Web appendix for Pinball boosting of regression quantiles

Ida Bauer[a,*], Harry Haupt[a], Stefan Linner[b]

[a] *University of Passau, Chair of Statistics and Data Analytics, D-94030 Passau, Germany*
[b] *TU Dortmund University, Department of Statistics, D-44221 Dortmund, Germany*

This appendix comprises two main parts: First, Section A promotes an in-depth understanding of the algorithm proposed in the main article for boosting of regression quantiles with pinball loss and score (QR-QR.Boost). It includes equivalent representations of the algorithms for "classical" boosting of regression means with L_2 loss and score (LS-LS.Boost) and the algorithm of Fenske et al. (2011) for boosting of regression quantiles with pinball loss and L_2 score (QR-LS.Boost). Subsection A.1 discusses similarities and differences summarized in Table 1. Detailed didactic descriptions and interpretations of the individual steps, along by informative visualizations, can be found in Subsection A.2. Subsection A.3 contains general implications for variable selection, model selection and functional form, interpretability and tuning parameters. Subsequently, Section B reproduces the main results and extends the simulation study from Fenske et al. (2011).

A. Algorithms

Step	LS-LS.Boost	QR-LS.Boost	QR-QR.Boost
0. Directive	regression mean	τ-regression quantile	
1. Initialize *fitted values*	sample mean	sample median	τ-sample quantile
base learners	simple regression means		simple regression quantiles
2. Fit	L_2 loss	pinball loss	
3. Update *with best fit*	L_2 score		pinball score
learning rate	ν		ν_τ

Table 1: Similarities and differences between the proposed algorithm and existing boosting algorithms.

A.1. Step-by-step comparison of the algorithms

0. Directive

The "directive" of both the QR-QR.Boost and the QR-LS.Boost algorithm is to estimate a generalized additive quantile regression model by functional gradient boosting, which is achieved by minimizing the well-known pinball loss function

$$L(y, \eta_\tau) = \rho_\tau(y - \eta_\tau) = \psi_\tau(y_i - \eta_\tau) \cdot (y_i - \eta_\tau),$$

where $\psi_\tau(z) := \tau - \mathbb{1}(z < 0)$. Both methods are particularly suitable for situations that demand interpretability and variable selection as well as model choice, e.g., in high-dimensional settings.

1. Initialize the fitted values

In the first step, the iteration counter m is set to zero, and the algorithms are set up. Initializing the component-wise gradient boosting algorithm with appropriate starting values can significantly influence the results. Therefore, the fitted values for the τth conditional quantile, $\widehat{\eta}^{[0]}_{\tau i}$, should be initialized with their best initial guess. Although

*Corresponding author
Email addresses: ida.bauer@uni-passau.de (Ida Bauer), harry.haupt@uni-passau.de (Harry Haupt), stefan-linner@gmx.de (Stefan Linner)

Algorithm LS-LS.Boost Boosting regression means with L_2 loss and score

1. **Initialize the fitted values** for the conditional mean, $\widehat{\eta}_i^{[0]}$, with the sample mean of the response. Set the iteration counter to $m := 0$.

2. **Fit the working residuals.**

   2a. Set $m := m + 1$.

   2b. Compute the working residuals using the subgradient of squared loss

$$u_i^{[m]} = \psi\left(y_i - \widehat{\eta}_i^{[m-1]}\right) = \left(y_i - \widehat{\eta}_i^{[m-1]}\right).$$

   2c. Compute a least squares regression fit for each base learner according to

$$E(u_i{}^{[m]}|x_{ij}) = a_j + b_j x_{ij} \qquad \text{for} \qquad j = 1, ..., p,$$

   and obtain $\widehat{a}_j^{[m]}$ and $\widehat{b}_j^{[m]}$ and $\widehat{u}_{ij}^{[m]} = \widehat{a}_j^{[m]} + \widehat{b}_j^{[m]} x_{ij}$.

3. **Update one component.**

   3a. Select the component x_{j*} that fits the working residuals best based on

$$\arg\min_j RSS_j = \arg\min_j \sum_{i=1}^{n} \left(u_i^{[m]} - \widehat{u}_{ij}^{[m]}\right)^2.$$

   3b. Update the estimate of the τth regression quantile for learning rate ν

$$\widehat{\boldsymbol{\beta}}^{[m]} = \widehat{\boldsymbol{\beta}}^{[m-1]} + \nu \cdot \widehat{\boldsymbol{b}}_{(j*)}^{[m]},$$

   where $\widehat{\boldsymbol{b}}_{(j*)}^{[m]}$ is a $((p+1) \times 1)$-vector with first entry $\widehat{a}_{j*}^{[m]}$, the $(j*+1)$-th entry $\widehat{b}_{j*}^{[m]}$ for the best-fitting component x_{j*}, and 0 for all remaining components.

Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

Algorithm QR-LS.Boost Boosting regression quantiles with pinball loss and L_2 score

1. **Initialize the fitted values** for the τth conditional quantile, $\widehat{\eta}_{\tau i}^{[0]}$, with sample median of the response. Set the iteration counter to $m := 0$.

2. **Fit the working residuals.**

   2a. Set $m := m + 1$.

   2b. Compute the working residuals using the subgradient of pinball loss

$$u_i^{[m]} = \psi_\tau\left(y_i - \widehat{\eta}_{\tau i}^{[m-1]}\right) = \begin{cases} \tau & y_i \geq \widehat{\eta}_{\tau i}^{[m-1]} \\ \tau - 1 & y_i < \widehat{\eta}_{\tau i}^{[m-1]} \end{cases}.$$

   2c. Compute a least squares regression fit for each base learner according to

$$E(u_i{}^{[m]}|x_{ij}) = a_j + b_j x_{ij} \qquad \text{for} \qquad j = 1, ..., p,$$

   and obtain $\widehat{a}_j^{[m]}$ and $\widehat{b}_j^{[m]}$ and $\widehat{u}_{ij}^{[m]} = \widehat{a}_j^{[m]} + \widehat{b}_j^{[m]} x_{ij}$.

3. **Update one component.**

   3a. Select the component x_{j*} that fits the working residuals best based on

$$\arg\min_j RSS_j = \arg\min_j \sum_{i=1}^{n} \left(u_i^{[m]} - \widehat{u}_{ij}^{[m]}\right)^2.$$

   3b. Update the estimate of the τth regression quantile for learning rate ν

$$\widehat{\boldsymbol{\beta}}_\tau^{[m]} = \widehat{\boldsymbol{\beta}}_\tau^{[m-1]} + \nu \cdot \widehat{\boldsymbol{b}}_{(j*)}^{[m]},$$

   where $\widehat{\boldsymbol{b}}_{(j*)}^{[m]}$ is a $((p+1) \times 1)$-vector with the first entry $\widehat{a}_{j*}^{[m]}$, the $(j*+1)$-th entry $\widehat{b}_{jj}^{[m]}$ for the best-fitting component x_{j*}, and 0 for all remaining components.

Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

the respective τth sample quantile of the response appears to be an obvious candidate, Fenske et al. (2011, p. 498 and p. 19 in the Electronic Supplementary Material) refer the reader to their empirical experience which suggests that initializing with the sample median leads to faster convergence of the algorithm, i.e., to a reduced optimal number of boosting iterations. Consequently, the QR-LS.Boost algorithm is initialized with the sample median regardless of the τth conditional quantile to be estimated.

Our simulation experiments suggest that for the QR-QR.Boost algorithm, the sample quantile used for initialization that leads to the fastest convergence of the algorithm depends on the underlying (unknown) data generating process. No single initialization-quantile universally leads to the fastest convergence. This property and its potential relevance to QR-LS.Boost are elaborated further in Section A.2. Consequently, QR-QR.Boost is initialized with the most intuitive choice, the respective τth sample quantile.

Stopping the algorithm prior to the first iteration delivers an estimate for the τth conditional quantile function, $\widehat{\eta}_{\tau}^{[0]}$, that contains only an intercept corresponding to the respective τth sample quantile of the response. This outcome aligns with our expectations when fitting a quantile regression model comprising only an intercept. It is important to note the difference in notation between the current fitted values in iteration m, $\widehat{\eta}_{\tau i}^{[m]}$, and the current estimate for the τth conditional quantile function, $\widehat{\eta}_{\tau}^{[m]}$, which consists of the regression coefficients in iteration m. Also, for clarity of notation, the quantile parameter referring to the sample quantile used for initialization, denoted as τ_{init}, is distinguished from the quantile parameter referring to the conditional quantile to be estimated, denoted as τ.

Along with the initialization of the fitted values, appropriate base learners must be specified to complete the algorithm setup. The choice of base learners effectively imposes structural assumptions on the functional form. Fenske et al. (2011, p. 498) argue that "least-squares base learners are a natural choice" for their proposed QR-LS.Boost algorithm and opt for simple linear regression means, $E(u_i|x_{ij}) = a_j + b_j x_{ij}$, for each predictor $j = 1, ..., p$. This statement originates from Friedman (2001, p. 1194), where least squares base learners arguably are a "natural choice" since the conditional expectation function is estimated by minimizing the squared loss.Given that in the present case the algorithm ultimately aims to estimate the conditional quantile function, simple linear regression quantiles, $Q_{u_i}(\tau|x_{ij}) = a_{\tau j} + b_{\tau j} x_{ij}$, for each predictor $j = 1, ..., p$, seem to be an intuitive choice for the base learners. Hence, the proposed QR-QR.Boost algorithm uses simple linear regression quantiles as base learners instead. In general, base learners are not limited to simple regression models. However, using multivariate regression models as base learners may compromise the variable selection property of the algorithm, as discussed in Subsection A.3. Theoretical considerations should underpin such a choice.

2. Fit the working residuals.

2a. Set $m := m + 1$.

2b. Compute the working residuals.

The negative gradient is the negative derivative of the pinball loss with respect to the τth conditional quantile function. For continuous variables, the pinball loss function is not differentiable at point $y_i = \eta_{\tau i}$, which can be neglected as this "exact fit" event occurs with zero probability, except in the first iteration. In the first iteration, the fitted values correspond to the sample quantile (QR-QR.Boost) or sample median (QR-LS.Boost) of the response, such that at least one observation is fitted exactly. Our Monte Carlo study suggests that the results are robust to whether the working residuals[1] of those "exact fit" observations are set to τ or $\tau - 1$. Setting the working residuals of these observations to 0, as implemented in Fenske et al. (2009, p. 6), has no considerable impact either. Thus, there is no difference between the computation of the negative gradient in QR-QR.Boost and QR-LS.Boost. Due to the different initializations, however, the computed working residuals differ between the two algorithms (except for $\tau = 0.5$).

2c. Compute a least squares/quantile regression fit for each base learner.

Each base learner from Step 1 is now fitted to the working residuals of the current iteration, m. QR-LS.Boost separately fits each predictor, $j = 1, .., p$, to the working residuals as a simple linear least squares regression: $E(u_i^{[m]}|x_{ij}) = a_j + b_j x_{ij}$. QR-QR.Boost separately fits each predictor, $j = 1, .., p$, to the working residuals as a simple linear quantile regression: $Q_{u_i^{[m]}}(\tau|x_{ij}) = a_{\tau j} + b_{\tau j} x_{ij}$.

[1]Other publications use the term "negative gradients" for the working residuals. We prefer "working residuals" to avoid confusion between the negative gradient and the working residuals. The negative gradient is the negative derivative of the pinball loss w.r.t. the conditional τth quantile function. The working residuals are obtained by evaluating the negative gradient at the fitted values for the τth conditional quantile of the previous iteration, i.e., at $\widehat{\eta}_{\tau i}^{[m-1]}$, and are thus a vector of length n whose values take either τ or $\tau - 1$.

3. Update one component.

3a. Select the component x_{j*} that fits the working residuals best.

The algorithm selects the base learner, x_{j*}, that best fits the working residuals of the current iteration, $u_i^{[m]}$. QR-LS.Boost defines the best-fitting base learner as the one with the smallest residual sum of squares (RSS),

$$\mathrm{RSS}_j = \sum_{i=1}^{n} \left(u_i^{[m]} - \widehat{u}_{ij}^{[m]} \right)^2,$$

which is a suitable criterion for regression means as base learners. QR-QR.Boost defines the best-fitting base learner as the one with the smallest empirical risk (based on the quantile score)

$$\begin{aligned} \mathrm{R}_{\tau j}\left(u^{[m]}, \widehat{u}_{\tau j}^{[m]}\right) &= \sum_{i=1}^{n} \rho_\tau \left(u_i^{[m]} - \widehat{u}_{\tau ij}^{[m]} \right) \\ &= \sum_{i=1}^{n} \begin{cases} \tau(u_i^{[m]} - \widehat{u}_{\tau ij}^{[m]}) & u_i^{[m]} > \widehat{u}_{\tau ij}^{[m]} \\ (\tau - 1)(u_i^{[m]} - \widehat{u}_{\tau ij}^{[m]}) & u_i^{[m]} \leq \widehat{u}_{\tau ij}^{[m]}. \end{cases} \end{aligned}$$

3b. Update the estimate of the τth regression quantile.

The coefficient vector of the best-fitting component of the τth conditional quantile function, $\widehat{\eta}_\tau^{[m]}$, and the fitted values, $\widehat{\eta}_{\tau i}^{[m]}$, are additively updated. Note that in the present setup the single components correspond to the predictors. A typically small and pre-specified learning rate, ν, acts as a shrinkage factor for the coefficient estimate. This regularizes the single estimations and reduces their influence on the final estimate (the choice of a suitable learning rate is discussed in Subsection A.3). QR-LS.Boost fixes the learning rate to the frequently employed value of $\nu = 0.1$. Since the accuracy of quantile regression critically depends on how informative the design is over the distribution of the response, for QR-QR.Boost, we opt for a quantile-specific learning rate ν_τ that is lower for quantiles at the tails of the distribution, where data is typically more sparse, and higher for the center of the distribution.

Thus, QR-QR.Boost employs a τ-specific learning rate ν_τ, while QR-LS.Boost fixes the learning rate to $\nu = 0.1$.

For QR-LS.Boost, the τth effect estimate is updated by

$$\widehat{\boldsymbol{\beta}}_\tau^{[m]} = \widehat{\boldsymbol{\beta}}_\tau^{[m-1]} + \nu \cdot \widehat{\boldsymbol{b}}_{(j*)}^{[m]},$$

where ν is the fixed learning rate and $\widehat{\boldsymbol{b}}_{(j*)}^{[m]}$ is a $((p+1) \times 1)$-vector with first entry $\widehat{a}_{j*}^{[m]}$, the $(j*+1)$-th entry $\widehat{b}_{j*}^{[m]}$ for the best-fitting component x_{j*}, and 0 for all remaining components.

The τth conditional quantile function and the fitted values are updated by

$$\begin{aligned} \widehat{\eta}_\tau^{[m]} &= \widehat{\eta}_\tau^{[m-1]} + \nu \widehat{E}(u^{[m]} | x_{j*}), \\ \widehat{\eta}_{\tau i}^{[m]} &= \widehat{\eta}_{\tau i}^{[m-1]} + \nu \widehat{u}_{ij}^{[m]}, \end{aligned}$$

where $\widehat{E}(u^{[m]} | x_{j*})$ represents the estimate for the conditional mean of the current working residuals as a function of the best-fitting predictor, x_{j*}. The respective fitted values for the conditional mean of the current working residuals are denoted by $\widehat{u}_{ij}^{[m]}$.

For QR-QR.Boost, the τth effect estimate is updated by

$$\widehat{\boldsymbol{\beta}}_\tau^{[m]} = \widehat{\boldsymbol{\beta}}_\tau^{[m-1]} + \nu_\tau \cdot \widehat{\boldsymbol{b}}_{\tau(j*)}^{[m]},$$

where ν_τ is the learning rate and $\widehat{\boldsymbol{b}}_{\tau(j*)}^{[m]}$ is a $((p+1) \times 1)$-vector, with first entry $\widehat{a}_{\tau j*}^{[m]}$, the $(j*+1)$-th entry $\widehat{b}_{\tau j*}^{[m]}$ for the best-fitting component x_{j*}, and 0 for all remaining components.

The τth conditional quantile function and the fitted values are updated by

$$\begin{aligned} \widehat{\eta}_\tau^{[m]} &= \widehat{\eta}_\tau^{[m-1]} + \nu_\tau \widehat{Q}_{u^{[m]}}(\tau | x_{j*}), \\ \widehat{\eta}_{\tau i}^{[m]} &= \widehat{\eta}_{\tau i}^{[m-1]} + \nu_\tau \widehat{u}_{\tau ij}^{[m]}, \end{aligned}$$

where $\widehat{Q}_{u^{[m]}}(\tau|x_{j*})$ represents the estimate for the τth conditional quantile of the current working residuals as a function of the best-fitting predictor, x_j. The respective fitted values for the τth conditional quantile of the current working residuals are denoted by $\widehat{u}^{[m]}_{\tau ij}$.

After convergence of QR-QR.Boost and QR-LS.Boost, the residual vectors are asymmetrically split into two parts, depending on τ, similar to classical quantile regression. Yet, the estimated regression relationships between y and x are not equivalent to classical quantile regressions: They do not satisfy typical quantile regression fit properties like exact fit, but they come arbitrarily close to this state.

4. Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

A.2. Step-by-step interpretation of the algorithms

Just as with the component-wise gradient boosting algorithm (Friedman, 2001), the modular nature of QR-QR.Boost and QR-LS.Boost makes individual steps and interim calculations traceable. A comprehensive explanation of the steps of these white box algorithms follows below. Graphical insight is given using an example data set generated from the simple model

$$y_i = 3 + 1x_i + 4u_i \qquad \text{with} \qquad u_i \sim N(0,1) \qquad \text{and} \qquad x_i \sim U\,[0,10]\,. \tag{1}$$

The "directive" for both algorithms is to estimate the 10% conditional quantile function, $\eta_{0.1}$. All results discussed in the following also apply to a multivariate setup.

1. Initialize the fitted values.

Because QR-QR.Boost (QR-LS.Boost) is initialized with the respective τth sample quantile (median), the initial estimate for the τth conditional quantile function, $\widehat{\eta}^{[0]}_{0.1}$, comprises only an intercept, equal to the sample quantile (median) of the response. Other predictor effects are initially set to zero. QR-QR.Boost initialized with the median serves as an intermediate between both approaches and is additionally reported (see Figure 1).

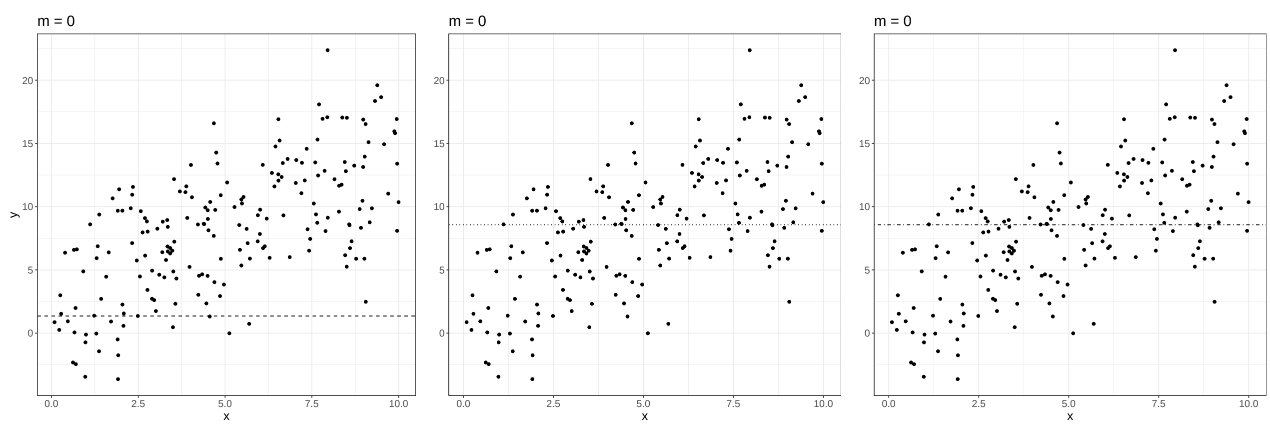

Figure 1: Scatterplot for example data drawn from the model in Equation 1. Left: QR-QR.Boost initialized with 10% sample quantile (dashed line). Middle: QR-QR.Boost initialized with the sample median (dotted line). Right: QR-LS.Boost initialized with the sample median (dashdotted line).

The example at hand contains only one predictor and therefore only one base learner in the respective algorithm: a simple linear 10% regression quantile, $Q_u(\tau = 0.1|x) = a_{0.1,1} + b_{0.1,1}x$, in the QR-QR.Boost algorithm and a simple linear regression mean, $E(u|x) = a_1 + b_1 x$, in the QR-LS.Boost algorithm.

2. Fit the working residuals.

2a. Set $m := m + 1$.

2b. Compute the working residuals.

Having initialized the fitted values for the QR-QR.Boost algorithm with the respective sample quantile in Step 1, all initial fitted values, $\widehat{\eta}^{[0]}_{0.1,i}$, are equal to the 10% sample quantile of the response. Therefore, 90% of the response's observations are greater than $\widehat{\eta}^{[0]}_{0.1,i}$ and 10% are smaller. Hence, 90% of the working residuals of the first iteration take a value of $\tau = 0.1$ and 10% take a value of $\tau - 1 = -0.9$ (Figure 2, left panel).

If, analogous to the initialization in the QR-LS.Boost algorithm, the fitted values in the QR-QR.Boost algorithm are initialized with the response's median instead, 50% of the response's observations are greater than the initial fitted values, $\widehat{\eta}^{[0]}_{0.1,i}$, and 50% are smaller. This results in half of the working residuals of the first iteration taking on a value of $\tau = 0.1$ and half taking a value of $\tau - 1 = -0.9$ (Figure 2, middle and right panel).

All plots of Figure 2 show a positive correlation between response and predictor, as larger values of x tend to correspond to larger values of y, and therefore to working residuals of the value 0.1.

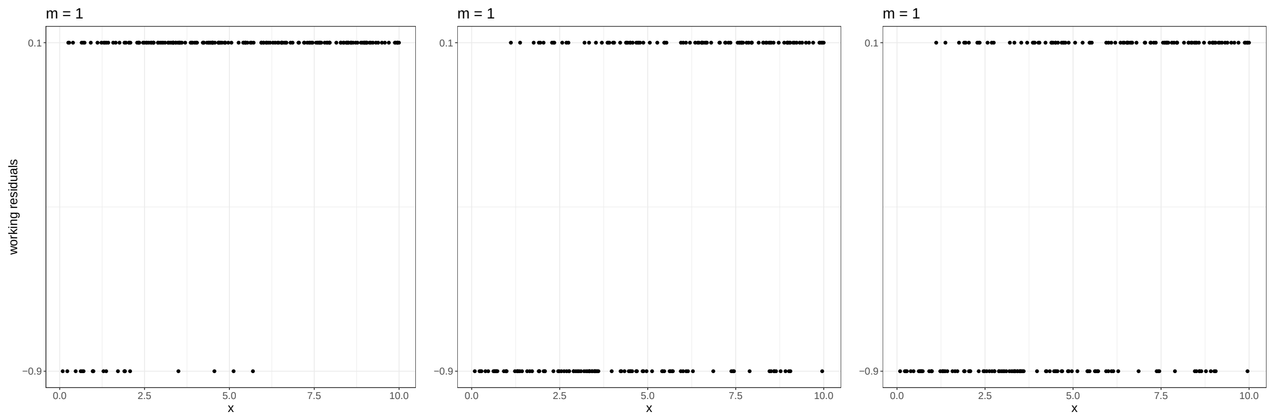

Figure 2: Working residuals of the first iteration against predictor x. Left: QR-QR.Boost initialized with 10% sample quantile (dashed line). Middle: QR-QR.Boost initialized with the sample median. Right: QR-LS.Boost initialized with the sample median.

2c. Compute a least squares/quantile regression fit for each base learner.

Given that the illustrative example at hand contains only one predictor, only one linear quantile regression fit, $\widehat{Q}_{u_i^{[m]}}(\tau = 0.1|x_i) = \widehat{a}_{0.1,1} + \widehat{b}_{0.1,1}x_i$, is obtained in each iteration in the QR-QR.Boost algorithm. Figure 3 (bottom left and middle panel) illustrates the linear quantile regression fit for both initializations (10% sample quantile vs. sample median) for iteration $m = 31$, which is the first iteration for which both slope estimates are nonzero.

If we were to initialize QR-QR.Boost with the sample median (representing the intermediate between both approaches), 50% of the observations are smaller than the initial fitted values, $\widehat{\eta}^{[0]}_{0.1,i}$, resulting in a steeper slope for the linear quantile regression. This, in turn, can lead to faster convergence of the algorithm. In the present case, 31 iterations are still required until a quantile regression model with a nonzero slope is fitted to the working residuals. The first 30 iterations estimate quantile regression models with only an intercept (as displayed in Figure 3, upper middle panel), aiming to convert negative to positive residuals to eventually fit a quantile regression model with a nonzero slope.

When QR-QR.Boost is initialized as specified with the 10% sample quantile, a quantile regression model with a nonzero slope can already be fitted in the first iteration (Figure 3, upper left panel), certainly with a flatter slope compared to the bottom middle panel of Figure 3.

