

## Highlights

### **Fortifying Critical Infrastructure Networks with Multicriteria Portfolio Decision Analysis: An Application to Railway Stations in Finland**

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- Topological metrics can miss alignment with network performance objectives.
- Multicriteria analysis supports efficient resource allocation for risk management.
- Search algorithms proposed to identify cost-efficient portfolios.
- Illustrative case study on fortifying switches at railway stations in Finland.

# Fortifying Critical Infrastructure Networks with Multicriteria Portfolio Decision Analysis: An Application to Railway Stations in Finland

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## Abstract

Advanced societies are crucially dependent on critical infrastructure networks for the reliable delivery of essential goods and services. Hence, well-founded analyses concerning disruptions are needed to guide decisions that seek to ensure the performance of these networks in the face of failures caused by vulnerabilities to external hazards or technical malfunctions. In this setting, we develop an approach based on multicriteria decision analysis to support the identification of cost-efficient portfolios of preventive fortification actions. Our approach (i) allows for multiple objectives, such as those that represent the traffic volume that is enabled between alternative origin-destination pairs in a transportation network, (ii) uses methods of probabilistic risk assessment to quantify the expected performance of the network, and (iii) uses a search algorithm combined with an optimization model to identify those combinations of fortification actions that are cost-efficient in improving the performance of the network, given the available, possibly incomplete information about the relative importance of objectives and minimum performance requirements on them. Our methodological contributions are illustrated by a case study on the analysis of railway switches at a representative Finnish railway station.

**Keywords:** Critical infrastructure, Transportation systems, Cost-efficiency analysis, Portfolio analysis, Risk analysis, Performance requirements

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## 1. Introduction

Critical infrastructures comprise all the assets, systems, and networks that provide essential functions for society [1]. These infrastructures are essential for ensuring the continuity of operations in sectors such as energy, water supply, transportation, and telecommunications. Because disruption or destruction of these infrastructures can significantly affect public health, safety, security, and economic well-being, they must function adequately to ensure that the goals related to economic productivity, sustainability, and social development can be achieved [2].

In Europe, rail networks amounted to 202,000 km in 2022, with considerable recent growth in high-speed trains [3]. Disruptions in rail networks can undermine related performance objectives, such as ensuring connectivity between strategic points or providing reliable connections for passengers and goods delivery. These disruptions can be caused by failures due to the usual wear of technical systems or by vulnerabilities to external hazards, including extreme weather conditions and intentional attacks. Consequently, there is a need to understand what types of disruption can affect the rail network, what impacts these disruptions can cause, and what actions are needed to mitigate them subject to the requirement that these actions must be selected as cost-effectively as possible given resource limitations [see, e.g. 4, 5, 6].

Not all components in transportation networks are equally important because disruptions in some parts of the network cause its performance to degrade more than those in other parts. Moreover, because impacts on network performance depend on the states of all network components, there is a need to evaluate the impacts of *combinations* of disruptions (e.g., a component may not be so important on its own, but its disruption simultaneously with another component may have drastic consequences for the performance of the network). Furthermore, it is pertinent to examine how the expected impacts of disruptions depend on alternative assumptions about the underlying probabilities of component failures, given that the information about these probabilities may be incomplete.

Fortification actions reduce the probability of disruptions that can undermine the performance of the network. Because many fortification actions are typically implemented jointly, we analyze *portfolios* consisting of many actions. These analyses support the cost-efficient allocation of resources to those portfolios of fortification actions, which, taken together, provide the highest expected performance relative to the costs of implementing these ac-

tions [7]. Furthermore, such analyses can be produced for a range of budget levels for aggregate implementation costs, for example, to prepare plans for the cost-efficient allocation of additional resources or, in case of budget cuts, to identify which one of the previously planned fortification actions can be left out so that the reduction on expected network performance will remain as small as possible.

In this paper, we develop an approach based on multicriteria portfolio decision analysis in order (i) to assess and aggregate multiple objectives that reflect the services provided by infrastructure networks and (ii) to guide the allocation of resources to actions that are cost-efficient in mitigating disruptions in different parts of the network. Specifically, we represent the network by nodes (components subject to failure) and edges (connections between these components), and our formulation can be used to identify which nodes (or combinations of nodes) are most important to network performance. Moreover, it helps identify which portfolios of fortification actions are cost-efficient in ensuring the desired level of network performance. Examples of such fortification actions include fortifying selected nodes against disruptions.

The remainder of this paper is organized as follows. Section 2 discusses earlier approaches to analyzing disruptions and their impacts on the performance of transportation networks. Section 3 develops an approach that combines probabilistic risk assessment with multicriteria portfolio decision analysis to quantify the expected performance of the network with regard to several objectives and, in addition, guides the allocation of resources to those fortification actions that fortify the network cost-efficiently. Section 4 presents a case study on the fortification of switches to improve the reliability of connections at a representative railway station in Finland. Section 5 discusses the numerical results, and Section 6 outlines future research areas.

## 2. Background

Early studies of railway networks focused mainly on their topological configuration due to limited data and computational power [8]. Later, there was a proliferation of specific models developed to reduce travel times [9], design schedules [10], plan new lines [11], reduce operating costs [12], and ensure track functionality, among other objectives. In many countries, infrastructure decisions (e.g., ensuring that the railway network is safe and functional) and operational decisions (e.g., ensuring that train timetables are being main-

tained) are made by entirely different entities. This is reflected in the fact that infrastructure and operational research have historically been somewhat decoupled. Within these streams of research, we focus on analyzing the network as a critical infrastructure in providing services.

Most railway system analysis relies on network theory to identify critical components, evaluate network performance, and develop strategies to reinforce or disrupt them [see, e.g., 13, 14, 15, 16]. Latora and Marchiori [17] present a method to find the critical components of an infrastructure network represented by nodes and edges. They also analyze how improvements, such as adding edges between nodes, enhance network performance. In their model, performance is measured using a topological metric, i.e., a metric that relies purely on how nodes are positioned and connected in the network, to quantify how efficiently information can be exchanged over the network. Ip and Wang [18] propose a methodology to evaluate the resilience of railway networks based solely on topological metrics, such as the number of independent paths. They also propose an algorithm to fortify the network to increase its resilience. Fecarotti et al. [19] propose a non-linear integer programming model that considers topological metrics to select maintenance strategies.

Although several studies have used topological metrics to assess network performance, few have evaluated the quality of these assessments [20]. In this regard, a general drawback of topological metrics is that these metrics are not necessarily closely related to network performance objectives. For example, Hao et al. [21] propose a multi-objective optimization approach to identify critical components in a network, accounting for their interactions with other systems. One of their key findings is that the criticality of the nodes is not necessarily related to their topological importance. LaRocca et al. [22] conclude that many topological metrics are of limited value in analyzing the robustness of power systems in different disruption scenarios. Moreover, Alderson et al. [23] show that the criticality of a given component can depend more on the set of disrupted components than on the component itself.

