Dinamica dei Veicoli

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1 Introduction

The aim of this project was to analyze the telemetry data provided by prof. M.Guiggiani taken from a lap of Circuit of Barcelona run by a GP2 race car. This circuit has long straights and sixteen curves. The circuit is shown in Fig. 1.

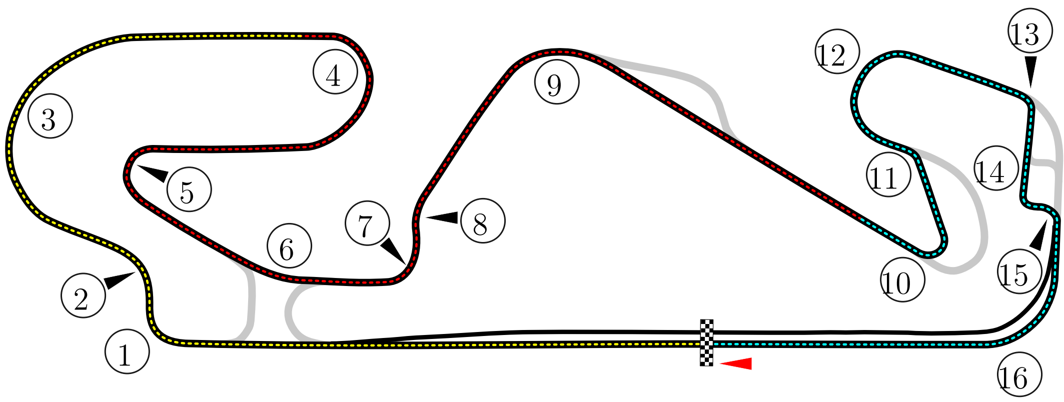


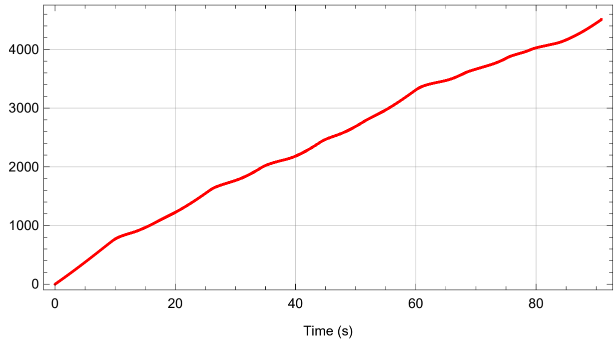
Figure 1: Circuit of Barcelona. Source: Wikipedia

The telemetry provided includes the following data: time, lap distance, longitudinal velocity, longitudinal acceleration, lateral acceleration, yaw rate, front wheel steer angle and body slip angle. The next few sections will explain how this data was filtered and converted into SI units. Some quantities necessary to analyze the telemetry will be calculated. These are the lateral velocity, the yaw angle and the angular acceleration. The last step was the telemetry analysis, which includes the drawing of the center of mass trajectory, the plot of all centrodes with and without the inflection circle, the g-g plot and the computation of the vehicle slip angle integrating numerically the lateral velocity.

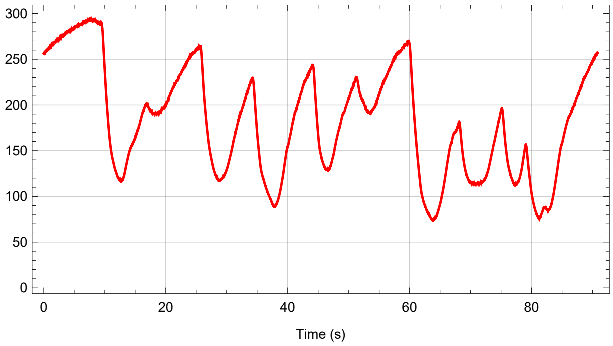
All the formulas used in this report were taken from [1, The Science of Vehicle Dynamics], therefore they will just be referenced here.

2 Raw Data

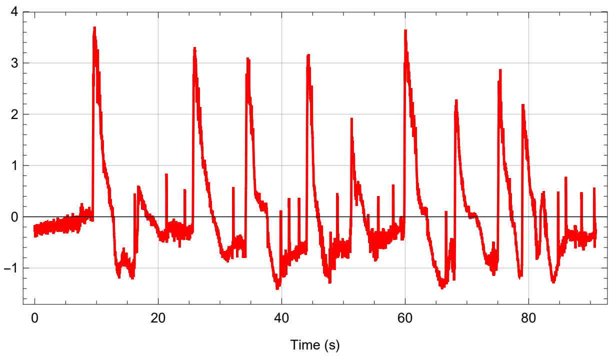
The raw data are shown in Fig. 2. These data are obtained directly from the sensors, therefore they include noise. Some of them need to be filtered appropriately, because they will be used to compute other signals, e.g. the lateral acceleration. The following graphs show the unfiltered signals.



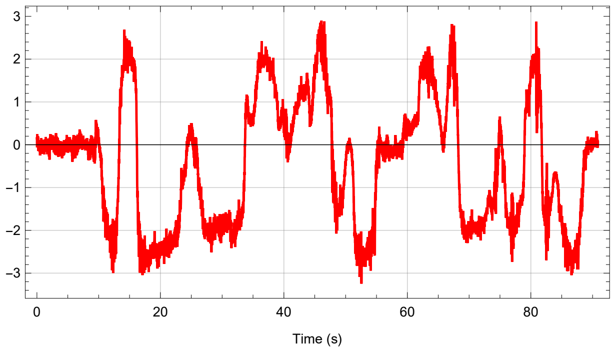
(a) Lap distance in m, versus time in s



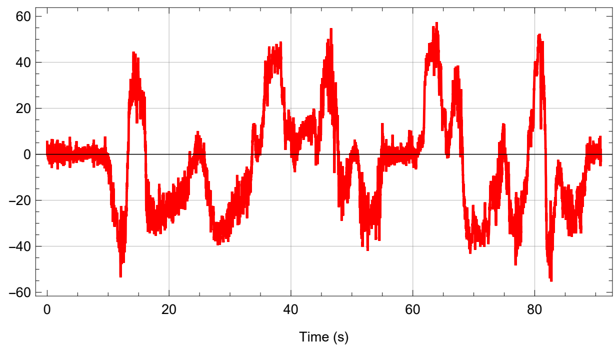
(b) Forward velocity in km/h, versus time



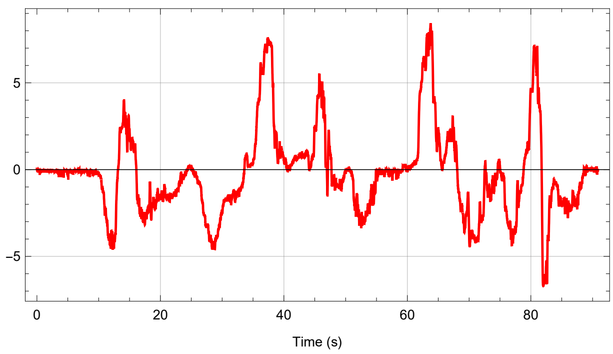
(c) Longitudinal acceleration in g, versus time



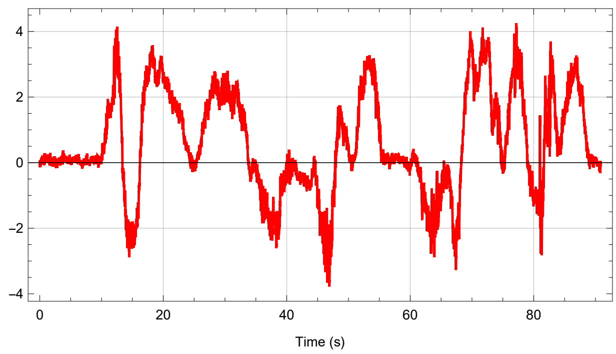
(d) Lateral acceleration in g, versus time



(e) Vehicle yaw rate in deg/s, versus time



(f) Front wheel steer angle in deg, versus time



(g) Vehicle slip angle in deg, versus time

Figure 2: Telemetry of a lap run by a GP2 race car

Some signs need to be adjusted and some conversions in the international system units need to be made.

