Optimization and equilibrium problems with discrete choice models

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Choices

- People have **different socioeconomic characteristics and tastes** that influence their **choices**.
- **Disaggregate models** of demand can capture this heterogeneity.

Discrete choice models

- **Discrete choice models** are used to **predict and forecast** choices of decision makers who select one out of a finite set of alternatives.
- They are probabilistic models, generally **non-linear and non-convex**.
- Choice probabilities of advanced DCMs cannot be expressed with a closed form.
- Difficult to integrate into **supply optimization** and **market equilibrium** models.

Choice-based optimization and choice-based equilibrium

- *Dominant paradigm:*

  Sacrifice complexity at the demand level to obtain tractable optimization and equilibrium problems.

- *Complementary view:*

  Models and algorithms that **accommodate advanced discrete choice models**.

Value of advanced discrete choice models

Some reasons to take this alternative point of view:

1. **Specification tests**: quantifiable trade-off between different demand models.
2. Estimation from increasingly large data sets $\rightarrow$ **richer specifications**
3. Disaggregate choice-based models $\rightarrow$ **differentiated offers and policies** to target specific groups of the population.

Positioning the doctoral thesis

Research directions

1. Development of mathematical models and algorithms for **choice-based equilibrium problems**.
2. Identification of problem reformulations and efficient algorithmic approaches for **choice-based optimization problems**.

Outline of the thesis

Chapter 2

Based on the article

Bortolomiol, S., Lurkin, V., Bierlaire, M. (2021). **A simulation-based heuristic to find approximate equilibria with disaggregate demand models**. *Transportation Science*, 55(5):1025–1045.

Chapter 3

Based on the article

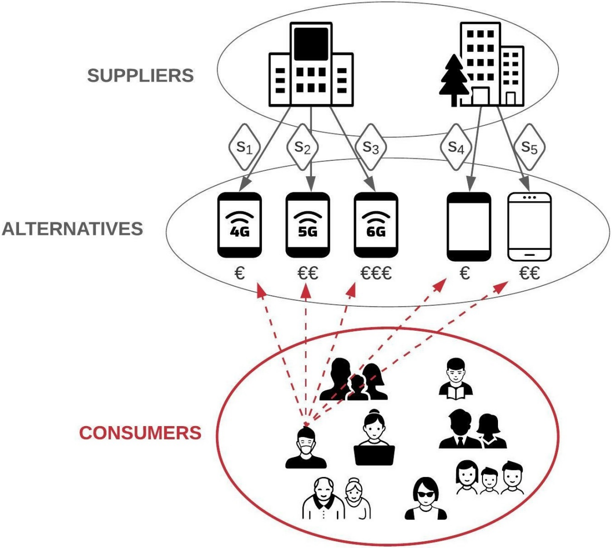
Bortolomiol, S., Lurkin, V., Bierlaire, M. (2021). **Price-based regulation of oligopolistic markets under discrete choice models of demand**. *Transportation*.

Chapter 4

Part of the work is included in the conference paper

Bortolomiol, S., Lurkin, V., Bierlaire, M., Bongiovanni, C. (2021). **Benders decomposition for choice-based optimization problems with discrete upper-level variables**. In *Proceedings of the 21st Swiss Transport Research Conference, Ascona, Switzerland*.

Oligopolistic competition



Oligopolies in transportation

The modeling framework: disaggregate demand

- Random utility models:

$$U_{in} = V_{in} + \varepsilon_{in}$$

$$P_{in} = \Pr[U_{in} + \varepsilon_{in} = \max_{j \in I}(U_{jn} + \varepsilon_{jn})]$$

- Simulation can be used to linearize the choice probabilities[1].
- In each simulation scenario, the alternative with the highest utility is chosen.
- Choice probabilities are obtained by sample average approximation.

[1] Pacheco Paneque et al., "Integrating advanced discrete choice models in mixed integer linear optimization" (2021).

