Optimization and equilibrium problems with discrete choice models

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Prof. Virginie Lurkin (Thesis co-director)
Prof. Francesco Corman (Internal jury member)
Prof. Emma Frejinger (External jury member)
Prof. Maria Grazia Speranza (External jury member)

1. Introduction
2. A simulation-based heuristic to find approximate equilibria with disaggregate demand models
3. Price-based regulation of oligopolistic markets under discrete choice models of demand
4. Benders decomposition for choice-based optimization problems
5. Conclusion

Discrete choice models

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- People have **different tastes and socioeconomic characteristics** that influence their choices.
- **Disaggregate models** of demand can capture this heterogeneity.
- **Discrete choice models** are a state-of-the-art approach when decision makers select one out of a finite set of alternatives.

Discrete choice models

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- Widely used for **prediction and forecasting**.
- Discrete choice probabilities are generally **non-linear and non-convex**.
- Choice probabilities of many advanced discrete choice models cannot be expressed with a closed form.
- Difficult to integrate into **supply optimization** and **market equilibrium** models.

Choice-based optimization and choice-based equilibrium

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- Dominant paradigm: **sacrifice complexity at the demand level** to obtain tractable optimization problems.
- **Simplistic specifications** to integrate discrete choice models into equilibrium problems.
- Complementary stance: optimization and equilibrium models that **accommodate advanced discrete choice models**.

Value of advanced discrete choice models

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Value of advanced discrete choice models

Some reasons to take this alternative point of view:

1. **Specification tests**: quantifiable trade-off between different demand models.
2. Estimation from increasingly large data sets (e.g. exploiting machine learning) $\rightarrow$ **richer and more complex specifications**
3. Disaggregate choice-based models $\rightarrow$ **differentiated offers and policies** to target specific groups of the population.

Positioning the doctoral thesis

Research directions

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1. Development of mathematical models and algorithms for **choice-based equilibrium problems**.

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1. Development of mathematical models and algorithms for **choice-based equilibrium problems**.
2. Identification of meaningful problem reformulations and efficient algorithmic approaches for **choice-based optimization problems**.

Positioning the doctoral thesis

Contributions

Positioning the doctoral thesis

Contributions

1. **Modeling:**
   - Optimization and equilibrium models which accommodate advanced discrete choice models of demand.
   - Aggregate and disaggregate metrics to evaluate the decisions of customers, suppliers and regulator within a competitive market.

Positioning the doctoral thesis

Contributions

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Positioning the doctoral thesis

Contributions

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2. **Algorithms:**
   - Exact algorithms for choice-based optimization and heuristic algorithms for choice-based equilibrium problems.
3. **Applications:**
   - Experiments on realistic case studies with non-trivial choice models.
   - Analysis of how disaggregate models can support decision making.

Outline of the thesis

Chapter 2

Based on the article

Bortolomiol, S., Lurkin, V., Bierlaire, M. (2021). **A simulation-based heuristic to find approximate equilibria with disaggregate demand models**. *Transportation Science*, 55(5):1025–1045.

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Chapter 4

Part of the work is included in the conference paper

Bortolomiol, S., Lurkin, V., Bierlaire, M., Bongiovanni, C. (2021). **Benders decomposition for choice-based optimization problems with discrete upper-level variables**. In *Proceedings of the 21st Swiss Transport Research Conference, Ascona, Switzerland*.

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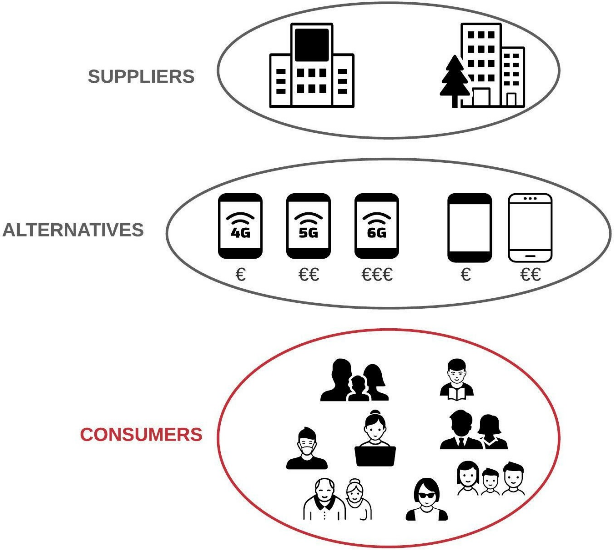
Oligopolistic competition



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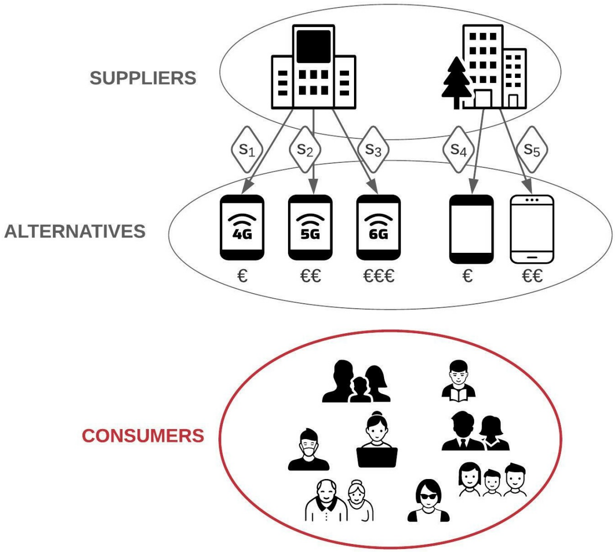

