg} = \{\}$; hence, P is not preserved.

Remark. Both implicative and disjunctive are necessary conditions such that the aggregation rule that preserves P must be dictatorial.

Notice that the rules R_1, R_2, and R_3 can only be in the form of $\alpha^c \leftarrow \beta$ for any $\alpha, \beta \in \mathcal{A}$ that denote attacks between assumptions. They cannot be in the form of $\alpha \leftarrow \beta$ that denote supports between assumptions because then some agents have a different set of closures of assumptions from the other agents. As a consequence, some agents' frameworks may satisfy P, while some other frameworks do not satisfy P because of the closures. In this case, the concept of implicative and disjunctive fails.

The preservation result of the acceptability of an assumption in Theorem 6 is an extension from [**Theorem 1**, 8]. The preservation result is true for all five semantics: preferred, complete, set-stable, well-founded, or ideal.

Theorem 6. *For $|\mathcal{A}| \geq 4$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, the aggregation rule that preserves the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics must be dictatorial.*

Proof. Let a Bipolar ABA framework property P be the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics. It needs to be proven that for $|\mathcal{A}| \geq 4$, P is implicative and disjunctive. The proof has the same structure for each of the five semantics. Take Bipolar ABA frameworks with at least four assumptions $\mathcal{A} = \{A, B, C, D, \cdots\}$.

To show that P is implicative, let assumption B be the accepted assumption to be checked. Let $\mathcal{R} = \{C^c \leftarrow A, \quad D \leftarrow A\}$, $R_1 = \{B^c \leftarrow C\}$, $R_2 = \{A^c \leftarrow B\}$, and $R_3 = \{C^c \leftarrow D\}$; illustrated in the left graph of Figure 3.5. Consider aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, where $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$ with $\mathcal{S} \subseteq \{R_1, R_2, R_3\}$. If $\mathcal{S} = \{\}$, $\{R_2\}$, $\{R_3\}$, or $\{R_2, R_3\}$; the assumption B is unattacked. If $\mathcal{S} = \{R_1\}$, $\{R_1, R_3\}$, or $\{R_1, R_2, R_3\}$; the assumption B is defended by other assumption. Therefore, assumption B is either unattacked or defended in all seven cases, i.e., B is acceptable under preferred, complete, set-stable, well-founded, and ideal semantics. However, if $\mathcal{S} = \{R_1, R_2\}$, the set of assumptions $\{A, B, C\}$ forms cyclic attacks so that the assumption A, B, and C is not acceptable under preferred, complete, set-stable, well-founded, and ideal semantics. Thus, there exists a set of rules $\mathcal{R}$ and three rules R_1, R_2, R_3 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, \overline{} \rangle$ if and only if $\mathcal{S} \neq \{R_1, R_2\}$. Accordingly, P is an implicative Bipolar ABA framework property.

To show that P is disjunctive, let assumption B be the accepted assumption to be checked. Let $\mathcal{R} = \{B^c \leftarrow A, \quad D \leftarrow C\}$, $R_1 = \{A^c \leftarrow C\}$, and $R_2 = \{A^c \leftarrow D\}$; illustrated in the right graph of Figure 3.5. Consider aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, where $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$ with $\mathcal{S} \subseteq \{R_1, R_2\}$. If $\mathcal{S} = \{R_1\}$, $\{R_2\}$, or $\{R_1, R_2\}$; the assumption B is defended. Therefore, B is acceptable under the five semantics. However, if $\mathcal{S} = \{\}$, the assumption B is attacked by A and is not defended, i.e., B is unacceptable under preferred, complete, set-stable, well-founded, and ideal semantics. Thus, there exists a set of rules $\mathcal{R}$ and two rules R_1, R_2 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, \overline{} \rangle$ if and only if $\mathcal{S} \neq \{\}$. Therefore, P is a disjunctive Bipolar ABA property.

As P is proven to be both implicative and disjunctive, then by Lemma 2, the aggregation rule that preserves P must be dictatorial. $\square$

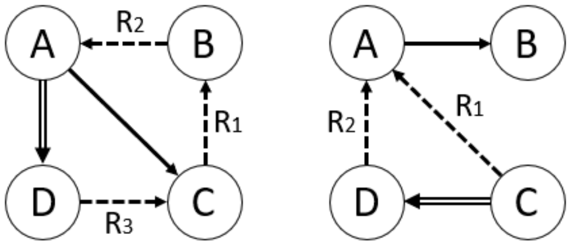

Figure 3.5: Acceptability of an Assumption: Implicative (Left) and Disjunctive (Right)

If it is restricted to not having any supports in the framework, then Theorem 6 is the same as [**Theorem 1**, 8]. The corner cases for the preservation of the acceptability of an assumption are not provided in [8]. Thus, Theorem 7 and Theorem 8 give the preservation result of the acceptability of an assumption if the number of assumptions $|\mathcal{A}| \leq 3$. Then, it is easy to see that the theorems can be generalised into Abstract Argumentation framework as well.

Theorem 7. *For $|\mathcal{A}| \leq 2$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, every quota rules and oligarchic rules preserve the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics.*

Proof. If $|\mathcal{A}| = 1$, the result holds vacuously as $\alpha \in \mathcal{A}$ must be accepted in all five semantics. If $|\mathcal{A}| = 2$ and $\alpha \in \mathcal{A}$ is accepted under those semantics, then α must be accepted in the aggregated framework as there cannot exist a rule $\alpha^c \leftarrow \beta$ in $\mathcal{R}_{agg}$. By the characteristic of quota rules and oligarchic rules, then that rule must not exist in the agents' framework as well. Thus, α is accepted in all agents' framework. □

Theorem 8. *For $|\mathcal{A}| = 3$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, majority rule, unanimity rule, and oligarchic rules preserve assumption acceptability under preferred, complete, set-stable, well-founded, or ideal semantics.*

Proof. Let a Bipolar ABA framework property P be the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics. Take a number of agents N and P holds in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$, where $\mathcal{A} = \{\alpha, \beta, \gamma\}$ and assume that α is acceptable under preferred, complete, set-stable, well-founded, and ideal semantics in all frameworks.

To prove by contradiction, assume P does not hold in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$. In other words, there is a rule $\alpha^c \leftarrow \beta$ that denotes an attack from assumption β to α, but there does not exist a rule $\beta^c \leftarrow \gamma$ that defends α. As a result, α is not acceptable under preferred, complete, set-stable, well-founded, and ideal semantics. From the characteristic of majority rule, unanimity rule, and oligarchic rules; the rule $\alpha^c \leftarrow \beta$ must exist in the majority (majority rule), all (unanimity rule), or veto powered (oligarchic rules) individual agents' frameworks, but there is at least one framework $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for some $i \in N$ without the rule $\beta^c \leftarrow \gamma$, i.e., α is not acceptable under preferred, complete, set-stable, well-founded, and ideal semantics in the agents' frameworks as well. It contradicts the initial assumption that α is acceptable in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$.

To show that for $|\mathcal{A}| = 3$, nomination rule do not preserve the assumption acceptability under preferred, complete, set-stable, well-founded, or ideal semantics, a counter example is given. Take three Bipolar ABA frameworks with rules $\mathcal{R}_1 = \{A^c \leftarrow C, \quad B^c \leftarrow C\}$, $\mathcal{R}_2 = \{B^c \leftarrow A, \quad C^c \leftarrow B\}$, and $\mathcal{R}_3 = \{C^c \leftarrow A, \quad A^c \leftarrow B\}$ as illustrated in Figure 3.6. Let assumption C be the accepted assumption in check. From the first framework, a set of assumptions $\{C\}$ is preferred, complete, set-stable, well-founded, and ideal. On the second framework is $\{A, C\}$ and third framework is $\{B, C\}$, both extensions are preferred, complete, set-stable, well-founded, and ideal as well. In all three frameworks, the assumption C is acceptable. It is still acceptable using unanimity rule as the aggregated rule is $\mathcal{R} = \{\}$. However, using nomination rule the preferred, complete, and set-stable extensions are $\{A\}, \{B\}$, and $\{C\}$; while the well-founded and ideal extensions are $\{\}$. Hence, the assumption $\{C\}$ is not acceptable in those five semantics. □

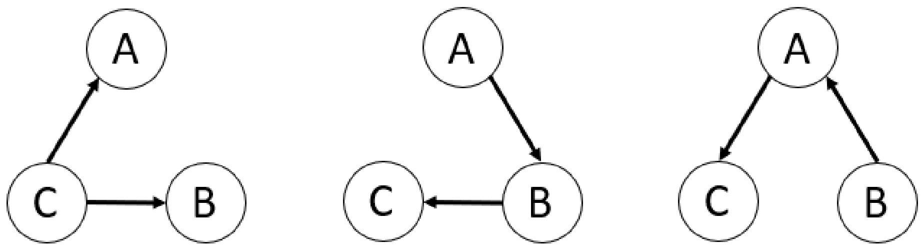

Figure 3.6: Counter Example of the Acceptability of an Assumption For $|\mathcal{A}| = 3$

To generalise Theorem 7 and Theorem 8 for Abstract Argumentation framework, it is easy to see that the presence of supports does not affect the preservation of the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics. There is no support if $|\mathcal{A}| = 1$. If $|\mathcal{A}| = 2$, support between assumption does not affect the acceptability of an assumption as the formed closure then must be accepted as well. If $|\mathcal{A}| = 3$, the proof on Theorem 8 has no support in the rules $\mathcal{R}_{agg}$ and can be directly translated into Abstract Argumentation framework equivalent.

