Web appendix for
Pinball boosting of regression quantiles

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This appendix consists of two main parts: First, the section A promotes an in-depth understanding of the algorithm proposed in the main article for the pinball boosting of regression quantiles (AL1BRQ). It contains equivalent representations of the algorithms Least Square Boosting of Regression Means (L2BRM) and Least Squares Boosting of Regression Quantiles (L2BRQ) and discusses similarities and differences in the subsection A.1. A detailed didactic description and interpretation of the individual steps of the algorithms, accompanied by informative visualizations, can be found in the subsection A.2. The subsection A.3 contains general implications for variable selection, model selection and functional form, interpretability and tuning parameters. Subsequently, section B deals with the reproduction of the main results and the extension of the simulation study from Fenske et al. (2011).

A. Algorithms

A.1. Step-by-step comparison of the algorithms

0. Directive

The directive of both the AL1BRQ and the L2BRQ algorithm is to estimate a generalized additive quantile regression model by functional gradient boosting, which is achieved by minimizing the well-known pinball loss function

$$\rho_\tau(y - \eta_\tau) = \psi_\tau(y_i - \eta_\tau) \cdot (y_i - \eta_\tau),$$

where $\psi_\tau(z) := \tau - \mathbb{1}(z < 0)$. Both methods are particularly suitable for situations that demand interpretability and variable selection as well as model choice, e.g., in high-dimensional settings.

1. Initialize the fitted values

In the first step, the iteration counter m is set to zero and the algorithms are set up. Initializing the component-wise gradient boosting algorithm with appropriate starting values can significantly influence the results. Therefore, the fitted values for the τth conditional quantile, $\hat{\eta}_{\tau i}^{[0]}$, should be initialized with their best initial guess. Although the respective τth sample quantile of the response appears to be an obvious candidate, Fenske et al. (2011, p. 498 and p. 19 in the Electronic Supplementary Material) refer the reader to their empirical experience which suggests that initializing with the sample median leads to faster convergence of the algorithm, i.e., to a reduced optimal number of boosting iterations. Consequently, the L2BRQ algorithm is initialized with the sample median regardless of the τth conditional quantile to be estimated.

Our simulation experiments suggest that for the AL1BRQ algorithm, the sample quantile used for initialization that leads to the fastest convergence of the algorithm depends on the (unknown) data generating process. No single initialization-quantile universally leads to the fastest convergence. This property and its potential relevance to L2BRQ are elaborated further in Section A.2. Consequently, AL1BRQ is initialized with the most intuitive choice, the respective τth sample quantile.

Stopping the algorithm prior to the first iteration delivers an estimate for the τth conditional quantile function, $\hat{\eta}_{\tau}^{[0]}$, that contains only an intercept corresponding to the respective τth sample quantile of the response, which coincides with what we expect when fitting a quantile regression model comprising only an intercept[1]. For clarity of notation, the quantile parameter referring to the sample quantile used for initialization, denoted as τ_{init}, is distinguished from the quantile parameter referring to the conditional quantile to be estimated, denoted as τ.

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[1]Note the difference in notation between the current fitted values in iteration m, $\hat{\eta}_{\tau i}^{[m]}$, and the current estimate for the τth conditional quantile function, $\hat{\eta}_{\tau}^{[m]}$, which consists of the regression coefficients in iteration m.

Algorithm L2BRM L_2 boosting of regression means

Directive: $\hat{\eta} = \arg\min_\eta \sum_{i=1}^n L_{2(}y_i, \eta_i) = \boldsymbol{x}^\top \hat{\boldsymbol{\beta}}$

1. **Initialize the fitted values** for the conditional mean, $\hat{\eta}_i^{[0]}$, with the sample mean of the response. Set the iteration counter to $m := 0$.

2. **Fit the working residuals.**

   2a. Set $\boldsymbol{m := m+1}$.

   2b. Compute the working residuals of the squared loss function

$$u_i^{[m]} = \left(y_i - \hat{\eta}_i^{[m-1]}\right).$$

   2c. Compute a least squares regression fit for each base learner according to the mean assumption

$$E(u_i{}^{[m]}|x_{ij}) = a_j + b_j x_{ij} \qquad \text{for} \qquad j = 1, ..., p,$$

   and obtain $\hat{a}_j^{[m]}$ and $\hat{b}_j^{[m]}$.

3. **Update one component.**

   3a. Select the component x_j that fits the working residuals best based on

$$\arg\min_j \sum_{i=1}^n \underline{\left(u_i^{[m]} - \hat{u}_{ij}^{[m]}\right)^2},$$

   where $\hat{u}_{ij}^{[m]} = \hat{a}_j^{[m]} + \hat{b}_j^{[m]} x_{ij}$.

   3b. Update the estimate of the τth regression quantile

$$\hat{\boldsymbol{\beta}}^{[m]} = \hat{\boldsymbol{\beta}}^{[m-1]} + \nu \cdot \hat{\boldsymbol{b}}_j^{[m]}$$

   where ν is the learning rate and $\hat{\boldsymbol{b}}_j^{[m]}$ is a $((p+1)\times 1)$-vector with the first entry $\hat{a}_j^{[m]}$, the $(j+1)$th entry $\hat{b}_j^{[m]}$ for the best-fitting component x_j and 0 for all remaining components.

Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

Algorithm L2BRQ L_2 boosting of regression quantiles

Directive: $\hat{\eta}_\tau = \arg\min_{\eta_\tau} \sum_{i=1}^n \rho_\tau(y_i - \eta_{\tau i}) = \boldsymbol{x}^\top \hat{\boldsymbol{\beta}}_\tau$

1. **Initialize the fitted values** for the τth conditional quantile, $\hat{\eta}_{\tau i}^{[0]}$, with sample median of the response. Set the iteration counter to $m := 0$.

2. **Fit the working residuals.**

   2a. Set $m := m + 1$.

   2b. Compute the working residuals of the pinball loss function

$$u_i^{[m]} = \psi_\tau\left(y_i - \hat{\eta}_{\tau i}^{[m-1]}\right) = \begin{cases} \tau & y_i \geq \hat{\eta}_{\tau i}^{[m-1]} \\ \tau - 1 & y_i < \hat{\eta}_{\tau i}^{[m-1]} \end{cases}.$$

   2c. Compute a least squares regression fit for each base learner according to the mean assumption

$$E(u_i{}^{[m]}|x_{ij}) = a_j + b_j x_{ij} \qquad \text{for} \qquad j = 1, ..., p,$$

   and obtain $\hat{a}_j^{[m]}$ and $\hat{b}_j^{[m]}$.

3. **Update one component.**

   3a. Select the component x_j that fits the working residuals best based on

$$\arg\min_j \sum_{i=1}^n \underline{\left(u_i^{[m]} - \hat{u}_{ij}^{[m]}\right)^2},$$

   where $\hat{u}_{ij}^{[m]} = \hat{a}_j^{[m]} + \hat{b}_j^{[m]} x_{ij}$.

   3b. Update the estimate of the τth regression quantile

$$\hat{\boldsymbol{\beta}}_\tau^{[m]} = \hat{\boldsymbol{\beta}}_\tau^{[m-1]} + \nu \cdot \hat{\boldsymbol{b}}_j^{[m]}$$

   where ν is the learning rate and $\hat{\boldsymbol{b}}_j^{[m]}$ is a $((p+1) \times 1)$-vector with the first entry $\hat{a}_j^{[m]}$, the $(j+1)$th entry $\hat{b}_j^{[m]}$ for the best-fitting component x_j and 0 for all remaining components.

Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

Along with the initialization of the fitted values, appropriate base learners must be specified to complete the algorithm setup. The choice of base learners effectively imposes structural assumptions on the functional form. Fenske et al. (2011, p. 498) argue that "least-squares base learners are a natural choice" for their proposed L2BRQ algorithm and choose simple linear regression means, $E(u_i|x_{ij}) = a_j + b_j x_{ij}$, for each predictor $j = 1, ..., p$. This statement originates from Friedman (2001, p. 1194), where least squares base learners arguably are a "natural choice" since the conditional expectation function is estimated by minimizing the squared loss (L_2 boosting). Given that in the present case the algorithm ultimately aims to estimate the conditional quantile function, simple linear regression quantiles, $Q_{u_i}(\tau|x_{ij}) = a_{\tau j} + b_{\tau j} x_{ij}$, for each predictor $j = 1, ..., p$, seem to be an intuitive choice for the base learners. Hence, the proposed AL1BRQ algorithm uses simple linear regression quantiles as base learners instead. In general, base learners are not limited to simple regression models. However, using multivariate regression models as base learners may compromise the variable selection property of the algorithm, as discussed in Subsection A.3. Theoretical considerations should be the underpinning of such a choice.

2. Fit the working residuals.

2a. Set $m := m + 1$.

2b. Compute the working residuals.

The negative gradient is the negative derivative of the pinball loss with respect to the τth conditional quantile function. For continuous variables, the pinball loss function is not differentiable at point $y_i = \eta_{\tau i}$, which can be neglected as this "exact fit" event occurs with zero probability, except in the first iteration. In the first iteration, the fitted values correspond to the sample quantile (AL1BRQ) or sample median (L2BRQ) of the response, such that at least one observation is fitted exactly. Our Monte Carlo study suggest that the results are robust to whether the working residuals[2] of those "exact fit" observations are set to τ or $\tau - 1$. Setting the working residuals of these observations to 0, as implemented in Fenske et al. (2009, p. 6), has no considerable impact either. Thus, there is no difference between the computation of the negative gradient in AL1BRQ and L2BRQ. Due to the different initializations, however, the computed working residuals differ between the two algorithms (except for $\tau = 0.5$).

2c. Compute a least squares/quantile regression fit for each base learner.

Each base learner from Step 1 is now fitted to the working residuals of the current iteration, m. L2BRQ separately fits each predictor, $j = 1, .., p$, to the working residuals as a simple linear least squares regression: $E(u_i^{[m]}|x_{ij}) = a_j + b_j x_{ij}$. AL1BRQ separately fits each predictor, $j = 1, .., p$, to the working residuals as a simple linear quantile regression: $Q_{u_i^{[m]}}(\tau|x_{ij}) = a_{\tau j} + b_{\tau j} x_{ij}$.

3. Update one component.

3a. Select the component x_j that fits the working residuals best.

The algorithm selects the base learner, x_j, that best fits the working residuals of the current iteration, $u_i^{[m]}$. L2BRQ defines the best fitting base learner as the one with the smallest residual sum of squares (RSS),

$$\mathrm{RSS}_j = \sum_{i=1}^{n} \left(u_i^{[m]} - \hat{u}_{ij}^{[m]} \right)^2,$$

which is a suitable criterion for regression means as base learners. AL1BRQ defines the best fitting base learner as the one with the smallest empirical risk (based on the quantile score)

$$\begin{aligned} \mathrm{R}_{\tau j} &= \sum_{i=1}^{n} \rho_\tau \left(u_i^{[m]} - \hat{u}_{\tau ij}^{[m]} \right) \\ &= \sum_{i=1}^{n} \begin{cases} \tau(u_i^{[m]} - \hat{u}_{\tau ij}^{[m]}) & u_i^{[m]} > \hat{u}_{\tau ij}^{[m]} \\ (\tau - 1)(u_i^{[m]} - \hat{u}_{\tau ij}^{[m]}) & u_i^{[m]} \leq \hat{u}_{\tau ij}^{[m]}. \end{cases} \end{aligned}$$

[2]Other publications use the term "negative gradients" for the working residuals. We prefer "working residuals" to avoid confusion between the negative gradient and the working residuals. The negative gradient is the negative derivative of the pinball loss w.r.t. the conditional τth quantile function. The working residuals are obtained by evaluating the negative gradient at the fitted values for the τth conditional quantile of the previous iteration, i.e., at $\hat{\eta}_{\tau i}^{[m-1]}$, and are thus a vector of length n whose values take either τ or $\tau - 1$.

3b. Update the estimate of the τth regression quantile function.

The coefficient vector of the best-fitting component of the τth conditional quantile function, $\hat{\eta}_{\tau}^{[m]}$, and the fitted values, $\hat{\eta}_{\tau i}^{[m]}$, are additively updated. Note that in the present setup the single components correspond to the predictors. A typically small and pre-specified learning rate, ν, acts as a shrinkage factor for the coefficient estimate. This regularizes the single estimations and discounts their influence on the final estimate (the choice of a suitable learning rate is discussed in Subsection A.3). L2BRQ fixes the learning rate to the frequently employed learning rate of $\nu = 0.1$. Since the accuracy of quantile regression depends critically on how informative the design is over the distribution of the response, for AL1BRQ, we opt for a quantile-specific learning rate ν_τ that is lower for quantiles at the tails of the distribution, where data is typically more sparse, and higher for the center of the distribution.

Thus, AL1BRQ employs a quantile-specific learning rate ν_τ, while L2BRQ fixes the learning rate to $\nu = 0.1$.

For L2BRQ, the τth effect estimate is updated by

$$\hat{\boldsymbol{\beta}}_{\tau}^{[m]} = \hat{\boldsymbol{\beta}}_{\tau}^{[m-1]} + \nu \cdot \hat{\boldsymbol{b}}_{j}^{[m]},$$

where ν is the fixed learning rate and $\hat{\boldsymbol{b}}_{j}^{[m]}$ is a $((p+1) \times 1)$-vector, with the first entry $\hat{a}_{j}^{[m]}$, the $(j+1)$-th entry $\hat{b}_{j}^{[m]}$ for the best-fitting component x_j and 0 for all remaining components.

The τth conditional quantile function and the fitted values are updated by

$$\begin{aligned}\hat{\eta}_{\tau}^{[m]} &= \hat{\eta}_{\tau}^{[m-1]} + \nu \hat{E}(u^{[m]}|x_j),\\ \hat{\eta}_{\tau i}^{[m]} &= \hat{\eta}_{\tau i}^{[m-1]} + \nu \hat{u}_{ij}^{[m]},\end{aligned}$$

where $\hat{E}(u^{[m]}|x_j)$ represents the estimate for the conditional mean of the current working residuals as a function of the best fitting predictor, x_j. The respective fitted values for the conditional mean of the current working residuals are denoted by $\hat{u}_{ij}^{[m]}$.

For AL1BRQ, the τth effect estimate is updated by

$$\hat{\boldsymbol{\beta}}_{\tau}^{[m]} = \hat{\boldsymbol{\beta}}_{\tau}^{[m-1]} + \nu_\tau \cdot \hat{\boldsymbol{b}}_{\tau j}^{[m]},$$

where ν_τ is the learning rate and $\hat{\boldsymbol{b}}_{\tau j}^{[m]}$ is a $((p+1) \times 1)$-vector, with the first entry $\hat{a}_{\tau j}^{[m]}$, the $(j+1)$-th entry $\hat{b}_{\tau j}^{[m]}$ for the best-fitting component x_j and 0 for all remaining components.

The τth conditional quantile function and the fitted values are updated by

$$\begin{aligned}\hat{\eta}_{\tau}^{[m]} &= \hat{\eta}_{\tau}^{[m-1]} + \nu_\tau \hat{Q}_{u^{[m]}}(\tau|x_j),\\ \hat{\eta}_{\tau i}^{[m]} &= \hat{\eta}_{\tau i}^{[m-1]} + \nu_\tau \hat{u}_{\tau ij}^{[m]},\end{aligned}$$

where $\hat{Q}_{u^{[m]}}(\tau|x_j)$ represents the estimate for the τth conditional quantile of the current working residuals as a function of the best fitting predictor, x_j. The respective fitted values for the τth conditional quantile of the current working residuals are denoted by $\hat{u}_{\tau ij}^{[m]}$.

After convergence of AL1BRQ and L2BRQ, the residual vectors are asymmetrically split into two parts, depending on τ, as is the case for classical quantile regression. Yet, the estimated regression relationships between y and x are not equivalent to classical quantile regressions: They do not satisfy typical quantile regression fit properties like exact fit, but they come arbitrarily close to this state.

4. Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

A.2. Step-by-step interpretation of the algorithms

Just as with the component-wise gradient boosting algorithm (Friedman, 2001), the modular nature of AL1BRQ and L2BRQ makes individual steps and interim calculations traceable. A comprehensive explanation of the steps of these white boxes follows below. Graphical insight is given using an example data set generated from the simple model

$$y_i = 3 + 1x_i + 4u_i \qquad \text{with} \qquad u_i \sim N(0,1) \qquad \text{and} \qquad x_i \sim U\left[0, 10\right]. \tag{1}$$

The directive for both algorithms is to estimate the 10% conditional quantile function, $\eta_{0.1}$. All results discussed in the following also apply to a multivariate setup.

1. Initialize the fitted values.

Because AL1BRQ (L2BRQ) is initialized with the respective τth sample quantile (median), the initial estimate for the τth conditional quantile function, $\hat{\eta}_{0.1}^{[0]}$, comprises only an intercept, equal to the sample quantile (median) of the response. Other predictor effects are initially set to zero. AL1BRQ initialized with the median serves as an intermediate between both approaches and is additionally reported (see Figure 1).

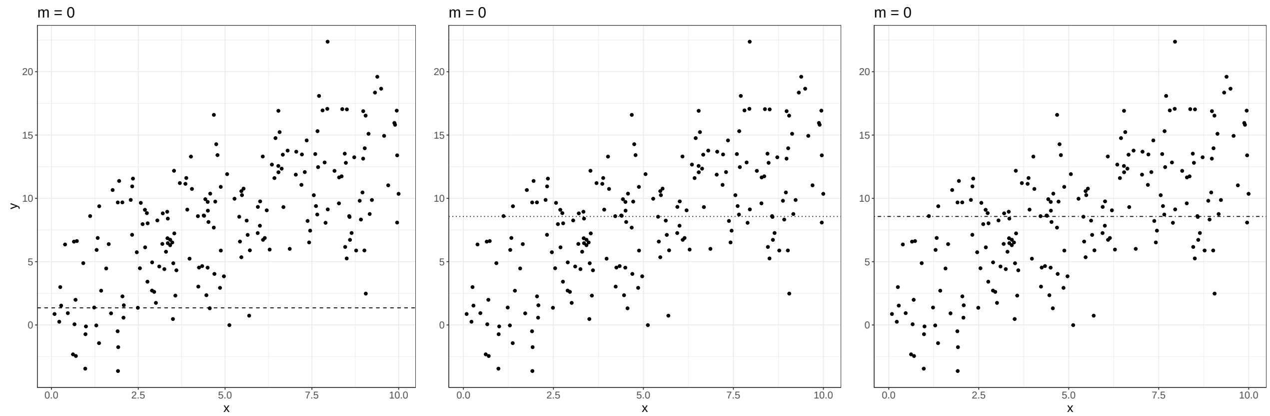

Figure 1: Scatterplot for example data drawn from the model in Equation 1. Left: AL1BRQ initialized with 10% sample quantile (dashed line). Middle: AL1BRQ initialized with the sample median (dotted line). Right: L2BRQ initialized with the sample median (dashdotted line).

The example at hand contains only one predictor and therefore only one base learner in the respective algorithm: a simple linear 10% regression quantile, $Q_u(\tau = 0.1|x) = a_{0.1,1} + b_{0.1,1}x$, in the AL1BRQ algorithm and a simple linear regression means, $E(u|x) = a_1 + b_1 x$, in the L2BRQ algorithm.

2. Fit the working residuals.

2a. Set $\boldsymbol{m := m + 1}$.

2b. Compute the working residuals.

Having initialized the fitted values for the AL1BRQ algorithm with the respective sample quantile in Step 1, all initial fitted values, $\hat{\eta}_{0.1,i}^{[0]}$, are equal to the 10% sample quantile of the response. Therefore, 90% of the response's observations are greater than $\hat{\eta}_{0.1,i}^{[0]}$ and 10% are smaller. Hence, 90% of the working residuals of the first iteration take a value of $\tau = 0.1$ and 10% take a value of $\tau - 1 = -0.9$ (Figure 2, left panel).

If, analogous to the initialization in the L2BRQ algorithm, the fitted values in the AL1BRQ algorithm are initialized with the response's median instead, 50% of the response's observations are greater than the initial fitted values, $\hat{\eta}_{0.1,i}^{[0]}$, and 50% are smaller. This results in half of the working residuals of the first iteration taking on a value of $\tau = 0.1$ and half taking a value of $\tau - 1 = -0.9$ (Figure 2, middle and right panel).

All plots of Figure 2 show a positive correlation between response and predictor, as larger values of x tend to correspond to larger values of y and therefore to working residuals of the value 0.1.

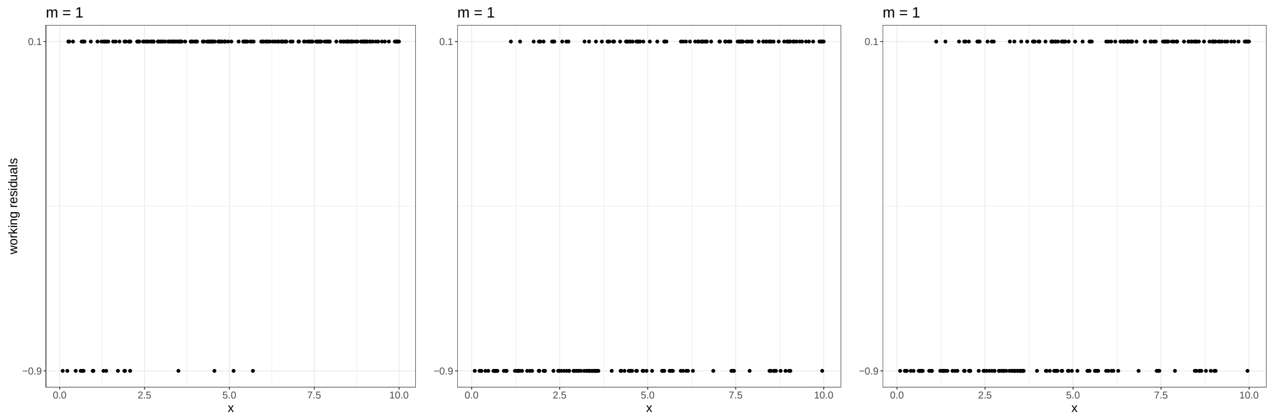

Figure 2: Working residuals of the first iteration against predictor x. Left: AL1BRQ initialized with 10% sample quantile (dashed line). Middle: AL1BRQ initialized with the sample median. Right: L2BRQ initialized with the sample median.

2c. Compute a least squares/quantile regression fit for each base learner.

Given that the illustrative example at hand contains only one predictor, only one linear quantile regression fit, $\hat{Q}_{u_i^{[m]}}(\tau = 0.1|x_i) = \hat{a}_{0.1,1} + \hat{b}_{0.1,1}x_i$, is obtained in each iteration in the AL1BRQ algorithm. Figure 3 (bottom left and middle panel) illustrates the linear quantile regression fit for both initializations (10% sample quantile vs. sample median) for iteration $m = 31$, which is the first iteration for which both slope estimates are nonzero.

If we were to initialize AL1BRQ with the sample median (representing the intermediate between both approaches), 50% of the observations are smaller than the initial fitted values, $\hat{\eta}^{[0]}_{0.1,i}$, resulting in a steeper slope for the linear quantile regression. This, in turn, can lead to a faster convergence of the algorithm. In the present case, 31 iterations are still required until a quantile regression model with a nonzero slope is fitted to the working residuals. The first 30 iterations estimate quantile regression models with only an intercept (as displayed in Figure 3, upper middle panel), aiming to convert negative to positive residuals to eventually fit a quantile regression model with a nonzero slope.

When AL1BRQ is initialized as specified with the 10% sample quantile, a quantile regression model with a nonzero slope can already be fitted in the first iteration (Figure 3, upper left panel), certainly with a flatter slope compared to the bottom middle panel of Figure 3.

