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Spectroscopy of Charmed Hadrons: Facing the Latest Experimental Results with the Theory

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Spectroscopy of Charmed Hadrons: Facing the Latest Experimental Results with the Theory

Codeste ambiguità, ridondanze e deficienze ricordano quelle che il dottor Franz Kuhn attribuisce a un'enciclopedia cinese che s'intitola *Emporio celeste di conoscimenti benevoli.* Nelle sue remote pagine è scritto che gli animali si dividono in (a) appartenenti all'Imperatore, (b) imbalsamati, (c) ammaestrati, (d) lattonzoli, (e) sirene, (f) favolosi, (g) cani randagi, (h) inclusi in questa classificazione, (i) che s'agitano come pazzi, (j) innumerevoli, (k) disegnati con un pennello finissimo di pelo di cammello, (l) eccetera, (m) che hanno rotto il vaso, (n) che da lontano sembrano mosche. [...] L'impossibilità di penetrare il disegno divino dell'universo non può, tuttavia, dissuaderci dal tracciare disegni umani, anche se li sappiamo provvisorî.

(Jorge Luis Borges, *El idioma analítico de John Wilkins*)

Abstract

One of the frontiers of Physics is the understanding of strong interactions, which is a challenging subject both theoretically and experimentally. Indeed, it is a remarkable achievement how the theoretical research has been able so far to extract many pieces of information from the experimental evidence, which under many aspects is very limited. On the other hand, a great deal of efforts has been put in this area of experimental research during the last decades and, as more data become available, there is hope that it will be possible to shed light on this subject, just like atomic spectroscopy was among the propellants of the quantum revolution of the 20th century.

One of these theoretical challenges is the following. Because of its success in the description of many aspects of hadronic interactions, the quantum chromodynamics is believed to be the theory behind them. However, it is unclear how to describe relativistic bound states in a quantum field theory in general and in quantum chromodynamics in particular. In other words, there are no analytic methods to calculate the spectrum of a given hadronic system.

However, as it will be shown in this thesis, mesons composed by a heavy quark and a light antiquark (*heavy-light* mesons) have peculiar features that allow to describe them using approximations within the quantum chromodynamics that can be elegantly cast in the form of effective theories. One of the applications of these theories is the classification of charmed mesons, the subject of this thesis, which is an important part of the endeavour of theoretical research in hadron spectroscopy.

Thesis Outline

The thesis is organized as follows:

- In the first chapter I give an overview of the recent experimental results on the subject of heavy meson spectroscopy, focusing on the open-charm sector, which has seen many improvements in the last years. I also mention other important findings in QCD exotica and in the baryonic sector, which are essential to get the whole picture of the present state of our understanding of the hadronic structure. In this regard, it should be stressed the importance of ordinary hadronic spectroscopy in order to individuate or exclude possible exotic candidates.

- In the second chapter I overview the main features of the effective theories used in the study performed in this thesis, i.e. the heavy quark effective theory and the

chiral perturbation theory. Such theories leverage one of the most beautiful and ubiquitous concepts of modern theoretical physics, the notion of symmetry. These effective theories emerge, respectively in the limit in which heavy quarks have infinite mass and light ones are massless, exploiting the symmetries that emerge in such limits. Combining the two approaches allows to make calculations of quantities which are sensitive to the quantum numbers of heavy-light hadrons and thus are suitable to classify such states. The predictions obtained in this framework are very sound, since they do not rely on approximate models, such as, for example, potential models. Instead, they stem from a theory that is obtained from quantum chromodynamics in a well defined limit (large mass of the heavy quarks, zero mass for the light quarks).

- In the third chapter all observed open-charm mesons are classified according to this theory. The classification of non-established states is discussed, presenting arguments based on their quantum numbers, masses and widths. Finally, it is presented an original contribution aiming at the identification of the $D_2^*(3000)$, an excited D meson recently observed by the LHCb collaboration.

Chapter 1

Advances in Hadron Spectroscopy

When the Nobel Prizes were first awarded in 1901, physicists knew of just two objects which are now called "elementary particles": the electron and the proton. A deluge of other "elementary" particles appeared after 1930; neutron, neutrino, μ meson (sic), π meson, heavier mesons, and various hyperions. I have heard it said that "the finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a \$ 10'000 fine"

(Willis Lamb, *Nobel Prize acceptance speech*)

The foundation of our present understanding of the hadronic spectroscopy is the quark model, independently formulated by Gell-Mann and Zweig in 1964. The model described the hadrons known at the time as composite objects of charged fermions, named "quarks", having baryon number $B = 1/3$ and electric charge 2/3 or $-1/3$. The hadrons were organized in multiplets of the $SU(3)$ symmetry group. However, the model implied violations of the Pauli exclusion principle and hence an additional quantum number, later known as *colour*, was attributed to the quarks. Han and Nambu (1965) proposed a model with a supplementary $SU(3)$ group of colour and the idea was promoted to a gauge theory by Bardeen, Fritzsch and Gell-Mann in 1972, with the gluons as gauge bosons of the theory. Therefore, in less than ten years the theory now known as Quantum ChromoDynamics (QCD) took shape (see section 2.1). Combinations of quarks corresponding to colour singlets could be formed with either $B = 0$ (mesons) or $B = 1$ (baryons). The simplest among such structures are $q\bar{q}$ and qqq, q being a quark and $\bar{q}$ an anti-quark.

Since the inception of the quark model, it was pointed out that more complex structures with the same baryon number of mesons and baryons could exist, such as the "tetraquark" mesons $qq\overline{qq}$ and the "pentaquark" baryons $qqq\bar{q}q$. Moreover, even more structures are possible once taken into account the possibility of valence gluons. A large number of hadrons has been discovered since the formulation of the quark model that fits very well the classification in baryons and mesons (*standard hadrons*). On the other hand, search for the above mentioned different structures (*exotic hadrons*) yielded negative or ambiguous results until recently.

The goal of this chapter is to briefly review some of the advances in exotic as well as ordinary hadron spectroscopy. Particular emphasis will be put on open-charm and open-

beauty mesons.

I shall first briefly consider some of the recent progresses in the search for exotica. The status of open-charm and open-beauty mesons will be reported afterwards.

It should be noticed the importance of ordinary hadron spectroscopy in order to exclude the identification of non-standard hadrons with standard ones or identify them as *supernumerary* states, a condition verified when more states with given quantum numbers than expected are experimentally observed.

1.1   Exotic Spectroscopy: Observations and Physical Pictures

Many of the particles discovered in the current era are candidates to be non-standard hadrons [35, 40]. Nearly all of these states are charmoniumlike particles, meaning that they contain a $c\bar{c}$ pair and have masses in the same region as conventional charmonium states, while the remaining are bottomoniumlike (defined in a similar way) and pentaquark candidates.

Such exotic hadron candidates were observed by different experiments devoted to heavy flavour physics and covering a wide range of energies. At the low end, with a c.m. energy between 2 and 4.6 GeV and thus including the production threshold for $c\bar{c}$ pairs, there is the BESIII experiment at the Institute of High Energy Physics in Beijing that operates at the BEPCII e^+e^- collider. The so-called B-factory experiments Belle and BABAR[1], mainly devoted to the production of particles containing b quarks and $b\bar{b}$ quark pairs, operated respectively at the KEKB e^+e^- collider at KEK and at the PEP-II e^+e^- collider at SLAC Linear Accelerator Laboratory with c.m. energies at the $\Upsilon(4S)$ mass, i.e. 10.58 GeV. The CDF and D0 experiments operated at the 1.96 TeV Tevatron $p\bar{p}$ collider. Finally, at the high end, the ATLAS, CMS and LHCb (the first hadron collider experiment dedicated to heavy-flavour physics) operate at the LHC pp collider at CERN, with c.m. energies between 7 and 13 TeV.

The fact that most exotic hadron candidates are quarkoniumlike particles is a posteriori not unexpected. Indeed, the number of different signatures available for such candidates makes them a natural target of this kind of research. A possible signature of exotic structure would be a combination of quantum numbers J^{PC} not allowed by non-relativistic quark models, where J is the spin, P the parity and C the charge conjugation parity. In such models a quarkoniumlike meson with total quark spin S and quark orbital angular momentum L has $J = L + S$, $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, and therefore 0^{--} and the series $J^{(-1)^J(-1)^{J+1}}$ cannot be obtained for any value of L and S. Moreover, charged quarkoniumlike states are necessarily exotic, since they must contain more valence quarks than just the neutral pair $c\bar{c}$ or $b\bar{b}$. Finally, the rather good understanding of the quarkonium spectrum (by means of potential models) and the limited availability of unassigned states with masses below 4.5 GeV, makes possible to pinpoint supernumerary states.

Another reason of the importance of quarkoniumlike particles in the search of unobserved exotic states is the fact that the large mass of the heavy quarks suppress the production

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[1]The Belle experiment ran from 1998 to 2010, data analysis is still ongoing. In 2010 the Belle II experiment was approved, which should operate at SuperKEKB accelerator, an upgrade of the KEKB accelerator intended to provide a peak luminosity 40 times larger than KEKB. Similarly, the BABAR experiment run from 1999 to 2008 but the analyses on collected data continue.

of quark anti-quark pairs from vacuum during quark-to-hadron fragmentation processes. Therefore, if a heavy quark and its antiparticle are found in the decay products of an unforeseen meson resonance, then it is likely that they are among the constituents of such a resonance and if they are the only constituents then the resonance is a quarkoniumlike state.

Analogously to the standard hadronic spectrum, in absence of any analytical method for making first-principle calculations of the non-standard one, simplified models have been developed. QCD suggests a colour-motivated classification of non-relativistic models in multiquark hadrons, such as tetraquark mesons and pentaquark baryons, in hybrid hadrons, which are formed by colourless combinations of quarks and valence gluons, and in glueballs, i.e. mesons composite only of gluons. Other models which do not fit or extend this classification are

QCD diquarks: it is possible to form coloured combinations of quark pairs that in many respects act as single particles and are called diquarks. These objects are not colour singlets, therefore cannot exist as free particles. However, they could combine with other quarks or diquarks to form colourless particles. This abstraction can be exploited to elaborate models for ordinary as well as exotic hadrons.

QCD hybrids: it is known that the colour-force field lines within a meson do not spread out in space, but are tightly confined in "flux tubes" that run between the quark and the anti-quark. In excited states it is possible to assign quantum numbers consistent with one or more gluons to the flux tube. In this way some combinations of J^{PC} quantum numbers not allowed to conventional mesons become accessible and such states are called hybrids, meaning that they are formed by a combination of quarks and (valence) gluons.

Hadronic Molecules: in this model remnants of the strong force, mediated by Yukawa-type meson exchange, are able to produce bound states between hadrons. Given the analogy with their electromagnetic counterpart, these systems are called hadronic molecules and are expected to have masses near the constituent particles threshold and the quantum numbers of an s-wave combination of them.

Hadrocharmonium: in this model, a compact colour-singlet charmonium core state is embedded in a spatially extended "blob" or "cloud" of light hadronic matter. These two components interact via QCD version of the van der Waals force. It can be shown that the mutual forces in this configuration are strong enough to form bound states if the light hadronic matter is a highly excited resonant state.

Kinematically Induced Mass Peaks: in some conditions, kinematic effects can produce resonance-like peaks in the invariant mass distribution of the reaction products and these peaks can be confused with the signal of an unstable hadron.

Many of the charmoniumlike states where first observed in e^+e^- collisions at c.m. energy of the $\Upsilon(4S)$, hence it is worth reviewing some of the production modes that occur in such processes.

Charmed quark production in B-meson decays: $\overline{B}$ mesons have a quark content $b\overline{q}$, where $\overline{q}$ is a light anti-quark[2]. When $\overline{q} = \overline{d}, \overline{u}$ their mass is approximately 5.28 GeV. Therefore, in e^+e^- annihilations at c.m. energy of 10.58 GeV, they are produced in $B\overline{B}$ pairs that are nearly at rest and with no accompanying particles. The primary decay channel of such mesons is by weak decay of $b \to c$ though the emission of a virtual W^- boson, which approximately 15% of the times materializes as a $\overline{c}$ and a s quarks. Therefore the s quark can form a K meson with the spectator anti-quark $\overline{q}$ while the $c\overline{c}$ pair can form a charmoniumlike state. This state must have J^{PC} equals to 0^{-+}, 1^{--} or 1^{++}.

Two-photon fusion process: both the incoming e^+ and e^- radiate photons, which then fuse to form a $q\overline{q}$ pair. This is a fourth order QED process and thus its probability is proportional to e_q^4, where e_q is the electric charge of q. Hence this fact favours the production of $c\overline{c}$ pairs against, say s or b quark anti-quark pairs. It can be shown that the dominant contribution to this process comes from events in which e^+ and e^- scatter at small angles and are therefore undetectable. Moreover, for such events the transverse momentum of $c\overline{c}$ is small and these facts all together provide a useful experimental signature. The allowed J^{PC} for $c\overline{c}$ in these processes are $0^{\pm+}$ and $2^{\pm+}$.

Double charmonium production: The production of J/ψ plus a $c\overline{c}$ following e^+e^- annihilation accounts for approximately 60% of the events with a J/ψ in the final state (prior to the operation of the B factories, this ratio was predicted to be less than 10%). This production method favours charmoniumlike systems with $J = 0$.

Initial State Radiation (ISR): occasionally one of the two collision partner radiates a high energy photon and subsequently annihilates at reduced c.m. energy. This allows to probe a broad range of reduced c.m. energies and the direct production of charmonium (or charmonium-like) states. Charmoniumlike particles produced in this way have $J^{PC} = 1^{--}$.

The nomenclature for exotic states is not entirely settled nor consistent. The name currently employed for charmoniumlike states are X, Y and Z, while bottomoniumlike ones are labelled with a subscript (i.e. Y_b and Z_b). Usually, charmoniumlike states first seen in B-meson decays are called X, those first observed in ISR processes are called Y, while charged charmoniumlike states are called Z. However, it should be noticed that the Particle Data Group calls all charmoniumlike particles X and bottomoniumlike ones X_b. The pentaquark candidates are more consistently indicated in the literature as P_c. I now list a few of the observed exotic candidates that are most significant to show the progress in this field.