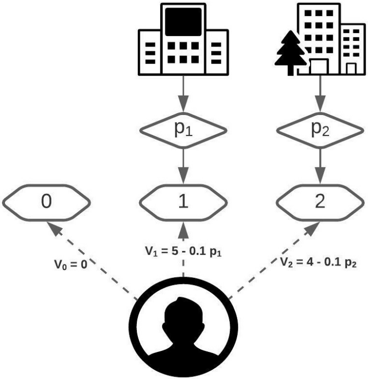
The modeling framework: supply

- Firms aim at maximizing profits and/or other objectives.
- Firms make decisions accounting for the expected behavior of customers.
- Decisions can be related to pricing, level of service, capacity, availability, etc.

Market interactions

- Oligopolistic market: firms interact strategically.
- **Nash equilibrium**: no firm can improve its payoff by unilaterally changing strategy.
  - Continuous demand functions + convex objective functions $\rightarrow$ first-order conditions.
  - Disaggregate demand $\rightarrow$ **no theoretical guarantees** of equilibrium existence.
- A simulation-based heuristic to find **approximate equilibria**.

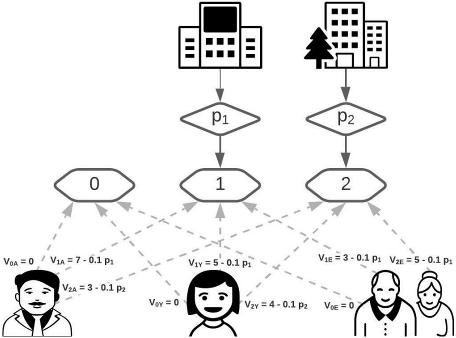
Small example: logit with unique Nash equilibrium[2]



Equilibrium		Prices		Profits		Market shares		
#	ε	1	2	1	2	1	2	3
Heuristic	0.9%	21.77	17.63	12.89	6.54	0.037	0.592	0.371
Analytical	0	23.02	16.57	13.02	6.57	0.038	0.566	0.396

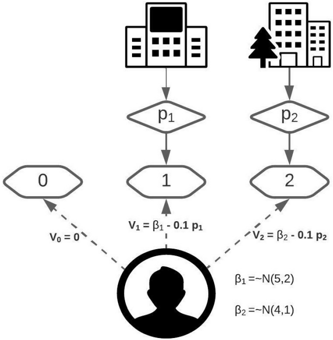
[2] Lin and Sibdari, "Dynamic price competition with discrete customer choices" (2009).

Numerical experiments: accounting for observed heterogeneity



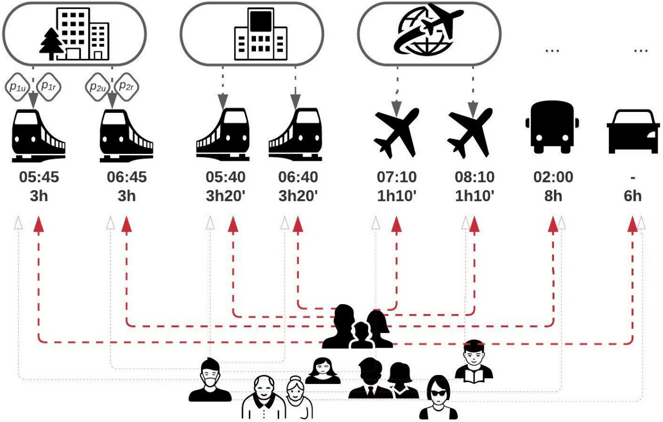
Equilibrium		Prices		Profits		Market shares		
#	ε	1	2	1	2	1	2	3
Heuristic	0.8%	33.85	26.04	16.92	11.02	0.077	0.500	0.423
(Analytical)	0	23.02	16.57	13.02	6.57	0.038	0.566	0.396