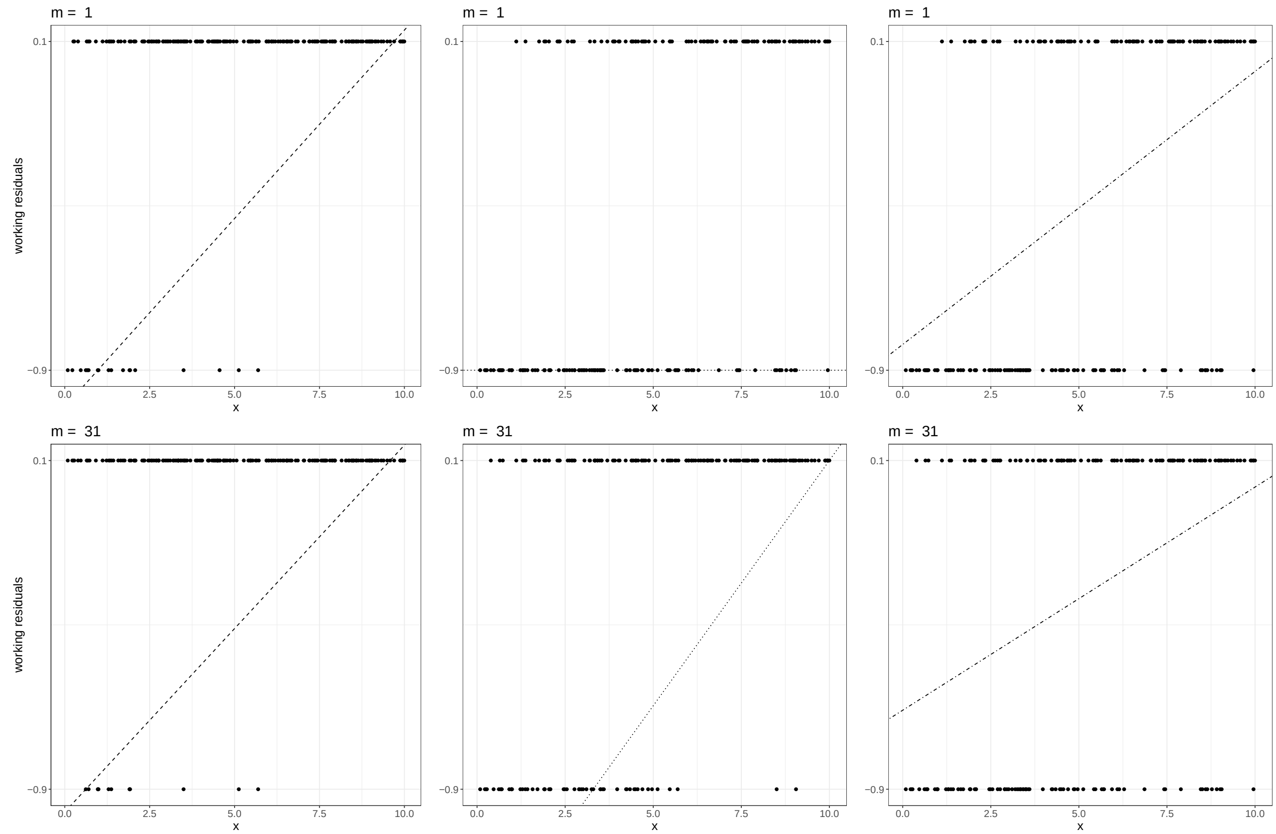

Figure 3: Working residuals of the 1st (top row) 31th (bottom row) iteration are plotted against the predictor x. Left: AL1BRQ initialized with the 10% sample quantile. Middle: AL1BRQ initialized with the sample median. Right: L2BRQ initialized with the sample median. Dashed, dotted and dash-dotted lines indicate the respective linear quantile regression fit for $\tau = 0.1$ or the respective linear LS regression fit.

Our example shows that it is not per se beneficial to initialize AL1BRQ with a higher sample quantile than the respective sample quantile (here 10%). While this leads to a steeper slope, it comes at the cost of longer lead time until a first quantile regression model with a nonzero slope estimate is fitted: In the present case, e.g., initializing the algorithm with an extreme sample quantile such as the 90% requires 100 iterations until a nonzero slope is fitted for the first time.

The algorithm converges fastest for an initialization which respects this tradeoff, i.e., leads to a steep slope while only requiring a short lead time. Which sample quantile represents the optimal initialization for the AL1BRQ algorithm, depends on the following three attributes of the true underlying data generating process.

(i) The *magnitude of the true underlying coefficient effect* does not affect the magnitude of the estimated slope in the individual iterations – as long as the sign of the residuals remains the same and thus the working residuals do not change. The magnitude of the coefficient effects is therefore proportional to the number of iterations required until the AL1BRQ algorithm converges.

Consequently, if the true underlying coefficient effect is relatively small, it takes not too many iterations until the algorithm converges. In that case, long lead times are especially costly and may not be compensated by a steeper resulting slope when the algorithm is initialized with a sample quantile in the direction of the sample median[3]. Hence, the AL1BRQ algorithm should be initialized with a sample quantile near τ. A change in the magnitude of the intercept, however, has no impact on the number of iterations required as this is captured by the initialization.

On the other hand, if the true underlying coefficient effect is large, it takes a considerable amount of time for the algorithm to converge. In that case, a steeper slope may compensate for longer lead times in the long run. Thus, the AL1BRQ algorithm should be initialized with a sample quantile in the direction of the sample median.

[3]Choosing a sample quantile in the opposite direction of the sample median always leads to inferior results, as the estimated slope becomes flatter and lead times increase.

(ii) The *value range of the predictor* affects the magnitude of the estimated slope in the individual iterations of the AL1BRQ algorithm: The estimated slope in the single iterations is larger for smaller value ranges of the predictor.

An argument similar to (i) applies: If the value range of the predictor is relatively small, the estimated slope in the individual iterations is larger, resulting in a fast convergence of the algorithm. In that case, long lead times are especially costly and may not be compensated by a steeper slope. Thus, the AL1BRQ algorithm should be initialized with a sample quantile near τ.

On the other hand, if the value range of the predictor is large, the estimated slope in the individual iterations is rather small, resulting in a slowly converging algorithm. In that case, a steeper slope may compensate for longer lead times in the long run and the AL1BRQ algorithm should be initialized with a sample quantile in the direction of the sample median.

(iii) If the *variance of the error term* is large, the individual observations scatter widely, thus, many iterations may be required until the sign of a sufficient number of residuals is changed to eventually fit a quantile regression with a nonzero slope estimate, resulting in long lead times. The resulting steeper slope in each iteration, when the AL1BRQ algorithm is initialized with a sample quantile in the direction of the sample median, may not compensate for the resulting longer lead times. As a result, the AL1BRQ algorithm should be initialized with a sample quantile near τ.

On the other hand, if the variance of the error term is small, not many iterations are required until the sign of a sufficient number of residuals is changed to fit a quantile regression with a nonzero slope estimate. Thus, a steep slope may compensate for longer lead times and the AL1BRQ algorithm should be initialized with a sample quantile in the direction of the sample median.

In practice, since both the error variance and the magnitude of the coefficient effects are unknown, the sample median of the response is a reasonable initial estimate in terms of fast convergence. As center of the distribution, the sample median balances the effects of a steep slope against long lead times quite well and is never the worst choice in terms of the number of iterations required. However, as the true underlying data generating process is unknown, it is impossible to predict which initialization value will produce the best results. In our opinion, the most intuitive choice for initialization is the respective τth sample quantile, leading to an interpretation of $\hat{\eta}_\tau^{[0]}$ that is consistent with the interpretation when fitting a quantile regression model with an intercept only.

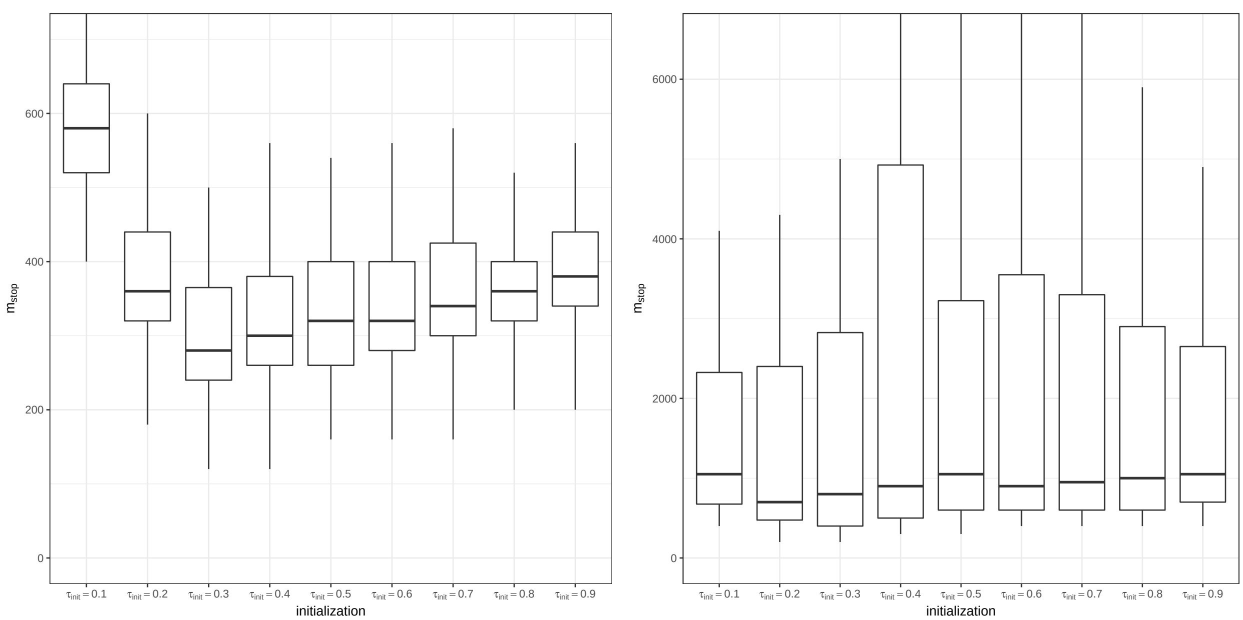

Figure 4: Boxplots of the empirical distribution of the optimal number of iterations for different initializations of the AL1BRQ (L2BRQ) algorithm in the left (right) panel. The abscissa indicates the respective sample quantile $\tau_{\text{init}} \in (0.1, 0.2, ..., 0.9)$ used for initialization. The results are obtained from $K = 100$ simulation runs estimating the 10% regression quantile by AL1BRQ (L2BRQ) for the data generating process of Equation 1. Outliers are removed from each boxplot for visualization purposes.

Figure 4 illustrates the empirical distribution of the optimal number of iterations for different initializations of the AL1BRQ algorithm, which follows a U-shaped course reflecting the tradeoff between a steeper slope and long lead times. In the underlying example, the AL1BRQ algorithm converges fastest (i.e., requirest the smallest median number of iterations) when initialized with the 30% sample quantile.

Moreover, the initialization of the algorithm only affects the number of iterations required, but not the estimation accuracy: AL1BRQ achieves similar empirical risk with different initializations (see Table 1).

The L2BRQ results are similar: Given that the example at hand contains only one predictor, only one least squares regression fit, $\hat{E}(u_i^{[m]}|x_i) = \hat{a} + \hat{b}x_i$, is obtained in each iteration in the L2BRQ algorithm. The right column of Figure 3 illustrates the linear LS regression fit for iterations $m = 1$ and $m = 31$.

Compared to the left column of Figure 3 for AL1BRQ, two facts are evident: First, the slope obtained in L2BRQ for iteration $m = 31$ is flatter than the slope in AL1BRQ, which may indicate that AL1BRQ converges faster than L2BRQ. Second, although the L2BRQ algorithm is initialized with the sample median, a regression model with a nonzero slope is fitted in the first iteration. As simple linear regression means are chosen as the base learners, the slope estimate is always (at least marginally) nonzero.

Consequently, initializing with a more extreme sample quantile (in the direction of the median) and obtaining a steeper slope are not directly related. There is no equivalent to the clear tradeoff between a steeper slope and long lead times observed for AL1BRQ. Figure 4 reflects this finding: The empirical distribution of the optimal number of iterations for different initializations of the L2BRQ algorithm does not follow the same U-shaped course as for AL1BRQ. On the contrary, no relationship between τ_{init} and the required number of iterations is visible.

Equivalently to AL1BRQ, L2BRQ achieves similar estimation accuracy with different initializations and shows no effect of the initialization of the algorithm on estimation accuracy (see Table 1).

	τ_{init}				
Method	0.1	0.3	0.5	0.7	0.9
AL1BRQ	0.682	0.683	0.682	0.681	0.680
L2BRQ	0.682	0.678	0.674	0.674	0.675

Table 1: Estimation accuracy of the AL1BRQ and L2BRQ algorithm measured by the empirical risk R_τ for different initializations. τ_{init} represents the sample quantile used for initialization in Step 1 of the algorithms. The results are obtained from $K = 100$ simulation runs estimating the 10% quantile regression by AL1BRQ and L2BRQ for the data generating process of Equation 1.

3. Update one component.

3a. Select the component x_j that fits the working residuals best.

In our example, only one predictor (ergo, only one base learner) is considered. As a result, this predictor is selected as the best-fitting component in every iteration for both algorithms, AL1BRQ and L2BRQ.

3b. Update the estimate of the τth regression quantile function.

Multiplying the coefficient estimates by a pre-specified learning rate ensures that the effect estimates are adjusted only slightly in each iteration m (see Figure 5).

As outlined above, the slope in the quantile regression fit of Step 2c can be estimated to be zero (Figure 3, upper middle panel). Selecting predictor x_j as the best-fitting base learner does not necessarily imply that its coefficient estimate is updated in that iteration. In fact, although the predictor x_j has been selected as the best-fitting base learner a few times, its effect estimate may never be updated and thus equal zero (see the estimate for the 10% conditional quantile function after 30 iterations in Figure 5, middle panel).

Iterations with a nonzero slope estimate for the best-fitting base learner, i.e., where the predictor makes an explanatory contribution to the model, should be distinguished from iterations with a zero slope estimate, i.e., where the predictor does not contribute to the model, but the intercept does. For L2BRQ, such differentiation is not possible, since simple linear regression means are chosen as base learners and therefore the slope estimate is always nonzero. When a predictor is selected as the best-fitting component in L2BRQ, its effect estimate is always updated (see dashdotted lines in Figure 6).

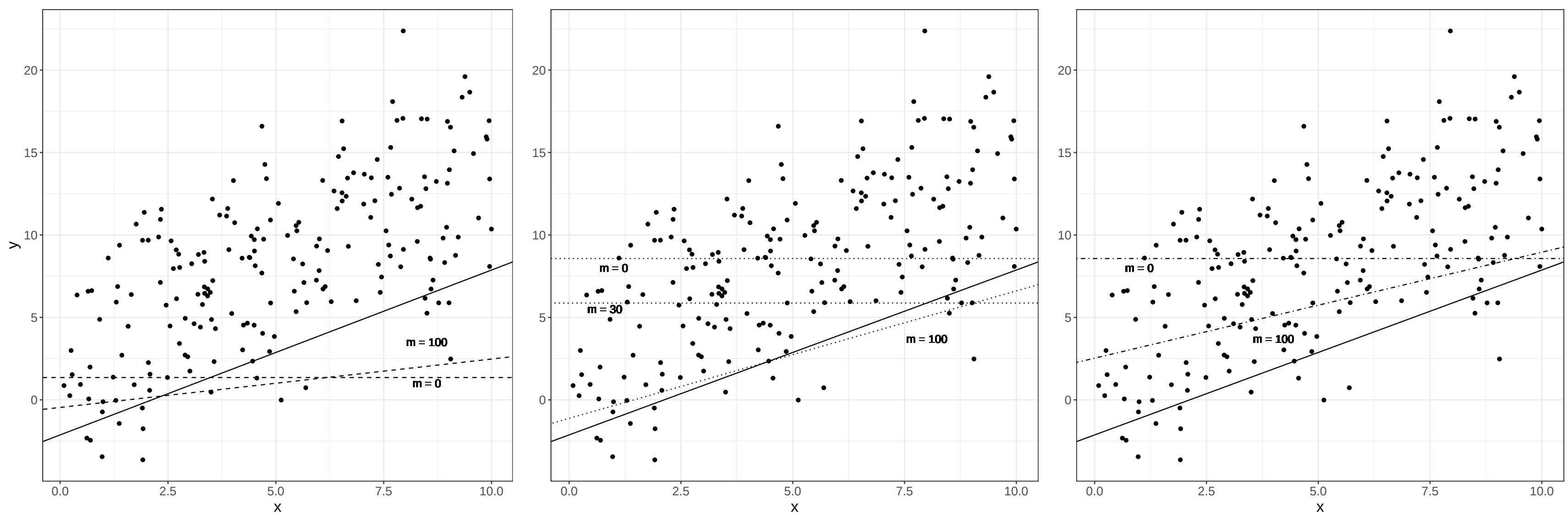

Figure 5: Evolution of the slope coefficient. Left: Dashed lines indicate the estimates for the 10% conditional quantile function after 0 and 100 iterations for the AL1BRQ algorithm initialized with the 10% sample quantile. Middle: Dotted lines indicate the estimates for the 10% conditional quantile function after 0, 30, and 100 iterations for the AL1BRQ algorithm initialized with the sample median. Right: Dashdotted lines indicate the estimate for the 10% conditional quantile function after 0 and 100 iterations for the L2BRQ algorithm initialized with the sample median. Black lines indicate the true underlying 10% quantile curve.

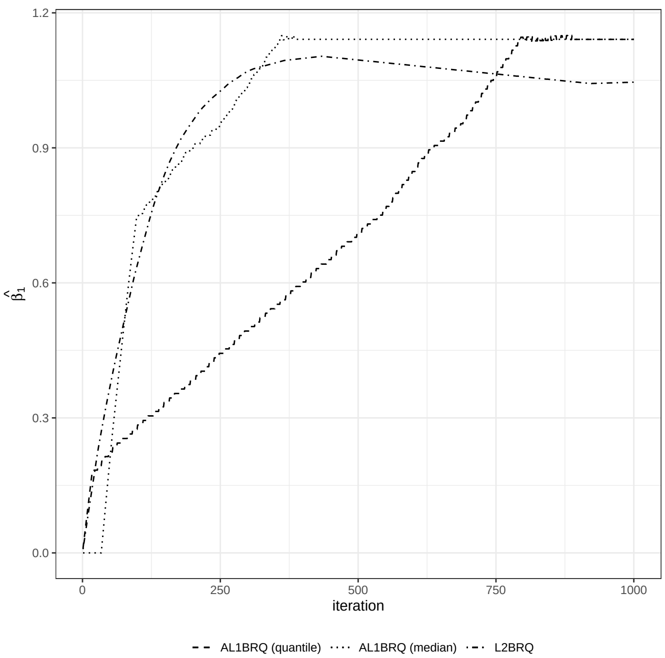

Figure 6: The dashed, dotted, and dashdotted lines represent the coefficient paths of the estimate $\hat{\beta}_{0.1,1}$ of AL1BRQ initialized with the 10% sample quantile, AL1BRQ initialized with the sample median, and L2BRQ, respectively, for the first 1000 iterations. The algorithms converge after 640, 200, and 4600 iterations, respectively.

4. Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

A.3. General implications of the component-wise functional gradient boosting algorithm

Since AL1BRQ and L2BRQ are adaptations of the component-wise functional gradient boosting algorithm, the following implications regarding simultaneous model estimation and variable selection, as well as model choice/functional form, apply to them as well. Additionally, the choice of the two tuning parameters (number of iterations and learning rate) is motivated.

A.3.1. Variable selection

In Step 1, simple linear regression means are chosen as the base learners for all predictors. In Step 3a, only the best-fitting base learner is selected. As a result, some predictors may never be selected during the m_{stop} iterations. The remaining predictors were initially set to zero in Step 1 and are never updated (Hofner et al., 2014, p. 6), hence, omitted from the final model. The component-wise functional gradient boosting algorithm can generally perform variable selection, provided the base learners are adequately specified and the algorithm is stopped before convergence (“early stopping”) (Mayr et al., 2014a, p. 425).

In summary, model estimation and variable selection are conducted simultaneously during the boosting iterations, leading to significant reductions in computation time compared to the exhaustive all subset selection in classical quantile regression. This feature proves especially useful in high-dimensional settings (Bühlmann and Hothorn, 2007, p. 491), i.e., in situations where the number of predictor is much larger than the number of observations ($p \gg n$). In these settings, many classical statistical learning algorithms, which do not conduct inherent variable selection, become infeasible (Mayr et al., 2014b, p. 429).

Nonetheless, component-wise functional gradient boosting using squared error loss may still include too many irrelevant predictors in the final model. Bühlmann and Yu (2006) propose so-called “sparse boosting”, which uses a penalized squared error loss criterion for selection in Step 3a. However, considering that predictive performance and estimation accuracy (obtaining sparse and interpretable models) are different analysis goals, it is difficult to do justice to both at the same time. Especially since predictors may be irrelevant to the interpretation of the model but relevant to improving predictive performance (Mayr et al., 2014b, p. 431).

A.3.2. Model choice and functional form

Besides variable selection, the component-wise functional gradient boosting algorithm also proves useful for model selection. Defining multiple functional forms of base learners for one predictor, e.g., linear and nonlinear, the component-wise functional gradient boosting algorithm chooses the best-fitting component in each iteration and thus decides not only whether to include the predictor, but also in which functional form: linear, nonlinear, or both. To warrant unbiased variable selection, one should ensure that the complexity (i.t.o. degrees of freedom) of not only the base learners defined for the same predictors, but all base learners is comparable, otherwise the component-wise functional gradient algorithm systematically prefers more complex base learners (Hofner et al., 2011, p. 956).

A.3.3. Interpretability

Owing to the additive updating of the coefficient estimates in Step 3b, the base learners bequeath their structure to the resulting estimate for the conditional quantile function, $\hat{\eta}_\tau^{[m_{\text{stop}}]}$ (Bühlmann and Hothorn, 2007, p. 484). If in Step 1, simple linear regression means are chosen as base learners, that results in a linear estimate for the conditional quantile function. Therefore, the individual quantile-specific predictor effects are interpretable.

A.3.4. Stage-wise and component-wise nature

From Step 3, it is apparent why the algorithm is termed a "forward stage-wise and component-wise additive gradient" boosting algorithm: The coefficient estimates of only one component, the best-fitting one, are additively updated in each iteration. Moreover, in each iteration, an estimate for the negative gradient of the loss function is added to the current fitted values, resulting in a stage-wise reduction of empirical loss (Hofner et al., 2014, p. 6). The component-wise functional gradient boosting algorithm is also described as a "greedy stage-wise approach" (Friedman, 2001, p. 1192). This characteristic can be seen in Step 3b, where the selection of the best-fitting base learner results in the steepest descent in the empirical loss in each iteration.

A.3.5. Tuning parameters

The two tuning parameters of component-wise functional gradient boosting, the learning rate, ν, and the number of iterations, m_{stop}, appear in Steps 3b and 4, respectively. The learning rate leads to only slowly increasing coefficient estimates during the boosting process. This ensures that the algorithm does not overshoot the minimum of the empirical risk and that individual estimations are regularized, such that they do not heavily influence the final outcome. Combining this fact with early stopping results in shrunk coefficient estimates. Consequently, the bias of the estimate is slightly increased while its variance is decreased, which often improves predictive performance and is known as the bias-variance tradeoff (Hofner et al., 2014, p. 7).

Choosing a relatively small value for ν (e.g., $\nu = 0.1$) is standard practice and yields reasonable results (Bühlmann and Hothorn, 2007, p. 480). Our simulation results suggest that this is also the case for AL1BRQ and L2BRQ. In turn, small values of ν require a larger number of iterations m_{stop} which are proportional to computation time (Hastie et al., 2009, p. 365). In addition, the learning rate for algorithms estimating a conditional quantile should be tied to the sparseness of the observations near the quantile of interest, since the precision of quantile regression depends on this quantity (Koenker, 2005, p. 77).

As variable selection and shrinkage can only result from early stopping, the tuning parameter m_{stop} controls both. The maximum number of iterations, m_{stop}, reflects the bias-variance tradeoff: More iterations lead to more flexible models, accompanied by greater variance but less model bias, whereas fewer iterations lead to more shrinkage and variable selection, resulting in less flexible models (Mayr et al., 2012, p. 197). One should carefully choose the right number of iterations to prevent the algorithm from overfitting the data. The optimal number of iterations m_{stop} can be determined by cross-validation, where it is crucial to use the same loss function, that the algorithm seeks to minimize (Mayr et al., 2014a, p. 425). For AL1BRQ and L2BRQ, the pinball loss function should be used.

B. Replication of the results from Fenske et al. (2011)

Estimation accuracy and variable selection properties of pinball boosting of regression quantiles, least squares boosting of regression quantiles, classical quantile regression without (RQ), and with all subset selection (RQAic) are studied in a simulation study outlined in Table 4 of the main document. Particular focus is placed on estimation accuracy, the ability to correctly identify and exclude irrelevant predictors and differences in computational time.

B.1. Evaluation criteria

AL1BRQ and L2BRQ are compared with respect to three aspects: estimation accuracy, computational efficiency and variable selection. Additional to the measures introduced in the main document, estimation accuracy is

Criterion	Parameter setup				
	Homo-skedastic	Hetero-skedastic	Multi-variate	Multi-variate2	High-dimensional
Estimation accuracy					
MSE	×	×	×	×	
τ-fit	×	×	×	×	×
R_τ	×	×	×	×	×
Bias	×	×	×	×	
Computational efficiency					
Median number of iterations	×	×	×	×	×
Variable selection					
Sensitivity					×
Specificity					×
MFI			×	×	
MPI			×	×	
PER			×	×	

Table 2: Overview of the evaluation criteria used for each parameter setup

mesaured by the Bias for each quantile-specific parameter $(\beta_{\tau 0}, \beta_{\tau 1}, ..., \beta_{\tau p})^\top$,

$$\text{Bias}(\hat{\beta}_{\tau j,k}) = \hat{\beta}_{\tau j,k} - \beta_{\tau j},$$

where $j = 0, ..., p$ denotes the respective predictor and $k = 1, ..., K$ the simulation replication (Fenske et al., 2009, p. 10).

