Künstliche Intelligenz

Game Theory

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Game Theory

– Decision often depends on the actions of other actors

– **Game Theory** studies decision making of multiple actors, it tells us
  1. How an agent should decide
  2. How the decision situation should be designed to let actors make good decisions

Single Move Games

- The simplest form of game is where each actor makes exactly one move

- A game is defined by:
  - Players
  - Possible Actions
  - Payoff function

Example: Two Finger Morra

- Players O and E simultaneously display one or two fingers

- Let f be the total number of fingers shown
  - If f is odd, O gets f Dollar from E
  - If f is even, E gets f Dollar from O

- What are good strategies for O and E ?

Two Finger Morra

– Single Move Game in Normal Form:

Two-Finger Morra	O: one Finger	O: two Fingers
E: One Finger	E: +2, O: -2	E: -3, O: +3
E: Two fingers	E: -3, O: +3	E: +4, O: -4

– Strategies:

- Pure: always make the same choice,
  e.g. E: one
- Mixed: A probability distribution over choices,
  e.g. [0.5: one, 0.5: two]

The prisoner‘s Dillemma

– Imagine the following decision situation: two prisoners are accused of a crime. Based on their willingness to cooperate, they will do more or less time.

– The corresponding normal form game is:

Prisoner‘s Dillema	A: testify	A: refuse
B: testify	A: -5, B: -5	A: -10, B: 0
B: refuse	A: 0, B: -10	A: -1, B: -1

– What should the prisoners do ?

Dominant Strategies

– From A‘s perspektive, testify is the best option, because no matter, what B does, it is always the better choice:

Prisoner‘s Dillema	A: testify	A: refuse
B: testify	A: -5, B: -5	A: -10, B: 0
B: refuse	A: 0, B: -10	A: -1, B: -1

→ ‚testify‘ **strongly dominates** ‚refuse‘

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→‚testify‘ **strongly dominates** ‚refuse‘

– As the same holfs for B, both actors will chose to testify, as there is no other outcome that both would prefer.

→ Both prisones testifying is **Pareto optimal**

Nash Equilibria

- When each player has a dominant strategy, their combination is called a **Nash equilibrium.**
- In general a combination of strategies is a **Nash equilibrium**, if no player can gain by changing the strategy.
- Attention: Equilibria are not necessarily the best solution for all players!
- Both prisoners would be better off, if they would both refuse, still this is not the best rational strategy

Decision Support

Zero Sum Games

Zero Sum Games

- Two Finger Morra is a zero-sum game

- As it has no dominant strategy, there is no pure strategy, but we can determine an optimal mixed strategy