Another challenge in evaluating the performance of networks under uncertainty is the identification of possible hazards and their impacts. In some cases, it is unclear what hazards can affect the network [24] or to what extent the infrastructure will withstand expected / unexpected hazards [25]. Zhang et al. [26] present a summary of recent studies to quantify the loss of functionality of railway systems due to various hazards, such as extreme

weather or seismic events. They propose a framework for combined hazards to simulate functionality loss in a coupled railway and airline system. One of the key aspects in quantifying this loss is to account for the propagation of the disruptions. In general, the assumption of independent failure rates can lead to overestimating the reliability of the network, leading to poor design and unacceptable performance. Nazarizadeh et al. [27] propose a model which includes common cause failures and interactive failures. They apply the model to the Iranian railway system and show that it can provide better reliability estimates than other methodologies.

Several authors proceed by elaborating scenarios characterizing the realization of hazards and estimating their probabilities. For example, Joshi et al. [28] and Yang et al. [29] consider scenarios of rainfall and tornadoes to assess the risks affecting the railway systems in India and China, respectively. Turoff et al. [30] propose a collaborative dynamic scenario model based on expert judgments to estimate the cascading effects of critical infrastructure interactions.

Zio [25] and Sedghi et al. [31] call for the development of frameworks that help railway infrastructure managers understand and quantify the complexity of the network and, by doing so, help them prepare for hazards to ensure acceptable network performance. In this context, we develop a general assessment approach and accompanying search algorithms that help assess the importance of components in a rail network, considering disruption probabilities, multiple objectives associated with the services provided by the network, and possible minimum performance requirements on these objectives. This assessment of the components' importance is further used to provide fortification recommendations.

### 3. Proposed Approach for Fortifying Networks

#### 3.1. Network Representation

Let  $G(V; E)$  denote a network consisting of a set of nodes  $V = \{1, \dots, n\}$  and a set of undirected edges  $E \subseteq \{(i, i') \mid i, i' \in V\}$  between the nodes. A path is a sequence of nodes and edges that connect two nodes in the network. The state of a node is operational or disrupted. If a node is disrupted, none of the paths containing it can be traversed. Thus, if the nodes in  $D \subseteq V$  are disrupted, the remaining network is  $G(V^D; E^D)$ , where  $V^D = V \setminus D$  and  $E^D = \{(i, i') \in E \mid i, i' \in V^D\} \subseteq E$ .

The state  $x_k$  of node  $k$  is modeled as a realization of a binary random variable  $X_k$  such that  $x_k = 0$  if node  $k \in V$  is disrupted and  $x_k = 1$  if it is operational. The state of the network  $x = (x_1, \dots, x_n) \in \mathcal{X} = \{0, 1\}^n$  is the joint realization of the random variables that represent the states of nodes. Thus, there are  $2^n$  network states. We assume that the disruption events at the nodes occur independently and that  $p_k = \mathbb{P}[X_k = 0]$  is the probability that node  $k$  is disrupted. Due to this independence assumption, the probability distribution over network states is fully characterized by the vector  $p = (p_1, \dots, p_n)$ .

### 3.2. Assessing Network Performance

Critical infrastructure networks can enable multiple services to their users, so their performance is generally measured by considering several objectives. For example, the performance of a transportation network can be assessed with regard to its ability to provide connections between relevant origin-destination pairs. The objectives can also be assessed in terms of topological metrics such as the average of the shortest distances between the nodes of the network [see, e.g., 32, 33].

The performance of the network depends on its state because if some nodes are disrupted, the network may not be able to provide its intended services. The attainment of  $m$  multiple objectives can be measured by corresponding criteria and, more technically, by utility functions  $u_j(\cdot) : \mathcal{X} \mapsto [0, 1]$  such that  $u_j(x)$  represents the utility that is associated with the performance on criterion  $j = 1, \dots, m$  when the network is in state  $x \in \mathcal{X}$ . We assume that the network is coherent in that these utilities cannot increase with the disruption of a node. The utilities are normalized so that the minimum  $u_j(x) = 0$  is associated with the states that give the worst performance on criterion  $j$  (i.e., all nodes are disrupted  $x_k^0 = 0, k = 1, \dots, n$ ). The maximum  $u_j(x) = 1$  is attained when the network performs at its best on the  $j$ -th criterion (i.e., all nodes are operational  $x_k^* = 1, k = 1, \dots, n$ ).

The utility functions  $u_j(\cdot)$  can be aggregated by employing the additive multicriteria utility function in (1), assuming that the criteria are mutually preferentially independent (i.e., preferences for a given criterion do not depend on those for any other criterion), and every criterion is additive independent (i.e., there are no preferences for how the realizations for a given criterion coincide with realizations with other criteria, provided that the probabilities of all realizations on the different criteria remain unchanged)

[see 34]. Specifically, in the utility function

$$u(x, w) = \sum_{j=1}^m w_j u_j(x) \in [0, 1], \quad (1)$$

the weight of the criterion  $w_j \in [0, 1]$ ,  $j = 1, \dots, m$  reflects the relative overall utility increase gained as a result of the improvement on criterion  $j$  when the state of the network changes from its worst state (all nodes are disrupted) to its best state (all nodes are operational). Following the usual convention, these weights can be normalized so that  $\sum_{j=1}^m w_j = 1$ .

However, obtaining a complete preference characterization through criteria weights can be challenging. For example, if the different criteria represent the planned volume of traffic between different origin-destination pairs in the network, it is possible that this volume is not fully known at the time of planning the fortification actions. In other cases, decision-makers (DM) may be reluctant to specify the importance of criteria, or there can be multiple DMs with different priorities. In response to this recognition, and also to support extensive sensitivity analyses, we characterize information about the relative importance of criteria through an *information set*  $\mathcal{S}$  consisting of all the weights that are compatible with the elicited preference statements [35]. These statements can be elicited by asking the DM to express statements concerning the relative importance of criteria. Thus, for example, if criterion 1 is at least as important as criterion 2 but not more than two times more important, the constraints  $w_2 \leq w_1 \leq 2w_2$  hold. Assuming that the preferences are elicited through statements that correspond to linear constraints, then the resulting weight set  $\mathcal{S}$  is a subset of the non-informative weight set  $\mathcal{S}^0$

$$\mathcal{S} = \{w \in \mathbb{R}^m \mid Aw \leq b\} \subseteq \left\{ w \in \mathbb{R}^m \mid w_j \geq 0 \ \forall j, \sum_{j=1}^m w_j = 1 \right\} = \mathcal{S}^0, \quad (2)$$

where the constraint matrix  $A \in \mathbb{R}$  and the vector  $b \in \mathbb{R}$  contains the coefficients of the constraints.

### 3.3. Fortifying the Network

The expected performance of the network can be improved by implementing fortification actions that, for example, lead to lower disruption probabilities at the nodes where they are implemented. Figure 1 illustrates the decision to implement an action to fortify a network node. Without this action,

the probability of disruption of node  $k$  is  $p_k$ . If the corresponding action is implemented, this probability is reduced to  $p'_k < p_k$ .