$X(3872)$ **and** $Y(4260)$ **Neutral Charmoniumlike States**   The $X(3872)$ is the first charmoniumlike non-standard candidate that was observed. It was observed in 2003 by the Belle collaboration and appeared as an unforeseen peak in the $\pi^+\pi^-J/\psi$ invariant mass

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[2]$B_c^\pm$ ($b\overline{c}$ or $\overline{b}c$) mesons have also been observed, however they do not fit in the category of heavy-light mesons this thesis is concerned with.

distribution in $B \to K\pi^+\pi^- J/\psi$ decays, and later confirmed by many other experiments. Its most peculiar feature is its mass $M_{X(3872)} = 3871.69 \pm 0.17$ MeV [41], which at the current level of precision is indistinguishable from the $D^0 \overline{D}^{*0}$ threshold. There is strong evidence that this particle has even charge conjugation parity and the LHCb collaboration found that the assignment $J^{PC} = 1^{++}$ has the highest likelihood value for $J \leq 4$. Among the states predicted by the quark model, the only one with $J^{PC} = 1^{++}$ is the state called χ'_{c1}, the first radial exitation of the χ_{c1}. This assignment is debated, in particular since in this case the observed decay $X(3872) \to \rho J/\psi$ would violate isospin. These facts led to the speculation that this particle could be indeed a non-standard hadron. In particular its mass suggested that the $X(3872)$ could be a hadronic molecule made by a D^0 and a D^{*0}. However, also this picture is controversial and the identification of the $X(3872)$ is still uncertain.

After the discovery of the $X(3872)$ but before its quantum numbers were established, the BABAR collaboration considered the possibility it were a 1^{--} state and searced for its direct production in the ISR process $e^+ e^- \to \gamma_{\text{ISR}}\pi^+\pi^- J/\psi$. This analysis led to the conclusion that the quantum numbers of $X(3872)$ are not $J^{PC} = 1^{--}$. However, during the same analysis it was found an accumulation of events with $\pi^+\pi^- J/\psi$ invariant masses with a peak near 4.26 GeV. This resonance was called $Y(4260)$ and its existence was later confirmed by other experiments, CLEO and Belle in the first place. Its production mode ensures that $Y(4260)$ has the same quantum numbers of the photon, i.e. $J^{PC} = 1^{--}$. However, all the 1^{--} charmonium levels below 4.5 GeV are assigned to well-established resonances and hence the $Y(4260)$ qualifies as an exotic candidate. Other properties of the $Y(4260)$ which contrast with the identification with an ordinary charmonium state are the absence of any sign of decays to open-charm mesons and its strong signal in the $\pi^+\pi^- J/\psi$ decay channel. These facts led to the speculation that the $Y(4260)$ might be a multi-quark meson or a hybrid meson.

$Z(4430)^+$, $Z(4200)^+$ **and** $\chi_{c1}\pi^+$ **Charged Resonances**   The $Z(4430)^+$ is the first charged charmonium candidate established and it was observed by Belle in 2007 as a peak in the invariant mass of the $\psi'\pi^+$ system in $\overline{B} \to \psi'\pi^+ K$. At the beginning, BABAR did not find strong evidence of such a resonance but was not able to rule it out either. Subsequent analyses by Belle confirmed the existence of the $Z(4430)^+$, although with larger mass and total width with respect to what suggested by the first analysis. Later its existence was confirmed independently by LHCb and both experiments agree on the $J^P = 1^+$ assignment.

In 2014 the Belle Collaboration found that a satisfactory description of the decays $\overline{B}^0 \to J/\psi\pi^+ K^-$ was possible only including contribution of two resonances in the $J/\psi\pi^+$ channel, one with quantum numbers $J^P = 1^+$ which was called $Z(4200)^+$ and the other corresponding to the $Z(4430)^+$. However, at present, the $Z(4420)^+$ still needs independent confirmation.

In the same analyses the Belle collaboration also reported evidence for two charged $\chi_{c1}\pi^+$ resonances in a Dalitz plot analysis of $\overline{B}^0 \to \chi_{c1}\pi^+ K^-$ decays with axis $M^2(K\pi)$ vs $M^2(\chi_{c1}\pi)$. These resonances were called $Z(4050)^+$ and $Z(4250)^+$ but, as for the $Z(4200)^+$, still await for independent confirmation and a more complete analysis of the six-dimensional phase space (three-body decay).

$Z_b(10610)^+$ **and** $Z_b(10650)^+$ **Charged Bottomoniumlike Resonances**   In 2006 the Belle collaboration found an anomalously large $\pi^+\pi^-\Upsilon(n_rS)$ ($r = 1, 2, 3$) production rate, while searching for a behaviour in the bottomonium system similar to the large $Y(4260) \rightarrow \pi^+\pi^-J/\psi$ signal found by BABAR. This peak in the production rate near c.m. energy of 10.89 GeV is very close to the resonance known as $\Upsilon(10860)$, which is at present identified with $\Upsilon(5S)$. In 2012 further analysis of the same collaboration in the same region led to the discovery of two previously unseen bottomonium states, the $h_b(1P)$ and $h_b(2P)$, and of two charged bottomoniumlike states $Z_b(10610)^+$ and $Z_b(10650)^+$, which appeared in intermediate processes $e^+e^- \rightarrow \pi^\pm Z_b(10610)^\mp$ and $e^+e^- \rightarrow \pi^\pm Z_b(10650)^\mp$. Later studies assigned $J^P = 1^+$ to both $Z_b(10610)^+$ and $Z_b(10650)^+$ and observed a neutral partner of the $Z_b(10610)^+$, where the ratio of cross sections of the charged and associated neutral processes are consistent with expectations for an isovector $Z_b(10610)^+$. The Z_b^+ masses are just above the production threshold for $B\bar{B}^*$ and $B^*\bar{B}^*$ respectively. This fact, together with their quantum numbers, motivated the experimental research of decays in such channels and theoretical speculations about their interpretation as s-wave hadronic molecules. Indeed, in 2016 the Belle collaboration observed the decays $Z_b(10610) \rightarrow B\bar{B}^*$ and $Z_b(10650) \rightarrow B^*\bar{B}^*$.

$P_c(4380)^+$ **and** $P_c(4450)^+$ **Pentaquark Candidates**   In 2015 the LHCb collaboration reported evidence for pentaquarklike structures with a minimal quark content of $uudc\bar{c}$ in the context of a study of the process $\Lambda_b^0 \rightarrow J/\psi pK^-$ resonances. In addition to contributes from many established $\Lambda^* \rightarrow K^-p$ intermediate resonances, the data contained a narrow peak in the $J/\psi p$ invariant mass. It was performed a kinematically complete six-dimensional amplitude fit finding that it was necessary to add two non-standard $P_c^+ \rightarrow J/\psi p$ pentaquark contributions to reproduce this structure in the $M(J/\psi p)$ invariant mass. These two states were called $P_c(4380)^+$ and $P_c(4450)^+$. Although the quantum numbers of these states were not uniquely determined, LHCb was able to restrict their value to the combinations $J_{P_c^+(4380)} = 3/2$ and $J_{P_c^+(4450)} = 5/2$ or $J_{P_c^+(4380)} = 5/2$ and $J_{P_c^+(4450)} = 3/2$, with $P_c^+(4380)$ and $P_c^+(4380)$ having of opposite parity in the two cases.

1.2   The Present State of Heavy Meson Spectroscopy

The expression charmed hadrons, or open-charm hadrons, means hadrons containing a single charm quark c. Hadrons containing more than one c quark are called doubly charmed, while hadrons in which c appears in quark anti-quark pairs $c\bar{c}$ are said to have hidden charm. Hadrons in which the other constituents are light quarks (i.e. u, d or s) are of particular interest. Indeed, given the great difference between the masses of heavy and light quarks, several approximations can be made in the description of such hadrons that can be consistently formulated as effective theories. Among the baryons there are many different known charmed states, which are usually labelled with a subscript c (e.g. Λ_c, Σ_c, etc.), while among the mesons one can distinguish the D and D_s mesons, which respectively have strangeness $S = 0$ or $S = 1$. Analogous considerations hold for beauty hadrons, in which case the mesons with open beauty are the B and B_s mesons. Together the $D_{(s)}$ and $B_{(s)}$ mesons are referred to as heavy mesons, or sometimes as heavy-light mesons, when one

$c\bar{q}$	mass (MeV)	τ (s)	J^P
D^0	1864.83 ± 0.05	$(4.101 \pm 0.08) \times 10^{-13}$	0^-
D^+	1869.59 ± 0.09	$(1.040 \pm 0.007) \times 10^{-12}$	0^-

$c\bar{q}$	mass (MeV)	Γ (MeV)	J^P
D^{*0}	2006.85 ± 0.05	< 2.1	1^-
D^{*+}	2010.26 ± 0.05	0.083 ± 0.002	1^-
$D_0^*(2400)^0$	2318 ± 29	267 ± 40	0^+
$D_0^*(2400)^+$	2351 ± 7	230 ± 17	0^+
$D_1'(2430)^0$	2427 ± 40	384^{+130}_{-110}	1^+
$D_1(2420)^0$	2420.8 ± 0.5	31.7 ± 2.5	1^+
$D_1(2420)^+$	2423.2 ± 2.4	25 ± 6	1^+
$D_2^*(2460)^0$	2460.7 ± 0.4	47.5 ± 1.1	2^+
$D_2^*(2460)^+$	2465.4 ± 1.3	46.7 ± 1.2	2^+

Table 1.1: Masses, widths and spin-parity of the established open-charm mesons [41].

wants to stress the fact that the other quark is a light one. The following of this section will be concerned with an overview of the present state of heavy meson spectroscopy, with a focus on charmed mesons, the subject of this thesis.

Open-charm Spectroscopy

Well established charmed mesons are the pseudoscalar D meson, the vector D^*, the axial vector $D_1(2420)$ and the $J^P = 2^+$ $D_2^*(2460)$. However, today the spectrum of observed states is far richer. The modern history of charmed meson spectroscopy can be made to begin in the 2000, with the evidence gained by the CLEO collaboration of two new broad states [12].

In 2003 the Belle collaboration confirmed the latter, with the observation of two broad states near 2.4 GeV obtained through the Dalitz plot analysis of charged B decays $B^\pm \rightarrow D^{(*)\mp}\pi^\pm\pi^\pm$ [10], where the selection of final states with pions with the same sign prevented them from forming bound states (thus making the analysis simpler). Indeed, contributions from the D_2^{*0} and a broad scalar resonance, the $D_0^*(2400)^0$, were needed in order to describe the data in the channel $D\pi\pi$, while contribution from the D_2^{*0}, the D_1^0 and a broad axial excited D meson, the $D_1'(2430)^0$, were needed in the channel $D^*\pi\pi$. It was also proposed that there could be a mixing of the two 1^+ states, i.e. D_1 and D_1', and indeed Belle measured a small mixing angle $\omega = 0.10 \pm 0.04$ rad ≈ 6 deg, suggesting that such mixing can be safely neglected. Particles with quantum numbers and broad widths[3] of these new states were expected in the quark model, but in a different mass region.

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[3]Associated to s-wave decays, in contrast with the narrow D_2^* and D_1, which decay through d-wave.

Resonance	mass (MeV)	Γ (MeV)	J^P
$D(2550)^0$	$2539.4 \pm 4.5 \pm 6.8$	$130 \pm 12 \pm 13$	0^-
$D^*(2600)^0$	$2608.7 \pm 2.4 \pm 2.5$	$93 \pm 6 \pm 13$	natural
$D^*(2600)^\pm$	$2621.3 \pm 3.7 \pm 4.2$	93 (fixed)	natural
$D(2750)^0$	$2752.4 \pm 1.7 \pm 2.7$	$71 \pm 6 \pm 11$	
$D^*(2760)^0$	$2763.3 \pm 2.3 \pm 2.3$	$60.9 \pm 5.1 \pm 3.6$	natural
$D^*(2760)^\pm$	$2769.7 \pm 3.8 \pm 1.5$	60.9 (fixed)	natural

Table 1.2: Mass, width and spin-parity of new open-charm meson candidates observed by BaBar in 2010 [11].

Shortly after, in the 2004, another confirmation of the existence of the $D_0^*(2400)$ (both in the neutral and charged version) came from the analysis of data collected by the FOCUS experiment[4] at Fermilab in the 1996-1997 fixed-target run. Finally, the $D_0^*(2400)^0$ was observed also by BaBar [15] in 2009 and the $D_0^*(2400)^-$ by LHCb in 2015 [4, 5] while the $D_1'(2430)$ by BaBar [18] in 2006. In light of these observations, the existence of these states can be safely considered as established (thus are included in table 1.1).

In 2010 the BaBar collaboration made an inclusive production study [11] of processes of the type $e^+e^- \to c\bar{c} \to D^{(*)}\pi X$ at SLAC-PEPII, where X is any additional system. The size of the event sample was approximately 6×10^8 events at c.m. energy of 10.58 Gev, corresponding to $\approx$ 450fb^{-1}. The $D\pi$ system was investigated in the neutral $D^+\pi^-$ and charged $D^0\pi^+$ channels, were the D^+ was reconstructed through the decay $D^+ \to K^-\pi^+\pi^+$ and the D^0 through $D^0 \to K^-\pi^+$. The $D^*\pi$ system was investigated only in the neutral channel $D^{*+}\pi^-$ and reconstructed through the decay $D^{*+} \to D^0\pi_s^+$, were π_s is a slow pion constrained to having a production vertex in proximity of the D^0, which was reconstructed through $D^0 \to K^-\pi^+$ and $D^0 \to K^-\pi^+\pi^-\pi^+$ decay modes.

The $D\pi$ system was studied looking for structures for the mass variables $M(D^+\pi^-) = m(K^-\pi^+\pi^+\pi^-) - m(K^-\pi^+\pi^+) + m_{D^+}$ and $M(D^0\pi^+) = m(K^-\pi^+\pi^+) - m(K^-\pi^+) + m_{D^0}$. In this amplitude analysis, the peaks for the $D_2^*(2460)$ isospin triplet, and two unforeseen structures around 2.6 GeV and 2.75 GeV, the $D^*(2600)$ and the $D^*(2760)$, were observed. It should be noticed that spin-parity conservation implies that states strongly decaying to $D\pi$ must have natural parity, moreover, if they also decay to $D^*\pi$ then the $J^P = 0^+$ assignment is ruled out. Also, because the observed $D^0\pi^+$ mass spectrum was similar to the one of $D^+\pi^-$, but the background was larger and the statistical precision lower, assuming isospin symmetry, the width of the new resonances were fixed to the values measured in the $D^+\pi^-$ channel (see table 1.2).