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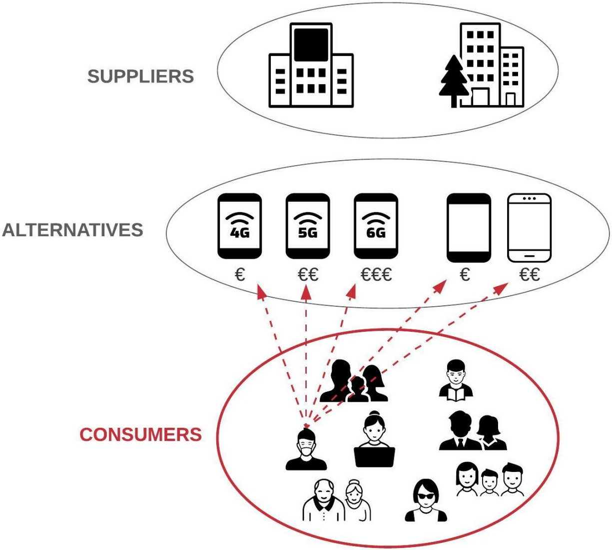

Oligopolies in transportation

Disaggregate demand models

- Random utility models[1,2]:

$$U_{in} = V_{in} + \varepsilon_{in} \qquad \forall i \in I, \forall n \in N.$$

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- Choice probabilities are expressed as a function of the explanatory variables:

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- Simulation-based linearization of choice probabilities[3]:

$$U_{inr} = V_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R,$$

$$x_{inr} = \begin{cases} 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \\ 0 \text{ otherwise} \end{cases} \qquad \forall i \in I, \forall n \in N, \forall r \in R.$$

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- Sample average approximation:

$$P_{in} = \frac{1}{|R|} \sum_{r \in R} x_{inr} \qquad \forall i \in I, \forall n \in N.$$

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- Utility maximization conditions are enforced as constraints of the choice-based optimization problem:

$$\begin{aligned}
s.t. \quad & U_{inr} = \beta_{p,in} p_{in} + \beta_{in} Y_{in} + q_{in} + \xi_{inr} && \forall i \in I_k, \forall n \in N, \forall r \in R, \\
& U_{inr} \leq U_{nr} && \forall i \in I, \forall n \in N, \forall r \in R, \\
& U_{nr} \leq U_{inr} + M_{U_{nr}}(1 - x_{inr}) && \forall i \in I, \forall n \in N, \forall r \in R, \\
& \sum_{i\in I} x_{inr} = 1 && \forall n \in N, \forall r \in R, \\
& x_{inr} \in \{0,1\} && \forall i \in I, \forall n \in N, \forall r \in R.
\end{aligned}$$

Equilibrium

- **Nash equilibrium**:

$$\pi_k^{max} = \pi_s = \max_{s \in \mathcal{S}_k} \pi_s(s, s_{K \setminus \{k\}}) \qquad \forall k \in K.$$

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- Disaggregate demand → **no theoretical guarantees** of equilibrium existence.

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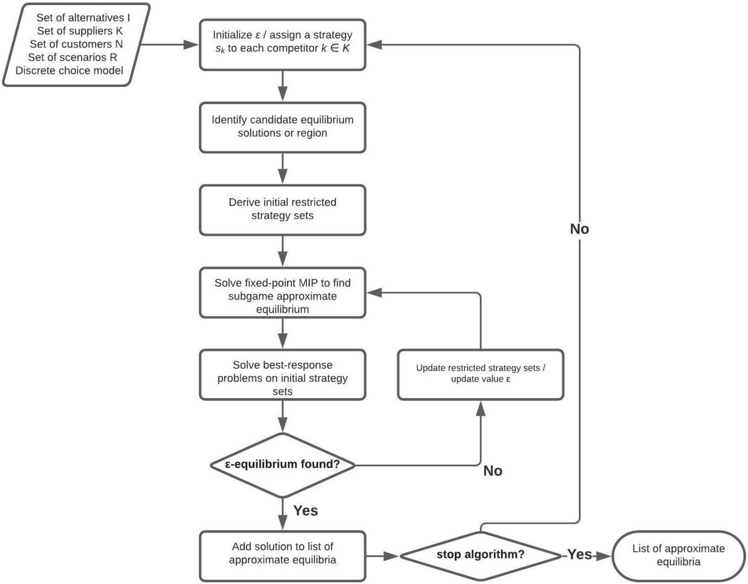
- Continuous demand + convex objective functions $\rightarrow$ first-order conditions.
- Disaggregate demand $\rightarrow$ **no theoretical guarantees** of equilibrium existence.
- Fixed-point iteration algorithm as a heuristic $\rightarrow$ fixed-point MIP model: **subgame near-equilibrium** using (small) finite restricted strategy sets.

$$\min \quad \sum_{k \in K} (\pi_k^{''} - \pi_k^{'}).$$

A simulation-based heuristic to find approximate equilibria



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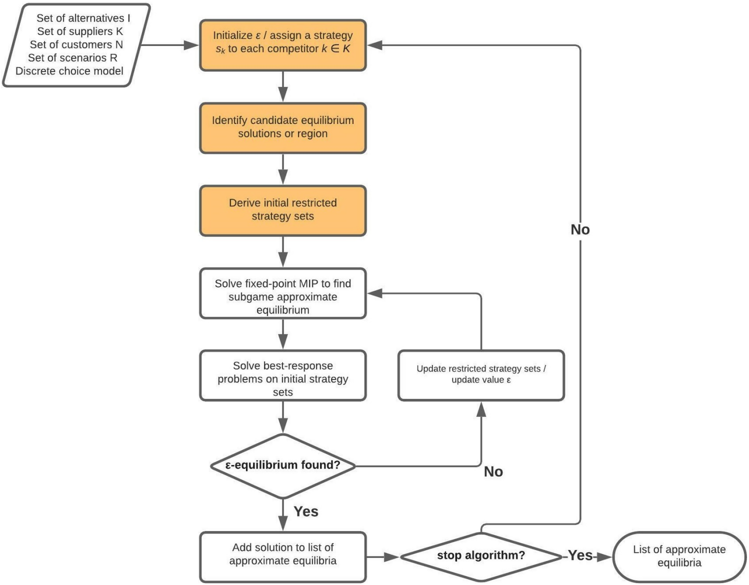