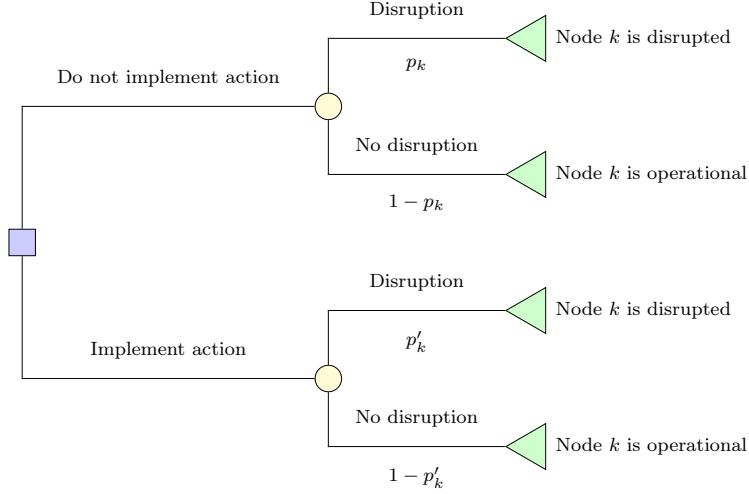


Figure 1: Decision tree for implementing an action to fortify network node  $k$ .

Usually, there are many possible fortification actions for improving network performance. As a result, the probability of different levels of network performance depends on what is the *portfolio* (i.e., subset) of actions that are implemented. We assume that there are  $h$  possible fortification actions, represented by the binary variables  $q_1, \dots, q_h$  such that  $q_l = 1$  if action  $l$  is implemented and  $q_l = 0$  if not,  $l = 1, \dots, h$ . The portfolio of fortification actions is given by a vector  $q = (q_1, \dots, q_h) \in \mathcal{Q} = \{0, 1\}^h$ . The total cost of a portfolio  $q$  is  $c_{tot}(q) \in \mathbb{R}_+$ . This total cost  $c_{tot}$  is taken to be the sum of the costs of the individual actions  $c_l$  in the portfolio so that  $c_{tot}(q) = \sum_{l=1}^h c_l q_l$ . More complex cost structures can be easily introduced to account for cost synergies among actions.

A portfolio of fortification actions  $q$  is *feasible* if (i) its total cost  $c_{tot}(q)$  does not exceed the level of available resources  $r$  (i.e.,  $c_{tot}(q) \leq r$ ) and (ii) satisfies relevant logical constraints (e.g., if actions 1 and 2 are mutually exclusive, allowing at most one of them to be implemented, the constraint  $q_1 + q_2 \leq 1$  holds). The set of feasible portfolios is denoted by  $\mathcal{Q}_F \subseteq \mathcal{Q}$ . The aim is to determine those sets of feasible portfolios that satisfy the relevant constraints and contribute the most to the attainment of objectives, such as

maximizing expected performance.

The probability of disruption  $p_k(q)$  at node  $k$  depends on the portfolio  $q \in \mathcal{Q}_F$  of actions. Thus, assuming that the disruption events at each node depend on these actions only, the probability of the network state  $x \in \mathcal{X}$  is

$$p(x | q) = \prod_{k=1}^n [x_k(1 - p_k(q)) + (1 - x_k)p_k(q)] \in [0, 1]. \quad (3)$$

This formulation is general in that the probability of the disruption of each node can depend on the entire portfolio of implemented actions. Thus, if two actions affect the probability of disruption of a given node, and these two actions can be taken jointly, it would be necessary to estimate the disruption probability of this node for the situation in which both actions are implemented or if only one is taken. Still, the formulation considers that the disruption events are independent in that the probability of disruption at a given node is not impacted by whether or not there has been a disruption at some other node(s).

### 3.4. Non-Dominated and Cost-Efficient Portfolios

When the DM seeks to maximize the expected network performance, the objective is to determine which feasible portfolios outperform others for all feasible weights that represent the importance of the different objectives. For further insights, such analyses can be produced by comparing portfolios based on the concept of *dominance* at different cost levels that are feasible with the available budget.

**Definition 1.** Portfolio  $q^1 \in \mathcal{Q}_F$  is dominated by portfolio  $q^2 \in \mathcal{Q}_F$  in the information set  $\mathcal{S}$ , denoted by  $q^2 \succ^{\mathcal{S}} q^1$ , if and only if  $\mathbb{E}[u(x, w) | q^1] \leq \mathbb{E}[u(x, w) | q^2]$  for all  $w \in \mathcal{S}$  and (ii)  $\mathbb{E}[u(x, w) | q^1] < \mathbb{E}[u(x, w) | q^2]$  for some  $w \in \mathcal{S}$ .

The dominance between two portfolios can be determined by comparing the expected utility of these portfolios at the extreme points  $w_e \in \mathcal{S}^{ext}$  of the information set  $\mathcal{S}$  [36]. Because this information set is a polyhedral set, these points can be computed with linear programming techniques [see, e.g., 37]. Moreover, if  $\mathbb{E}[u(x, w) | q^1] = \mathbb{E}[u(x, w) | q^2] \forall w \in \mathcal{S}$ , then portfolios  $q^1$  and  $q^2$  have the same expected performance, which is denoted by  $q^1 \sim^{\mathcal{S}} q^2$ . A feasible portfolio is cost-efficient if (i) it is not dominated by any feasible

portfolio of equal or lower cost and (ii) there is no other portfolio with lower cost with equal expected performance.

**Definition 2.** *Portfolio  $q^1 \in Q_F$  is cost-efficient with respect to another portfolio  $q^2 \in Q_F$  in the information set  $\mathcal{S}$ , denoted by  $q^1 \succ_C^{\mathcal{S}} q^2$ , if and only if (i)  $q^1 \succ^{\mathcal{S}} q^2, c(q^1) \leq c(q^2)$  or (ii)  $q^1 \sim^{\mathcal{S}} q^2$  and  $c(q^1) < c(q^2)$ .*

**Definition 3.** *Portfolio  $q^1 \in Q_F$  is cost-efficient in the information set  $\mathcal{S}$ , denoted by  $q^1 \in Q_{CE} \subseteq Q_F$  if and only if  $\nexists q^2 \in Q_F$  such that  $q^2 \succ_C^{\mathcal{S}} q^1$ .*

Although Definitions 1, 2, and 3 refer to the maximization of expected network performance measured by the utility function, these definitions can be extended to represent feasibility constraints that may arise from the consideration of risk measures such as value at risk (VaR) and conditional value at risk (CVaR) [38].

### 3.5. Determination of Non-Dominated Portfolios

#### 3.5.1. Algorithm to Determine Cost-efficient Portfolios

Algorithm 1 is presented to determine cost-effective portfolios  $\mathcal{Q}_{CE}$ . At each iteration, it generates new portfolios by adding a fortification action to the previously computed set of cost-efficient portfolios and the *basic portfolios* (i.e., portfolios that are not cost-efficient but can potentially be extended to cost-efficient portfolios by adding fortification actions). In particular, the portfolios that cannot belong to the set of basic portfolios are removed, which avoids the complete enumeration of feasible portfolios.