Numerical experiments: accounting for unobserved heterogeneity

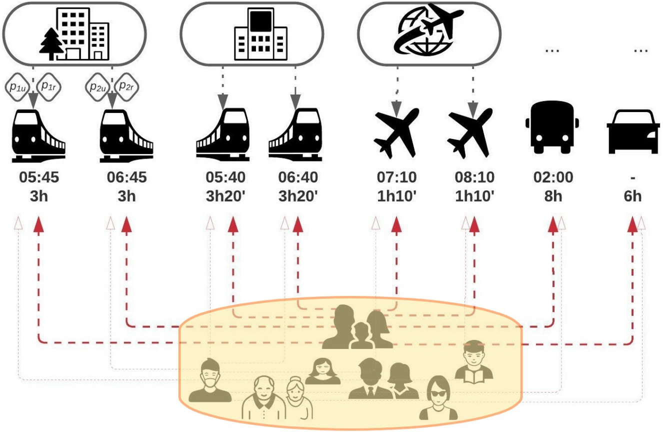


Equilibrium		Prices		Profits		Market shares		
#	ε	1	2	1	2			
Heuristic	0.9%	33.69	25.68	17.55	9.86	0.095	0.521	0.384
(Analytical)	0	23.02	16.57	13.02	6.57	0.038	0.566	0.396

Case study: schedule-based high-speed rail competition

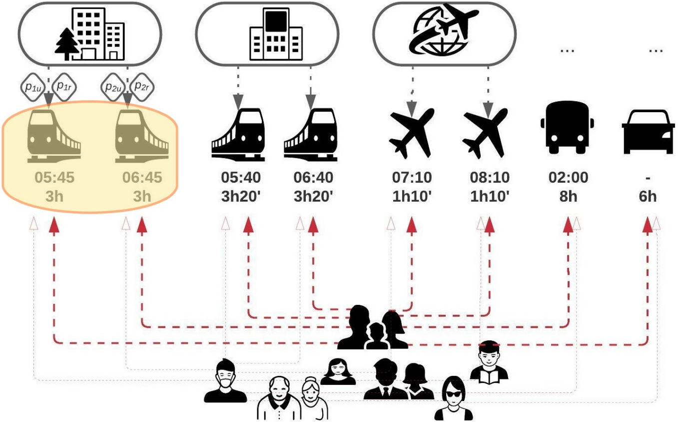


Case study: schedule-based high-speed rail competition



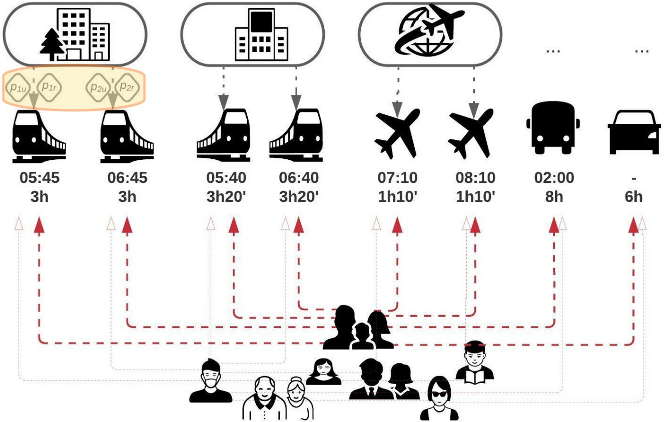
- Heterogeneous demand

Case study: schedule-based high-speed rail competition



- Heterogeneous demand
- Multi-product offer by suppliers

Case study: schedule-based high-speed rail competition



- Heterogeneous demand
- Multi-product offer by suppliers
- Price differentiation

Conclusion of the section

Summary

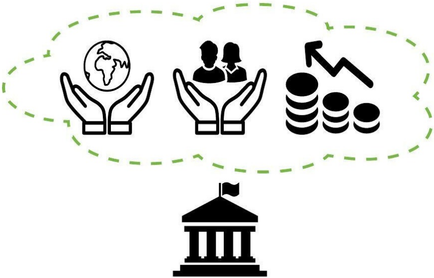
- Integration of discrete choice models into choice-based equilibrium problems.
- Simulation-based heuristic to find approximate equilibrium solutions.
- Application to transportation case studies.