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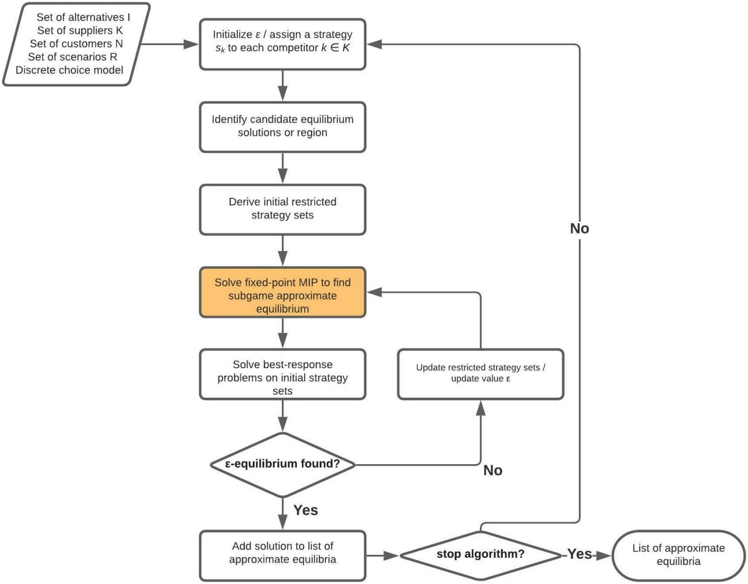

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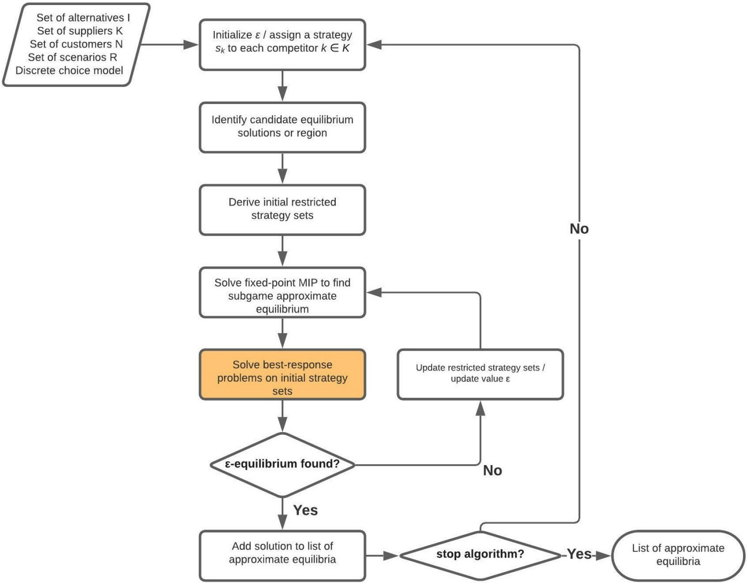

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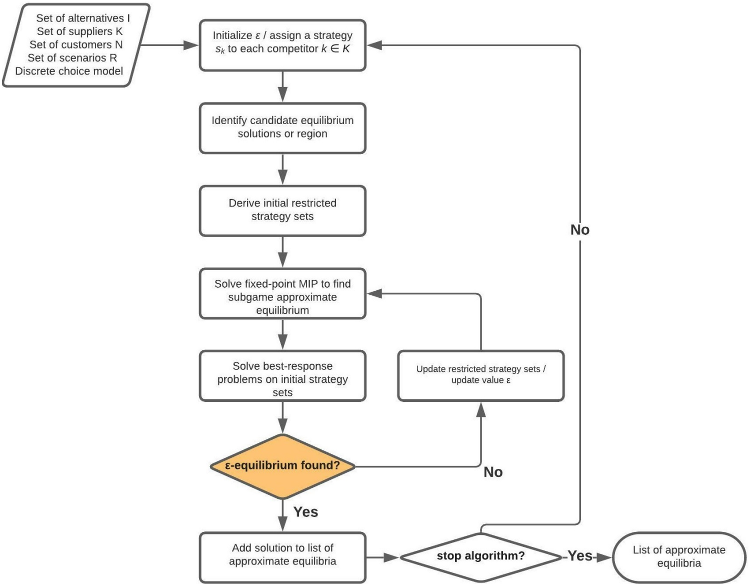