Corollary 2. *For $|Arg| \leq 2$, where Arg is the arguments in Abstract Argumentation framework, every quota rules and oligarchic rules preserve the acceptability of an argument under preferred, complete, stable, grounded, or ideal semantics. For $|Arg| = 3$, majority rule, unanimity rule, and oligarchic rule preserves argument acceptability under preferred, complete, stable, grounded, or ideal semantics.*

3.6 Preferred, Complete, Well-founded, and Ideal

A set of assumptions is preferred if it is maximally admissible with respect to the subset of assumptions. A set of assumptions is complete if it is admissible and all assumptions it defends are included in it. A set of assumptions is well-founded if it is the intersection of all complete extensions. A set of assumptions is ideal if it is maximally admissible with respect to the subset of assumptions and it is contained in all preferred extensions. The proof for the preservation of preferred, complete, well-founded, and ideal semantics uses the concept of implicativeness and disjunctiveness as explained in Section 3.5. The preservation result is an extension of [**Theorem 4**, 8] with an addition of ideal semantic, which is not included in it.

Theorem 9. *For $|\mathcal{A}| \geq 5$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, the aggregation rule that preserves preferred, complete, well-founded, and ideal semantics must be dictatorial.*

Proof. Let a Bipolar ABA framework property P be the preferred, complete, well-founded, and ideal semantics. It needs to be proven that for $|\mathcal{A}| \geq 5$, P is implicative and disjunctive. The proof has the same structure for each of the four semantics. Take Bipolar ABA frameworks with at least five assumptions $\mathcal{A} = \{A, B, C, D, E, \cdots\}$.

To show that P is implicative, let $\Delta = \{B, D, E\}$ be the extension to be checked. Let $\mathcal{R} = \{C^c \leftarrow D, \quad A^c \leftarrow B, \quad E \leftarrow D\}$, $R_1 = \{B^c \leftarrow C\}$, $R_2 = \{D^c \leftarrow A\}$, and $R_3 = \{A^c \leftarrow E\}$; as illustrated in the left graph of Figure 3.7. Consider aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, where $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$ with $\mathcal{S} \subseteq \{R_1, R_2, R_3\}$. If $\mathcal{S} = \{\}, \{R_1\}, \{R_2\}, \{R_3\}, \{R_1, R_3\}, \{R_2, R_3\}$, or $\{R_1, R_2, R_3\}$, then Δ is preferred, complete, well-founded, and ideal. However, if $\mathcal{S} = \{R_1, R_2\}$, the set of assumptions $\{A, B, C, D\}$ forms cyclic attacks such that Δ is not preferred, complete, well-founded, or ideal. Thus, there exists a set of rules $\mathcal{R}$ and three rules R_1, R_2, R_3 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, \overline{} \rangle$ if and only if $\mathcal{S} \neq \{R_1, R_2\}$. Hence, P is an implicative Bipolar ABA framework property.

To show that P is disjunctive, let $\Delta = \{B, D, E\}$ be the extension to be checked. Let $\mathcal{R} = \{C^c \leftarrow D, \quad B^c \leftarrow C, \quad A^c \leftarrow B, \quad D^c \leftarrow A, \quad D \leftarrow E\}$, $R_1 = \{C^c \leftarrow E\}$, and $R_2 = \{A^c \leftarrow E\}$; illustrated in the right graph of Figure 3.7. Consider aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, where $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$ with $\mathcal{S} \subseteq \{R_1, R_2\}$. If $\mathcal{S} = \{R_1\}, \{R_2\}$, or $\{R_1, R_2\}$, then Δ is preferred, complete, well-founded, and ideal. However, if $\mathcal{S} = \{\}$, the set of assumptions $\{A, B, C, D\}$ forms cyclic attacks such that Δ is not preferred, complete, well-founded, or ideal. Therefore, there exists a set of rules $\mathcal{R}$ and two rules R_1, R_2 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, \overline{} \rangle$ if and only if $\mathcal{S} \neq \{\}$. Thus, P is a disjunctive Bipolar ABA property.

As P is proven to be both implicative and disjunctive, then by Lemma 2, the aggregation rule that preserves P must be dictatorial. □

Although preferred and complete semantics may accept multiple extensions, as long as all agents agree on the extensions, then Theorem 9 still holds. The only restriction is the presence of supports in the frameworks. If all agents agree on the supports, i.e., the supports are included in $\mathcal{R}$, then it does not affect the preservation. However, if supports exist in the additional rules $\mathcal{S}$, then some agents will have different sets of closed assumptions from the other agents. This may lead into different calculation for the accepted assumption under preferred, complete, well-founded, and ideal semantics.

To show that quota rules do not preserve preferred, complete, well-founded, and ideal semantics for $|\mathcal{A}| \geq 5$, a counter example is given. Assume three Bipolar ABA

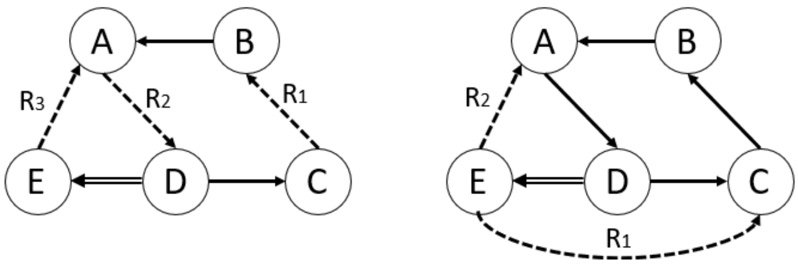

Figure 3.7: Preferred, Complete, Well-founded, and Ideal: Implicative (Left) and Disjunctive (Right)

frameworks with rules $\mathcal{R}_1 = \{A^c \leftarrow D, \quad D^c \leftarrow B, \quad C^c \leftarrow D\}$, $\mathcal{R}_2 = \{A^c \leftarrow D, \quad B^c \leftarrow D, \quad D^c \leftarrow C\}$, and $\mathcal{R}_3 = \{D^c \leftarrow A\}$. It is depicted in Figure 3.8. The set of assumptions $\Delta = \{A, B, C, E\}$ is preferred, complete, well-founded, and ideal on each framework. However, using unanimity rule ($q = 3$), the set of assumption $\{A, B, C, D, E\}$ is preferred, complete, well-founded, and ideal instead as the aggregated rule $\mathcal{R}_{agg} = \{\}$. With majority rule ($q = 2$), the preferred, complete, well-founded, and ideal extensions are $\{B, C, D, E\}$ with aggregated rule $\mathcal{R}_{agg} = \{A^c \leftarrow D\}$. Lastly, using nomination rule gives $\{A, B, C, E\}$ and $\{D, E\}$ as the preferred extensions, $\{A, B, C, E\}$, $\{D, E\}$, and $\{E\}$ as the complete extensions, and $\{E\}$ as the well-founded and ideal extensions. Therefore, as Δ is not preserved, quota rules do not preserve preferred, complete, well-founded, or ideal semantics for $|\mathcal{A}| \geq 5$.

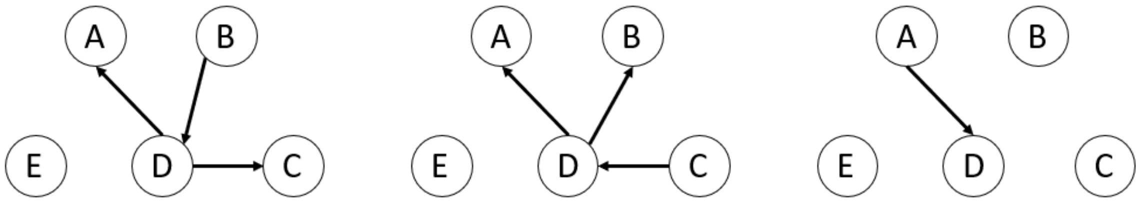

Figure 3.8: Counter Example of the Preservation of Preferred, Complete, Well-founded, and Ideal Semantics for $|\mathcal{A}| \geq 5$

The corner cases interestingly show an impossibility result for $|\mathcal{A}| = 3$ and $|\mathcal{A}| = 4$. It shows that preserving the whole extension is a more difficult task than preserving the acceptability of only one assumption as in Section 3.5. The other corner cases, for $|\mathcal{A}| \leq 2$, show a positive result as the calculation to get the extensions is more limited.