The $D^*\pi$ system was analysed looking for structures for the mass variable $M(D^{*+}\pi^-) =$

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[4]FOCUS, aka E831, was an upgraded version of the experiment E687, where a forward multi-particle spectrometer was used to to investigate the interactions of high energy photons on a segmented BeO target. The photon beam was derived from the bremmstrahlung of secondary electrons (< 250 GeV) produced from the Tevatron proton beam.

Resonance	mass (MeV)	Γ (MeV)	J^P
$D_J(2580)^0$	$2579.5 \pm 3.4 \pm 5.5$	$177.5 \pm 17.7 \pm 46.0$	unnatural
$D_J^*(2650)^0$	$2649.2 \pm 3.5 \pm 3.5$	$140.2 \pm 17.1 \pm 18.6$	natural
$D_J(2740)^0$	$2737.0 \pm 3.5 \pm 11.2$	$73.2 \pm 13.4 \pm 25.0$	unnatural
$D_J^*(2760)^0$	$2761.1 \pm 5.1 \pm 6.5$	$74.4 \pm 4.3 \pm 37.0$	natural
$D_J^*(2760)^0$	$2760.1 \pm 1.1 \pm 3.7$	$74.4 \pm 3.4 \pm 19.1$	natural
$D_J^*(2760)^\pm$	$2771.7 \pm 1.7 \pm 3.8$	$66.7 \pm 6.6 \pm 10.5$	natural
$D_J(3000)^0$	2971.8 ± 8.7	188.1 ± 44.8	unnatural
$D_J^*(3000)^0$	3008.1 ± 4.0	110.5 ± 11.5	natural
$D_J^*(3000)^\pm$	3008.1 (fixed)	110.5 (fixed)	natural

Table 1.3: Masses, widths and spin-parity of new open-charm meson candidates observed by LHCb in 2013 [1].

$m\left(K^-\pi^+\pi^+\pi^-\pi_s^+\pi^-\right) - \left(K^-\pi^+\pi^+\pi^-\pi_s^+\right) + m_{D^{*+}}$, which brought to the observation of the peaks corresponding $D_1(2420)^0$ and $D_2^*(2460)^0$; a peak near 2.6 GeV, identified with the $D^*(2600)$ found in the $D\pi$ system, and another peak near 2.75 GeV, called the $D(2750)^0$.

In order to fix the spin-parity of the $D^*(2600)^0$ and $D^*(2750)^0$ a study of the helicity angle θ_h was conducted. The latter is defined as the angle between the flight of the fast primary pion in the c.m. frame of $D^*\pi$ and the flight of the slow secondary pion, produced in the decay of the D^*, in the frame in which D^* is at rest. Signals from natural parity states are proportional to $\sin^2(\theta_h)$ (except for $J^P = 0^+$, which is ruled out), the ones from 0^- states are proportional to $\cos^2(\theta_h)$ and signals from all other unnatural parity states are proportional to $1 - h\cos^2(\theta_h)$, where $h > 0$ is a free parameter.

The analysis was initially conducted assuming only two signals at 2.6 GeV and 2.75 GeV. However, these hypothesis failed to describe well the data and this led to the inclusion of another state in the mass region of the $D^*(2660)$, the $D(2550)^0$. This analysis yielded results compatible with natural parity assignment to the $D^*(2600)^0$, which supported the identification of the latter as the same state found in the $D\pi$ channel. The $D(2550)^0$ was found to be consistent with $J^P = 0^-$, while the assignment of natural or unnatural parity to the $D(2750)^0$ could not be decided.

Finally, BABAR measured the following relative branching ratios

$$\frac{\mathcal{B}\left(D^*(2600)^0 \to D^+\pi^-\right)}{\mathcal{B}\left(D^*(2600)^0 \to D^{*+}\pi^-\right)} = 0.32 \pm 0.02 \pm 0.09\,, \tag{1.1a}$$

$$\frac{\mathcal{B}\left(D^*(2760)^0 \to D^+\pi^-\right)}{\mathcal{B}\left(D^*(2750)^0 \to D^{*+}\pi^-\right)} = 0.42 \pm 0.05 \pm 0.11\,. \tag{1.1b}$$

The assignment $J^P = 3^-$ for the $D_J^*(2760)$ was subsequently fixed by a Dalitz plot analysis of the LHCb collaboration [5].

Resonance	mass (MeV)	Γ (MeV)	J^P
$D_1^*(2680)^0$	$2681.1 \pm 5.6 \pm 4.9 \pm 13.1$	$186.7 \pm 8.5 \pm 8.6 \pm 8.2$	1^-
$D_3^*(2760)^0$	$2775.5 \pm 4.5 \pm 4.5 \pm 4.7$	$95.3 \pm 9.6 \pm 7.9 \pm 33.1$	3^-
$D_2^*(3000)^0$	$3214 \pm 29 \pm 33 \pm 36$	$186 \pm 39 \pm 34 \pm 63$	2^+

Table 1.4: Masses, widths and spin-parity of new open-charm meson candidates observed by LHCb in 2016 [3].

In 2013 the LHCb collaboration performed a search for excited D mesons in pp collisions at c.m. energy of 7 TeV. The data sample consisted of 1 fb^{-1} of events corresponding to inclusive reactions of the type $pp \rightarrow D^+\pi^-X$, $pp \rightarrow D^0\pi^+X$ and $pp \rightarrow D^{*+}\pi^-X$. These processes were investigated through the same decay channels and mass variables already outlined for the BABAR study of 2010.

In order to analyse the $D^{*+}\pi^-$ mass spectrum three sub-samples were extracted: a *natural parity sample* with $\cos(\theta_h) < 0.5$ (68.8% of the initial sample), an *unnatural parity sample* with $\cos(\theta_h) > 0.5$ (31.2%) and an *enhanced unnatural parity sample* with $\cos(\theta_h) > 0.75$ (8.6%). In the enhanced unnatural sample a resonance consistent with the $D_1(2420)^0$ was observed as well as other three resonances: the $D_J(2580)^0$, the $D_J(2740)^0$ and the $D_J(3000)$, hence all with unnatural parity. The masses and widths of these resonances are fixed in the fit for the natural parity sample, which exhibits the prominent contribution of the $D_2^*(2460)^0$ and the minor ones of the $D_J(2580)^0$, $D_J(2740)^0$ and $D_J(3000)^0$. Within this sample two new resonances were observed, the $D_J^*(2650)^0$ and the $D_J^*(2760)^0$, which thus have natural parity. The unnatural parity sample and the total sample were used as a cross-check, with parameters fixed by the previous fits. Finally, the spin-parity assignments were checked against the analysis of the helicity angle (similar to those performed in the BABAR study) confirming previous result (the $J^P = 0^-$ hypothesis was also considered for the $D_J(2580)^0$).

In the charged and neutral $D\pi$ systems, the charged partner of the $D_J^*(2760)^0$ was observed together with two other nonestablished states, the $D_J^*(3000)^0$ and the $D_J^*(3000)^\pm$. The mass and width of the latter were fixed to the value of the former, due to large background and poor statistics, assuming isospin symmetry. However, the LHCb collaborations warns that the properties of these last states are uncertain and that they may correspond to superposition of several states in that mass region.

In 2016 the LHCb collaboration made a Dalitz plot analysis of $B^- \rightarrow D^+\pi^-\pi^-$ decays [3] looking for structure in the neutral $D^+\pi^-$ channel, where the D^+ was reconstructed through the decay $D^+ \rightarrow K^-\pi^+\pi^+$, using a data sample correspondint to 3 fb^{-1} of pp collisions at c.m. energy of 7 TeV (2011) and 8 TeV (2012). Apart from the $D_2^*(2460)^0$, three other resonances with masses near 2.68, 2.76 and 3.0 GeV were observed. The analyses allowed to determine the spin of these resonances and, because they must belong to the natural parity series, to fix the parity. These resonances are the $D_1^*(2680)$, the $D_3^*(2760)^0$ and the $D_2^*(3000)$ and their properties are reported in table 1.4. It is not clear if the $D_1^*(2680)$ or the $D_2^*(3000)$ can be identified with previously observed states, while the $D_3^*(2760)$ seems to be consistent with previous measurement [11, 1, 3].

$c\bar{s}$	mass (MeV)	τ (s)	J^P
D_s^+	1968.28 ± 0.10	$(5.00 \pm 0.07) \times 10^{-13}$	0^-

$c\bar{s}$	mass (MeV)	Γ (MeV)	J^P
D_s^{*+}	2112.1 ± 0.4	< 1.9	1^-
$D_{s0}^*(2317)^+$	2317.7 ± 0.6	< 3.8	0^+
$D_{s1}'(2460)^+$	2459.5 ± 0.6	< 3.5	1^+
$D_{s1}(2536)^+$	2535.10 ± 0.06	0.92 ± 0.05	1^+
$D_{s2}^*(2573)^+$	2569.1 ± 0.8	16.9 ± 0.8	2^+
$D_{s1}^*(2700)^+$	$2708.3^{+4.0}_{-3.4}$	120 ± 11	1^-
$D_{s1}^*(2860)^+$	2859 ± 27	159 ± 80	1^-
$D_{s3}^*(2860)^+$	2860 ± 7	53 ± 10	3^-
$D_{sJ}^*(3040)^+$	3044^{+31}_{-9}	239 ± 60	

Table 1.5: Masses, widths and spin-parity of observed $c\bar{s}$ mesons [41].

The spectrum of observed strange open-charm mesons has seen as well many improvements in recent years. Before the operation of the B-factories only four charmed mesons with strangeness were known: the pseudoscalar D_s meson, the vector D_s^*, the axial $D_{s1}(2536)$ and the $J^P = 2^+ D_{s2}^*(2573)$. The D_s^* has a narrow width and decays mainly through $D_s\gamma$. The $D_{s1}(2536)$ and the $D_{s2}(2573)$ decay respectively in the D^*K and DK channels.

In 2003 the BABAR collaboration observed a very narrow state with mass near 2.32 GeV [13] in e^+e^- collisions at c.m. energy of the $\Upsilon(4s)$ ($\approx$ 10.6 GeV), which was called the $D_{sJ}^*(2317)$. The data sample consisted of 91 fb^{-1} of events related to inclusive processes $e^+e^- \to D_s\pi^0X$, where X is an arbitrary collection of particles. The D_s^+ was reconstructed through the decay $D_s^+ \to K^+K^-\pi^+$ while the π^0 through $\pi \to \gamma\gamma$. The decay mode indicated natural parity and hence its low mass suggested the assignment 0^+ (i.e. the lowest available state). The narrow width was consistent with decays violating isospin conservation. However, if tentative $J^P = 0^+$ assignment is confirmed, the low mass, small width, and decay mode of $D_{sJ}^*(2317)$ are quite different from those predicted by potential models.

Shortly after, the CLEO experiment confirmed the existence of this state and reported of another one with mass near 2.46 GeV [21], which was called the $D_{sJ}(2463)$ (a.k.a. $D_{sJ}(2460)$). The search was conducted on data collected by the CLEOII detector and relative to the same inclusive processes and c.m. energy of the BABAR study. Indeed, this search was motivated by the latter. The reconstruction of the D_s^+ in these case was through the chain of decays $D_s^+ \to \phi\pi^+$ and $\phi \to K^+K^-$. In the same year, the Belle collaboration observed these states in inclusive processes [9] and in B decays [34] ($B \to \overline{D}D_{sJ}(2317)$ and $B \to \overline{D}D_{sJ}(2457)$, in the article indicated as $D_{sJ}(2457)$).

The spin-parity of the $D_{sJ}^*(2317)$ and of the $D_{sJ}(2460)$, or $D_{s0}^*(2320)$ and $D_{s1}^*(2460)$ as the PDG calls these states, is not firmly established. For the D_{s0}^*, experimental data have not ruled out higher spin from the natural series. There is also evidence for assignment of $J^P = 1^+$ to the $D_{s1}(2460)$, which nonetheless cannot be considered definitive [14]. However, their width

and mass splitting is consistent with what is expected from their supposed identification.

It is remarkable that their masses are much smaller than suggested by the quark model, as can be easily seen comparing other corresponding pairs of heavy mesons with and without strangeness. Indeed, they are almost degenerate with their non-strange partners whereas others differ by about 100 MeV, which accounts for the mass difference between the strange and the down or up quarks.

In 2006 the BABAR collaboration observed a new resonance in $D^0 K^+$ and $D^+ K_S^0$ systems [16] during amplitude analysis of $e^+ e^-$ collisions corresponding to 240 fb^{-1} at the c.m. energy of 10.6 GeV, which was called $D_{sJ}^*(2860)^+$. Moreover, in the same analysis it was observed a signal which could not be described by enhancements produced by two or three body decays of known narrow resonances where one of the decay products was missed (so-called *reflections*), which was indicated as $X(2690)^+$ and had a rather large width ($\approx$ 100 MeV). The latter was later identified with a vector D_s meson excitation, called $D_{s1}^*(2700)$, observed by Belle in 2008 [22] during the analysis of approximately 5×10^8 $B\overline{B}$ events at 10.6 GeV, through $B^+ \to \overline{D}^0 D_s^{**} \to \overline{D}^0 D^0 K^+$ decays (where D_s^{**} is a generic state).