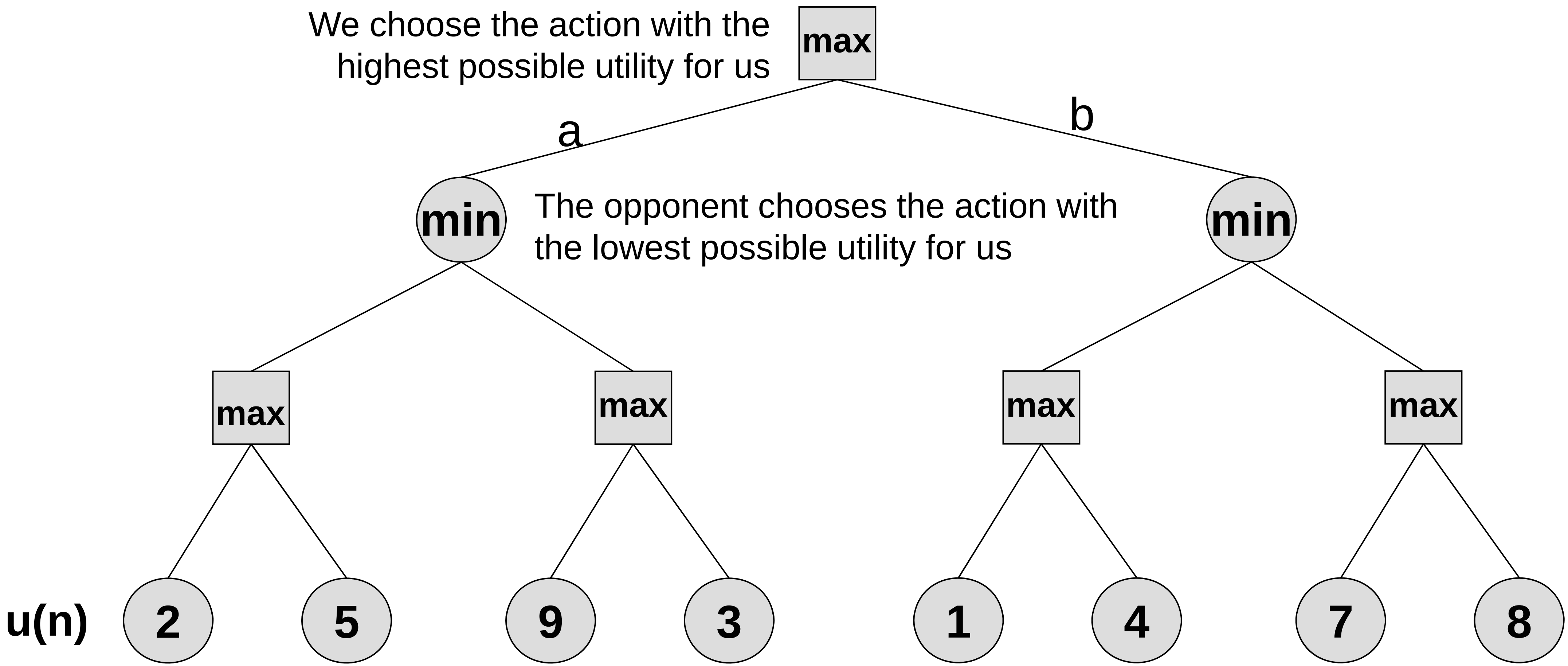
MinMax Strategy

– Assumption: players act rationally
  - Player chooses the move that maximizes its utility
  - Player chosses the move that minimizes the utility of the opponent (zero sum game)

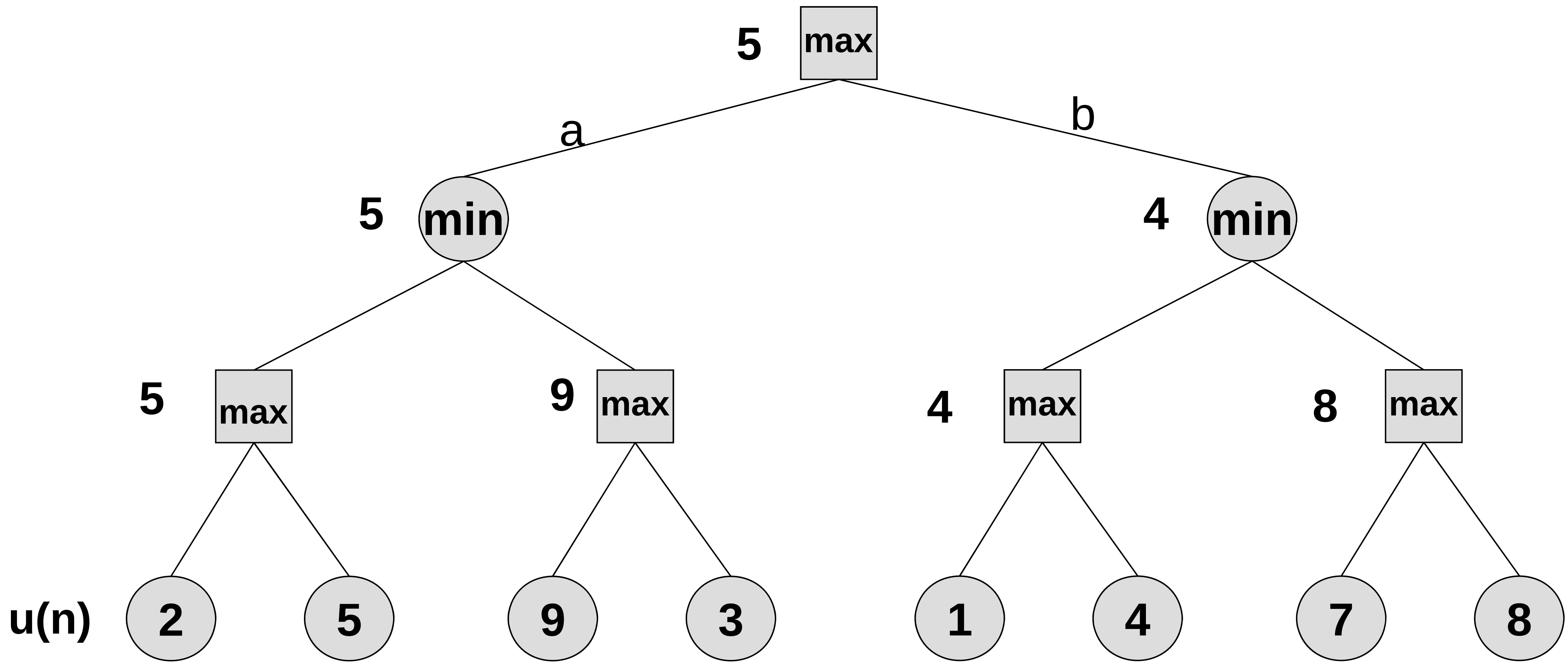
– This leads to the minimax strategy:
  - Strategy is determined by choosing the maximal value for the function