Directions for future work

- Non-linear congestion effects
- Capacity constraints $\rightarrow$ simulation of arrival of customers

Regulated competitive markets in transportation

- Imperfect competition, barriers to entry, externalities
- Government as welfare maximizer



- Regulation according to competition and antitrust laws
- Economic instruments: subsidies and taxes

Price-based regulation

- *Deregulated competition*:

  The price paid for product i by customer n is equal to the revenue received by the supplier selling product i to customer n.

$$p_{in} = r_{in}$$

- *Regulated competition*:

$$p_{in} = r_{in} + t_{in},$$

  where t_{in} is a tax (> 0) or subsidy (< 0) set by the regulator.

Optimization problem of the regulator

Objective function

Maximize a **social welfare function** that can include utilities of the customers, profits of the suppliers, environmental externalities and public budget.

Constraints

Market equilibrium

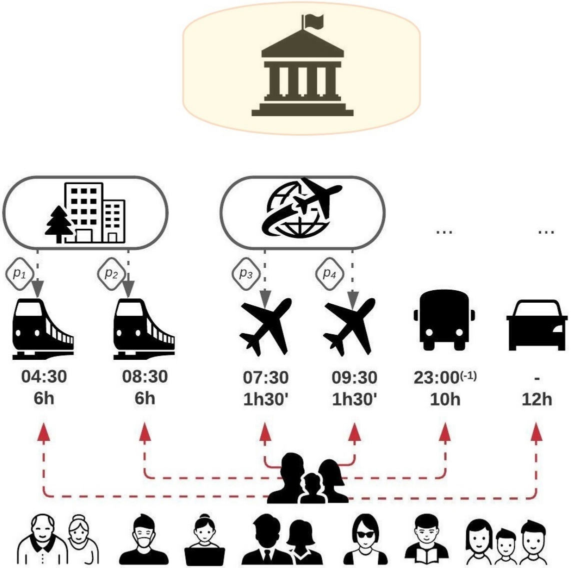
Profit maximization

Utility maximization

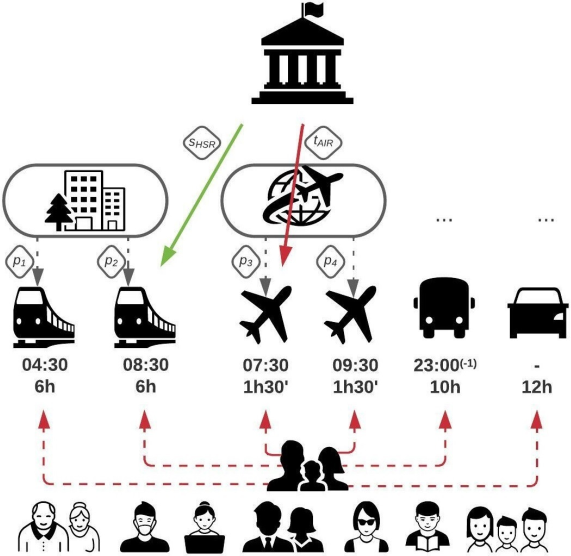
Problem-specific constraints

- Budget
- Policy fairness
- Capacities
- ...

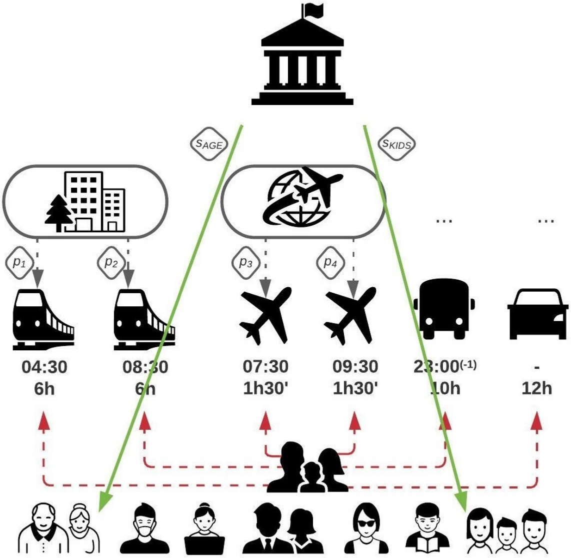
Case study: disaggregate policies



Case study: disaggregate policies



Case study: disaggregate policies



Social cost of carbon and numerical experiments

- Monetary value of the damage caused by emitting one unit of $CO2$.
- Economic indicator used for climate policy and cost-benefit analyses.