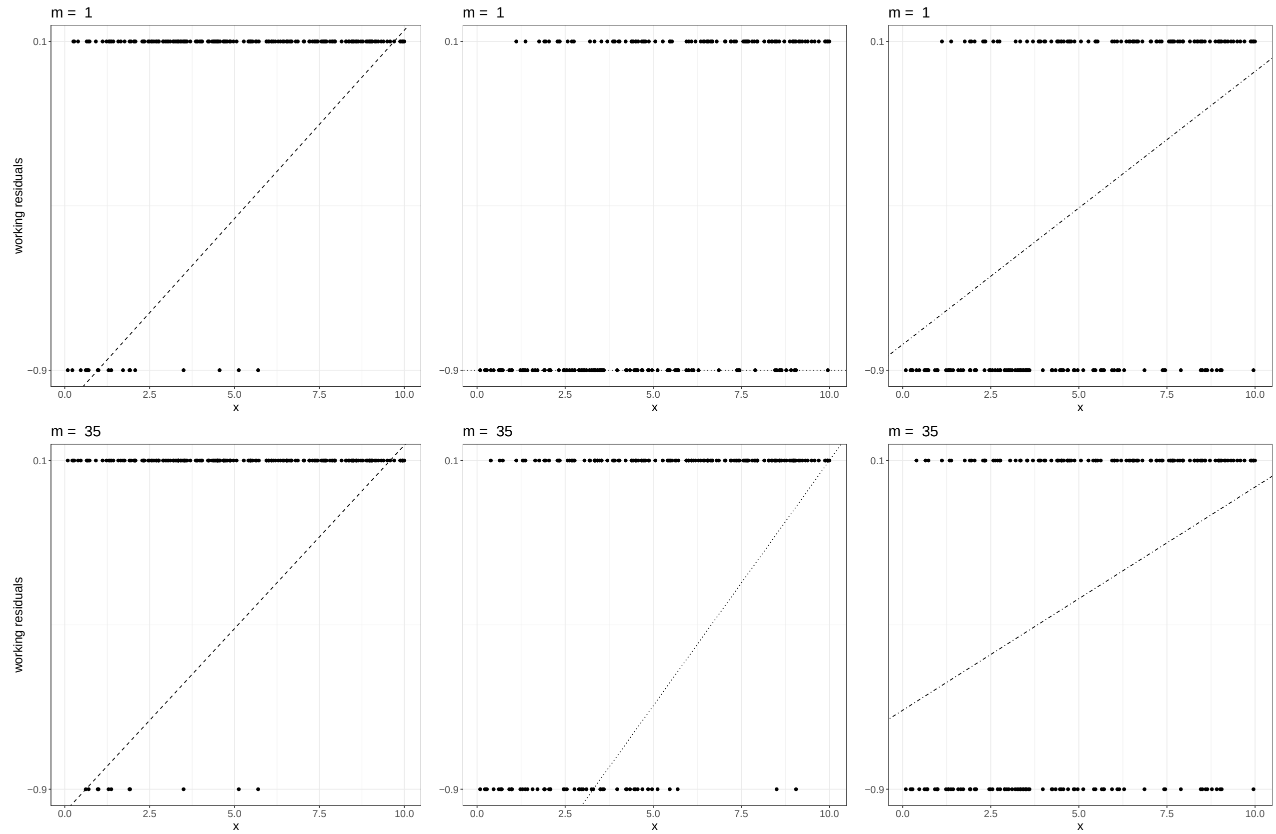

Figure 3: Working residuals of the 1st (top row) 31th (bottom row) iteration are plotted against the predictor x. Left: QR-QR.Boost initialized with the 10% sample quantile. Middle: QR-QR.Boost initialized with the sample median. Right: QR-LS.Boost initialized with the sample median. Dashed, dotted and dash-dotted lines indicate the respective linear quantile regression fit for $\tau = 0.1$ or the respective linear LS regression fit.

Our example shows that it is not per se advantageous to initialize QR-QR.Boost with a higher sample quantile than the respective sample quantile (here 10%). While this leads to a steeper slope, it comes at the cost of longer lead time until a first quantile regression model with a nonzero slope estimate is fitted: In the present case, e.g., initializing the algorithm with an extreme sample quantile such as the 90% quantile requires 100 iterations until a nonzero slope is fitted for the first time.

The algorithm converges fastest for an initialization which respects this tradeoff, i.e., leads to a steep slope while only requiring a short lead time. Which sample quantile represents the optimal initialization for the QR-QR.Boost algorithm, depends on the following three attributes of the true underlying data generating process.

(i) The *magnitude of the true underlying coefficient effect* does not affect the magnitude of the estimated slope in the individual iterations – as long as the sign of the residuals remains unchanged and thus the working residuals do not change. The magnitude of the coefficient effects is therefore proportional to the number of iterations required until the QR-QR.Boost algorithm converges.

Consequently, if the true underlying coefficient effect is relatively small, it takes not too many iterations until the algorithm converges. In that case, long lead times are especially costly and may not be compensated by a steeper resulting slope when the algorithm is initialized with a sample quantile in the direction of the sample median[2]. Hence, the QR-QR.Boost algorithm should be initialized with a sample quantile near τ. A change in the magnitude of the intercept, however, has no impact on the number of iterations required as this is captured by the initialization.

On the other hand, if the true underlying coefficient effect is large, it takes a considerable amount of time for the algorithm to converge. In that case, a steeper slope may compensate for longer lead times in the long run. Thus, the QR-QR.Boost algorithm should be initialized with a sample quantile in the direction of the sample median.

[2]Choosing a sample quantile in the opposite direction of the sample median always leads to inferior results, as the estimated slope becomes flatter and lead times increase.

(ii) The *value range of the predictor* affects the magnitude of the estimated slope in the individual iterations of the QR-QR.Boost algorithm: The estimated slope in the single iterations is larger for smaller value ranges of the predictor.

An argument similar to (i) applies: If the value range of the predictor is relatively small, the estimated slope in individual iterations is larger, facilitating fast convergence of the algorithm. In that case, long lead times are especially costly and may not be compensated by a steeper slope. Thus, the QR-QR.Boost algorithm should be initialized with a sample quantile near τ.

On the other hand, if the value range of the predictor is large, the estimated slope in the individual iterations is rather small, resulting in a slowly converging algorithm. In that case, a steeper slope may compensate for longer lead times in the long run and the QR-QR.Boost algorithm should be initialized with a sample quantile in the direction of the sample median.

(iii) If the *variance of the error term* is large, individual observations scatter widely, necessitating many iterations until a sufficient number of residuals change sign to eventually fit a quantile regression with a nonzero slope estimate, resulting in long lead times. The resulting steeper slope in each iteration, when the QR-QR.Boost algorithm is initialized with a sample quantile in the direction of the sample median, may not compensate for the resulting longer lead times. As a result, the QR-QR.Boost algorithm should be initialized with a sample quantile near τ.

On the other hand, if the variance of the error term is small, fewer iterations are required untila sufficient number of residuals change sign to fit a quantile regression with a nonzero slope estimate. Thus, a steep slope may compensate for longer lead times and the QR-QR.Boost algorithm should be initialized with a sample quantile in the direction of the sample median.

In practice, given the unknown nature of both the error variance and the magnitude of the coefficient effects, the sample median of the response is a reasonable initial estimate in terms of fast convergence. As center of the distribution, the sample median balances the effects of a steep slope against long lead times quite well and is never the worst choice in terms of the number of iterations required. However, as the true underlying data generating process is unknown, it is impossible to predict which initialization value will produce the best results. In our opinion, the most intuitive choice for initialization is the respective τth sample quantile, leading to an interpretation of $\widehat{\eta}_\tau^{[0]}$ that is consistent with the interpretation when fitting a quantile regression model with an intercept only.

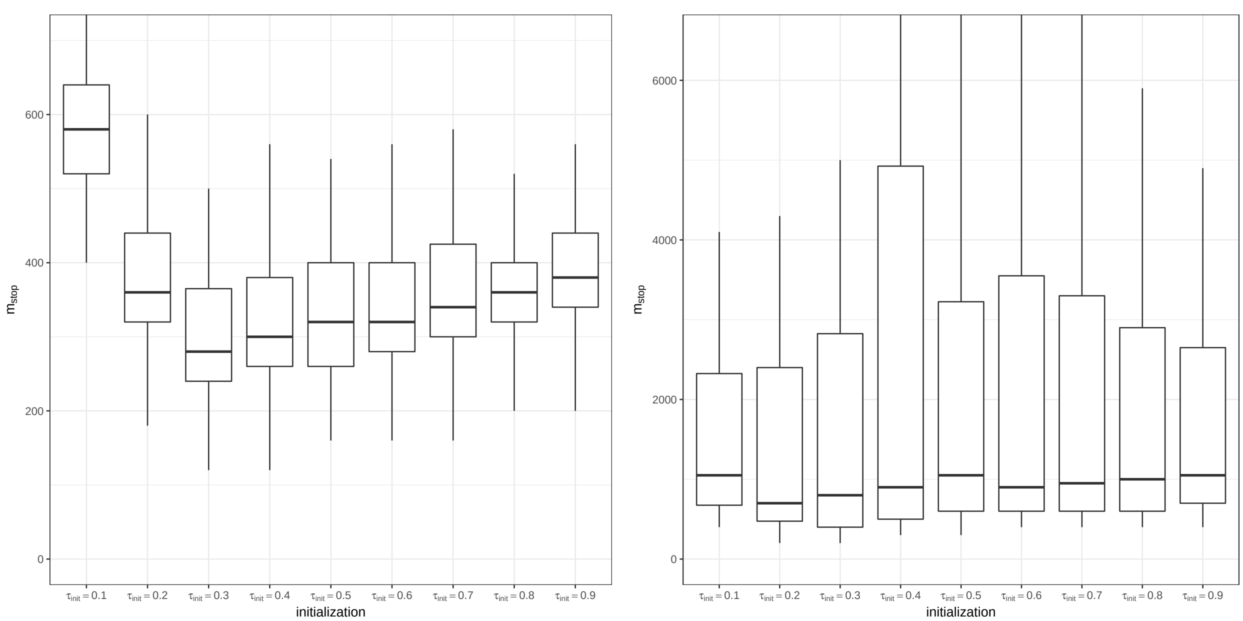

Figure 4: Boxplots of the empirical distribution of the optimal number of iterations for different initializations of the QR-QR.Boost (QR-LS.Boost) algorithm in the left (right) panel. The abscissa indicates the respective sample quantile $\tau_{\text{init}} \in (0.1, 0.2, ..., 0.9)$ used for initialization. The results are obtained from $K = 100$ simulation runs estimating the 10% regression quantile by QR-QR.Boost (QR-LS.Boost) for the data generating process of Equation 1. Outliers are removed from each boxplot for visualization purposes.

Figure 4 illustrates the empirical distribution of the optimal number of iterations for different initializations of the QR-QR.Boost algorithm, which follows a U-shaped course reflecting the tradeoff between a steeper slope and long lead times. In the underlying example, the QR-QR.Boost algorithm converges fastest (i.e., requires the smallest median number of iterations) when initialized with the 30% sample quantile.

Moreover, the initialization of the algorithm only affects the number of iterations required, but not the estimation accuracy: QR-QR.Boost achieves similar empirical risk regardless of the initialization (see Table 2).

The QR-LS.Boost results are similar: Given that the example at hand contains only one predictor, each iteration in the QR-LS.Boost algorithm yields only one least squares regression fit, $\widehat{E}(u_i^{[m]}|x_i) = \widehat{a} + \widehat{b}x_i$. The right column of Figure 3 illustrates the linear LS regression fit for iterations $m = 1$ and $m = 31$.

Comparison with the left column of Figure 3 for QR-QR.Boost reveals two observations: First, the slope obtained in QR-LS.Boost for iteration $m = 31$ is flatter than the slope in QR-QR.Boost, which may indicate that QR-QR.Boost converges faster than QR-LS.Boost. Second, although the QR-LS.Boost algorithm is initialized with the sample median, a regression model with a nonzero slope is fitted in the first iteration. As simple linear regression means are chosen as the base learners, the slope estimate is always (at least marginally) nonzero.

Consequently, initializing with a more extreme sample quantile (in the direction of the median) and obtaining a steeper slope are not directly related. There is no equivalent to the clear tradeoff between a steeper slope and long lead times observed for QR-QR.Boost. Figure 4 reflects this finding: The empirical distribution of the optimal number of iterations for different initializations of the QR-LS.Boost algorithm does not follow the same U-shaped course as for QR-QR.Boost. On the contrary, no relationship between τ_{init} and the required number of iterations is visible.

Equivalently to QR-QR.Boost, QR-LS.Boost achieves similar estimation accuracy with different initializations and shows no effect of the initialization of the algorithm on estimation accuracy (see Table 2).

	τ_{init}				
Method	0.1	0.3	0.5	0.7	0.9
QR-QR.Boost	0.682	0.683	0.682	0.681	0.680
QR-LS.Boost	0.682	0.678	0.674	0.674	0.675

Table 2: Estimation accuracy of the QR-QR.Boost and QR-LS.Boost algorithm measured by the empirical risk R_τ for different initializations. τ_{init} represents the sample quantile used for initialization in Step 1 of the algorithms. The results are obtained from $K = 100$ simulation runs estimating the 10% quantile regression by QR-QR.Boost and QR-LS.Boost for the data generating process of Equation 1.

3. Update one component.

3a. Select the component x_{j*} that fits the working residuals best.

In our example, only one predictor (i.e., only one base learner) is considered. As a result, this predictor is selected as the best-fitting component in every iteration for both algorithms, QR-QR.Boost and QR-LS.Boost.

3b. Update the estimate of the τth regression quantile.

Multiplying the coefficient estimates by a pre-specified learning rate ensures that the effect estimates are adjusted only slightly in each iteration m (see Figure 5).

As outlined above, the slope in the quantile regression fit of Step 2c can be estimated to be zero (Figure 3, upper middle panel). Selecting predictor x_j as the best-fitting base learner does not necessarily imply that its coefficient estimate is updated in that iteration. In fact, although the predictor x_j has been selected as the best-fitting base learner a few times, its effect estimate may never be updated and thus equal zero (see the estimate for the 10% conditional quantile function after 30 iterations in Figure 5, middle panel).

Iterations with a nonzero slope estimate for the best-fitting base learner, i.e., where the predictor makes an explanatory contribution to the model, should be distinguished from iterations with a zero slope estimate, i.e., where the predictor does not contribute to the model, but the intercept does. For QR-LS.Boost, such differentiation is not possible, since simple linear regression means are chosen as base learners and therefore the slope estimate is always nonzero. When a predictor is selected as the best-fitting component in QR-LS.Boost, its effect estimate is always updated (see dashdotted lines in Figure 6).

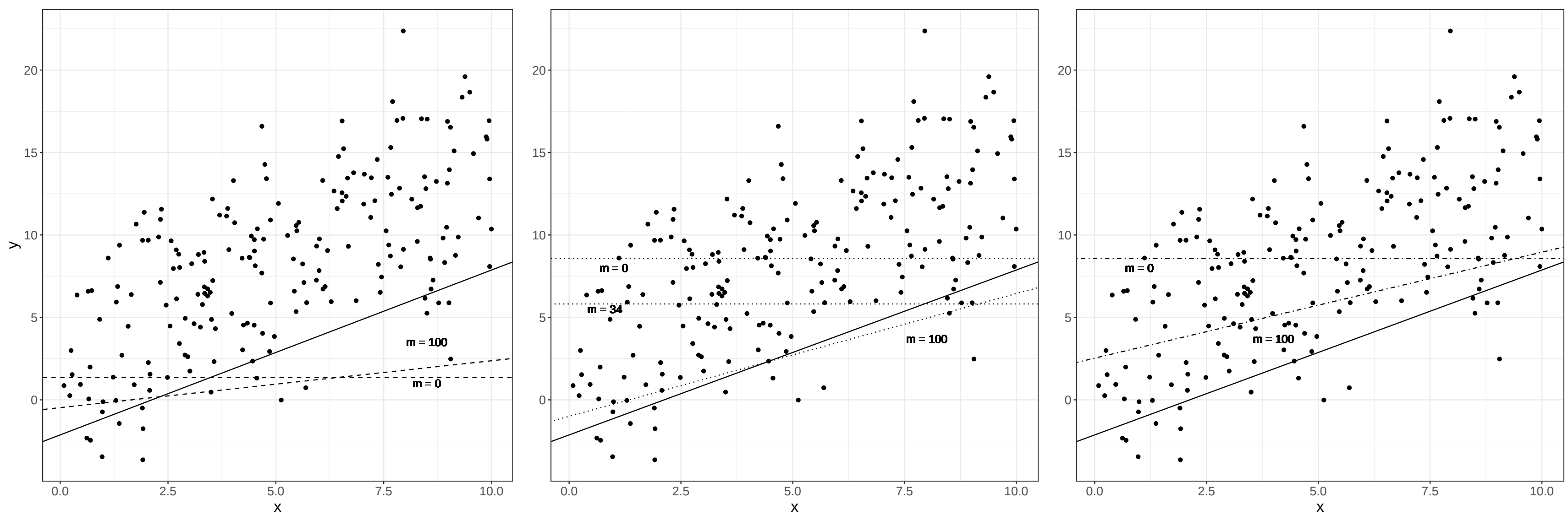

Figure 5: Evolution of the slope coefficient. Left: Dashed lines indicate the estimates for the 10% conditional quantile function after 0 and 100 iterations for the QR-QR.Boost algorithm initialized with the 10% sample quantile. Middle: Dotted lines indicate the estimates for the 10% conditional quantile function after 0, 30, and 100 iterations for the QR-QR.Boost algorithm initialized with the sample median. Right: Dashdotted lines indicate the estimate for the 10% conditional quantile function after 0 and 100 iterations for the QR-LS.Boost algorithm initialized with the sample median. Black lines indicate the true underlying 10% quantile curve.

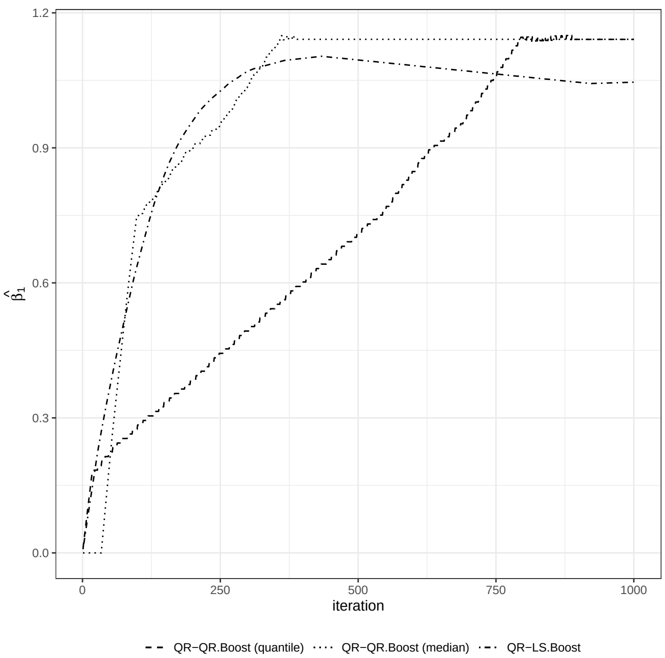

Figure 6: The dashed, dotted, and dashdotted lines represent the coefficient paths of the estimate $\widehat{\beta}_{0.1,1}$ of QR-QR.Boost initialized with the 10% sample quantile, QR-QR.Boost initialized with the sample median, and QR-LS.Boost, respectively, for the first 1000 iterations. The algorithms converge after 640, 200, and 4600 iterations, respectively.

4. Iterate Steps 2 and 3 until $m = m_{\text{stop}}$.

A.3. General implications of the component-wise functional gradient boosting algorithm

Since QR-QR.Boost and QR-LS.Boost are adaptations of the component-wise functional gradient boosting algorithm, the following implications regarding simultaneous model estimation and variable selection, as well as model choice/functional form, apply to them as well. Additionally, the choice of the two tuning parameters (number of iterations and learning rate) is motivated.

A.3.1. Variable selection

In Step 1, simple linear regression means are chosen as the base learners for all predictors. In Step 3a, only the best-fitting base learner is selected. As a result, some predictors may never be selected during the m_{stop} iterations. The remaining predictors were initially set to zero in Step 1 and are never updated (Hofner et al., 2014, p. 6), hence, omitted from the final model. The component-wise functional gradient boosting algorithm can generally perform variable selection, provided the base learners are adequately specified and the algorithm is stopped before convergence ("early stopping") (Mayr et al., 2014a, p. 425).

In summary, model estimation and variable selection are conducted simultaneously during the boosting iterations, leading to significant reductions in computation time compared to the exhaustive all subset selection in classical quantile regression. This feature proves especially useful in high-dimensional settings (Bühlmann and Hothorn, 2007, p. 491), i.e., in situations where the number of predictor is much larger than the number of observations ($p \gg n$). In these settings, many classical statistical learning algorithms, which do not conduct inherent variable selection, become infeasible (Mayr et al., 2014b, p. 429).

Nonetheless, component-wise functional gradient boosting using squared error loss may still include too many irrelevant predictors in the final model. Bühlmann and Yu (2006) propose so-called "sparse boosting", which uses a penalized squared error loss criterion for selection in Step 3a. However, considering that predictive performance and estimation accuracy (obtaining sparse and interpretable models) are different analysis goals, it is difficult to do justice to both at the same time. Especially since predictors may be irrelevant to the interpretation of the model but relevant to improving predictive performance (Mayr et al., 2014b, p. 431).

A.3.2. Model choice and functional form

Besides variable selection, the component-wise functional gradient boosting algorithm also proves useful for model selection. Defining multiple functional forms of base learners for one predictor, e.g., linear and nonlinear, the component-wise functional gradient boosting algorithm chooses the best-fitting component in each iteration and thus decides not only whether to include the predictor, but also in which functional form: linear, nonlinear, or both. To warrant unbiased variable selection, one should ensure that the complexity (i.t.o. degrees of freedom) of not only the base learners defined for the same predictors, but all base learners is comparable, otherwise the component-wise functional gradient algorithm systematically prefers more complex base learners (Hofner et al., 2011, p. 956).

A.3.3. Interpretability

Owing to the additive updating of the coefficient estimates in Step 3b, the base learners bequeath their structure to the resulting estimate for the conditional quantile function, $\widehat{\eta}_{\tau}^{[m_{\text{stop}}]}$ (Bühlmann and Hothorn, 2007, p. 484). If in Step 1, simple linear regression means are chosen as base learners, that results in a linear estimate for the conditional quantile function. Therefore, the individual quantile-specific predictor effects are interpretable.

A.3.4. Stage-wise and component-wise nature

From Step 3, it is apparent why the algorithm is termed a "forward stage-wise and component-wise additive gradient" boosting algorithm: The coefficient estimates of only one component, the best-fitting one, are additively updated in each iteration. Moreover, in each iteration, an estimate for the negative gradient of the loss function is added to the current fitted values, resulting in a stage-wise reduction of empirical loss (Hofner et al., 2014, p. 6). The component-wise functional gradient boosting algorithm is also described as a "greedy stage-wise approach" (Friedman, 2001, p. 1192). This characteristic can be seen in Step 3b, where the selection of the best-fitting base learner results in the steepest descent in the empirical loss in each iteration.

A.3.5. Tuning parameters

The two tuning parameters of component-wise functional gradient boosting, the learning rate, ν, and the number of iterations, m_{stop}, appear in Steps 3b and 4, respectively. The learning rate leads to only slowly increasing coefficient estimates during the boosting process. This ensures that the algorithm does not overshoot the minimum of the empirical risk and that individual estimations are regularized, such that they do not heavily influence the final outcome. Combining this fact with early stopping results in shrunk coefficient estimates. Consequently, the bias of the estimate is slightly increased while its variance is decreased, which often improves predictive performance and is known as the bias-variance tradeoff (Hofner et al., 2014, p. 7).

Choosing a relatively small value for ν (e.g., $\nu = 0.1$) is standard practice and yields reasonable results (Bühlmann and Hothorn, 2007, p. 480). Our simulation results suggest that this is also the case for QR-QR.Boost and QR-LS.Boost. In turn, small values of ν require a larger number of iterations m_{stop} which are proportional to computation time (Hastie et al., 2009, p. 365). In addition, the learning rate for algorithms estimating a conditional quantile should be tied to the sparseness of the observations near the quantile of interest, since the precision of quantile regression depends on this quantity (Koenker, 2005, p. 77).

As variable selection and shrinkage can only result from early stopping, the tuning parameter m_{stop} controls both. The maximum number of iterations, m_{stop}, reflects the bias-variance tradeoff: More iterations lead to more flexible models, accompanied by greater variance but less model bias, whereas fewer iterations lead to more shrinkage and variable selection, resulting in less flexible models (Mayr et al., 2012, p. 197). One should carefully choose the right number of iterations to prevent the algorithm from overfitting the data. The optimal number of iterations m_{stop} can be determined by cross-validation, where it is crucial to use the same loss function, that the algorithm seeks to minimize (Mayr et al., 2014a, p. 425). For QR-QR.Boost and QR-LS.Boost, the pinball loss function should be used.

B. Replication of the results from Fenske et al. (2011)

Estimation accuracy and variable selection properties of QR-QR.Boost, QR-LS.Boost, classical quantile regression without (RQ), and with all subset selection (RQAic) are studied in a simulation study outlined in Table 3. Particular focus is placed on estimation accuracy, the ability to correctly identify and exclude irrelevant predictors and differences in computational time.