In Step 1, the utility function is computed for all network states and extreme points of the set of feasible weights. If the computation of the utility function is time-consuming, a subset of the states can be evaluated. In Step 2, the set of cost-efficient portfolios is initialized by including only the portfolio with no fortification actions. In Step 3, the set of *basic portfolios* is initialized as an empty set. In Steps 4 to 12, the index  $l = 1, \dots, h$  iterates through all fortification actions. In Step 5, a set  $\mathcal{Q}^l$  is constructed by taking every portfolio in the set  $\mathcal{Q}^{l-1} \cup \mathcal{Q}_B^{l-1}$  and modifying its  $l$ -th action. In Step 6, the portfolios in  $\mathcal{Q}^l$  are compared to portfolios in  $\mathcal{Q}^{l-1}$ , and cost-inefficient portfolios are stored in the set  $\mathcal{Q}_D^1$ . In Step 7, the cost-inefficient portfolios in  $\mathcal{Q}_D^1$  are removed from the set  $\mathcal{Q}^l$ . In Step 8, the portfolios in  $\mathcal{Q}^{l-1}$  are compared to portfolios in  $\mathcal{Q}^l$ , and cost-inefficient portfolios are stored in the set  $\mathcal{Q}_D^2$ . In Step 9, the cost-inefficient portfolios in  $\mathcal{Q}_D^2$  are

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**Algorithm 1** Compute  $\mathcal{Q}_{CE}$ 


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1: Compute  $u(x, w)$  for all  $x \in \mathcal{X}$  and  $w \in \mathcal{S}^{ext}$ 
2:  $\mathcal{Q}^0 \leftarrow \{(0, \dots, 0)\}$ 
3:  $\mathcal{Q}_B^0 \leftarrow \{\}$ 
4: for  $l = 1$  to  $h$  do
5:    $\mathcal{Q}^l \leftarrow \{q^1 \in \mathcal{Q}_F \mid q_l^1 = 1 \wedge \exists q^2 \in \mathcal{Q}_B^{l-1} \cup \mathcal{Q}^{l-1} : q_k^1 = q_k^2, \forall k \neq l\}$ 
6:    $\mathcal{Q}_D^1 \leftarrow \left\{ q^1 \in \mathcal{Q}^l \mid \exists q^2 \in \mathcal{Q}^l \cup \mathcal{Q}^{l-1} : q^2 \xsucc{S^{ext}}_C q^1 \right\}$ 
7:    $\mathcal{Q}^l \leftarrow \mathcal{Q}^l \setminus \mathcal{Q}_D^1$ 
8:    $\mathcal{Q}_D^2 \leftarrow \left\{ q^1 \in \mathcal{Q}^{l-1} \mid \exists q^2 \in \mathcal{Q}^l : q^2 \xsucc{S^{ext}}_C q^1 \right\}$ 
9:    $\mathcal{Q}^{l-1} \leftarrow \mathcal{Q}^{l-1} \setminus \mathcal{Q}_D^2$ 
10:   $\mathcal{Q}^l \leftarrow \mathcal{Q}^l \cup \mathcal{Q}^{l-1}$ 
11:   $\mathcal{Q}_B^l \leftarrow \left\{ q \in \mathcal{Q}_D^1 \cup \mathcal{Q}_D^2 \cup \mathcal{Q}_B^{l-1} \mid \nexists q^1 \in \mathcal{Q}^l : q^1 \xsucc{S^{ext}}_C q^a \right\}$ 
    with  $q_k^a = q_k, \forall k \leq l \wedge q_k^a = 1, \forall k > l$ 
12: end for
13:  $\mathcal{Q}_{CE} \leftarrow \mathcal{Q}^h$ 

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removed from the set  $\mathcal{Q}^{l-1}$ . In Step 10, the set  $\mathcal{Q}^l$  is updated, including the cost-efficient portfolios from  $\mathcal{Q}^{l-1}$ . In Step 11, the set of basic portfolios is updated by including the previous basic portfolios and the cost-inefficient portfolios identified in the iteration that can be extended to cost-efficient portfolios. To determine if a portfolio  $q$  is included in basic portfolios, the algorithm constructs its extended portfolio  $q^a$  by adding all the remaining fortification actions to  $q$ . If the extended portfolio  $q^a$  of a portfolio  $q$  is not cost-efficient, then  $q$  is not in the set of basic portfolios. The extended portfolios  $q^a$  provide an upper bound of the expected utility that can be achieved by extending a portfolio. Still, they are not necessarily feasible (e.g., their cost can be higher than the budget). If a portfolio  $q^a$  is infeasible, the provided upper bound does not accurately reflect the value of the maximum expected utility. As a result, the algorithm can keep portfolios in the set basic portfolios that should be discarded. In Appendix A, we provide one complementary approach that can generate tighter bounds, improving the removal of portfolios. The algorithm terminates in Step 13, returning the set of cost-efficient portfolios  $\mathcal{Q}^h$ .

### 3.5.2. Binary Utility Function for Individual Objectives

Utility functions such as (4) can be used to represent whether or not the performance of the network satisfies a given condition. For example, in a transportation network, the DM may be interested in having more than two operational edges connected to a specific node for evacuation purposes or in having a connection between two nodes to transport sensitive cargo. In such cases, the resulting objective can be represented by the binary utility function

$$u_j(x) = \begin{cases} 1, & \text{if objective } j \text{ is met in network state } x \in \mathcal{X}. \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The expected value of the binary utility function (4) associated with objective  $j$  is the same as the probability of achieving the objective, i.e.,

$$\mathbb{E}[u_j(x)] = 1 \cdot \mathbb{P}[u_j(x) = 1] + 0 \cdot \mathbb{P}[u_j(x) = 0] = \mathbb{P}[u_j(x) = 1] \quad (5)$$

### 3.5.3. Minimum Performance Requirements for Individual Objectives

Depending on the context, there may be a requirement such that the expected utility related to binary objective  $j$  is greater than or equal to  $\alpha_j$ . This requirement can be represented by the chance constraint  $\mathbb{E}[u_j(x)] \geq \alpha_j$ . The vector of such binary performance requirements is denoted by  $\alpha = (\alpha_1, \dots, \alpha_m)$ , where  $\alpha_j = 0$  if there is no such requirement.

Algorithm 1 for determining cost-efficient portfolios does not consider such minimum performance requirements for individual objectives because it does not consider the utility functions for individual objectives directly. Instead, it uses the overall utility function of the network performance (1), which is a weighted sum of utility functions for the objectives.

The non-informative set  $S^0$  is a special case, defined in (2). In this case, there are no stated preferences for individual objectives. The extreme points of the set  $S^0$  are the canonical vectors  $e_j = (w_1, \dots, w_j, \dots, w_m) \in S^{0,ext}$ , where  $w_j = 1$  and  $w_i = 0$  for all  $i \neq j$ . At every extreme point  $e_j \in S^{0,ext}$ , the utility function of the network is equal to the utility function of the corresponding objective  $j$ , i.e.,  $u(x, e_j) = u_j(x)$ . Thus, for each objective, there is a cost-efficient portfolio in the non-informative set that gives the maximum performance on this objective. This is formalized in Lemma 1.

**Lemma 1.**  $\forall j = 1, \dots, h \exists q^1 \in \mathcal{Q}_{CE}$  in the information set  $\mathcal{S}^0$  such that  $\mathbb{E}[u_j(x) | q^1] \geq \mathbb{E}[u_j(x) | q^2] \forall q^2 \in \mathcal{Q}_F \setminus \mathcal{Q}_{CE}$ .