		Air Prices		HSR Prices		Regulation	
SCC	tCO_2	r_2	r_3	r_4	r_5	t_{TRAIN}	t_{AIR}
100	150.05	128.82	124.27	93.80	80.95	-0.03	0.00
200	132.69	97.12	99.48	84.90	83.71	-22.34	14.35
300	124.17	79.02	80.25	85.75	79.55	-30.00	30.00

Table: Social welfare maximization problem with marginal cost of public funds.

		Air Prices		HSR Prices		Regulation			
SCC	tCO_2	r_2	r_3	r_4	r_5	t^H_{TRAIN}	t^L_{TRAIN}	t^H_{AIR}	t^L_{AIR}
100	151.37	113.83	116.45	81.16	81.16	30.00	-29.96	30.00	-8.93
200	141.63	95.88	103.98	87.11	84.47	28.42	-30.00	30.00	-0.28
300	120.05	84.72	107.15	88.24	82.70	3.33	-30.00	30.00	28.95

Table: Social welfare maximization problem with disaggregate policy.

Conclusion of the section

Summary

- Framework to find optimal price-based regulation of oligopolistic markets under discrete choice models of demand.
- Definition of a social welfare function which includes disaggregate indicators.
- Application to an intercity travel market showing the value of disaggregate demand models.

Directions for future work

- Investigating the role of value judgements when optimizing social welfare: distributional preferences, policy acceptability, perceived fairness, etc.

Choice-based assortment and price optimization

- The use of simulation comes with open questions on scalability and flexibility of the framework.
- We investigate tradeoffs related to the decision variables and their domains.
- Assortment and pricing are two common supply problems.

Continuous (CPP) vs Discrete (DPP) Pricing Problem

Prices p are the only set of decision variables for supplier k.
Binary variables x capture the choices of the customers.

Continuous formulation:

- The utilities of the customers are price-dependent variables.

$$U_{inr} = \beta_{p,inr} p_i + \hat{q}_{inr} + \xi_{inr} \qquad \forall i \in I_k, \forall n \in N, \forall r \in R.$$

- The linearization of the product $p_i \cdot x_{inr}$ (continuous and binary) can be done using big-M constraints.

Discrete formulation:

- Prices p_i of each alternative $i \in I_k$ are chosen from a finite set.
- Utilities are parameters of the optimization model: $\hat{U}_{inr} = \beta_{p,inr} \hat{p}_i + \hat{q}_{inr} + \xi_{inr}$.
- Binary variables y capture the choice of the price level by the supplier.

Numerical experiments

$\|R\|$	CPP		DPP			Gap
	Time	Opt	$\|I_i^{exp}\|$	Time	Opt	
100	101.64	66452.18	21	34.48	66118.40	0.50%
			51	161.03	66255.90	0.30%
			101	395.86	66341.32	0.17%
200	288.89	70788.17	21	139.17	69859.60	1.31%
			51	415.90	70489.95	0.42%
			101	1829.24	70571.67	0.31%

Table: High-speed rail pricing: solving CPP and DPP to optimality with CPLEX.

Assortment and Pricing Problem

- We include the binary decision variables y about offering or not any product to customers.
- The continuous formulation (ACPP) requires an additional set of big-M constraints.
- The discrete formulation (ADPP) remains pretty much unchanged.