minmax(n) = u(n) , if n is TERMINAL
max{ minmax(s)| s is successor of n} on each own turn
min{minmax(s)| s is successor of n} on each opponents turn

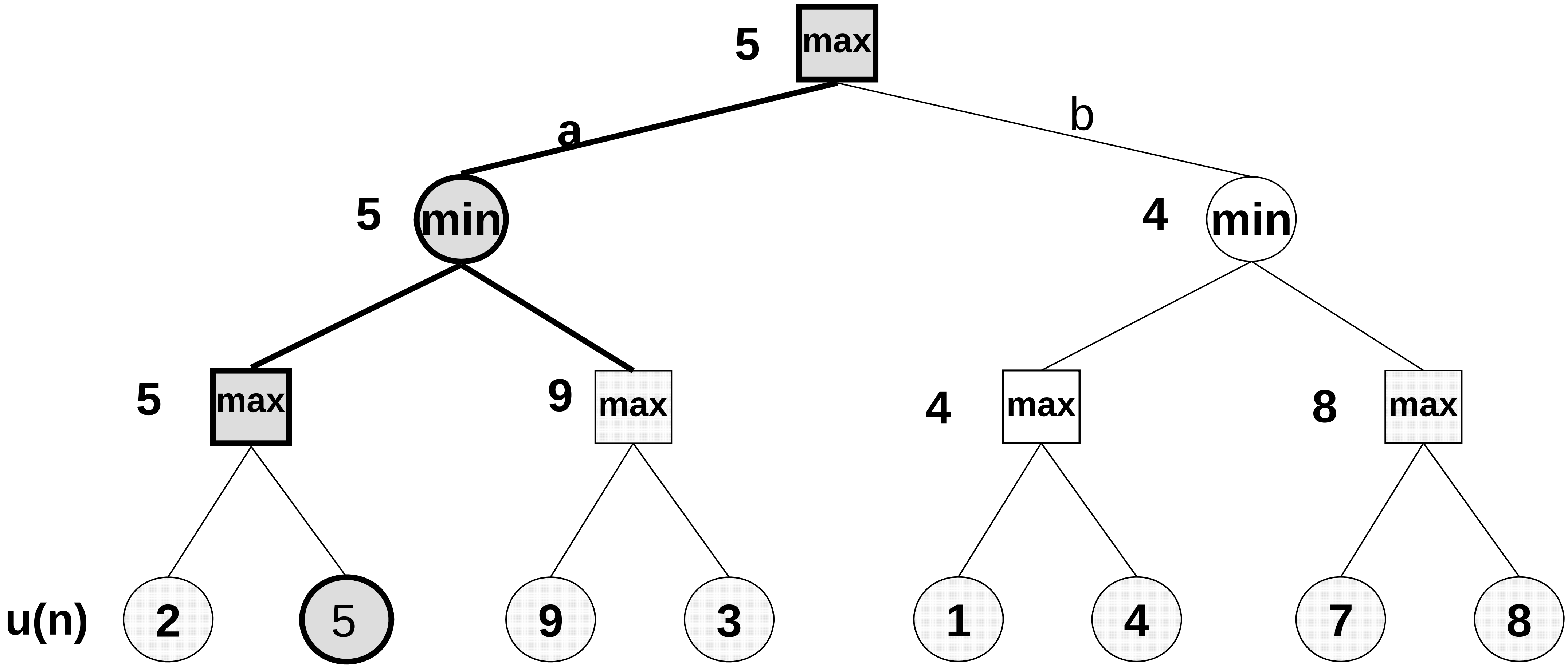
Game Trees


We choose the action with the highest possible utility for us
max
a
b
min
The opponent chooses the action with the lowest possible utility for us
min
max
max
max
max
u(n)
2
5
9
3
1
4
7
8