For the multivariate setups, the variable selection properties of the boosting algorithms are compared by three additional measures: By the proportion of simulation iterations in which the respective predictor x_j is never updated (PER), by the mean proportion of boosting iterations in which the respective predictor x_j is selected with a nonzero slope estimate (MPI), and by the mean first boosting iteration in which the respective predictor x_j is first selected with a nonzero slope estimate (MFI). For L2BRQ, the number of boosting iterations with a nonzero slope estimate is equal to the total number of boosting iterations, since the slope estimate is always nonzero.

B.2. Homoskedastic and heteroskedastic setup

100 location-scale models with only one predictor for each $\tau \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, for both the homoskedastic and heteroskedastic setup, and for all error distributions are simulated. In both cases, the estimation results for $\beta_{\tau 0}$ and $\beta_{\tau 1}$ of AL1BRQ and L2BRQ share the characteristic of an increased bias (see Tables 9 and 10) and a lower variance as compared to the RQ estimates (see Figure 7).

Both boosting procedures hit the bias-variance tradeoff better and outperform RQ in terms of estimation accuracy. Consequently, the regularization has the intended effect, which is to trade a slight increase in the model's bias for a significant reduction in variance, thereby minimizing the MSE. L2BRQ performs best in terms of MSE, closely followed by AL1BRQ (see Table 7).

Across all error distributions, in the homoskedastic and heteroskedastic setup displayed in Table 5 (Table 6 for the contaminated cases), L2BRQ exhibits the smallest R_τ for the major part of setups, often closely followed by AL1BRQ. Ultimately, L2BRQ and AL1BRQ perform similar in terms of estimation accuracy, although L2BRQ is more often in the lead, albeit just barely. RQ performs weakest, but still shows competitive results.

In terms of in-sample accuracy, as measured by τ-fit, RQ performs best, while AL1BRQ and L2BRQ are on par (compare Tables 3 and 4).

B.3. Multivariate setup

Equivalent to the bivariate case, both boosting algorithms lead to an increased estimation bias for $(\beta_{\tau 0}, \beta_{\tau 1}, ..., \beta_{\tau 6})^\top$ (see Tables 9 and 10). In terms of estimation accuracy measured by MSE, AL1BRQ, L2BRQ, and RQAic equally well. Comparing the results of RQAic and RQ, the panel for the multivariate setup in Table 7 shows that the inclusion of the predictors five and six leads to poorer estimation accuracy. This effect is expected to be even more significant with a large number of irrelevant predictors (see Subsections B.5). As long as all subset selection is feasible, it represents a competitive approach in terms of estimation accuracy measured by the MSE compared to the boosting algorithms. In high dimensional data settings with a large number of predictors (and a possibly large

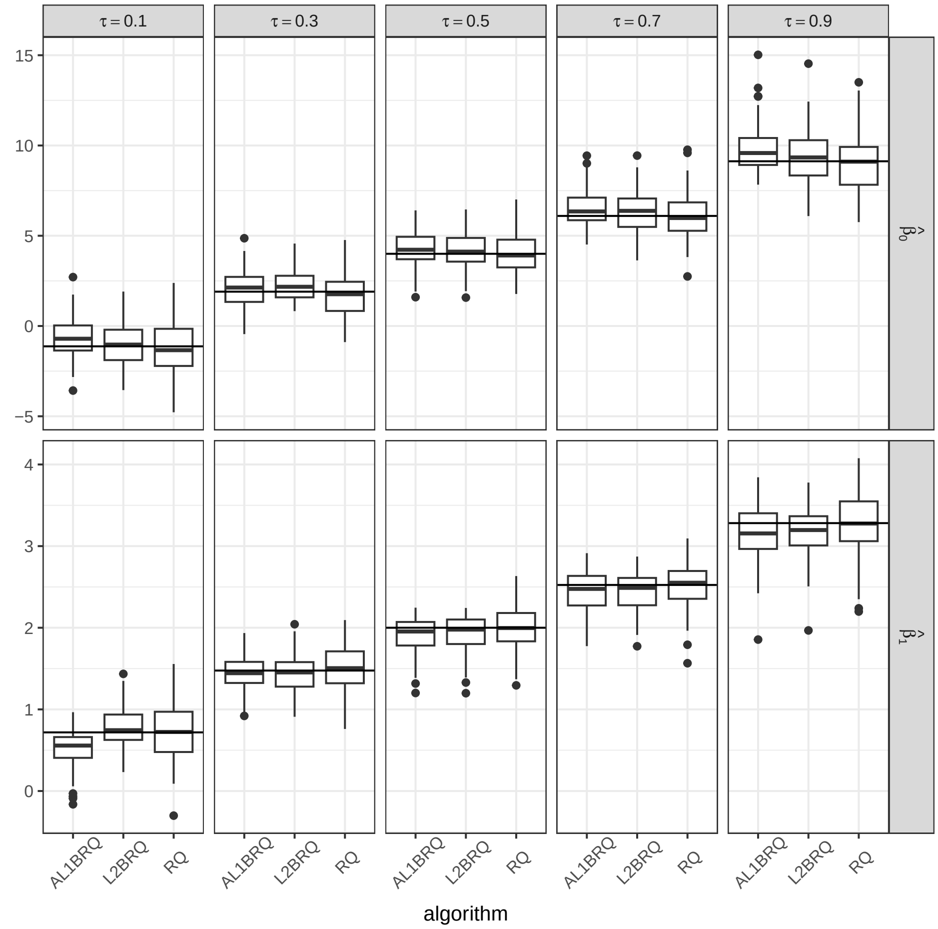

Figure 7: Boxplots of the empirical distribution of the estimated parameters $\hat{\beta}_{\tau 0}$ and $\hat{\beta}_{\tau 1}$ from $K = 100$ simulation runs (heteroskedastic setup, normal errors, not contaminated), for each τ and estimation procedure (AL1BRQ, L2BRQ, and RQ).

number of irrelevant predictors), all subset selection is computationally infeasible and the boosting algorithms are expected to outperform classical quantile regression.

The empirical risk results for the multivariate setup mimic those obtained previously for the homoskedastic and heteroskedastic setup: L2BRQ performs better than AL1BRQ, although the margin over AL1BRQ is peripheral (see Table 5 and Table 6).

In terms of in-sample accuracy, as measured by τ-fit, AL1BRQ, L2BRQ, and RQ perform equally well (see Tables 3 and 4).

In terms of variable importance, AL1BRQ provides superior interpretability regarding the importance of predictors to the model compared to L2BRQ. As all predictors are drawn from the same distribution, the magnitude of the respective predictor effects $(\beta_{\tau 1}, ..., \beta_{\tau 6})^\top$ indicates its importance. The predictor effects for different τ and error distribution are displayed in Table 11. Given $\tau = 0.5$ and normal errors, the first predictor is most important, while the fifth and sixth are not relevant for the model. This fact can be reflected in MPI and MFI: More important predictors should be selected more frequently during the boosting iterations – resulting in a larger MPI – and less important predictors should be selected in, if any, later stages of the boosting iterations – translating to a larger MFI.

The panel for the multivariate setup in Table 12 shows that AL1BRQ manages to clearly rank the predictors according to their importance in the model: Exemplary for $\tau = 0.5$ and normal errors, x_1 is the most important one with an MPI of 0.449, whereas x_5 and x_6 are least important with an MPI of 0.09 and 0.012 each. The ranking is less pronounced for L2BRQ as x_1 and x_2 show similiar MPIs (0.276 vs. 0.256) and x_5 and x_6 as the least important predictors receive MPIs of 0.061 each. Moreover, the MPIs for L2BRQ are subject to greater uncertainty compared to AL1BRQ, as evidenced by a larger variance and a larger number of outliers across the 100 simulation runs (see Figure 8).

Regarding the ability to correctly identify and exclude irrelevant predictors from the final model, both boosting algorithms perform poorly especially in comparison to RQAic. AL1BRQ excludes the irrelevant predictors x_5 and

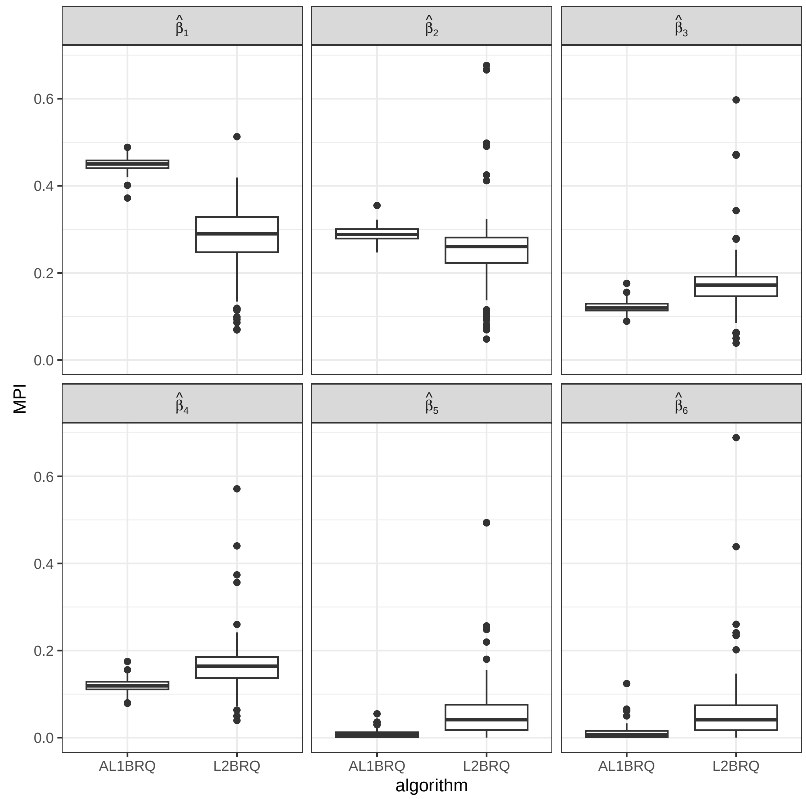

Figure 8: Boxplots for the empirical distribution of the proportion of selection iterations (from mslope iterations, where mslope indicates the number of iterations where the selected component has a nonzero slope estimate) for each predictor, obtained from 100 simulation runs (multivariate setup, normal errors, not contaminated, and $\tau = 0.5$).

x_6 only 13 and 11 times, respectively, out of 100 cases, and L2BRQ only six and eight times, respectively. In contrast, RQAic manages to exclude x_5 and x_6 58 and 97 times.

However still, AL1BRQ is able to identify those predictors as irrelevant, as x_5 and x_6 are selected for the first time after 93.5% and 93.3% of the boosting algorithm are completed, while L2BRQ selected them for the first time after 57.7% and 60.7% of the boosting algorithm are completed (see Table 14 (exemplary for $\tau = 0.5$ and normal errors). Moreover, MFI_5 and MFI_6 for L2BRQ are accompanied by great uncertainty, indicated by high variance across the 100 simulation runs (see Figure 9). The ability to unambiguously rank the predictors by their importance could favor AL1BRQ compared to L2BRQ i.t.o. variable selection in a high-dimensional setup with many irrelevant predictors. This setting is further discussed in the following section B.5.

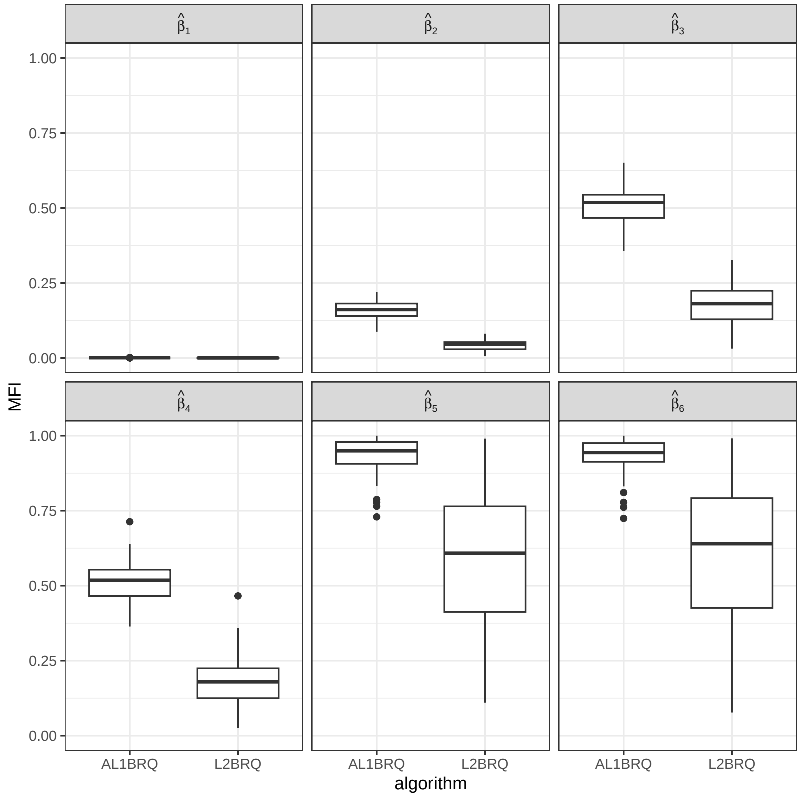

Figure 9: Boxplots for the empirical distribution of the first selection iteration (from *mslope* iterations) for each predictor, obtained from 100 simulation runs (multivariate setup, normal errors, not contaminated, and $\tau = 0.5$).

B.4. Multivariate2 setup

In terms of the empirical risk, L2BRQ also performs best in the multivariate2 setup, again closely followed by AL1BRQ (see Table 5 and Table 6).

Recall that predictor x_1 only influences lower quantiles $\tau \in \{0.1, 0.3\}$ and predictor x_3 only influences higher quantiles $\tau \in \{0.7, 0.9\}$. We would suspect the method at hand to exclude x_1 (x_3) in upper (lower) quantiles and to include them in the remaining quantiles.

Indeed AL1BRQ, L2BRQ and RQAic manage to include predictor x_1 for the two lower quantiles for all error distributions. However, RQAic includes the predictor for *all* quantiles under consideration, even the remaining quantiles, where all methods should exclude the predictor. Furthermore, L2BRQ excludes x_1 for the remaining quantiles more frequently. Similar results can be observed for x_3: RQAic never excludes x_3 in the lower quantiles, while L2BRQ and AL1BRQ manage to do so (see Table 16).

As for the irrelevant predictors x_5 and x_6, the results remain essentially the same as in the multivariate setup. Overall, our simulation results suggest that RQAic reliably excludes irrelevant predictors if they are irrelevant for the entire conditional distribution of y, but not if they are irrelevant only for parts of it. In contrast, L2BRQ and AL1BRQ do not exclude predictors, that are irrelevant for the entire conditional distribution as reliably as RQAic, but exclude predictors that are only irrelevant for parts of the conditional distribution to a similar degree.

In fact, across all error distributions, RQAic excludes many relevant predictors from the final quantile regression model for all quantiles, while AL1BRQ and L2BRQ rarely do so (see Table 16).

B.5. High-dimensional setup

In the high-dimensional setup, L2BRQ achieves the best results i.t.o. the empirical risk, again, closely followed by AL1BRQ. Both boosting algorithms clearly outperform classical quantile regression (RQ) (see Table 5). Thus, amid the risk of potentially including a large number of irrelevant predictors in the model, RQ is no longer a competitive approach.

Regarding variable selection, AL1BRQ and L2BRQ show similar results for more details please refer to the main document.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	AL1BRQ	0.185	0.195	0.197	0.199	0.181
		L2BRQ	0.188	0.194	0.197	0.2	0.182
		RQ	0.183	0.194	0.199	0.199	0.188
	tdist	AL1BRQ	0.051	0.098	0.111	0.092	0.044
		L2BRQ	0.049	0.099	0.111	0.093	0.047
		RQ	0.055	0.097	0.112	0.097	0.054
	gamma	AL1BRQ	0.251	0.18	0.14	0.097	0.06
		L2BRQ	0.252	0.182	0.14	0.096	0.059
		RQ	0.251	0.183	0.132	0.103	0.071
	mixed	AL1BRQ	0.123	0.202	0.239	0.235	0.162
		L2BRQ	0.128	0.201	0.239	0.236	0.164
		RQ	0.116	0.207	0.246	0.229	0.165
heteroskedastic	norm	AL1BRQ	0.028	0.11	0.17	0.231	0.288
		L2BRQ	0.031	0.11	0.17	0.231	0.287
		RQ	0.034	0.11	0.176	0.235	0.289
	tdist	AL1BRQ	0.003	0.045	0.087	0.126	0.115
		L2BRQ	0.002	0.048	0.087	0.127	0.115
		RQ	0.009	0.047	0.097	0.13	0.125
	gamma	AL1BRQ	0.319	0.295	0.287	0.269	0.273
		L2BRQ	0.321	0.296	0.287	0.268	0.271
		RQ	0.321	0.3	0.284	0.275	0.279
	mixed	AL1BRQ	0.052	0.174	0.258	0.298	0.266
		L2BRQ	0.059	0.174	0.258	0.299	0.284
		RQ	0.061	0.177	0.261	0.299	0.284
multivariate	norm	AL1BRQ	0.546	0.534	0.53	0.517	0.486
		L2BRQ	0.546	0.534	0.53	0.517	0.487
		RQ	0.545	0.533	0.529	0.517	0.486
		RQAic	0.543	0.532	0.528	0.516	0.483
	tdist	AL1BRQ	0.272	0.351	0.356	0.317	0.196
		L2BRQ	0.272	0.351	0.356	0.316	0.193
		RQ	0.283	0.348	0.361	0.328	0.194
		RQAic	0.272	0.347	0.36	0.327	0.172
	gamma	AL1BRQ	0.51	0.44	0.385	0.314	0.238
		L2BRQ	0.51	0.44	0.385	0.314	0.238
		RQ	0.515	0.444	0.379	0.316	0.236
		RQAic	0.514	0.443	0.377	0.274	0.231
	mixed	AL1BRQ	0.31	0.378	0.388	0.346	0.254
		L2BRQ	0.309	0.378	0.389	0.346	0.256
		RQ	0.307	0.376	0.389	0.35	0.254
		RQAic	0.299	0.375	0.388	0.347	0.24
multivariate2	norm	AL1BRQ	0.542	0.531	0.31	0.272	0.183
		L2BRQ	0.541	0.531	0.31	0.272	0.182
		RQ	0.542	0.526	0.313	0.27	0.197
		RQAic	0.433	0.453	-0.04	-0.257	-0.288
	tdist	AL1BRQ	0.274	0.339	0.181	0.132	0.032
		L2BRQ	0.275	0.338	0.18	0.132	0.03
		RQ	0.272	0.338	0.183	0.138	0.04
		RQAic	0.213	0.281	-0.036	-0.182	-0.069
	gamma	AL1BRQ	0.506	0.428	0.033	0.037	0.071
		L2BRQ	0.506	0.428	0.033	0.039	0.072
		RQ	0.507	0.432	0.043	0.041	0.076
		RQAic	0.455	0.414	-0.022	-0.035	0.03
	mixed	AL1BRQ	0.316	0.375	0.129	0.107	0.095
		L2BRQ	0.316	0.375	0.129	0.108	0.098
		RQ	0.309	0.377	0.129	0.103	0.101
		RQAic	0.27	0.351	0.112	0.1	0.081
high-dimensional	norm	AL1BRQ	0.595	0.571	0.561	0.564	0.526
		L2BRQ	0.575	0.571	0.559	0.556	0.515
		RQ	0.801	0.722	0.699	0.715	0.776
	tdist	AL1BRQ	0.351	0.39	0.392	0.361	0.246
		L2BRQ	0.322	0.387	0.39	0.352	0.19
		RQ	0.665	0.561	0.544	0.549	0.612
	gamma	AL1BRQ	0.543	0.474	0.396	0.347	0.284
		L2BRQ	0.538	0.469	0.394	0.333	0.261
		RQ	0.733	0.642	0.6	0.603	0.707
	mixed	AL1BRQ	0.379	0.416	0.415	0.367	0.319
		L2BRQ	0.354	0.411	0.416	0.361	0.276
		RQ	0.68	0.594	0.576	0.586	0.678

Table 3: τ-fit of AL1BRQ, L2BRQ, RQ and RQAic for all parameter setups and all error distributions for each τ. Extension of Table 8 from the main document. Blue values indicate the superior result in the respective category.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	AL1BRQ	0.164	0.165	0.157	0.144	0.093
		L2BRQ	0.166	0.165	0.157	0.144	0.094
		RQ	0.167	0.173	0.158	0.146	0.091
	tdist	AL1BRQ	0.041	0.071	0.074	0.058	0.02
		L2BRQ	0.042	0.072	0.074	0.059	0.021
		RQ	0.049	0.07	0.078	0.056	0.022
	gamma	AL1BRQ	0.209	0.141	0.093	0.061	0.028
		L2BRQ	0.21	0.142	0.093	0.06	0.027
		RQ	0.209	0.146	0.096	0.063	0.031
	mixed	AL1BRQ	0.099	0.157	0.181	0.154	0.078
		L2BRQ	0.101	0.158	0.181	0.155	0.078
		RQ	0.11	0.161	0.173	0.15	0.074
heteroskedastic	norm	AL1BRQ	0.029	0.087	0.13	0.175	0.146
		L2BRQ	0.036	0.087	0.13	0.174	0.146
		RQ	0.036	0.089	0.137	0.167	0.152
	tdist	AL1BRQ	0.003	0.031	0.065	0.073	0.046
		L2BRQ	0.002	0.032	0.065	0.073	0.046
		RQ	0.007	0.034	0.065	0.075	0.054
	gamma	AL1BRQ	0.257	0.228	0.204	0.181	0.132
		L2BRQ	0.258	0.229	0.204	0.18	0.131
		RQ	0.257	0.233	0.214	0.183	0.132
	mixed	AL1BRQ	0.041	0.133	0.197	0.198	0.117
		L2BRQ	0.044	0.133	0.197	0.198	0.124
		RQ	0.054	0.138	0.189	0.185	0.131
multivariate	norm	AL1BRQ	0.528	0.51	0.494	0.471	0.374
		L2BRQ	0.529	0.51	0.494	0.471	0.374
		RQ	0.533	0.509	0.491	0.466	0.379
		RQAic	0.531	0.508	0.49	0.465	0.376
	tdist	AL1BRQ	0.253	0.312	0.295	0.244	0.111
		L2BRQ	0.253	0.312	0.295	0.244	0.11
		RQ	0.263	0.307	0.294	0.245	0.112
		RQAic	0.251	0.306	0.293	0.244	0.098
	gamma	AL1BRQ	0.483	0.401	0.332	0.258	0.158
		L2BRQ	0.483	0.401	0.333	0.259	0.158
		RQ	0.485	0.404	0.334	0.258	0.155
		RQAic	0.484	0.403	0.331	0.22	0.151
	mixed	AL1BRQ	0.283	0.343	0.329	0.267	0.155
		L2BRQ	0.283	0.343	0.33	0.267	0.156
		RQ	0.289	0.342	0.328	0.267	0.169
		RQAic	0.284	0.342	0.327	0.263	0.157
multivariate2	norm	AL1BRQ	0.532	0.504	0.3	0.257	0.167
		L2BRQ	0.532	0.504	0.3	0.257	0.166
		RQ	0.53	0.509	0.309	0.268	0.162
		RQAic	0.42	0.435	-0.021	-0.243	-0.326
	tdist	AL1BRQ	0.252	0.296	0.149	0.096	0.019
		L2BRQ	0.252	0.296	0.149	0.097	0.018
		RQ	0.249	0.298	0.147	0.1	0.022
		RQAic	0.195	0.244	-0.032	-0.149	-0.053
	gamma	AL1BRQ	0.474	0.391	0.029	0.03	0.052
		L2BRQ	0.474	0.391	0.03	0.032	0.052
		RQ	0.474	0.394	0.035	0.036	0.054
		RQAic	0.42	0.379	-0.021	-0.035	0.017
	mixed	AL1BRQ	0.288	0.334	0.1	0.077	0.054
		L2BRQ	0.287	0.334	0.1	0.077	0.056
		RQ	0.285	0.34	0.106	0.08	0.062
		RQAic	0.247	0.318	0.09	0.077	0.047
high-dimensional	norm	AL1BRQ	0.558	0.508	0.482	0.445	0.311
		L2BRQ	0.547	0.505	0.483	0.443	0.308
		RQ	0.75	0.655	0.61	0.568	0.483
	tdist	AL1BRQ	0.304	0.314	0.286	0.235	0.123
		L2BRQ	0.291	0.308	0.286	0.231	0.097
		RQ	0.592	0.461	0.402	0.359	0.303
	gamma	AL1BRQ	0.479	0.389	0.316	0.233	0.135
		L2BRQ	0.472	0.386	0.314	0.224	0.128
		RQ	0.636	0.525	0.462	0.409	0.347
	mixed	AL1BRQ	0.338	0.345	0.325	0.254	0.154
		L2BRQ	0.318	0.343	0.326	0.247	0.13
		RQ	0.591	0.479	0.435	0.413	0.338

Table 4: τ-fit of AL1BRQ, L2BRQ, RQ and RQAic for the contaminated cases of all parameter setups and all error distributions for each τ. Extension of Table 9 from the main document. Blue values indicate the superior result in the respective category.