$\|R\|$	ACPP		ADPP			Gap
	Time	Opt	$\|I_i^{exp}\|$	Time	Opt	
10	11706	907.8	16	132	864.0	4.82%
			31	800	876.0	3.50%
20	129600*	877.0*	16	429	842.0	3.99%
			31	2778	862.5	1.65%

Table: Parking assortment and pricing: solving ACPP and ADPP to optimality with CPLEX.

Structure of the ADPP

- The discrete upper-level variables **y** of the supplier represent a joint decision on assortment and discrete price.
- If we fix these variables to $\mathbf{y}^*$, the lower-level utility maximization problem for a single customer n and scenario r is as follows:

$$\begin{aligned}
\max_{x} \quad & U = \sum_{i \in I} \hat{U}_i x_i, & \\
s.t. \quad & \sum_{i \in I} x_i = 1, & \\
& x_i \leq y_i^* & \forall i \in I, \\
& x_i \geq 0 & \forall i \in I.
\end{aligned}$$

- Continuous knapsack problem:
  - knapsack's capacity $= 1$;
  - weight of each item i (alternative) $= 1$;
  - value of each item i (alternative) $= \hat{U}_i$.
- Benders decomposition to exploit duality.

Benders decomposition Branch-and-Benders-cut implementation

- Classical approach of solving the *master problem* at each iteration is inefficient.
- Benders cuts can be inserted while processing the branch-and-bound tree of the master problem.[3]
- Efficient cut generation is key to the success of this approach.
- Some attempts:
  - Pareto-optimal cuts;[4]
  - minimal infeasible subset cuts;
  - partial Benders decomposition;[5]
  - etc.

[3] Fischetti, Ljubić, and Sinnl, "Redesigning Benders decomposition for large-scale facility location" (2017).

[4] Magnanti and Wong, "Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria" (1981).

[5] Crainic et al., "Partial Benders decomposition: General methodology and application to stochastic network design" (2021).

Computational performance: facility location and pricing

$\|R\|$	$\|I_i^{exp}\|$	Best	CPLEX (s)	BBC (s)	DualTime (%)
50	3	2625.00	**4.92**	33.78	0.84
50	6	2814.00	**47.69**	108.52	0.79
50	12	2892.00	684.46	**383.59**	0.52
100	3	2567.00	**17.85**	62.96	0.78
100	6	2857.00	258.60	**237.33**	0.61
100	12	2865.00	2047.72	**1476.88**	0.32
200	3	2588.50	**39.53**	131.01	0.75
200	6	2861.50	**215.05**	515.43	0.63
200	12	2861.50	4025.04	**3268.00**	0.25
500	3	2572.20	**221.06**	369.10	0.68
500	6	2824.30	**1753.52**	1784.22	0.38
500	12	2835.65	46166.88	**20903.66**	0.10
1000	3	2580.80	**677.68**	720.91	0.41
1000	6	2809.45	7913.22	**6988.75**	0.13
1000	12	2820.25	172000.00*	**100862.04**	0.03

Table: *N8I8* instances: running time for the ADPP using CPLEX and the BBC algorithm with disaggregate Benders cuts.

Conclusion of the section

Summary

- Using simulation, we can exploit **decomposition techniques** to solve choice-based optimization problems with discrete upper-level variables.
- **Branch-and-Benders-cut** is competitive against a black-box MILP solver on instances of a facility location and pricing problem with disaggregate demand.

Directions for future work

- Column generation to avoid large discretized strategy sets.
- Clustering at scenario and at customer level to optimize the generation of cuts.
- Testing simulation and decomposition on problems with complex interactions at the alternative and at the customer level (e.g. network design, scheduling).

Final remarks

- The increasing availability of demand data and **precise behavioral models** make choice-based models more appealing.
- The **trade-off** between the realism of the demand model and the complexity of the resulting optimization problem must be evaluated case by case.
- The possibility to **personalize offers and differentiate policies** provides a strong case for disaggregate demand models.
- This thesis contributes with **exact and heuristic algorithms** for realistic optimization and equilibrium problems with **disaggregate demand**.

Discussion