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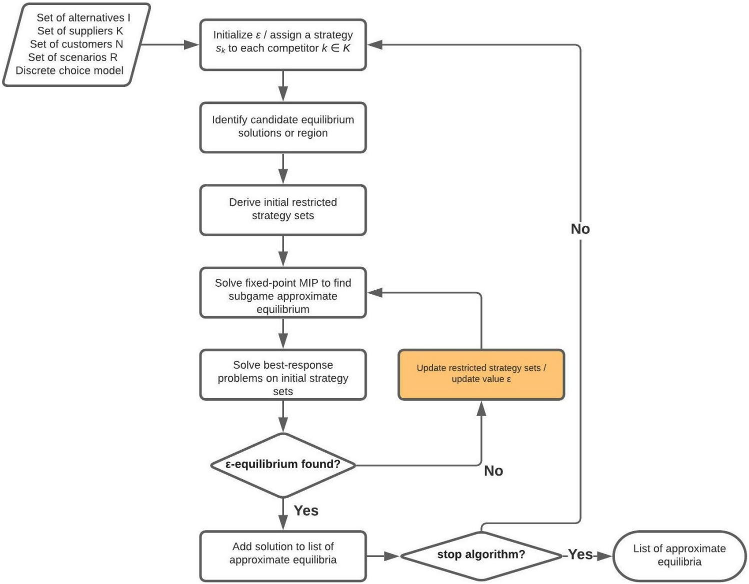

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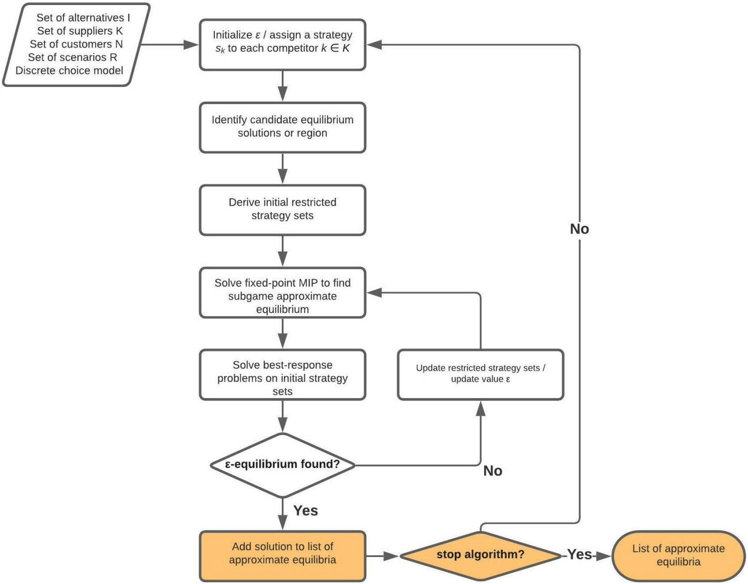
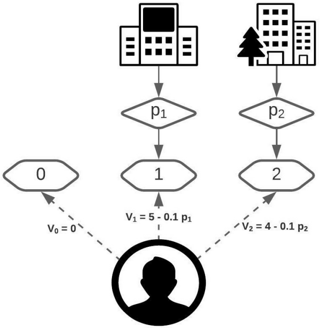


Numerical experiments: logit with unique Nash equilibrium[4]



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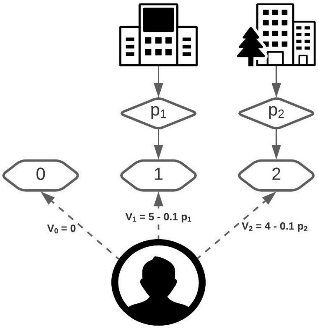

Equilibrium		Prices		Profits		Market shares		
#	ε	1	2	1	2	1	2	3
1	0.9%	21.77	17.63	12.89	6.54	0.037	0.592	0.371
2	0.7%	21.83	16.57	12.40	6.56	0.036	0.568	0.396
3	0.7%	22.56	17.94	13.06	6.83	0.040	0.579	0.381
4	0.7%	23.38	17.96	13.07	7.17	0.042	0.559	0.399
5	0.7%	21.90	16.65	12.44	6.59	0.036	0.568	0.396
Analytical	0	23.02	16.57	13.02	6.57	0.038	0.566	0.396

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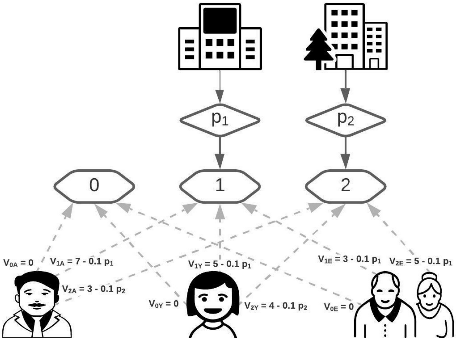
Numerical experiments: accounting for observed heterogeneity



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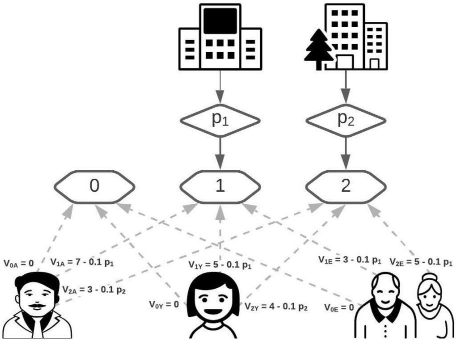

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#	ε	1	2	1	2	1	2	3
1	0.8%	33.85	26.04	16.92	11.02	0.077	0.500	0.423
2	0.8%	34.15	27.14	17.36	11.08	0.083	0.508	0.408
3	0.6%	34.14	26.15	17.01	11.12	0.077	0.498	0.425
4	0.7%	34.14	26.18	17.07	11.08	0.077	0.500	0.423
5	0.9%	34.19	27.15	17.38	11.09	0.083	0.508	0.408
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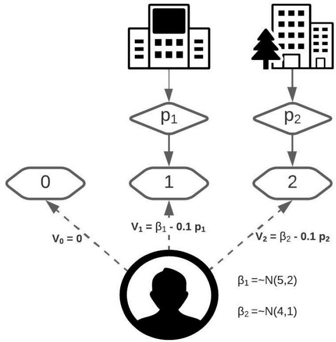