			τ				
Parameter setup	Error distribution	Method	0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	AL1BRQ	0.680	1.371	1.583	1.399	0.723
		L2BRQ	0.675	1.370	1.583	1.399	0.723
		RQ	0.682	1.372	1.583	1.403	0.728
	tdist	AL1BRQ	1.298	2.149	2.418	2.204	1.351
		L2BRQ	1.289	2.149	2.417	2.204	1.347
		RQ	1.300	2.153	2.422	2.211	1.361
	gamma	AL1BRQ	0.673	1.630	2.118	2.128	1.304
		L2BRQ	0.673	1.628	2.117	2.126	1.299
		RQ	0.676	1.629	2.121	2.133	1.301
	mixed	AL1BRQ	1.453	2.592	3.017	2.815	1.708
		L2BRQ	1.443	2.590	3.017	2.811	1.701
		RQ	1.456	2.596	3.018	2.817	1.707
heteroskedastic	norm	AL1BRQ	1.512	3.042	3.538	3.125	1.637
		L2BRQ	1.493	3.040	3.537	3.123	1.637
		RQ	1.507	3.054	3.541	3.136	1.644
	tdist	AL1BRQ	2.954	4.827	5.411	4.948	3.045
		L2BRQ	2.937	4.826	5.411	4.941	3.032
		RQ	2.968	4.848	5.418	4.950	3.062
	gamma	AL1BRQ	1.520	3.675	4.779	4.794	2.908
		L2BRQ	1.521	3.673	4.779	4.787	2.902
		RQ	1.525	3.676	4.782	4.796	2.917
	mixed	AL1BRQ	3.321	6.071	7.114	6.672	4.127
		L2BRQ	3.297	6.069	7.110	6.661	3.987
		RQ	3.330	6.070	7.125	6.668	4.021
multivariate	norm	AL1BRQ	2.823	5.466	6.231	5.400	2.709
		L2BRQ	2.821	5.463	6.231	5.400	2.707
		RQ	2.832	5.484	6.243	5.418	2.721
		RQAic	2.831	5.479	6.240	5.410	2.720
	tdist	AL1BRQ	5.960	9.404	10.472	9.819	6.316
		L2BRQ	5.942	9.397	10.472	9.817	6.311
		RQ	5.969	9.401	10.472	9.818	6.365
		RQAic	6.067	9.396	10.465	9.810	6.458
	gamma	AL1BRQ	2.664	6.418	8.313	7.982	4.617
		L2BRQ	2.663	6.414	8.314	7.987	4.612
		RQ	2.664	6.415	8.329	8.016	4.637
		RQAic	2.661	6.409	8.327	8.351	4.641
	mixed	AL1BRQ	6.453	11.181	13.194	12.693	7.854
		L2BRQ	6.434	11.177	13.189	12.688	7.863
		RQ	6.466	11.202	13.177	12.676	7.802
		RQAic	6.511	11.188	13.168	12.706	7.841
multivariate2	norm	AL1BRQ	2.825	5.463	6.218	5.399	2.706
		L2BRQ	2.822	5.457	6.217	5.398	2.695
		RQ	2.836	5.484	6.255	5.414	2.719
		RQAic	3.349	6.162	9.058	8.875	4.006
	tdist	AL1BRQ	5.936	9.389	10.449	9.828	6.310
		L2BRQ	5.921	9.380	10.447	9.825	6.375
		RQ	5.963	9.420	10.482	9.817	6.374
		RQAic	6.446	10.283	13.157	13.058	6.806
	gamma	AL1BRQ	2.665	6.408	8.246	7.952	4.598
		L2BRQ	2.663	6.402	8.243	7.977	4.587
		RQ	2.665	6.413	8.338	8.004	4.613
		RQAic	2.943	6.609	8.765	8.392	4.644
	mixed	AL1BRQ	6.439	11.197	13.180	12.658	7.839
		L2BRQ	6.419	11.197	13.176	12.647	7.843
		RQ	6.466	11.196	13.183	12.676	7.818
		RQAic	6.658	11.551	13.371	12.667	7.888
high-dimensional	norm	AL1BRQ	3.429	6.078	6.905	6.002	3.077
		L2BRQ	3.406	6.104	6.890	5.963	3.036
		RQ	8.041	9.843	10.386	9.640	7.958
	tdist	AL1BRQ	6.459	9.568	10.606	9.877	6.475
		L2BRQ	6.198	9.553	10.581	9.799	6.196
		RQ	18.194	16.043	16.625	16.468	18.602
	gamma	AL1BRQ	2.945	6.619	8.567	8.345	5.358
		L2BRQ	2.933	6.638	8.541	8.265	5.254
		RQ	8.107	11.143	13.290	14.145	13.252
	mixed	AL1BRQ	6.786	11.268	13.131	12.387	7.969
		L2BRQ	6.560	11.232	13.127	12.330	7.829
		RQ	19.744	19.579	20.723	21.207	21.364

Table 5: Empirical risk R_τ of AL1BRQ, L2BRQ, RQ and RQAic for all parameter setups and all error distributions for each τ. Extension of Table 10 from the main document. Blue values indicate the superior result in the respective category.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	AL1BRQ	0.703	1.432	1.689	1.545	0.913
		L2BRQ	0.697	1.430	1.689	1.545	0.915
		RQ	0.704	1.436	1.693	1.550	0.917
	tdist	AL1BRQ	1.342	2.289	2.648	2.530	1.765
		L2BRQ	1.336	2.290	2.648	2.527	1.763
		RQ	1.350	2.294	2.652	2.540	1.775
	gamma	AL1BRQ	0.715	1.754	2.327	2.421	1.673
		L2BRQ	0.715	1.753	2.327	2.415	1.667
		RQ	0.719	1.752	2.330	2.419	1.670
	mixed	AL1BRQ	1.510	2.760	3.293	3.211	2.206
		L2BRQ	1.498	2.760	3.292	3.207	2.198
		RQ	1.513	2.761	3.298	3.208	2.215
heteroskedastic	norm	AL1BRQ	1.560	3.205	3.807	3.506	2.123
		L2BRQ	1.549	3.203	3.806	3.501	2.124
		RQ	1.567	3.207	3.816	3.510	2.126
	tdist	AL1BRQ	3.063	5.157	5.953	5.704	4.033
		L2BRQ	3.050	5.151	5.953	5.697	4.020
		RQ	3.081	5.163	5.965	5.716	4.039
	gamma	AL1BRQ	1.646	4.048	5.401	5.650	4.021
		L2BRQ	1.646	4.046	5.401	5.643	4.015
		RQ	1.649	4.044	5.406	5.657	4.030
	mixed	AL1BRQ	3.499	6.583	7.965	7.861	5.679
		L2BRQ	3.468	6.584	7.963	7.853	5.532
		RQ	3.487	6.584	7.983	7.881	5.560
multivariate	norm	AL1BRQ	2.917	5.730	6.669	6.030	3.520
		L2BRQ	2.915	5.725	6.670	6.029	3.517
		RQ	2.922	5.755	6.695	6.044	3.522
		RQAic	2.920	5.748	6.689	6.038	3.523
	tdist	AL1BRQ	6.250	10.205	11.817	11.695	8.729
		L2BRQ	6.231	10.199	11.814	11.693	8.724
		RQ	6.253	10.224	11.817	11.704	8.787
		RQAic	6.344	10.210	11.805	11.693	8.921
	gamma	AL1BRQ	2.904	7.125	9.497	9.626	6.757
		L2BRQ	2.901	7.120	9.500	9.633	6.744
		RQ	2.901	7.122	9.511	9.669	6.769
		RQAic	2.900	7.115	9.506	10.040	6.770
	mixed	AL1BRQ	6.752	12.096	14.673	14.778	10.567
		L2BRQ	6.736	12.094	14.668	14.773	10.580
		RQ	6.766	12.092	14.676	14.768	10.507
		RQAic	6.795	12.081	14.663	14.816	10.626
multivariate2	norm	AL1BRQ	2.899	5.705	6.347	5.634	3.008
		L2BRQ	2.897	5.697	6.343	5.632	2.993
		RQ	2.916	5.723	6.359	5.653	3.020
		RQAic	3.457	6.426	9.015	9.150	4.559
	tdist	AL1BRQ	6.210	10.160	11.589	11.476	8.439
		L2BRQ	6.194	10.154	11.586	11.474	8.510
		RQ	6.245	10.180	11.601	11.478	8.495
		RQAic	6.734	11.100	14.341	15.046	9.029
	gamma	AL1BRQ	2.890	7.104	9.090	9.173	6.170
		L2BRQ	2.887	7.099	9.090	9.203	6.154
		RQ	2.898	7.111	9.169	9.232	6.187
		RQAic	3.204	7.294	9.585	9.677	6.206
	mixed	AL1BRQ	6.749	12.066	14.325	14.292	9.951
		L2BRQ	6.724	12.063	14.320	14.285	9.952
		RQ	6.755	12.072	14.307	14.292	9.888
		RQAic	6.972	12.418	14.510	14.315	10.006
high-dimensional	norm	AL1BRQ	3.465	6.332	7.358	6.575	3.799
		L2BRQ	3.415	6.350	7.350	6.557	3.766
		RQ	8.170	10.112	10.886	10.430	11.650
	tdist	AL1BRQ	7.016	11.385	13.480	13.881	11.651
		L2BRQ	6.765	11.370	13.473	13.792	11.395
		RQ	19.203	18.035	19.362	20.399	26.984
	gamma	AL1BRQ	3.120	7.199	9.440	9.612	6.929
		L2BRQ	3.106	7.213	9.419	9.534	6.843
		RQ	8.117	11.832	14.338	15.417	18.503
	mixed	AL1BRQ	7.242	12.545	15.262	15.398	11.840
		L2BRQ	6.984	12.528	15.260	15.329	11.667
		RQ	19.355	20.611	22.737	23.998	31.412

Table 6: Empirical risk R_τ of AL1BRQ, L2BRQ, RQ and RQAic for contaminated cases for all parameter setups and error distributions and each τ. Extension of Table 11 in main document. Blue values indicate superior result in the respective category.

Parameter setup	Error distr.	MSE(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.594	0.528	0.429	0.436	0.668
			L2BRQ	0.484	0.419	0.395	0.543	0.638
			RQ	1.124	0.546	0.432	0.804	1.009
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.017	0.015	0.012	0.012	0.021
			L2BRQ	0.016	0.014	0.011	0.013	0.019
			RQ	0.031	0.018	0.013	0.024	0.026
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	4.119	0.71	0.361	0.556	4.398
			L2BRQ	3.213	0.474	0.315	0.763	2.913
			RQ	5.444	0.992	0.583	0.894	4.713
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.155	0.016	0.01	0.019	0.099
			L2BRQ	0.092	0.014	0.008	0.021	0.071
			RQ	0.141	0.027	0.017	0.028	0.158
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.219	0.47	0.428	0.741	2.822
			L2BRQ	0.18	0.298	0.371	0.939	2.153
			RQ	0.333	0.509	0.784	1.712	4.748
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.006	0.012	0.012	0.024	0.119
			L2BRQ	0.006	0.009	0.009	0.027	0.073
			RQ	0.01	0.017	0.02	0.049	0.132
heteroskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	1.464	1.003	1.073	1.05	1.981
			L2BRQ	1.366	0.727	1.038	1.32	2.191
			RQ	1.894	1.44	1.31	1.49	2.975
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.091	0.05	0.059	0.066	0.127
			L2BRQ	0.069	0.052	0.057	0.061	0.1
			RQ	0.116	0.088	0.063	0.088	0.159
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	5.815	1.385	1.516	1.777	11.924
			L2BRQ	5.673	0.822	1.375	2.237	9.25
			RQ	13.137	2.165	1.299	1.846	10.292
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.101	0.089	0.084	0.108	0.49
			L2BRQ	0.358	0.069	0.074	0.095	0.37
			RQ	0.809	0.14	0.084	0.135	0.751
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.476	0.958	0.959	2.913	8.122
			L2BRQ	0.467	0.873	0.917	3.394	7.531
			RQ	0.707	1.289	1.533	4.128	6.822
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.021	0.05	0.054	0.154	0.498
			L2BRQ	0.029	0.045	0.049	0.128	0.373
			RQ	0.044	0.067	0.091	0.227	0.586
multivariate	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	12.551	7.946	7.545	8.104	9.523
			L2BRQ	11.8	8.088	6.974	7.594	9.61
			RQ	11.433	8.879	8.26	7.244	14.66
			RQAic	10.867	7.48	7.555	7.083	12.512
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.119	0.062	0.058	0.072	0.106
			L2BRQ	0.106	0.062	0.055	0.07	0.106
			RQ	0.145	0.077	0.06	0.08	0.12
			RQAic	0.15	0.076	0.061	0.08	0.11
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.136	0.113	0.077	0.054	0.119
			L2BRQ	0.139	0.104	0.075	0.055	0.123
			RQ	0.177	0.103	0.073	0.081	0.143
			RQAic	0.18	0.098	0.071	0.081	0.144
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.109	0.058	0.042	0.073	0.099
			L2BRQ	0.116	0.056	0.041	0.065	0.095
			RQ	0.1	0.067	0.066	0.075	0.114
			RQAic	0.105	0.068	0.069	0.07	0.112
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.113	0.089	0.091	0.062	0.106
			L2BRQ	0.104	0.083	0.086	0.061	0.098
			RQ	0.118	0.07	0.071	0.072	0.139
			RQAic	0.118	0.071	0.073	0.073	0.2
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.078	0.03	0.035	0.045	0.076
			L2BRQ	0.074	0.032	0.032	0.046	0.074
			RQ	0.117	0.079	0.054	0.092	0.144
			RQAic	0.149	0.085	0.085	0.112	0.175
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.062	0.042	0.044	0.048	0.069
			L2BRQ	0.069	0.048	0.042	0.046	0.066
			RQ	0.133	0.062	0.076	0.084	0.134
			RQAic	0.046	0.017	0.004	0.016	0.032
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	86.98	9.32	7.707	8.957	72.538
			L2BRQ	82.905	8.964	7.13	8.575	67.218
			RQ	54.131	13.055	8.297	14.977	70.468
			RQAic	46.746	9.849	6.483	11.222	62.547
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.833	0.119	0.081	0.139	0.614

			L2BRQ	0.682	0.1	0.073	0.146	0.719
			RQ	0.697	0.12	0.099	0.123	0.556
			RQAic	0.637	0.127	0.098	0.12	0.566
		$\hat{\beta}_{\tau 2}$	AL1BRQ	1.018	0.147	0.111	0.137	0.466
			L2BRQ	0.971	0.159	0.102	0.135	0.374
			RQ	0.651	0.111	0.082	0.149	0.783
			RQAic	0.69	0.123	0.082	0.146	3.133
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.678	0.121	0.078	0.158	0.763
			L2BRQ	0.67	0.114	0.081	0.157	0.791
			RQ	0.685	0.117	0.087	0.11	0.495
			RQAic	3.1	0.114	0.089	0.108	1.796
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.76	0.126	0.077	0.176	0.254
			L2BRQ	0.657	0.125	0.08	0.179	0.18
			RQ	0.655	0.106	0.076	0.118	0.861
			RQAic	2.367	0.108	0.076	0.145	0.961
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.3	0.058	0.03	0.043	0.217
			L2BRQ	0.276	0.057	0.027	0.037	0.177
			RQ	0.63	0.102	0.08	0.107	0.552
			RQAic	0.609	0.093	0.051	0.112	0.283
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.316	0.062	0.04	0.039	0.263
			L2BRQ	0.344	0.066	0.039	0.032	0.144
			RQ	0.403	0.124	0.087	0.101	0.571
			RQAic	0.14	0.006	0.006	0.01	0.033
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	4.431	8.404	8.01	17.129	71.855
			L2BRQ	4.73	8.76	8.254	15.548	65.968
			RQ	4.243	6.228	14.092	23.722	80.628
			RQAic	3.311	5.068	11.932	18.59	84.278
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.042	0.07	0.085	0.197	0.468
			L2BRQ	0.037	0.066	0.074	0.185	0.457
			RQ	0.037	0.075	0.109	0.193	0.534
			RQAic	0.037	0.072	0.105	0.191	0.515
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.059	0.09	0.127	0.114	0.554
			L2BRQ	0.057	0.086	0.133	0.105	0.581
			RQ	0.048	0.077	0.109	0.244	0.694
			RQAic	0.041	0.077	0.112	3.478	0.645
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.033	0.07	0.089	0.16	0.505
			L2BRQ	0.038	0.068	0.083	0.149	0.516
			RQ	0.041	0.05	0.1	0.169	0.5
			RQAic	0.041	0.052	0.098	2.138	0.412
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.037	0.072	0.08	0.115	0.699
			L2BRQ	0.039	0.065	0.077	0.119	0.624
			RQ	0.037	0.066	0.108	0.186	0.655
			RQAic	0.037	0.066	0.173	0.343	1.275
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.022	0.043	0.062	0.079	0.364
			L2BRQ	0.023	0.041	0.063	0.075	0.388
			RQ	0.037	0.045	0.075	0.167	0.557
			RQAic	0.036	0.054	0.036	0.044	0.613
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.026	0.039	0.035	0.06	0.318
			L2BRQ	0.029	0.042	0.04	0.05	0.348
			RQ	0.03	0.055	0.097	0.203	0.475
			RQAic	0.007	0	0	0	0.171
multivariate2	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	11.803	4.781	4.626	7.117	7.373
			L2BRQ	12.386	4.551	4.538	6.475	6.415
			RQ	16.192	12.094	9.486	8.234	18.349
			RQAic	14.026	10.031	7.169	7.113	15.572
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.122	0.053	0.024	0.028	0.066
			L2BRQ	0.122	0.052	0.022	0.026	0.037
			RQ	0.096	0.051	0.058	0.082	0.146
			RQAic	0.097	0.059	20.242	11.382	3.949
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.134	0.091	0.066	0.089	0.132
			L2BRQ	0.125	0.084	0.058	0.087	0.106
			RQ	0.162	0.097	0.115	0.107	0.182
			RQAic	0.168	0.101	10.059	26.049	10.838
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.065	0.035	0.03	0.065	0.114
			L2BRQ	0.067	0.034	0.029	0.064	0.136
			RQ	0.119	0.078	0.068	0.06	0.085
			RQAic	6.521	4.709	1.619	8.987	4.416
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.136	0.072	0.071	0.072	0.123
			L2BRQ	0.129	0.062	0.067	0.072	0.111
			RQ	0.107	0.085	0.067	0.073	0.113
			RQAic	6.489	4.434	3.851	1.718	0.424
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.077	0.033	0.037	0.048	0.033

			L2BRQ	0.077	0.035	0.039	0.048	0.021
			RQ	0.119	0.07	0.076	0.067	0.122
			RQAic	0.105	0.04	0.02	0.051	0.119
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.096	0.035	0.028	0.035	0.056
			L2BRQ	0.093	0.036	0.029	0.031	0.039
			RQ	0.146	0.082	0.055	0.078	0.126
			RQAic	0.017	0.001	0	0.007	0.012
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	58.154	8.655	4.63	7.436	38.811
			L2BRQ	60.366	8.53	4.342	6.712	42.836
			RQ	54.725	13.92	9.964	14.667	85.288
			RQAic	53.274	11.317	6.708	9.807	80.642
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.875	0.126	0.025	0.027	0.247
			L2BRQ	0.782	0.111	0.02	0.023	0.169
			RQ	0.481	0.13	0.088	0.107	0.579
			RQAic	0.505	0.138	21.832	12.278	2.752
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.768	0.198	0.102	0.181	0.488
			L2BRQ	0.744	0.18	0.095	0.169	0.716
			RQ	0.828	0.176	0.098	0.132	0.781
			RQAic	0.883	0.166	10.232	27.439	7.14
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.177	0.043	0.018	0.221	0.681
			L2BRQ	0.219	0.047	0.016	0.21	0.519
			RQ	0.606	0.098	0.098	0.127	0.797
			RQAic	9.94	5.825	1.107	9.554	4.242
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.65	0.139	0.095	0.187	0.188
			L2BRQ	0.659	0.147	0.098	0.19	0.151
			RQ	0.648	0.144	0.092	0.113	0.581
			RQAic	9.899	5.937	4.078	1.642	0.53
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.224	0.046	0.019	0.026	0.225
			L2BRQ	0.214	0.048	0.018	0.021	0.135
			RQ	0.766	0.101	0.096	0.109	0.708
			RQAic	0.51	0.025	0	0.024	0.135
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.308	0.044	0.02	0.019	0.151
			L2BRQ	0.301	0.044	0.018	0.017	0.094
			RQ	0.558	0.137	0.07	0.104	0.521
			RQAic	0.063	0	0	0	0
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	4.331	4.033	5.63	17.304	75.589
			L2BRQ	4.295	3.795	4.752	17.39	60.707
			RQ	4.405	6.923	10.38	26.641	55.751
			RQAic	3.678	5.016	8.835	19.209	55.818
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.03	0.048	0.017	0.046	0.288
			L2BRQ	0.028	0.048	0.017	0.057	0.363
			RQ	0.04	0.048	0.097	0.212	0.483
			RQAic	0.035	0.052	2.519	2.466	4.46
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.06	0.097	0.235	0.09	0.779
			L2BRQ	0.06	0.081	0.189	0.136	0.694
			RQ	0.041	0.067	0.112	0.209	0.7
			RQAic	0.039	0.063	1.926	1.934	0.806
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.02	0.034	0.013	0.227	0.462
			L2BRQ	0.023	0.037	0.009	0.19	0.355
			RQ	0.036	0.052	0.092	0.15	0.51
			RQAic	1.635	0.768	0.118	3.338	0.514
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.044	0.053	0.069	0.148	0.698
			L2BRQ	0.044	0.057	0.062	0.134	0.614
			RQ	0.037	0.065	0.107	0.175	0.541
			RQAic	1.749	0.78	0.12	0.254	3.211
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.027	0.029	0.016	0.053	0.34
			L2BRQ	0.03	0.032	0.009	0.073	0.332
			RQ	0.032	0.063	0.094	0.229	0.574
			RQAic	0.015	0.008	0.002	0.009	0.361
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.017	0.025	0.018	0.065	0.228
			L2BRQ	0.019	0.024	0.014	0.071	0.263
			RQ	0.042	0.056	0.096	0.135	0.426
			RQAic	0.005	0	0	0.008	0.041
high-dimensional	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	126.872	52.14	40.42	68.218	78.306
			L2BRQ	94.147	57.74	36.91	50.335	70.553
			RQ	2322.473	1490.684	964.144	1546.512	1963.622
		$\hat{\beta}_{\tau 1}$	AL1BRQ	2.826	0.657	0.477	0.534	1.168
			L2BRQ	2.755	0.616	0.497	0.561	1.169
			RQ	0.78	0.511	0.459	0.535	0.759
		$\hat{\beta}_{\tau 2}$	AL1BRQ	3.648	0.768	0.633	0.612	1.12
			L2BRQ	3.784	0.958	0.628	0.513	0.791
			RQ	3.731	0.864	0.495	0.732	3.585