Theorem 10. *For $|\mathcal{A}| \leq 2$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, every quota rules and oligarchic rules preserve preferred, complete, well-founded, and ideal semantics.*

Proof. If $|\mathcal{A}| = 1$, the result holds vacuously as the only extension is $\Delta = \{\alpha\}$ for $\alpha \in \mathcal{A}$. If $|\mathcal{A}| = 2$, if $\Delta = \{\alpha\}$ is preferred, complete, well-founded, or ideal in all the agents' frameworks; then they all must have rule $\mathcal{R}_i = \{\beta \leftarrow \alpha\}$ for $i \in N$. There does not exist any other alternatives as there are only two assumptions. It is the same case if $\Delta = \{\beta\}$ and $\Delta = \{\alpha, \beta\}$. Thus, any quota rules and oligarchic rules preserve Δ as the agents' rules are all the same. □

Theorem 11. *For $|\mathcal{A}| = 3$ and $|\mathcal{A}| = 4$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, quota rules and oligarchic rules do not preserve preferred, complete, well-founded, and ideal semantics.*

Proof. Let a Bipolar ABA framework property P be the preferred, complete, well-founded, and ideal semantics. To show that they are not preserved by quota rules, counter examples are given. For $|\mathcal{A}| = 3$, assume four Bipolar ABA frameworks with two frameworks having rules $\mathcal{R}_{1,2} = \{B^c \leftarrow A, \quad C^c \leftarrow B\}$ and the other two frameworks having rules $\mathcal{R}_{3,4} = \{A^c \leftarrow B, \quad B^c \leftarrow C\}$. It is depicted in Figure 3.9. The set of assumptions $\Delta = \{A, C\}$ is preferred, complete, well-founded, and ideal on each framework. However, using the unanimity rule ($q = 4$) or oligarchic rule with veto powers given to all frameworks, the preferred, complete, well-founded, and ideal extensions are $\{A, B, C\}$ as the aggregated rule is empty $\mathcal{R}_{agg} = \{\}$. With majority rule or nomination rule, the extension $\{A, C\}$ and $\{B\}$ is preferred, $\{A, C\}$, $\{B\}$, and $\{\}$ is complete, and the empty set $\{\}$ is well-founded and ideal with aggregated rule $\mathcal{R}_{agg} = \{B^c \leftarrow A, \quad C^c \leftarrow B, \quad A^c \leftarrow B, \quad B^c \leftarrow C\}$. Thus, Δ is not preserved using quota rules and oligarchic rules, i.e., P is not preserved.

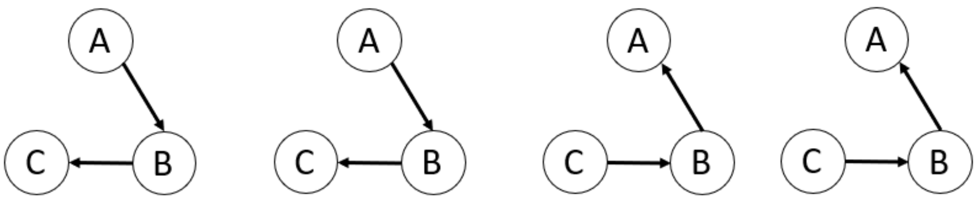

Figure 3.9: Counter Example of the Preservation of Preferred, Complete, Well-founded, and Ideal Semantics for $|\mathcal{A}| = 3$

Another counter example for $|\mathcal{A}| = 4$ follows the same reasoning as above and is depicted in Figure 3.10. Assume three Bipolar ABA frameworks with rules $\mathcal{R}_1 = \{A^c \leftarrow D, \quad D^c \leftarrow B, \quad C^c \leftarrow D\}$, $\mathcal{R}_2 = \{A^c \leftarrow D, \quad B^c \leftarrow D, \quad D^c \leftarrow C\}$, and $\mathcal{R}_3 = \{D \leftarrow A\}$. The set of assumptions $\Delta = \{A, B, C\}$ is preferred, complete, well-founded, and ideal on each framework. However, using unanimity rule ($q = 3$) or oligarchic rules with veto powers given to all frameworks, the preferred, complete, well-founded, and ideal extensions is $\{A, B, C, D\}$ as the aggregated rule is empty $\mathcal{R}_{agg} = \{\}$. With majority rule ($q = 2$), the extension $\{B, C, D\}$ is preferred, complete, well-founded, and ideal with aggregated rule $\mathcal{R}_{agg} = \{D^c \leftarrow A\}$. Lastly, using nomination rule gives $\{A, B, C\}$ and $\{D\}$ as the preferred extensions, $\{A, B, C\}$, $\{D\}$, and $\{\}$ as the complete extensions, and the empty set $\{\}$ as the well-founded and ideal extension. Thus, Δ is not preserved using quota rules and oligarchic rules, i.e., P is not preserved. □

Theorem 10 and Theorem 11 can also be generalised into the Abstract Argumentation framework. If $|Arg| \leq 2$, it is easy to see that every quota rules and oligarchic rules preserve preferred, complete, grounded, and ideal semantics. If $|Arg| = 3$ and

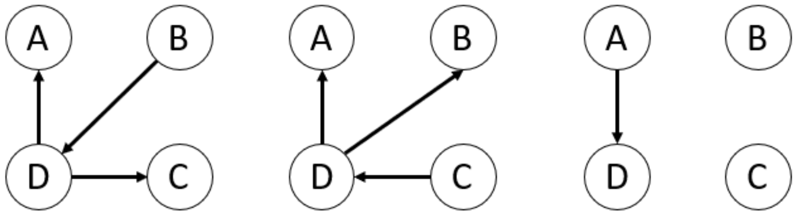

Figure 3.10: Counter Example of the Preservation of Preferred, Complete, Well-founded, and Ideal Semantics for $|\mathcal{A}| = 4$

$|Arg| = 4$, Figure 3.9 and Figure 3.10 also serve as the counter example for Abstract Argumentation framework.

Corollary 3. *For $|Arg| \leq 2$, where Arg is the arguments in Abstract Argumentation framework, every quota rules and oligarchic rules preserve preferred, complete, grounded, and ideal semantics. For $|Arg| = 3$ and $|Arg| = 4$, quota rules and oligarchic rules do not preserve preferred, complete, grounded, and ideal semantics.*

3.7 The Non-emptiness of Well-founded Extension

The well-founded extension is guaranteed to exist in a Bipolar ABA framework. However, to make sure that the well-founded extension is not empty, then the framework must have at least one unattacked assumption. In this way, the unattacked assumption must be included in all complete extensions, and the intersection always has the unattacked assumption in it. Thus, the preservation of non-emptiness of the well-founded extension deals with the existence of unattacked assumption. This is a concept called *k-exclusive* [8].

Definition 3.7.1. (k-exclusive). Let P be a property of Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$. P is *k-exclusive* if there exists rules $\mathcal{S} = \{R_1, \cdots, R_k\}$ such that if $\mathcal{R} \supseteq \mathcal{S}$ then P does not hold, but if $\mathcal{R} \subset \mathcal{S}$ then P holds.

Thus, to preserve P, the rules $\mathcal{S}$ cannot be adopted together, but only a subset of them.

Lemma 3. *Let P be a k-exclusive property of Bipolar ABA framework. For $k \geq N$, where N is the number of agents, P is preserved if at least one agent must have veto power.*

Proof. It needs to be showed that if an aggregation rule preserves P, then it has to give at least one agent with veto powers. Notice that if all agents accept a rule R, then it must be accepted in the aggregated rules, i.e., $R \in \mathcal{R}_{agg}$ if and only if $R \in \mathcal{R}_i$ for all $i \in N$.

For some agents $i \in N$ to have veto powers means that $\mathcal{R}_{agg} = (\bigcap \mathcal{R}_i)$. In other words, some agents have veto power, if the intersection of the agents' rules in $\bigcap \mathcal{R}_i$ are all accepted in $\mathcal{R}_{agg}$. Then, take any rule $R \in \mathcal{R}_{agg}$; as R is accepted in the aggregated framework, then all agents with veto powers must accept R as well such that the intersection of the set of rules $\bigcap \mathcal{R}_i$ is not empty.

Thus, the next step is to show that if an aggregation rule preserves P, then the intersection of k set of rules must be non-empty, i.e., $\mathcal{R}_1 \cap \cdots \cap \mathcal{R}_k \neq \{\}$. To prove by contradiction, assume there exist a profile of set of rules $\{\mathcal{R}_1 \cup \cdots \cup \mathcal{R}_k\} \subseteq \mathcal{R}_{agg}$ such that $\mathcal{R}_1 \cap \cdots \cap \mathcal{R}_k = \{\}$. Then, it means that for every $j \in \{1, \cdots, k\}$, exactly $\mathcal{R}_j$ accept a rule R_j. As no rule exist in all $\mathcal{R}_i$ for $i \in N$, no agents accept all k rules. However, as each of the k rules is accepted by an agent and $\{\mathcal{R}_1 \cup \cdots \cup \mathcal{R}_k\} \subseteq \mathcal{R}_{agg}$, they are all accepted in the aggregated framework, i.e., $\{R_1, \cdots, R_k\} \subseteq \mathcal{R}_{agg}$, such that P does not hold due to it being an *k-exclusive* property. This contradicts the initial assumption that the aggregation rule preserves P.