Utility Propagation in Search Trees

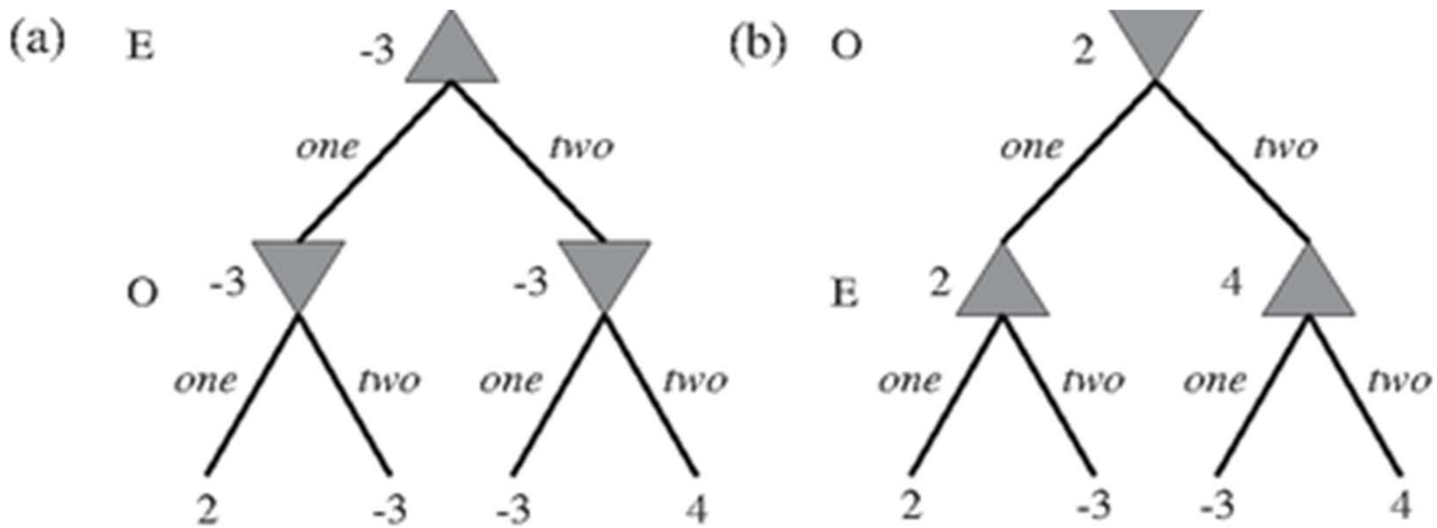


Choosing the best strategy



Utility Bounds for Two-Finger Morra

- Convert into turn-based games and determine the payoff
  a) If E has the first turn → -3
  b) If O has the first turn → 2