	$\hat{\beta}_{\tau 3}$	AL1BRQ	1.685	0.945	0.55	0.496	1.157
		L2BRQ	1.782	0.857	0.558	0.52	1.101
		RQ	0.775	0.54	0.37	0.46	0.555
	$\hat{\beta}_{\tau 4}$	AL1BRQ	2.718	0.823	0.641	0.443	0.307
		L2BRQ	2.729	0.913	0.623	0.422	0.306
		RQ	1.597	0.65	0.546	0.528	1.801
tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	678.469	90.836	61.399	106.118	758.03
		L2BRQ	378.22	70.128	58.21	83.487	240.843
		RQ	13122.689	3433.946	2078.63	3108.946	9107.436
	$\hat{\beta}_{\tau 1}$	AL1BRQ	13.529	1.475	0.947	1.106	9.912
		L2BRQ	8.251	1.297	0.897	1.135	4.422
		RQ	5.313	1.401	0.942	1.27	4.086
	$\hat{\beta}_{\tau 2}$	AL1BRQ	14.526	1.352	0.828	1.555	1.159
		L2BRQ	12.153	1.463	0.768	1.436	0.579
		RQ	10.243	1.732	1.092	1.454	10.833
	$\hat{\beta}_{\tau 3}$	AL1BRQ	2.816	1.202	0.929	1.145	2.793
		L2BRQ	2.912	1.121	0.956	1.176	2.763
		RQ	5.278	1.529	1.238	1.437	3.126
	$\hat{\beta}_{\tau 4}$	AL1BRQ	8.978	1.111	0.826	0.883	0.113
		L2BRQ	8.287	1.123	0.809	0.875	0.052
		RQ	6.964	1.688	1.254	1.62	5.224
gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	43.698	35.923	77.858	297.068	2327.665
		L2BRQ	36.898	34.621	72.091	228.123	1310.847
		RQ	1316.583	1853.465	1657.46	3396.255	5474.548
	$\hat{\beta}_{\tau 1}$	AL1BRQ	0.89	0.469	1.082	1.666	11.557
		L2BRQ	0.767	0.489	1.056	1.584	5.822
		RQ	0.751	0.641	0.714	1.145	1.735
	$\hat{\beta}_{\tau 2}$	AL1BRQ	1.266	0.625	1.157	0.046	5.042
		L2BRQ	1.208	0.831	1.076	0.043	5.655
		RQ	3.69	1.145	1.185	1.513	11.811
	$\hat{\beta}_{\tau 3}$	AL1BRQ	0.953	0.592	1.009	1.734	2.928
		L2BRQ	0.91	0.606	1.021	1.807	2.595
		RQ	0.578	0.614	0.845	1.273	1.476
	$\hat{\beta}_{\tau 4}$	AL1BRQ	0.81	0.428	0.089	0.163	2.889
		L2BRQ	0.801	0.497	0.092	0.177	2.984
		RQ	1.444	0.743	0.925	1.36	4.367

Table 7: Estimation accuracy measured by the MSE for AL1BRQ, L2BRQ, RQ, and RQAic for all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distr.	MSE(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.802	0.446	0.39	0.27	0.642
			L2BRQ	0.688	0.325	0.383	0.409	0.829
			RQ	0.958	0.635	0.553	0.474	0.975
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.021	0.009	0.012	0.008	0.019
			L2BRQ	0.022	0.009	0.01	0.009	0.022
			RQ	0.027	0.019	0.019	0.015	0.031
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	4.247	0.649	0.305	0.709	3.769
			L2BRQ	3.34	0.439	0.288	0.725	2.551
			RQ	6.483	1.119	0.504	1.184	5.042
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.11	0.018	0.01	0.024	0.092
			L2BRQ	0.081	0.017	0.009	0.022	0.074
			RQ	0.177	0.028	0.016	0.029	0.164
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.246	0.445	0.56	0.967	3.403
			L2BRQ	0.192	0.366	0.518	1.094	2.31
			RQ	0.361	0.462	0.944	1.458	4.347
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.006	0.013	0.018	0.03	0.121
			L2BRQ	0.007	0.011	0.016	0.029	0.079
			RQ	0.013	0.013	0.028	0.046	0.121
heteroskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	1.442	1.033	0.977	0.541	1.643
			L2BRQ	1.502	0.819	0.938	0.846	2.025
			RQ	2.422	1.045	1.283	1.025	1.815
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.062	0.053	0.048	0.055	0.1
			L2BRQ	0.097	0.053	0.045	0.043	0.094
			RQ	0.188	0.066	0.083	0.069	0.127
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	5.85	1.679	1.145	1.658	15.099
			L2BRQ	8.83	0.907	1.114	1.979	8.943
			RQ	15.069	2.479	1.364	1.554	14.22
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.1	0.094	0.053	0.114	0.796
			L2BRQ	0.487	0.066	0.049	0.094	0.56
			RQ	0.709	0.127	0.078	0.094	0.747
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.692	1.288	1.509	1.933	6.807
			L2BRQ	0.621	0.918	1.438	2.562	7.556
			RQ	0.798	1.018	1.94	3.131	13.388
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.03	0.058	0.082	0.132	0.453
			L2BRQ	0.034	0.057	0.075	0.106	0.365
			RQ	0.04	0.067	0.134	0.186	0.774
multivariate	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	15.772	6.654	5.205	6.833	14.418
			L2BRQ	15.453	7.014	4.973	6.667	13.834
			RQ	14.533	9.334	7.249	9.247	15.618
			RQAic	13.881	7.957	6.258	7.536	15.942
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.119	0.063	0.063	0.055	0.12
			L2BRQ	0.112	0.057	0.06	0.054	0.121
			RQ	0.082	0.053	0.059	0.073	0.107
			RQAic	0.08	0.05	0.06	0.071	0.115
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.125	0.084	0.068	0.078	0.107
			L2BRQ	0.119	0.076	0.067	0.077	0.097
			RQ	0.158	0.096	0.06	0.111	0.139
			RQAic	0.166	0.102	0.06	0.109	0.134
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.098	0.062	0.054	0.067	0.091
			L2BRQ	0.094	0.055	0.051	0.068	0.091
			RQ	0.133	0.065	0.06	0.069	0.109
			RQAic	0.127	0.063	0.06	0.076	0.101
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.124	0.057	0.057	0.075	0.126
			L2BRQ	0.124	0.059	0.051	0.075	0.128
			RQ	0.108	0.096	0.072	0.073	0.164
			RQAic	0.115	0.091	0.065	0.081	0.223
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.09	0.044	0.043	0.036	0.059
			L2BRQ	0.089	0.044	0.041	0.032	0.059
			RQ	0.124	0.059	0.073	0.072	0.124
			RQAic	0.179	0.071	0.085	0.081	0.157
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.084	0.044	0.04	0.055	0.053
			L2BRQ	0.079	0.045	0.042	0.053	0.05
			RQ	0.116	0.077	0.074	0.084	0.123
			RQAic	0.021	0.01	0.014	0.013	0.032
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	78.282	11.708	7.457	11.473	65.83
			L2BRQ	70.308	11.864	7.036	11.869	65.869
			RQ	80.756	12.512	9.827	11.142	68.627
			RQAic	73.475	10.159	8.001	8.734	64.743
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.871	0.115	0.092	0.159	0.581

			L2BRQ	0.784	0.104	0.085	0.17	0.734
			RQ	0.599	0.126	0.102	0.106	0.339
			RQAic	0.639	0.129	0.093	0.1	0.356
		$\hat{\beta}_{\tau 2}$	AL1BRQ	1.093	0.188	0.113	0.154	0.505
			L2BRQ	0.972	0.192	0.097	0.144	0.363
			RQ	1.032	0.135	0.098	0.175	0.918
			RQAic	0.968	0.133	0.099	0.18	3.954
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.886	0.144	0.098	0.163	0.626
			L2BRQ	0.849	0.128	0.091	0.165	0.721
			RQ	0.664	0.117	0.078	0.086	0.641
			RQAic	3.311	0.114	0.069	0.091	2.673
		$\hat{\beta}_{\tau 4}$	AL1BRQ	1.015	0.141	0.075	0.119	0.274
			L2BRQ	0.973	0.144	0.067	0.12	0.184
			RQ	0.61	0.111	0.082	0.122	0.757
			RQAic	1.85	0.107	0.088	0.135	0.783
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.223	0.045	0.031	0.024	0.262
			L2BRQ	0.205	0.046	0.028	0.022	0.168
			RQ	0.39	0.126	0.102	0.112	0.512
			RQAic	0.607	0.114	0.047	0.081	0.097
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.27	0.023	0.032	0.032	0.201
			L2BRQ	0.271	0.028	0.034	0.03	0.166
			RQ	0.803	0.13	0.064	0.125	0.637
			RQAic	0.098	0	0	0.017	0
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	4.154	7.611	8.394	14.749	88.253
			L2BRQ	4.114	8.272	8.409	13.71	74.509
			RQ	4.254	7.691	11.884	23.101	69.48
			RQAic	3.475	5.665	9.811	21.561	75.221
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.03	0.053	0.13	0.149	0.597
			L2BRQ	0.029	0.06	0.114	0.13	0.533
			RQ	0.031	0.053	0.085	0.171	0.772
			RQAic	0.032	0.051	0.082	0.188	0.76
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.053	0.099	0.133	0.12	0.736
			L2BRQ	0.053	0.089	0.138	0.125	0.697
			RQ	0.053	0.095	0.12	0.207	0.879
			RQAic	0.051	0.087	0.121	3.658	0.823
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.033	0.061	0.094	0.132	0.535
			L2BRQ	0.031	0.057	0.092	0.118	0.511
			RQ	0.035	0.058	0.115	0.17	0.401
			RQAic	0.036	0.057	0.112	2.432	0.369
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.037	0.074	0.08	0.13	0.833
			L2BRQ	0.038	0.066	0.08	0.132	0.732
			RQ	0.039	0.075	0.105	0.209	0.637
			RQAic	0.038	0.08	0.147	0.264	1.077
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.029	0.043	0.064	0.075	0.365
			L2BRQ	0.029	0.048	0.068	0.078	0.369
			RQ	0.037	0.067	0.098	0.156	0.512
			RQAic	0.041	0.072	0.043	0.04	0.548
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.031	0.039	0.039	0.059	0.447
			L2BRQ	0.03	0.039	0.045	0.064	0.478
			RQ	0.037	0.069	0.094	0.154	0.441
			RQAic	0.008	0.009	0.009	0	0.055
multivariate2	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	11.891	5.589	5.089	6.944	6.053
			L2BRQ	11.566	5.039	4.642	6.816	5.013
			RQ	17.481	9.039	7.421	12.648	15.455
			RQAic	16.441	7.199	4.01	8.923	14.388
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.102	0.05	0.052	0.035	0.055
			L2BRQ	0.101	0.054	0.045	0.031	0.027
			RQ	0.12	0.076	0.068	0.085	0.112
			RQAic	0.118	0.077	19.751	11.677	4.392
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.157	0.082	0.075	0.065	0.135
			L2BRQ	0.156	0.073	0.07	0.066	0.095
			RQ	0.139	0.091	0.085	0.092	0.134
			RQAic	0.138	0.091	8.475	26.43	13.911
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.049	0.039	0.037	0.086	0.064
			L2BRQ	0.056	0.036	0.03	0.085	0.071
			RQ	0.118	0.073	0.048	0.068	0.12
			RQAic	6.52	4.835	1.124	9.008	5.399
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.132	0.066	0.079	0.057	0.107
			L2BRQ	0.129	0.063	0.078	0.058	0.084
			RQ	0.164	0.05	0.067	0.086	0.141
			RQAic	6.888	4.723	3.831	1.681	0.572
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.05	0.029	0.046	0.041	0.048

			L2BRQ	0.048	0.026	0.04	0.035	0.032
			RQ	0.105	0.07	0.049	0.073	0.108
			RQAic	0.086	0.028	0.007	0.022	0.069
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.063	0.033	0.036	0.038	0.064
			L2BRQ	0.066	0.034	0.03	0.032	0.038
			RQ	0.114	0.054	0.058	0.061	0.122
			RQAic	0.015	0	0	0	0.006
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	61.411	8.785	6.406	6.598	34.568
			L2BRQ	52.262	8.742	5.765	6.626	30.811
			RQ	86.369	12.241	9.265	14.957	68.874
			RQAic	78.725	8.13	5.429	10.476	61.584
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.977	0.111	0.022	0.028	0.215
			L2BRQ	0.963	0.108	0.022	0.021	0.121
			RQ	0.719	0.104	0.068	0.112	0.453
			RQAic	0.75	0.102	23.193	13.417	3.569
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.854	0.187	0.116	0.196	0.485
			L2BRQ	0.74	0.188	0.104	0.207	0.693
			RQ	0.828	0.139	0.097	0.2	0.798
			RQAic	0.798	0.122	10.155	30.112	7.622
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.171	0.042	0.023	0.191	0.653
			L2BRQ	0.169	0.046	0.021	0.196	0.49
			RQ	0.753	0.111	0.072	0.122	0.608
			RQAic	10.213	6.227	0.615	10.521	4.623
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.907	0.142	0.092	0.193	0.145
			L2BRQ	0.855	0.143	0.092	0.175	0.181
			RQ	0.754	0.121	0.074	0.132	0.787
			RQAic	11.14	6.27	4.061	1.83	0.204
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.355	0.037	0.018	0.025	0.273
			L2BRQ	0.321	0.04	0.015	0.018	0.174
			RQ	0.565	0.091	0.095	0.119	0.552
			RQAic	0.283	0.003	0	0.033	0.048
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.197	0.041	0.035	0.036	0.138
			L2BRQ	0.187	0.041	0.034	0.031	0.098
			RQ	0.586	0.118	0.075	0.096	0.625
			RQAic	0.028	0	0	0.003	0.006
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	2.873	6.544	6.011	14.74	96.843
			L2BRQ	2.548	5.99	5.203	15.087	77.692
			RQ	4.549	7.469	13.199	27.644	64.93
			RQAic	3.326	6.077	8.16	21.208	43.844
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.031	0.069	0.014	0.057	0.387
			L2BRQ	0.029	0.07	0.015	0.074	0.314
			RQ	0.031	0.062	0.083	0.162	0.478
			RQAic	0.034	0.064	2.465	2.946	5.764
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.053	0.057	0.243	0.086	0.848
			L2BRQ	0.052	0.053	0.194	0.102	0.722
			RQ	0.051	0.082	0.09	0.266	0.601
			RQAic	0.05	0.077	1.978	1.961	0.954
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.018	0.04	0.023	0.228	0.641
			L2BRQ	0.017	0.039	0.021	0.158	0.547
			RQ	0.031	0.055	0.071	0.211	0.522
			RQAic	1.872	0.735	0.1	3.707	0.636
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.027	0.07	0.065	0.128	0.772
			L2BRQ	0.024	0.074	0.064	0.134	0.693
			RQ	0.038	0.063	0.124	0.198	0.697
			RQAic	1.914	0.736	0.12	0.246	2.975
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.013	0.035	0.009	0.057	0.299
			L2BRQ	0.013	0.035	0.008	0.079	0.302
			RQ	0.038	0.051	0.087	0.189	0.507
			RQAic	0.011	0.008	0.004	0.007	0.252
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.022	0.041	0.02	0.074	0.34
			L2BRQ	0.021	0.038	0.019	0.096	0.337
			RQ	0.035	0.068	0.155	0.156	0.492
			RQAic	0	0	0	0	0.068
high-dimensional	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	154.766	64.835	45.379	44.367	75.441
			L2BRQ	118.182	49.784	44.491	36.9	74.125
			RQ	1503.297	1182.88	1175.152	1313.203	5386.729
		$\hat{\beta}_{\tau 1}$	AL1BRQ	2.361	0.906	0.622	0.475	1.127
			L2BRQ	2.032	0.838	0.597	0.518	1.109
			RQ	0.694	0.665	0.517	0.733	2.195
		$\hat{\beta}_{\tau 2}$	AL1BRQ	3.118	0.751	0.673	0.543	1.045
			L2BRQ	3.017	0.919	0.664	0.469	0.672
			RQ	4.483	0.674	0.538	0.827	6.649

	$\hat{\beta}_{\tau 3}$	AL1BRQ	1.69	0.923	0.597	0.505	0.926
		L2BRQ	1.628	0.884	0.569	0.541	0.912
		RQ	0.693	0.528	0.477	0.703	1.591
	$\hat{\beta}_{\tau 4}$	AL1BRQ	2.434	0.773	0.646	0.486	0.31
		L2BRQ	2.163	0.825	0.652	0.434	0.304
		RQ	1.145	0.6	0.429	0.795	3.111
tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	637.1	100.177	66.997	113.506	698.861
		L2BRQ	326.512	79.459	64.732	82.235	242.049
		RQ	10187.038	3789.789	2462.623	2857.387	16307.736
	$\hat{\beta}_{\tau 1}$	AL1BRQ	12.251	1.422	0.88	1.308	7.796
		L2BRQ	6.709	1.395	0.879	1.303	4.116
		RQ	5.175	1.541	1.208	1.515	8.884
	$\hat{\beta}_{\tau 2}$	AL1BRQ	17.02	1.689	0.959	1.234	1.215
		L2BRQ	13.119	1.86	0.95	1.032	0.573
		RQ	10.602	1.636	1.143	1.58	14.69
	$\hat{\beta}_{\tau 3}$	AL1BRQ	3.163	1.295	0.905	0.929	2.766
		L2BRQ	2.91	1.217	0.878	0.884	2.74
		RQ	4.867	1.386	0.962	1.37	8.301
	$\hat{\beta}_{\tau 4}$	AL1BRQ	8.613	1.433	0.805	0.758	0.107
		L2BRQ	7.851	1.47	0.812	0.736	0.027
		RQ	5.623	1.77	1.266	1.935	9.011
gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	60.714	42.884	56.682	312.435	2317.729
		L2BRQ	50.037	41.524	56.67	233.131	1281.172
		RQ	1544.137	1513.149	2096.571	3108.821	10811.629
	$\hat{\beta}_{\tau 1}$	AL1BRQ	0.856	0.696	1.098	2.224	11.379
		L2BRQ	0.798	0.697	1.084	2.005	5.627
		RQ	0.645	0.542	0.881	0.896	4.067
	$\hat{\beta}_{\tau 2}$	AL1BRQ	1.213	0.821	0.961	0.043	5.48
		L2BRQ	1.196	1.061	0.916	0.032	6.207
		RQ	3.312	1.287	0.861	1.3	9.218
	$\hat{\beta}_{\tau 3}$	AL1BRQ	0.9	0.778	0.783	1.521	3.117
		L2BRQ	0.885	0.779	0.795	1.605	2.771
		RQ	0.584	0.577	0.715	1.352	4.163
	$\hat{\beta}_{\tau 4}$	AL1BRQ	0.8	0.375	0.101	0.175	2.873
		L2BRQ	0.807	0.434	0.101	0.183	2.92
		RQ	1.337	0.764	1.106	1.324	3.34

Table 8: Estimation accuracy measured by the MSE for AL1BRQ, L2BRQ, RQ, and RQAic for the contaminated cases of all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distr.	Bias(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.348	0.024	0.158	0.312	0.408
			L2BRQ	0.329	0.142	0.124	0.169	-0.004
			RQ	0.072	-0.037	-0.039	0.062	-0.025
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.087	-0.011	-0.028	-0.046	-0.055
			L2BRQ	-0.02	-0.017	-0.021	-0.041	-0.024
			RQ	-0.012	-0.005	0.012	0.001	-0.004
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.357	0.2	0.21	0.491	1.492
			L2BRQ	0.381	0.355	0.204	0.126	-0.237
			RQ	-0.21	-0.105	-0.04	-0.122	0.208
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.117	-0.072	-0.041	-0.076	-0.202
			L2BRQ	-0.001	-0.044	-0.041	-0.041	-0.022
			RQ	0.025	0.017	0.009	0.015	0.026
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.173	0.252	0.352	0.479	0.922
			L2BRQ	0.178	0.302	0.328	0.272	-0.3
			RQ	0.013	0.065	-0.068	0.005	0.061
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.042	-0.072	-0.05	-0.076	-0.144
			L2BRQ	-0.011	-0.034	-0.051	-0.088	-0.041
			RQ	-0.005	0.002	0.001	0.009	0.006
heteroskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.53	0.112	0.312	0.426	0.702
			L2BRQ	0.097	0.32	0.253	0.22	0.202
			RQ	-0.109	-0.129	0	-0.043	-0.068
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.209	-0.035	-0.097	-0.081	-0.124
			L2BRQ	0.058	-0.039	-0.083	-0.087	-0.114
			RQ	-0.005	0.038	-0.01	0.009	-0.007
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.613	0.507	0.478	0.771	2.533
			L2BRQ	-0.267	0.468	0.371	0.227	0.837
			RQ	-0.78	0.009	0.043	0.019	-0.012
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.001	-0.229	-0.175	-0.089	-0.358
			L2BRQ	0.295	-0.037	-0.15	-0.05	-0.266
			RQ	0.169	-0.035	-0.022	0.02	0.044
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.244	0.496	0.609	0.931	1.63
			L2BRQ	0.176	0.659	0.575	0.761	0.817
			RQ	0.026	0.158	-0.061	-0.037	-0.195
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.086	-0.181	-0.168	-0.142	-0.217
			L2BRQ	0.021	-0.142	-0.181	-0.224	-0.28
			RQ	0.001	-0.013	0.011	-0.052	-0.001
multivariate	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.323	0.317	-0.135	0.378	1.067
			L2BRQ	-0.437	0.263	-0.095	0.436	1.037
			RQ	-0.122	0.15	0.364	-0.044	0.347
			RQAic	-0.061	-0.034	0.229	-0.144	0.075
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.071	-0.07	-0.064	-0.035	-0.099
			L2BRQ	-0.055	-0.061	-0.054	-0.04	-0.097
			RQ	-0.009	-0.007	0.009	0.001	-0.016
			RQAic	-0.003	-0.003	0.016	0.015	-0.009
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.085	0.092	0.072	0.011	0.08
			L2BRQ	0.118	0.11	0.053	0.004	0.076
			RQ	0.079	0.021	-0.067	-0.011	-0.091
			RQAic	0.074	0.022	-0.068	-0.007	-0.091
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.058	-0.054	-0.029	-0.04	-0.131
			L2BRQ	-0.052	-0.05	-0.022	-0.042	-0.14
			RQ	-0.051	-0.047	-0.044	-0.003	-0.007
			RQAic	-0.058	-0.045	-0.044	0.003	-0.002
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.128	0.055	0.038	0.04	0.009
			L2BRQ	0.129	0.064	0.037	0.039	0.007
			RQ	0.082	0.029	0.012	0.012	-0.042
			RQAic	0.096	0.03	0.007	0.014	-0.012
		$\hat{\beta}_{\tau 5}$	AL1BRQ	-0.014	-0.016	0.018	-0.003	-0.008
			L2BRQ	-0.009	-0.012	0.009	-0.009	-0.01
			RQ	0.044	0.015	0.019	0.011	0.006
			RQAic	-0.044	0.033	0.026	0.02	0.024
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.002	-0.036	0.031	-0.028	0.014
			L2BRQ	0.004	-0.029	0.033	-0.026	0.014
			RQ	-0.081	-0.012	-0.023	0.009	0.046
			RQAic	-0.019	-0.008	-0.005	-0.005	0.042
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-1.831	0.206	0.271	0.339	4.296
			L2BRQ	-1.769	0.096	0.344	0.358	5.21
			RQ	-0.46	-0.537	-0.782	0.197	2.508
			RQAic	-0.156	-0.336	-0.491	0.207	1.72
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.52	-0.149	-0.099	-0.193	-0.362