B.1. Evaluation criteria

QR-QR.Boost and QR-LS.Boost are compared with respect to three aspects: estimation accuracy, computational efficiency and variable selection. Additional to the measures introduced in the main document, estimation

Model			
Configuration of the location-scale model depends on the parameter setup and the error distribution $Q_{y_i}(\tau\|\boldsymbol{x}_i) = \boldsymbol{x}_i^\top[\boldsymbol{\beta} + \boldsymbol{\alpha} Q_{\epsilon_i}(\tau\|\boldsymbol{x}_i)]$			
Parameter setups			
Homoskedastic	$n = 200$	$\boldsymbol{\beta} = (3,1)^\top$	$\boldsymbol{\alpha} = (4,0)^\top$
Heteroskedastic	$n = 200$	$\boldsymbol{\beta} = (4,2)^\top$	$\boldsymbol{\alpha} = (4,1)^\top$
Multivariate	$n = 500$	$\boldsymbol{\beta} = (5,8,-5,2,-2,0,0)^\top$	$\boldsymbol{\alpha} = (1,0,2,0,1,0,0)^\top$
Extension			
Multivariate2	$n = 500$	$\boldsymbol{\beta} = \begin{array}{c} {\scriptstyle \tau=0.1} \\ {\scriptstyle \tau=0.3} \\ {\scriptstyle \tau=0.5} \\ {\scriptstyle \tau=0.7} \\ {\scriptstyle \tau=0.9} \end{array} \begin{pmatrix} 5 & 8 & -5 & 0 & -2 & 0 & 0 \\ 5 & 8 & -5 & 0 & -2 & 0 & 0 \\ 5 & 0 & -5 & 0 & -2 & 0 & 0 \\ 5 & 0 & -5 & 2 & -2 & 0 & 0 \\ 5 & 0 & -5 & 2 & -2 & 0 & 0 \end{pmatrix}$	$\boldsymbol{\alpha} = (1,0,2,0,1,0,0)^\top$
High-dimensional	$n = 200$	$\boldsymbol{\beta} = (5,8,-5,2,-2,\mathbf{0}_{95}^\top)^\top$	$\boldsymbol{\alpha} = (1,0,2,0,1,\mathbf{0}_{95}^\top)^\top$,
Error distributions			
Standard normal distribution		$\epsilon \sim \mathcal{N}(0,1)$	
t-distribution		$\epsilon \sim t(2)$	
Gamma distribution		$\epsilon \sim \Gamma(2,1)$	
Extension			
Mixed distribution		$\begin{cases} x_1 < median(x_1) & \epsilon \sim t(2) \\ x_1 \geq median(x_1) & \epsilon \sim t(2) + \Gamma(2,1) \end{cases}$	
Methods			
QR-LS.Boost		Boosting of regression quantiles with pinball loss and L_2 score	
Extension			
QR-QR.Boost		Boosting of regression quantiles with pinball loss and score	

Table 3: Simulation setup of Fenske et al. (2009) and extensions with respect to parameter setups, error distributions, and considered boosting algorithms. For the data generating process, the listed parameter setups and error distributions are each plugged into the model and estimated by the respective methods. Each setup is then simulated again for a contaminated version of the error term. This leaves us with 20*2 = 40 simulation setups.

	Parameter setup				
	homoskedastic	heteroskedastic	multivariate	multivariate2	high-dimensional
Estimation accuracy					
MSE	×	×	×	×	
τ-fit	×	×	×	×	×
R_τ	×	×	×	×	×
Bias	×	×	×	×	
Computational efficiency					
Median number of iterations	×	×	×	×	×
Variable selection					
Sensitivity					×
Specificity					×
MFI			×	×	
MPI			×	×	
PER			×	×	

Table 4: Overview of the evaluation criteria used for each parameter setup.

accuracy is measured by the Bias for each quantile-specific parameter $(\beta_{\tau 0}, \beta_{\tau 1}, ..., \beta_{\tau p})^\top$,

$$\text{Bias}(\widehat{\beta}_{\tau j,k}) = \widehat{\beta}_{\tau j,k} - \beta_{\tau j},$$

where $j = 0, ..., p$ denotes the respective predictor and $k = 1, ..., K$ the simulation replication (Fenske et al., 2009, p. 10).

For the multivariate setups, the variable selection properties of the boosting algorithms are compared by three additional measures: By the proportion of simulation iterations in which the respective predictor x_j is never updated (PER), by the mean proportion of boosting iterations in which the respective predictor x_j is selected with a nonzero slope estimate (MPI), and by the mean first boosting iteration in which the respective predictor x_j is first selected with a nonzero slope estimate (MFI). For QR-LS.Boost, the number of boosting iterations with a nonzero slope estimate is equal to the total number of boosting iterations, since the slope estimate is always nonzero.

B.2. Homoskedastic and heteroskedastic setup

100 location-scale models with only one predictor for each $\tau \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, for both the homoskedastic and heteroskedastic setup, and for all error distributions are simulated. In both cases, the estimation results for $\beta_{\tau 0}$ and $\beta_{\tau 1}$ of QR-QR.Boost and QR-LS.Boost share the characteristic of an increased bias (see Tables 11 and 12) and a lower variance as compared to the RQ estimates (see Figure 7).

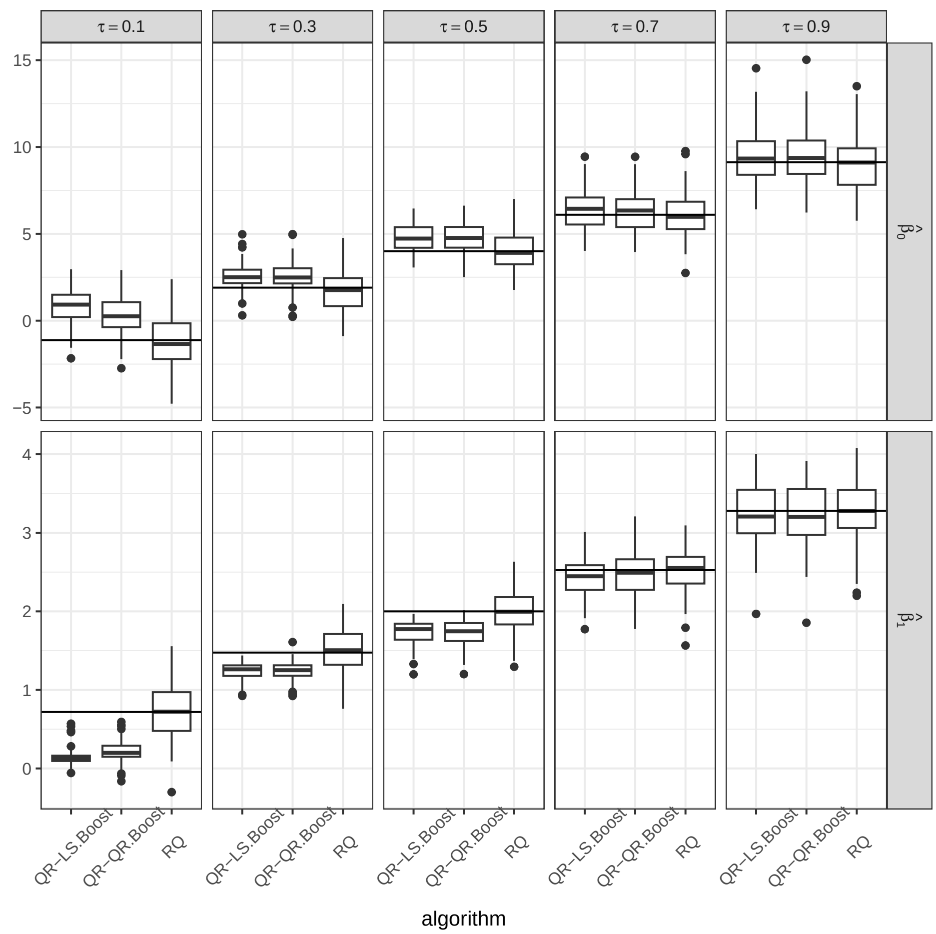

Figure 7: Boxplots of the empirical distribution of the estimated parameters $\widehat{\beta}_{\tau 0}$ and $\widehat{\beta}_{\tau 1}$ from $K = 100$ simulation runs (heteroskedastic setup, normal errors, not contaminated), for each τ and estimation procedure (QR-QR.Boost, QR-LS.Boost, and RQ).

Both boosting procedures hit the bias-variance tradeoff better and outperform RQ in terms of estimation accuracy. Consequently, the regularization has the intended effect, which is to trade a slight increase in the model's bias for a significant reduction in variance, thereby minimizing the MSE. QR-LS.Boost performs best in terms of MSE, closely followed by QR-QR.Boost (see Table 9).

Across all error distributions, in the homoskedastic and heteroskedastic setup displayed in Table 7 (Table 8 for the contaminated cases), QR-LS.Boost exhibits the smallest R_τ for the major part of setups, often closely followed by QR-QR.Boost. Ultimately, QR-LS.Boost and QR-QR.Boost perform similar in terms of estimation accuracy, although QR-LS.Boost is more often in the lead, albeit just barely. RQ performs weakest, but still shows competitive results.

In terms of in-sample accuracy, as measured by τ-fit, RQ performs best, while QR-QR.Boost and QR-LS.Boost are on par (compare Tables 5 and 6).

B.3. Multivariate setup

Similar to the bivariate case, both boosting algorithms lead to an increased estimation bias for $(\beta_{\tau 0}, \beta_{\tau 1}, ..., \beta_{\tau 6})^\top$ (see Tables 11 and 12). In terms of estimation accuracy measured by MSE, QR-QR.Boost, QR-LS.Boost, and RQAic equally well. Comparing the results of RQAic and RQ, the panel for the multivariate setup in Table 9 shows that the inclusion of the predictors five and six leads to poorer estimation accuracy. This effect is expected to be even more significant with a large number of irrelevant predictors (see Subsections B.5). As long as all subset selection is feasible, it represents a competitive approach in terms of estimation accuracy measured by the MSE compared to the boosting algorithms. In high dimensional data settings with a large number of predictors (and a possibly large number of irrelevant predictors), all subset selection is computationally infeasible and the boosting algorithms are expected to outperform classical quantile regression.

The empirical risk results for the multivariate setup mimic those obtained previously for the homoskedastic and heteroskedastic setup: QR-LS.Boost performs better than QR-QR.Boost, although the margin over QR-QR.Boost is peripheral (see Table 7 and Table 8).

In terms of in-sample accuracy, as measured by τ-fit, QR-QR.Boost, QR-LS.Boost, and RQ perform equally well (see Tables 5 and 6).

In terms of variable importance, QR-QR.Boost provides superior interpretability regarding the importance of predictors to the model compared to QR-LS.Boost. As all predictors are drawn from the same distribution, the magnitude of the respective predictor effects $(\beta_{\tau 1}, ..., \beta_{\tau 6})^\top$ indicates its importance. The predictor effects for different τ and error distribution are displayed in Table 13. Given $\tau = 0.5$ and normal errors, the first predictor is most important, while the fifth and sixth are not relevant for the model. This fact can be reflected in MPI and MFI: More important predictors should be selected more frequently during the boosting iterations – resulting in a larger MPI – and less important predictors should be selected in, if any, later stages of the boosting iterations – translating to a larger MFI.

The panel for the multivariate setup in Table 14 shows that QR-QR.Boost manages to clearly rank the predictors according to their importance in the model: Exemplary for $\tau = 0.5$ and normal errors, x_1 is the most important one with an MPI of 0.476, whereas x_5 and x_6 are least important with an MPI of 0.002 and 0.003 each. The ranking is less pronounced for QR-LS.Boost as x_1 and x_2 show similiar MPIs (0.363 vs. 0.283) and x_5 and x_6 as the least important predictors receive MPIs of 0.009 and 0.017. Moreover, the MPIs for QR-LS.Boost are subject to greater uncertainty compared to QR-QR.Boost, as evidenced by a larger variance and a larger number of outliers across the 100 simulation runs (see Figure 8).

Regarding the ability to correctly identify and exclude irrelevant predictors from the final model, both boosting algorithms perform similar. QR-QR.Boost excludes the irrelevant predictors x_5 and x_6 74 and 72 times, respectively, out of 100 cases, and QR-LS.Boost 75 and 77 times, respectively. In contrast, RQAic manages to exclude x_5 and x_6 54 and 91 times.

QR-QR.Boost is able to identify those predictors as irrelevant, as x_5 and x_6 are selected for the first time after 95.4% and 93.8% of the boosting algorithm are completed, while QR-LS.Boost selected them for the first time after 74.1% and 65.7% of the boosting algorithm are completed (see Table 16 (exemplary for $\tau = 0.5$ and normal errors). Moreover, MFI_5 and MFI_6 for QR-LS.Boost are accompanied by great uncertainty, indicated by high variance across the 100 simulation runs (see Figure 9). The ability to unambiguously rank the predictors by their importance could favor QR-QR.Boost compared to QR-LS.Boost i.t.o. variable selection in a high-dimensional setup with many irrelevant predictors. This setting is further discussed in the following Section B.5.

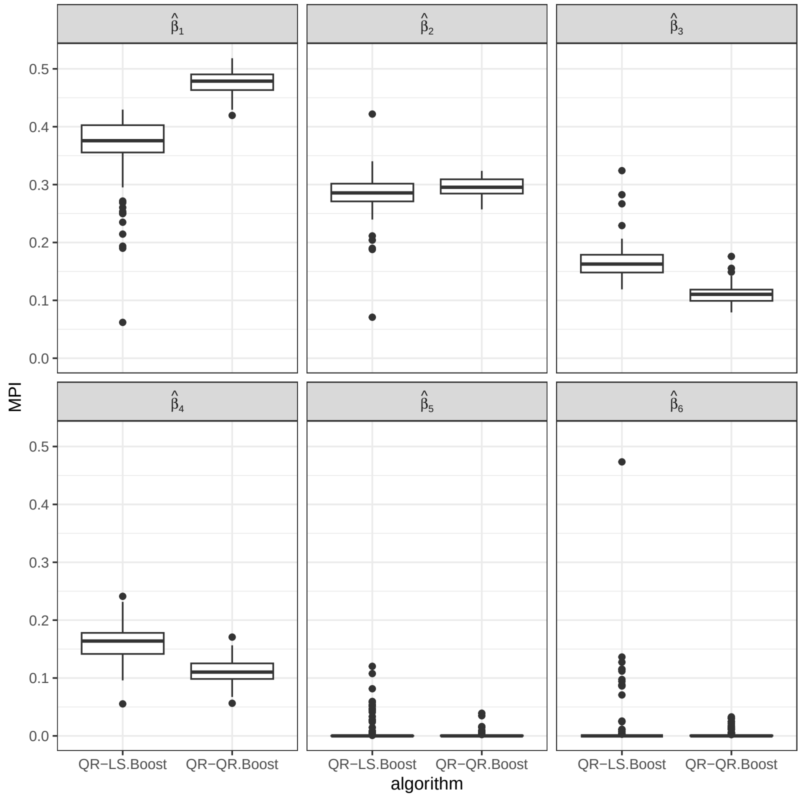

Figure 8: Boxplots for the empirical distribution of the proportion of selection iterations (from mslope iterations, where mslope indicates the number of iterations where the selected component has a nonzero slope estimate) for each predictor, obtained from 100 simulation runs (multivariate setup, normal errors, not contaminated, and $\tau = 0.5$).

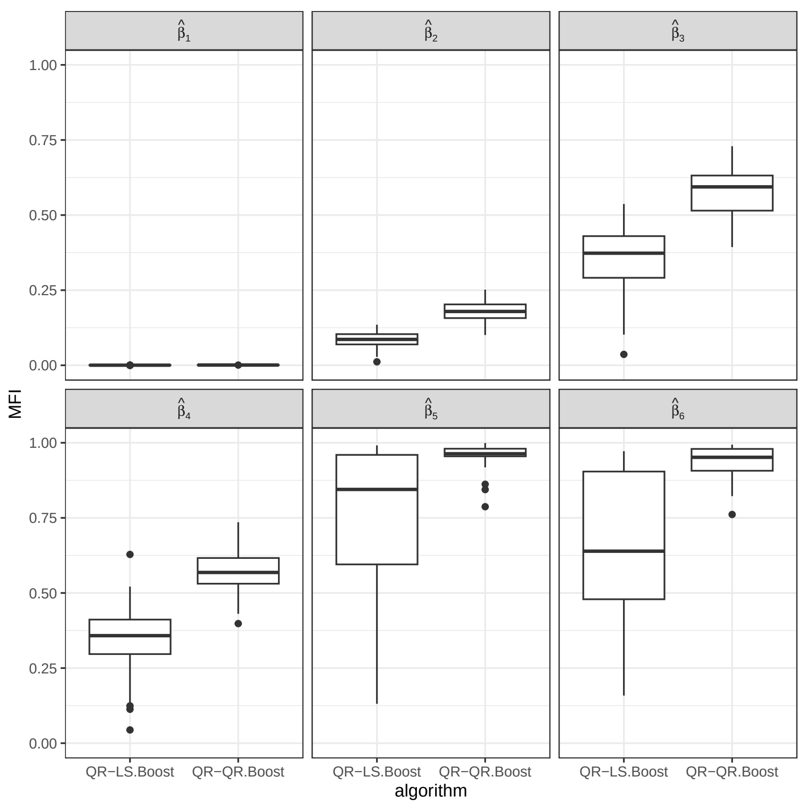

Figure 9: Boxplots for the empirical distribution of the first selection iteration (from *mslope* iterations) for each predictor, obtained from 100 simulation runs (multivariate setup, normal errors, not contaminated, and $\tau = 0.5$).

B.4. Multivariate2 setup

In terms of the empirical risk, QR-LS.Boost also performs best in the multivariate2 setup, again closely followed by QR-QR.Boost (see Table 7 and Table 8).

Recall that predictor x_1 only influences lower quantiles $\tau \in \{0.1, 0.3\}$ and predictor x_3 only influences higher quantiles $\tau \in \{0.7, 0.9\}$. We would suspect the method at hand to exclude x_1 (x_3) in upper (lower) quantiles and to include them in the remaining quantiles.

Indeed QR-QR.Boost, QR-LS.Boost and RQAic manage to include predictor x_1 for the two lower quantiles for all error distributions. However, RQAic includes the predictor for *all* quantiles under consideration, even the remaining quantiles, where all methods should exclude the predictor. Furthermore, QR-LS.Boost excludes x_1 for the remaining quantiles more frequently. Similar results can be observed for x_3: RQAic never excludes x_3 in the lower quantiles, while QR-LS.Boost and QR-QR.Boost manage to do so (see Table 18).

As for the irrelevant predictors x_5 and x_6, the results remain essentially the same as in the multivariate setup. Overall, our simulation results suggest that RQAic reliably excludes irrelevant predictors if they are irrelevant for the entire conditional distribution of y, but not if they are irrelevant only for parts of it. In contrast, QR-LS.Boost and QR-QR.Boost do not exclude predictors, that are irrelevant for the entire conditional distribution as reliably as RQAic, but exclude predictors that are only irrelevant for parts of the conditional distribution to a similar degree.

In fact, across all error distributions, RQAic excludes many relevant predictors from the final quantile regression model for all quantiles, while QR-QR.Boost and QR-LS.Boost rarely do so (see Table 18).

B.5. High-dimensional setup

In the high-dimensional setup, QR-LS.Boost achieves the best results i.t.o. the empirical risk, again, closely followed by QR-QR.Boost. Both boosting algorithms clearly outperform classical quantile regression (RQ) (see Table 7). Thus, amid the risk of potentially including a large number of irrelevant predictors in the model, RQ is no longer a competitive approach.

Regarding variable selection, QR-QR.Boost and QR-LS.Boost show similar results for more details please refer to the main document.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	QR-QR.Boost	0.170	0.189	0.195	0.200	0.183
		QR-LS.Boost	0.167	0.190	0.196	0.200	0.183
		RQ	0.183	0.194	0.199	0.199	0.188
	tdist	QR-QR.Boost	0.052	0.100	0.111	0.091	0.047
		QR-LS.Boost	0.052	0.100	0.112	0.093	0.047
		RQ	0.055	0.097	0.112	0.097	0.054
	gamma	QR-QR.Boost	0.252	0.171	0.100	0.050	0.014
		QR-LS.Boost	0.252	0.163	0.113	0.061	0.017
		RQ	0.251	0.183	0.132	0.103	0.071
	mixed	QR-QR.Boost	0.128	0.202	0.240	0.235	0.164
		QR-LS.Boost	0.128	0.202	0.240	0.235	0.164
		RQ	0.116	0.207	0.246	0.229	0.165
heteroskedastic	norm	QR-QR.Boost	0.014	0.103	0.165	0.232	0.289
		QR-LS.Boost	0.010	0.104	0.165	0.231	0.289
		RQ	0.034	0.110	0.176	0.235	0.289
	tdist	QR-QR.Boost	0.005	0.050	0.088	0.127	0.117
		QR-LS.Boost	0.005	0.050	0.088	0.127	0.117
		RQ	0.009	0.047	0.097	0.130	0.125
	gamma	QR-QR.Boost	0.321	0.284	0.253	0.250	0.186
		QR-LS.Boost	0.321	0.283	0.257	0.245	0.161
		RQ	0.321	0.300	0.284	0.275	0.279
	mixed	QR-QR.Boost	0.061	0.175	0.258	0.299	0.266
		QR-LS.Boost	0.061	0.175	0.258	0.299	0.286
		RQ	0.061	0.177	0.261	0.299	0.284
multivariate	norm	QR-QR.Boost	0.539	0.532	0.521	0.500	0.475
		QR-LS.Boost	0.538	0.532	0.522	0.500	0.475
		RQ	0.545	0.533	0.529	0.517	0.486
		RQAic	0.543	0.532	0.528	0.516	0.483
	tdist	QR-QR.Boost	0.260	0.348	0.357	0.317	0.191
		QR-LS.Boost	0.261	0.350	0.357	0.317	0.191
		RQ	0.283	0.348	0.361	0.328	0.194
		RQAic	0.272	0.347	0.360	0.327	0.172
	gamma	QR-QR.Boost	0.502	0.439	0.382	0.310	0.226
		QR-LS.Boost	0.502	0.440	0.381	0.310	0.228
		RQ	0.515	0.444	0.379	0.316	0.236
		RQAic	0.514	0.443	0.377	0.274	0.231
	mixed	QR-QR.Boost	0.311	0.377	0.388	0.343	0.260
		QR-LS.Boost	0.311	0.378	0.388	0.343	0.260
		RQ	0.307	0.376	0.389	0.350	0.254
		RQAic	0.299	0.375	0.388	0.347	0.240
multivariate2	norm	QR-QR.Boost	0.540	0.526	0.306	0.252	0.176
		QR-LS.Boost	0.539	0.526	0.306	0.253	0.177
		RQ	0.542	0.526	0.313	0.270	0.197
		RQAic	0.433	0.453	-0.040	-0.257	-0.288
	tdist	QR-QR.Boost	0.272	0.329	0.182	0.133	0.027
		QR-LS.Boost	0.273	0.337	0.182	0.134	0.028
		RQ	0.272	0.338	0.183	0.138	0.040
		RQAic	0.213	0.281	-0.036	-0.182	-0.069
	gamma	QR-QR.Boost	0.498	0.421	0.032	0.033	0.055
		QR-LS.Boost	0.495	0.426	0.033	0.034	0.062
		RQ	0.507	0.432	0.043	0.041	0.076
		RQAic	0.455	0.414	-0.022	-0.035	0.030
	mixed	QR-QR.Boost	0.317	0.375	0.129	0.102	0.102
		QR-LS.Boost	0.318	0.375	0.129	0.103	0.102
		RQ	0.309	0.377	0.129	0.103	0.101
		RQAic	0.270	0.351	0.112	0.100	0.081
high-dimensional	norm	QR-QR.Boost	0.578	0.583	0.564	0.547	0.526
		QR-LS.Boost	0.570	0.577	0.565	0.544	0.514
		RQ	0.801	0.722	0.699	0.715	0.776
	tdist	QR-QR.Boost	0.391	0.415	0.395	0.339	0.226
		QR-LS.Boost	0.399	0.411	0.396	0.344	0.225
		RQ	0.665	0.561	0.544	0.549	0.612
	gamma	QR-QR.Boost	0.539	0.479	0.398	0.349	0.255
		QR-LS.Boost	0.534	0.475	0.398	0.344	0.245
		RQ	0.733	0.642	0.600	0.603	0.707
	mixed	QR-QR.Boost	0.386	0.393	0.387	0.357	0.301
		QR-LS.Boost	0.385	0.388	0.385	0.353	0.293
		RQ	0.680	0.594	0.576	0.586	0.678

Table 5: τ-fit of QR-QR.Boost, QR-LS.Boost, RQ and RQAic for all parameter setups and all error distributions for each τ. Extension of Table 5 from the main document. Blue values indicate the superior result in the respective category.