3 Filtering

The filtering operation was done, when necessary, in a way that preserved most of the information, using a low pass filter for each filtered signal.

The longitudinal velocity shows a small noise that is removable with a low pass filter with $\omega_c = 0.1$ rad/s. This filtered signal was used to compute the lateral velocity, as it will be discussed later.

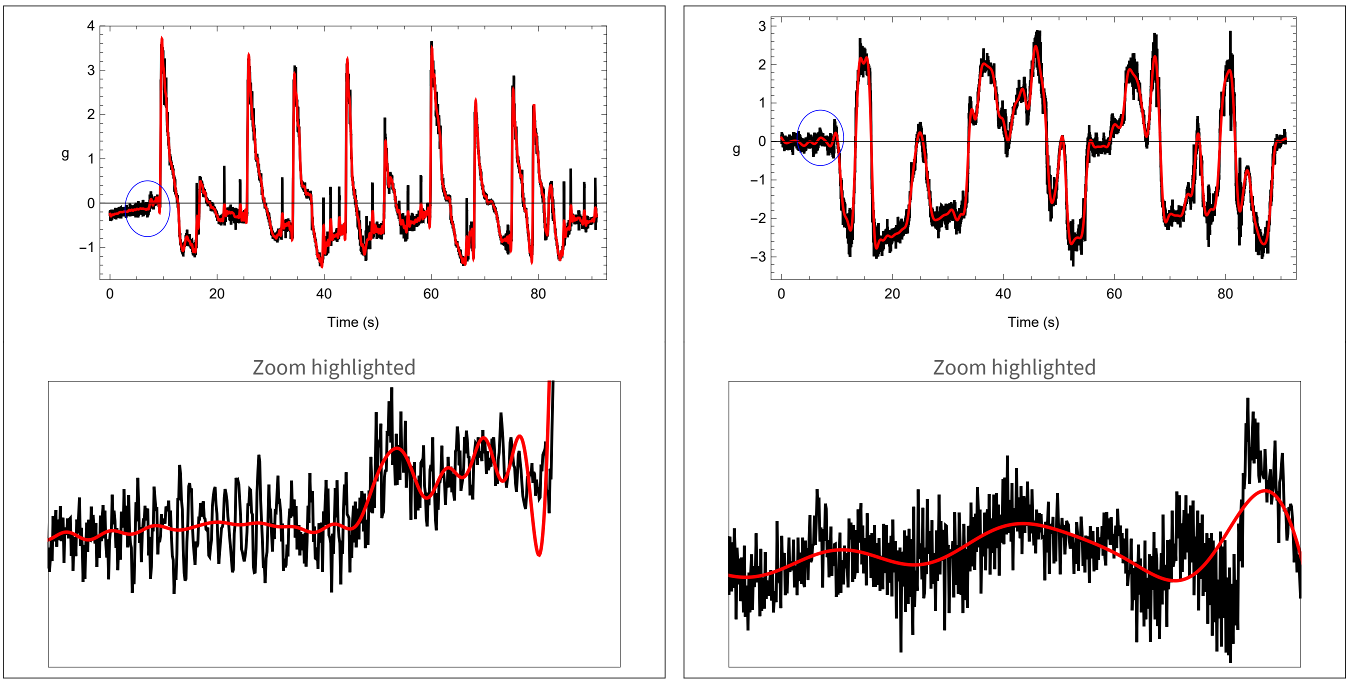
Both accelerations are noisy and since they are very important data they were filtered using the following parameters:

- Longitudinal acceleration: $\omega_c = 0.3$ rad/s. The result is shown in Fig. 3a, where the highlighted area in blue in the upper figure is zoomed below.
- Lateral acceleration: $\omega_c = 0.08$ rad/s. The result is shown in Fig. 3b. Again, the highlighted area in the upper figure is zoomed below.

Both signals may appear raw, but the results obtained proved to be sufficient. Note that the longitudinal acceleration still needs to be flipped. This operation, like other conversions, will be performed later.

The yaw rate is integrated in order to obtain the yaw angle and, for this purpose, the raw signal is used, but it will be filtered in order to compute the angular acceleration, all cetrodes and the inflection circle. Therefore, after the integration to obtain the yaw angle, this signal was filtered with a low pass filter having $\omega_c = 0.1$ rad/s. The result is shown in Fig. 4c.

The steer angle is not used to calculate other signals, but it has been filtered anyway because it was used to verify what the driver did during the lap. It was filtered with $\omega_c = 0.19$ rad/s. The final signal is shown in Fig. 4a.



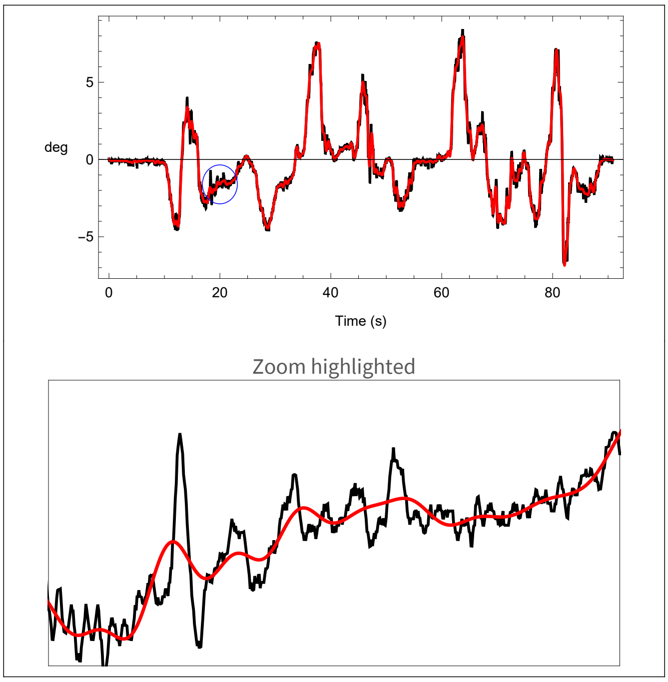
(a) Longitudinal acceleration, original versus filtered

(b) Lateral acceleration, original versus filtered

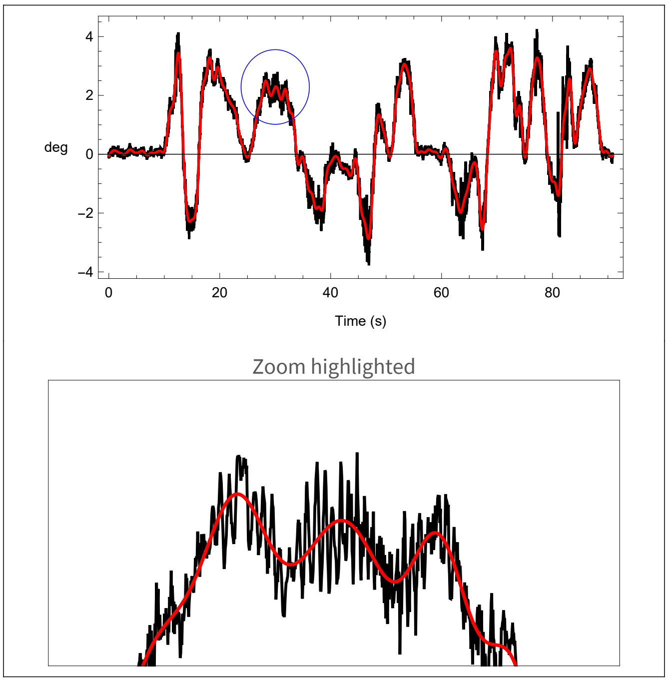
Figure 3: Comparisons between the original signals (black lines) and the filtered data (red lines)

The vehicle slip angle, the last filtered signal, will be used to compute the lateral velocity with (1), therefore it has been filtered with $\omega_c = 0.1$ rad/s. The result is shown in Fig. 4b.