*Proof.* We prove Lemma 1 by contradiction. Consider the objective  $j$ . Let  $q^1 \in \mathcal{Q}_{CE}$  denote the portfolio with the maximum expected performance in  $j$ . Assume, for the sake of contradiction, that there exists  $q^2 \in \mathcal{Q}_F \setminus \mathcal{Q}_{CE}$  such that  $\mathbb{E}[u_j(x) | q^2] > \mathbb{E}[u_j(x) | q^1]$ . Since  $q^1$  is chosen as the portfolio with the maximum expected performance in  $j$ , it follows that  $\nexists q \in \mathcal{Q}_{CE}$  such that  $\mathbb{E}[u(x, e_j) | q^2] < \mathbb{E}[u(x, e_j) | q]$ , where  $e_j \in \mathcal{S}^0$  represents a canonical vector.

By Definitions 1 and 2, this implies that  $q^2$  is non-dominated in  $\mathcal{S}^0$ . Consequently,  $q^2 \notin \mathcal{Q}_F \setminus \mathcal{Q}_{CE}$ , contradicting the initial assumption. Thus, the lemma is proven.  $\square$

Moreover, the maximum possible performance on objective  $j$  is attained by some cost-efficient portfolio in the information set  $S$  that contains the vector  $e_j \in S$ . Therefore, Algorithm 1 for determining the cost-efficient portfolios can be used in the presence of minimum performance requirements for individual objectives, provided that the corresponding extreme point is in the information set ( $\alpha_j \neq 0 \Rightarrow e_j \in S^{ext}, \forall j = 1, \dots, m$ ). After determining the cost-efficient portfolios, the set must be filtered to remove those that do not fulfill requirements on expected performance.

### 3.5.4. Algorithm to Determine Cost-efficient Portfolios with Minimum Performance Requirements

Algorithm 2 can be deployed to identify cost-efficient portfolios that fulfill minimum performance requirements. Toward this end, the information set is extended to include all extreme points associated with the minimum performance requirements.

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#### Algorithm 2 Compute $\mathcal{Q}_{CE}$ with minimum performance requirements $\alpha$

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- 1:  $\mathcal{S}^\alpha \leftarrow \{e_j \mid j = 1, \dots, m \mid \alpha_j \neq 0\}$
  - 2:  $\mathcal{S}^* \leftarrow \mathcal{S}^{ext} \cup \mathcal{S}^\alpha$
  - 3:  $\mathcal{Q}_{CE}^* \leftarrow \text{Algorithm 1 with } \mathcal{S}^*$
  - 4:  $\mathcal{Q}_{CE}^\alpha \leftarrow \{q \in \mathcal{Q}_{CE}^* \mid \mathbb{E}[u_j(x) | q] \geq \alpha_j \forall j = 1, \dots, m\}$
  - 5:  $\mathcal{Q}_{CE} \leftarrow \mathcal{Q}_{CE}^\alpha \setminus \left\{ q^1 \in \mathcal{Q}_{CE}^\alpha \mid \exists q^2 \in \mathcal{Q}_{CE}^\alpha : q^2 \succ_C q^1 \right\}$
- 

In Step 1, the weight set  $\mathcal{S}^\alpha$  is constructed to contain one canonical vector  $e_j$  for every minimum performance requirement  $\alpha_j$ . In Step 2, the extended

set  $\mathcal{S}^*$  is obtained by combining  $\mathcal{S}^\alpha$  with the extreme points of the set representing the DM's preferences  $\mathcal{S}^{ext}$ . In Step 3, the cost-efficient portfolios in the extended information set  $\mathcal{S}^*$  are determined with Algorithm 1. In Step 4, portfolios that do not fulfill the minimum performance requirements are removed. In Step 5, the cost-efficient portfolios in the information set  $\mathcal{S}^{ext}$  are computed. The set of portfolios  $\mathcal{Q}_{CE}^\alpha$  is usually much smaller than the set of feasible portfolios, so it is possible to use algorithms such as the one proposed by [39] to identify the set  $\mathcal{Q}_{CE}$ .

#### 4. Case Study

We illustrate our methodological contributions by considering the Siilinjärvi train station in Northern Savonia, Finland, depicted in Figure 2a. The network representation of the structure of this station is in Figure 2b. Here, the small squares represent the 22 rail switches, which are mechanical devices that allow trains to change from one track to another. Specifically, these switches correspond to the nodes of the network, while the rail segments between the switches correspond to the edges of the network. Nodes A, B, and C are the border nodes that define the geographic boundaries of the station.

The station serves the rail commuting needs of the municipality of Siilinjärvi and a significant amount of passenger and freight traffic from the south, east, and west. A precondition for enabling this traffic is that the trains can pass through the station. To do that, trains change tracks to go in different directions. For example, a train from the south can enter the station at border node C and change direction to exit the station at border node B to go to the east. For the train to be able to perform that maneuver, some of the switches at the station must operate. The switches required to change direction depend on the origin and end direction, so some combinations are still possible, even when some switches fail.

The rail segments connecting switches are much less prone to possible disruptions than switches, which are relatively complex systems composed of numerous components whose functioning may be thwarted due to technical failures or adverse weather conditions. From this perspective, it is interesting to determine which switches are particularly important to ensure that the transportation objectives related to providing connections between the three directions defined by the three border nodes can be met.

Some guidance for estimating the probabilities of switch failures can be inferred from the information offered by condition-based monitoring systems [40]. However, in this example, we illustrate the portfolio optimization approach by assuming that switches are equally prone to failure, i.e., the probability of each switch failure is  $p_k = 0.01$ , for the imminent period of operational maintenance. Furthermore, the fortification actions that can be applied to switches are equally costly and equally effective in reducing this probability by 50% ( $p'_k = 0.005$ ). Both assumptions can be readily altered to better reflect the prevailing realities. Yet, the assumption of equal parameters allows for deriving results to better understand the relationship between the switches' relative importance and their relative position within the station.

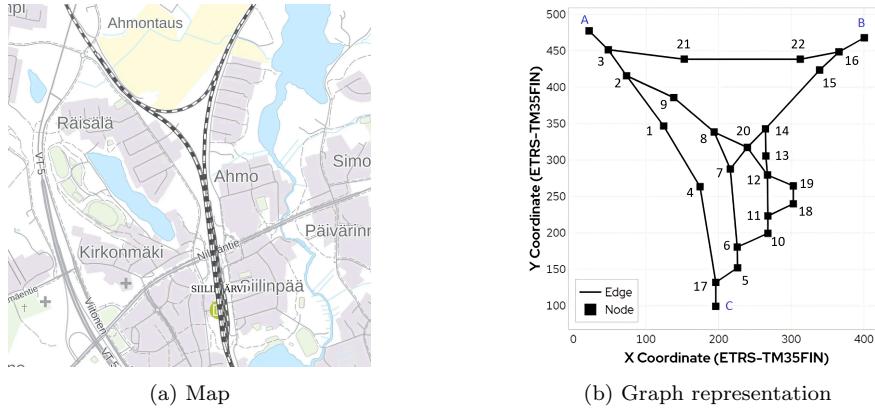


Figure 2: Representation of the Siilinjärvi railway station.

#### 4.1. Station Performance

We evaluate the performance of the station based on the reliability of the connections between pairs of border nodes A, B, and C. The connections are bidirectional, and we name them (A, B), (B, C), and (A, C). A connection  $(X, Y)$  is operational if at least one path connects X with Y without disrupted switches. The reliability of a connection is its probability of being operational. For each connection  $j$ , there is an associated binary utility function (4) representing the objective "connection  $j$  is operational." These three utility functions are combined by the additive utility function (1) to measure the overall performance of the station.