Parameter setup	Error distribution	Method	τ				
			0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	QR-QR.Boost	0.153	0.162	0.156	0.145	0.094
		QR-LS.Boost	0.150	0.163	0.156	0.145	0.094
		RQ	0.167	0.173	0.158	0.146	0.091
	tdist	QR-QR.Boost	0.043	0.074	0.074	0.058	0.021
		QR-LS.Boost	0.043	0.073	0.075	0.059	0.021
		RQ	0.049	0.070	0.078	0.056	0.022
	gamma	QR-QR.Boost	0.210	0.135	0.068	0.030	0.006
		QR-LS.Boost	0.210	0.129	0.074	0.038	0.008
		RQ	0.209	0.146	0.096	0.063	0.031
	mixed	QR-QR.Boost	0.103	0.158	0.182	0.154	0.079
		QR-LS.Boost	0.102	0.158	0.182	0.155	0.078
		RQ	0.110	0.161	0.173	0.150	0.074
heteroskedastic	norm	QR-QR.Boost	0.014	0.083	0.126	0.175	0.146
		QR-LS.Boost	0.011	0.083	0.127	0.174	0.146
		RQ	0.036	0.089	0.137	0.167	0.152
	tdist	QR-QR.Boost	0.005	0.033	0.066	0.072	0.046
		QR-LS.Boost	0.004	0.033	0.066	0.073	0.046
		RQ	0.007	0.034	0.065	0.075	0.054
	gamma	QR-QR.Boost	0.259	0.219	0.179	0.168	0.089
		QR-LS.Boost	0.259	0.219	0.182	0.164	0.076
		RQ	0.257	0.233	0.214	0.183	0.132
	mixed	QR-QR.Boost	0.046	0.134	0.198	0.198	0.117
		QR-LS.Boost	0.046	0.134	0.198	0.198	0.125
		RQ	0.054	0.138	0.189	0.185	0.131
multivariate	norm	QR-QR.Boost	0.520	0.508	0.486	0.456	0.361
		QR-LS.Boost	0.521	0.508	0.487	0.456	0.363
		RQ	0.533	0.509	0.491	0.466	0.379
		RQAic	0.531	0.508	0.490	0.465	0.376
	tdist	QR-QR.Boost	0.242	0.310	0.296	0.245	0.110
		QR-LS.Boost	0.241	0.312	0.296	0.245	0.110
		RQ	0.263	0.307	0.294	0.245	0.112
		RQAic	0.251	0.306	0.293	0.244	0.098
	gamma	QR-QR.Boost	0.477	0.400	0.330	0.255	0.151
		QR-LS.Boost	0.476	0.401	0.330	0.255	0.150
		RQ	0.485	0.404	0.334	0.258	0.155
		RQAic	0.484	0.403	0.331	0.220	0.151
	mixed	QR-QR.Boost	0.284	0.343	0.329	0.264	0.159
		QR-LS.Boost	0.285	0.343	0.329	0.264	0.159
		RQ	0.289	0.342	0.328	0.267	0.169
		RQAic	0.284	0.342	0.327	0.263	0.157
multivariate2	norm	QR-QR.Boost	0.530	0.499	0.297	0.237	0.159
		QR-LS.Boost	0.530	0.499	0.298	0.238	0.159
		RQ	0.530	0.509	0.309	0.268	0.162
		RQAic	0.420	0.435	-0.021	-0.243	-0.326
	tdist	QR-QR.Boost	0.251	0.288	0.150	0.097	0.015
		QR-LS.Boost	0.250	0.293	0.150	0.098	0.015
		RQ	0.249	0.298	0.147	0.100	0.022
		RQAic	0.195	0.244	-0.032	-0.149	-0.053
	gamma	QR-QR.Boost	0.466	0.385	0.029	0.028	0.039
		QR-LS.Boost	0.463	0.389	0.029	0.028	0.048
		RQ	0.474	0.394	0.035	0.036	0.054
		RQAic	0.420	0.379	-0.021	-0.035	0.017
	mixed	QR-QR.Boost	0.289	0.334	0.100	0.074	0.059
		QR-LS.Boost	0.289	0.334	0.100	0.075	0.059
		RQ	0.285	0.340	0.106	0.080	0.062
		RQAic	0.247	0.318	0.090	0.077	0.047
high-dimensional	norm	QR-QR.Boost	0.556	0.522	0.482	0.430	0.313
		QR-LS.Boost	0.549	0.518	0.482	0.427	0.311
		RQ	0.750	0.655	0.610	0.568	0.483
	tdist	QR-QR.Boost	0.342	0.331	0.289	0.222	0.108
		QR-LS.Boost	0.347	0.327	0.289	0.225	0.105
		RQ	0.592	0.461	0.402	0.359	0.303
	gamma	QR-QR.Boost	0.476	0.403	0.313	0.235	0.122
		QR-LS.Boost	0.471	0.397	0.313	0.234	0.111
		RQ	0.636	0.525	0.462	0.409	0.347
	mixed	QR-QR.Boost	0.335	0.327	0.306	0.246	0.143
		QR-LS.Boost	0.350	0.321	0.304	0.246	0.140
		RQ	0.591	0.479	0.435	0.413	0.338

Table 6: τ-fit of QR-QR.Boost, QR-LS.Boost, RQ and RQAic for the contaminated cases of all parameter setups and all error distributions for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	QR-QR.Boost	0.696	1.370	1.585	1.402	0.725
		QR-LS.Boost	0.699	1.369	1.583	1.402	0.724
		RQ	0.682	1.372	1.583	1.403	0.728
	tdist	QR-QR.Boost	1.298	2.151	2.420	2.207	1.353
		QR-LS.Boost	1.297	2.150	2.420	2.206	1.351
		RQ	1.300	2.153	2.422	2.211	1.361
	gamma	QR-QR.Boost	0.674	1.645	2.222	2.234	1.370
		QR-LS.Boost	0.673	1.661	2.193	2.215	1.374
		RQ	0.676	1.629	2.121	2.133	1.301
	mixed	QR-QR.Boost	1.452	2.592	3.020	2.817	1.707
		QR-LS.Boost	1.450	2.592	3.019	2.816	1.706
		RQ	1.456	2.596	3.018	2.817	1.707
heteroskedastic	norm	QR-QR.Boost	1.540	3.039	3.546	3.128	1.641
		QR-LS.Boost	1.546	3.039	3.545	3.125	1.640
		RQ	1.507	3.054	3.541	3.136	1.644
	tdist	QR-QR.Boost	2.958	4.836	5.423	4.957	3.048
		QR-LS.Boost	2.952	4.836	5.423	4.957	3.047
		RQ	2.968	4.848	5.418	4.950	3.062
	gamma	QR-QR.Boost	1.521	3.730	5.032	4.920	3.278
		QR-LS.Boost	1.521	3.735	5.010	4.943	3.385
		RQ	1.525	3.676	4.782	4.796	2.917
	mixed	QR-QR.Boost	3.315	6.073	7.122	6.675	4.127
		QR-LS.Boost	3.313	6.072	7.119	6.674	4.018
		RQ	3.330	6.070	7.125	6.668	4.021
multivariate	norm	QR-QR.Boost	2.838	5.460	6.264	5.522	2.760
		QR-LS.Boost	2.839	5.455	6.258	5.518	2.760
		RQ	2.832	5.484	6.243	5.418	2.721
		RQAic	2.831	5.479	6.240	5.410	2.720
	tdist	QR-QR.Boost	6.062	9.436	10.473	9.820	6.302
		QR-LS.Boost	6.039	9.402	10.469	9.817	6.303
		RQ	5.969	9.401	10.472	9.818	6.365
		RQAic	6.067	9.396	10.465	9.810	6.458
	gamma	QR-QR.Boost	2.711	6.432	8.324	7.998	4.717
		QR-LS.Boost	2.707	6.421	8.335	8.002	4.699
		RQ	2.664	6.415	8.329	8.016	4.637
		RQAic	2.661	6.409	8.327	8.351	4.641
	mixed	QR-QR.Boost	6.466	11.187	13.214	12.760	7.795
		QR-LS.Boost	6.460	11.185	13.205	12.766	7.804
		RQ	6.466	11.202	13.177	12.676	7.802
		RQAic	6.511	11.188	13.168	12.706	7.841
multivariate2	norm	QR-QR.Boost	2.826	5.479	6.224	5.487	2.721
		QR-LS.Boost	2.822	5.463	6.217	5.483	2.721
		RQ	2.836	5.484	6.255	5.414	2.719
		RQAic	3.349	6.162	9.058	8.875	4.006
	tdist	QR-QR.Boost	5.965	9.477	10.454	9.827	6.304
		QR-LS.Boost	5.951	9.407	10.452	9.824	6.303
		RQ	5.963	9.420	10.482	9.817	6.374
		RQAic	6.446	10.283	13.157	13.058	6.806
	gamma	QR-QR.Boost	2.709	6.479	8.265	7.959	4.668
		QR-LS.Boost	2.719	6.423	8.268	7.965	4.644
		RQ	2.665	6.413	8.338	8.004	4.613
		RQAic	2.943	6.609	8.765	8.392	4.644
	mixed	QR-QR.Boost	6.446	11.202	13.192	12.743	7.783
		QR-LS.Boost	6.440	11.201	13.195	12.725	7.792
		RQ	6.466	11.196	13.183	12.676	7.818
		RQAic	6.658	11.551	13.371	12.667	7.888
high-dimensional	norm	QR-QR.Boost	3.466	6.121	6.932	6.046	3.106
		QR-LS.Boost	3.467	6.143	6.932	6.013	3.106
		RQ	8.041	9.843	10.386	9.640	7.958
	tdist	QR-QR.Boost	6.531	9.692	10.640	9.969	6.540
		QR-LS.Boost	6.482	9.664	10.641	9.893	6.455
		RQ	18.194	16.043	16.625	16.468	18.602
	gamma	QR-QR.Boost	2.969	6.657	8.597	8.382	5.447
		QR-LS.Boost	2.966	6.668	8.583	8.344	5.387
		RQ	8.107	11.143	13.290	14.145	13.252
	mixed	QR-QR.Boost	6.817	11.345	13.226	12.433	8.049
		QR-LS.Boost	6.757	11.329	13.232	12.394	7.985
		RQ	19.744	19.579	20.723	21.207	21.364

Table 7: Empirical risk R_τ of QR-QR.Boost, QR-LS.Boost, RQ and RQAic for all parameter setups and all error distributions for each τ. Extension of Table 6 from the main document. Blue values indicate the superior result in the respective category.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	QR-QR.Boost	1.144	2.710	3.822	4.531	4.751
		QR-LS.Boost	1.146	2.710	3.821	4.531	4.751
		RQ	1.130	2.715	3.825	4.534	4.753
	tdist	QR-QR.Boost	2.021	4.323	6.036	7.273	7.858
		QR-LS.Boost	2.020	4.323	6.036	7.273	7.858
		RQ	2.027	4.325	6.037	7.279	7.868
	gamma	QR-QR.Boost	1.498	4.118	6.340	8.010	8.788
		QR-LS.Boost	1.498	4.130	6.322	7.988	8.790
		RQ	1.502	4.101	6.246	7.901	8.719
	mixed	QR-QR.Boost	2.411	5.472	7.811	9.537	10.334
		QR-LS.Boost	2.411	5.471	7.811	9.536	10.333
		RQ	2.416	5.470	7.814	9.531	10.343
heteroskedastic	norm	QR-QR.Boost	2.595	6.212	8.830	10.529	11.147
		QR-LS.Boost	2.600	6.211	8.829	10.521	11.148
		RQ	2.570	6.215	8.829	10.529	11.147
	tdist	QR-QR.Boost	4.652	9.921	13.898	16.815	18.312
		QR-LS.Boost	4.647	9.922	13.897	16.817	18.307
		RQ	4.667	9.921	13.896	16.819	18.314
	gamma	QR-QR.Boost	3.623	10.031	15.533	19.598	22.158
		QR-LS.Boost	3.623	10.034	15.506	19.624	22.277
		RQ	3.624	9.970	15.283	19.485	21.804
	mixed	QR-QR.Boost	5.849	13.648	19.744	24.339	26.859
		QR-LS.Boost	5.848	13.648	19.744	24.339	26.736
		RQ	5.840	13.644	19.751	24.356	26.739
multivariate	norm	QR-QR.Boost	4.746	11.152	15.751	18.825	19.867
		QR-LS.Boost	4.736	11.145	15.748	18.824	19.846
		RQ	4.731	11.181	15.738	18.701	19.763
		RQAic	4.729	11.174	15.732	18.695	19.764
	tdist	QR-QR.Boost	9.164	18.681	25.898	31.400	34.063
		QR-LS.Boost	9.150	18.652	25.894	31.401	34.065
		RQ	9.068	18.671	25.894	31.412	34.126
		RQAic	9.160	18.657	25.882	31.401	34.260
	gamma	QR-QR.Boost	6.129	16.705	25.457	31.967	35.534
		QR-LS.Boost	6.129	16.696	25.460	31.971	35.534
		RQ	6.090	16.690	25.457	31.994	35.466
		RQAic	6.089	16.682	25.452	32.365	35.467
	mixed	QR-QR.Boost	10.919	24.571	35.467	43.946	47.918
		QR-LS.Boost	10.914	24.570	35.471	43.944	47.924
		RQ	10.922	24.561	35.458	43.862	47.914
		RQAic	10.952	24.550	35.445	43.910	48.033
multivariate2	norm	QR-QR.Boost	4.454	10.397	6.997	8.654	6.785
		QR-LS.Boost	4.453	10.380	6.993	8.649	6.781
		RQ	4.474	10.397	7.010	8.572	6.752
		RQAic	5.016	11.099	9.667	12.077	8.316
	tdist	QR-QR.Boost	8.793	17.937	20.391	25.539	26.510
		QR-LS.Boost	8.783	17.878	20.388	25.536	26.514
		RQ	8.808	17.868	20.397	25.543	26.579
		RQAic	9.297	18.788	23.136	29.111	27.113
	gamma	QR-QR.Boost	5.865	15.952	17.099	22.176	22.953
		QR-LS.Boost	5.874	15.901	17.101	22.182	22.920
		RQ	5.824	15.891	17.163	22.233	22.901
		RQAic	6.131	16.073	17.579	22.678	22.917
	mixed	QR-QR.Boost	10.632	23.699	27.406	34.826	36.201
		QR-LS.Boost	10.629	23.698	27.408	34.810	36.207
		RQ	10.630	23.699	27.381	34.752	36.194
		RQAic	10.848	24.045	27.584	34.775	36.312
high-dimensional	norm	QR-QR.Boost	5.270	11.676	16.199	18.963	19.670
		QR-LS.Boost	5.246	11.685	16.194	18.937	19.639
		RQ	9.863	15.328	19.621	22.615	27.137
	tdist	QR-QR.Boost	9.819	19.660	27.145	33.073	36.254
		QR-LS.Boost	9.732	19.636	27.137	32.970	36.166
		RQ	21.886	26.209	32.987	39.464	51.225
	gamma	QR-QR.Boost	6.167	16.323	24.575	30.782	34.251
		QR-LS.Boost	6.159	16.322	24.560	30.736	34.192
		RQ	11.138	20.896	29.442	36.547	45.480
	mixed	QR-QR.Boost	11.000	23.765	33.929	41.461	45.343
		QR-LS.Boost	10.901	23.764	33.944	41.410	45.303
		RQ	23.048	31.753	41.307	49.981	64.464

Table 8: Empirical risk R_τ of QR-QR.Boost, QR-LS.Boost, RQ and RQAic for contaminated cases for all parameter setups and error distributions and each τ. Blue values indicate superior result in the respective category.

Parameter setup	Error distr.	MSE(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	1.490	0.523	0.493	0.577	0.812
			QR-LS.Boost	1.655	0.454	0.441	0.602	0.770
			RQ	1.124	0.546	0.432	0.804	1.009
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.074	0.016	0.017	0.017	0.026
			QR-LS.Boost	0.084	0.015	0.013	0.017	0.026
			RQ	0.031	0.018	0.013	0.024	0.026
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	4.128	0.993	0.402	0.855	4.083
			QR-LS.Boost	3.962	0.864	0.383	0.745	4.073
			RQ	5.444	0.992	0.583	0.894	4.713
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.166	0.024	0.013	0.026	0.097
			QR-LS.Boost	0.162	0.021	0.012	0.023	0.091
			RQ	0.141	0.027	0.017	0.028	0.158
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.211	0.859	8.191	16.114	30.111
			QR-LS.Boost	0.215	2.827	5.614	12.375	25.602
			RQ	0.333	0.509	0.784	1.712	4.748
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.006	0.040	0.247	0.426	0.680
			QR-LS.Boost	0.006	0.090	0.172	0.357	0.734
			RQ	0.010	0.017	0.020	0.049	0.132
heteroskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	2.968	1.062	1.426	1.346	2.491
			QR-LS.Boost	4.959	0.969	1.311	1.346	2.226
			RQ	1.894	1.440	1.310	1.490	2.975
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.262	0.069	0.099	0.080	0.152
			QR-LS.Boost	0.344	0.066	0.094	0.065	0.141
			RQ	0.116	0.088	0.063	0.088	0.159
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	6.195	1.699	1.711	2.465	12.298
			QR-LS.Boost	5.218	1.740	1.596	2.572	11.503
			RQ	13.137	2.165	1.299	1.846	10.292
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.232	0.109	0.103	0.145	0.576
			QR-LS.Boost	0.196	0.116	0.099	0.150	0.535
			RQ	0.809	0.140	0.084	0.135	0.751
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.477	2.963	28.833	35.342	350.318
			QR-LS.Boost	0.531	3.528	24.098	39.273	443.881
			RQ	0.707	1.289	1.533	4.128	6.822
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.021	0.280	1.309	1.046	8.238
			QR-LS.Boost	0.023	0.302	1.227	1.393	10.837
			RQ	0.044	0.067	0.091	0.227	0.586
multivariate	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	12.316	6.786	7.917	8.642	17.207
			QR-LS.Boost	11.883	6.424	7.651	8.812	16.720
			RQ	11.433	8.879	8.260	7.244	14.660
			RQAic	10.867	7.480	7.555	7.083	12.512
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.314	0.111	0.242	0.341	0.350
			QR-LS.Boost	0.326	0.111	0.261	0.392	0.351
			RQ	0.145	0.077	0.060	0.080	0.120
			RQAic	0.150	0.076	0.061	0.080	0.110
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.265	0.140	0.254	0.433	0.279
			QR-LS.Boost	0.325	0.140	0.249	0.410	0.280
			RQ	0.177	0.103	0.073	0.081	0.143
			RQAic	0.180	0.098	0.071	0.081	0.144
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.337	0.085	0.214	0.335	0.285
			QR-LS.Boost	0.367	0.081	0.203	0.387	0.282
			RQ	0.100	0.067	0.066	0.075	0.114
			RQAic	0.105	0.068	0.069	0.070	0.112
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.297	0.119	0.235	0.375	0.187
			QR-LS.Boost	0.345	0.116	0.211	0.363	0.181
			RQ	0.118	0.070	0.071	0.072	0.139
			RQAic	0.118	0.071	0.073	0.073	0.200
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.036	0.016	0.009	0.002	0.028
			QR-LS.Boost	0.027	0.017	0.009	0.002	0.026
			RQ	0.117	0.079	0.054	0.092	0.144
			RQAic	0.149	0.085	0.085	0.112	0.175
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.043	0.026	0.014	0.005	0.025
			QR-LS.Boost	0.046	0.029	0.014	0.003	0.022
			RQ	0.133	0.062	0.076	0.084	0.134
			RQAic	0.046	0.017	0.004	0.016	0.032
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	96.038	6.901	9.515	9.547	95.086
			QR-LS.Boost	89.091	7.565	8.980	9.658	95.566
			RQ	54.131	13.055	8.297	14.977	70.468
			RQAic	46.746	9.849	6.483	11.222	62.547

		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	1.984	0.244	0.080	0.125	0.730
			QR-LS.Boost	1.829	0.121	0.076	0.124	0.712
			RQ	0.697	0.120	0.099	0.123	0.556
			RQAic	0.637	0.127	0.098	0.120	0.566
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	2.480	0.270	0.095	0.140	0.630
			QR-LS.Boost	2.306	0.188	0.091	0.139	0.640
			RQ	0.651	0.111	0.082	0.149	0.783
			RQAic	0.690	0.123	0.082	0.146	3.133
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	1.841	0.213	0.079	0.145	0.986
			QR-LS.Boost	1.844	0.108	0.070	0.143	0.985
			RQ	0.685	0.117	0.087	0.110	0.495
			RQAic	3.100	0.114	0.089	0.108	1.796
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	2.072	0.179	0.078	0.164	0.212
			QR-LS.Boost	1.974	0.144	0.073	0.167	0.261
			RQ	0.655	0.106	0.076	0.118	0.861
			RQAic	2.367	0.108	0.076	0.145	0.961
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.120	0.042	0.053	0.066	0.162
			QR-LS.Boost	0.111	0.062	0.048	0.062	0.173
			RQ	0.630	0.102	0.080	0.107	0.552
			RQAic	0.609	0.093	0.051	0.112	0.283
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.146	0.034	0.057	0.055	0.215
			QR-LS.Boost	0.193	0.066	0.059	0.052	0.221
			RQ	0.403	0.124	0.087	0.101	0.571
			RQAic	0.140	0.006	0.006	0.010	0.033
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	4.567	7.982	8.289	39.340	358.474
			QR-LS.Boost	4.900	8.923	8.248	37.929	318.259
			RQ	4.243	6.228	14.092	23.722	80.628
			RQAic	3.311	5.068	11.932	18.590	84.278
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.154	0.080	0.144	0.326	1.238
			QR-LS.Boost	0.151	0.075	0.160	0.346	1.036
			RQ	0.037	0.075	0.109	0.193	0.534
			RQAic	0.037	0.072	0.105	0.191	0.515
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.153	0.109	0.204	0.090	1.153
			QR-LS.Boost	0.202	0.098	0.214	0.084	1.211
			RQ	0.048	0.077	0.109	0.244	0.694
			RQAic	0.041	0.077	0.112	3.478	0.645
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.134	0.070	0.140	0.302	0.985
			QR-LS.Boost	0.124	0.065	0.150	0.298	0.954
			RQ	0.041	0.050	0.100	0.169	0.500
			RQAic	0.041	0.052	0.098	2.138	0.412
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.180	0.091	0.082	0.137	1.331
			QR-LS.Boost	0.205	0.079	0.087	0.127	1.303
			RQ	0.037	0.066	0.108	0.186	0.655
			RQAic	0.037	0.066	0.173	0.343	1.275
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.010	0.033	0.041	0.046	0.189
			QR-LS.Boost	0.013	0.041	0.039	0.040	0.289
			RQ	0.037	0.045	0.075	0.167	0.557
			RQAic	0.036	0.054	0.036	0.044	0.613
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.009	0.036	0.027	0.037	0.222
			QR-LS.Boost	0.013	0.044	0.027	0.027	0.256
			RQ	0.030	0.055	0.097	0.203	0.475
			RQAic	0.007	0.000	0.000	0.000	0.171
multivariate2	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	11.757	6.470	7.396	4.633	6.685
			QR-LS.Boost	11.924	5.069	7.008	5.138	6.636
			RQ	16.192	12.094	9.486	8.234	18.349
			RQAic	14.026	10.031	7.169	7.113	15.572
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.207	0.202	0.013	0.002	0.037
			QR-LS.Boost	0.177	0.176	0.013	0.002	0.040
			RQ	0.096	0.051	0.058	0.082	0.146
			RQAic	0.097	0.059	20.242	11.382	3.949
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.176	0.212	0.131	0.442	0.211
			QR-LS.Boost	0.181	0.249	0.119	0.434	0.215
			RQ	0.162	0.097	0.115	0.107	0.182
			RQAic	0.168	0.101	10.059	26.049	10.838
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.053	0.007	0.017	0.336	0.227
			QR-LS.Boost	0.054	0.009	0.016	0.349	0.210
			RQ	0.119	0.078	0.068	0.060	0.085
			RQAic	6.521	4.709	1.619	8.987	4.416
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.201	0.184	0.164	0.399	0.148
			QR-LS.Boost	0.221	0.190	0.130	0.398	0.148
			RQ	0.107	0.085	0.067	0.073	0.113

			RQAic	6.489	4.434	3.851	1.718	0.424
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.065	0.010	0.027	0.004	0.020
			QR-LS.Boost	0.061	0.012	0.027	0.007	0.024
			RQ	0.119	0.070	0.076	0.067	0.122
			RQAic	0.105	0.040	0.020	0.051	0.119
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.073	0.015	0.019	0.002	0.034
			QR-LS.Boost	0.071	0.012	0.022	0.002	0.037
			RQ	0.146	0.082	0.055	0.078	0.126
			RQAic	0.017	0.001	0.000	0.007	0.012
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	75.888	17.120	5.431	8.016	47.875
			QR-LS.Boost	64.064	10.868	5.608	7.927	45.354
			RQ	54.725	13.920	9.964	14.667	85.288
			RQAic	53.274	11.317	6.708	9.807	80.642
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	1.154	0.688	0.050	0.043	0.201
			QR-LS.Boost	1.249	0.203	0.048	0.046	0.193
			RQ	0.481	0.130	0.088	0.107	0.579
			RQAic	0.505	0.138	21.832	12.278	2.752
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	1.150	0.642	0.087	0.161	0.792
			QR-LS.Boost	1.168	0.354	0.074	0.152	0.753
			RQ	0.828	0.176	0.098	0.132	0.781
			RQAic	0.883	0.166	10.232	27.439	7.140
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.176	0.014	0.046	0.204	0.822
			QR-LS.Boost	0.205	0.044	0.042	0.183	0.850
			RQ	0.606	0.098	0.098	0.127	0.797
			RQAic	9.940	5.825	1.107	9.554	4.242
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	1.010	0.480	0.082	0.197	0.136
			QR-LS.Boost	1.017	0.241	0.084	0.189	0.140
			RQ	0.648	0.144	0.092	0.113	0.581
			RQAic	9.899	5.937	4.078	1.642	0.530
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.225	0.008	0.040	0.037	0.188
			QR-LS.Boost	0.215	0.042	0.041	0.039	0.147
			RQ	0.766	0.101	0.096	0.109	0.708
			RQAic	0.510	0.025	0.000	0.024	0.135
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.303	0.008	0.041	0.028	0.133
			QR-LS.Boost	0.295	0.039	0.042	0.028	0.128
			RQ	0.558	0.137	0.070	0.104	0.521
			RQAic	0.063	0.000	0.000	0.000	0.000
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	6.506	6.053	7.004	30.768	312.094
			QR-LS.Boost	8.841	4.668	6.927	24.897	211.235
			RQ	4.405	6.923	10.380	26.641	55.751
			RQAic	3.678	5.016	8.835	19.209	55.818
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.182	0.262	0.037	0.041	0.139
			QR-LS.Boost	0.219	0.108	0.036	0.039	0.248
			RQ	0.040	0.048	0.097	0.212	0.483
			RQAic	0.035	0.052	2.519	2.466	4.460
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.178	0.206	0.307	0.092	1.795
			QR-LS.Boost	0.279	0.153	0.281	0.091	1.411
			RQ	0.041	0.067	0.112	0.209	0.700
			RQAic	0.039	0.063	1.926	1.934	0.806
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.005	0.015	0.018	0.412	1.387
			QR-LS.Boost	0.006	0.023	0.017	0.409	1.109
			RQ	0.036	0.052	0.092	0.150	0.510
			RQAic	1.635	0.768	0.118	3.338	0.514
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.169	0.192	0.090	0.150	1.607
			QR-LS.Boost	0.254	0.120	0.096	0.141	1.284
			RQ	0.037	0.065	0.107	0.175	0.541
			RQAic	1.749	0.780	0.120	0.254	3.211
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.006	0.007	0.015	0.043	0.140
			QR-LS.Boost	0.008	0.015	0.010	0.044	0.244
			RQ	0.032	0.063	0.094	0.229	0.574
			RQAic	0.015	0.008	0.002	0.009	0.361
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.003	0.006	0.033	0.056	0.154
			QR-LS.Boost	0.004	0.011	0.031	0.053	0.222
			RQ	0.042	0.056	0.096	0.135	0.426
			RQAic	0.005	0.000	0.000	0.008	0.041
high-dimensional	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	129.299	71.062	56.139	69.611	95.093
			QR-LS.Boost	108.053	71.538	54.595	54.287	90.205
			RQ	2322.473	1490.684	964.144	1546.512	1963.622
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	3.228	0.561	0.480	0.673	1.261
			QR-LS.Boost	3.174	0.588	0.487	0.698	1.331
			RQ	0.780	0.511	0.459	0.535	0.759