The existence of these two new D_s mesons was confirmed by BABAR in 2009 [17] in a study of the decays $D_{s1}^*(2710)^+ \to D^*K$ and $D_{sJ}^*(2860)^+ \to D^*K$. An helicity angle analysis of these decays allowed to confirm the natural spin-parity of the $D_{s1}^*(2700)$ and of the $D_{sJ}^*(2860)$. Moreover, it was possible to calculated the relative branching ratios

$$\frac{\mathcal{B}\left(D_{s1}^*(2700)^+ \to D^*K\right)}{\mathcal{B}\left(D_{s1}^*(2700)^+ \to DK\right)} = 0.91 \pm 0.13 \pm 0.12 \,, \tag{1.2a}$$

$$\frac{\mathcal{B}\left(D_{sJ}^*(2860)^+ \to D^*K\right)}{\mathcal{B}\left(D_{sJ}^*(2850)^+ \to DK\right)} = 1.10 \pm 0.15 \pm 0.19 \,, \tag{1.2b}$$

where $\frac{\mathcal{B}(D_s^{**+} \to D^*K)}{\mathcal{B}(D_s^{**+} \to DK)}$ is the weighted average of $\frac{\mathcal{B}(D_s^{**+} \to D^{*0}K^+)}{\mathcal{B}(D_s^{**+} \to D^0 K^+)}$ and $\frac{\mathcal{B}(D_s^{**+} \to D^{*+}K_S^0)}{\mathcal{B}(D_s^{**+} \to D^+ K_S^0)}$. Finally, in this study it was observed another broad state near 3 GeV, the $D_{sJ}(3040)$.

The existence of the $D_{s1}^*(2700)^+$ and of the $D_{sJ}^*(2860)^+$ was confirmed by LHCb in 2012 [2] in inclusive amplitude analysis of $D^+ K_S^0$ and $D^0 K^+$ systems in pp collisions relative to a data sample of 1 fb^{-1} at 7 TeV. However, they did not observed any statistically significant D_{sJ} resonance in the mass region above 3 GeV. In addition, a Dalitz plot analysis of $B^0 \to D^- D^0 K^+$ and $B^+ \to \overline{D}^0 D^0 K^+$ decays conducted by BABAR in 2014 [36] confirmed the assignment $J^P = 1^-$ to the $D_{s1}^*(2700)^+$ (but did not observed the $D_{sJ}^*(2860)$ nor the $D_{sJ}(3040)$).

The spin-parity assignment of the $D_{sJ}^*(2860)^+$ remained an open question until the 2014, when the LHCb collaboration performed a Dalitz plot analysis of the $B_s^0 \to \overline{D}^0 K^- \pi^+$ decay [7] using a sample corresponding to 3.0 fb^{-1} of pp collision data at c.m. energy of 7 TeV. The signal in the $m(\overline{D}^0 K^-) \approx 2.86$ GeV was found to be an admixture of spin 1 and spin 3 resonances. Therefore the $D_{sJ}^*(2860)$ state previously seen consisted of at least two particles, the $D_{s1}^*(2860)$ and the $D_{s3}^*(2860)$. The latter is the first state among heavy mesons to which experimental evidence assigned spin 3.

$b\bar{q}$	mass (MeV)	τ (s)	J^P
B^0	5279.63 ± 0.15	$(153.0 \pm 0.4) \times 10^{-14}$	0^-
B^-	5279.32 ± 0.14	$(163.8 \pm 0.4) \times 10^{-14}$	0^-

$b\bar{q}$	mass (MeV)	Γ (MeV)	J^P
B^*	5324.65 ± 0.25		1^-
$B_1(5721)^0$	5726.0 ± 1.3	27.5 ± 3.4	1^+
$B_1(5721)^-$	$5725.9^{+2.5}_{-2.7}$	31 ± 6	1^+
$B_2^*(5747)^0$	5739.5 ± 0.7	24.2 ± 1.7	2^+
$B_2^*(5747)^-$	5737.2 ± 0.7	20 ± 5	2^+

(a)

$b\bar{s}$	mass (MeV)	τ (s)	J^P
B_s^0	5366.89 ± 0.19	$(1.505 \pm 0.005) \times 10^{-12}$	0^-

$b\bar{s}$	mass (MeV)	Γ (MeV)	J^P
B_s^{*0}	$5415.4^{+1.8}_{-1.5}$		1^-
$B_{s1}(5830)^0$	5828.63 ± 0.27	0.5 ± 0.4	1^+
$B_{s2}^*(5840)^0$	5839.85 ± 0.17	1.47 ± 0.33	2^+

(b)

Table 1.6: Measured masses and widths of the observed open-beauty mesons [41]

Open-beauty spectroscopy

The spectrum of open-beauty mesons is less rich than the charmed one. The fundamental state B as well as the vector B^* are well established. The first evidences of open-beauty resonances were gained by several collaborations at LEP (OPAL, DELPHI, ALEPH) in the second half of the 90's. However, in these analyses of inclusive or semi-exclusive B decays, the separation of states was impossible. In 2007 the D0 collaboration observed two $J^P = 1^+$ and 2^+ states, the B_1 and the B_2^*, later confirmed by CDF. Observations in the strange and non-strange sector are reported in table 1.6. Moreover, all these states were confirmed by LHCb.

1.3 Afterword and a Note on the Baryon Sector

The present wealth of observed heavy mesons motivates phenomenological studies to classify them. This kind of studies corroborates and supports first principle calculations. Moreover the experimental analyses are not always able to bring a complete description of observed states, like in prompt production studies, that can only determine if the spin-parity of a given meson belongs to the natural or unnatural series, but cannot fix the precise value of J^P

quantum numbers. Furthermore in such cases, the phenomenological approach can predict or give criteria for assignment of those quantum numbers. However, these studies not only can guide in the identification of observed states, but also can suggest where to look for yet unobserved ones.

Even though this thesis is only concerned with the meson case and thus I will not review the general state of the baryon spectroscopy, I would like to conclude this overview by mentioning the latest important results in the baryon sector. Indeed, although the heavy mesons spectroscopy counts a larger number of observed states, the spectroscopy of baryons containing a heavy quark has seen great improvements in recent years and, notwithstanding its intrinsic experimental challenges, it is a promising field of experimental research due to the work at the LHCb and Belle II facilities.

The spectrum of Λ_c^+, Σ_c and Ξ_c particles, in spite of considerable experimental progress, is largely unknown. Moreover, until the 2017, the only Ω_c baryon states (with quark content *css* and isospin zero) known were the Ω_c^0 and the $\Omega_c^0(2770)$, presumed to be the $J^P = 1/2^+$ and $3/2^+$ states. In 2017, the $\Xi_c^+ K^-$ mass spectrum was investigated by the LHCb experiment [6]. A large high-purity sample of Ξ_c baryons was reconstructed in the Cabibbo-suppressed decay mode $pK^-\pi^+$ and five new, narrow excited states were observed: the $\Omega_c(3000)^0$, the $\Omega_c(3050)^0$, the $\Omega_c(3066)^0$, the $\Omega^c(3090)^0$ and the $\Omega_c(3119)^0$. In the same year a highly significant structure was observed in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum [8], where the Λ_c^+ baryon was reconstructed in the decay mode $pK^-\pi^+$. The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon Ξ_{cc}^+. Its mass was determined to be 3261 MeV.

As already noticed, many more observations in the near future are expected and their importance resides, other than in the enriching of our knowledge of hadron spectroscopy *per se*, in the hope that they could shed light on the nature of strong interactions.

Chapter 2

Heavy Quark and Chiral Symmetries

> The difficulties in quantum theory are of two kinds. I might call them Class One difficulties and Class Two difficulties. Class One difficulties are the difficulties I have already mentioned: How can one form a consistent picture behind the rules for the present quantum theory? These Class One difficulties do not really worry the physicist. If the physicist knows how to calculate results and compare them with experiment, he is quite happy if the results agree with his experiments, and that is all he needs. It is only the philosopher, wanting to have a satisfying description of nature, who is bothered by Class One difficulties. There are, in addition to the Class One difficulties, the Class Two difficulties, which stem from the fact that the present laws of quantum theory are not always adequate to give any results.
>
> (Paul A. M. Dirac, *The Evolution of the Physicist's Picture of Nature*)

In this chapter I introduce the theoretical framework in which the calculations that will be described in the next chapter have been performed. I first review the fundamentals of QCD and the reasons that lead to the use of effective theories. After a general introduction to them, I focus on the effective theories that are relevant for the research work presented in this thesis: the heavy quark effective theory and the chiral perturbation theory. Finally I introduce the chiral heavy quark theory, in which it is possible to study strong interactions of heavy hadrons with light pseudoscalar mesons.

2.1 Elements of Quantum Chromodynamics

Quantum ChromoDynamics (QCD) is the theory of strong interactions, that is the interactions which bind together the constituents of the hadrons. This theory originated in the 60's from the work of Gell-Mann, who used group theoretical ideas to classify hadrons in almost degenerate multiplets (the so-called *Eightfold Way*) and since the 70's is the most accredited theory describing hadronic physics. The fundamental particles of this theory are the *quarks*, which come in six *flavours* and can be organized in three *families* or *generations* of two flavours each (table 2.1).

First Generation		Second Generation		Third Generation	
Flavour	Mass [MeV]	Flavour	Mass [MeV]	Flavour	Mass [MeV]
d	4.7	s	96	b	4 180
u	2.2	c	1 280	t	173 100

Table 2.1: Quark generations and masses [41].

QCD is gauge field theory, i.e. a theory described by a Lagrangian density invariant under local transformations of a (gauge) symmetry group. In the case of QCD, this group is $SU(3)$, that is the group of unitary unimodular 3×3 matrices. Since $SU(3)$ is a non-Abelian group, QCD belongs to the class of Yang-Mills theories. In the following this group will be denoted as $SU(3)_C$, with the subscript C meaning *colour*, the quantum number of quarks.

The transformations of a finite dimensional representation of $SU(3)$ have the form (sum over repeated indices is understood)

$$U(\theta) = e^{-i\theta^a T^a} \tag{2.1}$$

where the matrices T^a are the eight generators of $SU(3)$ (which depend on the representation) while the scalars θ^a are the parameters of the transformation. The generators satisfy the commutation relation

$$[T^a, T^b] = if^{abc}T^c, \tag{2.2}$$

with f^{abc} structure constants of $SU(3)$. In the fundamental representation holds

$$T^a = \lambda^a/2, \tag{2.3}$$

where λ^a are the Gell-Mann matrices, while in the adjoint

$$(T^a)_{bc} = -if^{abc}. \tag{2.4}$$

Moreover, the generators always satisfy the relation

$$\text{Tr}\left[T^a T^b\right] = C(r)\,\delta^{ab}, \tag{2.5}$$

where $C(r)$ is a constant that depends on the representation r. In particular, for the fundamental representation one has $C(r) = 1/2$, while in the adjoint $C(r) = 3$.

Quarks belong to the fundamental representation of $SU(3)$. Therefore they can be written as triplets, the components of which are conventionally labelled red, green and blue. Their transformation law under a finite, global $SU(3)_C$ transformation is $q_a \mapsto U_{ab}(\theta)q_b$.

The free Dirac Lagrangian

$$\mathcal{L}_{\text{Dirac}} = \bar{q}\,(i\slashed{\partial} - m)\,q \tag{2.6}$$

is invariant under such transformations but not under local ones, i.e. transformations in which the parameters depend on coordinates. However, *if we manage to substitute the partial derivative in* (2.6) *with an operator which transforms in a covariant manner, then the Dirac*

Lagrangian would become automatically invariant under local transformations. Such an operator is called *covariant derivative* and is defined in the following way

$$D_\mu q = \left(\partial_\mu - ig\mathbf{A}_\mu\right) q, \tag{2.7}$$

where $\mathbf{A}_\mu$ are gauge fields with transformation law

$$\mathbf{A}_\mu \mapsto U\mathbf{A}_\mu U^{-1} + \frac{i}{g}U\partial_\mu U^{-1} \tag{2.8}$$

and g is the coupling constant of the theory.

$\mathbf{A}_\mu$ can be decomposed with respect to the generators $\mathbf{A}_\mu = A_\mu^a T^a$. After quantization, the quanta of the vector fields A_μ^a are called gluons. The variation of the gluon field with respect to a local infinitesimal transformation is

$$\delta A_\mu^a = f^{abc}\theta^b A_\mu^c - \frac{1}{g}\partial_\mu \theta^a, \tag{2.9}$$

which shows that it transforms according the adjoint representation of $SU(3)$.

After the use of the covariant derivative in place of the partial derivative in (2.6), a new Lagrangian density is obtained, consisting of the kinetic term for quarks plus a term describing quark-gluon interactions.

However, at this point, the gauge field is a background field with no dynamics. In order to account for the latter, the Lagrangian needs to be supplemented by a pure-gauge term. For this purpose, in analogy with Quantum ElectroDynamics (QED), a field strength tensor $\mathbf{G}_{\mu\nu}$ is introduced through the relation

$$\left[iD_\mu, iD_\nu\right] q = ig\mathbf{G}_{\mu\nu}q = igG_{\mu\nu}^a T^a q, \tag{2.10}$$

finding that

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c. \tag{2.11}$$

$\mathbf{G}_{\mu\nu}$ is covariant, i.e. it transforms with law $\mathbf{G}_{\mu\nu} \mapsto U\mathbf{G}_{\mu\nu}U^{-1}$, as well as every product of tensors $\mathbf{G}_{\mu\nu}\cdots\mathbf{G}_{\alpha\beta}$. Hence, the traces in colour space of these objects are gauge invariant. The simplest Lorentz and parity invariant term of this type is $\text{Tr}\left[\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right]$. Therefore, a natural choice for the pure gauge Lagrangian is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\text{Tr}\left[\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right] = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}, \tag{2.12}$$

where the prefactor is such that, in the Abelian case, (2.12) reduces to the free Maxwell Lagrangian. Finally, we can write

$$\mathcal{L}_{\text{QCD}} = \sum_{\substack{f=d,u,s,\\c,b,t}} \overline{q}_f\left(i\slashed{D} - m_f\right)q_f - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}. \tag{2.13}$$

Once the Lagrangian is known, the generating functional of the theory can be used to calculate perturbatively the n-point correlation functions, from which, using the LSZ reduction formula, it is finally possible to calculate the scattering matrix elements.

This procedure has been very successful for the electrodynamics and for the unified theory of electroweak interactions, however, in the case of QCD it has to be applied with care. Indeed, as for all field theories, the coupling constant of QCD is a *running* quantity, i.e. it depends on the scale at which it is measured. At odds with what happens with QED, the coupling constant is small in correspondence of large momentum scales or, analogously, at short distances. This phenomenon is called *asymptotic freedom* and implies that in the ultraviolet limit the quarks propagate almost freely.