The task of evaluating the utility function (4), which represents the objective of having an operational connection between a pair of border nodes,

is known in the literature as the *terminal-pair reliability problem* [41]. This problem is NP-hard and, therefore, cannot be solved in polynomial time [42]. It can be solved for small networks by checking if a path exists between the pair border nodes for each network state. However, when the number of nodes  $n$  is large, this is impractical because there are  $2^n$  network states, which can become a huge number.

To reduce complexity, we use the minimum cut upper bound approximation [43], recognizing that there are algorithms for computing the exact reliability efficiently even for large networks (see [44] for a survey of recent exact algorithms). If the network size cannot be handled by exact algorithms, or the computation time is critical, one can use approaches such as decomposition approaches [45], Monte Carlo simulation in combination with neural networks [46] or Dijkstra's algorithm [47], or deep neural networks [48], among others.

#### 4.2. Preferences Regarding Connections

We consider two situations representing different specifications of preferences regarding the connections. In the first, the DM does not state preferences about the three connections. This situation is represented by the set  $\mathcal{S}^0 = \{w \in \mathbb{R}^3 \mid w_j \geq 0 \forall j, \sum_{j=1}^3 w_j = 1\}$ .

In the second, the DM provides a ranking of the connections based on their yearly traffic volume: (A,B) at 500 trains/year, (A,C) at 500 trains/year, and (B,C) at 100 trains/year<sup>1</sup>. This is represented by the set  $\mathcal{S}^1 = \{w \in \mathbb{R}^3 \mid w_j \geq 0 \forall j, \sum_{j=1}^3 w_j = 1, w_2 \geq 5w_1, w_3 \geq 5w_1\}$ . For this last situation, we explore how the DM can ensure that the reliability of the individual connections will exceed 95% and 96%. We denote these reliability requirements by  $\mathcal{S}^1 \wedge \alpha = 0.95$  and  $\mathcal{S}^1 \wedge \alpha = 0.96$  respectively.

#### 4.3. Results

##### 4.3.1. Cost-efficient Portfolios

We evaluated portfolios of up to 20 fortification actions, assuming that the cost of each action is the same. The corresponding cost-efficient portfolios, computed with Algorithm 1 for different information sets and minimum reliability requirements, are in Table 1. As a reference, the number of feasible portfolios is provided for different numbers of fortified switches.

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<sup>1</sup>These values are provided for illustrative purposes and are not actual traffic volumes.

Table 1: Number of cost-efficient portfolios for different information sets and reliability requirements.

<b>Fortified switches</b>	1	5	10	15	20
<b>Feasible portfolios</b>	23	35.4k	1.7M	4M	4.2M
<b>Non-informative <math>S^0</math></b>	5	163	972	1.7k	1.9k
<b>Incomplete information <math>S^1</math></b>	2	51	434	673	738
$\mathcal{S}^1 \wedge \alpha = 0.95$	0	27	337	575	640
$\mathcal{S}^1 \wedge \alpha = 0.96$	0	4	116	266	325

When preferences about traffic volume and minimum reliability requirements are provided, the number of cost-efficient portfolios becomes much smaller. Thus, there are fewer viable portfolios of fortification actions, which makes it easier to provide recommendations about these actions.

#### 4.3.2. Reliability of the Connections

The reliability of the connections for the cost-efficient portfolios based on the set  $S^0$  is in Figure 3. Each point corresponds to a different portfolio for a given connection. The marginal reliability improvement gained by one additional switch decreases with the number of actions. This is because the most impactful actions are implemented first and because the actions are of equal cost. However, in general, if the actions are not of equal cost, the marginal improvement may not be decreasing because there could be very impactful actions of high cost that can be implemented only if there are sufficient resources to do so.

#### 4.3.3. Minimum Performance for a Single Connection

The reliability of the connections for all cost-efficient portfolios containing up to five fortified switches is in Figure 4 for the cases of non-informative and incomplete information. In the case of incomplete information, there is relatively little improvement in the reliability of the connection (B, C) because this connection has less traffic than those that include the border node A.

The non-informative set contains the weights  $(1,0,0)$ ,  $(0,1,0)$ , and  $(0,0,1)$ . These weights prioritize a single connection; for example, for the weight  $(1,0,0)$ , the reliability of the first connection is the only one that contributes to the utility function. Therefore, the maximum reliability that can be achieved

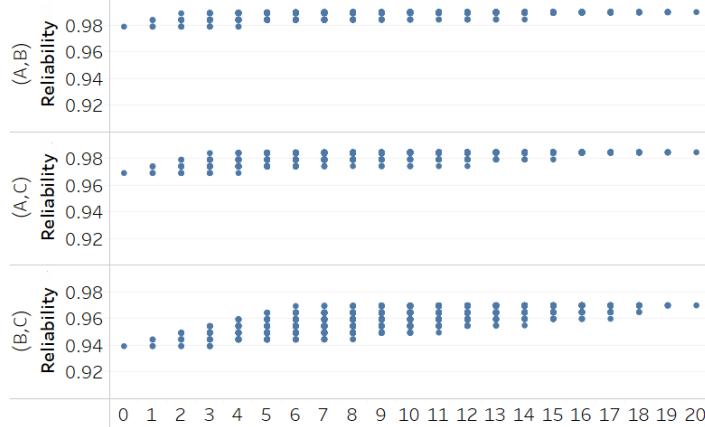


Figure 3: Reliability of the connections as a result of implementing cost-efficient portfolios in the information set  $S^0$ .

for every single connection is given by cost-efficient portfolios for the non-informative set (see Lemma 1). For example, the maximum reliability of the connection (B, C) achieved by fortifying up to five switches is around 0.96.

Moreover, if the minimum reliability requirement is set at 0.95, there are some cost-efficient portfolios in the incomplete information set  $S^1$  that do not meet this requirement, treated as a separate constraint on top of the aggregate cost of fortification actions, which is included in the characterization of otherwise feasible portfolios. If this requirement is tightened to 0.96, it can be met only by portfolios that are not cost-efficient in  $S^1$ .

The analysis of cost-efficient portfolios (two last rows of Table 1), computed with Algorithm 2 subject to the reliability requirements at 0.95 and 0.96, provides information about how many switches must be fortified to fulfill reliability requirements. For example, at least three switches must be fortified to achieve a reliability level of 0.95 and five for 0.96.