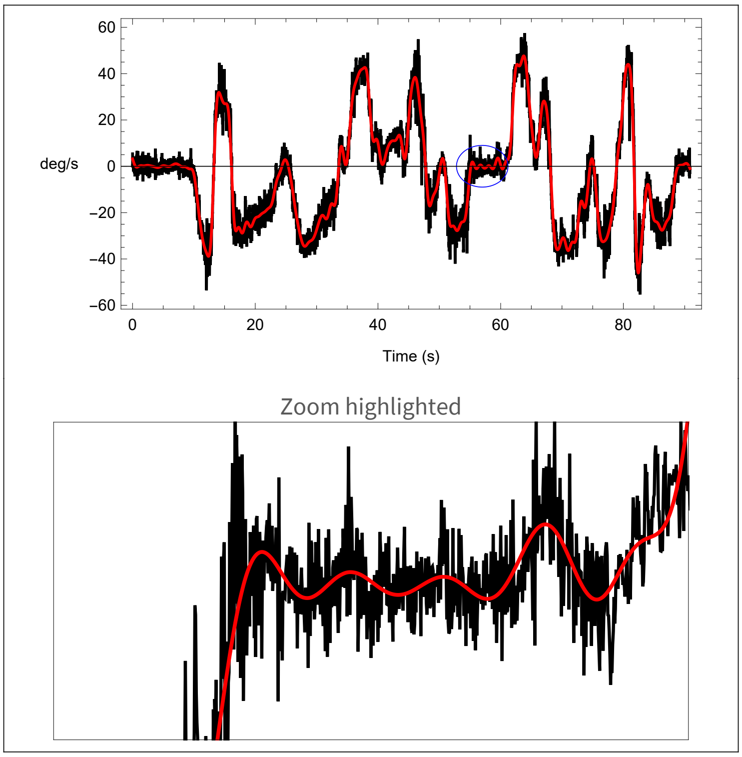
The other signals do not need to be filtered because they do not have noise.



(a) Front wheel steer angle, original versus filtered



(b) Vehicle slip angle in deg, original versus filtered



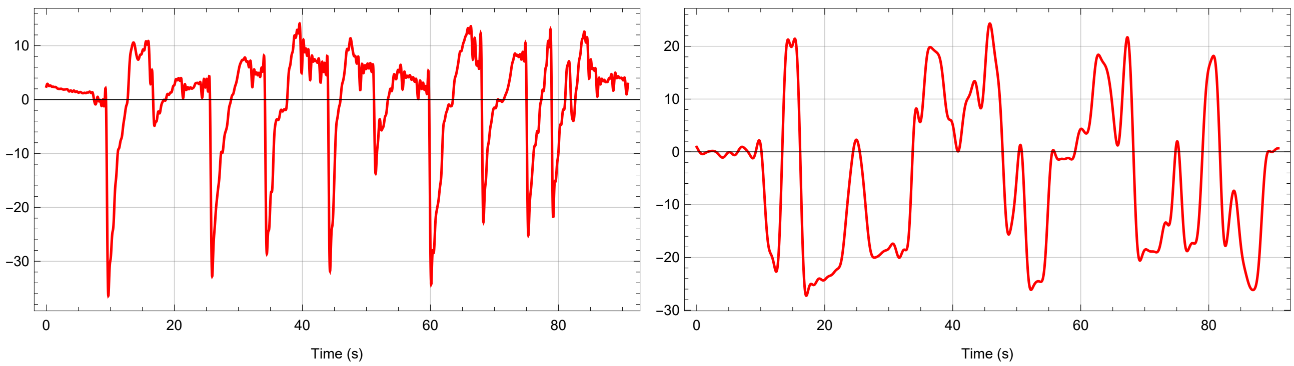
(c) Vehicle yaw rate, original versus filtered

Figure 4: Comparisons between the original signals (black lines) and the filtered data (red lines)

4 Conversion into SI

After filtering the signals, the next step was to convert the data. The longitudinal acceleration formula was therefore reversed, resulting in the braking having a negative sign. The unit of measurement in both accelerations was changed from g to $\mathrm{m/s^2}$. The longitudinal velocity was converted from km/h to m/s. The front wheel steer angle and the vehicle slip angle have been converted from degrees to radians. The yaw rate was converted from deg/s to rad/s.

The result is shown in Fig. 5, where only the accelerations are reported for brevity. The other signals, as well as the accelerations, have been simply converted, so there has been no alteration of the signals, apart from their module being scaled.



(a) Longitudinal acceleration in $\mathrm{m/s^2}$ (b) Lateral acceleration in $\mathrm{m/s^2}$

Figure 5: Acceleration conversion

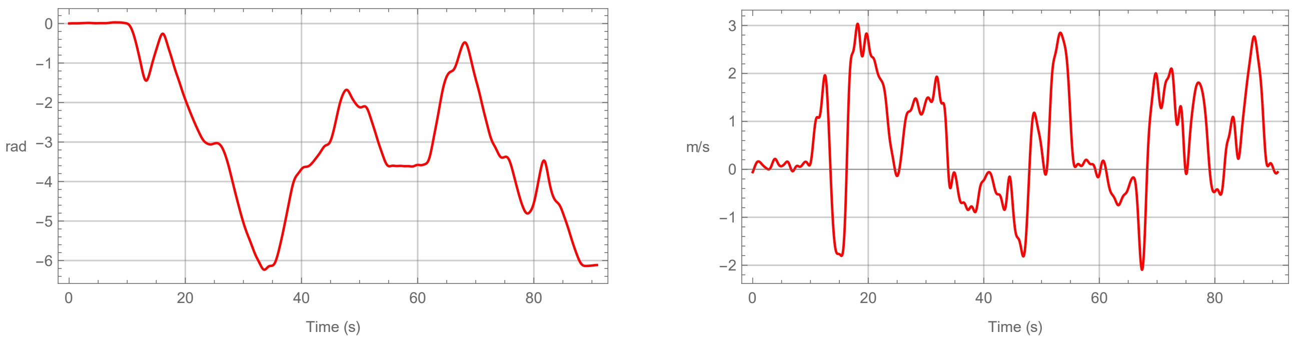
5 Telemetry Analysis

5.1 Yaw Angle and Lateral Velocity

The yaw angle, as by [1, eqn. 3.8], is the integration of the yaw rate, which is provided in the telemetry. It is shown in Fig. 6a.

The lateral velocity was computed by inverting [1, eqn. 3.16] and [1, eqn. 3.18]. Therefore the formula used is (1).