	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	4.168	0.738	0.664	0.872	1.162
		QR-LS.Boost	4.199	0.970	0.652	0.726	1.157
		RQ	3.731	0.864	0.495	0.732	3.585
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	1.834	0.855	0.549	0.580	1.149
		QR-LS.Boost	1.865	0.828	0.523	0.610	1.260
		RQ	0.775	0.540	0.370	0.460	0.555
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	3.054	0.832	0.671	0.586	0.309
		QR-LS.Boost	3.150	0.942	0.648	0.569	0.316
		RQ	1.597	0.650	0.546	0.528	1.801
tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	707.248	129.622	75.478	91.641	908.792
		QR-LS.Boost	587.858	96.069	75.387	90.419	721.033
		RQ	13122.689	3433.946	2078.630	3108.946	9107.436
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	12.176	1.146	0.954	1.366	12.071
		QR-LS.Boost	9.601	1.066	0.921	1.315	9.585
		RQ	5.313	1.401	0.942	1.270	4.086
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	13.403	1.205	0.883	2.198	1.191
		QR-LS.Boost	11.503	1.327	0.824	1.878	1.205
		RQ	10.243	1.732	1.092	1.454	10.833
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	2.555	1.003	0.879	1.382	2.909
		QR-LS.Boost	2.439	0.960	0.835	1.325	2.808
		RQ	5.278	1.529	1.238	1.437	3.126
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	8.476	1.010	0.850	1.066	0.072
		QR-LS.Boost	7.576	1.024	0.857	0.926	0.044
		RQ	6.964	1.688	1.254	1.620	5.224
gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	38.895	48.867	89.970	330.314	2858.328
		QR-LS.Boost	36.045	53.792	91.709	288.956	2622.956
		RQ	1316.583	1853.465	1657.460	3396.255	5474.548
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.942	0.466	1.099	1.775	16.132
		QR-LS.Boost	0.960	0.490	1.069	1.708	15.282
		RQ	0.751	0.641	0.714	1.145	1.735
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	1.365	0.672	1.211	0.044	5.174
		QR-LS.Boost	1.404	0.854	1.140	0.043	5.206
		RQ	3.690	1.145	1.185	1.513	11.811
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	1.071	0.580	1.120	1.868	3.223
		QR-LS.Boost	1.048	0.622	1.137	1.888	3.215
		RQ	0.578	0.614	0.845	1.273	1.476
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.888	0.429	0.089	0.162	3.001
		QR-LS.Boost	0.932	0.488	0.091	0.167	3.055
		RQ	1.444	0.743	0.925	1.360	4.367

Table 9: Estimation accuracy measured by the MSE for QR-QR.Boost, QR-LS.Boost, RQ, and RQAic for all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distr.	MSE(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	1.503	0.413	0.491	0.436	0.952
			QR-LS.Boost	1.525	0.435	0.489	0.461	0.939
			RQ	0.958	0.635	0.553	0.474	0.975
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.071	0.011	0.015	0.011	0.036
			QR-LS.Boost	0.076	0.012	0.013	0.012	0.035
			RQ	0.027	0.019	0.019	0.015	0.031
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	4.258	0.792	0.383	0.990	3.094
			QR-LS.Boost	4.551	0.780	0.336	0.970	2.727
			RQ	6.483	1.119	0.504	1.184	5.042
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.117	0.022	0.014	0.032	0.089
			QR-LS.Boost	0.138	0.022	0.012	0.034	0.080
			RQ	0.177	0.028	0.016	0.029	0.164
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.242	0.789	7.847	17.170	30.772
			QR-LS.Boost	0.243	2.449	6.239	12.722	25.858
			RQ	0.361	0.462	0.944	1.458	4.347
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.006	0.040	0.241	0.440	0.697
			QR-LS.Boost	0.006	0.081	0.193	0.357	0.733
			RQ	0.013	0.013	0.028	0.046	0.121
heteroskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	2.886	1.034	1.302	0.901	1.946
			QR-LS.Boost	5.053	0.984	1.146	0.742	2.013
			RQ	2.422	1.045	1.283	1.025	1.815
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.243	0.074	0.086	0.069	0.120
			QR-LS.Boost	0.326	0.068	0.076	0.039	0.119
			RQ	0.188	0.066	0.083	0.069	0.127
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	6.352	2.137	1.266	2.199	15.792
			QR-LS.Boost	5.202	2.203	1.141	2.391	14.052
			RQ	15.069	2.479	1.364	1.554	14.220
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.233	0.107	0.073	0.144	0.888
			QR-LS.Boost	0.208	0.116	0.069	0.150	0.825
			RQ	0.709	0.127	0.078	0.094	0.747
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.679	3.355	29.173	30.305	336.852
			QR-LS.Boost	0.709	3.929	24.295	34.673	444.223
			RQ	0.798	1.018	1.940	3.131	13.388
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.032	0.298	1.363	0.944	7.891
			QR-LS.Boost	0.031	0.319	1.239	1.345	11.178
			RQ	0.040	0.067	0.134	0.186	0.774
multivariate	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	12.208	5.728	6.138	6.902	17.706
			QR-LS.Boost	11.706	5.883	6.285	7.417	15.244
			RQ	14.533	9.334	7.249	9.247	15.618
			RQAic	13.881	7.957	6.258	7.536	15.942
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.412	0.084	0.252	0.296	0.261
			QR-LS.Boost	0.342	0.083	0.267	0.356	0.227
			RQ	0.082	0.053	0.059	0.073	0.107
			RQAic	0.080	0.050	0.060	0.071	0.115
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.247	0.104	0.239	0.479	0.316
			QR-LS.Boost	0.265	0.116	0.222	0.461	0.264
			RQ	0.158	0.096	0.060	0.111	0.139
			RQAic	0.166	0.102	0.060	0.109	0.134
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.337	0.108	0.227	0.341	0.314
			QR-LS.Boost	0.295	0.101	0.218	0.407	0.246
			RQ	0.133	0.065	0.060	0.069	0.109
			RQAic	0.127	0.063	0.060	0.076	0.101
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.345	0.099	0.193	0.423	0.329
			QR-LS.Boost	0.356	0.091	0.194	0.420	0.306
			RQ	0.108	0.096	0.072	0.073	0.164
			RQAic	0.115	0.091	0.065	0.081	0.223
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.031	0.027	0.006	0.000	0.013
			QR-LS.Boost	0.027	0.028	0.006	0.000	0.009
			RQ	0.124	0.059	0.073	0.072	0.124
			RQAic	0.179	0.071	0.085	0.081	0.157
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.048	0.028	0.014	0.002	0.023
			QR-LS.Boost	0.045	0.030	0.014	0.002	0.018
			RQ	0.116	0.077	0.074	0.084	0.123
			RQAic	0.021	0.010	0.014	0.013	0.032
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	86.060	10.348	8.074	11.546	77.238
			QR-LS.Boost	73.334	11.489	7.953	11.133	74.508
			RQ	80.756	12.512	9.827	11.142	68.627
			RQAic	73.475	10.159	8.001	8.734	64.743

		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	1.936	0.201	0.096	0.118	0.609
			QR-LS.Boost	2.032	0.114	0.093	0.127	0.612
			RQ	0.599	0.126	0.102	0.106	0.339
			RQAic	0.639	0.129	0.093	0.100	0.356
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	2.104	0.348	0.106	0.139	0.627
			QR-LS.Boost	2.357	0.239	0.098	0.142	0.620
			RQ	1.032	0.135	0.098	0.175	0.918
			RQAic	0.968	0.133	0.099	0.180	3.954
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	2.145	0.181	0.095	0.128	0.703
			QR-LS.Boost	2.057	0.121	0.090	0.122	0.700
			RQ	0.664	0.117	0.078	0.086	0.641
			RQAic	3.311	0.114	0.069	0.091	2.673
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	2.362	0.199	0.075	0.085	0.262
			QR-LS.Boost	2.503	0.139	0.067	0.100	0.274
			RQ	0.610	0.111	0.082	0.122	0.757
			RQAic	1.850	0.107	0.088	0.135	0.783
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.180	0.034	0.048	0.060	0.227
			QR-LS.Boost	0.187	0.061	0.047	0.059	0.230
			RQ	0.390	0.126	0.102	0.112	0.512
			RQAic	0.607	0.114	0.047	0.081	0.097
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.166	0.020	0.050	0.065	0.208
			QR-LS.Boost	0.124	0.044	0.057	0.059	0.222
			RQ	0.803	0.130	0.064	0.125	0.637
			RQAic	0.098	0.000	0.000	0.017	0.000
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	3.455	7.384	9.626	34.175	297.702
			QR-LS.Boost	3.674	8.064	8.250	31.363	337.693
			RQ	4.254	7.691	11.884	23.101	69.480
			RQAic	3.475	5.665	9.811	21.561	75.221
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.124	0.081	0.181	0.285	0.956
			QR-LS.Boost	0.145	0.069	0.196	0.276	0.950
			RQ	0.031	0.053	0.085	0.171	0.772
			RQAic	0.032	0.051	0.082	0.188	0.760
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.100	0.104	0.250	0.106	1.308
			QR-LS.Boost	0.142	0.101	0.230	0.092	1.439
			RQ	0.053	0.095	0.120	0.207	0.879
			RQAic	0.051	0.087	0.121	3.658	0.823
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.118	0.060	0.148	0.274	1.048
			QR-LS.Boost	0.126	0.062	0.138	0.270	1.103
			RQ	0.035	0.058	0.115	0.170	0.401
			RQAic	0.036	0.057	0.112	2.432	0.369
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.140	0.077	0.081	0.143	1.455
			QR-LS.Boost	0.168	0.078	0.084	0.142	1.512
			RQ	0.039	0.075	0.105	0.209	0.637
			RQAic	0.038	0.080	0.147	0.264	1.077
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.013	0.038	0.037	0.035	0.242
			QR-LS.Boost	0.015	0.045	0.033	0.033	0.269
			RQ	0.037	0.067	0.098	0.156	0.512
			RQAic	0.041	0.072	0.043	0.040	0.548
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.014	0.029	0.027	0.035	0.305
			QR-LS.Boost	0.016	0.034	0.022	0.036	0.345
			RQ	0.037	0.069	0.094	0.154	0.441
			RQAic	0.008	0.009	0.009	0.000	0.055
multivariate2	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	12.841	5.508	6.316	5.434	12.643
			QR-LS.Boost	12.681	4.830	6.490	5.567	11.635
			RQ	17.481	9.039	7.421	12.648	15.455
			RQAic	16.441	7.199	4.010	8.923	14.388
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.129	0.202	0.030	0.006	0.084
			QR-LS.Boost	0.133	0.194	0.029	0.006	0.085
			RQ	0.120	0.076	0.068	0.085	0.112
			RQAic	0.118	0.077	19.751	11.677	4.392
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.224	0.183	0.123	0.481	0.213
			QR-LS.Boost	0.213	0.229	0.122	0.448	0.196
			RQ	0.139	0.091	0.085	0.092	0.134
			RQAic	0.138	0.091	8.475	26.430	13.911
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.031	0.011	0.024	0.330	0.200
			QR-LS.Boost	0.042	0.007	0.024	0.367	0.177
			RQ	0.118	0.073	0.048	0.068	0.120
			RQAic	6.520	4.835	1.124	9.008	5.399
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.174	0.173	0.134	0.385	0.206
			QR-LS.Boost	0.168	0.178	0.122	0.403	0.187
			RQ	0.164	0.050	0.067	0.086	0.141

			RQAic	6.888	4.723	3.831	1.681	0.572
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.042	0.009	0.034	0.003	0.047
			QR-LS.Boost	0.043	0.009	0.029	0.002	0.045
			RQ	0.105	0.070	0.049	0.073	0.108
			RQAic	0.086	0.028	0.007	0.022	0.069
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.054	0.012	0.025	0.003	0.060
			QR-LS.Boost	0.065	0.016	0.026	0.003	0.069
			RQ	0.114	0.054	0.058	0.061	0.122
			RQAic	0.015	0.000	0.000	0.000	0.006
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	63.247	16.889	7.982	7.344	29.702
			QR-LS.Boost	60.889	10.586	7.576	7.012	27.203
			RQ	86.369	12.241	9.265	14.957	68.874
			RQAic	78.725	8.130	5.429	10.476	61.584
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	1.208	0.651	0.045	0.038	0.127
			QR-LS.Boost	1.198	0.331	0.047	0.036	0.122
			RQ	0.719	0.104	0.068	0.112	0.453
			RQAic	0.750	0.102	23.193	13.417	3.569
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.952	0.558	0.098	0.192	0.859
			QR-LS.Boost	1.046	0.390	0.091	0.179	0.768
			RQ	0.828	0.139	0.097	0.200	0.798
			RQAic	0.798	0.122	10.155	30.112	7.622
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.170	0.013	0.048	0.176	0.782
			QR-LS.Boost	0.188	0.039	0.045	0.179	0.764
			RQ	0.753	0.111	0.072	0.122	0.608
			RQAic	10.213	6.227	0.615	10.521	4.623
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	1.365	0.485	0.077	0.165	0.105
			QR-LS.Boost	1.338	0.292	0.074	0.164	0.121
			RQ	0.754	0.121	0.074	0.132	0.787
			RQAic	11.140	6.270	4.061	1.830	0.204
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.377	0.016	0.036	0.035	0.126
			QR-LS.Boost	0.362	0.032	0.037	0.035	0.104
			RQ	0.565	0.091	0.095	0.119	0.552
			RQAic	0.283	0.003	0.000	0.033	0.048
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.224	0.022	0.058	0.049	0.216
			QR-LS.Boost	0.233	0.037	0.052	0.046	0.215
			RQ	0.586	0.118	0.075	0.096	0.625
			RQAic	0.028	0.000	0.000	0.003	0.006
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	4.645	5.963	6.949	22.276	323.826
			QR-LS.Boost	6.144	5.481	7.321	19.580	178.979
			RQ	4.549	7.469	13.199	27.644	64.930
			RQAic	3.326	6.077	8.160	21.208	43.844
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.181	0.295	0.024	0.042	0.150
			QR-LS.Boost	0.234	0.148	0.023	0.033	0.338
			RQ	0.031	0.062	0.083	0.162	0.478
			RQAic	0.034	0.064	2.465	2.946	5.764
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.154	0.170	0.313	0.082	1.615
			QR-LS.Boost	0.231	0.113	0.299	0.063	1.172
			RQ	0.051	0.082	0.090	0.266	0.601
			RQAic	0.050	0.077	1.978	1.961	0.954
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.003	0.017	0.020	0.352	1.414
			QR-LS.Boost	0.002	0.028	0.017	0.359	1.070
			RQ	0.031	0.055	0.071	0.211	0.522
			RQAic	1.872	0.735	0.100	3.707	0.636
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.148	0.236	0.085	0.140	1.705
			QR-LS.Boost	0.227	0.152	0.097	0.130	1.198
			RQ	0.038	0.063	0.124	0.198	0.697
			RQAic	1.914	0.736	0.120	0.246	2.975
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.002	0.014	0.012	0.040	0.175
			QR-LS.Boost	0.002	0.029	0.014	0.039	0.294
			RQ	0.038	0.051	0.087	0.189	0.507
			RQAic	0.011	0.008	0.004	0.007	0.252
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.004	0.015	0.015	0.057	0.213
			QR-LS.Boost	0.003	0.033	0.013	0.057	0.299
			RQ	0.035	0.068	0.155	0.156	0.492
			RQAic	0.000	0.000	0.000	0.000	0.068
high-dimensional	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	155.932	74.786	49.982	35.505	103.798
			QR-LS.Boost	126.772	68.996	44.938	39.331	76.931
			RQ	1503.297	1182.880	1175.152	1313.203	5386.729
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	2.453	0.729	0.607	0.532	0.773
			QR-LS.Boost	2.347	0.729	0.593	0.607	0.738
			RQ	0.694	0.665	0.517	0.733	2.195

	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	3.365	0.704	0.730	0.794	0.907
		QR-LS.Boost	3.345	0.844	0.729	0.684	0.828
		RQ	4.483	0.674	0.538	0.827	6.649
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	1.765	0.854	0.595	0.599	0.956
		QR-LS.Boost	1.723	0.810	0.590	0.651	0.930
		RQ	0.693	0.528	0.477	0.703	1.591
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	2.435	0.777	0.682	0.677	0.364
		QR-LS.Boost	2.416	0.840	0.691	0.618	0.357
		RQ	1.145	0.600	0.429	0.795	3.111
tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	799.894	143.693	72.215	98.046	885.070
		QR-LS.Boost	622.084	112.396	64.122	83.792	755.001
		RQ	10187.038	3789.789	2462.623	2857.387	16307.736
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	10.292	1.197	0.834	1.524	9.953
		QR-LS.Boost	8.093	1.149	0.856	1.402	8.968
		RQ	5.175	1.541	1.208	1.515	8.884
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	15.982	1.457	0.997	1.927	1.279
		QR-LS.Boost	13.256	1.644	1.005	1.596	1.248
		RQ	10.602	1.636	1.143	1.580	14.690
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	2.819	1.056	0.843	1.189	3.093
		QR-LS.Boost	2.702	1.023	0.815	1.083	3.060
		RQ	4.867	1.386	0.962	1.370	8.301
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	7.701	1.264	0.882	0.918	0.069
		QR-LS.Boost	7.020	1.358	0.855	0.823	0.107
		RQ	5.623	1.770	1.266	1.935	9.011
gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	65.129	65.384	69.594	316.569	3103.930
		QR-LS.Boost	52.817	54.812	68.403	282.127	2929.207
		RQ	1544.137	1513.149	2096.571	3108.821	10811.629
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.893	0.578	1.198	2.213	16.295
		QR-LS.Boost	0.914	0.606	1.120	1.987	16.273
		RQ	0.645	0.542	0.881	0.896	4.067
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	1.287	0.782	1.069	0.039	5.662
		QR-LS.Boost	1.299	1.014	1.028	0.034	5.859
		RQ	3.312	1.287	0.861	1.300	9.218
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.992	0.707	0.934	1.579	3.349
		QR-LS.Boost	0.979	0.740	0.949	1.653	3.435
		RQ	0.584	0.577	0.715	1.352	4.163
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.860	0.371	0.092	0.182	3.058
		QR-LS.Boost	0.878	0.405	0.099	0.189	3.095
		RQ	1.337	0.764	1.106	1.324	3.340

Table 10: Estimation accuracy measured by the MSE for QR-QR.Boost, QR-LS.Boost, RQ, and RQAic for the contaminated cases of all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distr.	Bias(·)	Method	τ 0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.939	0.281	0.216	0.113	0.249
			QR-LS.Boost	1.018	0.398	0.219	0.103	0.214
			RQ	0.072	-0.037	-0.039	0.062	-0.025
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.247	-0.093	-0.051	-0.017	-0.028
			QR-LS.Boost	-0.267	-0.098	-0.049	-0.019	-0.026
			RQ	-0.012	-0.005	0.012	0.001	-0.004
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.324	-0.008	0.171	0.603	0.954
			QR-LS.Boost	0.440	0.069	0.096	0.383	0.848
			RQ	-0.210	-0.105	-0.040	-0.122	0.208
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.079	0.004	-0.024	-0.085	-0.112
			QR-LS.Boost	-0.089	-0.008	-0.012	-0.059	-0.113
			RQ	0.025	0.017	0.009	0.015	0.026
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.182	0.626	2.774	3.871	5.381
			QR-LS.Boost	0.165	1.409	2.296	3.447	4.934
			RQ	0.013	0.065	-0.068	0.005	0.061
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.034	-0.182	-0.490	-0.628	-0.823
			QR-LS.Boost	-0.029	-0.274	-0.408	-0.592	-0.855
			RQ	-0.005	0.002	0.001	0.009	0.006
heteroskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	1.378	0.613	0.764	0.204	0.358
			QR-LS.Boost	2.012	0.630	0.742	0.294	0.297
			RQ	-0.109	-0.129	0.000	-0.043	-0.068
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.494	-0.236	-0.274	-0.051	-0.057
			QR-LS.Boost	-0.579	-0.235	-0.268	-0.103	-0.048
			RQ	-0.005	0.038	-0.010	0.009	-0.007
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-0.708	-0.148	0.246	0.599	1.741
			QR-LS.Boost	-0.598	-0.205	0.154	0.124	1.408
			RQ	-0.780	0.009	0.043	0.019	-0.012
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.101	0.034	-0.083	-0.042	-0.205
			QR-LS.Boost	0.143	0.050	-0.062	0.021	-0.171
			RQ	0.169	-0.035	-0.022	0.020	0.044
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.207	1.403	5.200	5.208	17.914
			QR-LS.Boost	0.134	1.531	4.787	5.982	20.988
			RQ	0.026	0.158	-0.061	-0.037	-0.195
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.041	-0.514	-1.133	-0.878	-2.675
			QR-LS.Boost	-0.023	-0.534	-1.100	-1.141	-3.232
			RQ	0.001	-0.013	0.011	-0.052	-0.001
multivariate	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.526	0.579	1.182	1.837	3.028
			QR-LS.Boost	0.712	0.580	1.206	1.865	2.954
			RQ	-0.122	0.150	0.364	-0.044	0.347
			RQAic	-0.061	-0.034	0.229	-0.144	0.075
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.359	-0.212	-0.410	-0.536	-0.469
			QR-LS.Boost	-0.402	-0.201	-0.431	-0.583	-0.469
			RQ	-0.009	-0.007	0.009	0.001	-0.016
			RQAic	-0.003	-0.003	0.016	0.015	-0.009
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.284	0.184	0.427	0.614	0.432
			QR-LS.Boost	0.361	0.225	0.416	0.595	0.447
			RQ	0.079	0.021	-0.067	-0.011	-0.091
			RQAic	0.074	0.022	-0.068	-0.007	-0.091
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.400	-0.162	-0.356	-0.511	-0.446
			QR-LS.Boost	-0.425	-0.139	-0.354	-0.550	-0.445
			RQ	-0.051	-0.047	-0.044	-0.003	-0.007
			RQAic	-0.058	-0.045	-0.044	0.003	-0.002
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.367	0.160	0.341	0.540	0.295
			QR-LS.Boost	0.420	0.172	0.338	0.549	0.296
			RQ	0.082	0.029	0.012	0.012	-0.042
			RQAic	0.096	0.030	0.007	0.014	-0.012
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	-0.010	0.004	0.011	-0.007	-0.018
			QR-LS.Boost	-0.017	0.001	0.010	-0.009	-0.018
			RQ	0.044	0.015	0.019	0.011	0.006
			RQAic	-0.044	0.033	0.026	0.020	0.024
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	-0.071	-0.069	-0.033	-0.017	-0.024
			QR-LS.Boost	-0.077	-0.066	-0.037	-0.012	-0.016
			RQ	-0.081	-0.012	-0.023	0.009	0.046
			RQAic	-0.019	-0.008	-0.005	-0.005	0.042
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-1.982	0.339	-0.025	0.069	6.222
			QR-LS.Boost	-1.826	0.227	-0.030	-0.021	6.184
			RQ	-0.460	-0.537	-0.782	0.197	2.508
			RQAic	-0.156	-0.336	-0.491	0.207	1.720