			L2BRQ	-0.468	-0.131	-0.106	-0.213	-0.537
			RQ	-0.058	-0.004	0.031	0.021	-0.123
			RQAic	-0.072	0	0.022	0.016	-0.12
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.506	0.194	0.143	0.237	0.204
			L2BRQ	0.568	0.228	0.13	0.235	0.117
			RQ	0.219	0.018	-0.024	-0.072	-0.147
			RQAic	0.193	0.008	-0.017	-0.066	0.391
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.281	-0.195	-0.112	-0.252	-0.464
			L2BRQ	-0.274	-0.177	-0.12	-0.269	-0.654
			RQ	-0.104	-0.01	0.036	0.026	0.017
			RQAic	-0.449	-0.019	0.034	0.03	-0.555
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.419	0.133	0.108	0.32	-0.093
			L2BRQ	0.444	0.147	0.111	0.324	-0.067
			RQ	0.063	0.023	0.002	-0.017	-0.134
			RQAic	0.426	0.03	-0.014	0	0.028
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.103	0.034	0.001	0.051	0.13
			L2BRQ	0.084	0.037	0.013	0.058	0.166
			RQ	-0.169	-0.014	-0.014	-0.016	-0.023
			RQAic	-0.192	0.035	0.042	-0.031	0.03
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.063	-0.038	-0.02	0.006	-0.018
			L2BRQ	0.069	-0.033	-0.036	-0.002	-0.017
			RQ	-0.007	0.074	0.103	0.002	0.024
			RQAic	-0.041	-0.001	0.011	-0.002	0.016
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.101	0.39	0.468	2.21	4.16
			L2BRQ	-0.177	0.401	0.341	2.096	3.141
			RQ	-0.089	0.014	0.387	-0.01	2.789
			RQAic	-0.101	-0.03	0.207	0.055	3.171
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.067	-0.068	-0.09	-0.202	-0.221
			L2BRQ	-0.056	-0.065	-0.068	-0.201	-0.175
			RQ	0.007	0.029	-0.054	0.029	-0.135
			RQAic	0.01	0.025	-0.066	0.036	-0.131
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.065	0.092	0.115	0.102	-0.249
			L2BRQ	0.079	0.107	0.091	0.066	-0.262
			RQ	0.052	0.011	-0.005	-0.062	-0.031
			RQAic	0.035	0.017	-0.026	1.407	-0.01
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.038	-0.088	-0.114	-0.153	-0.097
			L2BRQ	-0.024	-0.087	-0.098	-0.15	-0.022
			RQ	-0.008	-0.054	0.01	0.011	-0.144
			RQAic	-0.006	-0.05	0.007	-1.152	-0.098
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.044	0.05	0.034	-0.11	-0.258
			L2BRQ	0.044	0.063	0.027	-0.139	-0.263
			RQ	0	-0.033	0.027	-0.003	-0.148
			RQAic	0.004	-0.03	0.082	-0.354	-0.292
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.019	-0.039	-0.006	0.011	0.035
			L2BRQ	0.021	-0.034	-0.005	0.015	0.024
			RQ	0.017	-0.005	-0.025	0.005	-0.054
			RQAic	0.012	0.041	-0.012	0.037	-0.061
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.029	0.022	0.028	0.011	-0.013
			L2BRQ	0.031	0.016	0.028	0.023	0.01
			RQ	-0.008	0.044	-0.011	0.014	0.033
			RQAic	0.005	0	0	0	0.054
multivariate2	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	-1.248	-0.297	-0.61	-0.174	0.644
			L2BRQ	-1.233	-0.34	-0.516	-0.157	0.883
			RQ	-0.109	-0.01	0.33	-0.33	0.689
			RQAic	-0.237	-0.237	0.35	-0.192	0.71
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.034	-0.045	0.035	0.032	-0.009
			L2BRQ	-0.023	-0.034	0.033	0.029	-0.015
			RQ	0.035	0.051	-0.009	0.024	-0.013
			RQAic	0.045	0.044	-3.991	-2.858	-1.465
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.041	0.059	0.079	0.056	0.137
			L2BRQ	0.079	0.088	0.06	0.051	0.02
			RQ	0.033	0.038	-0.053	0.022	-0.148
			RQAic	0.032	0.032	2.703	4.363	2.383
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.002	-0.021	-0.021	-0.082	-0.179
			L2BRQ	0.001	-0.013	-0.023	-0.087	-0.251
			RQ	-0.011	-0.001	0.016	-0.002	0.006
			RQAic	-1.923	-1.806	-0.732	-2.562	-1.516
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.174	0.104	0.078	0.084	0.048
			L2BRQ	0.178	0.116	0.074	0.084	0.083
			RQ	0.023	0.011	-0.083	-0.022	-0.086
			RQAic	1.936	1.781	1.913	1.101	0.366
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.014	-0.018	-0.009	0.001	-0.012

			L2BRQ	0.024	-0.016	-0.007	0.004	-0.007
			RQ	-0.023	-0.013	-0.01	0.004	0.025
			RQAic	-0.033	0.01	-0.014	0.015	-0.005
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.034	-0.013	-0.017	0.003	-0.002
			L2BRQ	0.027	-0.011	-0.016	0.003	0.009
			RQ	-0.041	-0.07	0.019	0.054	0.009
			RQAic	-0.024	-0.004	0	-0.004	0.017
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-2.889	-1.063	-1.282	-0.231	2.695
			L2BRQ	-2.802	-1.059	-1.203	-0.241	3.44
			RQ	-1.957	-0.44	0.233	0.664	2.837
			RQAic	-2.148	-0.591	-0.073	0.796	2.416
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.603	-0.192	0.001	-0.004	-0.094
			L2BRQ	-0.554	-0.185	-0.011	-0.011	-0.028
			RQ	-0.03	0.01	0.026	0.005	0.088
			RQAic	-0.006	0.013	-4.349	-3.171	-0.607
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.496	0.161	0.19	0.3	0.254
			L2BRQ	0.537	0.208	0.181	0.277	-0.718
			RQ	0.265	0.096	0.016	-0.087	-0.155
			RQAic	0.255	0.102	2.903	4.739	1.921
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.069	0.021	0	-0.33	-0.513
			L2BRQ	0.062	0.028	-0.001	-0.351	-0.418
			RQ	0.047	-0.077	0.009	-0.021	0.04
			RQAic	-2.367	-2.251	-0.535	-2.796	-1.565
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.447	0.176	0.186	0.281	-0.014
			L2BRQ	0.513	0.196	0.191	0.292	-0.125
			RQ	0.133	0.064	-0.009	-0.006	-0.147
			RQAic	2.487	2.246	2.016	1.113	-0.109
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.077	0.063	-0.009	0.038	0.128
			L2BRQ	0.082	0.07	-0.002	0.036	0.063
			RQ	0.076	0.027	-0.054	-0.024	-0.206
			RQAic	0.035	0.005	0	-0.03	0.017
		$\hat{\beta}_{\tau 6}$	AL1BRQ	-0.062	-0.033	0.003	-0.013	-0.039
			L2BRQ	-0.023	-0.035	-0.012	-0.016	-0.037
			RQ	-0.121	-0.041	-0.017	0.013	-0.066
			RQAic	0.005	0	0	0	0
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.522	-0.192	-1.657	1.973	3.9
			L2BRQ	-0.385	-0.249	-1.693	1.44	2.833
			RQ	-0.245	0.124	0.01	0.446	0.299
			RQAic	-0.185	-0.238	-0.009	0.745	0.666
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.061	-0.071	0.013	0.056	0.04
			L2BRQ	-0.055	-0.075	0.023	0.042	0.038
			RQ	0.007	0.027	-0.007	-0.024	0.078
			RQAic	-0.007	0.034	-1.4	1.052	1.571
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.094	0.049	0.406	0.129	-0.317
			L2BRQ	0.105	0.082	0.37	-0.01	-0.369
			RQ	0.013	0.007	-0.061	0.044	-0.283
			RQAic	0.008	0.009	1.167	1.027	-0.523
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.03	0.005	0.012	-0.334	-0.228
			L2BRQ	0.025	0.003	0.017	-0.229	-0.119
			RQ	0.009	-0.005	0.044	-0.058	0.062
			RQAic	-1.109	-0.762	-0.047	-1.715	-0.054
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.057	0.04	0.153	-0.151	-0.28
			L2BRQ	0.054	0.063	0.165	-0.158	-0.275
			RQ	0.044	0.024	-0.021	-0.082	-0.11
			RQAic	1.16	0.793	0.268	-0.431	-1.343
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.021	0.001	-0.003	0.05	0.071
			L2BRQ	0.018	0.007	0	0.041	0.049
			RQ	0.006	-0.009	0.004	0.08	0.094
			RQAic	0.017	-0.013	0.004	-0.016	0.113
		$\hat{\beta}_{\tau 6}$	AL1BRQ	-0.01	0.014	0.007	0.01	-0.058
			L2BRQ	-0.004	0.02	0.006	-0.011	-0.062
			RQ	0.009	-0.047	0.025	0.01	0.003
			RQAic	0.003	0	0	-0.009	0.015
high-dimensional	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	-2.662	0.625	0.923	1.334	4.297
			L2BRQ	-2.396	0.48	0.808	2.122	4.294
			RQ	-8.828	-6.17	4.834	4.72	9.374
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-1.487	-0.689	-0.571	-0.589	-0.917
			L2BRQ	-1.478	-0.659	-0.585	-0.616	-0.923
			RQ	0.067	-0.051	0.152	0.073	-0.141
		$\hat{\beta}_{\tau 2}$	AL1BRQ	1.705	0.714	0.636	0.641	0.84
			L2BRQ	1.773	0.83	0.638	0.561	0.667
			RQ	1.729	0.58	0.059	-0.528	-1.722

	$\hat{\beta}_{\tau 3}$	AL1BRQ	-1.129	-0.856	-0.628	-0.561	-0.937
		L2BRQ	-1.169	-0.809	-0.644	-0.595	-0.923
		RQ	-0.032	0.076	-0.026	-0.047	-0.075
	$\hat{\beta}_{\tau 4}$	AL1BRQ	1.405	0.78	0.604	0.541	0.465
		L2BRQ	1.414	0.836	0.606	0.525	0.461
		RQ	0.899	0.299	-0.076	-0.16	-1.014
tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-15.026	-0.673	3.029	3.009	21.638
		L2BRQ	-9.565	-0.156	3.165	2.936	13.506
		RQ	3.187	6.232	-8.189	-0.373	17.293
	$\hat{\beta}_{\tau 1}$	AL1BRQ	-3.105	-1.041	-0.8	-0.825	-2.626
		L2BRQ	-2.551	-0.991	-0.776	-0.852	-1.941
		RQ	-0.054	-0.217	-0.188	0.086	-0.014
	$\hat{\beta}_{\tau 2}$	AL1BRQ	3.29	0.933	0.744	1.039	0.941
		L2BRQ	3.155	1.025	0.717	0.973	0.484
		RQ	2.407	0.617	-0.144	-0.46	-2.337
	$\hat{\beta}_{\tau 3}$	AL1BRQ	-1.441	-0.895	-0.797	-0.877	-1.527
		L2BRQ	-1.553	-0.879	-0.815	-0.896	-1.546
		RQ	-0.016	-0.21	0.052	-0.172	-0.109
	$\hat{\beta}_{\tau 4}$	AL1BRQ	2.76	0.824	0.738	0.757	0.073
		L2BRQ	2.635	0.854	0.743	0.743	0.043
		RQ	1.394	0.122	-0.031	-0.419	-0.91
gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.379	0.387	4.925	14.394	44.042
		L2BRQ	0.579	0.57	4.939	13.608	34.157
		RQ	-0.944	-1.752	2.672	0.58	9.598
	$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.792	-0.575	-0.901	-1.126	-3.027
		L2BRQ	-0.744	-0.591	-0.89	-1.125	-2.146
		RQ	-0.045	-0.037	0.041	-0.104	-0.013
	$\hat{\beta}_{\tau 2}$	AL1BRQ	0.956	0.639	0.94	0.125	-1.963
		L2BRQ	0.957	0.788	0.891	0.093	-2.288
		RQ	1.696	0.655	0.136	-0.462	-3.06
	$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.846	-0.643	-0.856	-1.16	-1.569
		L2BRQ	-0.831	-0.652	-0.87	-1.211	-1.468
		RQ	-0.114	-0.037	0.001	-0.057	0.202
	$\hat{\beta}_{\tau 4}$	AL1BRQ	0.791	0.582	0.27	-0.343	-1.584
		L2BRQ	0.792	0.654	0.265	-0.4	-1.671
		RQ	0.864	0.348	0.106	-0.247	-1.402

Table 9: Estimation accuracy measured by the Bias for AL1BRQ, L2BRQ, RQ, and RQAic of all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distr.	Bias(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.324	0.063	0.227	0.164	0.324
			L2BRQ	0.303	0.19	0.191	0.004	-0.088
			RQ	0.013	-0.077	0.094	-0.127	0.194
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.091	-0.016	-0.03	-0.013	-0.043
			L2BRQ	-0.023	-0.028	-0.023	-0.004	-0.007
			RQ	-0.023	0.012	-0.023	0.03	-0.024
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.376	0.271	0.162	0.556	1.472
			L2BRQ	0.38	0.404	0.126	0.199	-0.276
			RQ	-0.306	0.027	0.024	0.012	-0.345
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.098	-0.088	-0.043	-0.077	-0.208
			L2BRQ	0.01	-0.055	-0.036	-0.044	-0.028
			RQ	0.05	0.002	-0.015	0.011	0.049
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.154	0.279	0.419	0.54	1.09
			L2BRQ	0.18	0.346	0.445	0.326	-0.185
			RQ	0.133	-0.126	0.071	-0.085	-0.123
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.04	-0.078	-0.073	-0.064	-0.162
			L2BRQ	-0.011	-0.049	-0.08	-0.091	-0.05
			RQ	-0.008	0.028	0.015	0.028	0.029
heteroskedastic	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.475	0.04	0.256	0.115	0.584
			L2BRQ	-0.181	0.207	0.179	-0.092	0.267
			RQ	-0.357	0.155	-0.089	-0.188	0.174
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.17	-0.048	-0.089	-0.001	-0.117
			L2BRQ	0.137	-0.04	-0.074	-0.018	-0.119
			RQ	0.056	-0.062	0.014	0.026	-0.042
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.906	0.418	0.476	0.697	2.741
			L2BRQ	-0.711	0.557	0.42	0.21	0.822
			RQ	-0.315	0.048	-0.001	0.065	0.28
		$\hat{\beta}_{\tau 1}$	AL1BRQ	0.045	-0.231	-0.131	-0.109	-0.497
			L2BRQ	0.376	-0.085	-0.117	-0.081	-0.368
			RQ	0.068	-0.005	-0.009	0.026	0.041
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.29	0.598	0.801	0.699	1.438
			L2BRQ	0.249	0.678	0.784	0.403	0.43
			RQ	0.015	-0.028	-0.063	-0.025	-0.143
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.099	-0.195	-0.2	-0.12	-0.212
			L2BRQ	0.008	-0.134	-0.208	-0.189	-0.253
			RQ	0.003	0.024	0.035	0.02	0.042
multivariate	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.457	0.147	0.267	0.067	0.845
			L2BRQ	-0.41	0.156	0.244	0.174	0.798
			RQ	-0.371	-0.067	-0.228	-0.698	0.249
			RQAic	-0.506	-0.126	-0.313	-0.594	0.419
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.075	-0.042	-0.087	-0.009	-0.127
			L2BRQ	-0.064	-0.03	-0.071	-0.018	-0.124
			RQ	0.003	0	-0.034	0.026	0.031
			RQAic	-0.006	-0.001	-0.022	0.035	0.031
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.054	0.069	0.066	0.01	0.039
			L2BRQ	0.064	0.093	0.043	0.01	0.022
			RQ	0.065	0.012	0.028	0.019	-0.094
			RQAic	0.08	0.015	0.027	0.023	-0.099
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.023	-0.08	-0.101	-0.048	-0.084
			L2BRQ	-0.008	-0.078	-0.086	-0.052	-0.081
			RQ	-0.005	0.009	0.053	0.026	0.006
			RQAic	-0.01	0.008	0.057	0.015	-0.004
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.094	0.079	0.041	0.046	0.108
			L2BRQ	0.102	0.087	0.036	0.042	0.101
			RQ	0.058	0.035	-0.008	0.029	-0.111
			RQAic	0.053	0.032	-0.001	0.038	-0.066
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.039	-0.04	0.037	-0.01	0.008
			L2BRQ	0.026	-0.044	0.031	-0.015	-0.001
			RQ	0.008	-0.029	0.017	0.07	0.089
			RQAic	0.024	-0.027	0.002	0.051	0.028
		$\hat{\beta}_{\tau 6}$	AL1BRQ	-0.045	0.011	0.019	0.002	0
			L2BRQ	-0.045	0.011	0.016	-0.002	0.008
			RQ	-0.012	-0.01	-0.019	-0.005	-0.01
			RQAic	0.006	0	-0.006	-0.016	-0.006
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-1.264	-0.448	0.434	1.047	4.334
			L2BRQ	-1.047	-0.494	0.444	1.083	5.389
			RQ	-0.572	-0.381	0.276	0.686	1.521
			RQAic	-0.354	-0.41	-0.042	0.7	1.79
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.429	-0.088	-0.131	-0.249	-0.343

			L2BRQ	-0.423	-0.072	-0.125	-0.269	-0.536
			RQ	-0.162	0.049	0.006	0.009	0.109
			RQAic	-0.153	0.053	0.004	0.007	0.083
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.446	0.206	0.117	0.257	0.15
			L2BRQ	0.505	0.239	0.101	0.242	0.095
			RQ	0.176	0.026	-0.04	-0.046	-0.284
			RQAic	0.124	0.03	-0.032	-0.024	0.611
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.457	-0.183	-0.142	-0.235	-0.357
			L2BRQ	-0.403	-0.162	-0.132	-0.246	-0.533
			RQ	-0.158	-0.038	-0.033	-0.003	0.065
			RQAic	-0.526	-0.046	-0.036	0.011	-0.783
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.459	0.209	0.128	0.206	-0.156
			L2BRQ	0.509	0.219	0.119	0.214	-0.109
			RQ	0.133	0.052	0.047	-0.071	-0.002
			RQAic	0.419	0.028	0.051	-0.073	-0.039
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.044	-0.003	0.032	0.045	0.154
			L2BRQ	0.041	0.003	0.036	0.041	0.16
			RQ	0.095	0.008	-0.06	0.007	0.035
			RQAic	0.106	-0.026	0.012	-0.022	-0.022
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.002	-0.005	-0.007	-0.006	-0.084
			L2BRQ	-0.041	0.001	-0.018	-0.011	-0.089
			RQ	-0.13	-0.063	0.01	-0.022	-0.094
			RQAic	-0.046	0	0	-0.022	0
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	0.42	-0.242	0.398	1.309	4.326
			L2BRQ	0.384	-0.26	0.194	0.785	3.227
			RQ	0.226	-0.087	0.675	0.241	1.322
			RQAic	-0.009	-0.089	0.045	0.558	1.622
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.07	-0.036	-0.179	-0.104	-0.103
			L2BRQ	-0.063	-0.034	-0.149	-0.078	-0.031
			RQ	0.008	-0.001	-0.058	-0.086	-0.11
			RQAic	0.014	0	-0.05	-0.084	-0.125
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.022	0.06	0.187	0.153	-0.189
			L2BRQ	0.038	0.086	0.143	0.086	-0.206
			RQ	0.022	0.083	0.021	-0.03	-0.204
			RQAic	0.016	0.069	0.01	1.534	-0.171
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.042	-0.019	-0.058	-0.103	-0.181
			L2BRQ	-0.042	-0.015	-0.03	-0.069	-0.145
			RQ	-0.039	0.029	-0.028	-0.004	-0.012
			RQAic	-0.041	0.031	-0.026	-1.285	0.059
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.017	0.057	0.033	-0.161	-0.344
			L2BRQ	0.021	0.06	0.021	-0.165	-0.328
			RQ	0.031	-0.027	0.038	0.029	-0.046
			RQAic	0.04	-0.027	0.089	-0.286	-0.188
		$\hat{\beta}_{\tau 5}$	AL1BRQ	-0.007	0.005	0.014	-0.007	0.063
			L2BRQ	-0.004	0.006	0.016	0.021	0.065
			RQ	-0.023	0.007	-0.076	0.021	0.065
			RQAic	-0.001	-0.001	0.01	0.023	0.176
		$\hat{\beta}_{\tau 6}$	AL1BRQ	-0.011	0.002	0.027	0.055	-0.035
			L2BRQ	0	0.003	0.036	0.062	-0.017
			RQ	-0.013	-0.01	-0.002	0.041	0.117
			RQAic	0	0.005	0	0	0.009
multivariate2	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.852	0.045	-0.227	0.22	0.384
			L2BRQ	-0.752	-0.05	-0.257	0.203	0.517
			RQ	-0.551	-0.366	0.323	0.199	0.832
			RQAic	-0.677	-0.265	0.25	0.132	0.801
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.024	-0.087	-0.014	-0.014	0.008
			L2BRQ	-0.001	-0.088	-0.014	-0.01	-0.002
			RQ	-0.016	0.039	0.003	-0.039	-0.009
			RQAic	-0.004	0.044	-3.91	-2.916	-1.748
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.112	0.046	0.087	0.011	0.09
			L2BRQ	0.134	0.086	0.085	0.012	0.007
			RQ	0.083	0.045	-0.003	-0.032	-0.065
			RQAic	0.089	0.041	2.523	4.394	3.073
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.003	-0.008	0.006	-0.055	-0.075
			L2BRQ	0.003	-0.005	0.007	-0.068	-0.168
			RQ	-0.024	-0.001	-0.007	0.034	-0.033
			RQAic	-2.009	-1.891	-0.546	-2.565	-1.958
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.102	0.078	0.089	0.057	0.066
			L2BRQ	0.104	0.099	0.095	0.064	0.124
			RQ	0.101	-0.002	-0.002	-0.049	-0.072
			RQAic	2.103	1.862	1.901	1.034	0.512
		$\hat{\beta}_{\tau 5}$	AL1BRQ	-0.002	-0.033	-0.025	-0.004	-0.026

			L2BRQ	-0.016	-0.026	-0.021	-0.004	-0.03
			RQ	0.035	-0.021	-0.031	-0.009	-0.005
			RQAic	0.027	0	-0.011	-0.003	-0.022
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.003	-0.027	-0.034	-0.005	-0.037
			L2BRQ	-0.014	-0.016	-0.037	-0.004	-0.022
			RQ	0.01	0.015	-0.015	0.022	0.033
			RQAic	0.014	0	0	0	0.006
	tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-2.138	-1.099	-1.243	-0.384	1.237
			L2BRQ	-1.728	-0.959	-1.196	-0.498	2.734
			RQ	-3.316	-0.564	-0.515	0.17	1.739
			RQAic	-3.057	-0.311	-0.536	0.308	1.037
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.571	-0.179	0.02	-0.008	-0.012
			L2BRQ	-0.554	-0.151	0.014	-0.013	0.014
			RQ	-0.008	0.006	0.015	-0.013	-0.068
			RQAic	0.007	-0.002	-4.614	-3.488	-0.655
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.366	0.213	0.218	0.313	0.272
			L2BRQ	0.456	0.258	0.201	0.296	-0.706
			RQ	0.173	0.059	0.009	-0.077	-0.3
			RQAic	0.155	0.039	3.006	5.194	2.212
		$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.013	-0.027	-0.024	-0.303	-0.462
			L2BRQ	-0.03	-0.033	-0.019	-0.323	-0.436
			RQ	0.055	0.01	-0.011	-0.019	0.082
			RQAic	-2.334	-2.347	-0.29	-3.072	-1.824
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.616	0.188	0.166	0.303	-0.048
			L2BRQ	0.606	0.192	0.168	0.296	-0.188
			RQ	0.148	0.025	0.041	-0.003	-0.12
			RQAic	2.634	2.383	2.011	1.255	0.116
		$\hat{\beta}_{\tau 5}$	AL1BRQ	0.005	0.027	0.036	0.02	0.207
			L2BRQ	0.004	0.033	0.037	0.035	0.13
			RQ	0.094	0.022	0.028	0.042	0.03
			RQAic	0.002	0.006	0	0.027	0.031
		$\hat{\beta}_{\tau 6}$	AL1BRQ	-0.079	0.005	-0.032	0.005	0.008
			L2BRQ	-0.063	-0.003	-0.042	0.006	0.012
			RQ	0.044	0	0.022	0.025	0.086
			RQAic	-0.009	0	0	0.006	0.008
	gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.135	-0.21	-2.018	1.757	4.974
			L2BRQ	-0.146	-0.218	-1.951	1.008	3.834
			RQ	0.076	0.602	0.329	0.769	1.494
			RQAic	0.114	0.441	-0.152	0.709	0.552
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.052	-0.077	0.031	0.052	0.068
			L2BRQ	-0.049	-0.083	0.027	0.03	0.092
			RQ	-0.018	-0.034	0.016	-0.001	0.014
			RQAic	-0.015	-0.038	-1.426	1.163	1.913
		$\hat{\beta}_{\tau 2}$	AL1BRQ	0.023	0.044	0.397	0.141	-0.257
			L2BRQ	0.049	0.073	0.358	-0.027	-0.359
			RQ	0.005	-0.007	0.011	-0.032	-0.108
			RQAic	0.001	-0.002	1.243	1.006	-0.604
		$\hat{\beta}_{\tau 3}$	AL1BRQ	0.004	0.006	0.01	-0.316	-0.268
			L2BRQ	0.004	0.006	0.007	-0.204	-0.138
			RQ	0.003	-0.038	0	0.003	-0.044
			RQAic	-1.231	-0.763	-0.08	-1.815	-0.08
		$\hat{\beta}_{\tau 4}$	AL1BRQ	0.032	0.116	0.161	-0.217	-0.376
			L2BRQ	0.036	0.136	0.171	-0.191	-0.408
			RQ	0.006	0.016	0.001	-0.001	-0.026
			RQAic	1.254	0.762	0.33	-0.432	-1.311
		$\hat{\beta}_{\tau 5}$	AL1BRQ	-0.011	-0.013	0.004	0.01	-0.035
			L2BRQ	-0.008	-0.016	0.006	0.018	-0.047
			RQ	0.015	-0.016	-0.004	-0.046	-0.094
			RQAic	-0.019	-0.011	0.006	-0.012	-0.007
		$\hat{\beta}_{\tau 6}$	AL1BRQ	0.039	0.008	0.031	0.074	-0.129
			L2BRQ	0.04	0.015	0.036	0.078	-0.121
			RQ	-0.015	-0.013	-0.065	-0.023	-0.009
			RQAic	0	0	0	0	-0.005
high-dimensional	norm	$\hat{\beta}_{\tau 0}$	AL1BRQ	-3.545	0.738	1.153	1.175	5.352
			L2BRQ	-1.783	0.881	1.06	1.447	5.618
			RQ	-4.776	-7.64	-0.179	3.238	13.497
		$\hat{\beta}_{\tau 1}$	AL1BRQ	-1.325	-0.824	-0.63	-0.555	-0.876
			L2BRQ	-1.227	-0.8	-0.621	-0.583	-0.879
			RQ	-0.266	0.067	-0.051	0.209	0.652
		$\hat{\beta}_{\tau 2}$	AL1BRQ	1.536	0.678	0.708	0.617	0.84
			L2BRQ	1.527	0.798	0.693	0.53	0.631
			RQ	1.885	0.44	-0.011	-0.527	-2.348