However, for small momenta, the coupling constant is large, so that quarks are confined inside hadrons, a phenomenon known as *colour confinement*. This is why only colour singlet states are experimentally observed.

Therefore, QCD has a phase diagram which shows at least two phases: one in which the Degrees of Freedom (DoF's) are the quarks and gluons and one in which they are the hadrons. These are the only true relativistic bound states known (for example, in nucleons the binding energy accounts for more than 99% of the mass) and, at the present, there is no analytic tool to describe such states in terms of their fundamental constituents. However, it is still possible to use field theoretical techniques to attempt an approximate description of strong interactions at low energies through effective field theories.

2.2 Some General Remarks About Effective Theories

Effective theories give an approximate description of phenomenological observations conducted in certain conditions or regimes. In this thesis, the focus is on low-energy effective theories, which fall under two categories: decoupling or non-decoupling [31]. The effective theory for strong interactions which will be introduced in this chapter and used in the following, make use and combine aspects of both.

Decoupling Effective Theories

In decoupling effective theories, below some energy scale Λ, the DoF's of the (the sector of) the fundamental theory which one would like to describe remain the same, except that particles with mass $M \gg \Lambda$ are integrated out.

This fact can be stated more formally saying that, in the considered kinematic region, the n-point functions with heavy DoF's on external legs vanish. Such n-point functions can be obtained from a new generating functional Z', which is obtained from the old one Z, by a marginalization procedure. This procedure consists in decoupling the heavy DoF's from external currents in the expression of Z and integrating them out. In many cases of interest, the functional integral defining Z is Gaussian with respect to the heavy DoF's and hence Z' can be computed analytically.

Let be ϕ the heavy DoF of the theory with equation of motion

$$K_0 \phi = \chi, \tag{2.14}$$

where K_0 is a linear operator and χ is a function of the other fields. It follows that (neglecting a surface term) the relevant part of the generating functional Z' can be cast in the following

form

$$Z'_\phi = \int [\mathrm{d}\,\phi]\; e^{-\frac{i}{2}(\phi, K_0\phi)+i(\chi,\phi)} \propto \det \Delta_0 \exp\left(\frac{i}{2}(\chi, \Delta_0 \chi)\right) \qquad (2.15)$$

where $\Delta_0 = K_0^{-1}$.

If $\det \Delta_0$ does not introduce quantum corrections, then the whole procedure is equivalent to using the equation of motion (2.14) in the Lagrangian.

Finally, Z'_ϕ corresponds to a non-local action and hence, in order to apply the usual perturbation theory techniques, it is necessary to perform a so-called *OPE* (Operator Product Expansion) of Δ_0, i.e. expanding it in a series of local operators with increasing powers of $1/M$. These terms contain the dynamics at energy scales greater than Λ and may be treated perturbatively.

Examples of decoupling effective theories are the Fermi theory of weak interactions, the Euler-Heisenberg non-linear electrodynamics, the non-relativistic Electrodynamics and the heavy quark effective theory.

The most notorious example of decoupling effective theory is the Fermi theory of beta decay. In this case, the heavy DoF which decouples at low energy is the charged intermediate vector boson W. The relevant part of its Lagrangian is

$$\mathcal{L}_W = -\frac{1}{2}\left(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu\right)\left(\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu}\right)$$
$$+ M_W^2 W^-_\mu W^{+\mu} + \frac{g}{2\sqrt{2}}\left(J^-_\mu W^{+\mu} + W^-_\mu J^{+\mu}\right), \quad (2.16)$$

with

$$W^- = (W^+)^\dagger \qquad J^- = (J^+)^\dagger, \qquad (2.17)$$

where, in the Weinberg-Glashow-Salam theory of electroweak interactions, g is the coupling constant of the $SU(2)$ gauge field and, in the hadronic sector

$$J^+_\mu = V_{pn}\,\overline{p}\gamma_\mu(1-\gamma^5)n \qquad p = (u, c, t) \qquad n = (d, s, b), \qquad (2.18)$$

where V_{pn} is the relevant element of the Cabibbo-Kobayashi-Maskawa matrix, while in the leptonic one

$$J^+_\mu = \overline{l}\gamma_\mu(1-\gamma^5)\nu_l \qquad l = (e, \mu, \tau). \qquad (2.19)$$

Independently of the nature of the current, the equation of motion for W is

$$-(\partial^2 + M_W^2)W^+_\mu + \partial_\mu(\partial_\nu W^{+\nu}) = \frac{g}{2\sqrt{2}}J^+_\mu \qquad (2.20)$$

and hence, formally

$$W^+_\mu = \frac{g}{2\sqrt{2}}\Delta_\mu{}^\nu J^+_\nu, \qquad (2.21)$$

where the propagator Δ is the inverse of

$$K^{\mu\nu}(x, y) = \delta(x-y)\left(-\left(\partial^2 + M_W^2\right)g^{\mu\nu} + \partial^\mu\partial^\nu\right), \qquad (2.22)$$

i.e. the propagator for a massive vector field. In the momentum space the latter is

$$\Delta_{\mu\nu}(k) = \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2}. \tag{2.23}$$

Finally, interactions between charged current in the effective theory are described by the non-local action

$$S_{\text{NL}} = \frac{g^2}{8} \int \mathrm{d}^4 x\, \mathrm{d}^4 y\, J^{-\mu}(x)\Delta_{\mu\nu}(x,y)J^{+\nu}(y). \tag{2.24}$$

The propagator can be expanded in a series of powers of k^2/M_W^2

$$\Delta_{\mu\nu}(k) = -\frac{1}{M_W^2} g_{\mu\nu} + \mathcal{O}\left(\frac{k^2}{M_W^2}\right) \tag{2.25}$$

and to the leading order the interaction term of the effective Lagrangian is

$$\mathcal{L}_{\text{I}} = -\frac{g^2}{8M_W^2} J_\mu^- J^{+\mu}, \tag{2.26}$$

which correspond to the current-current or point interactions of the Fermi theory. The Fermi constant $G_F = 1.16632\ 10^{-5}\ \text{GeV}^{-2}$ can be expressed in terms of the coupling g through the relation

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}. \tag{2.27}$$

For the sake of this discussion, there are other two points worth noting. The first is that no new particles are generated in the transition from the fundamental to the effective level, indeed the heavy ones disappear from the theory. The second is that the coupling constants appearing in the effective Lagrangian can be calculated from the original theory through an analytic procedure.

Non-decoupling Effective Theories

In this case, the original theory is subjected to a phase transition at energies lower than Λ and its DoF's are not the fundamental fields but (possibly relativistic) bound states, which is not know how to describe in terms of their fundamental constituents. Non-decoupling effective theories are built thanks to a theorem stated by Weinberg (but never proved [44])

> [...] Quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible *S*-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles.

One wants to *model* bound states using effective fields, with ad-hoc internal DoF's and transformation properties, and describe their interactions in the framework of local quantum field theory, but does not know the Lagrangian for such effective fields. However, Weinberg's theorem guarantees that the most general Lagrangian will reproduce the most general scattering matrix consistent with the assumed symmetry principles. Therefore we can construct such a Lagrangian by exhaustion, including all allowed terms. The result will contain (possibly numerous) coupling constants, referred in the literature as Low Energy Constants (LEC's), which have to be deduced experimentally or calculated in some other way (e.g. using numerical lattice methods).

2.3   Heavy Quark Effective Theory

The Heavy Quark Effective Theory (HQET) describes strong interactions of heavy quarks, typically within hadrons, at energy scales much lower than their mass (for a review see [39]). A quark with mass m_Q is said to be a heavy quark if $m_Q \gg \Lambda_{\text{QCD}}$, where Λ_{QCD} is the scale parameter appearing in the formula of the running of the strong coupling

$$\alpha_S\left(q^2\right) = \frac{12\pi}{\left(33 - 2n_f\right)\ln\left(\frac{q^2}{\Lambda_{\text{QCD}}^2}\right)}, \tag{2.28}$$

where n_f is the number of flavours and q is the momentum scale. At large distances (small q^2) the coupling becomes large, leading to the confinement of quarks and gluons. Therefore the typical hadronic radius $R_{\text{had}} \approx 1$ fm is of the order of $1/\Lambda_{\text{QCD}}$, which means $\Lambda_{\text{QCD}} \approx 200$ MeV. Hence, the heavy quarks are c, b and t. However, t decays too quickly to form bound states. Conversely the light quarks are d, u and s.

The HQET is obtained from QCD in the limit in which the mass of the heavy quarks tends to infinity and, at the same time, their velocity is kept fixed. This limit applies to systems containing a heavy quark interacting with other light quarks by exchanging soft gluons ($E \lessgtr \Lambda_{QCD}$). In this case, its momentum fluctuates by an amount of the order of Λ_{QCD} and hence its velocity by Λ_{QCD}/m_Q, a quantity which vanishes in the limit $m_Q \rightarrow \infty$. Therefore, in such systems the velocity of the heavy quark is no more a DoF and becomes a conserved quantity.

The Light Degrees of Freedom (LDoF's) of such systems (light quarks and gluons) experience only the static colour field of the heavy quark and are blind to its flavour (that is, its mass) and spin orientation. This is because they interact with the heavy quark by means of soft gluons which cannot resolve distances of the order of the Compton length of the heavy quark $\lambda_Q \approx 1/m_Q$.

The HQET is a decoupling effective theory, however, the procedure outlined in the last section requires some comments. If conservation laws forbid the decay of some heavy particles, they cannot be removed from the theory at low scales and indeed, in this scenario, they act like external sources. This is the case of HQET (moreover, if one want to describe systems containing them then there is no interest in eliminating them completely from the theory). Having the physical picture in mind, the goal is to decompose the heavy quark field

Q in a light component h_v, which will survive, and a heavy one H_v, which will be integrated out.

There is a strong analogy between the HQET and the non-relativistic limit of a spin-1/2 particle interacting with the electromagnetic field. Lets briefly review the latter. A particle of such, with mass m_0 and charge e, is described by a Dirac spinorial field ψ and its Lagrangian is

$$\mathcal{L}_{\text{QED}} = \psi^\dagger \left(i\frac{\partial}{\partial t} - e\phi - \boldsymbol{\alpha} \cdot (-i\boldsymbol{\nabla} - e\mathbf{A}) - \beta m_0 \right) \psi, \tag{2.29}$$

where $(\phi, \mathbf{A})$ is the electromagnetic four-potential and $(\beta, \boldsymbol{\alpha})$ are the Dirac matrices. In order to take the non-relativistic limit, we consider a particle with four-momentum $p_\mu = m_0 \delta_{\mu 0} + k_\mu$, where k_μ is small compared to m_0 . It is convenient to drop out the rest-frame oscillating phase and write explicitly the antiparticle and particle bispinor components

$$\psi = \exp\left(-im_0 t\right) \begin{pmatrix} \varphi \\ \chi \end{pmatrix}. \tag{2.30}$$

In terms of φ and χ, the Lagrangian becomes

$$\mathcal{L}_{\text{QED}} = \varphi^\dagger \left(i\frac{\partial}{\partial t} - e\phi \right) \varphi + \chi^\dagger \left(i\frac{\partial}{\partial t} - e\phi + 2m_0 \right) \chi$$
$$- \varphi^\dagger c\boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla} - e\mathbf{A}) \chi - \chi^\dagger c\boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla} - e\mathbf{A}) \varphi, \tag{2.31}$$

from which it is apparent that χ represent a DoF with mass $2m_0$ coupled to ϕ (which is massless) and thus we identify the negative energy solution with the heavy DoF that has to be removed.

The equation of motion for χ is

$$\left(i\frac{\partial}{\partial t} - e\phi + 2m_0 \right) \chi = \boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla} - e\mathbf{A}) \varphi \tag{2.32}$$

and its solution can be plugged in the action to integrate out χ. However, because the propagator

$$\Delta_\chi = \left(i\frac{\partial}{\partial t} - e\phi + 2m_0 \right)^{-1} \tag{2.33}$$

depends on the electric potential, we expect quantum corrections to arise. Nonetheless, it turns out that $\det \Delta_\chi$ is a constant, as can be easily seen in the temporal gauge where ϕ vanish, and hence does not introduce additional terms in the effective action.

Finally, using the relation

$$(\boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla} - e\mathbf{A}))^2 = (-i\boldsymbol{\nabla} - e\mathbf{A})^2 - e\boldsymbol{\sigma} \cdot \mathbf{B}, \tag{2.34}$$

at order $1/m_0$, the Lagrangian becomes

$$\mathcal{L}_{\text{Pauli}} = \varphi^\dagger \left(i\frac{\partial}{\partial t} - e\phi - \frac{1}{2m_0}(-i\boldsymbol{\nabla} - e\mathbf{A})^2 + \frac{e}{2m_0}\boldsymbol{\sigma} \cdot \mathbf{B} \right) \varphi \tag{2.35}$$

and the corresponding equation of motion is the famous Pauli equation

$$i\frac{\partial}{\partial t}\varphi = \left(e\phi + \frac{1}{2m_0}\left(-i\boldsymbol{\nabla} - e\mathbf{A}\right)^2 - \frac{e}{2m_0}\boldsymbol{\sigma}\cdot\mathbf{B}\right)\varphi. \tag{2.36}$$

As expected, the kinetic term and the coupling with the magnetic field appear as corrections of order $1/m_0$.

In the HQET, one follows exactly the same steps in a formally covariant fashion and considers a perturbation k_μ (with $k_\mu \ll m_Q$) about the four-momentum $p_\mu = m_Q v_\mu$. The phase $-im_Q v^\mu x_\mu$ can be isolated and the positive and negative energy contributions can be made explicit by means of the energy projectors

$$\mathrm{P}_\pm = \frac{m_Q \pm \not{p}}{2m_Q} = \frac{1 \pm \not{v}}{2}. \tag{2.37}$$

As a matter of fact, the heavy quark field Q can be written in the following way

$$Q(x) = e^{-im_Q v^\mu x_\mu}\left(h_v(x) + H_v(x)\right), \tag{2.38}$$

$$\mathrm{P}_+ h_v = h_v, \quad \mathrm{P}_- H_v = H_v. \tag{2.39}$$

This decomposition, which is the boosted version of (2.30), requires some comments. The four-velocity v which appear in the energy projectors is a phenomenological quantity involved in the description of a particular configuration. It is not a dynamical quantity and does **not** transform covariantly under Lorentz transformations. Its meaning in HQET is that of an index.