$$v = u \tan \hat{\beta} \tag{1}$$



(a) Yaw angle (b) Lateral velocity

Figure 6: Computed signals

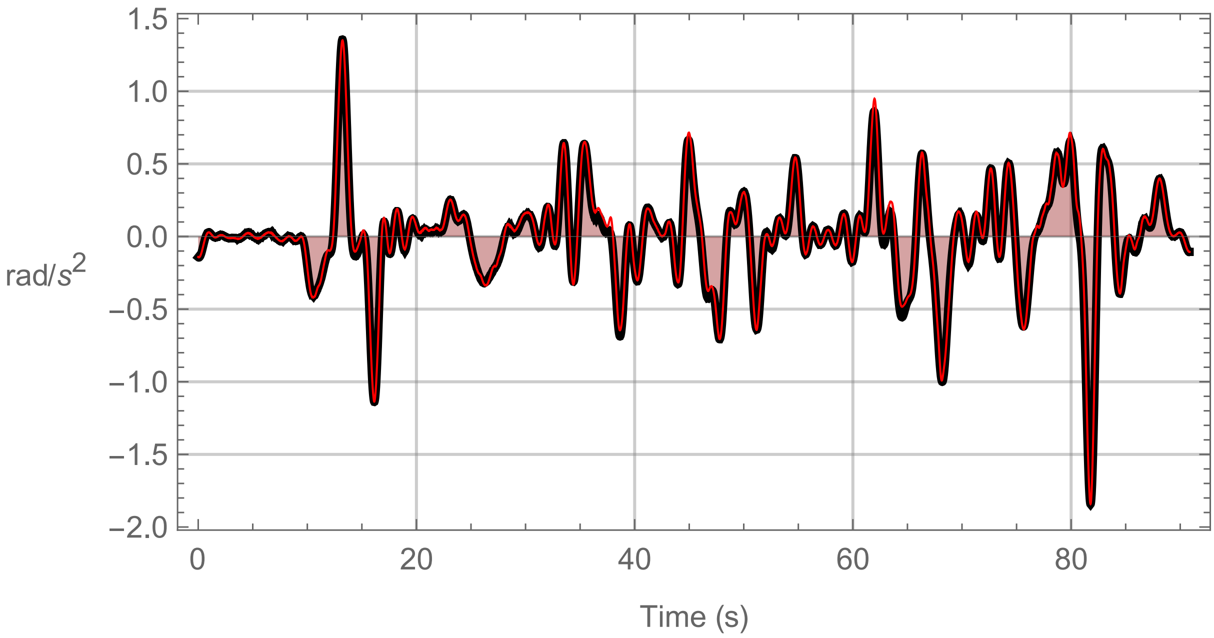
5.2 Angular Acceleration

For the angular acceleration [1, eqn. 3.23] was used. From [1, eqn. 3.26], $\dot{u}$ was computed. The ratio ρ was define as $\rho = r/u$, while $\dot{\rho}$ has been differentiated numerically. This way, it is possible compute the angular acceleration.

There is an easier way to get the angular acceleration, that is differentiating as (2).

$$\dot{r} = \ddot{\psi} = \frac{\mathrm{d}}{\mathrm{d}t} r \tag{2}$$

The comparison between the two methods is shown in Fig. 7. The signals have been graphed with different thicknesses so that they are both visible, since they are very similar.



(a) Comparison between $\dot{r} = u\dot{\rho} + \dot{u}\rho$ (red line) and the differentiate of the yaw rate $\dot{r} = \ddot{\psi}$ (black line)

Figure 7: Angular acceleration

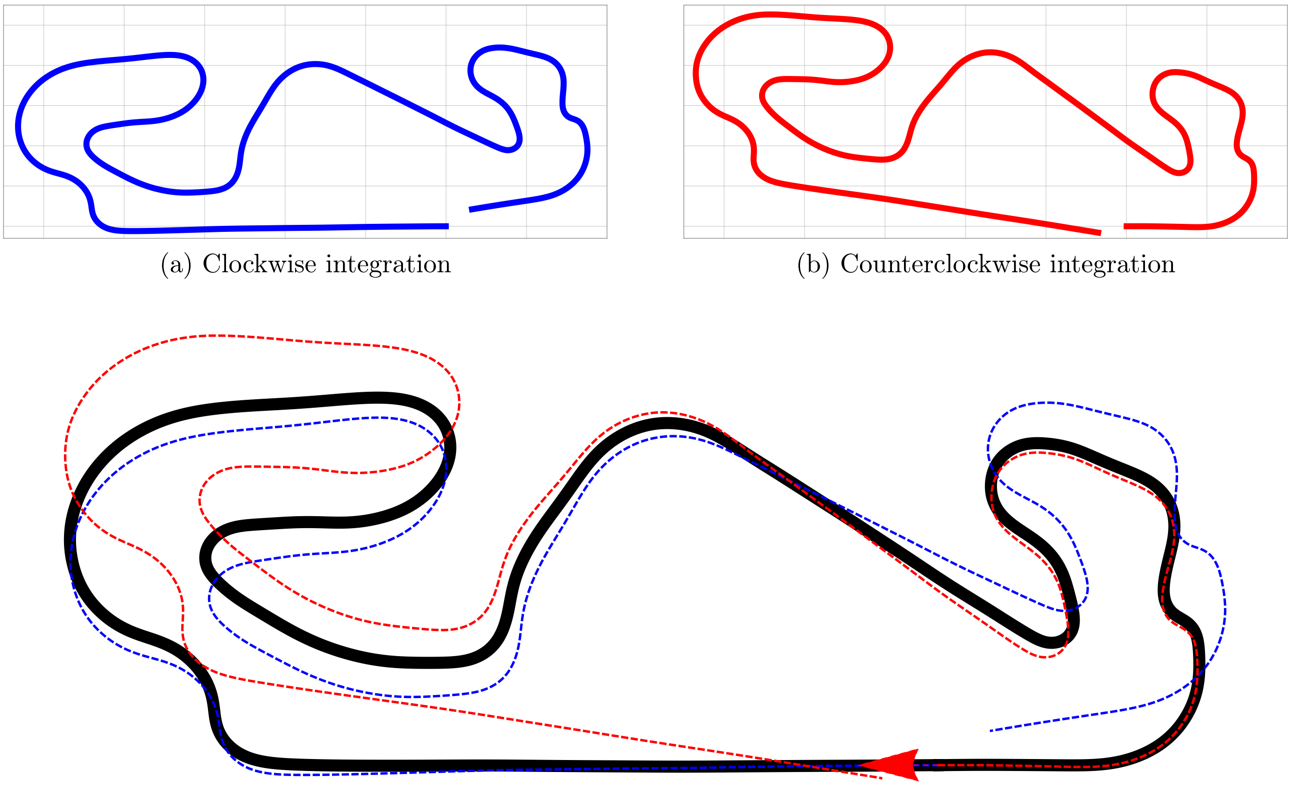
5.3 Trajectory of the Center of Mass G

Having the yaw angle, the longitudinal and the lateral velocity, it was possible to calculate the velocity of the center of mass $\mathbf{V}_G$ with (3) or, in an equivalent way, with [1, eqn.3.7]. This equation provides the velocity of the G point, expressed in the ground-fixed reference frame.

$$\mathbf{V}_G = \begin{pmatrix} \dot{x}_0^G \\ \dot{y}_0^G \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{v} \end{pmatrix} \tag{3}$$

Now integrating the vector, according to [1, eq. 3.9], the points of the trajectory performed by the vehicle were obtained. But, as shown in Fig 8a, the error grows as the integration progresses. This phenomenon has been treated in [1, Sect. 7.7.1] and it represents an error due to the sensors quality. If the integration is made in the opposite direction, the result obtained is the same: the error increases with the progression of the integration, but from the opposite direction, as shown in Fig. 8b. By making a weighted average of the two trajectories it is possible to obtain the correct (and closed) trajectory.

The final trajectory is shown in Fig. 8c, where the dashed lines represent the two trajectories obtained by integration (clockwise and counterclockwise), the black one is the weighted average of the two integrations.



(a) Clockwise integration

(b) Counterclockwise integration

(c) Weighted average of the two trajectories

Figure 8: Trajectory of the center of mass

5.4 Curve Identification

The curves were obtained using the lateral acceleration. The procedure has been:

1. Further lateral acceleration filtering
2. Absolute value computation
3. Scaling by an empirical value

This procedure is represented by (4), where the value 10.75 is an empirical threshold expressed in $\mathrm{m/s^2}$, $\tilde{a}_\mathrm{y}$ symbolizes the further lateral acceleration filtering, $\hat{a}_\mathrm{y}$ is the result. After this procedure, the values above this threshold identify all the curves. It is possible to discriminate the points belonging to a curve with (5), applying it to $\hat{a}_\mathrm{y}$. This way, a function $f(t)$ (6) can be built that will be worth 1 when G is in a curve, zero otherwise.

The output of this function is represented in Fig. 9(top) with a yellow line, which is high if the lateral acceleration is greater than the threshold, and therefore we can assume we are cornering, low otherwise.

In the bottom part of the same figure, the previously found trajectory is shown and the identified curves are marked in red.