	$\hat{\beta}_{\tau 3}$	AL1BRQ	-1.111	-0.807	-0.667	-0.576	-0.809
		L2BRQ	-1.098	-0.791	-0.658	-0.611	-0.804
		RQ	-0.061	-0.03	0.014	0.103	0.156
	$\hat{\beta}_{\tau 4}$	AL1BRQ	1.311	0.704	0.665	0.596	0.447
		L2BRQ	1.277	0.75	0.666	0.553	0.448
		RQ	0.72	0.217	-0.038	-0.345	-1.139
tdist	$\hat{\beta}_{\tau 0}$	AL1BRQ	-9.881	-0.802	2.911	4.828	19.826
		L2BRQ	-6.955	-0.541	2.498	3.998	12.866
		RQ	-21.505	-10.278	6.453	-1.371	14.751
	$\hat{\beta}_{\tau 1}$	AL1BRQ	-2.873	-0.969	-0.789	-0.948	-2.416
		L2BRQ	-2.182	-0.964	-0.795	-0.939	-1.836
		RQ	0.057	0.137	-0.1	-0.033	-0.108
	$\hat{\beta}_{\tau 2}$	AL1BRQ	3.662	1.096	0.772	0.912	0.93
		L2BRQ	3.335	1.198	0.751	0.807	0.413
		RQ	2.169	0.517	-0.022	-0.424	-2.525
	$\hat{\beta}_{\tau 3}$	AL1BRQ	-1.633	-0.984	-0.794	-0.808	-1.554
		L2BRQ	-1.57	-0.949	-0.784	-0.79	-1.561
		RQ	-0.09	0.271	0.03	0.065	0.156
	$\hat{\beta}_{\tau 4}$	AL1BRQ	2.667	1.005	0.734	0.718	0.137
		L2BRQ	2.537	1.025	0.742	0.706	0.061
		RQ	1.322	0.286	-0.147	-0.307	-1.38
gamma	$\hat{\beta}_{\tau 0}$	AL1BRQ	-0.469	1.554	4.484	14.843	44.499
		L2BRQ	-0.293	1.517	4.635	13.611	34.056
		RQ	-4.609	1.386	-5.313	8.291	17.085
	$\hat{\beta}_{\tau 1}$	AL1BRQ	-0.773	-0.698	-0.933	-1.24	-3.06
		L2BRQ	-0.748	-0.7	-0.932	-1.201	-2.113
		RQ	0.178	0.097	0.092	-0.011	0.504
	$\hat{\beta}_{\tau 2}$	AL1BRQ	0.939	0.673	0.851	0.119	-2.186
		L2BRQ	0.935	0.843	0.821	0.075	-2.44
		RQ	1.58	0.82	0.192	-0.459	-2.429
	$\hat{\beta}_{\tau 3}$	AL1BRQ	-0.831	-0.767	-0.739	-1.091	-1.573
		L2BRQ	-0.824	-0.776	-0.749	-1.141	-1.531
		RQ	-0.053	-0.098	0.154	-0.194	0.294
	$\hat{\beta}_{\tau 4}$	AL1BRQ	0.799	0.511	0.261	-0.381	-1.495
		L2BRQ	0.823	0.576	0.269	-0.411	-1.559
		RQ	0.785	0.352	0.168	-0.325	-0.637

Table 10: Estimation accuracy measured by the Bias for AL1BRQ, L2BRQ, RQ, and RQAic for the contaminated cases of all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	τ	Error distr.	$\beta_{\tau 0}$	True population parameters $\beta_{\tau 1}$	$\beta_{\tau 2}$	$\beta_{\tau 3}$	$\beta_{\tau 4}$	$\beta_{\tau 5}$	$\beta_{\tau 6}$
multivariate	0.1	norm	3.718	8	-7.563	2	-3.282	0	0
		tdist	3.114	8	-8.771	2	-3.886	0	0
		gamma	5.532	8	-3.936	2	-1.468	0	0
	0.3	norm	4.476	8	-6.049	2	-2.524	0	0
		tdist	4.383	8	-6.234	2	-2.617	0	0
		gamma	6.097	8	-2.805	2	-0.903	0	0
	0.5	norm	5.000	8	-5.000	2	-2.000	0	0
		tdist	5.000	8	-5.000	2	-2.000	0	0
		gamma	6.678	8	-1.643	2	-0.322	0	0
	0.7	norm	5.524	8	-3.951	2	-1.476	0	0
		tdist	5.617	8	-3.766	2	-1.383	0	0
		gamma	7.439	8	-0.122	2	0.439	0	0
	0.9	norm	6.282	8	-2.437	2	-0.718	0	0
		tdist	6.886	8	-1.229	2	-0.114	0	0
		gamma	8.890	8	2.779	2	1.890	0	0
multivariate2	0.1	norm	3.718	8	-7.563	0	-3.282	0	0
		tdist	3.114	8	-8.771	0	-3.886	0	0
		gamma	5.532	8	-3.936	0	-1.468	0	0
	0.3	norm	4.476	8	-6.049	0	-2.524	0	0
		tdist	4.383	8	-6.234	0	-2.617	0	0
		gamma	6.097	8	-2.805	0	-0.903	0	0
	0.5	norm	5.000	0	-5.000	0	-2.000	0	0
		tdist	5.000	0	-5.000	0	-2.000	0	0
		gamma	6.678	0	-1.643	0	-0.322	0	0
	0.7	norm	5.524	0	-3.951	2	-1.476	0	0
		tdist	5.617	0	-3.766	2	-1.383	0	0
		gamma	7.439	0	-0.122	2	0.439	0	0
	0.9	norm	6.282	0	-2.437	2	-0.718	0	0
		tdist	6.886	0	-1.229	2	-0.114	0	0
		gamma	8.890	0	2.779	2	1.890	0	0

Table 11: True population parameters for the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates					
				x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	AL1BRQ	0.337	0.326	0.097	0.160	0.041	0.038
			L2BRQ	0.257	0.267	0.171	0.189	0.062	0.054
		tdist	AL1BRQ	0.288	0.334	0.091	0.172	0.055	0.060
			L2BRQ	0.247	0.298	0.143	0.207	0.051	0.054
		gamma	AL1BRQ	0.434	0.252	0.139	0.109	0.035	0.032
			L2BRQ	0.262	0.262	0.196	0.181	0.048	0.051
	0.3	norm	AL1BRQ	0.394	0.316	0.102	0.154	0.019	0.014
			L2BRQ	0.275	0.277	0.161	0.178	0.035	0.074
		tdist	AL1BRQ	0.411	0.327	0.099	0.137	0.012	0.014
			L2BRQ	0.294	0.277	0.163	0.187	0.041	0.037
		gamma	AL1BRQ	0.549	0.198	0.146	0.070	0.019	0.018
			L2BRQ	0.305	0.230	0.193	0.137	0.062	0.072
	0.5	norm	AL1BRQ	0.449	0.290	0.121	0.120	0.009	0.012
			L2BRQ	0.276	0.256	0.178	0.168	0.061	0.061
		tdist	AL1BRQ	0.458	0.290	0.118	0.118	0.008	0.009
			L2BRQ	0.322	0.281	0.174	0.167	0.030	0.026
		gamma	AL1BRQ	0.638	0.137	0.167	0.028	0.017	0.013
			L2BRQ	0.370	0.219	0.223	0.080	0.060	0.049
	0.7	norm	AL1BRQ	0.483	0.256	0.130	0.093	0.014	0.024
			L2BRQ	0.293	0.236	0.175	0.173	0.053	0.070
		tdist	AL1BRQ	0.527	0.254	0.126	0.076	0.008	0.009
			L2BRQ	0.368	0.253	0.192	0.138	0.026	0.022
		gamma	AL1BRQ	0.716	0.026	0.187	0.034	0.020	0.017
			L2BRQ	0.432	0.090	0.271	0.089	0.063	0.055
	0.9	norm	AL1BRQ	0.472	0.181	0.160	0.087	0.052	0.047
			L2BRQ	0.307	0.229	0.206	0.135	0.060	0.063
		tdist	AL1BRQ	0.536	0.107	0.151	0.066	0.069	0.071
			L2BRQ	0.457	0.113	0.243	0.065	0.055	0.066
		gamma	AL1BRQ	0.384	0.186	0.143	0.137	0.072	0.079
			L2BRQ	0.227	0.219	0.190	0.173	0.093	0.098
multivariate2	0.1	norm	AL1BRQ	0.381	0.362	0.027	0.163	0.033	0.033
			L2BRQ	0.292	0.310	0.051	0.234	0.057	0.056
		tdist	AL1BRQ	0.329	0.371	0.044	0.173	0.037	0.045
			L2BRQ	0.276	0.349	0.045	0.236	0.047	0.047
		gamma	AL1BRQ	0.471	0.320	0.028	0.117	0.031	0.034
			L2BRQ	0.327	0.309	0.048	0.196	0.056	0.064
	0.3	norm	AL1BRQ	0.450	0.352	0.016	0.145	0.012	0.025
			L2BRQ	0.333	0.318	0.047	0.207	0.050	0.045
		tdist	AL1BRQ	0.461	0.360	0.009	0.148	0.012	0.011
			L2BRQ	0.352	0.328	0.034	0.206	0.042	0.038
		gamma	AL1BRQ	0.622	0.243	0.019	0.082	0.017	0.018
			L2BRQ	0.348	0.280	0.072	0.170	0.065	0.065
	0.5	norm	AL1BRQ	0.014	0.658	0.017	0.272	0.020	0.018
			L2BRQ	0.058	0.459	0.064	0.284	0.057	0.078
		tdist	AL1BRQ	0.015	0.676	0.014	0.269	0.014	0.013
			L2BRQ	0.043	0.525	0.035	0.317	0.038	0.043
		gamma	AL1BRQ	0.040	0.763	0.030	0.095	0.033	0.040
			L2BRQ	0.046	0.704	0.033	0.124	0.040	0.053
	0.7	norm	AL1BRQ	0.032	0.455	0.253	0.180	0.032	0.048
			L2BRQ	0.064	0.317	0.246	0.217	0.085	0.071
		tdist	AL1BRQ	0.015	0.523	0.262	0.174	0.015	0.011
			L2BRQ	0.030	0.377	0.312	0.227	0.027	0.028
		gamma	AL1BRQ	0.045	0.076	0.663	0.104	0.053	0.060
			L2BRQ	0.105	0.128	0.399	0.165	0.103	0.099
	0.9	norm	AL1BRQ	0.089	0.315	0.278	0.155	0.087	0.075
			L2BRQ	0.078	0.209	0.385	0.199	0.064	0.064
		tdist	AL1BRQ	0.121	0.179	0.300	0.128	0.129	0.143
			L2BRQ	0.123	0.234	0.345	0.109	0.101	0.088
		gamma	AL1BRQ	0.103	0.267	0.191	0.213	0.126	0.100
			L2BRQ	0.099	0.334	0.169	0.221	0.099	0.077

Table 12: MPI for AL1BRQ and L2BRQ for the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	AL1BRQ	0.328	0.344	0.105	0.146	0.044	0.034
			L2BRQ	0.257	0.268	0.174	0.193	0.058	0.050
		tdist	AL1BRQ	0.292	0.317	0.091	0.176	0.061	0.063
			L2BRQ	0.253	0.302	0.143	0.211	0.047	0.043
		gamma	AL1BRQ	0.429	0.260	0.146	0.100	0.034	0.031
			L2BRQ	0.285	0.267	0.181	0.157	0.056	0.053
	0.3	norm	AL1BRQ	0.401	0.309	0.115	0.140	0.010	0.024
			L2BRQ	0.266	0.275	0.173	0.192	0.051	0.043
		tdist	AL1BRQ	0.418	0.323	0.107	0.135	0.009	0.007
			L2BRQ	0.300	0.285	0.171	0.180	0.033	0.031
		gamma	AL1BRQ	0.531	0.211	0.143	0.062	0.032	0.021
			L2BRQ	0.288	0.232	0.193	0.142	0.072	0.072
	0.5	norm	AL1BRQ	0.444	0.289	0.116	0.124	0.015	0.011
			L2BRQ	0.269	0.236	0.179	0.161	0.089	0.066
		tdist	AL1BRQ	0.458	0.293	0.117	0.117	0.007	0.008
			L2BRQ	0.309	0.256	0.174	0.178	0.033	0.050
		gamma	AL1BRQ	0.633	0.131	0.173	0.032	0.018	0.013
			L2BRQ	0.341	0.219	0.226	0.096	0.057	0.061
	0.7	norm	AL1BRQ	0.471	0.251	0.129	0.109	0.014	0.025
			L2BRQ	0.286	0.258	0.183	0.159	0.055	0.058
		tdist	AL1BRQ	0.523	0.247	0.126	0.087	0.007	0.009
			L2BRQ	0.365	0.255	0.186	0.150	0.021	0.024
		gamma	AL1BRQ	0.705	0.029	0.173	0.044	0.022	0.026
			L2BRQ	0.422	0.088	0.268	0.088	0.065	0.069
	0.9	norm	AL1BRQ	0.471	0.195	0.173	0.072	0.048	0.042
			L2BRQ	0.308	0.227	0.212	0.131	0.062	0.060
		tdist	AL1BRQ	0.518	0.118	0.160	0.075	0.060	0.069
			L2BRQ	0.461	0.122	0.249	0.060	0.058	0.050
		gamma	AL1BRQ	0.411	0.181	0.126	0.124	0.075	0.084
			L2BRQ	0.251	0.241	0.180	0.179	0.071	0.079
multivariate2	0.1	norm	AL1BRQ	0.395	0.364	0.029	0.156	0.028	0.028
			L2BRQ	0.299	0.316	0.048	0.236	0.051	0.050
		tdist	AL1BRQ	0.313	0.380	0.042	0.178	0.043	0.045
			L2BRQ	0.280	0.346	0.046	0.233	0.055	0.042
		gamma	AL1BRQ	0.469	0.331	0.029	0.120	0.022	0.028
			L2BRQ	0.347	0.288	0.049	0.206	0.058	0.051
	0.3	norm	AL1BRQ	0.464	0.359	0.011	0.149	0.009	0.009
			L2BRQ	0.344	0.317	0.042	0.213	0.038	0.046
		tdist	AL1BRQ	0.459	0.367	0.010	0.150	0.007	0.008
			L2BRQ	0.359	0.325	0.032	0.217	0.032	0.035
		gamma	AL1BRQ	0.613	0.241	0.018	0.078	0.025	0.026
			L2BRQ	0.354	0.282	0.055	0.166	0.072	0.071
	0.5	norm	AL1BRQ	0.027	0.643	0.020	0.266	0.023	0.021
			L2BRQ	0.087	0.423	0.064	0.281	0.071	0.074
		tdist	AL1BRQ	0.015	0.666	0.020	0.270	0.012	0.017
			L2BRQ	0.039	0.513	0.046	0.312	0.035	0.055
		gamma	AL1BRQ	0.034	0.772	0.044	0.084	0.027	0.039
			L2BRQ	0.048	0.683	0.064	0.107	0.037	0.061
	0.7	norm	AL1BRQ	0.040	0.454	0.253	0.176	0.033	0.044
			L2BRQ	0.071	0.323	0.255	0.214	0.064	0.072
		tdist	AL1BRQ	0.012	0.519	0.263	0.182	0.010	0.014
			L2BRQ	0.020	0.380	0.324	0.227	0.020	0.029
		gamma	AL1BRQ	0.046	0.068	0.674	0.096	0.055	0.061
			L2BRQ	0.106	0.104	0.418	0.150	0.105	0.117
	0.9	norm	AL1BRQ	0.085	0.318	0.325	0.119	0.071	0.083
			L2BRQ	0.074	0.191	0.406	0.196	0.064	0.069
		tdist	AL1BRQ	0.116	0.201	0.339	0.097	0.119	0.129
			L2BRQ	0.100	0.217	0.365	0.104	0.119	0.095
		gamma	AL1BRQ	0.125	0.279	0.190	0.186	0.106	0.114
			L2BRQ	0.083	0.343	0.167	0.225	0.093	0.089

Table 13: MPI for AL1BRQ and L2BRQ for the contaminated cases of the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates					
				x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	AL1BRQ	0.004	0.024	0.486	0.349	0.719	0.720
			L2BRQ	0.000	0.015	0.187	0.096	0.561	0.558
		tdist	AL1BRQ	0.044	0.015	0.502	0.278	0.629	0.600
			L2BRQ	0.000	0.015	0.298	0.102	0.553	0.571
		gamma	AL1BRQ	0.000	0.176	0.392	0.513	0.764	0.772
			L2BRQ	0.000	0.054	0.112	0.199	0.593	0.599
	0.3	norm	AL1BRQ	0.001	0.084	0.516	0.427	0.886	0.869
			L2BRQ	0.000	0.036	0.206	0.167	0.647	0.623
		tdist	AL1BRQ	0.001	0.078	0.560	0.445	0.896	0.898
			L2BRQ	0.000	0.039	0.243	0.178	0.635	0.619
		gamma	AL1BRQ	0.001	0.314	0.439	0.646	0.839	0.823
			L2BRQ	0.000	0.097	0.120	0.273	0.564	0.538
	0.5	norm	AL1BRQ	0.001	0.160	0.511	0.510	0.935	0.933
			L2BRQ	0.000	0.043	0.178	0.179	0.577	0.607
		tdist	AL1BRQ	0.001	0.164	0.526	0.513	0.927	0.924
			L2BRQ	0.000	0.059	0.252	0.244	0.716	0.677
		gamma	AL1BRQ	0.001	0.545	0.473	0.831	0.884	0.903
			L2BRQ	0.000	0.180	0.145	0.463	0.542	0.580
	0.7	norm	AL1BRQ	0.001	0.234	0.436	0.548	0.841	0.859
			L2BRQ	0.000	0.047	0.154	0.196	0.552	0.560
		tdist	AL1BRQ	0.001	0.305	0.495	0.649	0.884	0.883
			L2BRQ	0.000	0.078	0.250	0.352	0.720	0.720
		gamma	AL1BRQ	0.001	0.831	0.502	0.788	0.827	0.847
			L2BRQ	0.000	0.535	0.166	0.434	0.531	0.577
	0.9	norm	AL1BRQ	0.000	0.287	0.349	0.553	0.693	0.682
			L2BRQ	0.000	0.046	0.125	0.271	0.521	0.535
		tdist	AL1BRQ	0.000	0.494	0.433	0.581	0.605	0.578
			L2BRQ	0.000	0.112	0.253	0.393	0.504	0.500
		gamma	AL1BRQ	0.000	0.215	0.341	0.339	0.520	0.550
			L2BRQ	0.000	0.077	0.077	0.134	0.377	0.348
multivariate2	0.1	norm	AL1BRQ	0.010	0.034	0.783	0.417	0.791	0.804
			L2BRQ	0.000	0.020	0.606	0.132	0.598	0.599
		tdist	AL1BRQ	0.054	0.010	0.684	0.333	0.686	0.694
			L2BRQ	0.000	0.018	0.598	0.130	0.582	0.548
		gamma	AL1BRQ	0.000	0.187	0.816	0.544	0.818	0.789
			L2BRQ	0.000	0.072	0.614	0.231	0.606	0.642
	0.3	norm	AL1BRQ	0.001	0.091	0.861	0.479	0.892	0.887
			L2BRQ	0.000	0.043	0.623	0.204	0.629	0.640
		tdist	AL1BRQ	0.001	0.080	0.899	0.475	0.915	0.901
			L2BRQ	0.000	0.045	0.619	0.217	0.651	0.622
		gamma	AL1BRQ	0.001	0.355	0.854	0.697	0.856	0.865
			L2BRQ	0.000	0.115	0.543	0.284	0.530	0.566
	0.5	norm	AL1BRQ	0.891	0.002	0.899	0.383	0.879	0.885
			L2BRQ	0.532	0.000	0.538	0.116	0.506	0.513
		tdist	AL1BRQ	0.879	0.002	0.903	0.394	0.890	0.891
			L2BRQ	0.644	0.000	0.691	0.167	0.640	0.664
		gamma	AL1BRQ	0.723	0.008	0.718	0.564	0.659	0.634
			L2BRQ	0.553	0.001	0.506	0.403	0.489	0.455
	0.7	norm	AL1BRQ	0.782	0.001	0.184	0.325	0.759	0.771
			L2BRQ	0.503	0.000	0.068	0.086	0.444	0.471
		tdist	AL1BRQ	0.846	0.002	0.233	0.421	0.848	0.858
			L2BRQ	0.629	0.000	0.149	0.198	0.657	0.681
		gamma	AL1BRQ	0.664	0.633	0.004	0.472	0.639	0.564
			L2BRQ	0.441	0.139	0.005	0.335	0.364	0.346
	0.9	norm	AL1BRQ	0.511	0.011	0.030	0.301	0.507	0.527
			L2BRQ	0.546	0.000	0.097	0.178	0.584	0.502
		tdist	AL1BRQ	0.340	0.185	0.037	0.334	0.345	0.398
			L2BRQ	0.451	0.000	0.104	0.235	0.442	0.485
		gamma	AL1BRQ	0.393	0.027	0.136	0.152	0.379	0.355
			L2BRQ	0.323	0.041	0.000	0.068	0.309	0.342

Table 14: MFI for AL1BRQ and L2BRQ for the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	AL1BRQ	0.004	0.020	0.495	0.349	0.730	0.752
			L2BRQ	0.000	0.014	0.185	0.100	0.573	0.613
		tdist	AL1BRQ	0.040	0.013	0.490	0.296	0.631	0.616
			L2BRQ	0.000	0.015	0.306	0.118	0.580	0.572
		gamma	AL1BRQ	0.000	0.173	0.414	0.509	0.750	0.768
			L2BRQ	0.000	0.059	0.125	0.212	0.620	0.624
	0.3	norm	AL1BRQ	0.001	0.090	0.546	0.435	0.883	0.858
			L2BRQ	0.000	0.035	0.218	0.166	0.618	0.633
		tdist	AL1BRQ	0.001	0.073	0.567	0.448	0.869	0.918
			L2BRQ	0.000	0.042	0.248	0.191	0.636	0.694
		gamma	AL1BRQ	0.001	0.317	0.439	0.664	0.852	0.857
			L2BRQ	0.000	0.095	0.120	0.283	0.514	0.534
	0.5	norm	AL1BRQ	0.001	0.155	0.510	0.494	0.914	0.917
			L2BRQ	0.000	0.038	0.163	0.154	0.523	0.530
		tdist	AL1BRQ	0.001	0.162	0.540	0.516	0.927	0.933
			L2BRQ	0.000	0.054	0.235	0.218	0.629	0.657
		gamma	AL1BRQ	0.001	0.563	0.460	0.820	0.878	0.888
			L2BRQ	0.000	0.182	0.131	0.436	0.534	0.552
	0.7	norm	AL1BRQ	0.001	0.245	0.428	0.532	0.830	0.827
			L2BRQ	0.000	0.048	0.158	0.198	0.585	0.544
		tdist	AL1BRQ	0.001	0.297	0.496	0.620	0.907	0.877
			L2BRQ	0.000	0.079	0.259	0.328	0.705	0.674
		gamma	AL1BRQ	0.001	0.804	0.503	0.745	0.807	0.807
			L2BRQ	0.000	0.473	0.152	0.453	0.510	0.545
	0.9	norm	AL1BRQ	0.000	0.284	0.336	0.589	0.680	0.681
			L2BRQ	0.000	0.044	0.114	0.300	0.508	0.516
		tdist	AL1BRQ	0.000	0.497	0.393	0.597	0.608	0.588
			L2BRQ	0.000	0.104	0.249	0.463	0.535	0.483
		gamma	AL1BRQ	0.000	0.242	0.334	0.364	0.524	0.508
			L2BRQ	0.000	0.090	0.083	0.156	0.392	0.371
multivariate2	0.1	norm	AL1BRQ	0.007	0.025	0.804	0.403	0.817	0.803
			L2BRQ	0.000	0.019	0.645	0.132	0.636	0.626
		tdist	AL1BRQ	0.062	0.005	0.694	0.339	0.697	0.687
			L2BRQ	0.000	0.017	0.595	0.135	0.599	0.579
		gamma	AL1BRQ	0.000	0.182	0.820	0.544	0.804	0.813
			L2BRQ	0.000	0.070	0.599	0.219	0.607	0.605
	0.3	norm	AL1BRQ	0.001	0.091	0.925	0.497	0.909	0.927
			L2BRQ	0.000	0.042	0.631	0.206	0.663	0.624
		tdist	AL1BRQ	0.002	0.078	0.932	0.498	0.924	0.924
			L2BRQ	0.000	0.048	0.697	0.235	0.704	0.712
		gamma	AL1BRQ	0.001	0.347	0.826	0.690	0.826	0.852
			L2BRQ	0.000	0.113	0.542	0.299	0.528	0.529
	0.5	norm	AL1BRQ	0.854	0.002	0.876	0.366	0.868	0.869
			L2BRQ	0.501	0.000	0.542	0.110	0.507	0.513
		tdist	AL1BRQ	0.887	0.002	0.889	0.383	0.890	0.865
			L2BRQ	0.651	0.000	0.626	0.155	0.641	0.594
		gamma	AL1BRQ	0.672	0.008	0.659	0.564	0.742	0.695
			L2BRQ	0.473	0.001	0.460	0.405	0.558	0.528
	0.7	norm	AL1BRQ	0.735	0.001	0.176	0.318	0.772	0.770
			L2BRQ	0.458	0.000	0.072	0.089	0.485	0.499
		tdist	AL1BRQ	0.814	0.002	0.229	0.419	0.814	0.795
			L2BRQ	0.674	0.000	0.161	0.217	0.687	0.636
		gamma	AL1BRQ	0.642	0.638	0.004	0.482	0.605	0.612
			L2BRQ	0.369	0.171	0.004	0.316	0.403	0.369
	0.9	norm	AL1BRQ	0.538	0.009	0.028	0.367	0.557	0.560
			L2BRQ	0.517	0.000	0.096	0.173	0.523	0.510
		tdist	AL1BRQ	0.324	0.207	0.028	0.387	0.319	0.348
			L2BRQ	0.476	0.000	0.101	0.208	0.430	0.466
		gamma	AL1BRQ	0.422	0.030	0.151	0.172	0.444	0.435
			L2BRQ	0.333	0.045	0.001	0.117	0.325	0.321