Therefore, as it can be showed that the intersection of the agents' rules is not empty, then some agents must have veto powers. □

To use the concept of *k-exclusive* in the preservation of the non-emptiness of well-founded extension, the number of assumptions $|\mathcal{A}|$ is used to represent k. The result is an extension of [**Theorem 7**, 8].

Theorem 12. *For $|\mathcal{A}| \geq N$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework and N is the number of agents, at least one agent must have veto power to preserve the non-emptiness of the well-founded extension.*

Proof. Let a Bipolar ABA framework property P be the non-emptiness of the well-founded extension. It needs to be showed that the P is a *k-exclusive*. Let $k = |\mathcal{A}|$ and $\{A_1, \cdots, A_k\} \in \mathcal{A}$. Assume that the rules are $\mathcal{S} = \{A^c_{i+1} \leftarrow A_i$ for $i < |\mathcal{A}|$ and $A^c_1 \leftarrow A_k\}$. $\mathcal{S}$ forms attacks between the assumptions, illustrated in Figure 3.11, and fits the definition of *k-exclusive*. If $\mathcal{S} \subseteq \mathcal{R}$, then in the case of $\mathcal{S} = \mathcal{R}$, the well-founded extension is empty due to the cyclic attacks. However, if only a subset of it is adopted $\mathcal{R} \subset \mathcal{S}$, the well-founded extension is not empty as at least one assumption is not attacked. Therefore, P is preserved when at least one agent has veto power to prevent each assumptions to be attacked. □

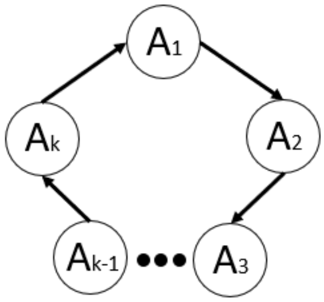

Figure 3.11: Graphical illustration of *k-exclusive* property

The supports in Bipolar ABA framework do not affect the preservation of the non-emptiness of well-founded extension. It is because supports between assumptions do not change the unattacked assumption. If $\exists(\alpha^c \leftarrow \beta)$ for $\alpha, \beta \in \mathcal{A}$ and β

is unattacked, then supports from and into β do not change the fact that β is unattacked; and supports from and into α also leave β to remain unattacked.

3.7.1 Acyclicity

It is clear that the *k-exclusive* deals with cyclic attacks. A Bipolar ABA framework is acyclic if there does not exist any cyclic attacks among the assumptions. It is extended from [**Theorem 8**, 8] in the way that supports are also considered.

Definition 3.7.2. (Cyclic). The rules $\mathcal{R}$ in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ contain cyclic attacks if there exist a chained connection between the assumptions such that $\mathcal{R} = \{\alpha_1^c \leftarrow \alpha_2, \alpha_2^c \leftarrow \alpha_3, \cdots, \alpha_k^c \leftarrow \alpha_1\}$ for $\alpha_i \in \mathcal{A}$ and they always maintain the cycle without being able to break it.

The preservation result of the acyclicity has similar proof structure as the preservation of the non-emptiness of the well-founded semantic in Theorem 12. Thus, it is presented as a corollary.

Corollary 4. *For $|\mathcal{A}| \geq N$ and $|\mathcal{A}| \geq 2$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework and N is the number of agents, at least one agent must have veto power to preserve acyclicity.*

Proof. Let a Bipolar ABA property P be the acyclicity. It needs to be showed that P is *k-exclusive*. To get a proper cycle, a minimum number of two assumptions are required. Thus, let $k = |\mathcal{A}|$ with $|\mathcal{A}| \geq 2$ and $\{A_1, \cdots, A_k\} \in \mathcal{A}$. Assume that the rules are $\mathcal{S} = \{A_{i+1}^c \leftarrow A_i$ for $i < |\mathcal{A}|$ and $A_1^c \leftarrow A_k\}$. $\mathcal{S}$ forms cyclic attacks between the assumptions as illustrated in Figure 3.11 and fits the definition of *k-exclusive*. If $\mathcal{S} \subseteq \mathcal{R}$, then in the case of $\mathcal{S} = \mathcal{R}$, the cyclic attacks remain in the framework. However, if only a subset of it is adopted $\mathcal{R} \subset \mathcal{S}$, the cyclic attacks are broken because one rule that connects the cycle disappear. Therefore, P is preserved when at least one agent must have veto power. □

The presence of supports does not make an acyclic framework to be cyclic, but instead breaks the existing cycle. Let $k = |\mathcal{A}|$ with $|\mathcal{A}| \geq 2$, $\{A_1, \cdots, A_k\} \in \mathcal{A}$, and the rules are $\mathcal{S} = \{A_{i+1}^c \leftarrow A_i$ for $i < |\mathcal{A}|\}$. The rules are acyclic and if a support $A_1 \leftarrow A_k$ or $A_k \leftarrow A_1$ is added, then they will remain acyclic. On the contrary, if there exists cyclic attacks, then by replacing one rule into support breaks the cycle due to the newly formed closure of assumptions.

3.8 Coherence

Coherence happens when two or more semantics coincide. In other words, given a Bipolar ABA framework, two or more semantics have identical extensions. For example, if a set of assumptions is set-stable, then it is preferred as well. The preservation result is extended from [**Theorem 9**, 8] and shows that to preserve coherence, the aggregation rule must be dictatorial. The proof for the result uses the concept of implicativeness and disjunctiveness explained in Section 3.5.

Theorem 13. *For $|\mathcal{A}| \geq 4$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, the aggregation rule that preserves coherence must be dictatorial.*

Proof. Let a Bipolar ABA framework property P be the coherence. It needs to be proven that for $|\mathcal{A}| \geq 4$, P is implicative and disjunctive property of the Bipolar ABA framework. Take Bipolar ABA frameworks with at least four assumptions $\mathcal{A} = \{A, B, C, D, \cdots\}$.

To show that P is implicative, let $\mathcal{R} = \{C^c \leftarrow A, \quad D \leftarrow A\}$, $R_1 = \{B^c \leftarrow C\}$, $R_2 = \{A^c \leftarrow B\}$, and $R_3 = \{C^c \leftarrow D\}$; as illustrated in the left graph of Figure 3.12. Consider aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, where $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$ with $\mathcal{S} \subseteq \{R_1, R_2, R_3\}$. If $\mathcal{S} = \{\}$, $\{R_1\}$, $\{R_3\}$, or $\{R_1, R_3\}$; the only preferred extension is $\{A, B, D\}$, which is stable as well. If $\mathcal{S} = \{R_2\}$ the set of assumptions $\{B, C, D\}$ is both preferred and stable. If $\mathcal{S} = \{R_2, R_3\}$ or $\{R_1, R_2, R_3\}$; then the set of assumptions $\{B, D\}$ is both preferred and stable as well. However, if $\mathcal{S} = \{R_1, R_2\}$, the only preferred extension is $\{D\}$ and it is not stable as the other assumptions are not attacked. Thus, there exists a set of rules $\mathcal{R}$ and three rules R_1, R_2, R_3 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, \overline{} \rangle$ if and only of $\mathcal{S} \neq \{R_1, R_2\}$. Accordingly, P is an implicative Bipolar ABA framework property.

To show that P is disjunctive, let $\mathcal{R} = \{A^c \leftarrow D, \quad B^c \leftarrow A, \quad D^c \leftarrow B, \quad C \leftarrow A\}$, $R_1 = \{D^c \leftarrow C\}$, and $R_2 = \{B^c \leftarrow C\}$; as illustrated in the right graph of Figure 3.12. Consider aggregated Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$, where $\mathcal{R}_{agg} = \mathcal{R} \cup \mathcal{S}$ with $\mathcal{S} \subseteq \{R_1, R_2\}$. If $\mathcal{S} = \{R_1\}$ or $\{R_1, R_2\}$; the set of assumptions $\{A, C\}$ is both preferred and set-stable. If $\mathcal{S} = \{R_2\}$, the set of assumptions $\{C, D\}$ is also preferred and set-stable. However, if $\mathcal{S} = \{\}$, the preferred extension is $\{C\}$ and it is not set-stable because the other assumptions are not attacked. Therefore, there exists a set of rules $\mathcal{R}$ and two rules R_1, R_2 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, \overline{} \rangle$ if and only if $\mathcal{S} \neq \{\}$. Hence, P is a disjunctive Bipolar ABA property.