		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-1.065	-0.300	-0.024	-0.160	-0.626
			QR-LS.Boost	-1.026	-0.142	-0.026	-0.169	-0.606
			RQ	-0.058	-0.004	0.031	0.021	-0.123
			RQAic	-0.072	0.000	0.022	0.016	-0.120
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	1.069	0.303	0.051	0.185	0.480
			QR-LS.Boost	1.097	0.228	0.044	0.169	0.522
			RQ	0.219	0.018	-0.024	-0.072	-0.147
			RQAic	0.193	0.008	-0.017	-0.066	0.391
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-1.024	-0.329	-0.028	-0.221	-0.638
			QR-LS.Boost	-0.975	-0.170	-0.023	-0.218	-0.637
			RQ	-0.104	-0.010	0.036	0.026	0.017
			RQAic	-0.449	-0.019	0.034	0.030	-0.555
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.961	0.225	0.017	0.280	-0.071
			QR-LS.Boost	0.994	0.130	0.013	0.275	-0.097
			RQ	0.063	0.023	0.002	-0.017	-0.134
			RQAic	0.426	0.030	-0.014	0.000	0.028
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	-0.069	-0.026	-0.005	0.077	-0.020
			QR-LS.Boost	-0.066	-0.029	-0.004	0.095	-0.022
			RQ	-0.169	-0.014	-0.014	-0.016	-0.023
			RQAic	-0.192	0.035	0.042	-0.031	0.030
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.099	0.004	0.003	0.031	0.121
			QR-LS.Boost	0.115	-0.015	0.004	0.025	0.109
			RQ	-0.007	0.074	0.103	0.002	0.024
			RQAic	-0.041	-0.001	0.011	-0.002	0.016
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-0.255	0.348	1.027	4.372	13.971
			QR-LS.Boost	-0.406	0.249	1.080	4.500	11.709
			RQ	-0.089	0.014	0.387	-0.010	2.789
			RQAic	-0.101	-0.030	0.207	0.055	3.171
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.315	-0.136	-0.254	-0.376	-0.747
			QR-LS.Boost	-0.300	-0.087	-0.285	-0.395	-0.629
			RQ	0.007	0.029	-0.054	0.029	-0.135
			RQAic	0.010	0.025	-0.066	0.036	-0.131
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.226	0.120	0.268	0.204	-0.529
			QR-LS.Boost	0.275	0.112	0.265	0.202	-0.515
			RQ	0.052	0.011	-0.005	-0.062	-0.031
			RQAic	0.035	0.017	-0.026	1.407	-0.010
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.248	-0.132	-0.250	-0.322	-0.531
			QR-LS.Boost	-0.228	-0.090	-0.278	-0.358	-0.386
			RQ	-0.008	-0.054	0.010	0.011	-0.144
			RQAic	-0.006	-0.050	0.007	-1.152	-0.098
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.332	0.123	0.184	-0.189	-0.746
			QR-LS.Boost	0.351	0.108	0.210	-0.234	-0.699
			RQ	0.000	-0.033	0.027	-0.003	-0.148
			RQAic	0.004	-0.030	0.082	-0.354	-0.292
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.002	-0.017	0.025	0.030	0.084
			QR-LS.Boost	-0.003	-0.021	0.033	0.031	0.075
			RQ	0.017	-0.005	-0.025	0.005	-0.054
			RQAic	0.012	0.041	-0.012	0.037	-0.061
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.018	0.002	-0.012	-0.052	-0.058
			QR-LS.Boost	0.033	0.008	-0.017	-0.044	-0.086
			RQ	-0.008	0.044	-0.011	0.014	0.033
			RQAic	0.005	0.000	0.000	0.000	0.054
multivariate2	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-0.662	-1.304	-1.542	-0.954	0.962
			QR-LS.Boost	-0.529	-1.135	-1.386	-1.032	0.971
			RQ	-0.109	-0.010	0.330	-0.330	0.689
			RQAic	-0.237	-0.237	0.350	-0.192	0.710
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.170	-0.377	-0.010	-0.009	-0.082
			QR-LS.Boost	-0.162	-0.358	-0.007	-0.011	-0.083
			RQ	0.035	0.051	-0.009	0.024	-0.013
			RQAic	0.045	0.044	-3.991	-2.858	-1.465
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.099	0.301	0.272	0.622	0.316
			QR-LS.Boost	0.128	0.372	0.251	0.609	0.301
			RQ	0.033	0.038	-0.053	0.022	-0.148
			RQAic	0.032	0.032	2.703	4.363	2.383
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.048	0.007	0.002	-0.507	-0.331
			QR-LS.Boost	-0.064	0.010	-0.002	-0.525	-0.298
			RQ	-0.011	-0.001	0.016	-0.002	0.006
			RQAic	-1.923	-1.806	-0.732	-2.562	-1.516
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.261	0.348	0.236	0.559	0.203
			QR-LS.Boost	0.271	0.381	0.213	0.560	0.184
			RQ	0.023	0.011	-0.083	-0.022	-0.086

			RQAic	1.936	1.781	1.913	1.101	0.366
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.006	0.002	0.017	-0.008	-0.004
			QR-LS.Boost	0.007	0.009	0.022	-0.013	-0.009
			RQ	-0.023	-0.013	-0.010	0.004	0.025
			RQAic	-0.033	0.010	-0.014	0.015	-0.005
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	-0.068	-0.039	-0.060	-0.005	-0.044
			QR-LS.Boost	-0.062	-0.039	-0.075	-0.008	-0.048
			RQ	-0.041	-0.070	0.019	0.054	0.009
			RQAic	-0.024	-0.004	0.000	-0.004	0.017
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-3.233	-2.975	-0.646	-0.464	3.875
			QR-LS.Boost	-2.900	-1.368	-0.663	-0.512	3.851
			RQ	-1.957	-0.440	0.233	0.664	2.837
			RQAic	-2.148	-0.591	-0.073	0.796	2.416
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.624	-0.692	0.002	-0.011	-0.170
			QR-LS.Boost	-0.594	-0.253	-0.010	-0.016	-0.198
			RQ	-0.030	0.010	0.026	0.005	0.088
			RQAic	-0.006	0.013	-4.349	-3.171	-0.607
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.571	0.546	0.068	0.226	0.598
			QR-LS.Boost	0.621	0.279	0.066	0.201	0.559
			RQ	0.265	0.096	0.016	-0.087	-0.155
			RQAic	0.255	0.102	2.903	4.739	1.921
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.131	0.009	0.018	-0.282	-0.687
			QR-LS.Boost	-0.142	0.018	0.024	-0.268	-0.701
			RQ	0.047	-0.077	0.009	-0.021	0.040
			RQAic	-2.367	-2.251	-0.535	-2.796	-1.565
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.585	0.571	0.081	0.245	-0.029
			QR-LS.Boost	0.604	0.225	0.078	0.229	-0.047
			RQ	0.133	0.064	-0.009	-0.006	-0.147
			RQAic	2.487	2.246	2.016	1.113	-0.109
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	-0.043	-0.009	-0.032	0.075	-0.081
			QR-LS.Boost	-0.038	0.003	-0.021	0.080	-0.076
			RQ	0.076	0.027	-0.054	-0.024	-0.206
			RQAic	0.035	0.005	0.000	-0.030	0.017
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.027	0.003	0.016	0.016	0.106
			QR-LS.Boost	0.049	0.017	0.014	0.020	0.110
			RQ	-0.121	-0.041	-0.017	0.013	-0.066
			RQAic	0.005	0.000	0.000	0.000	0.000
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-1.740	-1.562	-1.631	3.263	13.365
			QR-LS.Boost	-2.071	-0.924	-1.633	2.700	8.868
			RQ	-0.245	0.124	0.010	0.446	0.299
			RQAic	-0.185	-0.238	-0.009	0.745	0.666
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.358	-0.404	-0.053	0.009	0.010
			QR-LS.Boost	-0.406	-0.220	-0.056	0.001	-0.026
			RQ	0.007	0.027	-0.007	-0.024	0.078
			RQAic	-0.007	0.034	-1.400	1.052	1.571
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.314	0.265	0.386	0.207	-0.699
			QR-LS.Boost	0.420	0.202	0.355	0.199	-0.563
			RQ	0.013	0.007	-0.061	0.044	-0.283
			RQAic	0.008	0.009	1.167	1.027	-0.523
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.010	0.048	0.010	-0.469	-0.828
			QR-LS.Boost	0.015	0.044	0.007	-0.477	-0.548
			RQ	0.009	-0.005	0.044	-0.058	0.062
			RQAic	-1.109	-0.762	-0.047	-1.715	-0.054
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.343	0.362	0.263	-0.206	-0.911
			QR-LS.Boost	0.428	0.229	0.281	-0.236	-0.702
			RQ	0.044	0.024	-0.021	-0.082	-0.110
			RQAic	1.160	0.793	0.268	-0.431	-1.343
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.006	0.002	0.015	0.059	0.091
			QR-LS.Boost	0.006	0.016	0.015	0.087	0.124
			RQ	0.006	-0.009	0.004	0.080	0.094
			RQAic	0.017	-0.013	0.004	-0.016	0.113
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.002	-0.003	-0.025	-0.045	-0.083
			QR-LS.Boost	0.005	0.001	-0.030	-0.023	-0.107
			RQ	0.009	-0.047	0.025	0.010	0.003
			RQAic	0.003	0.000	0.000	-0.009	0.015
high-dimensional	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-4.565	0.457	0.758	1.912	4.495
			QR-LS.Boost	-4.183	0.279	0.300	1.941	4.883
			RQ	-8.828	-6.170	4.834	4.720	9.374
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-1.576	-0.612	-0.571	-0.681	-0.925
			QR-LS.Boost	-1.584	-0.630	-0.575	-0.705	-0.965
			RQ	0.067	-0.051	0.152	0.073	-0.141

	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	1.832	0.686	0.648	0.796	0.883
		QR-LS.Boost	1.860	0.824	0.641	0.711	0.920
		RQ	1.729	0.580	0.059	-0.528	-1.722
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-1.196	-0.789	-0.611	-0.617	-0.925
		QR-LS.Boost	-1.216	-0.781	-0.605	-0.640	-0.982
		RQ	-0.032	0.076	-0.026	-0.047	-0.075
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	1.525	0.774	0.610	0.653	0.470
		QR-LS.Boost	1.555	0.844	0.603	0.634	0.485
		RQ	0.899	0.299	-0.076	-0.160	-1.014
tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-14.247	-1.907	2.501	3.905	23.218
		QR-LS.Boost	-9.878	-1.382	2.269	3.647	21.769
		RQ	3.187	6.232	-8.189	-0.373	17.293
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-2.803	-0.864	-0.777	-0.947	-2.885
		QR-LS.Boost	-2.536	-0.835	-0.757	-0.920	-2.677
		RQ	-0.054	-0.217	-0.188	0.086	-0.014
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	3.113	0.830	0.758	1.269	1.008
		QR-LS.Boost	2.960	0.938	0.732	1.152	1.023
		RQ	2.407	0.617	-0.144	-0.460	-2.337
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-1.338	-0.745	-0.762	-0.992	-1.578
		QR-LS.Boost	-1.300	-0.744	-0.739	-0.962	-1.563
		RQ	-0.016	-0.210	0.052	-0.172	-0.109
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	2.652	0.742	0.733	0.884	0.071
		QR-LS.Boost	2.497	0.772	0.740	0.807	0.066
		RQ	1.394	0.122	-0.031	-0.419	-0.910
gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.259	0.490	5.218	14.993	48.827
		QR-LS.Boost	0.022	0.762	5.144	14.165	46.886
		RQ	-0.944	-1.752	2.672	0.580	9.598
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.801	-0.537	-0.894	-1.123	-3.459
		QR-LS.Boost	-0.810	-0.559	-0.873	-1.109	-3.409
		RQ	-0.045	-0.037	0.041	-0.104	-0.013
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.993	0.650	0.953	0.118	-1.969
		QR-LS.Boost	1.008	0.776	0.915	0.104	-2.089
		RQ	1.696	0.655	0.136	-0.462	-3.060
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.898	-0.640	-0.918	-1.205	-1.717
		QR-LS.Boost	-0.887	-0.658	-0.908	-1.223	-1.709
		RQ	-0.114	-0.037	0.001	-0.057	0.202
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.831	0.571	0.267	-0.329	-1.633
		QR-LS.Boost	0.842	0.632	0.256	-0.370	-1.661
		RQ	0.864	0.348	0.106	-0.247	-1.402

Table 11: Estimation accuracy measured by the Bias for QR-QR.Boost, QR-LS.Boost, RQ, and RQAic of all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	Error distr.	Bias(·)	Method	τ				
				0.1	0.3	0.5	0.7	0.9
homoskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.849	0.299	0.278	-0.014	0.050
			QR-LS.Boost	0.885	0.374	0.317	-0.036	0.016
			RQ	0.013	-0.077	0.094	-0.127	0.194
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.237	-0.087	-0.051	0.009	0.015
			QR-LS.Boost	-0.249	-0.093	-0.057	0.009	0.013
			RQ	-0.023	0.012	-0.023	0.030	-0.024
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.328	0.070	0.116	0.562	0.995
			QR-LS.Boost	0.406	0.104	0.068	0.348	0.704
			RQ	-0.306	0.027	0.024	0.012	-0.345
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.055	-0.013	-0.025	-0.066	-0.128
			QR-LS.Boost	-0.056	-0.016	-0.016	-0.040	-0.093
			RQ	0.050	0.002	-0.015	0.011	0.049
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.163	0.631	2.730	3.989	5.465
			QR-LS.Boost	0.154	1.323	2.448	3.498	4.981
			RQ	0.133	-0.126	0.071	-0.085	-0.123
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.032	-0.186	-0.484	-0.639	-0.834
			QR-LS.Boost	-0.027	-0.263	-0.434	-0.593	-0.854
			RQ	-0.008	0.028	0.015	0.028	0.029
heteroskedastic	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	1.360	0.561	0.646	-0.152	0.133
			QR-LS.Boost	2.025	0.532	0.586	0.008	0.021
			RQ	-0.357	0.155	-0.089	-0.188	0.174
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.476	-0.244	-0.253	0.039	0.005
			QR-LS.Boost	-0.563	-0.232	-0.235	-0.047	0.014
			RQ	0.056	-0.062	0.014	0.026	-0.042
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-1.000	-0.108	0.188	0.647	2.247
			QR-LS.Boost	-0.848	-0.126	0.141	0.121	1.826
			RQ	-0.315	0.048	-0.001	0.065	0.280
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	0.158	-0.029	-0.018	-0.084	-0.385
			QR-LS.Boost	0.181	-0.021	-0.009	-0.016	-0.349
			RQ	0.068	-0.005	-0.009	0.026	0.041
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.225	1.481	5.226	5.019	17.693
			QR-LS.Boost	0.146	1.605	4.783	5.662	20.978
			RQ	0.015	-0.028	-0.063	-0.025	-0.143
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.046	-0.531	-1.158	-0.855	-2.644
			QR-LS.Boost	-0.031	-0.546	-1.105	-1.128	-3.278
			RQ	0.003	0.024	0.035	0.020	0.042
multivariate	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.898	0.490	1.356	1.283	2.076
			QR-LS.Boost	0.990	0.482	1.407	1.469	1.836
			RQ	-0.371	-0.067	-0.228	-0.698	0.249
			RQAic	-0.506	-0.126	-0.313	-0.594	0.419
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.454	-0.167	-0.404	-0.493	-0.238
			QR-LS.Boost	-0.410	-0.147	-0.423	-0.553	-0.206
			RQ	0.003	0.000	-0.034	0.026	0.031
			RQAic	-0.006	-0.001	-0.022	0.035	0.031
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.285	0.166	0.416	0.645	0.426
			QR-LS.Boost	0.318	0.212	0.401	0.626	0.360
			RQ	0.065	0.012	0.028	0.019	-0.094
			RQAic	0.080	0.015	0.027	0.023	-0.099
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.424	-0.199	-0.395	-0.508	-0.378
			QR-LS.Boost	-0.400	-0.178	-0.392	-0.572	-0.347
			RQ	-0.005	0.009	0.053	0.026	0.006
			RQAic	-0.010	0.008	0.057	0.015	-0.004
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.352	0.179	0.349	0.589	0.503
			QR-LS.Boost	0.361	0.191	0.346	0.591	0.478
			RQ	0.058	0.035	-0.008	0.029	-0.111
			RQAic	0.053	0.032	-0.001	0.038	-0.066
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.011	-0.016	0.011	0.000	-0.009
			QR-LS.Boost	0.008	-0.025	0.004	0.000	-0.008
			RQ	0.008	-0.029	0.017	0.070	0.089
			RQAic	0.024	-0.027	0.002	0.051	0.028
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	-0.092	-0.048	-0.039	-0.007	-0.026
			QR-LS.Boost	-0.094	-0.046	-0.044	-0.006	-0.033
			RQ	-0.012	-0.010	-0.019	-0.005	-0.010
			RQAic	0.006	0.000	-0.006	-0.016	-0.006
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-1.831	-0.776	0.161	0.333	5.028
			QR-LS.Boost	-1.341	-0.705	0.045	0.280	4.979
			RQ	-0.572	-0.381	0.276	0.686	1.521
			RQAic	-0.354	-0.410	-0.042	0.700	1.790

		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.993	-0.216	-0.043	-0.125	-0.450
			QR-LS.Boost	-1.080	-0.038	-0.023	-0.141	-0.466
			RQ	-0.162	0.049	0.006	0.009	0.109
			RQAic	-0.153	0.053	0.004	0.007	0.083
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.980	0.320	0.037	0.113	0.315
			QR-LS.Boost	1.112	0.210	0.014	0.096	0.298
			RQ	0.176	0.026	-0.040	-0.046	-0.284
			RQAic	0.124	0.030	-0.032	-0.024	0.611
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-1.203	-0.276	-0.070	-0.113	-0.386
			QR-LS.Boost	-1.180	-0.120	-0.041	-0.112	-0.392
			RQ	-0.158	-0.038	-0.033	-0.003	0.065
			RQAic	-0.526	-0.046	-0.036	0.011	-0.783
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	1.084	0.295	0.046	0.079	-0.188
			QR-LS.Boost	1.130	0.169	0.029	0.074	-0.197
			RQ	0.133	0.052	0.047	-0.071	-0.002
			RQAic	0.419	0.028	0.051	-0.073	-0.039
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	-0.084	-0.028	0.028	0.082	-0.025
			QR-LS.Boost	-0.079	-0.057	0.031	0.108	-0.004
			RQ	0.095	0.008	-0.060	0.007	0.035
			RQAic	0.106	-0.026	0.012	-0.022	-0.022
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.079	0.041	-0.004	0.028	0.096
			QR-LS.Boost	0.057	0.059	-0.001	0.025	0.108
			RQ	-0.130	-0.063	0.010	-0.022	-0.094
			RQAic	-0.046	0.000	0.000	-0.022	0.000
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	0.001	-0.183	0.888	3.811	11.787
			QR-LS.Boost	-0.062	-0.309	0.948	3.685	11.396
			RQ	0.226	-0.087	0.675	0.241	1.322
			RQAic	-0.009	-0.089	0.045	0.558	1.622
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.277	-0.116	-0.311	-0.286	-0.471
			QR-LS.Boost	-0.280	-0.080	-0.333	-0.298	-0.427
			RQ	0.008	-0.001	-0.058	-0.086	-0.110
			RQAic	0.014	0.000	-0.050	-0.084	-0.125
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.160	0.086	0.308	0.237	-0.335
			QR-LS.Boost	0.212	0.101	0.294	0.221	-0.394
			RQ	0.022	0.083	0.021	-0.030	-0.204
			RQAic	0.016	0.069	0.010	1.534	-0.171
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.214	-0.073	-0.173	-0.298	-0.501
			QR-LS.Boost	-0.224	-0.035	-0.184	-0.306	-0.484
			RQ	-0.039	0.029	-0.028	-0.004	-0.012
			RQAic	-0.041	0.031	-0.026	-1.285	0.059
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.261	0.131	0.187	-0.239	-0.682
			QR-LS.Boost	0.300	0.122	0.198	-0.257	-0.708
			RQ	0.031	-0.027	0.038	0.029	-0.046
			RQAic	0.040	-0.027	0.089	-0.286	-0.188
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.000	0.018	0.014	0.024	0.085
			QR-LS.Boost	0.003	0.019	0.014	0.026	0.096
			RQ	-0.023	0.007	-0.076	0.021	0.065
			RQAic	-0.001	-0.001	0.010	0.023	0.176
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.008	-0.001	-0.007	-0.035	-0.111
			QR-LS.Boost	0.013	-0.003	-0.015	-0.033	-0.107
			RQ	-0.013	-0.010	-0.002	0.041	0.117
			RQAic	0.000	0.005	0.000	0.000	0.009
multivariate2	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-0.476	-1.066	-0.532	-0.984	1.435
			QR-LS.Boost	-0.222	-0.880	-0.590	-1.122	1.427
			RQ	-0.551	-0.366	0.323	0.199	0.832
			RQAic	-0.677	-0.265	0.250	0.132	0.801
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.110	-0.393	-0.063	-0.020	-0.112
			QR-LS.Boost	-0.105	-0.378	-0.073	-0.018	-0.107
			RQ	-0.016	0.039	0.003	-0.039	-0.009
			RQAic	-0.004	0.044	-3.910	-2.916	-1.748
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.172	0.254	0.230	0.631	0.192
			QR-LS.Boost	0.185	0.347	0.240	0.617	0.178
			RQ	0.083	0.045	-0.003	-0.032	-0.065
			RQAic	0.089	0.041	2.523	4.394	3.073
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.065	0.015	0.005	-0.506	-0.098
			QR-LS.Boost	-0.068	0.017	0.011	-0.540	-0.091
			RQ	-0.024	-0.001	-0.007	0.034	-0.033
			RQAic	-2.009	-1.891	-0.546	-2.565	-1.958
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.166	0.314	0.184	0.577	0.208
			QR-LS.Boost	0.170	0.342	0.187	0.586	0.197
			RQ	0.101	-0.002	-0.002	-0.049	-0.072

			RQAic	2.103	1.862	1.901	1.034	0.512
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.000	0.001	-0.017	-0.005	-0.060
			QR-LS.Boost	0.003	0.002	-0.014	-0.002	-0.057
			RQ	0.035	-0.021	-0.031	-0.009	-0.005
			RQAic	0.027	0.000	-0.011	-0.003	-0.022
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	-0.070	-0.037	-0.079	-0.008	-0.110
			QR-LS.Boost	-0.088	-0.048	-0.085	-0.009	-0.119
			RQ	0.010	0.015	-0.015	0.022	0.033
			RQAic	0.014	0.000	0.000	0.000	0.006
	tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-2.231	-3.019	-0.647	-0.559	2.491
			QR-LS.Boost	-1.660	-1.670	-0.714	-0.612	2.241
			RQ	-3.316	-0.564	-0.515	0.170	1.739
			RQAic	-3.057	-0.311	-0.536	0.308	1.037
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.592	-0.666	0.041	-0.004	-0.081
			QR-LS.Boost	-0.555	-0.323	0.036	-0.007	-0.080
			RQ	-0.008	0.006	0.015	-0.013	-0.068
			RQAic	0.007	-0.002	-4.614	-3.488	-0.655
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.459	0.562	0.088	0.231	0.660
			QR-LS.Boost	0.517	0.377	0.086	0.197	0.596
			RQ	0.173	0.059	0.009	-0.077	-0.300
			RQAic	0.155	0.039	3.006	5.194	2.212
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.145	-0.006	-0.031	-0.244	-0.666
			QR-LS.Boost	-0.188	-0.013	-0.016	-0.239	-0.680
			RQ	0.055	0.010	-0.011	-0.019	0.082
			RQAic	-2.334	-2.347	-0.290	-3.072	-1.824
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.695	0.536	0.067	0.236	-0.041
			QR-LS.Boost	0.685	0.310	0.065	0.226	-0.055
			RQ	0.148	0.025	0.041	-0.003	-0.120
			RQAic	2.634	2.383	2.011	1.255	0.116
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	-0.150	-0.029	0.037	0.062	-0.026
			QR-LS.Boost	-0.161	-0.040	0.036	0.068	-0.029
			RQ	0.094	0.022	0.028	0.042	0.030
			RQAic	0.002	0.006	0.000	0.027	0.031
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.009	0.044	-0.032	0.035	0.159
			QR-LS.Boost	0.004	0.052	-0.030	0.043	0.179
			RQ	0.044	0.000	0.022	0.025	0.086
			RQAic	-0.009	0.000	0.000	0.006	0.008
	gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-1.520	-1.371	-1.730	3.008	14.340
			QR-LS.Boost	-1.756	-0.747	-1.865	2.978	7.939
			RQ	0.076	0.602	0.329	0.769	1.494
			RQAic	0.114	0.441	-0.152	0.709	0.552
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.374	-0.422	-0.046	0.011	0.008
			QR-LS.Boost	-0.434	-0.227	-0.044	-0.001	0.034
			RQ	-0.018	-0.034	0.016	-0.001	0.014
			RQAic	-0.015	-0.038	-1.426	1.163	1.913
		$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.259	0.265	0.390	0.199	-0.635
			QR-LS.Boost	0.362	0.181	0.376	0.178	-0.366
			RQ	0.005	-0.007	0.011	-0.032	-0.108
			RQAic	0.001	-0.002	1.243	1.006	-0.604
		$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	0.009	0.045	-0.015	-0.435	-0.834
			QR-LS.Boost	0.008	0.054	-0.014	-0.468	-0.453
			RQ	0.003	-0.038	0.000	0.003	-0.044
			RQAic	-1.231	-0.763	-0.080	-1.815	-0.080
		$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.342	0.419	0.265	-0.255	-1.013
			QR-LS.Boost	0.435	0.290	0.293	-0.275	-0.662
			RQ	0.006	0.016	0.001	-0.001	-0.026
			RQAic	1.254	0.762	0.330	-0.432	-1.311
		$\widehat{\beta}_{\tau 5}$	QR-QR.Boost	0.001	-0.019	0.006	0.029	-0.025
			QR-LS.Boost	0.000	-0.022	0.002	0.031	-0.019
			RQ	0.015	-0.016	-0.004	-0.046	-0.094
			RQAic	-0.019	-0.011	0.006	-0.012	-0.007
		$\widehat{\beta}_{\tau 6}$	QR-QR.Boost	0.014	-0.009	-0.014	-0.016	-0.100
			QR-LS.Boost	0.015	-0.009	-0.010	-0.021	-0.169
			RQ	-0.015	-0.013	-0.065	-0.023	-0.009
			RQAic	0.000	0.000	0.000	0.000	-0.005
high-dimensional	norm	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-5.031	0.743	1.048	1.194	3.952
			QR-LS.Boost	-3.910	0.802	1.193	1.337	4.083
			RQ	-4.776	-7.640	-0.179	3.238	13.497
		$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-1.326	-0.724	-0.613	-0.608	-0.394
			QR-LS.Boost	-1.322	-0.726	-0.616	-0.654	-0.482
			RQ	-0.266	0.067	-0.051	0.209	0.652

	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	1.597	0.639	0.732	0.765	0.698
		QR-LS.Boost	1.611	0.753	0.718	0.711	0.681
		RQ	1.885	0.440	-0.011	-0.527	-2.348
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-1.116	-0.739	-0.663	-0.626	-0.694
		QR-LS.Boost	-1.108	-0.723	-0.663	-0.664	-0.715
		RQ	-0.061	-0.030	0.014	0.103	0.156
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	1.333	0.685	0.688	0.731	0.423
		QR-LS.Boost	1.344	0.723	0.686	0.697	0.448
		RQ	0.720	0.217	-0.038	-0.345	-1.139
tdist	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-7.314	-1.731	1.881	4.399	24.207
		QR-LS.Boost	-5.137	-1.106	1.390	4.090	22.270
		RQ	-21.505	-10.278	6.453	-1.371	14.751
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-2.575	-0.843	-0.760	-1.044	-2.677
		QR-LS.Boost	-2.277	-0.835	-0.778	-0.996	-2.575
		RQ	0.057	0.137	-0.100	-0.033	-0.108
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	3.505	0.999	0.789	1.174	1.013
		QR-LS.Boost	3.321	1.100	0.771	1.019	1.035
		RQ	2.169	0.517	-0.022	-0.424	-2.525
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-1.498	-0.850	-0.748	-0.904	-1.685
		QR-LS.Boost	-1.460	-0.822	-0.733	-0.861	-1.665
		RQ	-0.090	0.271	0.030	0.065	0.156
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	2.502	0.919	0.742	0.833	0.121
		QR-LS.Boost	2.380	0.964	0.742	0.767	0.118
		RQ	1.322	0.286	-0.147	-0.307	-1.380
gamma	$\widehat{\beta}_{\tau 0}$	QR-QR.Boost	-0.293	1.006	5.256	14.879	50.386
		QR-LS.Boost	0.109	1.247	5.119	13.937	50.422
		RQ	-4.609	1.386	-5.313	8.291	17.085
	$\widehat{\beta}_{\tau 1}$	QR-QR.Boost	-0.779	-0.602	-0.970	-1.220	-3.472
		QR-LS.Boost	-0.794	-0.627	-0.939	-1.161	-3.569
		RQ	0.178	0.097	0.092	-0.011	0.504
	$\widehat{\beta}_{\tau 2}$	QR-QR.Boost	0.960	0.660	0.899	0.118	-2.235
		QR-LS.Boost	0.960	0.808	0.872	0.107	-2.297
		RQ	1.580	0.820	0.192	-0.459	-2.429
	$\widehat{\beta}_{\tau 3}$	QR-QR.Boost	-0.868	-0.725	-0.795	-1.116	-1.767
		QR-LS.Boost	-0.877	-0.740	-0.804	-1.137	-1.818
		RQ	-0.053	-0.098	0.154	-0.194	0.294
	$\widehat{\beta}_{\tau 4}$	QR-QR.Boost	0.834	0.490	0.279	-0.381	-1.558
		QR-LS.Boost	0.854	0.545	0.270	-0.407	-1.586
		RQ	0.785	0.352	0.168	-0.325	-0.637

Table 12: Estimation accuracy measured by the Bias for QR-QR.Boost, QR-LS.Boost, RQ, and RQAic for the contaminated cases of all parameter setups and each error distribution (except mixed) for each τ. Blue values indicate the superior result in the respective category.