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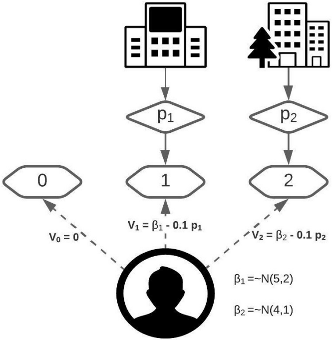

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2	0.7%	33.46	25.68	17.53	9.79	0.095	0.524	0.381
3	0.7%	33.86	25.68	17.61	9.86	0.096	0.520	0.384
4	0.7%	33.23	25.68	17.48	9.76	0.094	0.526	0.380
5	0.7%	33.94	25.43	17.41	9.97	0.095	0.513	0.392
Analytical	0	23.02	16.57	13.02	6.57	0.038	0.566	0.396

Case study: parking choice

Mixed logit model estimation[5]

β	Value
...	...
Fee (€)	$\sim \mathcal{N}(-32.328, 14.168)$
Fee PSP - low income (€)	-10.995
Fee PUP - low income (€)	-13.729
Fee PSP - resident (€)	-11.440
Fee PUP - resident (€)	-10.668
Access time to parking (min)	$\sim \mathcal{N}(-0.788, 1.06)$
Access time to destination (min)	-0.612
Age of vehicle (1/0)	4.037
Origin (1/0)	-5.762

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Base scenario

Equilibrium		Prices		Profits		Market shares residents			Market shares non-residents		
#	ε	PSP	PUP	PSP	PUP	FSP	PSP	PUP	FSP	PSP	PUP
1	1.2%	0.428	0.588	5.95	1.40	0.020	0.783	0.197	0.004	0.518	0.478

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1	3.3%	0.385	0.611	3.86	2.37	0.002	0.435	0.563	0.004	0.741	0.255

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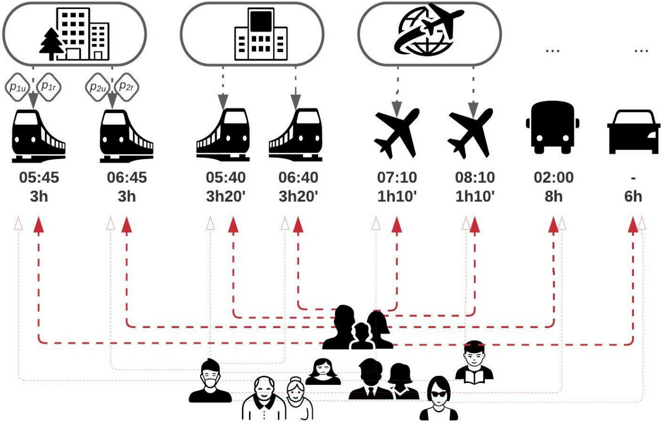
1	1.2%	0.428	0.588	5.95	1.40	0.020	0.783	0.197	0.004	0.518	0.478

Resident discount scenario

Equilibrium		Prices		Profits		Market shares residents			Market shares non-residents		
#	ε	PSP	PUP	PSP	PUP	FSP	PSP	PUP	FSP	PSP	PUP
1	3.3%	0.385	0.611	3.86	2.37	0.002	0.435	0.563	0.004	0.741	0.255

[5] Ibeas et al., “Modelling parking choices considering user heterogeneity” (2014).

Case study: schedule-based high-speed rail competition



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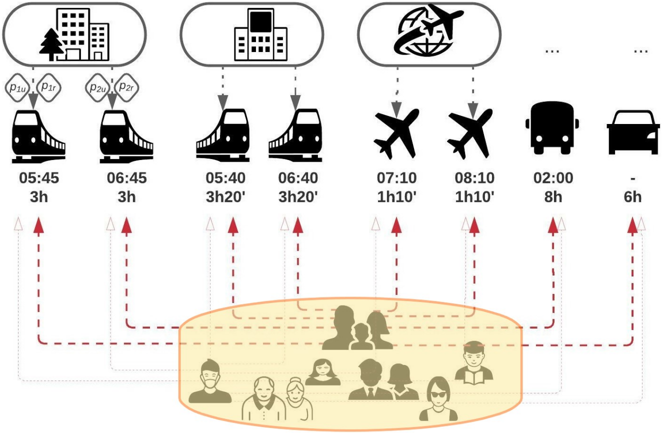

- **Heterogeneous demand**

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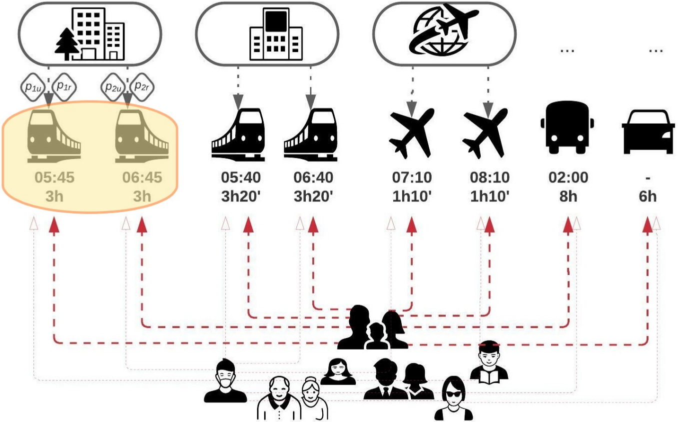