As P is proven to be both implicative and disjunctive, then by Lemma 2, the aggregation rule that preserves P must be dictatorial. □

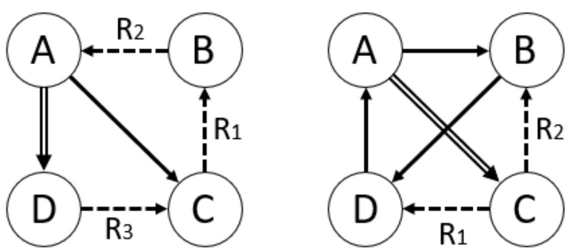

Figure 3.12: Coherence: Implicative (Left) and Disjunctive (Right)

Theorem 13 also works for other semantics. Actually in the proof, all the accepted sets of assumptions are not only preferred and set-stable, but also complete and well-founded. The presence of supports is acceptable in the preservation of coherence only if the supports are adopted by each agent, i.e., supports can only exist in $\mathcal{R}$, such that all agents have the same closure of assumptions. If supports join the additional rules in $\mathcal{S}$ as either $R_1, R_2,$ or R_3, then coherence is not preserved in the aggregated framework as some agents have different set of closures from the other agents.

The corner cases show more positive results with unanimity rule being the rule that preserves coherence.

Theorem 14. *For $|\mathcal{A}| = 1$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, every quota rules and oligarchic rules preserve coherence.*

Proof. If there is only an assumption α for $\alpha \in \mathcal{A}$ in the framework, then the rules $\mathcal{R}$ must be empty. The set of assumption $\{\alpha\}$ is coherence as it is closed, conflict-free, admissible, preferred, complete, set-stable, well-founded, and ideal. Therefore, using any quota rules and oligarchic rules, the aggregated framework is coherence as well. □

Theorem 15. *For $|\mathcal{A}| = 2$ or $|\mathcal{A}| = 3$, where $\mathcal{A}$ is a set of assumptions in Bipolar ABA framework, unanimity rule is the only quota rule that preserves coherence.*

Proof. Let a Bipolar ABA framework property P be the coherence. Assume that P holds in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. In other words, there exist a set of assumptions Δ in each framework, which is preferred, complete, set-stable, well-founded and ideal. To prove by contradiction, assume that Δ is not coherence in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$. By unanimity rule, all agents must have $\mathcal{R}_i \supseteq \mathcal{R}_{agg}$ for all $i \in N$. Thus, Δ is not coherence in the agents' frameworks $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ as well. This contradicts the initial assumption that P holds in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \overline{} \rangle$ for all $i \in N$. □

With the proofs above, it is easy to generalise the corner cases theorems into the Abstract Argumentation framework as clearly the presence of supports does not affect the preservation results.

Corollary 5. *For $|Arg| = 1$, where Arg is the arguments in Abstract Argumentation framework, every quota rules and oligarchic rules preserve coherence. For $|Arg| = 2$ or $|Arg| = 3$, unanimity rule is the only quota rule that preserves coherence.*

3.9 Summary

To sum up, all properties introduced in [8] are extended to also consider support relationships. Moreover, new properties, which are the closure and ideal semantics, are also introduced. The preservation study of the properties in Bipolar ABA framework gives several different results. For some results, the presence of supports in Bipolar ABA frameworks does not affect the preservation of the properties; while with other results, supports only restrict the preservation in a natural way, i.e., does not change the overall results. Table 3.1 gives a summary of the preservation results of the Bipolar ABA framework properties.

Compared with the results from [8], bipolarity in an argumentation framework does not greatly affect the preservation of the properties. This can happen because of the assumption that all agents have to agree on the property in each individual framework. Hence, if a property considers some supports, then all agents must considers those supports as well; resulting in a more natural constraint.

Property	Preservation Result
Conflict-free	Every quota rules and oligarchic rules preserve conflict-freeness.
Closure	Every quota rules and oligarchic rules preserve closedness of a set of assumptions.
Admissibility	For $\|\mathcal{A}\| \geq 4$, nomination rule is the only quota rule that preserves admissibility. For $\|\mathcal{A}\| \leq 3$, every quota rules and oligarchic rules preserve admissibility.
Set-Stable	The nomination rule is the only quota rule that preserves set-stable extension.
The Acceptability of an Assumption	For $\|\mathcal{A}\| \geq 4$, the aggregation rule that preserves the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics must be dictatorial. For $\|\mathcal{A}\| \leq 2$, every quota rules and oligarchic rules preserve the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics. For $\|\mathcal{A}\| = 3$, majority rule, unanimity rule, and oligarchic rules preserve assumption acceptability under preferred, complete, set-stable, well-founded, or ideal semantics.
Preferred, Complete, Well-founded, Ideal	For $\|\mathcal{A}\| \geq 5$, the aggregation rule that preserves preferred, complete, well-founded, and ideal semantics must be dictatorial. For $\|\mathcal{A}\| \leq 2$, every quota rules and oligarchic rules preserve preferred, complete, well-founded, and ideal semantics. For $\|\mathcal{A}\| = 3$ and $\|\mathcal{A}\| = 4$, quota rules and oligarchic rules do not preserve preferred, complete, well-founded, and ideal semantics.
The Non-emptiness of Well-founded Extension	For $\|\mathcal{A}\| \geq N$, at least one agent must have veto power to preserve the non-emptiness of the well-founded extension.
Acyclicity	For $\|\mathcal{A}\| \geq N$ and $\|\mathcal{A}\| \geq 2$, at least one agent must have veto power to preserve acyclicity.
Coherence	For $\|\mathcal{A}\| \geq 4$, the aggregation rule that preserves coherence must be dictatorial. For $\|\mathcal{A}\| = 1$, every quota rules and oligarchic rules preserve coherence. For $\|\mathcal{A}\| = 2$ or $\|\mathcal{A}\| = 3$, unanimity rule is the only quota rule that preserves coherence.

Table 3.1: Summary of Preservation Results

Chapter 4

Application: E-Polling

One application of the aggregation of Bipolar ABA frameworks is e-polling. It is a way to measure the opinion of the public. With a set of main topics or assumptions to be polled, e-polling is a process of collecting opinions of the agents that can be well represented as Bipolar ABA frameworks. Through a voting process, an agent chooses whether to agree, disagree, or be neutral with the assumptions. The goal is to get a final sentiment for the main topics. [17]

E-polling can be seen as a special type of Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$. With the main topics or assumptions $\mathcal{T} = \{\alpha_1, \cdots, \alpha_k \in \mathcal{A}\}$ as the centre, the remaining assumptions $\{\beta_1, \cdots, \beta_k \in \mathcal{A} \setminus \mathcal{T}\}$ form a tree-like structure with the main topics as the root such that either $\alpha_i^c \leftarrow \beta_j$, $\alpha_i \leftarrow \beta_j$, $\beta_i^c \leftarrow \beta_j$, or $\beta_i \leftarrow \beta_j$ for $\alpha, \beta \in \mathcal{A}$. In other words, the main topics can be attacked or supported by other assumptions, but cannot attack or support other assumptions. The assumptions that are not the main topics can attack or support both the main topics and the other assumptions. If an agent votes for an assumption α that attacks another assumption β such that $\beta^c \leftarrow \alpha$, then α gets more attacking power. On the other hand, if an agent votes against α, then α gets less attacking power. The same also applies for the assumptions that support other assumptions.

The method to calculate the power of the assumptions depending on the number of votes is given in [17], called QuAD for Voting (QuAD-V) frameworks. QuAD-V is used in the phase before and after aggregating the opinions of the agents. [17] also gives a procedure to get rational frameworks from each agent and collectively combined into a final framework. However, there are not any specific methods to aggregate the frameworks. Thus, this chapter tries to extend the procedure by giving analyses on using quota rules and oligarchic rules to aggregate the opinions of the agents in the form of Bipolar ABA frameworks. It completes the procedure by giving awareness on which rules should be used with their own strengths and weaknesses.

4.1 E-polling Scenario

A scenario is necessary to give an example of a real application of e-polling. As the core scenario, a Brexit debates is used and can be found in [17], which is similar to the background scenario in Section 2.1 with one less assumption and the use of support relationship in addition to the attack relationship. The scenario will be expanded further to fit e-polling procedures and analyses of aggregation rules in the next sections.

Assume several assumptions $\{A, B, C, D \in \mathcal{A}\}$. Every assumption represents one statement in the Brexit debates with assumption A as the main topic.

A: The UK should leave the EU.
B: The UK staying in the EU is good for its economy.
C: The EU's immigration policies are bad for the UK's economy.
D: EU membership fees are too high.

In this scenario, the main topic is A. Thus, the goal is to calculate the sentiment on whether the UK should leave the EU. There can be many interpretations of the relations between assumptions, one of which is given by the rule $\mathcal{R} = \{A^c \leftarrow B, \quad A \leftarrow C, \quad A \leftarrow D\}$ and can be represented as the graph in Figure 4.1. The rules might be different between the agents due to the e-polling procedures explained in the next section.

