Modelling competitive markets within a demand-based optimization framework

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Outline

1. Problem description

2. Oligopolistic market equilibrium

3. Applications

Microeconomic theory

- Demand: consumers as utility maximizers.
- Supply: producers as profit maximizers.
- Market: perfect competition vs market power.

Oligopolies

- Market power: suppliers make strategic decisions which take into account interactions between actors.
- Interactions:
  - Supply-demand
  - Supply-supply
- Game theory

Oligopolies in transportation

Oligopolies in transportation

Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.

Discrete choice modelling 101

- Customer n selects an alternative from the finite choice set I.
- The utility associated to each alternative $i \in I$ is

$$U_{in} = V_{in} + \varepsilon_{in} = \boldsymbol{\beta_{in} X_{in}} + \varepsilon_{in}.$$

- The probability that customer n chooses alternative i is defined as

$$P_{in} = \Pr[U_{in} = max_{j \in I} U_{jn}].$$

- For the logit model, there is a closed-form expression:

$$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}.$$

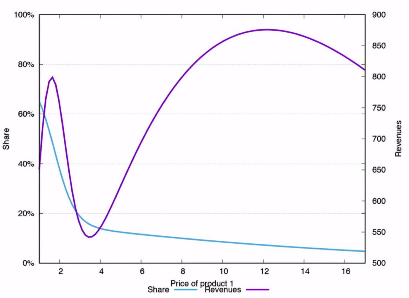
- For other discrete choice models, there is no closed-form expression and numerical approximation is needed.

Supply: optimization

- Suppliers take decisions that maximize their profits.
- Decisions include the pricing, quantity and quality of the offered products. The related variables could be continuous or discrete.
- Constrained optimization models can describe the supplier problem.

Demand-based optimization

- Stackelberg game: each supplier (leader) solves a profit maximization problem subject to the customers' (followers') utility maximization constraints.
- Discrete choice models into the optimization problem of the suppliers:



From the MOOC "Introduction to Discrete Choice Models" (Michel Bierlaire and Virginie Lurkin)

Supply-supply interactions

- We consider non-cooperative games.
- Nash equilibrium solutions, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.
- Existence, uniqueness, algorithms to find them.

Existence theorems

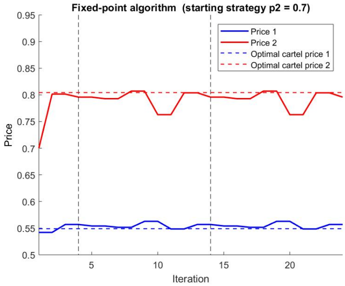
- Brouwer's fixed-point theorem (1941), extended by Kakutani (1941) and Glicksberg (1952).
- *Equilibrium points in n-person games* (Nash, 1950)
- *Existence of an equilibrium for a competitive economy* (Arrow and Debreu, 1954)
- General assumptions:
  - ✓ continuously differentiable demand curve;
  - ✓/ ✗ (convex) continuously differentiable supply curve;
  - ✗ concave profit function.

Finding market equilibria

- We have no existence or uniqueness proof, but we can still search for (pure strategy) equilibria...
  - Fixed-point iteration method
  - Fixed-point optimization model

The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
  - Initialization: players start from an initial feasible strategy.
  - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
  - Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.



The fixed-point optimization model

- We can minimize the *distance* between two consecutive iterations.
- A generic solution for a duopolistic market: $x_1^{'}, x_2^{'}$
- Optimization problems for the supplier:

$$x_1^{''} = \arg\max_{x_1 \in X_1} V_1(x_1, x_2^{'})$$
$$x_2^{''} = \arg\max_{x_2 \in X_2} V_2(x_1^{'}, x_2)$$

- Minimization problem:

$$z^* = \min |x_1^{''} - x_1^{'}| + |x_2^{''} - x_2^{'}|$$

- If $z^* = 0$, we have an equilibrium solution. What can we say about this equilibrium? If $z^* > 0$, can we conclude something?

Algorithmic framework: motivation

- Strategic problems have many variables and constraints, and heterogeneous demand makes the problem even more difficult.
- Neither traditional microeconomic nor game-theoretic approaches are applicable as such.
- Equilibrium problems $\neq$ optimization problems...
- But in real life problems equilibrium is quite loosely defined.

Algorithmic framework: solution approach

- Identify candidate (sub-game) equilibrium solutions or regions efficiently: sequential game, nonlinear formulation.
- Use exact methods on restricted strategy sets derived from candidate solutions: fixed-point optimization model, linearized formulation.
- Verify if best-response conditions are satisfied for the original game.
- Compare different equilibrium or near-equilibrium solutions: quasi-rationality, ε-equilibria, folk theorem.

Numerical experiments

- Parking choice case study: users choose among 3 options, 2 owned by 2 different operators and 1 opt-out option.
- Tests: nonlinear and linearized formulations with logit and mixed logit specifications.
- Computational results:
  - Stackelberg game: the nonlinear model is faster on all instances and it finds the optimal solution.
  - Fixed-point optimization model: on larger instances and mixed logit specification, the nonlinear model fails to converge.

Case study

- Demand: RP/SP survey collected in 2010 to forecast market shares in a competitive market.
- Supply: instance generation, starting with a single origin-destination pair and prices as only decision variables.

An applicable framework

Demand:

- Use advanced discrete choice models to better represent the demand.
- Consider endogenous decision variables (continuous or discrete) other than price.

Supply:

- Allow for product and price differentiation across customers.
- Test different objective functions for different operators.

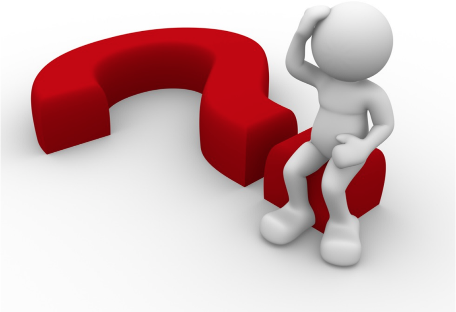
Competition:

- Define equilibrium or near-equilibrium in real-life case studies.

Summary

- Demand-based optimization: discrete choice models can be embedded in the optimization problem of the supplier (nonlinear and linearized formulation).
- Oligopolistic competition: it can be modelled with a sequential game or with a fixed-point optimization model.
- Our algorithmic approach (in development) should:
  - find candidate equilibrium solutions for the initial game;
  - solve restricted equilibrium problems with a demand-based optimization approach;
  - allow comparison between equilibrium or near-equilibrium solutions.
- Real-life case study: flexible and scalable framework, interpretable results.

Questions and discussion time



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