- Heterogeneous demand
- **Multi-product offer by suppliers**

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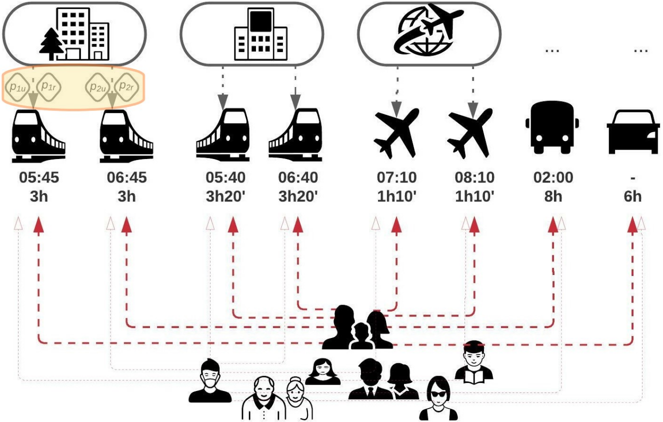

- Heterogeneous demand
- Multi-product offer by suppliers
- **Price differentiation**

Conclusion of the section

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- Capacity constraints $\rightarrow$ priority rules to simulate arrival process of customers (e.g. revenue management).

1 Introduction

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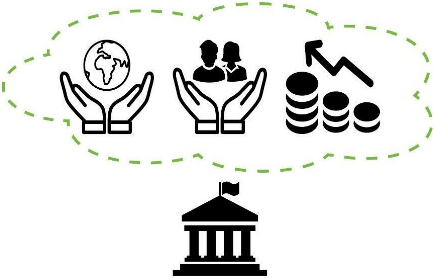
Regulated competitive markets in transportation



- Government as welfare maximizer

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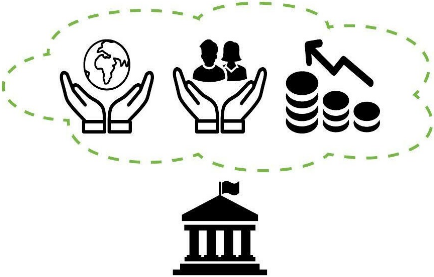

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- Regulation according to competition and antitrust laws

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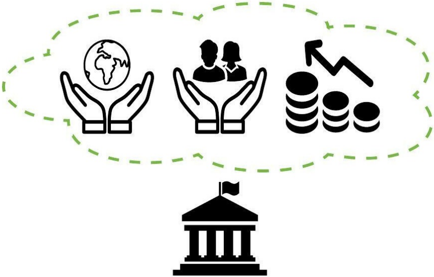

- Government as welfare maximizer
- Regulation according to competition and antitrust laws
- Economic instruments: subsidies and taxes

Price-based regulation

- The utility of alternative i for customer n depends on the price p_{in}.

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- With regulation, we can write

$$p_{in} = r_{in} + t_{in}.$$

Optimization problem of the regulator

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Objective function

Maximize a social welfare function

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Constraints

Market equilibrium

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Problem-specific constraints

- Budget
- Policy fairness
- Capacities
- ...

Social welfare function

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$$SWF_E = -\sum_{i \in I} c_i d_i,$$

with $c_i = \ell_i \cdot e_i \cdot SCC$ and $d_i = \sum_{n \in N} \theta_n P_{in}$.

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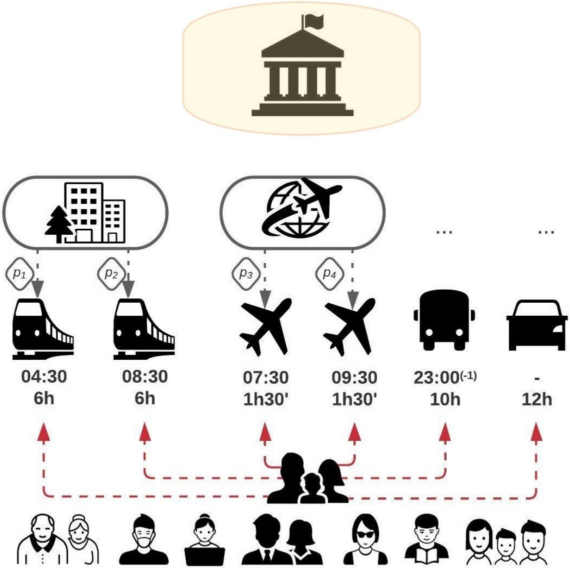
- Budget:

$$SWF_B = \sum_{i \in I} \sum_{n \in N} \theta_n \frac{\sum_{r \in R} x_{inr}}{\mid R \mid} t_{in}.$$

Case study: disaggregate policies



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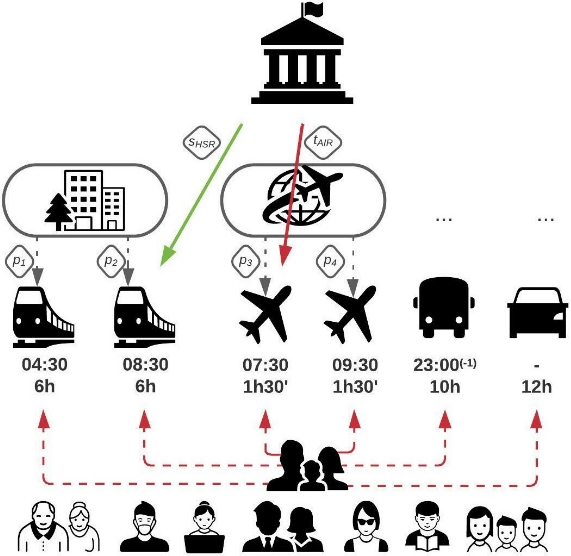