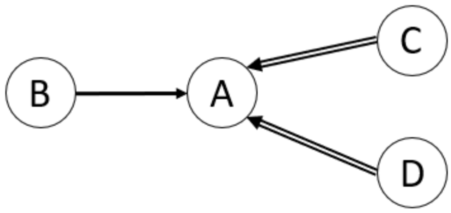

Figure 4.1: Brexit Scenario

4.2 E-Polling Procedures

Given the freedom of the agents to agree or disagree with the assumptions by voting for or against them, it is very likely that they give irrational opinions. The definition of an agent being irrational is adapted from [17].

Definition 4.2.1. (Irrational). An agent is irrational if he/she accepts a set of assumptions which is either not closed or not conflict-free.

In other words, the agent is irrational if he/she agrees with an assumption α and also agrees with one of the attacker of α, while disagreeing with the supporters of α. The opposite also holds, the agent is irrational if he/she disagrees with an assumption α and also disagrees with the attackers of α, while agreeing with one of the supporters of α. For example, using the Brexit scenario in Figure 4.1, an agent is irrational if he/she disagrees with assumption A, but agrees with C, i.e., accepting a not closed set of assumptions $\{C\}$, because agreeing with C means agreement with A as well because C supports A and this indicates an inconsistency on the sentiment of assumption A as the main topic. Another example, if an agent votes for both assumption A and B, then he/she is also being irrational because $\{A, B\}$ is not conflict-free with B attacks A.

Thus, [17] gives a systematical procedure to remove the irrationality of the agents as part of the QuAD-V framework. Then, after the agents all have rational opinions, an aggregation of those opinion takes place and finally, the final sentiment of the main topics is computed using QuAD framework formulas. Notice that being rational does not mean the agents always give relevant opinions or rules. A rule is deemed to be irrelevant if it depicts weak connections between two assumptions or does not concern the main topics.

Step 1. Take all the irrational agents' frameworks. To change the framework to be rational, either the votes should be changed or new assumptions and rules are added to the framework. However, an agent cannot remove the existing assumption or rule as it might affect other agents' opinions. This step is done through a process called dynamic questions. The agents are asked questions to validate their votes and given a chance to correct their votes or give additional assumptions and rules. The process is iterative until there is no more irrationality in all agents' framework. For example, assume the Brexit scenario in Section 4.1 and an irrational agent who agrees with assumption C but disagrees with assumption A. A question is then asked to the agent on why he/she disagrees with assumption A when he/she agree with the supporter C and none of its attackers. Depending on the response, the agent might change the opinion by agreeing with assumption A, disagreeing with assumption C, agreeing with one of the attackers of A, or add an assumption E : *The UK staying in the EU is good for world peace* that attacks assumption A. In the latter case, the framework is revised like in Figure 4.2 with rules $\mathcal{R} = \{A^c \leftarrow B, \quad A^c \leftarrow E, \quad A \leftarrow C, \quad A \leftarrow D\}$. Other agents might not be aware of the newly added assumption and they consider the assumption to be neutral of their own opinion, i.e., the additional assumption and rule do not affect their opinions.

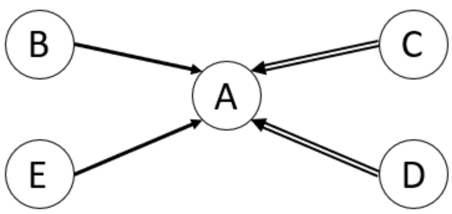

Figure 4.2: Example of Revised Framework

Step 2. By this step, all agents' frameworks are rational and ready to be aggregated. However, a simple aggregation method such as to join all assumptions, rules, and the votes might not work well as there might be contradiction after the aggregation or some irrelevant assumptions are added. Therefore, analyses of aggregation rules are given in Section 4.3 to explore the aggregation process deeper.

Step 3. Finally, all frameworks are combined into a final collective framework. To get the final set of accepted assumptions, various semantics can be computed. If focusing on the final sentiment value of the main topics, then quantitative computation by translating into QuAD-V framework is used. Take an aggregated Bipolar

ABA framework $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \overline{} \rangle$ and the votes of all agents on each assumption $\mathcal{V}$. The QuAD-V framework equivalent is $\langle \mathcal{A}, \mathcal{C}, \mathcal{P}, \mathcal{R}, \mathcal{U}, \mathcal{V} \rangle$, where $\mathcal{A}$ is the answer set or main topics (it is different from $\mathcal{A}$ in Bipolar ABA framework), $\mathcal{C}$ is the set of attacking assumptions α for $\forall\{\beta^c \leftarrow \alpha \in \mathcal{R}_{agg}\}$, $\mathcal{P}$ is the set of supporting assumptions α for $\forall\{\beta \leftarrow \alpha \in \mathcal{R}_{agg}\}$, $\mathcal{R}$ is the set of relations between assumptions (α, β) for $\forall\{\beta^c \leftarrow \alpha$ and $\beta \leftarrow \alpha \in \mathcal{R}_{agg}\}$, $\mathcal{U}$ is the set of agents, and $\mathcal{V}$ is the set of agents' votes. The base score for each assumption can be calculated by vote base score formula in Definition 2.6.10 of QuAD-V framework and the final score of the main topics can be calculated with strength function of QuAD framework in Definition 2.6.6. The main topics are accepted if the score is 1, rejected if the score is 0, and neutral if the score is 0.5. The scores between those thresholds can be seen as a confident rate of the assumption being accepted or rejected.

Although the e-polling procedures is finished with the final sentiment of the main topics, there still exist some problems with the aggregation on step 2. It is more desirable to get an aggregated framework which is able to consider as many of the agents' opinions as it can. This way, the final sentiment of the main topics can be confidently accepted as the opinions of the agents. Therefore, in the next section some analyses of aggregation rules is given and all of them are based on the preservation study in Chapter 3.

4.3 Analyses of Aggregation Rules

As mentioned in the previous section, to complete the e-polling procedures, especially on the second step, an appropriate aggregation rule is needed. It is important as different aggregation rules have different characteristics and computations; and thus, different preserved properties. Referring to Chapter 3, the preservation of the properties of Bipolar ABA frameworks also depends on the number of assumptions and the number of agents.

Some properties are also deemed stronger than the other in terms of getting the final accepted set of assumptions. For example, for a set of assumptions to be just conflict-free might not be enough, instead a stronger semantics such as preferred and well-founded are preferable. Taking the Bipolar ABA framework on Figure 4.2, a set of assumptions $\{B, C\}$ is conflict-free, but not enough to be the final set of assumptions as B attacks the main topic and C supports the main topic, i.e., both of them are contradicting. However, the well-founded extension $\{B, E\}$ is consistent to be the final accepted set of assumptions as both of them attack the main topic; in other words, taking the well-founded extension means disagreeing that the UK should leave the EU. Thus, depending on the context and desirable outcome, some properties are more suitable. Therefore, every quota rule and oligarchic rule are explored to give deeper understanding of their ability to preserve some properties.

Nomination rule. Using nomination rule means taking into account all of the agents' opinions. It preserves the property of being conflict-free, closed, admissible and set-stable without limitation. If the number of assumptions in the Bipolar ABA framework is less than two, then nomination rule also preserves the property of being preferred, complete, well-founded, and ideal; and the acceptability of an assumption under those semantics as well. However, for a lot of applications, such as e-polling, it is rare to have only two assumptions. The preservation of set-stable semantic is a good thing as it is one of the stronger property. Moreover, if a set of assumptions is set-stable, then it is preferred and complete. Therefore, having a set-stable extension guarantees the conflict-freeness, admissibility, and it considers all assumptions as either accepted or attacked.

One weakness of choosing nomination rule to aggregate the frameworks is the possibility of accepting irrelevant rules, which are not contributing to the main topic. These rules can appear as a result of the first step of e-polling procedures, in which the agents may add new assumptions in order to remove irrationality. As an example, take three revised Bipolar ABA frameworks belonging to three agents on the Brexit issue. They are depicted as the graph in Figure 4.3. The first framework on the left is the same as the revised framework in Figure 4.2. The middle framework has the core scenario revised with the addition of a new rule $C^c \leftarrow F$ for F : *The UK's tourism is improved* and the right framework adds $B \leftarrow G$ for G : *The partnership with the EU lower prices on trade and investment*.