Keeping this in mind, it is possible to calculate how the condition $\mathrm{P}_+ h_v = h_v$ is affected by Lorentz transformations, let $\mathrm{S}(\Lambda)$ be the action of such a transformation on bispinors

$$\frac{1 + \not{v}}{2}h_v \mapsto \frac{1 + \not{v}}{2}\mathrm{S}(\Lambda)h_v = \mathrm{S}(\Lambda)\frac{1 + v_\mu\overline{\mathrm{S}}(\Lambda)\gamma^\mu\mathrm{S}}{2}h_v = \mathrm{S}(\Lambda)\frac{1 + \not{v}'}{2}h_v, \tag{2.40}$$

with $v' = \Lambda^{-1}v$.

The relevant part with respect to Q of the QCD Lagrangian is

$$\mathcal{L}_{\mathrm{QCD}} = \overline{Q}\left(i\not{D} - m_Q\right)Q \tag{2.41}$$

and using the property

$$\mathrm{P}_\pm \not{D} = \not{D}\,\mathrm{P}_\mp \pm v^\mu D_\mu, \tag{2.42}$$

$\mathcal{L}_{\mathrm{QCD}}$ can be written in terms of h_v and H_v in the following way

$$\mathcal{L}_{\mathrm{QCD}} = \overline{h}_v\, i\, v^\mu D_\mu\, h_v - \overline{H}_v\left(i\, v^\mu D_\mu + 2m_Q\right)H_v + \overline{h}_v i\not{D}_\perp H_v + \overline{H}_v i\not{D}_\perp h_v, \tag{2.43}$$

where

$$D_\perp^\mu = D^\mu - v^\mu v^\nu D_\nu. \tag{2.44}$$

Hence H_v represents a DoF with mass $2m_Q$ coupled to h_v and will be integrated out. Its equation of motion is

$$\left(i\, v^\mu D_\mu + 2m_Q\right)H_v = i\not{D}_\perp h_v, \tag{2.45}$$

and the general considerations for a decoupling effective theory hold. Analogously to the non-relativistic QED case, there are no quantum corrections, as can be seen in the gauge with $v \cdot A^a = 0$.

Finally, we obtain the action

$$S = \int \mathrm{d}^4 x \; \overline{h}_v \; i \, v^\mu D_\mu \; h_v + \int \mathrm{d}^4 x \, \mathrm{d}^4 y \; \overline{h}_v i \slashed{D}_\perp \frac{1}{i \, v^\mu D_\mu + 2m_Q} i \slashed{D}_\perp h_v, \qquad (2.46)$$

which contains a non-local part.

An important observation is that having eliminated the negative energy solutions from the theory, this effective theory conserves the number of heavy quarks, that is pair creation is absent. De facto, the h_v operators annihilate positive energy quarks but do not create negative energy antiquarks (and vice-versa for $\overline{h}_v$). If one wants to describe processes involving heavy antiquarks, a different action should be used and it can be obtained from the previous one applying the substitution $v \mapsto -v$.

The terms of order $1/m_Q$ of the effective Lagrangian can be written in a more familiar way using the following identity

$$\mathrm{P}_+ \, i\slashed{D}_\perp i\slashed{D}_\perp \, \mathrm{P}_+ = \mathrm{P}_+ \left((iD_\perp)^2 + \frac{g}{2}\sigma_{\alpha\beta}G^{\alpha\beta} \right) \mathrm{P}_+, \qquad (2.47)$$

so that it holds

$$\mathcal{L} = \overline{h}_v \; i \, v^\mu D_\mu \; h_v + \frac{1}{2m_Q}\overline{h}_v(iD_\perp)^2 h_v + \frac{g}{4m_Q}\overline{h}_v\sigma_{\alpha\beta}G^{\alpha\beta}h_v + \mathcal{O}\left(\frac{1}{m_Q^2}\right). \qquad (2.48)$$

The interpretation of the two operators appearing at order $1/m_Q$ and their analogy with the non-relativistic electrodynamics is more evident in the rest frame. Within the latter, it holds that

$$\mathcal{O}_{\mathrm{kin}} = \frac{1}{2m_Q}\overline{h}_v(iD_\perp)^2 h_v \mapsto -\frac{1}{2m_Q}\overline{h}_v\|i\mathbf{D}\|^2 h_v, \qquad (2.49)$$

which is the non-Abelian generalization of the covariant kinetic energy $\|\mathbf{p} - e\mathbf{A}\|^2/2m_Q$, so that this operator is usually referred to as the HQ kinetic energy operator. Moreover,

$$\mathcal{O}_{\mathrm{mag}} = \frac{g}{4m_Q}\overline{h}_v\sigma_{\alpha\beta}G^{\alpha\beta}h_v \mapsto -\frac{g}{m_Q}\overline{h}_v\mathbf{S}\cdot\mathbf{B}_c h_v, \qquad (2.50)$$

which is the non-Abelian generalization of the Pauli term, where

$$B_c^i = -\frac{1}{2}\varepsilon^{ijk}G^{jk}, \quad S^i = \frac{1}{2}\gamma^5\slashed{v}\slashed{e}^i \qquad (2.51)$$

are respectively the color-magnetic field and the boosted spin operator ($\{e^i\}$ is a dreibein of four-vectors orthogonal to v). $\mathcal{O}_{\mathrm{mag}}$ is know as the chromo-magnetic operator.

The main point is that **at the leading order, the spin and flavour of heavy quarks are decoupled**. This can be put in a more formal statement saying that the theory exhibits a $SU(2N_h)$ symmetry, where N_h is the number of heavy flavours and means that, in the HQ

limit, the spin and flavour of heavy quarks are conserved quantum numbers. The analogy with the non-relativistic electrodynamics can be stressed observing that different isotopes have the same chemistry and that, in atomic physics, hyperfine levels are nearly degenerate.

Given an observable which contain products of the heavy quark field operators, one could, in principle, by means of the relation

$$Q(x) = e^{im_Q v^\mu x_\mu} \left(1 + \frac{1}{i \, v^\mu D_\mu + 2m_Q} i\slashed{D}_\perp \right) h_v(x) \tag{2.52}$$

expand it to the relevant order in $1/m_Q$. There is one caveat though, there is a residual m_Q dependence hidden in the h_v field, which indeed, at this moment, is the solution of the equations of motion for the Lagrangian (2.48). For example, the vector current

$$V^\mu(x) = e^{-im_Q v^\mu x_\mu} \, \overline{q}(x) \gamma^\mu \left(1 + \frac{i\slashed{D}_\perp}{2m_Q} + \cdots \right) h_v(x), \tag{2.53}$$

can be evaluated between a heavy meson state $M(v)$ and the vacuum

$$\langle 0|V^\mu|M(v)\rangle = \langle 0|\overline{q}\gamma^\mu h_v|M(v)\rangle + \frac{1}{2m_Q}\langle 0|\overline{q}\gamma^\mu i\slashed{D}_\perp h_v|M(v)\rangle + \cdots . \tag{2.54}$$

However, the second term of the RHS of the previous equation is not the leading correction in $1/m_Q$. Therefore, to make fully explicit the m_Q dependence and restore the correct power counting, the HQET Lagrangian is *defined* as (with $P_+ h_v = h_v$)

$$\mathcal{L}_{\text{HQET}} \equiv \overline{h}_v \, i \, v^\mu D_\mu \, h_v \tag{2.55}$$

and the terms containing higher powers of $1/m_Q$ are treated as perturbations

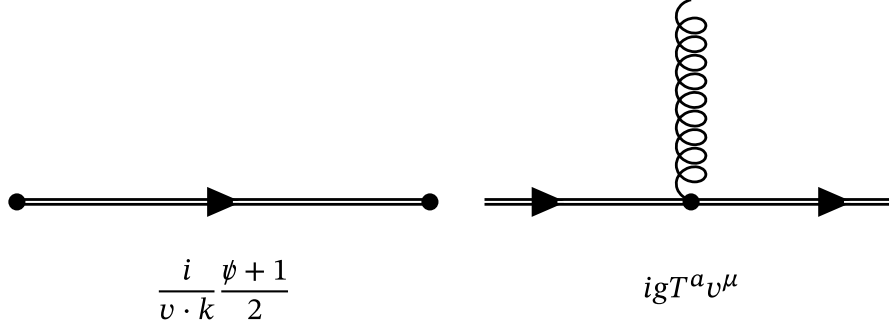$$\mathcal{L}_{\text{power}} = \frac{1}{2m_Q}\mathcal{L}_1 + \frac{1}{4m_Q^2}\mathcal{L}_2 + \cdots . \tag{2.56}$$

In the HQET, the equations of motion of h_v are

$$\begin{cases} v^\mu D_\mu h_v = 0 \\ \slashed{v} h_v = h_v \end{cases} , \tag{2.57}$$

which does not contain the heavy quark mass m_Q (and hence neither the states created acting on the vacuum with h_v). Taking into account these redefinitions, the previous matrix element of the vector current becomes

$$\langle 0|V^\mu|M(v)\rangle_{\text{QCD}} = \langle 0|\overline{q}\gamma^\mu h_v|M(v)\rangle_{\text{HQET}} + \frac{1}{2m_Q}\langle 0|\overline{q}\gamma^\mu i\slashed{D}_\perp h_v|M(v)\rangle_{\text{HQET}}$$
$$+ \frac{1}{2m_Q}\langle 0| \left(i \int d^4y \; T\{\overline{q}\gamma^\mu h_v(0), \mathcal{L}_1(y)\} \right) |M(v)\rangle_{\text{HQET}} + \mathcal{O}\left(\frac{1}{m_Q^2} \right). \tag{2.58}$$

$$\frac{i}{v \cdot k} \frac{\not{v} + 1}{2} \qquad\qquad\qquad igT^a v^\mu$$

Figure 2.1: Feynman rules in HQET.

The Feynman rules of HQET (displayed in figure 2.1) are simpler than those of QCD and coincide with their HQ limit. For example, for the heavy quark propagator holds that

$$\frac{i}{\not{P}_Q - m_Q} \to \frac{i}{v^\mu k_\mu} \frac{\not{v} + 1}{2} + \mathcal{O}\left(\frac{k}{m_Q}\right), \tag{2.59}$$

with $P_Q = m_Q v + k$, as $k/m_Q \to 0$. Also, the quark-gluon vertex simplifies, being evaluated between positive energy eigenstates

$$\mathrm{P}_+ \left(igT^a \gamma^\mu \right) \mathrm{P}_+ = \mathrm{P}_+ \left(igT^a v^\mu \right) \mathrm{P}_+, \tag{2.60}$$

so that the vertex between a heavy quark and a gluon becomes simply $igT^a v^\mu$, which does not contain gamma matrices.

It is important to observe that the HQET Lagrangian is not Lorentz invariant: under an infinitesimal transformation $\Lambda^\mu{}_\nu = \delta^\mu{}_\nu - i\omega^\mu{}_\nu$ (acting on h_v as $\mathrm{S} = \mathbb{1} + \frac{i}{4}\omega_{\mu\nu}\sigma^{\mu\nu}$)

$$\delta\mathcal{L}_{\mathrm{HQET}} = \omega^\mu{}_\nu \overline{h}_v v_\mu D^\nu h_v, \tag{2.61}$$

indeed the selection of a particular four-velocity v (i.e. a particular frame of reference, in which the heavy quark is at rest) breaks the Lorentz symmetry.

However, $\mathcal{L}_{\mathrm{HQET}}$ is invariant for transformations of $SU(2)$ acting on h_v in the following way (for infinitesimal transformations)

$$h_v \mapsto (1 + i\varepsilon \cdot \mathbf{S}) h_v, \tag{2.62}$$

which is the mathematical statement of the HQ spin symmetry of the HQET Lagrangian. Notice that these transformations preserve the positive energy condition indeed

$$\mathrm{P}_+ (1 + i\varepsilon \cdot \mathbf{S}) h_v = (1 + i\varepsilon \cdot \mathbf{S}) h_v. \tag{2.63}$$

It can be shown [37] that, if one consider the full QCD Lagrangian and studies the case in which QCD interactions change the velocity of the heavy quark, in the limit $m_Q \to \infty$,

the QCD Green functions involving one ingoing and one outgoing heavy quark vanish if the two velocity are different. This fact in the literature is sometimes referred to as *velocity superselection rule*. Since there is no coupling among quarks with different velocities one can restore the Lorentz invariance of the HQET summing in a covariant way over the velocities (which, in this case, transform in turn covariantly). This generalization is due to Georgi [33] and the Lagrangian reads

$$\mathcal{L}_{\text{HQET}} = \int \frac{\mathrm{d}^3 v}{(2\pi)^3} \frac{1}{2\sqrt{\|\mathbf{v}\|^2 + 1}} \, \overline{h}_v \, i \, v^\mu D_\mu \, h_v. \tag{2.64}$$

It is easy to see that now $\mathcal{L}_{\text{HQET}}$ is Lorentz-invariant, indeed for a finite transformation $\Lambda^\mu{}_\nu$

$$\mathcal{L}_{\text{HQET}} = \int \frac{\mathrm{d}^4 v}{(2\pi)^4} \, \delta^4(v^2 - 1) \, \theta(v_0) \overline{h}_v \, i \, v^\mu D_\mu \, h_v$$
$$\mapsto \int \frac{\mathrm{d}^4 u}{(2\pi)^4} \, |\det \Lambda| \, \delta^4(u^2 - 1) \, \theta(u_0) \overline{h}_u \, i \, u^\mu D_\mu \, h_u = \mathcal{L}_{\text{HQET}}, \tag{2.65}$$

using the substitution $v = \Lambda u$ and taking into account that for a orthochronus Lorentz transformations holds $u_0 > 0$ and $|\det \Lambda| = 1$.