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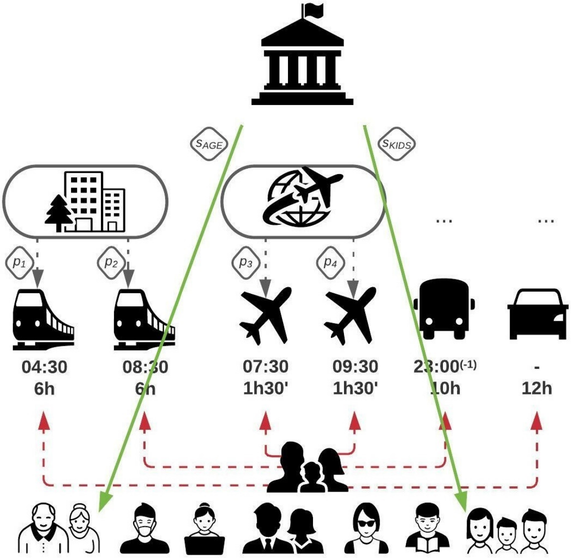

Social cost of carbon and numerical experiments

- Monetary value of the damage caused by emitting one more unit of carbon at some point of time. Literature values diverge considerably.
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		Air Prices		HSR Prices		Regulation	
SCC	$t\mathrm{CO}_2$	r_2	r_3	r_4	r_5	t_{TRAIN}	t_{AIR}
100	150.05	128.82	124.27	93.80	80.95	-0.03	0.00
150	143.84	103.09	108.98	79.68	80.82	-15.08	0.00
200	132.69	97.12	99.48	84.90	83.71	-22.34	14.35
250	123.74	79.91	89.67	87.05	85.74	-30.00	30.00
300	124.17	79.02	80.25	85.75	79.55	-30.00	30.00

Table: Social welfare maximization problem with marginal cost of public funds.

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150	143.51	97.60	106.60	97.76	84.82	29.80	-29.93	30.00	-2.18
200	141.63	95.88	103.98	87.11	84.47	28.42	-30.00	30.00	-0.28
250	131.18	93.36	94.79	87.52	79.52	7.88	-30.00	30.00	12.12
300	120.05	84.72	107.15	88.24	82.70	3.33	-30.00	30.00	28.95

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- Further investigating the role of value judgements when optimizing social welfare: distributional preferences, policy acceptability, perceived fairness, etc.

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Choice-based optimization

- Using simulation to linearize choice probabilities[6].

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- Open questions:
  - Scalability
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Continuous Pricing Problem (CPP)

- Supplier k wants to maximize profits obtained from alternatives $I_k \subset I$.
- The utilities of the customers are price-dependent variables:

$$U_{inr} = \beta_{p,inr} p_i + \hat{q}_{inr} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R.$$

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\max_{p} \quad & \pi = \sum_{i \in I_k} \sum_{n \in N} \sum_{r \in R} \frac{1}{|R|} \theta_n p_i x_{inr}, && \\
s.t. \quad & \sum_{i \in I} x_{inr} = 1 && \forall n \in N, \forall r \in R, \\
& \sum_{j \in I} U_{jnr} x_{jnr} \geq U_{inr} && \forall i \in I, \forall n \in N, \forall r \in R, \\
& 0 \leq p_i \leq M_i^p && \forall i \in I, \\
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\end{aligned}$$

- The linearization of the product $p_i \cdot x_{inr}$ (continuous and binary) can be done using big-M constraints.

Discrete Pricing Problem (DPP)

- We constrain prices p_i of each alternative $i \in I_k$ to the set $Q_i = \{p_i^1, p_i^2, ..., p_i^{|Q|}\}$.
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$$
\begin{aligned}
\max_{y} \quad & \pi = \sum_{i \in I_k^{exp}} \sum_{n \in N} \sum_{r \in R} \frac{1}{|R|} \theta_n \hat{p}_i x_{inr}, && \\
s.t. \quad & \sum_{j \in I_i^{exp}} y_j = 1 && \forall i \in I, \\
& \sum_{i \in I^{exp}} x_{inr} = 1 && \forall n \in N, \forall r \in R, \\
& x_{inr} \leq y_i && \forall i \in I^{exp}, \forall n \in N, \forall r \in R, \\
& \sum_{j \in I^{exp}} \hat{U}_{jnr} x_{jnr} \geq \hat{U}_{inr} y_i && \forall i \in I^{exp}, \forall n \in N, \forall r \in R, \\
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& y_i \in \{0, 1\} && \forall i \in I^{exp}.
\end{aligned}
$$

Numerical experiments

$\lvert R \rvert$	CPP		DPP			Gap
	Time	Opt	$\lvert I_i^{exp} \rvert$	Time	Opt	
			21	1.42	70390.50	1.93%
20	0.45	71774.95	51	7.18	71316.20	0.64%
			101	8.89	71379.90	0.55%
			21	14.59	71889.00	0.74%
50	10.46	72423.71	51	31.51	72106.36	0.44%
			101	89.91	72185.30	0.33%
			21	34.48	66118.40	0.50%
100	101.64	66452.18	51	161.03	66255.90	0.30%
			101	395.86	66341.32	0.17%
			21	139.17	69859.60	1.31%
200	288.89	70788.17	51	415.90	70489.95	0.42%
			101	1829.24	70571.67	0.31%

Table: High-speed rail pricing: solving CPP and DPP to optimality with CPLEX.

Assortment and Continuous Pricing Problem (ACPP)

- We include the decision about offering or not any product $i \in I_k$ to customers.
- The actual utility for the customer is $U^a_{inr} = U_{inr} \cdot y_i$.