This procedure depends on two parameters: the filtering value and the threshold. It is not possible to establish them a priori, but a trial and error operation is necessary. The filtering process, in this case as before, was made so that the fundamental information was preserved.

$$\hat{a}_{\mathrm{y}} = |\tilde{a}_{\mathrm{y}}| - 10.75 \tag{4}$$

$$F(x) = \frac{\operatorname{sgn}(x) + 1}{2} \tag{5}$$

$$f(t) = \begin{cases} 1 & \text{if } F(\hat{a}_y) > 0 \\ 0 & \text{else} \end{cases} \tag{6}$$

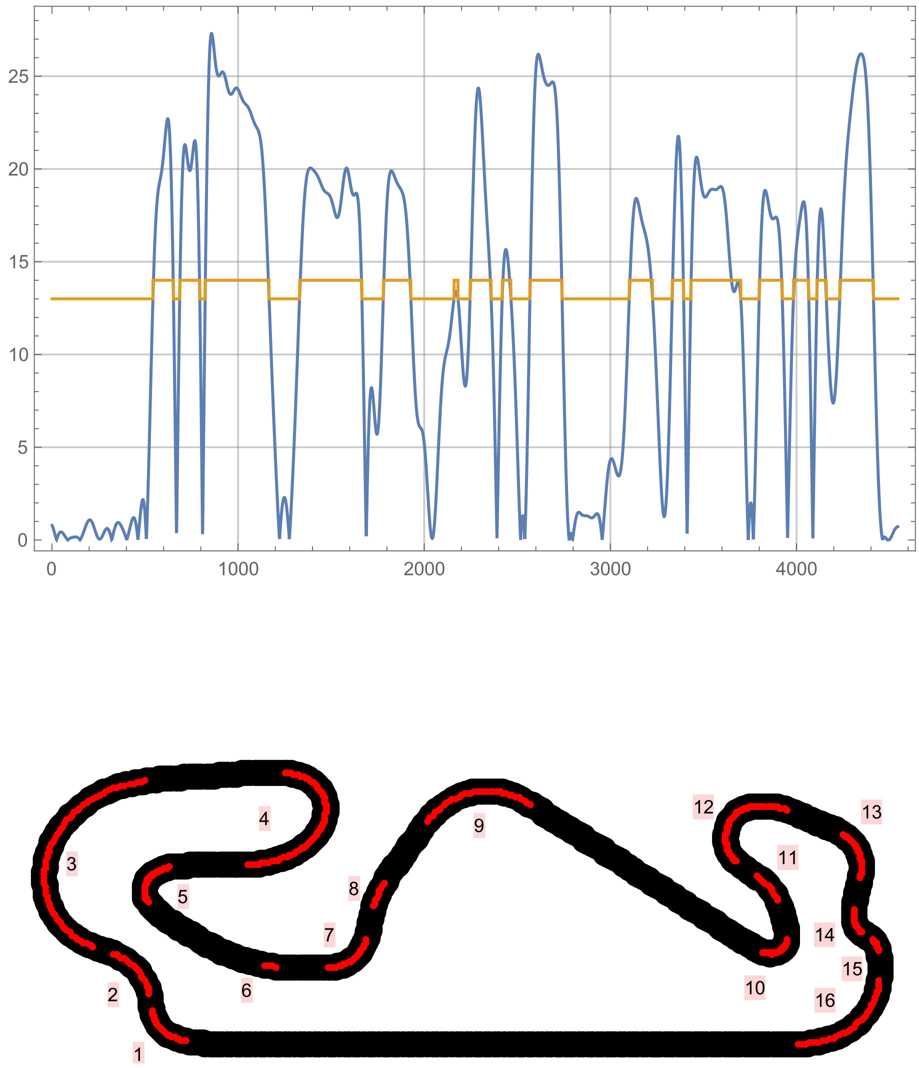


Figure 9: Curves identification

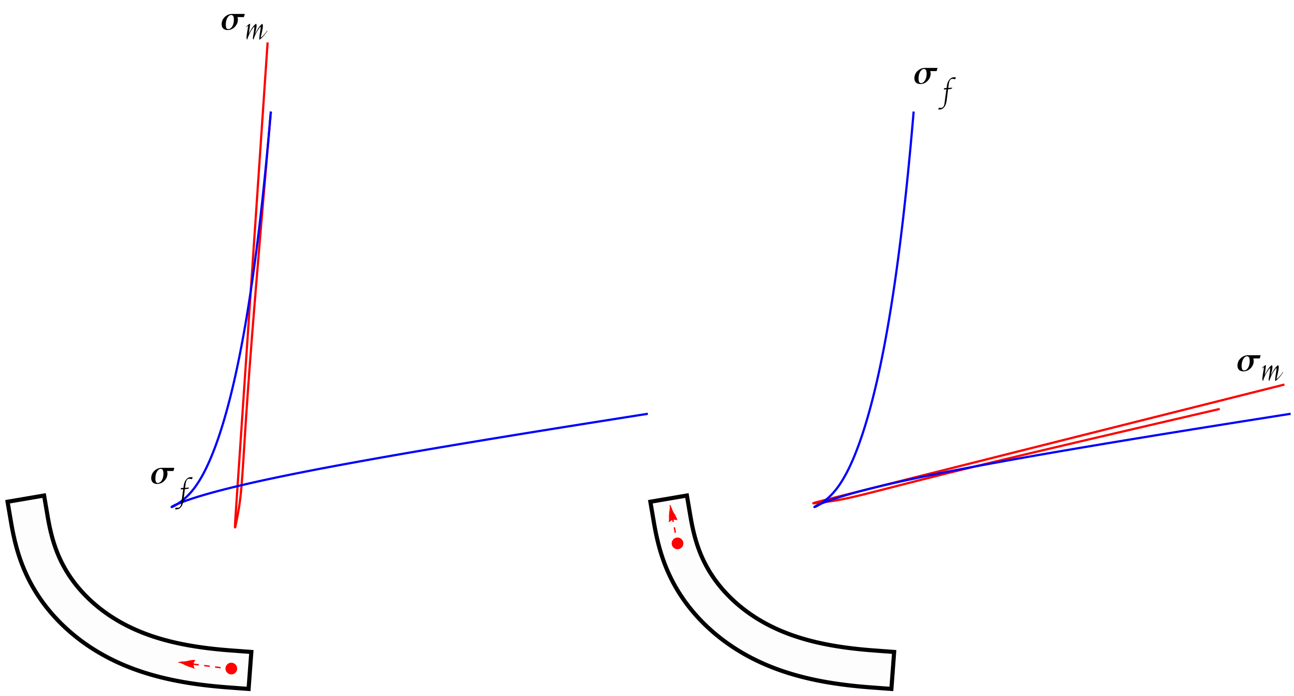
5.5 Inflection Circle, Moving and Fixed Centrodes

The equations used in this section were taken from [1, Sect. 5] of the reference book. The centrodes of all the curves have been calculated, however only two will be shown in order not to extend the report too much.

5.5.1 Moving and Fixed Centrodes

The moving and fixed centrodes, defined in [1, Sect. 5.2.1], were implemented by calculating the lengths R and S, which are defined in [1, eqn. (3.13)] and [1, eqn. (3.14)].

[1, eqn. (5.12)] was used for the fixed centrode σ_{f} and [1, eqn. (5.13)] for the moving centrode σ_{m} expressed in ground-fixed reference system. Both centrodes of turn 1 of the Barcelona circuit are shown in Fig. 10, where it is possible to gather from the shape of the fixed and moving centrodes that the driver addressed the curve correctly. Other curves will be shown with the inflection circle.



(a) Vehicle entering the curve

(b) Vehicle exiting the curve

Figure 10: Vehicle entering and exiting turn 1 of the Barcelona circuit

5.5.2 Inflection Circle

The inflection circle was defined in [1, Sect. 5.2.2]. The proposed equations allow various methods for calculating the inflection circle. For this reason, the equations used will be reported in this section. The procedure used here is the following:

1. Compute $\dot{R}$ and $\dot{S}$ with (7) and (8) and not with (9) and (10), according to what is stated in [1, Sect. (3.2.8)]
2. Use (11) and (12) to compute the diameter of the inflection circle and its vector
3. Get the radius of the inflection circle, here defined as r_{if}, and its center $\mathbf{c}_{\mathrm{if}}$

In (14) $\mathbf{R}_{\psi}$ is the transpose of a suitable rotation matrix dependent on the yaw angle, and $\binom{x_{\mathrm{f}}}{y_{\mathrm{f}}}$ is a point, time variant, belonging to the fixed centrode.