2.4 Spectroscopic Implications of HQET

The symmetries of HQET in the exact HQ limit provide a number of spectroscopic observable consequences:

- for each hadronic state containing a heavy quark and for each flavour of the latter there is a degenerate doublet of states differing by the orientation of the heavy quark spin $\mathbf{S}_Q$ and having the same full width;

- the sum of partial widths of strong decays between two doublets are the same for the two states of the initial doublet;

- the partial decay widths and mass splittings between different doublets are independent of the heavy quark flavour.

Moreover, due to the fact that, in the limit $m_Q \to \infty$, the heavy quark decouples from the LDoF, its spin $\mathbf{S}_Q$ and the total angular momentum of LDoF $\mathbf{S}_l$ are separately conserved. The total angular momentum of the LDoF's is the sum of the orbital angular momentum $\mathbf{L}$ of the light quark with respect to the heavy quark and its spin $\mathbf{S}_q$: $\mathbf{S}_l = \mathbf{L} + \mathbf{S}_q$. However, it should be noticed that nothing prevents the existence of different states with the same values of $\mathbf{S}_l$ and $\mathbf{S}_Q$. These states are interpreted as *radial excitations*, in analogy to non-relativistic systems (e.g. the hydrogen atom).

For the purpose of this work, from here on, I will concentrate on the heavy meson sector. Regarding the nomenclature for such particles, I will adhere to the convention of indicating the natural spin-parity series $(0^+, 1^-, 2^+, \cdots)$ with an asterisk. We remember that the parity

of a meson can be written as $P = (-1)^{L+1}$, having quarks and antiquarks opposite intrinsic parity. The spin of the particle will appear as a subscript, with the exception of the ground state with $S_\ell = (1/2)^-$, and the radial excitations will be indicated with a tilde.

In the ground state ($L = 0$) the quantum numbers of the LDoF's are those of the light antiquark, that is $S_\ell^P = (1/2)^-$. The corresponding doublet (P, P^*) will contain a singlet state P corresponding to a pseudoscalar particle with $J^P = 0^-$ and a triplet state P^* corresponding to a vector one with $J^P = 1^-$. When $L = 1$ two doublets can be built according to the two possible orientations of $\mathbf{S}_q$. The doublet (P_0^*, P_1) with $\mathbf{S}_q$ antiparallel to $\mathbf{L}$ will have the same angular momentum but opposite parity with respect to the ground state, that is $S_\ell = (1/2)^+$, and will contain a scalar P_0^* with $J^P = 0^+$ and a pseudovector P_1' with $J^P = 1^+$. The other doublet (P_1, P_2^*), with $S_\ell^P = (3/2)^+$, will contain a axial particle P_1 with $J^P = 1^+$ and a tensor one P_2^* with $J^P = 2^+$. In this way, doublets corresponding to higher values of L can be analogously defined.

Let us consider the implications for the masses of these states. The ground state doublet is experimentally confirmed to be nearly degenerate both in the charm and beauty sector, with and without strangeness (within few percentage points of the mass of the state). Moreover, being the small mass splitting a relativistic correction of order $1/m_Q$, one can predict that $m_{D^*}^2 - m_D^2 \approx m_{B^*}^2 - m_B^2$, which is also found to be true. For the same reason, one expects this relation to be true in the case of states with strangeness $m_{D_s^*}^2 - m_{D_s}^2 \approx m_{B_s^*}^2 - m_{B_s}^2$. However, in principle, this quantity is not the same in the two cases, because the quantum numbers of the LDoF's are different. Nonetheless, the data shows that these are very similar and therefore it can be deduced that hyperfine corrections are approximately independent of the light quark flavour.

2.5   Covariant Representation of Heavy Mesons

As seen in section 2.2, in non-decoupling effective theories fields with desired properties are introduced. These effective fields must belong to some representations of the symmetry group of the more fundamental theory in exam. However, these representations are in principle reducible and thus the effective fields can be decomposed in linear combinations of fields belonging to irreducible representations. Therefore, in general, effective fields represent multiplets of particles.

This is precisely the case of heavy mesons doublets. Because $\mathbf{S}_Q$ and $\mathbf{S}_l$ are separately conserved, as predicted by the HQET, the effective field M must belong to the tensor product of the representations of $SO(3, 1)$ to which belong the heavy quark and the LDoF's, i.e. the representation $2 \otimes (2 \times S_\ell + 1) = 2 \times S_\ell \oplus (2 \times S_\ell + 2)$. Moreover, it must have the right parity, it must have one light and one heavy flavour indices and it must belong to the trivial representation of the $SU(3)_C$ group (because observable states are colour singlets).

Given a field A_l with the quantum numbers of the LDoF's and a positive energy bispinor ψ_h with those of the heavy quark, an effective field with the required properties is obtained by considering their tensorial product

$$M_{hl} = \psi_h A_l. \tag{2.66}$$

In the case of mesons S_ℓ is half-integer and therefore A_l is a generalized Rarita-Schwinger vector-spinor transverse to v [32]. The task of decomposing the effective fields in fields belonging to irreducible representations (and thus representing physical states) can be carried on in a general way [32]. As an illustrative example, in the following I describe this decomposition in the case of the ground state mesons.

In the case of the ground state, i.e. the doublet with $S_\ell^P = 1/2^-$ (P indicates parity), the LDoF's transform under the Lorentz group as an antiquark $\overline{\psi}_l$ satisfying

$$\overline{\psi}_l \frac{1 - \slashed{v}}{2} = \overline{\psi}_l, \tag{2.67}$$

so that

$$H = \psi_h \overline{\psi}_l. \tag{2.68}$$

H is a linear combination of objects with spin 0 and 1. The identification of these components is simpler in the rest frame where the spin operator is

$$S^i = \frac{1}{2} \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} \tag{2.69}$$

and acts on H in the following way

$$\mathbf{S} H = (\mathbf{S}\, \psi_h)\, \overline{\psi}_l - \psi_h \left(\overline{\psi}_l\, \mathbf{S} \right). \tag{2.70}$$

The basis of H can be computed by definition as the tensorial product of the basis for ψ_h and ψ_l. In the rest frame the vectors of these spinor basis are

$$\Uparrow = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Downarrow = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \uparrow = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \downarrow = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{2.71}$$

Acting with $\mathbf{S}$ on $\{ i \otimes j \mid i = \Uparrow, \Downarrow, \ j = \uparrow, \downarrow \}$ one finds that the linear combination

$$\Uparrow\uparrow + \Downarrow\downarrow = \begin{pmatrix} 0 & -\mathbb{1} \\ 0 & 0 \end{pmatrix} \tag{2.72}$$

is invariant under transformations (2.70), i.e. it is a basis of a one dimensional invariant subspace, and similarly

$$\Uparrow\downarrow + \Downarrow\uparrow = \begin{pmatrix} 0 & \sigma^1 \\ 0 & 0 \end{pmatrix}, \tag{2.73a}$$

$$-i\left(\Uparrow\downarrow - \Downarrow\uparrow \right) = \begin{pmatrix} 0 & \sigma^2 \\ 0 & 0 \end{pmatrix}, \tag{2.73b}$$

$$\Uparrow\uparrow - \Downarrow\downarrow = \begin{pmatrix} 0 & \sigma^3 \\ 0 & 0 \end{pmatrix}, \tag{2.73c}$$

Doublet	H		S		T		X		X'		F	
(L, S_ℓ^P)	$\left(0, \frac{1}{2}^-\right)$		$\left(1, \frac{1}{2}^+\right)$		$\left(1, \frac{3}{2}^+\right)$		$\left(2, \frac{3}{2}^-\right)$		$\left(2, \frac{5}{2}^-\right)$		$\left(3, \frac{5}{2}^+\right)$	
Mesons	P	P^*	P_0^*	P_1'	P_1	P_2^*	$P_1^{*\prime}$	P_2	P_2'	P_3^*	$P_2^{*\prime}$	P_3
J^P	0^-	1^-	0^+	1^+	1^+	2^+	1^-	2^-	2^-	3^-	2^+	3^+

Table 2.2: Nomenclature, spin and parity of the heavy meson doublets up to $S_\ell = 5/2$

is the basis of an invariant subspace of dimension three.

Therefore in the rest frame the components of the ground state doublet can be written in the following way

$$P = -\frac{1+\gamma^0}{2}\gamma^5, \tag{2.74a}$$

$$P^{*i} = \frac{1+\gamma^0}{2}\gamma^i. \tag{2.74b}$$

Their expression can be boosted in a generic frame of reference and combined to give the expression of the ground state multiplet

$$H = \frac{1+\slashed{v}}{2}\left(P^*{}_\mu\gamma^\mu - P\gamma^5\right) \tag{2.75}$$

where P^* is transverse to v and the heavy and light flavour indices have been omitted.

In the case of the first excited doublet S, with $S_\ell^P = (1/2)^+$, the decomposition can be carried on in a similar fashion and the expression of S in terms of his axial and scalar component is

$$S = \frac{1+\slashed{v}}{2}\left(P_1'{}_\mu\gamma^\mu\gamma^5 - P_0^*\right). \tag{2.76}$$

The expression of H and S shows the general property that doublets of opposite parity differ by a factor γ^5. Therefore, given the expression of a doublet with L even and $S_\ell = L+1/2$, the one with $L' = L + 1$ and $S_\ell = L' - 1/2$ can be obtained. The member of the multiplet with L even and $S_\ell = L + 1$, that is V with $J = L + 1$ and P with $J = L$, have the following form

$$V^{\mu_1\cdots\mu_{L+1}} = \frac{1+\slashed{v}}{2}\epsilon_V^{\mu_1\cdots\mu_L}{}_{\mu_{L+1}}\gamma^{\mu_{L+1}}, \tag{2.77a}$$

$$P^{\mu_1\cdots\mu_L} = \frac{1+\slashed{v}}{2}\gamma^5\epsilon_P^{\nu_1\cdots\nu_L}\sqrt{\frac{2k+1}{k+1}}\left(\delta_{\nu_1}^{\mu_1}\cdots\delta_{\nu_L}^{\mu_L} - \frac{1}{2L+1}\gamma_{\nu_1}\left(\gamma^{\mu_1} - v^{\mu_1}\right)\delta_{\nu_2}^{\mu_2}\cdots\delta_{\nu_L}^{\mu_L}\right.$$
$$\left. -\cdots - \frac{1}{2L+1}\delta_{\nu_1}^{\mu_1}\cdots\delta_{\nu_{L-1}}^{\mu_{L-1}}\left(\gamma^{\mu_L} - v^{\mu_L}\right)\right), \tag{2.77b}$$

where $\epsilon_V^{\mu_1\cdots\mu_{L+1}}$ and $\epsilon_P^{\mu\cdots\mu_L}$ are polarization tensors.

The meson doublets of both parities with S_ℓ equals to 1/2, 3/2 and 5/2 are explicitly listed below, given their relevance within the research work presented in this thesis

$$T^\mu = \frac{1 + \not{v}}{2} \left(P_2^{*\,\mu}{}_\nu \gamma^\nu - P_1{}_\nu \sqrt{\frac{3}{2}} \gamma^5 \left(\eta^{\mu\nu} - \frac{1}{3}\gamma^\nu \left(\gamma^\mu - v^\mu \right) \right) \right) \tag{2.78a}$$

$$X^\mu = \frac{1 + \not{v}}{2} \left(P_2{}^\mu{}_\nu \gamma^\nu \gamma^5 - P_1^{*\prime}{}_\nu \sqrt{\frac{3}{2}} \left(\eta^{\mu\nu} - \frac{1}{3}\gamma^\nu \left(\gamma^\mu + v^\mu \right) \right) \right) \tag{2.78b}$$

$$X'^{\mu\nu} = \frac{1 + \not{v}}{2} \left(P_3^{*\,\mu\nu}{}_\rho \gamma^\rho - P_2'{}_{\alpha\beta} \sqrt{\frac{5}{3}} \gamma^5 \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{5}\gamma^\alpha\eta^{\nu\beta} \left(\gamma^\mu - v^\mu \right) \right. \right.$$
$$\left. \left. - \frac{1}{5}\gamma^\beta\eta^{\mu\alpha} \left(\gamma^\nu - v^\nu \right) \right) \right), \tag{2.78c}$$

$$F^{\mu\nu} = \frac{1 + \not{v}}{2} \left(P_3{}^{\mu\nu}{}_\rho \gamma^\rho \gamma^5 - P_2^{*\prime}{}_{\alpha\beta} \sqrt{\frac{5}{3}} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{5}\gamma^\alpha\eta^{\nu\beta} \left(\gamma^\mu + v^\mu \right) \right. \right.$$
$$\left. \left. - \frac{1}{5}\gamma^\beta\eta^{\mu\alpha} \left(\gamma^\nu + v^\nu \right) \right) \right). \tag{2.78d}$$

Their content in terms of meson states is reported in table 2.1. Moreover, for the same reason, I list below the formulas for the sum of polarization tensors of rank 1, 2 and 3

$$\sum_\lambda \epsilon_\mu^{(\lambda)} \epsilon_\nu^{*(\lambda)} = - K_{\mu\nu}, \tag{2.79a}$$

$$\sum_\lambda \epsilon_{\mu\nu}^{(\lambda)} \epsilon_{\rho\sigma}^{*(\lambda)} = \frac{1}{2} K_{\mu\rho} K_{\nu\sigma} + \frac{1}{2} K_{\mu\sigma} K_{\nu\rho} - \frac{1}{3} K_{\mu\nu} K_{\rho\sigma}, \tag{2.79b}$$

$$\sum_\lambda \epsilon_{\mu\nu\alpha}^{(\lambda)} \epsilon_{\rho\sigma\beta}^{*(\lambda)} = \frac{1}{15} \left(K_{\mu\alpha} K_{\sigma\nu} + K_{\mu\sigma} K_{\alpha\nu} + K_{\mu\nu} K_{\alpha\sigma} \right) K_{\rho\beta}$$
$$+ \frac{1}{15} \left(K_{\mu\alpha} K_{\sigma\rho} + K_{\mu\sigma} K_{\alpha\rho} + K_{\mu\rho} K_{\alpha\sigma} \right) K_{\nu\beta}$$
$$+ \frac{1}{15} \left(K_{\mu\alpha} K_{\sigma\beta} + K_{\mu\sigma} K_{\alpha\beta} + K_{\mu\beta} K_{\alpha\sigma} \right) K_{\nu\rho}$$
$$- \frac{1}{6} \left(K_{\mu\nu} K_{\alpha\rho} + K_{\mu\rho} K_{\nu\alpha} \right) K_{\sigma\beta}$$
$$- \frac{1}{6} \left(K_{\mu\rho} K_{\alpha\beta} + K_{\mu\beta} K_{\rho\alpha} \right) K_{\sigma\nu}$$
$$- \frac{1}{6} \left(K_{\mu\nu} K_{\alpha\beta} + K_{\mu\beta} K_{\nu\alpha} \right) K_{\sigma\rho}, \tag{2.79c}$$

where

$$K_{\mu\nu} = \eta_{\mu\nu} - v_\mu v_\nu. \tag{2.80}$$

2.6 Hadronic masses in HQET

In the strict HQ limit, the two member P and P^* of the ground state doublet H are exactly degenerate and their common mass can be written as

$$m_H = \bar{\Lambda} + m_Q. \tag{2.81}$$

Because $\bar{\Lambda}$ is independent of the heavy quark flavour, it can be interpreted as the mass of the LDoF's (in the ground state configuration).