#### 4.3.4. Selecting Switches to Fortify

The composition of cost-efficient portfolios can be studied to derive recommendations for the selection of fortification actions. If there is a single cost-efficient portfolio at a budget level (i.e., at a given number of fortified switches), the actions in this portfolio should be selected at this budget level. However, in general, there are many cost-efficient portfolios after

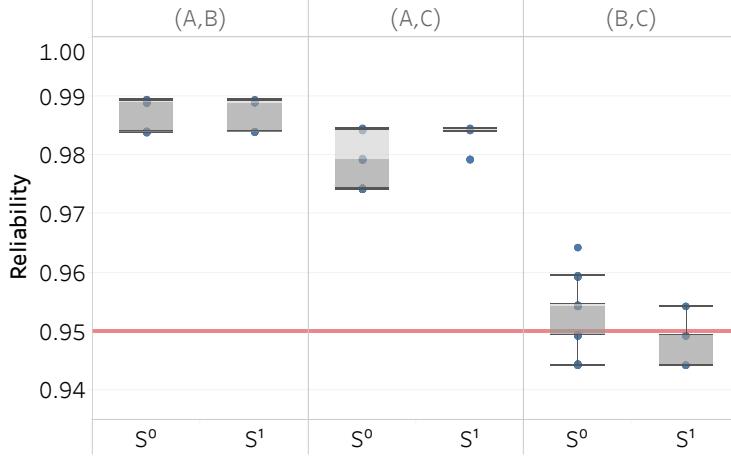


Figure 4: Reliability of connections for cost-efficient portfolios for up to five fortified switches. Values are presented for an incomplete information set ( $S^1$ ) and the non-informative set ( $S^0$ ).

eliciting preference information and introducing reliability requirements. In such cases, the *core index* can be computed to derive recommendations [49]. This index is defined as the relative share of the action in all non-dominated portfolios. By definition, all cost-efficient portfolios with the same cost are non-dominated. We denote by  $\mathcal{Q}_{ND}(c)$  the set of non-dominated portfolios of cost  $c$ . Then, the core index of an action  $q_k$  when there are  $c$  resources is given by

$$CI(q_k, c) = \frac{|\{q \in \mathcal{Q}_{ND}(c) \mid q_k \in q\}|}{|\mathcal{Q}_{ND}(c)|}, \quad (6)$$

where  $|\cdot|$  denotes the cardinality of the set.

At a given budget level, a fortification action whose core index is 1 can be recommended because it is in every non-dominated portfolio. If the core index is 0, the action can be discarded, and the focus can be placed on the other actions. Eliciting additional preferences about the importance of connections between border nodes tends to reduce the number of cost-efficient portfolios and, therefore, change the core index of the actions, too.

Figure 5 shows the core indices of the fortification actions for switches for the two different information sets with and without reliability requirements at different budget levels. If only a few switches can be fortified, there are

no actions with index 1 because these actions do not contribute to improving the reliability of all connections. At higher budget levels, which allow more switches to be fortified, some actions are in the portfolios that improve the reliability of all connections, and thus, their core index becomes 1.

If there are minimum performance requirements, there are far fewer cost-efficient portfolios that satisfy these constraints, too. Thus, the core indices tend to be 1 or 0. Some of the previously recommended actions will be discarded, such as fortifying switch 3 at a budget level of five switches. In contrast, some actions become relevant, such as fortifications of switches 16 and 17, even for small budgets. Thus, performance requirements should be introduced at the outset rather than afterward.

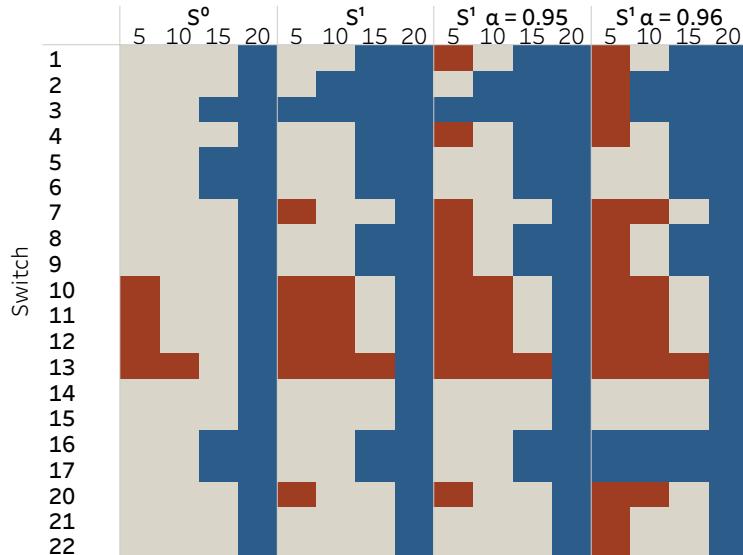


Figure 5: Core index for the individual fortification actions given different number of fortified switches and information sets. Red: core index 0, blue: core index 1, and gray: otherwise.

Centrality metrics such as *degree* or *betweenness* are widely used to quantify the relevance of nodes in a network. For example, Ghorbani-Renani et al. [50] examine five centrality metrics to guide the selection of nodes to be protected in interdependent water, gas, and power networks. However, a concern about using centrality metrics to guide fortification actions is that these metrics do not necessarily reflect the performance of the networks. As we have

shown, the relevance of different switches depends on the DM preferences for the importance of connections and the presence of minimum performance requirements. For example, switch 10 is ranked highly by some centrality metrics (closeness 2nd, degree 3rd, and betweenness 6th), but it is only relevant for fortifying the network if many switches can be fortified.

#### 4.3.5. Impact of Estimated Failure Probability

Although failure probabilities can be estimated from historical data or reliability models, this estimation task may be difficult due to sparse or inaccurate data. This notwithstanding, it can be instructive to provide recommendations based on approximate estimates. Therefore, we assess in the study the impact of the probability of failure on the composition of cost-efficient portfolios for different values of  $p_k$  and  $p'_k$ . The cost-efficient portfolios are here the same when considering failure probabilities<sup>2</sup>  $p_k \in \{0.01, 0.02, 0.03, 0.04, 0.05\}$  and failure probabilities after fortification  $p'_k \in \{p_k/2, p_k/3, p_k/4, p_k/5\}$ .

A special case occurs for perfect fortification at  $p'_k = 0$ , which means that the possibility of disruption is completely eliminated at the node where the action is implemented. In this case, for a given  $p_k$ , there are fewer cost-efficient portfolios than for  $p'_k > 0$ . If  $p'_k = 0$ , the cost-efficient portfolios are the same for  $p_k \in \{0.01, 0.02, 0.03, 0.04, 0.05\}$ .

To understand this, consider the transportation network in Figure 6, where switches 2 and 3 have a failure probability  $p_2 = p_3 = 0.1$ . The action portfolio is  $q = (q_2, q_3)$ . For the imperfect fortification ( $p'_k > 0$ ), there are four cost-efficient portfolios:  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$ ,  $(1, 1)$ . However, if the fortification is perfect, the fortification actions at switch 2 and switch 3 are individually sufficient to guarantee that there is a path between the border nodes. Then<sup>3</sup>,  $\mathbb{E}[u(x) | (1, 0)] = \mathbb{E}[u(x) | (0, 1)] = \mathbb{E}[u(x) | (1, 1)] = 1$ . As  $(1, 1)$  is more expensive than  $(1, 0)$  and  $(0, 1)$ , it is not cost-efficient.

The significance of this result is that recommendations about fortification actions can be made even if the estimated failure probabilities are not

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<sup>2</sup>We do not present higher failure probabilities because the computation of the utility function is based on a cut set method, whose precision decreases for high values. Nevertheless, if required by the data, the accuracy can be improved by adding more detail to this calculation.

<sup>3</sup>We omit the weight in the argument of the utility function because there is an individual objective.