Table 15: MFI for AL1BRQ and L2BRQ for the contaminated cases of the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	AL1BRQ	0	0	0	0	0.01	0.02
			L2BRQ	0	0	0	0	0.06	0.03
			RQAic	0	0	0	0	0.42	0.86
		tdist	AL1BRQ	0	0	0	0	0.03	0.04
			L2BRQ	0	0	0	0	0.23	0.22
			RQAic	0	0	0	0.02	0.49	0.88
		gamma	AL1BRQ	0	0	0	0	0.01	0.02
			L2BRQ	0	0	0	0	0.05	0.03
			RQAic	0	0	0	0	0.59	0.92
		mixed	AL1BRQ	0	0	0	0	0.07	0.04
			L2BRQ	0	0	0.01	0	0.2	0.18
			RQAic	0	0	0	0.04	0.38	0.88
	0.3	norm	AL1BRQ	0	0	0	0	0.13	0.14
			L2BRQ	0	0	0	0	0.07	0.12
			RQAic	0	0	0	0	0.54	0.91
		tdist	AL1BRQ	0	0	0	0	0.29	0.28
			L2BRQ	0	0	0	0	0.25	0.28
			RQAic	0	0	0	0	0.75	0.98
		gamma	AL1BRQ	0	0	0	0	0.12	0.14
			L2BRQ	0	0	0	0	0.08	0.11
			RQAic	0	0	0	0	0.69	1
		mixed	AL1BRQ	0	0	0	0	0.05	0.05
			L2BRQ	0	0	0	0	0.1	0.05
			RQAic	0	0	0	0	0.71	0.98
	0.5	norm	AL1BRQ	0	0	0	0	0.13	0.11
			L2BRQ	0	0	0	0	0.12	0.05
			RQAic	0	0	0	0	0.58	0.97
		tdist	AL1BRQ	0	0	0	0	0.26	0.27
			L2BRQ	0	0	0	0	0.28	0.36
			RQAic	0	0	0	0	0.84	0.98
		gamma	AL1BRQ	0	0	0	0.18	0.16	0.23
			L2BRQ	0	0	0	0.16	0.15	0.18
			RQAic	0	0	0	0.42	0.87	1
		mixed	AL1BRQ	0	0	0	0	0.13	0.11
			L2BRQ	0	0	0	0	0.08	0.09
			RQAic	0	0	0	0.03	0.79	0.97
	0.7	norm	AL1BRQ	0	0	0	0	0.08	0.08
			L2BRQ	0	0	0	0	0.09	0.06
			RQAic	0	0	0	0	0.51	0.92
		tdist	AL1BRQ	0	0	0	0	0.46	0.41
			L2BRQ	0	0	0	0	0.47	0.42
			RQAic	0	0	0	0	0.72	0.97
		gamma	AL1BRQ	0	0.2	0	0.15	0.22	0.23
			L2BRQ	0	0.17	0	0.21	0.2	0.19
			RQAic	0	0	0.22	0.65	0.93	1
		mixed	AL1BRQ	0	0	0.01	0.15	0.15	0.22
			L2BRQ	0	0.01	0.01	0.17	0.16	0.18
			RQAic	0	0	0.01	0.41	0.81	0.99
	0.9	norm	AL1BRQ	0	0	0	0.01	0.01	0.03
			L2BRQ	0	0	0	0.02	0.14	0.18
			RQAic	0	0	0	0.04	0.43	0.88
		tdist	AL1BRQ	0	0.02	0	0.07	0.08	0.1
			L2BRQ	0	0.03	0.02	0.46	0.46	0.42
			RQAic	0	0.02	0.12	0.34	0.81	0.97
		gamma	AL1BRQ	0	0	0	0.01	0.02	0
			L2BRQ	0	0	0	0.02	0.03	0.04
			RQAic	0	0	0.01	0.06	0.44	0.86
		mixed	AL1BRQ	0	0.15	0.11	0.15	0.28	0.3
			L2BRQ	0	0.17	0.04	0.23	0.36	0.34
			RQAic	0	0	0.07	0.34	0.72	0.95
multivariate2	0.1	norm	AL1BRQ	0	0	0.02	0	0.05	0.02
			L2BRQ	0	0	0.12	0	0.15	0.12
			RQAic	0	0	0	0.18	0.6	0.93
		tdist	AL1BRQ	0	0	0.07	0	0.12	0.11
			L2BRQ	0	0	0.24	0	0.31	0.33
			RQAic	0	0	0	0.28	0.63	0.94
		gamma	AL1BRQ	0	0	0.04	0	0.03	0.04
			L2BRQ	0	0	0.11	0	0.08	0.11
			RQAic	0	0	0	0.45	0.85	0.95
		mixed	AL1BRQ	0	0	0.07	0	0.03	0.07
			L2BRQ	0	0	0.24	0	0.2	0.18

		RQAic	0	0	0	0.29	0.73	0.96
0.3	norm	AL1BRQ	0	0	0.23	0	0.2	0.17
		L2BRQ	0	0	0.22	0	0.19	0.14
		RQAic	0	0	0	0.36	0.85	0.99
	tdist	AL1BRQ	0	0	0.29	0	0.28	0.38
		L2BRQ	0	0	0.34	0	0.31	0.37
		RQAic	0	0	0	0.64	0.97	1
	gamma	AL1BRQ	0	0	0.2	0	0.17	0.16
		L2BRQ	0	0	0.14	0	0.14	0.12
		RQAic	0	0	0.01	0.6	0.95	1
	mixed	AL1BRQ	0	0	0.11	0	0.12	0.08
		L2BRQ	0	0	0.11	0	0.08	0.12
		RQAic	0	0	0	0.66	0.94	1
0.5	norm	AL1BRQ	0.3	0	0.24	0	0.21	0.24
		L2BRQ	0.22	0	0.22	0	0.21	0.21
		RQAic	0	0	0.46	0.77	0.94	1
	tdist	AL1BRQ	0.4	0	0.36	0	0.39	0.37
		L2BRQ	0.39	0	0.33	0	0.4	0.37
		RQAic	0	0	0.61	0.96	1	1
	gamma	AL1BRQ	0.53	0	0.57	0.38	0.61	0.56
		L2BRQ	0.55	0	0.66	0.4	0.62	0.58
		RQAic	0	0.29	0.64	0.86	0.99	1
	mixed	AL1BRQ	0	0	0.22	0.01	0.25	0.21
		L2BRQ	0	0	0.16	0.01	0.21	0.18
		RQAic	0	0	0.06	0.71	0.97	1
0.7	norm	AL1BRQ	0.1	0	0	0	0.07	0.09
		L2BRQ	0.1	0	0	0	0.08	0.09
		RQAic	0	0	0	0.44	0.84	0.98
	tdist	AL1BRQ	0.49	0	0	0	0.55	0.45
		L2BRQ	0.51	0	0	0	0.53	0.51
		RQAic	0	0	0	0.65	0.93	1
	gamma	AL1BRQ	0.45	0.31	0	0.3	0.39	0.46
		L2BRQ	0.21	0.1	0	0.11	0.26	0.29
		RQAic	0	0.17	0.57	0.87	0.97	0.99
	mixed	AL1BRQ	0	0	0.01	0.22	0.22	0.18
		L2BRQ	0	0	0.01	0.19	0.2	0.21
		RQAic	0	0	0.02	0.5	0.83	0.98
0.9	norm	AL1BRQ	0.03	0	0	0.01	0.04	0.04
		L2BRQ	0.29	0	0	0.03	0.35	0.35
		RQAic	0	0	0.03	0.22	0.59	0.95
	tdist	AL1BRQ	0.1	0.08	0.04	0.13	0.15	0.14
		L2BRQ	0.31	0	0	0.13	0.35	0.37
		RQAic	0	0.04	0.26	0.55	0.89	1
	gamma	AL1BRQ	0.01	0	0.01	0	0.02	0.04
		L2BRQ	0.05	0	0	0.02	0.1	0.06
		RQAic	0	0	0.02	0.24	0.59	0.93
	mixed	AL1BRQ	0	0.13	0.12	0.23	0.17	0.26
		L2BRQ	0	0.07	0.01	0.18	0.28	0.32
		RQAic	0	0.01	0.12	0.38	0.82	0.94

Table 16: PER for AL1BRQ, L2BRQ, and RQAic for the multivariate and multivariate2 setup and all error distributions.

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	AL1BRQ	0	0	0	0	0	0
			L2BRQ	0	0	0	0	0.1	0.03
			RQAic	0	0	0	0	0.42	0.9
		tdist	AL1BRQ	0	0	0.01	0	0.02	0.04
			L2BRQ	0	0	0.05	0	0.26	0.23
			RQAic	0	0	0	0.06	0.57	0.94
		gamma	AL1BRQ	0	0	0	0	0.01	0.01
			L2BRQ	0	0	0	0	0.07	0.09
			RQAic	0	0	0	0	0.64	0.92
		mixed	AL1BRQ	0	0	0.01	0	0.04	0.06
			L2BRQ	0	0	0.01	0	0.13	0.18
			RQAic	0	0	0.01	0.03	0.44	0.88
	0.3	norm	AL1BRQ	0	0	0	0	0.14	0.2
			L2BRQ	0	0	0	0	0.12	0.14
			RQAic	0	0	0	0	0.64	0.95
		tdist	AL1BRQ	0	0	0	0	0.41	0.29
			L2BRQ	0	0	0	0	0.28	0.31
			RQAic	0	0	0	0	0.73	1
		gamma	AL1BRQ	0	0	0	0	0.06	0.11
			L2BRQ	0	0	0	0	0.09	0.11
			RQAic	0	0	0	0.01	0.68	0.96
		mixed	AL1BRQ	0	0	0	0	0.08	0.11
			L2BRQ	0	0	0	0	0.06	0.14
			RQAic	0	0	0	0	0.82	0.96
	0.5	norm	AL1BRQ	0	0	0	0	0.07	0.09
			L2BRQ	0	0	0	0	0.04	0.07
			RQAic	0	0	0	0	0.58	0.93
		tdist	AL1BRQ	0	0	0	0	0.32	0.28
			L2BRQ	0	0	0	0	0.27	0.22
			RQAic	0	0	0	0	0.88	1
		gamma	AL1BRQ	0	0	0	0.15	0.18	0.25
			L2BRQ	0	0	0	0.12	0.13	0.17
			RQAic	0	0	0	0.46	0.87	0.98
		mixed	AL1BRQ	0	0	0	0.02	0.14	0.16
			L2BRQ	0	0	0	0.02	0.07	0.07
			RQAic	0	0	0	0.05	0.8	0.98
	0.7	norm	AL1BRQ	0	0	0	0	0.13	0.09
			L2BRQ	0	0	0	0	0.12	0.13
			RQAic	0	0	0	0	0.66	0.94
		tdist	AL1BRQ	0	0	0	0	0.52	0.42
			L2BRQ	0	0	0	0	0.53	0.5
			RQAic	0	0	0	0.01	0.83	0.97
		gamma	AL1BRQ	0	0.23	0	0.18	0.2	0.33
			L2BRQ	0	0.22	0	0.17	0.19	0.23
			RQAic	0	0	0.33	0.73	0.91	1
		mixed	AL1BRQ	0	0.01	0	0.16	0.17	0.19
			L2BRQ	0	0	0	0.18	0.18	0.19
			RQAic	0	0	0.06	0.6	0.93	1
	0.9	norm	AL1BRQ	0	0	0	0.01	0.04	0.03
			L2BRQ	0	0	0	0.02	0.16	0.14
			RQAic	0	0	0	0.07	0.54	0.91
		tdist	AL1BRQ	0	0.05	0.01	0.02	0.09	0.1
			L2BRQ	0	0.02	0.01	0.36	0.44	0.47
			RQAic	0	0.03	0.29	0.61	0.93	1
		gamma	AL1BRQ	0	0.01	0	0.01	0.03	0
			L2BRQ	0	0	0.01	0	0.07	0.05
			RQAic	0	0	0	0.04	0.59	0.96
		mixed	AL1BRQ	0	0.16	0.13	0.14	0.27	0.27
			L2BRQ	0	0.2	0.05	0.16	0.25	0.37
			RQAic	0	0.02	0.3	0.69	0.91	1
multivariate2	0.1	norm	AL1BRQ	0	0	0.03	0	0.02	0.01
			L2BRQ	0	0	0.13	0	0.15	0.12
			RQAic	0	0	0	0.29	0.7	0.92
		tdist	AL1BRQ	0	0	0.09	0	0.06	0.09
			L2BRQ	0	0	0.27	0	0.22	0.32
			RQAic	0	0	0	0.34	0.85	0.98
		gamma	AL1BRQ	0	0	0.04	0	0.02	0.02
			L2BRQ	0	0	0.15	0	0.13	0.08
			RQAic	0	0	0	0.61	0.9	1
		mixed	AL1BRQ	0	0	0.1	0.01	0.08	0.1
			L2BRQ	0	0	0.31	0.01	0.22	0.21

		RQAic	0	0	0	0.39	0.8	0.97
0.3	norm	AL1BRQ	0	0	0.17	0	0.22	0.19
		L2BRQ	0	0	0.21	0	0.18	0.25
		RQAic	0	0	0	0.48	0.88	1
	tdist	AL1BRQ	0	0	0.35	0	0.48	0.38
		L2BRQ	0	0	0.35	0	0.38	0.33
		RQAic	0	0	0	0.71	0.99	1
	gamma	AL1BRQ	0	0	0.13	0.01	0.11	0.06
		L2BRQ	0	0	0.09	0	0.13	0.06
		RQAic	0	0	0.01	0.63	0.96	1
	mixed	AL1BRQ	0	0	0.14	0	0.08	0.2
		L2BRQ	0	0	0.14	0	0.11	0.15
		RQAic	0	0	0	0.68	0.95	0.99
0.5	norm	AL1BRQ	0.15	0	0.19	0	0.16	0.12
		L2BRQ	0.14	0	0.16	0	0.18	0.14
		RQAic	0	0	0.49	0.87	0.98	1
	tdist	AL1BRQ	0.31	0	0.32	0	0.44	0.41
		L2BRQ	0.32	0	0.34	0	0.42	0.38
		RQAic	0	0	0.71	0.94	1	1
	gamma	AL1BRQ	0.56	0	0.52	0.39	0.54	0.51
		L2BRQ	0.55	0	0.54	0.36	0.56	0.53
		RQAic	0	0.3	0.74	0.97	0.99	1
	mixed	AL1BRQ	0	0	0.19	0.03	0.23	0.21
		L2BRQ	0	0	0.17	0.02	0.18	0.16
		RQAic	0	0	0.07	0.73	0.98	1
0.7	norm	AL1BRQ	0.17	0	0	0	0.11	0.12
		L2BRQ	0.19	0	0	0	0.12	0.1
		RQAic	0	0	0	0.43	0.91	1
	tdist	AL1BRQ	0.61	0	0	0.01	0.61	0.6
		L2BRQ	0.64	0	0	0	0.62	0.59
		RQAic	0	0	0.01	0.8	0.93	0.99
	gamma	AL1BRQ	0.44	0.34	0	0.34	0.47	0.38
		L2BRQ	0.29	0.09	0	0.22	0.24	0.22
		RQAic	0	0.21	0.63	0.89	0.98	1
	mixed	AL1BRQ	0	0	0	0.22	0.19	0.17
		L2BRQ	0	0	0	0.18	0.15	0.12
		RQAic	0	0	0.04	0.43	0.92	0.99
0.9	norm	AL1BRQ	0.02	0	0	0	0.02	0.03
		L2BRQ	0.32	0	0	0.02	0.33	0.41
		RQAic	0	0	0.05	0.39	0.79	0.97
	tdist	AL1BRQ	0.16	0.11	0.02	0.12	0.14	0.17
		L2BRQ	0.35	0	0	0.15	0.28	0.37
		RQAic	0.01	0.19	0.54	0.87	0.98	0.99
	gamma	AL1BRQ	0.03	0.02	0.01	0.01	0.03	0.03
		L2BRQ	0.13	0	0	0	0.16	0.09
		RQAic	0	0.01	0.04	0.42	0.81	0.94
	mixed	AL1BRQ	0	0.18	0.17	0.25	0.37	0.28
		L2BRQ	0	0.05	0.04	0.14	0.2	0.34
		RQAic	0	0.03	0.19	0.62	0.9	1

Table 17: PER for AL1BRQ, L2BRQ, and RQAic for the contaminated cases for the multivariate and multivariate2 setup and all error distributions.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9	$\frac{\text{time}}{\text{iteration}}$
homoskedastic	norm	AL1BRQ	600	180	100	180	660	0.148
		L2BRQ	1100	1100	1800	1050	1250	0.054
	tdist	AL1BRQ	640	160	80	180	530	0.145
		L2BRQ	2550	700	650	1250	2100	0.054
	gamma	AL1BRQ	680	140	80	180	540	0.169
		L2BRQ	900	500	450	550	2000	0.055
	mixed	AL1BRQ	1090	360	180	400	1700	0.145
		L2BRQ	1700	1150	1350	1750	2450	0.054
heteroskedastic	norm	AL1BRQ	360	320	180	380	2110	0.157
		L2BRQ	2350	900	3050	2750	3050	0.054
	tdist	AL1BRQ	40	240	180	400	2230	0.146
		L2BRQ	3300	700	1900	2400	4850	0.054
	gamma	AL1BRQ	1910	500	280	700	3760	0.165
		L2BRQ	1500	850	950	1900	3950	0.054
	mixed	AL1BRQ	1440	800	460	960	5000	0.151
		L2BRQ	2950	2000	3150	5050	6750	0.054
multivariate	norm	AL1BRQ	13950	3000	1400	2700	9500	1.409
		L2BRQ	18000	8375	7250	7750	9250	0.287
	tdist	AL1BRQ	13950	3050	1400	2500	7950	1.303
		L2BRQ	20750	8250	6000	5500	5750	0.287
	gamma	AL1BRQ	10300	2200	1000	1900	9600	1.718
		L2BRQ	12000	7500	5750	5750	24875	0.289
	mixed	AL1BRQ	15300	3500	1500	3000	11350	1.372
		L2BRQ	20000	11500	10500	9625	11000	0.286
multivariate2	norm	AL1BRQ	12700	2700	700	1400	3700	1.689
		L2BRQ	13750	6500	3500	5375	3500	0.289
	tdist	AL1BRQ	12800	2700	600	1200	2200	1.567
		L2BRQ	16625	6875	2750	2750	3250	0.287
	gamma	AL1BRQ	9100	1900	200	400	4000	1.063
		L2BRQ	7875	5500	1000	2750	14000	0.286
	mixed	AL1BRQ	14300	3200	800	1500	4850	1.217
		L2BRQ	16250	9375	6500	8625	11625	0.286
high-dimensional	norm	AL1BRQ	14100	3150	1500	2850	9600	12.184
		L2BRQ	8500	5250	4250	4250	4500	2.975
	tdist	AL1BRQ	14925	3300	1500	2850	8550	11.878
		L2BRQ	8875	5750	4750	4750	2000	2.927
	gamma	AL1BRQ	10350	2250	900	1800	7950	13.487
		L2BRQ	5750	3750	2750	2250	3750	2.953
	mixed	AL1BRQ	16500	3450	1500	2700	12975	12.171
		L2BRQ	8750	5375	4500	3500	4375	2.944

Table 18: Median number of iterations required of AL1BRQ and L2BRQ for all parameter setups and error distributions for each τ. Last column indicates the the median computing time for one iteration in ms. Extension of Table 5 from the main document.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9	$\frac{\text{time}}{\text{iteration}}$
homoskedastic	norm	AL1BRQ	620	200	100	180	650	0.148
		L2BRQ	1350	1700	950	950	1200	0.053
	tdist	AL1BRQ	630	160	80	180	510	0.147
		L2BRQ	2100	600	650	700	1900	0.055
	gamma	AL1BRQ	660	140	80	180	560	0.167
		L2BRQ	900	400	600	500	1600	0.053
	mixed	AL1BRQ	1100	360	180	380	1740	0.142
		L2BRQ	1700	1050	1100	1700	2300	0.053
heteroskedastic	norm	AL1BRQ	400	320	180	380	2080	0.156
		L2BRQ	2650	1000	2800	1950	3450	0.054
	tdist	AL1BRQ	60	240	180	400	2180	0.144
		L2BRQ	3250	700	1050	3150	5300	0.053
	gamma	AL1BRQ	1920	500	280	700	3680	0.172
		L2BRQ	1250	750	950	1800	4700	0.057
	mixed	AL1BRQ	1410	800	460	970	5000	0.171
		L2BRQ	3600	1850	2750	5400	7600	0.063
multivariate	norm	AL1BRQ	13900	3000	1400	2700	9400	1.411
		L2BRQ	18000	8000	8250	7500	10375	0.287
	tdist	AL1BRQ	13950	3000	1400	2500	8100	1.304
		L2BRQ	19750	8125	6250	5250	6000	0.287
	gamma	AL1BRQ	10250	2200	1000	1900	9600	1.706
		L2BRQ	10125	7750	5500	5750	22625	0.287
	mixed	AL1BRQ	15300	3500	1500	3000	11200	1.375
		L2BRQ	20625	10750	11000	9750	10750	0.287
multivariate2	norm	AL1BRQ	12600	2700	700	1400	3700	1.686
		L2BRQ	13875	6500	3500	5375	3000	0.288
	tdist	AL1BRQ	12900	2700	600	1200	2400	1.550
		L2BRQ	16750	6250	3000	2500	3250	0.288
	gamma	AL1BRQ	9200	1900	200	400	3800	1.066
		L2BRQ	9125	6000	1000	3250	13250	0.287
	mixed	AL1BRQ	14400	3100	800	1500	4850	1.219
		L2BRQ	14250	8250	6500	9250	10250	0.287
high-dimensional	norm	AL1BRQ	14100	3000	1500	2700	9450	12.230
		L2BRQ	8625	5000	4500	4500	4500	2.942
	tdist	AL1BRQ	13425	3300	1500	2850	8550	11.927
		L2BRQ	8000	6000	4750	4750	2250	2.954
	gamma	AL1BRQ	10650	2250	900	1800	7575	13.418
		L2BRQ	6000	3500	2750	2250	3750	2.941
	mixed	AL1BRQ	16500	3525	1500	2850	12975	12.127
		L2BRQ	8125	5500	4750	3375	4000	2.933

Table 19: Median number of iterations required of AL1BRQ and L2BRQ for the contaminated cases of all parameter setups and error distributions for each τ. Last column indicates the the median computing time for one iteration in ms. Extension of Table 5 from the main document.

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