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- The actual utility for the customer is $U^a_{inr} = U_{inr} \cdot y_i$.
- Customers must choose the alternative with the highest utility among those that are made available by the supplier:

$$
\begin{aligned}
& U_{inr} = \beta_{p,inr} p_i + q_{inr} + \xi_{inr} && \forall i \in I, \forall n \in N, \forall r \in R, \\
& U^a_{inr} \leq U_{inr} && \forall i \in I, \forall n \in N, \forall r \in R, \\
& U_{inr} \leq U^a_{inr} + M^U_{inr}(1 - y_i) && \forall i \in I, \forall n \in N, \forall r \in R, \\
& U^a_{inr} \leq M^U_{inr} y_i && \forall i \in I, \forall n \in N, \forall r \in R,
\end{aligned}
$$

Assortment and Discrete Pricing Problem (ADPP)

- The formulation of the DPP **still applies**, with a small change:

$$
\begin{aligned}
\max_{y} \quad & \pi = \sum_{i \in I_k^{exp}} \sum_{n \in N} \sum_{r \in R} \frac{1}{|R|} \theta_n \hat{p}_i x_{inr}, & \\
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$$

Numerical experiments

$\|R\|$	ACPP		ADPP			Gap
	Time	Opt	$\|I_i^{exp}\|$	Time	Opt	
10	11706	907.8	16	132	864.0	4.82%
			31	800	876.0	3.50%
20	129600*	877.0*	16	429	842.0	3.99%
			31	2778	862.5	1.65%
50	129600*	842.8*	16	837	816.4	3.13%
			31	12191	830.4	1.47%
100	129600*	844.0*	16	3419	828.2	1.87%
			31	39425	831.8	1.45%

Table: Parking assortment and pricing: solving ACPP and ADPP to optimality with CPLEX.

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s.t. \quad & \sum_{i \in I} x_i = 1, & \\
& x_i \leq y_i^* & \forall i \in I, \\
& x_i \geq 0 & \forall i \in I.
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\max_{x} \quad & U = \sum_{i \in I} \hat{U}_i x_i, & \\
s.t. \quad & \sum_{i \in I} x_i = 1, & \\
& x_i \leq y_i^* & \forall i \in I, \\
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- Continuous knapsack problem:
  - knapsack's capacity $= 1$;
  - weight of each item i (alternative) $= 1$;
  - value of each item i (alternative) $= \hat{U}_i$.

Discrete upper-level variables

- Let us fix the discrete upper-level variables of the supplier to $\mathbf{y}^*$.
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- Benders decomposition to exploit duality.

Benders decomposition

1. Initialize $UB = \infty$ and $LB = -\infty$ of the master problem (MP).
2. Initialize the restricted master problem (RMP):

$$\begin{aligned} \min_{y,z} \quad & \sum_{i \in I_k^{exp}} f_i y_i^* + z \\ s.t. \quad & \textit{Domain constraints on the y variables} \\ & z \geq LB_z. \end{aligned}$$

3. Solve current RMP. Save the solution y^*, z^*. Let $f(y^*, z^*)$ be the optimal objective value. Update $LB = f(y^*, z^*)$.
4. Given y^*, compute $f(y^*)^{ADPP}$ for the original problem by deriving the choices for all customers and scenarios. Update $UB = \min\{UB, f(y^*)^{ADPP}\}$.
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Branch-and-Benders-cut implementation

- Solving the master problem at each iteration is inefficient.
- Benders cuts can be inserted while processing the branch-and-bound tree of the master problem.[7]
- Classical Benders cuts provide slow convergence $\rightarrow$ efficient cut generation is key to the success of this approach.

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Computational performance: facility location and pricing

$\lvert R \rvert$	$\lvert I_i^{exp} \rvert$	Best	CPLEX (s)	BBC (s)	DualTime (%)
50	3	2625.00	**4.92**	33.78	0.84
50	6	2814.00	**47.69**	108.52	0.79
50	12	2892.00	684.46	**383.59**	0.52
100	3	2567.00	**17.85**	62.96	0.78
100	6	2857.00	258.60	**237.33**	0.61
100	12	2865.00	2047.72	**1476.88**	0.32
200	3	2588.50	**39.53**	131.01	0.75
200	6	2861.50	**215.05**	515.43	0.63
200	12	2861.50	4025.04	**3268.00**	0.25
500	3	2572.20	**221.06**	369.10	0.68
500	6	2824.30	**1753.52**	1784.22	0.38
500	12	2835.65	46166.88	**20903.66**	0.10
1000	3	2580.80	**677.68**	720.91	0.41
1000	6	2809.45	7913.22	**6988.75**	0.13
1000	12	2820.25	172000.00*	**100862.04**	0.03

Table: *N8I8* instances: running time for the ADPP using CPLEX and the BBC algorithm with disaggregate Benders cuts.

Conclusion of the section

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- Column generation to avoid large discretized strategy sets.
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- Testing simulation and decomposition on problems with complex interactions at the alternative and at the customer level (e.g. network design, scheduling).

1 Introduction

2 A simulation-based heuristic to find approximate equilibria with disaggregate demand models

3 Price-based regulation of oligopolistic markets under discrete choice models of demand

4 Benders decomposition for choice-based optimization problems

5 Conclusion

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- The **trade-off** between the realism of the demand model and the complexity of the resulting optimization or equilibrium problem must be evaluated on a case-by-case basis.
- The possibility to **personalize offers and differentiate policies** provides a strong case for disaggregate demand models.
- This thesis contributes with **exact and heuristic algorithms** that are behaviorally sound and computationally tractable for realistic optimization and equilibrium problems with **disaggregate demand**.