Parameter setup	τ	Error distr.	$\beta_{\tau 0}$	True population parameters $\beta_{\tau 1}$	$\beta_{\tau 2}$	$\beta_{\tau 3}$	$\beta_{\tau 4}$	$\beta_{\tau 5}$	$\beta_{\tau 6}$
multivariate	0.1	norm	3.718	8	-7.563	2	-3.282	0	0
		tdist	3.114	8	-8.771	2	-3.886	0	0
		gamma	5.532	8	-3.936	2	-1.468	0	0
	0.3	norm	4.476	8	-6.049	2	-2.524	0	0
		tdist	4.383	8	-6.234	2	-2.617	0	0
		gamma	6.097	8	-2.805	2	-0.903	0	0
	0.5	norm	5.000	8	-5.000	2	-2.000	0	0
		tdist	5.000	8	-5.000	2	-2.000	0	0
		gamma	6.678	8	-1.643	2	-0.322	0	0
	0.7	norm	5.524	8	-3.951	2	-1.476	0	0
		tdist	5.617	8	-3.766	2	-1.383	0	0
		gamma	7.439	8	-0.122	2	0.439	0	0
	0.9	norm	6.282	8	-2.437	2	-0.718	0	0
		tdist	6.886	8	-1.229	2	-0.114	0	0
		gamma	8.890	8	2.779	2	1.890	0	0
multivariate2	0.1	norm	3.718	8	-7.563	0	-3.282	0	0
		tdist	3.114	8	-8.771	0	-3.886	0	0
		gamma	5.532	8	-3.936	0	-1.468	0	0
	0.3	norm	4.476	8	-6.049	0	-2.524	0	0
		tdist	4.383	8	-6.234	0	-2.617	0	0
		gamma	6.097	8	-2.805	0	-0.903	0	0
	0.5	norm	5.000	0	-5.000	0	-2.000	0	0
		tdist	5.000	0	-5.000	0	-2.000	0	0
		gamma	6.678	0	-1.643	0	-0.322	0	0
	0.7	norm	5.524	0	-3.951	2	-1.476	0	0
		tdist	5.617	0	-3.766	2	-1.383	0	0
		gamma	7.439	0	-0.122	2	0.439	0	0
	0.9	norm	6.282	0	-2.437	2	-0.718	0	0
		tdist	6.886	0	-1.229	2	-0.114	0	0
		gamma	8.890	0	2.779	2	1.890	0	0

Table 13: True population parameters for the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates					
				x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	QR-QR.Boost	0.364	0.348	0.094	0.155	0.021	0.019
			QR-LS.Boost	0.328	0.333	0.115	0.187	0.017	0.021
		tdist	QR-QR.Boost	0.324	0.370	0.068	0.163	0.035	0.040
			QR-LS.Boost	0.354	0.354	0.070	0.181	0.020	0.021
		gamma	QR-QR.Boost	0.480	0.275	0.134	0.084	0.014	0.012
			QR-LS.Boost	0.420	0.278	0.158	0.106	0.016	0.021
	0.3	norm	QR-QR.Boost	0.416	0.319	0.105	0.131	0.014	0.014
			QR-LS.Boost	0.321	0.288	0.164	0.177	0.021	0.030
		tdist	QR-QR.Boost	0.415	0.332	0.095	0.140	0.009	0.009
			QR-LS.Boost	0.294	0.267	0.159	0.180	0.045	0.055
		gamma	QR-QR.Boost	0.558	0.210	0.147	0.059	0.012	0.013
			QR-LS.Boost	0.329	0.238	0.208	0.122	0.051	0.053
	0.5	norm	QR-QR.Boost	0.476	0.296	0.111	0.112	0.002	0.003
			QR-LS.Boost	0.363	0.283	0.166	0.162	0.009	0.017
		tdist	QR-QR.Boost	0.448	0.287	0.120	0.121	0.011	0.013
			QR-LS.Boost	0.294	0.250	0.176	0.163	0.060	0.058
		gamma	QR-QR.Boost	0.668	0.131	0.166	0.016	0.011	0.008
			QR-LS.Boost	0.463	0.215	0.232	0.035	0.030	0.025
	0.7	norm	QR-QR.Boost	0.545	0.262	0.117	0.073	0.001	0.001
			QR-LS.Boost	0.438	0.279	0.161	0.117	0.002	0.003
		tdist	QR-QR.Boost	0.521	0.253	0.126	0.079	0.010	0.011
			QR-LS.Boost	0.349	0.264	0.186	0.134	0.034	0.033
		gamma	QR-QR.Boost	0.746	0.021	0.179	0.028	0.014	0.012
			QR-LS.Boost	0.573	0.028	0.270	0.055	0.038	0.037
	0.9	norm	QR-QR.Boost	0.570	0.189	0.150	0.053	0.017	0.021
			QR-LS.Boost	0.496	0.232	0.182	0.062	0.015	0.014
		tdist	QR-QR.Boost	0.589	0.095	0.140	0.059	0.058	0.059
			QR-LS.Boost	0.560	0.108	0.188	0.055	0.041	0.048
		gamma	QR-QR.Boost	0.484	0.179	0.122	0.123	0.041	0.052
			QR-LS.Boost	0.435	0.202	0.148	0.114	0.057	0.045
multivariate2	0.1	norm	QR-QR.Boost	0.390	0.372	0.025	0.164	0.026	0.023
			QR-LS.Boost	0.345	0.340	0.034	0.200	0.040	0.042
		tdist	QR-QR.Boost	0.338	0.377	0.040	0.172	0.035	0.039
			QR-LS.Boost	0.340	0.343	0.035	0.200	0.039	0.043
		gamma	QR-QR.Boost	0.527	0.339	0.008	0.104	0.010	0.012
			QR-LS.Boost	0.533	0.324	0.005	0.119	0.012	0.007
	0.3	norm	QR-QR.Boost	0.476	0.373	0.003	0.139	0.003	0.005
			QR-LS.Boost	0.414	0.344	0.018	0.200	0.012	0.012
		tdist	QR-QR.Boost	0.478	0.376	0.004	0.138	0.002	0.002
			QR-LS.Boost	0.354	0.307	0.041	0.215	0.040	0.042
		gamma	QR-QR.Boost	0.683	0.245	0.010	0.052	0.007	0.004
			QR-LS.Boost	0.474	0.298	0.034	0.137	0.032	0.024
	0.5	norm	QR-QR.Boost	0.009	0.688	0.009	0.269	0.013	0.011
			QR-LS.Boost	0.027	0.515	0.038	0.333	0.037	0.049
		tdist	QR-QR.Boost	0.027	0.642	0.022	0.263	0.024	0.022
			QR-LS.Boost	0.086	0.416	0.072	0.269	0.076	0.080
		gamma	QR-QR.Boost	0.042	0.793	0.028	0.069	0.028	0.041
			QR-LS.Boost	0.051	0.735	0.039	0.073	0.041	0.061
	0.7	norm	QR-QR.Boost	0.002	0.564	0.263	0.158	0.003	0.010
			QR-LS.Boost	0.005	0.481	0.297	0.204	0.007	0.006
		tdist	QR-QR.Boost	0.021	0.518	0.257	0.170	0.019	0.015
			QR-LS.Boost	0.041	0.397	0.273	0.210	0.041	0.038
		gamma	QR-QR.Boost	0.039	0.045	0.720	0.100	0.040	0.055
			QR-LS.Boost	0.051	0.061	0.630	0.118	0.067	0.072
	0.9	norm	QR-QR.Boost	0.068	0.373	0.313	0.139	0.057	0.050
			QR-LS.Boost	0.060	0.394	0.313	0.155	0.038	0.040
		tdist	QR-QR.Boost	0.115	0.180	0.383	0.114	0.097	0.110
			QR-LS.Boost	0.085	0.229	0.475	0.083	0.069	0.059
		gamma	QR-QR.Boost	0.065	0.356	0.192	0.235	0.076	0.076
			QR-LS.Boost	0.069	0.338	0.245	0.213	0.071	0.064

Table 14: MPI for QR-QR.Boost and QR-LS.Boost for the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	QR-QR.Boost	0.354	0.372	0.093	0.141	0.021	0.020
			QR-LS.Boost	0.330	0.333	0.117	0.179	0.019	0.021
		tdist	QR-QR.Boost	0.328	0.356	0.069	0.160	0.046	0.040
			QR-LS.Boost	0.360	0.364	0.061	0.178	0.020	0.017
		gamma	QR-QR.Boost	0.464	0.264	0.144	0.092	0.021	0.015
			QR-LS.Boost	0.424	0.276	0.163	0.108	0.015	0.015
	0.3	norm	QR-QR.Boost	0.422	0.326	0.101	0.140	0.006	0.006
			QR-LS.Boost	0.311	0.292	0.164	0.175	0.031	0.026
		tdist	QR-QR.Boost	0.423	0.329	0.101	0.134	0.007	0.005
			QR-LS.Boost	0.294	0.262	0.171	0.180	0.041	0.051
		gamma	QR-QR.Boost	0.554	0.217	0.143	0.059	0.015	0.012
			QR-LS.Boost	0.329	0.225	0.202	0.128	0.065	0.051
	0.5	norm	QR-QR.Boost	0.477	0.297	0.109	0.113	0.002	0.003
			QR-LS.Boost	0.363	0.281	0.170	0.157	0.012	0.017
		tdist	QR-QR.Boost	0.450	0.290	0.118	0.119	0.011	0.012
			QR-LS.Boost	0.255	0.250	0.166	0.176	0.075	0.077
		gamma	QR-QR.Boost	0.665	0.127	0.172	0.017	0.011	0.009
			QR-LS.Boost	0.435	0.214	0.241	0.049	0.028	0.033
	0.7	norm	QR-QR.Boost	0.553	0.257	0.118	0.070	0.000	0.001
			QR-LS.Boost	0.444	0.283	0.160	0.111	0.001	0.001
		tdist	QR-QR.Boost	0.480	0.236	0.130	0.094	0.029	0.030
			QR-LS.Boost	0.324	0.263	0.172	0.152	0.042	0.048
		gamma	QR-QR.Boost	0.761	0.017	0.169	0.029	0.013	0.011
			QR-LS.Boost	0.563	0.051	0.263	0.050	0.033	0.040
	0.9	norm	QR-QR.Boost	0.587	0.193	0.166	0.029	0.015	0.010
			QR-LS.Boost	0.544	0.228	0.179	0.036	0.007	0.006
		tdist	QR-QR.Boost	0.545	0.109	0.159	0.067	0.059	0.061
			QR-LS.Boost	0.508	0.137	0.198	0.058	0.045	0.054
		gamma	QR-QR.Boost	0.488	0.177	0.120	0.105	0.050	0.061
			QR-LS.Boost	0.450	0.200	0.138	0.109	0.043	0.060
multivariate2	0.1	norm	QR-QR.Boost	0.406	0.370	0.024	0.154	0.023	0.023
			QR-LS.Boost	0.353	0.336	0.029	0.210	0.032	0.040
		tdist	QR-QR.Boost	0.311	0.383	0.043	0.178	0.042	0.043
			QR-LS.Boost	0.338	0.339	0.038	0.192	0.048	0.045
		gamma	QR-QR.Boost	0.524	0.353	0.007	0.101	0.008	0.007
			QR-LS.Boost	0.541	0.329	0.003	0.119	0.003	0.004
	0.3	norm	QR-QR.Boost	0.475	0.372	0.003	0.142	0.003	0.004
			QR-LS.Boost	0.415	0.347	0.009	0.203	0.011	0.016
		tdist	QR-QR.Boost	0.469	0.380	0.006	0.138	0.003	0.004
			QR-LS.Boost	0.381	0.326	0.037	0.192	0.030	0.034
		gamma	QR-QR.Boost	0.682	0.249	0.006	0.052	0.005	0.006
			QR-LS.Boost	0.456	0.297	0.039	0.125	0.044	0.039
	0.5	norm	QR-QR.Boost	0.016	0.665	0.013	0.270	0.019	0.017
			QR-LS.Boost	0.050	0.481	0.049	0.318	0.046	0.055
		tdist	QR-QR.Boost	0.026	0.634	0.029	0.265	0.020	0.025
			QR-LS.Boost	0.087	0.412	0.073	0.272	0.074	0.083
		gamma	QR-QR.Boost	0.041	0.791	0.047	0.063	0.029	0.028
			QR-LS.Boost	0.054	0.742	0.061	0.059	0.041	0.044
	0.7	norm	QR-QR.Boost	0.005	0.566	0.266	0.160	0.003	0.002
			QR-LS.Boost	0.006	0.483	0.293	0.206	0.007	0.004
		tdist	QR-QR.Boost	0.018	0.499	0.259	0.174	0.022	0.029
			QR-LS.Boost	0.039	0.401	0.269	0.212	0.036	0.043
		gamma	QR-QR.Boost	0.037	0.037	0.737	0.084	0.049	0.056
			QR-LS.Boost	0.050	0.055	0.670	0.097	0.052	0.076
	0.9	norm	QR-QR.Boost	0.058	0.349	0.375	0.114	0.050	0.054
			QR-LS.Boost	0.043	0.398	0.342	0.130	0.039	0.048
		tdist	QR-QR.Boost	0.104	0.173	0.424	0.095	0.100	0.105
			QR-LS.Boost	0.060	0.208	0.517	0.080	0.074	0.060
		gamma	QR-QR.Boost	0.084	0.366	0.203	0.200	0.067	0.080
			QR-LS.Boost	0.079	0.319	0.228	0.199	0.077	0.097

Table 15: MPI for QR-QR.Boost and QR-LS.Boost for the contaminated cases of the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates					
				x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	QR-QR.Boost	0.005	0.028	0.573	0.410	0.804	0.800
			QR-LS.Boost	0.002	0.022	0.438	0.278	0.739	0.688
		tdist	QR-QR.Boost	0.054	0.020	0.617	0.347	0.668	0.636
			QR-LS.Boost	0.031	0.011	0.558	0.293	0.684	0.650
		gamma	QR-QR.Boost	0.000	0.211	0.464	0.607	0.817	0.826
			QR-LS.Boost	0.000	0.161	0.353	0.482	0.724	0.757
	0.3	norm	QR-QR.Boost	0.001	0.089	0.547	0.455	0.887	0.876
			QR-LS.Boost	0.000	0.041	0.302	0.245	0.733	0.719
		tdist	QR-QR.Boost	0.001	0.081	0.577	0.461	0.888	0.902
			QR-LS.Boost	0.000	0.027	0.251	0.190	0.576	0.585
		gamma	QR-QR.Boost	0.001	0.335	0.467	0.691	0.873	0.851
			QR-LS.Boost	0.000	0.110	0.169	0.331	0.620	0.577
	0.5	norm	QR-QR.Boost	0.001	0.179	0.573	0.570	0.954	0.938
			QR-LS.Boost	0.000	0.083	0.351	0.345	0.741	0.657
		tdist	QR-QR.Boost	0.001	0.159	0.510	0.497	0.915	0.911
			QR-LS.Boost	0.000	0.042	0.182	0.177	0.570	0.550
		gamma	QR-QR.Boost	0.001	0.585	0.508	0.883	0.891	0.902
			QR-LS.Boost	0.000	0.294	0.235	0.701	0.683	0.672
	0.7	norm	QR-QR.Boost	0.001	0.297	0.556	0.701	0.837	0.913
			QR-LS.Boost	0.000	0.157	0.396	0.523	0.823	0.834
		tdist	QR-QR.Boost	0.001	0.300	0.487	0.637	0.866	0.877
			QR-LS.Boost	0.000	0.105	0.234	0.350	0.632	0.647
		gamma	QR-QR.Boost	0.001	0.794	0.542	0.792	0.802	0.831
			QR-LS.Boost	0.000	0.616	0.301	0.581	0.625	0.657
	0.9	norm	QR-QR.Boost	0.000	0.379	0.459	0.706	0.814	0.823
			QR-LS.Boost	0.000	0.324	0.362	0.644	0.734	0.769
		tdist	QR-QR.Boost	0.000	0.566	0.468	0.637	0.632	0.618
			QR-LS.Boost	0.000	0.498	0.393	0.522	0.577	0.527
		gamma	QR-QR.Boost	0.000	0.283	0.441	0.463	0.578	0.657
			QR-LS.Boost	0.000	0.192	0.284	0.350	0.492	0.522
multivariate2	0.1	norm	QR-QR.Boost	0.011	0.035	0.801	0.438	0.823	0.837
			QR-LS.Boost	0.005	0.019	0.693	0.278	0.689	0.707
		tdist	QR-QR.Boost	0.056	0.011	0.693	0.347	0.701	0.709
			QR-LS.Boost	0.027	0.007	0.655	0.250	0.688	0.630
		gamma	QR-QR.Boost	0.000	0.222	0.861	0.642	0.884	0.866
			QR-LS.Boost	0.000	0.210	0.733	0.595	0.746	0.773
	0.3	norm	QR-QR.Boost	0.001	0.103	0.861	0.533	0.888	0.890
			QR-LS.Boost	0.000	0.061	0.743	0.350	0.729	0.721
		tdist	QR-QR.Boost	0.001	0.090	0.869	0.533	0.930	0.900
			QR-LS.Boost	0.000	0.037	0.602	0.246	0.630	0.605
		gamma	QR-QR.Boost	0.001	0.414	0.841	0.805	0.858	0.862
			QR-LS.Boost	0.000	0.181	0.607	0.467	0.673	0.653
	0.5	norm	QR-QR.Boost	0.901	0.002	0.903	0.422	0.880	0.886
			QR-LS.Boost	0.588	0.000	0.581	0.184	0.593	0.562
		tdist	QR-QR.Boost	0.858	0.002	0.857	0.364	0.863	0.852
			QR-LS.Boost	0.481	0.000	0.486	0.108	0.483	0.479
		gamma	QR-QR.Boost	0.679	0.008	0.716	0.674	0.699	0.585
			QR-LS.Boost	0.431	0.001	0.528	0.633	0.554	0.476
	0.7	norm	QR-QR.Boost	0.787	0.002	0.277	0.488	0.726	0.869
			QR-LS.Boost	0.600	0.001	0.183	0.335	0.668	0.734
		tdist	QR-QR.Boost	0.793	0.002	0.220	0.406	0.803	0.789
			QR-LS.Boost	0.597	0.000	0.103	0.209	0.604	0.596
		gamma	QR-QR.Boost	0.637	0.586	0.006	0.490	0.619	0.592
			QR-LS.Boost	0.468	0.447	0.001	0.340	0.403	0.390
	0.9	norm	QR-QR.Boost	0.602	0.017	0.042	0.415	0.608	0.636
			QR-LS.Boost	0.538	0.005	0.041	0.330	0.592	0.545
		tdist	QR-QR.Boost	0.355	0.247	0.043	0.348	0.409	0.362
			QR-LS.Boost	0.365	0.253	0.037	0.379	0.489	0.441
		gamma	QR-QR.Boost	0.439	0.032	0.183	0.224	0.510	0.466
			QR-LS.Boost	0.439	0.035	0.123	0.168	0.424	0.387

Table 16: MFI for QR-QR.Boost and QR-LS.Boost for the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	QR-QR.Boost	0.005	0.023	0.569	0.407	0.802	0.797
			QR-LS.Boost	0.005	0.021	0.430	0.280	0.735	0.705
		tdist	QR-QR.Boost	0.051	0.017	0.602	0.364	0.651	0.636
			QR-LS.Boost	0.032	0.013	0.583	0.310	0.656	0.688
		gamma	QR-QR.Boost	0.000	0.198	0.469	0.583	0.790	0.814
			QR-LS.Boost	0.000	0.155	0.363	0.476	0.768	0.766
	0.3	norm	QR-QR.Boost	0.001	0.096	0.586	0.466	0.905	0.873
			QR-LS.Boost	0.000	0.042	0.319	0.245	0.711	0.696
		tdist	QR-QR.Boost	0.001	0.078	0.584	0.463	0.868	0.895
			QR-LS.Boost	0.000	0.029	0.246	0.187	0.597	0.641
		gamma	QR-QR.Boost	0.001	0.336	0.465	0.703	0.878	0.888
			QR-LS.Boost	0.000	0.110	0.168	0.338	0.545	0.597
	0.5	norm	QR-QR.Boost	0.001	0.175	0.574	0.557	0.946	0.930
			QR-LS.Boost	0.000	0.079	0.344	0.330	0.698	0.660
		tdist	QR-QR.Boost	0.001	0.157	0.523	0.500	0.918	0.918
			QR-LS.Boost	0.000	0.037	0.160	0.148	0.507	0.511
		gamma	QR-QR.Boost	0.001	0.604	0.494	0.867	0.893	0.903
			QR-LS.Boost	0.000	0.292	0.210	0.642	0.653	0.658
	0.7	norm	QR-QR.Boost	0.001	0.320	0.560	0.701	0.862	0.819
			QR-LS.Boost	0.000	0.173	0.399	0.542	0.929	0.848
		tdist	QR-QR.Boost	0.001	0.268	0.446	0.555	0.836	0.804
			QR-LS.Boost	0.000	0.088	0.201	0.273	0.593	0.581
		gamma	QR-QR.Boost	0.001	0.839	0.562	0.783	0.858	0.844
			QR-LS.Boost	0.000	0.603	0.298	0.536	0.663	0.607
	0.9	norm	QR-QR.Boost	0.000	0.390	0.467	0.800	0.852	0.826
			QR-LS.Boost	0.000	0.347	0.390	0.741	0.818	0.777
		tdist	QR-QR.Boost	0.000	0.519	0.404	0.589	0.620	0.599
			QR-LS.Boost	0.000	0.445	0.326	0.521	0.514	0.530
		gamma	QR-QR.Boost	0.001	0.313	0.415	0.429	0.587	0.595
			QR-LS.Boost	0.000	0.209	0.276	0.337	0.487	0.472
multivariate2	0.1	norm	QR-QR.Boost	0.007	0.027	0.825	0.416	0.822	0.818
			QR-LS.Boost	0.003	0.021	0.697	0.257	0.709	0.692
		tdist	QR-QR.Boost	0.063	0.006	0.692	0.344	0.662	0.669
			QR-LS.Boost	0.031	0.005	0.604	0.243	0.622	0.594
		gamma	QR-QR.Boost	0.000	0.219	0.869	0.651	0.875	0.895
			QR-LS.Boost	0.000	0.218	0.846	0.604	0.825	0.866
	0.3	norm	QR-QR.Boost	0.001	0.099	0.933	0.540	0.912	0.932
			QR-LS.Boost	0.000	0.059	0.820	0.354	0.784	0.747
		tdist	QR-QR.Boost	0.002	0.086	0.911	0.544	0.903	0.862
			QR-LS.Boost	0.000	0.040	0.613	0.281	0.635	0.621
		gamma	QR-QR.Boost	0.001	0.409	0.805	0.809	0.793	0.842
			QR-LS.Boost	0.000	0.170	0.623	0.436	0.542	0.562
	0.5	norm	QR-QR.Boost	0.871	0.002	0.881	0.395	0.861	0.867
			QR-LS.Boost	0.577	0.000	0.542	0.162	0.550	0.548
		tdist	QR-QR.Boost	0.853	0.002	0.860	0.353	0.872	0.850
			QR-LS.Boost	0.489	0.000	0.524	0.100	0.520	0.495
		gamma	QR-QR.Boost	0.639	0.008	0.699	0.682	0.735	0.675
			QR-LS.Boost	0.431	0.001	0.510	0.584	0.573	0.506
	0.7	norm	QR-QR.Boost	0.773	0.002	0.265	0.483	0.853	0.749
			QR-LS.Boost	0.590	0.001	0.182	0.349	0.695	0.651
		tdist	QR-QR.Boost	0.770	0.002	0.216	0.391	0.782	0.758
			QR-LS.Boost	0.574	0.000	0.106	0.212	0.612	0.566
		gamma	QR-QR.Boost	0.639	0.666	0.005	0.527	0.621	0.630
			QR-LS.Boost	0.504	0.522	0.001	0.337	0.461	0.432
	0.9	norm	QR-QR.Boost	0.613	0.012	0.038	0.440	0.608	0.607
			QR-LS.Boost	0.548	0.003	0.040	0.362	0.510	0.497
		tdist	QR-QR.Boost	0.377	0.263	0.043	0.393	0.377	0.383
			QR-LS.Boost	0.394	0.241	0.035	0.310	0.437	0.335
		gamma	QR-QR.Boost	0.471	0.054	0.203	0.243	0.499	0.461
			QR-LS.Boost	0.377	0.017	0.132	0.151	0.393	0.375

Table 17: MFI for QR-QR.Boost and QR-LS.Boost for the contaminated cases of the multivariate and multivariate2 setup and all error distributions (except mixed).