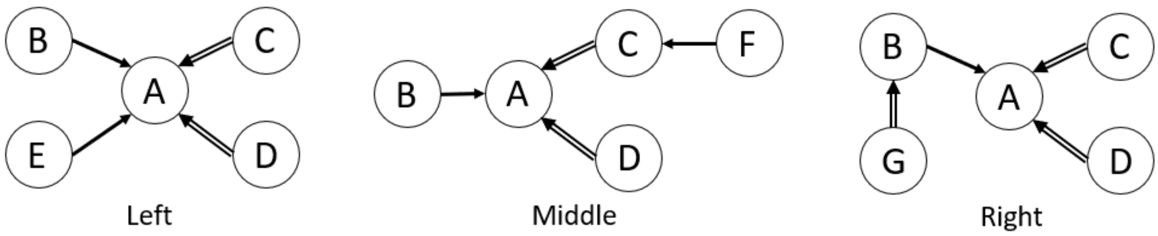

Figure 4.3: Brexit Scenario Frameworks Example for Nomination Rule

Assume that the rule $C^c \leftarrow F$ in the middle framework is not relevant to the main topic A. As each framework has a different set of assumptions, it is difficult to directly aggregate the frameworks. Hence, a modification is needed to make all frameworks have the same assumptions. It is done by adding the missing assumptions on each framework without adding the relation between the assumptions in order to keep the neutrality. Those frameworks are depicted in Figure 4.4. As a result, all of the Bipolar ABA frameworks have the same set-stable extension, which is $\{B, E, F, G\}$. Finally, as set-stable is preserved by the nomination rule, then the set-stable extension in the aggregated framework is $\{B, E, F, G\}$ as well. It includes assumption F that is deemed to be not relevant to the main topic A : *The UK should leave the EU*. As the set-stable extension is the strongest semantic preserved by nomination rule, $\{B, E, F, G\}$ is adopted as the final set of accepted assumptions, then there is one assumption that does not have the same importance as the other but still considered as the same. In this example, it does not introduce a problem as in the end, the sentiment towards the main assumption is still the same, which is to disagree that the UK should leave the EU. But it might be a problem for other scenarios.

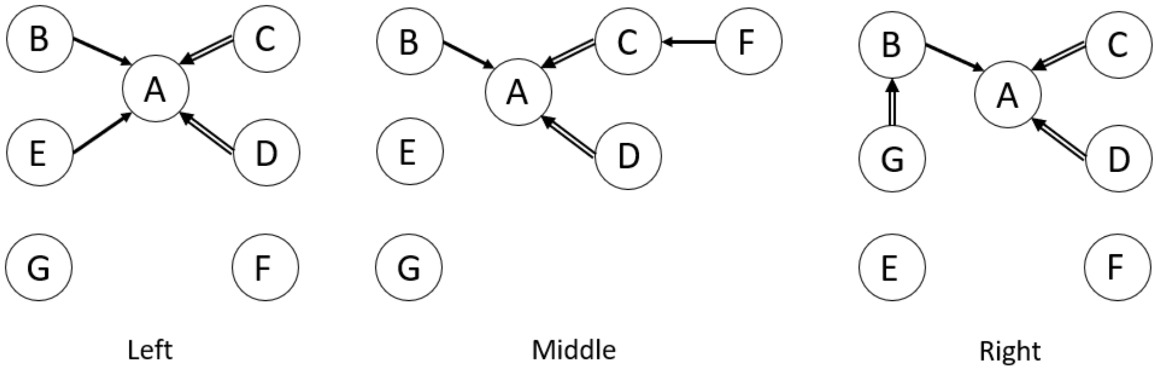

Figure 4.4: Revised Brexit Scenario Frameworks Example for Nomination Rule

Oligarchic rules / dictatorship. Oligarchic rules give veto powers on some agents, while dictatorship is an oligarchic rule with only one agent with veto power. Both of them are undesirable because they usually depict impossibility and only the selected agents' opinions are considered and ignoring the others. However, to preserve some of the stronger properties, the aggregation rule must be oligarchic or dictatorial.

The conflict-freeness and closure are preserved by oligarchic rules. Moreover, the aggregation rule must be dictatorial to preserve the acceptability of an assumption, preferred, complete, well-founded, ideal, and coherence. The preferred, complete, well-founded, and ideal semantics are all strong properties to get the final set of accepted assumptions. However, when the aggregation rule is dictatorial, then it means only the opinion of one agent is being considered. This might result to the inclusion of irrelevant rules or the ignorance of important rules. For example, still using the Brexit scenario from Section 4.1, take a number of agent $N \in \{1, \cdots, k\}$ with $k > 2$. Assume that 1 agent proposes his/her opinion depicted in the left framework of Figure 4.5, while k-1 agents propose their opinions depicted in the right framework of Figure 4.5. The assumptions F and G have the same statement as in Figure 4.3; assumption H is *The immigration policies are being improved with a plan to streamline all processes* and assumption I is *The development of a new transportation system to accommodate the UK and the EU citizens is discarded*.

This example also shows the disjunctiveness as defined in Definition 3.5.3 with $R_1 = \{C^c \leftarrow F\}$ and $R_2 = \{I^c \leftarrow F\}$. Assume that the rule $C^c \leftarrow F$ is not relevant, and the rule $B \leftarrow G$ is relevant. The set of assumptions $\{B, F, G, H\}$ is preferred, complete, well-founded, and ideal in all frameworks. Then, as to preserve those semantics, the aggregation must be dictatorial, only one agent can have veto power. If veto power is given to the agent with the left framework of Figure 4.5, then the irrelevant rule $C^c \leftarrow F$ is included, while the relevant rule $B \leftarrow G$ is ignored. Moreover, it considers the framework with only one agent who supports it; while in fact, it is better to give veto power to one of the agents of the right framework of Figure 4.5 as it is supported by k-1 agent.

The use of oligarchic rules are also able to preserve the non-emptiness of well-founded extension and acyclicity if at least one agent must have veto power. These results are useful to make sure some assumptions to be accepted.

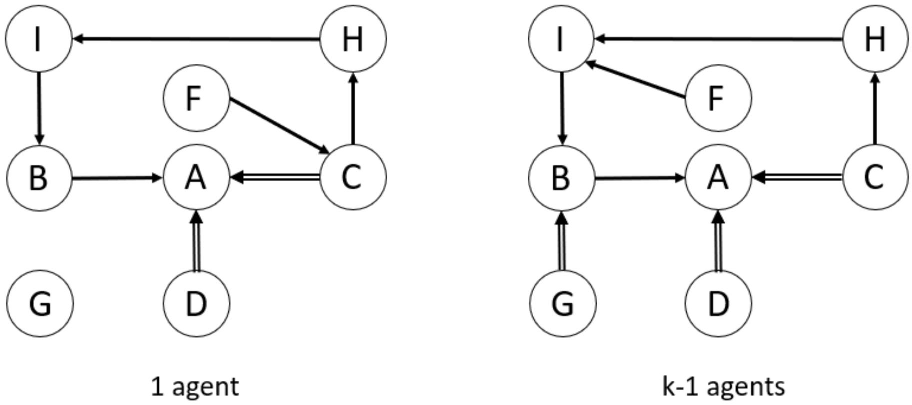

Figure 4.5: Brexit Scenario Frameworks Example for Oligarchic Rules / Dictatorship

Majority rule and unanimity rule. Using majority rule means to only consider the opinions of the majority, while using unanimity rule only accepts opinions that are supported by all agents. The properties that are preserved indefinitely by these aggregation rules are the conflict-freeness and closure. For a small number of assumptions in the framework, the admissibility, preferred, complete, well-founded, ideal, and coherence properties are preserved. However, such corner cases are not preferable in many applications. Hence, this is a bad news because conflict-freeness and closure are not strong properties and usually are not enough to determine the final accepted set of assumptions well. It is rather unfortunate because both aggregation rules, especially the unanimity rule, may offer a guarantee that only the most relevant arguments to be accepted.

4.4 Summary

Each quota rule and oligarchic rule have its own strengths and weaknesses. Table 4.1 shows comparisons of each of the aggregation rules in terms of the preserved properties, number of opinions considered, and weaknesses; while ignoring the corner cases. It can be concluded that nomination rule is the preferable aggregation rule. Nomination rule tries to consider as many opinions as it can, while preserving the set-stable semantic, which should be enough to determine the final set of accepted assumptions. Oligarchic rules and dictatorship are less preferable as they consider only the agents with veto powers and ignore the others, although many important properties are preserved only with these aggregation rules.

Aggregation Rule	Preserved Properties	Number of Opinions	Weaknesses
Nomination	- Conflict-free - Closure - Admissibility - Set-stable	All opinions	Inclusion of irrelevant rules

Aggregation Rule	Preserved Properties	Number of Opinions	Weaknesses
Oligarchic & Dictatorial	- Conflict-free - Closure - The acceptability of an assumption - Preferred - Complete - Well-founded - Ideal - The non-emptiness of well-founded extension - Acyclicity - Coherence	Only opinions from agents given with veto powers	Ignorance of opinions, inclusion of irrelevant rules, ignorance of important rules
Majority & Unanimity	- Conflict-free - Closure	Only the majority or unanimously accepted opinions	Weak preservation capability

Table 4.1: Summary of Aggregation Rules Analyses

It is also interesting to see that the preservation results do not change just because of the special form of Bipolar ABA framework for e-polling. Even though the frameworks have tree-like structure, the preservation of the properties still relies heavily on the agreement from all agents.