This procedure was done for each curve.

$$\dot{R} = \frac{a_{\mathrm{x}} - r^2 S - \dot{r} R}{r} \tag{7}$$

$$\dot{S} = -\frac{a_{\mathrm{y}} - r^2 S - \dot{r} R}{r} \tag{8}$$

$$\dot{R} = \frac{\dot{u} r - u \dot{r}}{r^2} \tag{9}$$

$$\dot{S} = -\frac{\dot{v} r - v \dot{r}}{r^2} \tag{10}$$

$$d = \sqrt{\frac{\dot{R}^2 + \dot{S}^2}{r^2}} \tag{11}$$

$$\mathbf{d} = \frac{\dot{R}\mathbf{i} + \dot{S}\mathbf{j}}{r} \tag{12}$$

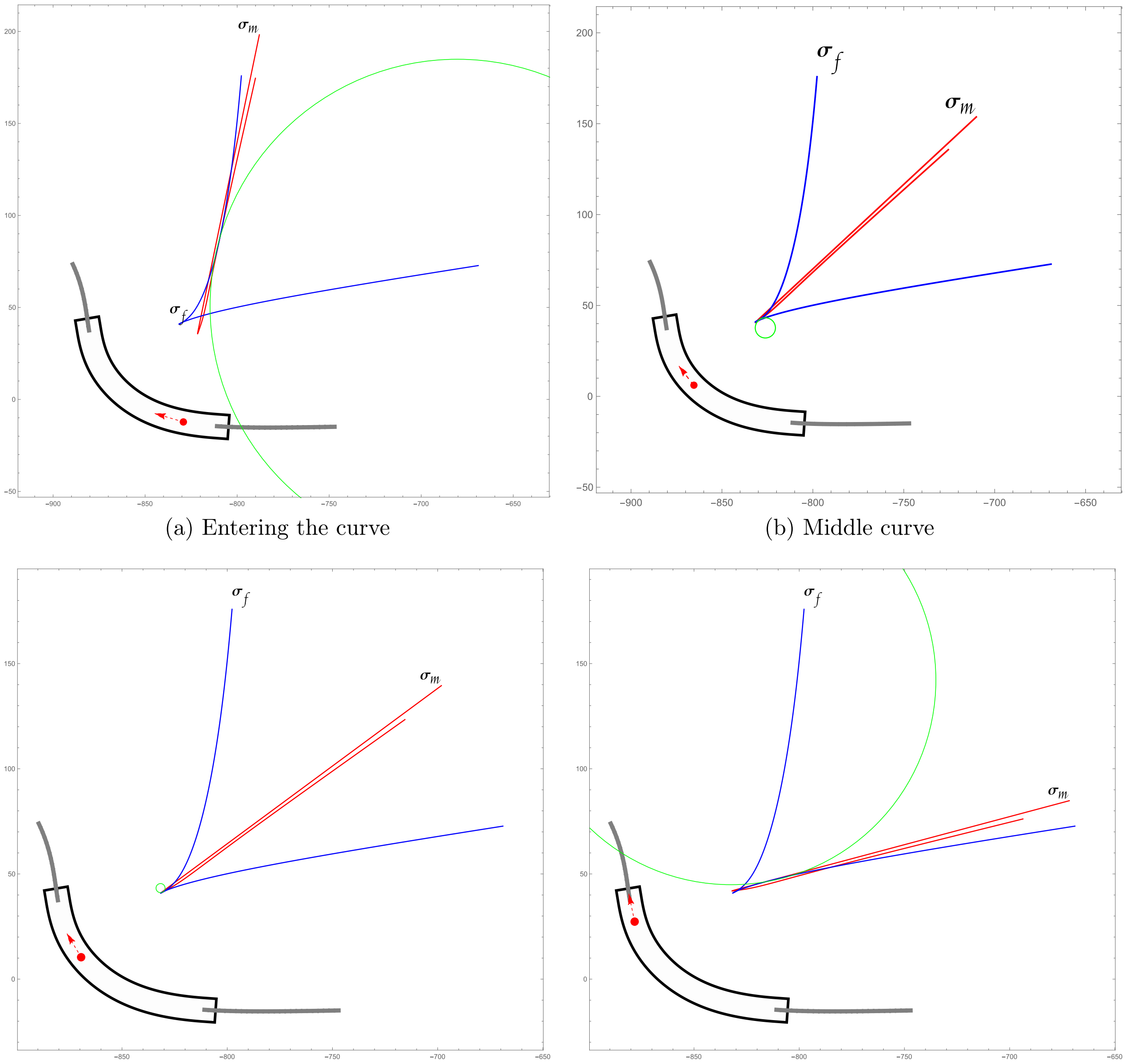
$$r_{\mathrm{if}} = \frac{d}{2} \tag{13}$$

$$\mathbf{c}_{\mathrm{if}} = \begin{pmatrix} x_{\mathrm{f}} \\ y_{\mathrm{f}} \end{pmatrix} + \frac{1}{2}\mathbf{R}_{\psi}\mathbf{d} \tag{14}$$

The results obtained are shown in Fig. 11, where the inflection circle is shown for the first corner, in entering and exiting the curve.

In Fig. 12, the centrodes and the inflection circle of turn 13 are shown. Only two curves are shown not to make this report too long.

Again, the inflection circle was calculated for all curves.