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.160	0.200
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.440	0.490
			RQAic	0.000	0.000	0.000	0.000	0.420	0.860
		tdist	QR-QR.Boost	0.000	0.000	0.050	0.020	0.290	0.290
			QR-LS.Boost	0.000	0.000	0.130	0.010	0.610	0.590
			RQAic	0.000	0.000	0.000	0.020	0.490	0.880
		gamma	QR-QR.Boost	0.000	0.000	0.000	0.000	0.390	0.390
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.580	0.550
			RQAic	0.000	0.000	0.000	0.000	0.590	0.920
		mixed	QR-QR.Boost	0.000	0.000	0.000	0.000	0.000	0.020
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.000	0.020
			RQAic	0.000	0.000	0.000	0.040	0.380	0.880
	0.3	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.380	0.440
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.360	0.350
			RQAic	0.000	0.000	0.000	0.000	0.540	0.910
		tdist	QR-QR.Boost	0.000	0.000	0.000	0.000	0.430	0.380
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.220	0.210
			RQAic	0.000	0.000	0.000	0.000	0.750	0.980
		gamma	QR-QR.Boost	0.000	0.000	0.000	0.000	0.230	0.320
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.130	0.200
			RQAic	0.000	0.000	0.000	0.000	0.690	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.000	0.000	0.160	0.180
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.050	0.090
			RQAic	0.000	0.000	0.000	0.000	0.710	0.980
	0.5	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.740	0.720
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.750	0.770
			RQAic	0.000	0.000	0.000	0.000	0.580	0.970
		tdist	QR-QR.Boost	0.000	0.000	0.000	0.000	0.080	0.120
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.080	0.120
			RQAic	0.000	0.000	0.000	0.000	0.840	0.980
		gamma	QR-QR.Boost	0.000	0.000	0.000	0.290	0.460	0.550
			QR-LS.Boost	0.000	0.000	0.000	0.350	0.470	0.600
			RQAic	0.000	0.000	0.000	0.420	0.870	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.000	0.010	0.230	0.250
			QR-LS.Boost	0.000	0.000	0.000	0.010	0.170	0.170
			RQAic	0.000	0.000	0.000	0.030	0.790	0.970
	0.7	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.860	0.900
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.930	0.930
			RQAic	0.000	0.000	0.000	0.000	0.510	0.920
		tdist	QR-QR.Boost	0.000	0.000	0.000	0.000	0.430	0.350
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.430	0.370
			RQAic	0.000	0.000	0.000	0.000	0.720	0.970
		gamma	QR-QR.Boost	0.000	0.510	0.000	0.300	0.440	0.470
			QR-LS.Boost	0.000	0.550	0.000	0.340	0.470	0.470
			RQAic	0.000	0.000	0.220	0.650	0.930	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.020	0.280	0.370	0.420
			QR-LS.Boost	0.000	0.010	0.020	0.340	0.440	0.440
			RQAic	0.000	0.000	0.010	0.410	0.810	0.990
	0.9	norm	QR-QR.Boost	0.000	0.000	0.000	0.080	0.330	0.290
			QR-LS.Boost	0.000	0.000	0.000	0.130	0.690	0.680
			RQAic	0.000	0.000	0.000	0.040	0.430	0.880
		tdist	QR-QR.Boost	0.000	0.060	0.070	0.180	0.220	0.220
			QR-LS.Boost	0.000	0.180	0.050	0.480	0.420	0.450
			RQAic	0.000	0.020	0.120	0.340	0.810	0.970
		gamma	QR-QR.Boost	0.000	0.040	0.030	0.030	0.240	0.160
			QR-LS.Boost	0.000	0.040	0.080	0.080	0.320	0.300
			RQAic	0.000	0.000	0.010	0.060	0.440	0.860
		mixed	QR-QR.Boost	0.000	0.040	0.010	0.040	0.030	0.030
			QR-LS.Boost	0.000	0.070	0.020	0.050	0.090	0.060
			RQAic	0.000	0.000	0.070	0.340	0.720	0.950
multivariate2	0.1	norm	QR-QR.Boost	0.000	0.000	0.130	0.000	0.120	0.090
			QR-LS.Boost	0.000	0.000	0.280	0.000	0.260	0.250
			RQAic	0.000	0.000	0.000	0.180	0.600	0.930
		tdist	QR-QR.Boost	0.000	0.000	0.130	0.000	0.140	0.150
			QR-LS.Boost	0.000	0.000	0.310	0.000	0.260	0.310
			RQAic	0.000	0.000	0.000	0.280	0.630	0.940
		gamma	QR-QR.Boost	0.000	0.000	0.520	0.000	0.490	0.400
			QR-LS.Boost	0.000	0.000	0.810	0.000	0.780	0.800
			RQAic	0.000	0.000	0.000	0.450	0.850	0.950
		mixed	QR-QR.Boost	0.000	0.000	0.030	0.000	0.030	0.030
			QR-LS.Boost	0.000	0.000	0.060	0.000	0.050	0.040

		RQAic	0.000	0.000	0.000	0.290	0.730	0.960
0.3	norm	QR-QR.Boost	0.000	0.000	0.660	0.000	0.710	0.720
		QR-LS.Boost	0.000	0.000	0.660	0.000	0.680	0.700
		RQAic	0.000	0.000	0.000	0.360	0.850	0.990
	tdist	QR-QR.Boost	0.000	0.000	0.730	0.000	0.750	0.790
		QR-LS.Boost	0.000	0.000	0.280	0.000	0.270	0.320
		RQAic	0.000	0.000	0.000	0.640	0.970	1.000
	gamma	QR-QR.Boost	0.000	0.000	0.700	0.020	0.650	0.680
		QR-LS.Boost	0.000	0.000	0.500	0.000	0.430	0.460
		RQAic	0.000	0.000	0.010	0.600	0.950	1.000
	mixed	QR-QR.Boost	0.000	0.000	0.200	0.000	0.180	0.180
		QR-LS.Boost	0.000	0.000	0.080	0.000	0.060	0.050
		RQAic	0.000	0.000	0.000	0.660	0.940	1.000
0.5	norm	QR-QR.Boost	0.570	0.000	0.560	0.000	0.530	0.540
		QR-LS.Boost	0.500	0.000	0.510	0.000	0.440	0.520
		RQAic	0.000	0.000	0.460	0.770	0.940	1.000
	tdist	QR-QR.Boost	0.110	0.000	0.160	0.000	0.150	0.150
		QR-LS.Boost	0.130	0.000	0.140	0.000	0.130	0.120
		RQAic	0.000	0.000	0.610	0.960	1.000	1.000
	gamma	QR-QR.Boost	0.540	0.000	0.570	0.420	0.620	0.620
		QR-LS.Boost	0.590	0.000	0.620	0.320	0.620	0.580
		RQAic	0.000	0.290	0.640	0.860	0.990	1.000
	mixed	QR-QR.Boost	0.000	0.000	0.230	0.020	0.260	0.230
		QR-LS.Boost	0.000	0.000	0.210	0.020	0.300	0.250
		RQAic	0.000	0.000	0.060	0.710	0.970	1.000
0.7	norm	QR-QR.Boost	0.890	0.000	0.000	0.000	0.890	0.870
		QR-LS.Boost	0.930	0.000	0.000	0.000	0.890	0.890
		RQAic	0.000	0.000	0.000	0.440	0.840	0.980
	tdist	QR-QR.Boost	0.430	0.000	0.000	0.000	0.470	0.460
		QR-LS.Boost	0.370	0.000	0.000	0.000	0.410	0.420
		RQAic	0.000	0.000	0.000	0.650	0.930	1.000
	gamma	QR-QR.Boost	0.570	0.530	0.000	0.370	0.530	0.510
		QR-LS.Boost	0.550	0.490	0.000	0.370	0.560	0.510
		RQAic	0.000	0.170	0.570	0.870	0.970	0.990
	mixed	QR-QR.Boost	0.000	0.010	0.010	0.430	0.460	0.490
		QR-LS.Boost	0.000	0.010	0.010	0.430	0.470	0.500
		RQAic	0.000	0.000	0.020	0.500	0.830	0.980
0.9	norm	QR-QR.Boost	0.200	0.000	0.000	0.020	0.200	0.200
		QR-LS.Boost	0.390	0.000	0.000	0.050	0.450	0.470
		RQAic	0.000	0.000	0.030	0.220	0.590	0.950
	tdist	QR-QR.Boost	0.300	0.170	0.040	0.280	0.260	0.350
		QR-LS.Boost	0.490	0.190	0.040	0.470	0.470	0.520
		RQAic	0.000	0.040	0.260	0.550	0.890	1.000
	gamma	QR-QR.Boost	0.340	0.090	0.140	0.110	0.300	0.310
		QR-LS.Boost	0.300	0.070	0.110	0.100	0.290	0.380
		RQAic	0.000	0.000	0.020	0.240	0.590	0.930
	mixed	QR-QR.Boost	0.000	0.020	0.010	0.010	0.010	0.040
		QR-LS.Boost	0.000	0.080	0.020	0.050	0.090	0.090
		RQAic	0.000	0.010	0.120	0.380	0.820	0.940

Table 18: PER for QR-QR.Boost, QR-LS.Boost, and RQAic for the multivariate and multivariate2 setup and all error distributions.

Parameter setup	τ	Error distr.	Method	Covariates x_1	x_2	x_3	x_4	x_5	x_6
multivariate	0.1	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.160	0.260
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.460	0.520
			RQAic	0.000	0.000	0.000	0.000	0.420	0.900
		tdist	QR-QR.Boost	0.000	0.000	0.060	0.020	0.270	0.300
			QR-LS.Boost	0.000	0.000	0.170	0.020	0.640	0.580
			RQAic	0.000	0.000	0.000	0.060	0.570	0.940
		gamma	QR-QR.Boost	0.000	0.000	0.000	0.000	0.260	0.260
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.550	0.560
			RQAic	0.000	0.000	0.000	0.000	0.640	0.920
		mixed	QR-QR.Boost	0.000	0.000	0.010	0.000	0.000	0.000
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.010	0.020
			RQAic	0.000	0.000	0.010	0.030	0.440	0.880
	0.3	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.470	0.470
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.350	0.410
			RQAic	0.000	0.000	0.000	0.000	0.640	0.950
		tdist	QR-QR.Boost	0.000	0.000	0.000	0.000	0.510	0.550
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.210	0.210
			RQAic	0.000	0.000	0.000	0.000	0.730	1.000
		gamma	QR-QR.Boost	0.000	0.000	0.000	0.000	0.220	0.250
			QR-LS.Boost	0.000	0.000	0.000	0.010	0.190	0.170
			RQAic	0.000	0.000	0.000	0.010	0.680	0.960
		mixed	QR-QR.Boost	0.000	0.000	0.000	0.000	0.170	0.210
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.060	0.100
			RQAic	0.000	0.000	0.000	0.000	0.820	0.960
	0.5	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.680	0.730
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.730	0.710
			RQAic	0.000	0.000	0.000	0.000	0.580	0.930
		tdist	QR-QR.Boost	0.000	0.000	0.000	0.000	0.130	0.140
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.020	0.020
			RQAic	0.000	0.000	0.000	0.000	0.880	1.000
		gamma	QR-QR.Boost	0.000	0.000	0.000	0.270	0.450	0.500
			QR-LS.Boost	0.000	0.000	0.000	0.230	0.470	0.530
			RQAic	0.000	0.000	0.000	0.460	0.870	0.980
		mixed	QR-QR.Boost	0.000	0.000	0.000	0.010	0.190	0.210
			QR-LS.Boost	0.000	0.000	0.000	0.030	0.230	0.270
			RQAic	0.000	0.000	0.000	0.050	0.800	0.980
	0.7	norm	QR-QR.Boost	0.000	0.000	0.000	0.000	0.940	0.910
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.930	0.960
			RQAic	0.000	0.000	0.000	0.000	0.660	0.940
		tdist	QR-QR.Boost	0.000	0.000	0.000	0.000	0.230	0.230
			QR-LS.Boost	0.000	0.000	0.000	0.000	0.280	0.260
			RQAic	0.000	0.000	0.000	0.010	0.830	0.970
		gamma	QR-QR.Boost	0.000	0.490	0.000	0.320	0.380	0.530
			QR-LS.Boost	0.000	0.500	0.000	0.370	0.380	0.510
			RQAic	0.000	0.000	0.330	0.730	0.910	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.000	0.360	0.340	0.400
			QR-LS.Boost	0.000	0.000	0.000	0.410	0.380	0.470
			RQAic	0.000	0.000	0.060	0.600	0.930	1.000
	0.9	norm	QR-QR.Boost	0.000	0.000	0.000	0.270	0.630	0.700
			QR-LS.Boost	0.000	0.000	0.000	0.270	0.850	0.880
			RQAic	0.000	0.000	0.000	0.070	0.540	0.910
		tdist	QR-QR.Boost	0.000	0.070	0.020	0.100	0.130	0.170
			QR-LS.Boost	0.000	0.130	0.010	0.310	0.390	0.360
			RQAic	0.000	0.030	0.290	0.610	0.930	1.000
		gamma	QR-QR.Boost	0.000	0.040	0.040	0.110	0.220	0.120
			QR-LS.Boost	0.000	0.050	0.120	0.120	0.350	0.340
			RQAic	0.000	0.000	0.000	0.040	0.590	0.960
		mixed	QR-QR.Boost	0.000	0.030	0.050	0.020	0.030	0.050
			QR-LS.Boost	0.000	0.120	0.050	0.070	0.090	0.100
			RQAic	0.000	0.020	0.300	0.690	0.910	1.000
multivariate2	0.1	norm	QR-QR.Boost	0.000	0.000	0.060	0.000	0.120	0.060
			QR-LS.Boost	0.000	0.000	0.250	0.000	0.210	0.210
			RQAic	0.000	0.000	0.000	0.290	0.700	0.920
		tdist	QR-QR.Boost	0.000	0.000	0.110	0.000	0.150	0.130
			QR-LS.Boost	0.000	0.000	0.360	0.000	0.260	0.350
			RQAic	0.000	0.000	0.000	0.340	0.850	0.980
		gamma	QR-QR.Boost	0.000	0.000	0.590	0.000	0.460	0.490
			QR-LS.Boost	0.000	0.000	0.850	0.000	0.880	0.820
			RQAic	0.000	0.000	0.000	0.610	0.900	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.040	0.010	0.020	0.020
			QR-LS.Boost	0.000	0.000	0.060	0.000	0.020	0.010

			RQAic	0.000	0.000	0.000	0.390	0.800	0.970
	0.3	norm	QR-QR.Boost	0.000	0.000	0.740	0.000	0.690	0.770
			QR-LS.Boost	0.000	0.000	0.680	0.000	0.710	0.710
			RQAic	0.000	0.000	0.000	0.480	0.880	1.000
		tdist	QR-QR.Boost	0.000	0.000	0.770	0.000	0.760	0.760
			QR-LS.Boost	0.000	0.000	0.450	0.000	0.420	0.440
			RQAic	0.000	0.000	0.000	0.710	0.990	1.000
		gamma	QR-QR.Boost	0.000	0.000	0.660	0.020	0.660	0.680
			QR-LS.Boost	0.000	0.000	0.370	0.030	0.430	0.400
			RQAic	0.000	0.000	0.010	0.630	0.960	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.260	0.000	0.120	0.220
			QR-LS.Boost	0.000	0.000	0.060	0.000	0.030	0.070
			RQAic	0.000	0.000	0.000	0.680	0.950	0.990
	0.5	norm	QR-QR.Boost	0.340	0.000	0.390	0.000	0.410	0.390
			QR-LS.Boost	0.360	0.000	0.450	0.000	0.410	0.420
			RQAic	0.000	0.000	0.490	0.870	0.980	1.000
		tdist	QR-QR.Boost	0.110	0.000	0.090	0.000	0.150	0.140
			QR-LS.Boost	0.130	0.000	0.060	0.000	0.110	0.120
			RQAic	0.000	0.000	0.710	0.940	1.000	1.000
		gamma	QR-QR.Boost	0.580	0.000	0.510	0.380	0.570	0.610
			QR-LS.Boost	0.570	0.000	0.530	0.400	0.590	0.610
			RQAic	0.000	0.300	0.740	0.970	0.990	1.000
		mixed	QR-QR.Boost	0.000	0.000	0.220	0.010	0.290	0.250
			QR-LS.Boost	0.000	0.000	0.240	0.010	0.250	0.240
			RQAic	0.000	0.000	0.070	0.730	0.980	1.000
	0.7	norm	QR-QR.Boost	0.850	0.000	0.000	0.000	0.880	0.900
			QR-LS.Boost	0.900	0.000	0.000	0.000	0.900	0.930
			RQAic	0.000	0.000	0.000	0.430	0.910	1.000
		tdist	QR-QR.Boost	0.530	0.000	0.000	0.010	0.520	0.490
			QR-LS.Boost	0.490	0.000	0.000	0.010	0.450	0.450
			RQAic	0.000	0.000	0.010	0.800	0.930	0.990
		gamma	QR-QR.Boost	0.600	0.520	0.000	0.420	0.590	0.480
			QR-LS.Boost	0.570	0.470	0.000	0.480	0.580	0.490
			RQAic	0.000	0.210	0.630	0.890	0.980	1.000
		mixed	QR-QR.Boost	0.000	0.010	0.000	0.400	0.460	0.420
			QR-LS.Boost	0.000	0.020	0.000	0.390	0.430	0.390
			RQAic	0.000	0.000	0.040	0.430	0.920	0.990
	0.9	norm	QR-QR.Boost	0.280	0.000	0.000	0.080	0.270	0.300
			QR-LS.Boost	0.450	0.000	0.000	0.070	0.470	0.500
			RQAic	0.000	0.000	0.050	0.390	0.790	0.970
		tdist	QR-QR.Boost	0.300	0.240	0.000	0.320	0.240	0.380
			QR-LS.Boost	0.490	0.190	0.020	0.500	0.430	0.600
			RQAic	0.010	0.190	0.540	0.870	0.980	0.990
		gamma	QR-QR.Boost	0.360	0.060	0.120	0.120	0.400	0.390
			QR-LS.Boost	0.270	0.070	0.060	0.040	0.240	0.210
			RQAic	0.000	0.010	0.040	0.420	0.810	0.940
		mixed	QR-QR.Boost	0.000	0.030	0.010	0.000	0.030	0.010
			QR-LS.Boost	0.000	0.060	0.020	0.040	0.050	0.090
			RQAic	0.000	0.030	0.190	0.620	0.900	1.000

Table 19: PER for QR-QR.Boost, QR-LS.Boost, and RQAic for the contaminated cases for the multivariate and multivariate2 setup and all error distributions.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9	$\frac{\text{time}}{\text{iteration}}$
homoskedastic	norm	QR-QR.Boost	480	140	80	180	680	0.148
		QR-LS.Boost	500	300	400	900	1700	0.054
	tdist	QR-QR.Boost	650	200	100	170	600	0.145
		QR-LS.Boost	2450	1000	800	800	2100	0.054
	gamma	QR-QR.Boost	690	100	40	20	20	0.169
		QR-LS.Boost	900	200	100	100	100	0.055
	mixed	QR-QR.Boost	1240	380	180	420	1740	0.145
		QR-LS.Boost	2500	1850	2050	1200	3000	0.054
heteroskedastic	norm	QR-QR.Boost	60	240	160	400	2160	0.157
		QR-LS.Boost	100	600	500	1500	4250	0.054
	tdist	QR-QR.Boost	160	340	200	420	2310	0.146
		QR-LS.Boost	300	2750	2650	3050	5850	0.054
	gamma	QR-QR.Boost	1950	420	190	560	1820	0.165
		QR-LS.Boost	2500	500	300	700	1100	0.054
	mixed	QR-QR.Boost	1980	860	480	980	5000	0.151
		QR-LS.Boost	6550	4650	4800	4150	6800	0.054
multivariate	norm	QR-QR.Boost	13000	2900	1200	2100	8300	1.409
		QR-LS.Boost	15750	6625	3250	3250	8125	0.287
	tdist	QR-QR.Boost	12400	3000	1500	2500	7300	1.303
		QR-LS.Boost	17750	10125	8000	6250	11375	0.287
	gamma	QR-QR.Boost	10250	6875	2875	4000	20375	1.718
		QR-LS.Boost	12000	7500	5750	5750	24875	0.289
	mixed	QR-QR.Boost	15700	3450	1500	2800	12700	1.372
		QR-LS.Boost	36375	13750	7750	6000	29000	0.286
multivariate2	norm	QR-QR.Boost	12400	2500	600	1000	3100	1.689
		QR-LS.Boost	16250	4250	1875	1750	5125	0.289
	tdist	QR-QR.Boost	12600	2400	700	1200	1550	1.567
		QR-LS.Boost	22000	8000	4500	3750	3250	0.287
	gamma	QR-QR.Boost	8500	1700	100	300	2400	1.063
		QR-LS.Boost	7500	4000	750	1375	14625	0.286
	mixed	QR-QR.Boost	14900	3100	800	1300	6500	1.217
		QR-LS.Boost	32875	11000	4750	4875	23750	0.286
high-dimensional	norm	QR-QR.Boost	13200	3300	1500	2400	9300	12.184
		QR-LS.Boost	12000	5500	4000	3750	8250	2.975
	tdist	QR-QR.Boost	18600	3975	1500	2250	7050	11.878
		QR-LS.Boost	20750	8250	4500	4500	6500	2.927
	gamma	QR-QR.Boost	10350	2250	900	1800	6300	13.487
		QR-LS.Boost	9000	4000	2250	3000	6500	2.953
	mixed	QR-QR.Boost	16950	3000	1200	2400	11325	12.171
		QR-LS.Boost	17500	4375	2500	3250	10750	2.944

Table 20: Median number of iterations required of QR-QR.Boost and QR-LS.Boost for all parameter setups and error distributions for each τ. Last column indicates the the median computing time for one iteration in ms. Extension of Table 5 from the main document.

Parameter setup	Error distribution	Method	τ 0.1	0.3	0.5	0.7	0.9	$\frac{\text{time}}{\text{iteration}}$
homoskedastic	norm	QR-QR.Boost	490	160	80	180	710	0.148
		QR-LS.Boost	500	400	300	1000	2100	0.053
	tdist	QR-QR.Boost	680	200	100	180	600	0.147
		QR-LS.Boost	2800	1350	800	950	2200	0.055
	gamma	QR-QR.Boost	680	100	40	20	20	0.167
		QR-LS.Boost	1000	200	100	100	100	0.053
	mixed	QR-QR.Boost	1260	380	180	400	1820	0.142
		QR-LS.Boost	2800	2000	2500	1550	3400	0.053
heteroskedastic	norm	QR-QR.Boost	60	220	160	400	2150	0.156
		QR-LS.Boost	100	600	550	1100	4950	0.054
	tdist	QR-QR.Boost	130	320	200	400	2250	0.144
		QR-LS.Boost	600	2400	2250	3250	5950	0.053
	gamma	QR-QR.Boost	1980	400	200	550	1760	0.172
		QR-LS.Boost	2700	500	300	700	1100	0.057
	mixed	QR-QR.Boost	1880	880	500	980	5000	0.171
		QR-LS.Boost	7800	4300	4500	5150	8100	0.063
multivariate	norm	QR-QR.Boost	12850	2900	1200	2100	8100	1.411
		QR-LS.Boost	16250	6500	3500	3250	7500	0.287
	tdist	QR-QR.Boost	12300	2900	1500	2600	7850	1.304
		QR-LS.Boost	17625	10375	9125	7625	13000	0.287
	gamma	QR-QR.Boost	9600	2200	900	1800	8800	1.706
		QR-LS.Boost	10250	7250	3250	3875	21000	0.287
	mixed	QR-QR.Boost	15900	3400	1500	2700	12600	1.375
		QR-LS.Boost	35750	12625	7125	6750	28000	0.287
multivariate2	norm	QR-QR.Boost	12400	2500	600	1000	2900	1.686
		QR-LS.Boost	17625	4250	2125	1750	4500	0.288
	tdist	QR-QR.Boost	13000	2400	700	1200	1300	1.550
		QR-LS.Boost	26625	6500	4125	3625	3000	0.288
	gamma	QR-QR.Boost	8500	1700	100	300	2300	1.066
		QR-LS.Boost	7500	4125	750	1250	17000	0.287
	mixed	QR-QR.Boost	14900	3100	750	1300	6300	1.219
		QR-LS.Boost	35250	11000	5125	5875	24375	0.287
high-dimensional	norm	QR-QR.Boost	14025	3300	1500	2400	10200	12.230
		QR-LS.Boost	13000	6000	4000	3750	8500	2.942
	tdist	QR-QR.Boost	17700	3900	1500	2400	7275	11.927
		QR-LS.Boost	18500	8000	4250	4875	7250	2.954
	gamma	QR-QR.Boost	10575	2400	900	1800	6750	13.418
		QR-LS.Boost	8875	4250	2250	3000	4750	2.941
	mixed	QR-QR.Boost	15900	3000	1200	2625	11475	12.127
		QR-LS.Boost	17250	4250	2500	3500	10500	2.933

Table 21: Median number of iterations required of QR-QR.Boost and QR-LS.Boost for the contaminated cases of all parameter setups and error distributions for each τ. Last column indicates the the median computing time for one iteration in ms. Extension of Table 7 from the main document.

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