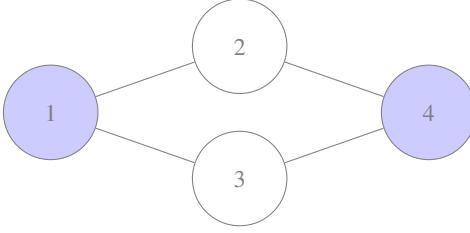


Figure 6: Transportation network with border nodes: 1 and 4; and non-border nodes: 2 and 3.

fully accurate. However, accurate estimates would be needed to assess the network's performance or to compute cost-efficient portfolios that are guaranteed to achieve minimum reliability requirements.

## 5. Discussion

Our methodology assumes that individual node disruptions do not depend on the disruption of other nodes. In some situations, however, the disruption probability of a node may depend on what other nodes have been disrupted: this would be the case, for example, if the failure of a node increases the load on some other node so that the disruption probability of the latter increases. Analyses of such interdependencies could be captured through Bayesian analyses [see, e.g., 51, 52]. Challenges arising from the resulting increase in the number of required parameters can be alleviated by admitting incomplete information about disruption probabilities. This information could be obtained, for example, by asking experts to express statements on verbal scales and mapping each statement to an interval of probabilities [see, e.g., 53].

Using binary variables to model disruptions assumes that nodes are operational or disrupted. To model different levels of node performance more comprehensively, one can introduce multi-valued state variables to capture different gradations of node performance and, for example, to model the capacity that nodes in a transportation network have during peak hours or in the presence of minor road accidents [see, e.g., 54, 55]. A potential challenge with multi-valued variables is that the number of network states grows quickly (e.g., if there are three rather than two states at each node in a network with  $n$  nodes, the number of network states grows from  $2^n$  to  $3^n$ ).

The proposed approach can be extended to build scenarios that characterize external operating conditions. For example, specifying different weather

scenarios could be instructive if the node disruption probabilities depend on the weather. This allows one to explore the robustness of cost-efficient portfolios by assessing which portfolios perform relatively well across all scenarios. Moreover, if probabilities are associated with scenarios, one can gain insight into the expected performance of the network in view of probabilities about scenarios and node disruption.

Although we have showcased the usefulness of the portfolio optimization approach by analyzing a single station, it can be applied to larger systems to study railway networks more comprehensively or even multiple interconnected networks. A critical step towards such extensions can be built through hierarchical models so that the railway network can be modeled at a higher level or aggregation by treating individual railway stations as nodes and railway tracks as edges that connect these stations. The cost-efficient portfolios for the stations can then be combined to explore how resources should be allocated among the stations to contribute to the performance objectives of the network.

Our illustrative example considers a single fortification action per node. However, the same methodology can be used to analyze choices among multiple actions at a single node or even cross-cutting actions that affect the reliability of multiple nodes. From the modeling perspective, these extensions are relatively straightforward in that they can be accommodated through logical constraints. For example, if a given cross-cutting fortification action improves the reliability of multiple nodes, then this is equivalent to our basic formulation subject to the constraint that the corresponding improvement in reliability is attained at all the nodes affected by this action. An additional extension would be to explicitly consider the sequential dynamics of implementing fortification actions. That is, if there are constraints on how many fortification actions can be implemented per unit of time within the planning horizon, the question becomes *in which order* the actions should be implemented so that the performance of the network can be improved as quickly as possible. One could also explore the restoring order of the nodes if the network's performance has been compromised and the aim is to improve its resilience by restoring performance as quickly as possible.

## 6. Conclusion

In this paper, we have developed a multi-objective portfolio optimization approach to support the fortification of infrastructure networks whose nodes

may be disrupted due to events such as natural hazards, technical failures, or intentional attacks. This optimization is based on probabilistic risk assessment in the quantification of disruption impacts and the identification of which nodes of the network should be fortified, given multiple objectives, resource constraints, and possible reliability requirements. Our approach explicitly accounts for the DM's preferences for the different performance objectives of the network. It also accommodates minimum performance requirements regarding these objectives.

The proposed approach opens avenues for further work on strengthening critical infrastructure systems through fortification actions. A relatively straightforward extension is to consider infrastructure networks for the transportation of multiple commodities (e.g., multicommodity rail networks [56, 57]), as such commodities could be prioritized in the same way as the connections between border nodes in this paper. A somewhat more complex extension would be to consider multiple interdependent networks for energy, transportation, and communication because the dependencies between these networks would need to be modeled. The approach could be extended to situations where the disruption probabilities are contingent not only on the selected fortification actions but also on the state of other nodes or even external conditions depicted by scenarios.

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### Appendix A. Upper Bound of the Portfolio Performance

The performance of Algorithm 1 depends on identifying portfolios that cannot be extended to cost-efficient ones. These portfolios should be discarded to avoid the complete enumeration of feasible portfolios. The earlier they are removed, the fewer portfolios that need to be evaluated. The method to compute the extended portfolios  $q^a$  presented in the algorithm is simple and provides an exact upper bound (i.e., the maximum expected utility that can be achieved by a feasible portfolio constructed from the one under analysis) if the portfolios  $q^a$  are feasible. On the other hand, if the portfolios are not feasible, the method can provide weak bounds.

An alternative approach to generate tighter bounds at a higher computational cost is presented. The solution of the optimization problem (A.1) provides an upper bound  $\hat{u}_{w_e}$  of the expected utility at the extreme  $w_e$ , achievable by implementing a feasible portfolio by adding more fortification actions to portfolio  $q$ . Where  $C$  and  $D$  are the indices of the actions implemented and not implemented in the portfolio  $q$ , i.e.,  $C = \{l \mid q_l = 1\}$  and  $D = \{l \mid q_l = 0\}$ .

$$(P_e) \quad \begin{aligned} & \text{Maximize: } u(q, w_e) \\ & \text{Subject to: } \sum_{l=1}^h q_l c_l \leq r \\ & \quad q_l = 1, \quad \forall l \in C \\ & \quad q_l = 0, \quad \forall l \in D \\ & \quad q_l \in \{0, 1\}, \quad l = 1, \dots, h. \end{aligned} \tag{A.1}$$

Let  $q^*$  denote a portfolio whose implementation provides the upper bounds of the expected utility at each extreme point, computed by solving (A.1). If any cost-efficient portfolio dominates portfolio  $q^*$ , the portfolio  $q$  can be discarded because no cost-efficient portfolio can be generated from it. The main advantage of this approach is that the bounds are computed by considering the remaining available resources. One can also incorporate logical constraints into the optimization problem. Nevertheless, solving the problem (A.1) can lead to a different portfolio for each extreme point. If that is the case, it means that  $q^*$  is not necessarily a feasible portfolio, and consequently, the upper bounds are not necessarily exact. Since  $q^a$  implies implementing all the remaining fortification actions, the upper bounds provided by  $q^*$  are always lower or equal (worst case) than the ones provided by  $q^a$ .

The efficiency of the methods depends on several parameters, such as the utility function, available budget, cost of the fortification actions, etc. For example, optimization problems with complex utility functions may be hard to solve, making the computation of  $\hat{u}_{w_e}$  impractical. In other cases, where the budget is small and just a few actions can be implemented, the bounds provided by  $q_a$  can be very loose. One could also complement the methodologies by applying Algorithm 1 as presented and switch to the optimization model on the points where the portfolios  $q^a$  are infeasible.

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