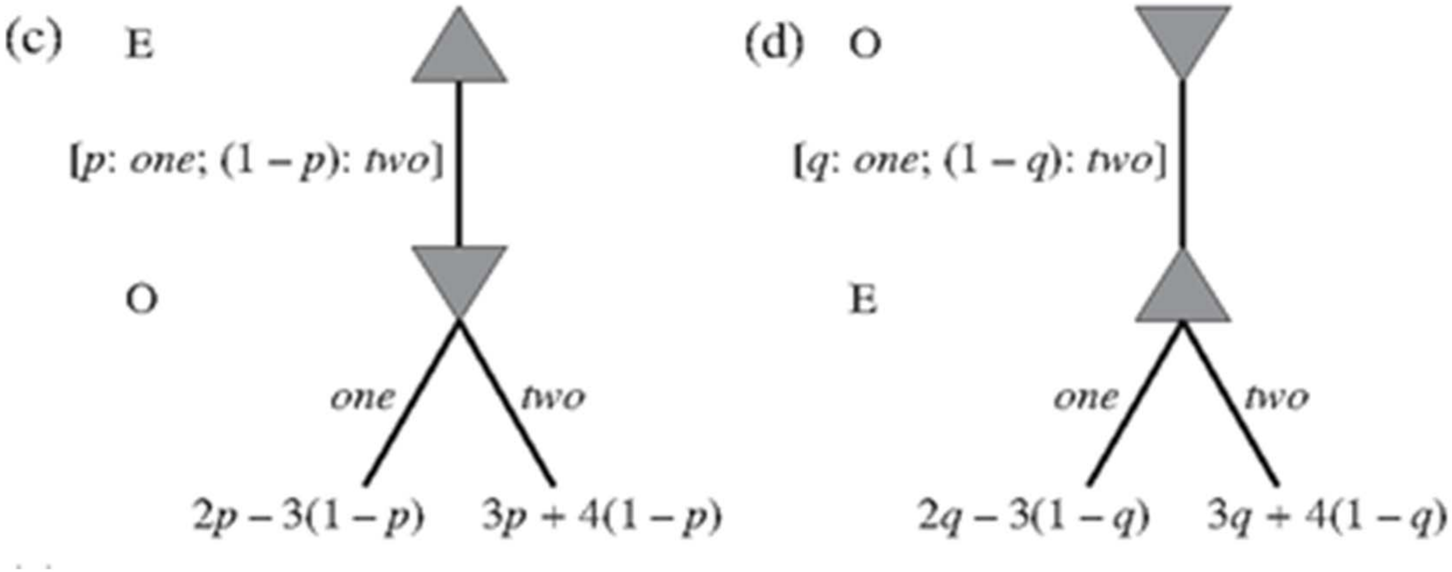


- This gives us upper and lower bounds for the utility of the optimal solution

Mixed strategies

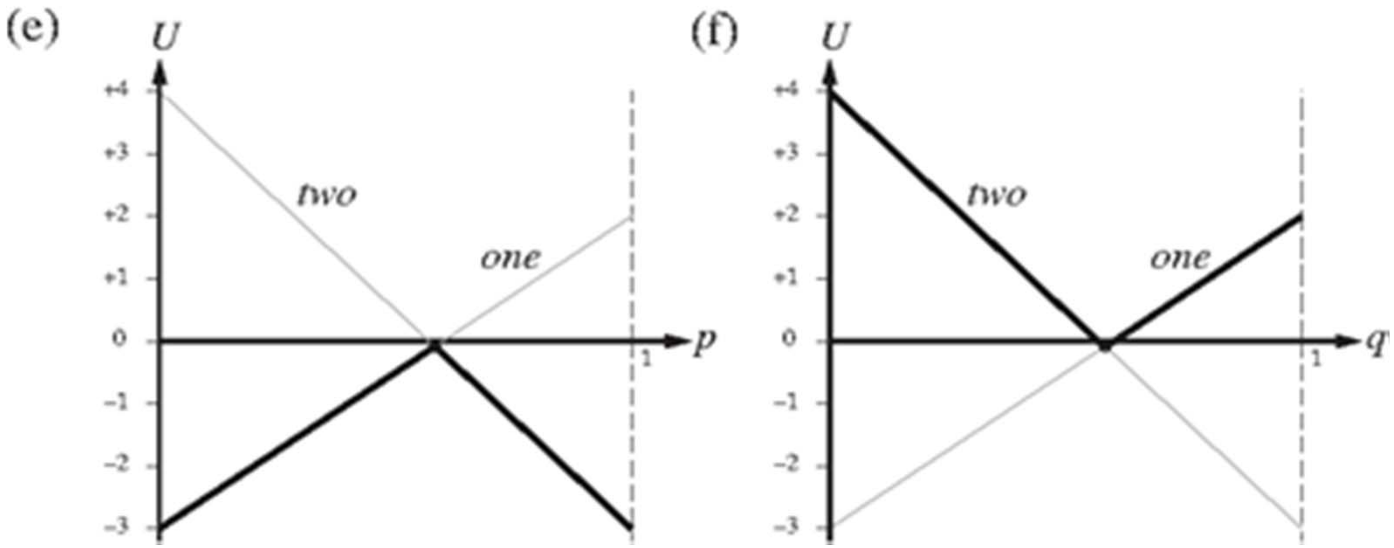
– Now we look at the case, where the first player has a mixed strategy [p: one, (1-p):two]

– This gives us utilities that depend on the choice of p:



Determining a mixed Strategy

– The optimal strategy can now be determined by solving a system of linear equations modelling the possible gains:



– The optimal strategy is the intersection of the two lines

Determining a mixed Strategy

– If E chooses first:
  - The expected payoff for ‚one‘ is $2p - 3(1-p) = 5p - 3$
  - The expected payoff for ‚two‘ is $-3p + 4(1-p) = 4 - 7p$
  - The expected payoff is the solution for:

$$5p - 3 = 4 - 7p \Rightarrow p = \frac{7}{12}$$

– If O chosses first:
  - The expected payoff for ‚one‘ is $2q - 3(1-q) = 5p - 3$
  - The expected payoff for ‚two‘ is $-3q + 4(1-q) = 4 - 7q$
  - The expected payoff is the solution for:

$$5q - 3 = 4 - 7q \Rightarrow q = \frac{7}{12}$$

– This means that the best mixed strategy for both players is [7/12: one, 5/12: two]