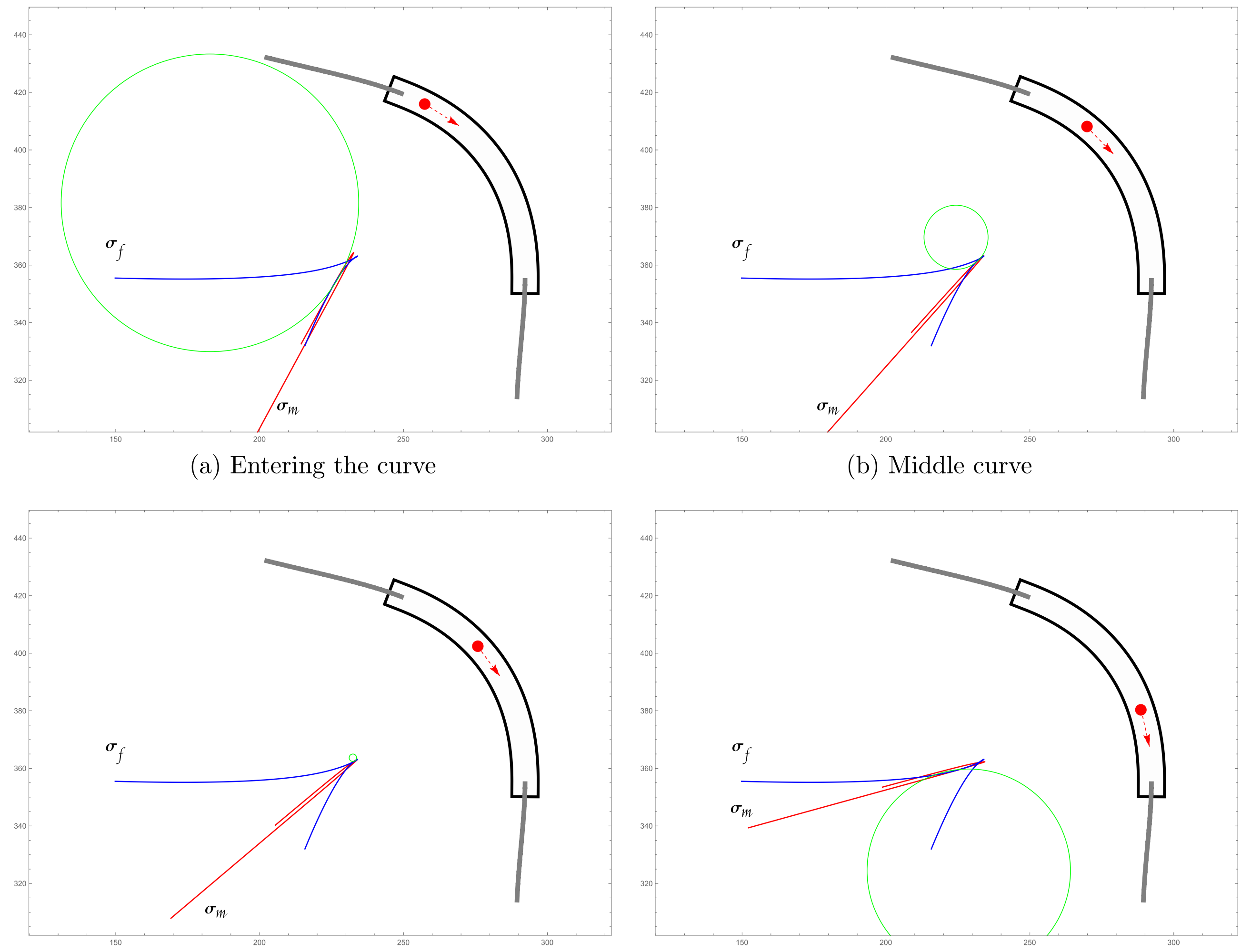
(a) Entering the curve

(b) Middle curve

(c) Middle curve

(d) Exiting the curve

Figure 11: Inflection circle, moving and fixed centrodes of the first curve of Barcelona



(a) Entering the curve

(b) Middle curve

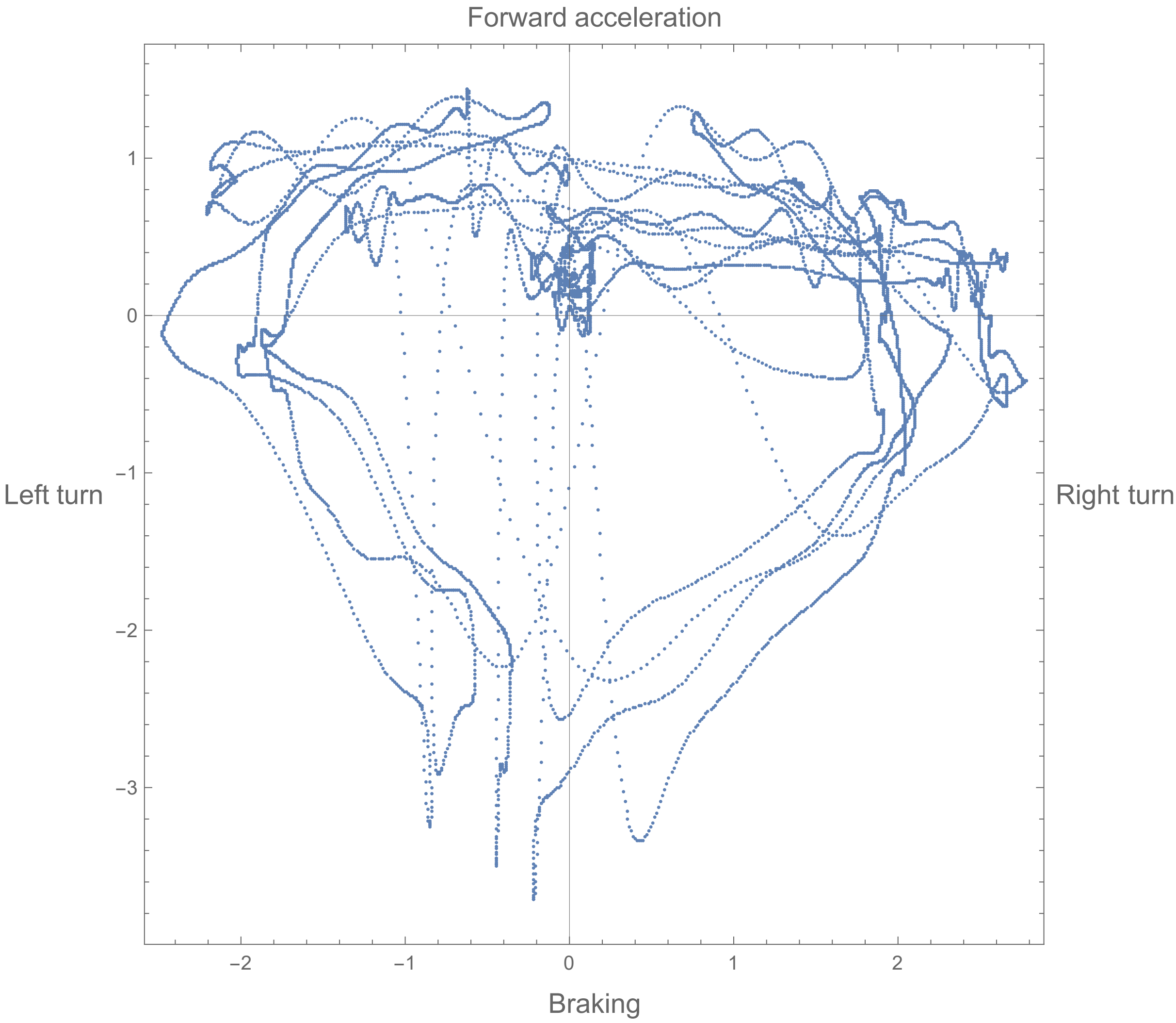
(c) Middle curve

(d) Exiting the curve

Figure 12: Inflection circle, moving and fixed centrodes of turn 13 of Barcelona

6 gg Plot

The gg-plot was graphed by inverting the sign of the lateral acceleration in order to have on the positive side of the x axis the right turns and on the left side the left turns. The gg-plot is shown in Fig. 6. As we expected there are more points on the right side because this circuit is run clockwise.



(a) gg-plot

7 Vehicle Slip Angle

To obtain the vehicle slip angle from other signals, [1, Sect. 3.2.4] and [1, Sect. 3.2.5] have been taken as a reference.

The vehicle slip angle is defined in [1, eqn. 3.18], where the ratio β is defined in [1, eqn. 3.16]. These formulas are reported for clarity in (15) and (16).

$$\hat{\beta} = \arctan \beta \tag{15}$$

$$\beta = \frac{v}{u} \tag{16}$$

$$a_{\mathrm{y}} = \dot{v} + ur \tag{17}$$

$$\dot{v} = a_{\mathrm{y}}^{\mathrm{tel}} - \hat{a}_{\mathrm{y}} = a_{\mathrm{y}}^{\mathrm{tel}} - ur \tag{18}$$

$$v^{\mathrm{diff}} = v_{\mathrm{d}} - \hat{v}_{\mathrm{d}} \tag{19}$$

Suppose that the lateral velocity is not available. It is possible to obtain it using [1, eqn. 3.27], here reported as (17) and rewritten as (18), where $a_{\mathrm{y}}^{\mathrm{tel}}$ represents the lateral accelerations a_{y} taken from the telemetry and $\hat{a}_{\mathrm{y}}$ is the lateral acceleration under steady-state conditions ur. The comparison between the lateral acceleration and ur is shown in Fig. 13.

Now, integrating numerically $\dot{v}$, a "drifted" v_{d} were obtained, as shown in Fig. 14a, where the desired lateral velocity computed with the telemetry data using (1), here called v^{tel}, the previously calculated $\dot{v}$ and the same signal suitably filtered $\hat{\dot{v}}$ are shown. The v^{diff} was computed as per (19), therefore this signal was used in (16).

Finally it is possible to apply (15). The result is shown in Fig. 14b. The disadvantage of this procedure is that it depends on the filtering operations applied to $\hat{v}_{\mathrm{d}}$.