To sum up, with nomination rule, the e-polling procedures is now complete. Although the weakness of nomination rule, which is the possibility of including irrelevant rules, still exists, but with the votes on the irrelevant rules must be insignificant, then they do not affect the final calculation of the strength of the main topics.

Chapter 5

Conclusion and Future Works

5.1 Conclusion

The opinions of the agents can be interpreted into argumentation frameworks. To consider both attack and support relationships within the frameworks, Bipolar ABA framework is chosen instead of Bipolar Argumentation framework. The reason is that in the Bipolar Argumentation framework, there are several definitions of the support relationships with each definition has its own set of semantics. It causes inconsistencies in the aggregation process. However, with Bipolar ABA framework, all definitions of support relationship in Bipolar Argumentation framework can be translated into Bipolar ABA framework, which has the same set of semantics.

The aggregation of Bipolar ABA frameworks combines the rules of all agents into a collective set of rules. The aggregation rules used in this project are taken from Social Choice Theory, specifically judgement aggregation, which are quota rules and oligarchic rules. Quota rules use a number of quota as a threshold to determine the accepted rules, while oligarchic rules give veto powers to the agents such that only those agents' opinions are considered. Then, by both extending the results from [8] and proposing new properties, the preservation study investigates whether the properties in the individual agents' frameworks are preserved in the aggregated framework. Hypothetically, if a property is true in all the individual agents' frameworks, then it should also be true in the aggregated framework. However, the preservation results show that not all properties are preserved without any limitation. It is also important to notice that aggregating the rules in Bipolar ABA framework means aggregating both attack and support relationship. The preservation results also show that the presence of supports in the frameworks does not greatly affect the performance of the aggregation rules. The constraints this support relationships produce are actually quite natural and in line with the properties. For example, if α supports β then intuitively if α is accepted, then so does β.

For the most positive results, the properties of being conflict-free and closed are preserved by any quota rules and oligarchic rules. These two properties are the most basic ones without many requirements to be fulfilled; and thus, easier to be preserved. More positive results are given by admissibility and set-stable. Admissibility is preserved by nomination rule if the number of assumptions is more or equal than four, otherwise it is preserved by every quota rule and oligarchic rule. The set-stable semantic is preserved by nomination rule. It can be seen that although both properties are preserved, but some limitations need to be put into place; for example, in the admissibility, the number of assumptions limits the aggregation rules that preserve it.

For the following preservation results, either oligarchic rules or dictatorship must be used in order to preserve the properties. The use of these particular aggregation rules is actually not ideal as many opinions are ignored. However, it is still better than not being able to preserve the properties at all. For the acceptability of an assumption and coherence properties, when the number of assumptions is greater or equal than four, the aggregation rule must be dictatorial. It is also the same with the preferred, complete, well-founded, and ideal semantics; but with the number of assumptions must be greater or equal than five. Unsurprisingly, the corner cases, i.e., when the number of assumptions is small, these properties are easier to be preserved with some quota rules are able to do so.

The last two preservation results involve the non-emptiness of the well-founded extension and the acyclicity properties. Both of them are preserved when at least one agent must have veto power and the number of assumptions must be greater or equal than the number of agents. This unique constraint is put to ensure the existence of cyclic relationships and a way to break it. To sum up, the preservation results of Bipolar ABA frameworks' properties show both positive results and more negative results in terms of the use of oligarchic rules and dictatorship.

To put the results into applications, e-polling scenario is used. With the goal to complete the e-polling procedure, some analyses of the aggregation rules are given to provide an intuition on which aggregation rule is preferable in this case. Nomination rule is the preferable aggregation rule because it preserves set-stable semantic, one of the stronger semantic in Bipolar ABA framework. Moreover, it considers all the agents' opinions such that they are fully reflected in the aggregated framework. However, the weakness of nomination rule is that the possibility of considering irrelevant opinions. Oligarchic rules and dictatorship are actually used in the preservation of many important properties such as preferred, complete, well-founded, and ideal semantics. However, the fatal weakness of these rules is the ignorance of many of the agents' opinions. Hence, it is not preferable. The majority and unanimity rules are unfortunately unable to preserve many important properties in general cases. They only excel in preserving the properties for the corner cases, which are not as common in many applications.

To conclude, aggregating Bipolar ABA frameworks can be done by combining the rules of all the agents using various aggregation rules, such as quota rules and oligarchic rules. Each aggregation rule has its own strength and weaknesses in terms of preserving the properties of Bipolar ABA framework. The preservation results provide good insights on which aggregation rule performs better. Then, those insights are developed more into analyses that fit into e-polling application and complete the whole procedures.

5.2 Future Works

There are still some possible directions for continuing this project in the future. First of all, in this project the preservation of the properties assumes that all agents agree on the properties. However, in real application, it is very likely to have some agents with different opinions than the others, i.e., some of them disagree with the properties. If the number of disagreeing agents are very small, then it is interesting to study the preservation results of the properties using various aggregation rules.

In this project, the aggregation of Bipolar ABA framework takes place in the rules of the frameworks. In other words, the agents are assumed to have the same language, assumptions, and contraries. Hence, another direction of future work is to study aggregation methods of Bipolar ABA framework if the agents have different knowledge about the environment.

Another possible direction is to expand the choice of aggregation rules. As the rules are basically mapping functions from several frameworks into a single framework, there are many ways to aggregate the frameworks. Quota rules and oligarchic rules are popular aggregation rules in Social Choice Theory domain, but they are not the only rules. It will give a nice supplement on the preservation study to have more aggregation rules to be chosen.

Finally, it should be worth to generalise the study of aggregating Bipolar ABA frameworks into aggregating the more general non-flat ABA frameworks. The possibility of having some rules with empty head might change the preservation results. Along with this, it is also interesting to study whether self-attacking assumptions affect the results. This is useful to have more applications that can be implemented using aggregation procedures.

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Appendix A

Ethics Checklist

	Yes	No
Section 1: HUMAN EMBRYOS/FOETUSES		
Does your project involve Human Embryonic Stem Cells?		x
Does your project involve the use of human embryos?		x
Does your project involve the use of human foetal tissues / cells?		x
Section 2: HUMANS		
Does your project involve human participants?		x
Section 3: HUMAN CELLS / TISSUES		
Does your project involve human cells or tissues? (Other than from "Human Embryos/Foetuses" i.e. Section 1)?		x
Section 4: PROTECTION OF PERSONAL DATA		
Does your project involve personal data collection and/or processing?		x
Does it involve the collection and/or processing of sensitive personal data (e.g. health, sexual lifestyle, ethnicity, political opinion, religious or philosophical conviction)?		x
Does it involve processing of genetic information?		x
Does it involve tracking or observation of participants? It should be noted that this issue is not limited to surveillance or localization data. It also applies to Wan data such as IP address, MACs, cookies etc.		x
Does your project involve further processing of previously collected personal data (secondary use)? For example Does your project involve merging existing data sets?		x
Section 5: ANIMALS		
Does your project involve animals?		x
Section 6: DEVELOPING COUNTRIES		
Does your project involve developing countries?		x
If your project involves low and/or lower-middle income countries, are any benefit-sharing actions planned?		x
Could the situation in the country put the individuals taking part in the project at risk?		x

Figure A.1: Ethics Checklist Part 1

	Yes	No
Section 7: ENVIRONMENTAL PROTECTION AND SAFETY		
Does your project involve the use of elements that may cause harm to the environment, animals or plants?		x
Does your project deal with endangered fauna and/or flora /protected areas?		x
Does your project involve the use of elements that may cause harm to humans, including project staff?		x
Does your project involve other harmful materials or equipment, e.g. high-powered laser systems?		x
Section 8: DUAL USE		
Does your project have the potential for military applications?		x
Does your project have an exclusive civilian application focus?		x
Will your project use or produce goods or information that will require export licenses in accordance with legislation on dual use items?		x
Does your project affect current standards in military ethics – e.g., global ban on weapons of mass destruction, issues of proportionality, discrimination of combatants and accountability in drone and autonomous robotics developments, incendiary or laser weapons?		x
Section 9: MISUSE		
Does your project have the potential for malevolent/criminal/terrorist abuse?		x
Does your project involve information on/or the use of biological-, chemical-, nuclear/radiological-security sensitive materials and explosives, and means of their delivery?		x
Does your project involve the development of technologies or the creation of information that could have severe negative impacts on human rights standards (e.g. privacy, stigmatization, discrimination), if misapplied?		x
Does your project have the potential for terrorist or criminal abuse e.g. infrastructural vulnerability studies, cybersecurity related project?		x
SECTION 10: LEGAL ISSUES		
Will your project use or produce software for which there are copyright licensing implications?		x
Will your project use or produce goods or information for which there are data protection, or other legal implications?		x
SECTION 11: OTHER ETHICS ISSUES		
Are there any other ethics issues that should be taken into consideration?		x

Figure A.2: Ethics Checklist Part 2