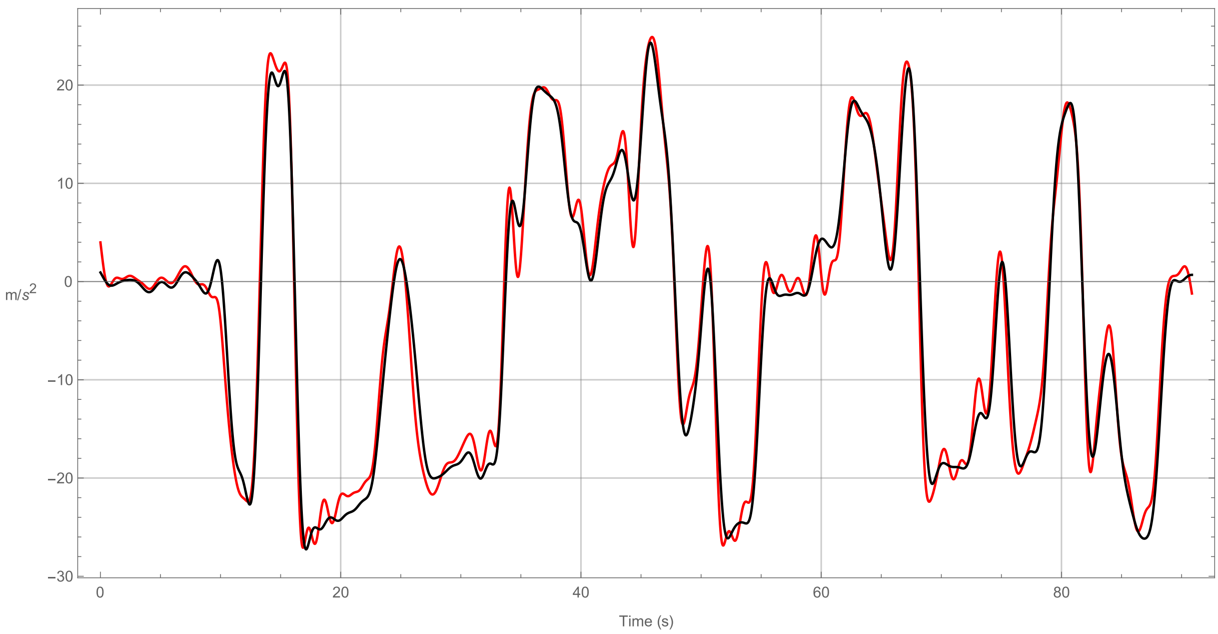
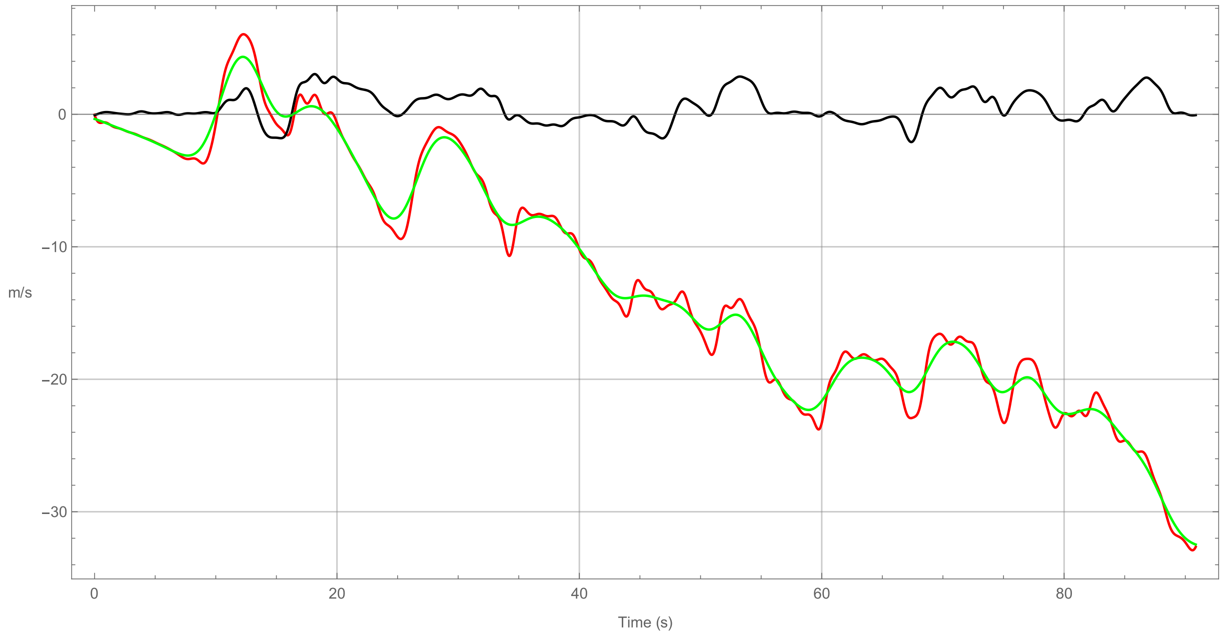
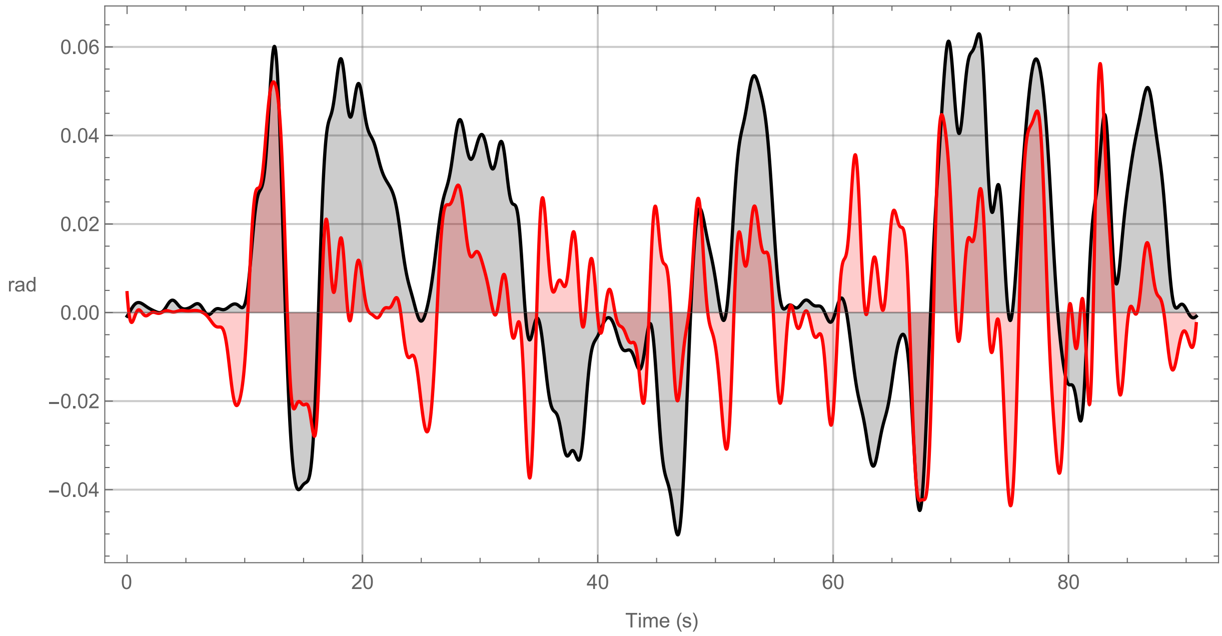


Figure 13: Comparison between the lateral acceleration given in the telemetry (red line) and lateral acceleration under steady state conditions (black line)



(a) Comparison between v^{tel} (black line), the computed v_{d} (red line) and $\hat{v}_{\mathrm{d}}$ (green)



(b) Comparison between $\hat{\beta}$ taken from telemetry (black line) and the computed $\hat{\beta}$ (red line)

Figure 14: Procedure to compute $\hat{\beta}$

References

[1] Massimo Guiggiani. *The Science of Vehicle Dynamics*. Springer, 2018.