However, because the heavy quark masses are finite, the physical mass of P and P^* will obey an expansion of the form

$$m - m_Q = \bar{\Lambda} + \frac{\Delta m^2}{2m_Q} + \mathcal{O}\left(\frac{1}{m_Q^2}\right). \tag{2.82}$$

The term of order $1/m_Q$ can be computed from

$$\langle M \,|M\rangle \, \Delta m^2 = -\langle M| \int \mathrm{d}^3x \, \mathcal{L}_1(x) \, |M\rangle, \tag{2.83}$$

where $\mathcal{L}_1$ is the first of the power corrections defined in (2.56).

It can be shown that

$$\Delta m^2 \equiv -\lambda_1 - d\lambda_2, \tag{2.84}$$

where λ_1 and λ_2 are parameters equal for P and P^*, representing the corrections respectively due to the kinetic and the chromo-magnetic operator, while d is a dimensionless coefficient different for the two states. Notice that the term responsible for the mass splitting is chromo-magnetic one, parametrized by λ_2. Using the mass independent normalization of states

$$\langle M(p') \, |M(p)\rangle = \frac{2p^0}{m_M} \, (2\pi)^3 \, \delta^3 \left(\mathbf{p} - \mathbf{p}'\right), \tag{2.85}$$

it is found

$$d_P = 3, \quad d_{P^*} = -1. \tag{2.86}$$

Therefore, the relation (2.84) can be inverted, and in particular

$$\lambda_2 = -\frac{1}{4}\left(\Delta m_{P^*}^2 - \Delta m_P^2\right) \approx -\frac{1}{4}\left(m_{P^*}^2 - m_P^2\right). \tag{2.87}$$

This considerations can be extended to all doublets. The mass splitting between the two members of a doublet is given by a term analogous to (2.50). The mass splitting Lagrangian terms can be written as

$$\mathcal{L}_1 = \frac{1}{2m_Q}\left(\lambda_H \operatorname{Tr}\left(\overline{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}\right) - \lambda_S \operatorname{Tr}\left(\overline{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}\right) + \lambda_T \operatorname{Tr}\left(\overline{T}_a^\alpha \sigma^{\mu\nu} T_{a\alpha}\sigma_{\mu\nu}\right)\right.$$
$$\left. -\lambda_X \operatorname{Tr}\left(\overline{X}_a^\alpha \sigma^{\mu\nu} X_{a\alpha}\sigma_{\mu\nu}\right) + \lambda_{X'} \operatorname{Tr}\left(\overline{X'}_a^{\alpha\beta} \sigma^{\mu\nu} X'_{a\alpha\beta}\sigma_{\mu\nu}\right) - \lambda_F \operatorname{Tr}\left(\overline{F'}_a^{\alpha\beta} \sigma^{\mu\nu} F_{a\alpha\beta}\sigma_{\mu\nu}\right)\right), \tag{2.88}$$

where the Dirac adjoint $\overline{M}$ of a heavy meson doublet M is defined

$$\overline{M} = \gamma^0 M \gamma^0 \tag{2.89}$$

and the constants λ are related to the hyperfine splitting of each doublet

$$\lambda_H = \frac{1}{8}\left(m_{P^*}^2 - m_P^2\right), \tag{2.90a}$$

$$\lambda_S = \frac{1}{8}\left(m_{P_1'}^2 - m_{P_0^*}^2\right), \tag{2.90b}$$

$$\lambda_T = \frac{3}{16}\left(m_{P_2^*}^2 - m_{P_1}^2\right), \tag{2.90c}$$

$$\lambda_X = \frac{3}{16}\left(m_{P_2}^2 - m_{P_1^{*\prime}}^2\right), \tag{2.90d}$$

$$\lambda_{X'} = \frac{5}{24}\left(m_{P_3^*}^2 - m_{P_2'}^2\right), \tag{2.90e}$$

$$\lambda_F = \frac{5}{24}\left(m_{P_3}^2 - m_{P_2^{*\prime}}^2\right). \tag{2.90f}$$

2.7 Chiral Symmetry Breaking

The Chiral Perturbation Theory (ChPT or χPT) is an effective theory of the non-decoupling type (for a review see [42]), which means that the low energy behaviour of (the sector of) theory which it models is not related to the decoupling of some heavy DoF but rather to the change of the nature of the DoF's of such theory. This effective theory relies on symmetry considerations and in particular exploits the concept of *spontaneous symmetry breaking*. Indeed, the ChPT can be seen as a systematic method for discussing the consequences of the approximate global flavour symmetries of QCD at low energies.

The masses of the light quarks (d, u and s) are much smaller than those of the heavy quarks and of lightest hadronic states ($m_\rho \approx 770$ MeV) with the exclusion of the pseudoscalar octet (π, K, η). This suggests on one hand to treat separately the light quarks and on the other that the mechanism that gives mass to the pseudoscalar octet may be different from that of other states.

The fermionic part of the QCD Lagrangian for the light quarks is

$$\mathcal{L}_{\text{light}} = \sum_{f=d,u,s} \overline{q}_f\left(i\slashed{D} - m_f\right)q_f. \tag{2.91}$$

However, in the light of previous considerations, the mass terms can be neglected and the Lagrangian becomes

$$\mathcal{L}_{\text{light}} = \sum_{f=d,u,s} \overline{q}_f\left(i\slashed{D}\right)q_f. \tag{2.92}$$

Because of the relations

$$\overline{q}\Gamma q = \begin{cases} \overline{q}_R\Gamma q_R + \overline{q}_L\Gamma q_L & \Gamma \in \{\gamma^\mu, \gamma^\mu\gamma^5\} \\ \overline{q}_R\Gamma q_L + \overline{q}_R\Gamma q_L & \Gamma \in \{1, \gamma^5, \sigma^{\mu\nu}\} \end{cases}, \tag{2.93}$$

$$q_R = P_R q, \quad q_L = P_L q, \tag{2.94}$$

the Lagrangian can be written explicitly in terms of the chiral components

$$\mathcal{L}_{\text{light}} = \sum_{f=d,u,s} \bar{q}_{R,f} \, (i\slashed{D}) \, q_{R,f} + \bar{q}_{L,f} \, (i\slashed{D}) \, q_{L,f}. \tag{2.95}$$

This Lagrangian has a larger symmetry group than the full theory. In absence of the mass term, it is possible to rotate separately the two chiral components of the quarks in flavour space, leaving the Lagrangian invariant. These transformations are the generalizations of (A.21) to $SU(3)$

$$U_R : q \mapsto e^{i\theta_R^a T^a} q, \tag{2.96a}$$

$$U_L : q \mapsto e^{i\theta_L^a T^a} q, \tag{2.96b}$$

where T^a are the generators of $SU(3)$ in the fundamental representation. The new symmetry group is then $SU(3)_R \times SU(3)_L \times U(1)_R \times U(1)_L$. The transformations of $U(1)_R \times U(1)_L$ can be re-parametrized as phase rotations of the chiral components respectively by the same or opposite angle. This leads to $U(1)_V \times U(1)_A$. However the latter is a symmetry group only of the classical theory and is broken in the quantum case (the so-called *chiral anomaly*). Therefore, ultimately the symmetry group of the light QCD Lagrangian, in the limit of massless quarks, is $SU(3)_R \times SU(3)_L \times U(1)_V$.

The $U(1)_V$ symmetry results in baryon number conservation. The charges of $SU(3)_R$ and $SU(3)_L$ are respectively Q_R^a and Q_L^a, which can be recombined in the same way as the vector and axial $U(1)$ charges. Therefore, the following ones can be defined

$$Q_V^a = Q_R^a + Q_L^a, \tag{2.97a}$$

$$Q_A^a = Q_R^a - Q_L^a. \tag{2.97b}$$

These charges commute with H_{QCD}^0, the Hamiltonian of the theory described by (2.92), and it can be shown that they have definite parity, i.e. the action of the parity operator P is

$$P \, Q_V^a \, P^{-1} = Q_V^a, \tag{2.98a}$$

$$P \, Q_A^a \, P^{-1} = -Q_A^a. \tag{2.98b}$$

From these premises, one would conclude that for any state of positive parity there must exist a degenerate state of negative parity (*parity doubling*). Indeed, if $|i, +\rangle$ is a state with energy E_i and positive parity, then $|i, -\rangle = Q_A^a \, |i, +\rangle$ is a state with the same energy and negative parity, as can be seen from the following

$$H_{\text{QCD}}^0 \, |i, -\rangle = H_{\text{QCD}}^0 Q_A^a \, |i, +\rangle = Q_A^a H_{\text{QCD}}^0 \, |i, +\rangle = E_i \, |E_i, -\rangle, \tag{2.99}$$

$$P \, |i, -\rangle = P \, Q_A^a \, P^{-1} \, P \, |i, +\rangle = -\, |i, -\rangle. \tag{2.100}$$

Finally such a state can be decomposed in terms of the members of the multiplet with energy E_i and negative parity

$$|i, -\rangle = Q_A^a \, |i, +\rangle = -(T^a)_{ij} \, |j, -\rangle. \tag{2.101}$$

However, the low-energy hadronic spectrum does not contain a degenerate baryon octet of negative parity. Therefore, it must be concluded that there is an inconsistency in the previous arguments.

Indeed it was tacitly assumed in the equation (2.101) that the ground state of QCD is annihilated by Q_A^a. Let be $a_i^\dagger$ and $b_i^\dagger$ creator operators of states with quantum numbers i and respectively positive and negative parity. Also assume that $|i, +\rangle$ and $|i, -\rangle$ can be decomposed with respect to two bases of irreducible representations of $SU(3)_L \times SU(3)_R$ and that under $SU(3)_L \times SU(3)_R$ the creation operators are related by

$$\left[Q_A^a, a_i^\dagger \right] = -(T^a)_{ij}\, b_j^\dagger, \tag{2.102}$$

then

$$Q_A^a |i, +\rangle = \left(\left[Q_A^a, a_i^\dagger \right] + a_i^\dagger Q_A^a \right) |0\rangle = -(T^a)_{ij}\, b_j^\dagger |0\rangle. \tag{2.103}$$

However, if the ground state is not annihilated by Q_A^a then (2.101) does no longer apply. In this case, the ground state is not invariant under the full symmetry group of the Lagrangian, that is the theory exhibits spontaneous symmetry breaking. In this scenario, given their small mass, the mesons of the pseudoscalar meson octet are the candidates for the Goldstone bosons of the chiral symmetry breaking.

In this regard, it is worthy recalling that a non-vanishing scalar quark condensate in the chiral limit is a sufficient (although not necessary) condition for spontaneous symmetry breaking [42]. This can be interpreted as follows. Quarks and antiquarks have strong attractive interactions, and, if they are massless, the energy cost of creating a new quark-antiquark pair is small. Thus one expects that the vacuum of QCD in the chiral limit will contain a condensate of quark-antiquark pairs, that is the vacuum expectation value of the scalar operator $\overline{Q}Q$ is non-zero

$$\langle 0| \overline{Q}Q |0\rangle = \langle 0| \overline{Q}_L Q_R + \overline{Q}_R Q_L |0\rangle \neq 0. \tag{2.104}$$

This scalar condensate is not invariant under chiral transformations, as it couples quark components with opposite chirality.

In the following, it will be shown, as an example, that the pions have indeed the right quantum numbers to be the Goldstone bosons of the chiral symmetry breaking of the isospin group $SU(2)$. This subgroup of the light flavour rotations does not involve the strange quark and hence the relevant part of the Lagrangian (2.92) does not contain its contribution.

The currents associated with $SU(2)_V \times SU(2)_A \times U(1)_V$ are

$$j^\mu = \overline{Q}\gamma^\mu Q, \tag{2.105a}$$

$$j_V^{a\mu} = \overline{Q}\gamma^\mu \tau^a Q, \tag{2.105b}$$

$$j_A^{a\mu} = \overline{Q}\gamma^\mu \gamma^5 \tau^a Q, \tag{2.105c}$$

where $\tau^a = \sigma^a/2$, $a = 1, 2, 3$ are the generators of $SU(2)$. If the pions are Goldstone bosons of the broken $SU(2)_A$ group, then it is possible to parametrize the matrix element of $j_A^{a\mu}$ between the vacuum and an on-shell pion by writing

$$\langle 0| j_A^{a\mu}(x) |\pi^b(p)\rangle = -ip^\mu f \delta^{ab} e^{-ip \cdot x}, \tag{2.106}$$

where f is a constant with the dimensions of a mass. The value of f can be determined from the weak decay rate of the charged pions (for this reason, f is referred to as the *pion decay constant*). Contracting (2.106) with p_μ and using the conservation of the axial current, it is found that the pion is massless $p^2 = 0$, as expected for Goldstone bosons.

If the mass terms are restored in (2.92), then the chiral components become coupled and using

$$i\slashed{D}Q = \mathbf{m}Q, \text{ with } \mathbf{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \tag{2.107}$$

the non-vanishing four-divergence of $j_A^{a\mu}$ can be calculated.

$$\partial_\mu j_A^{a\mu} = i\overline{Q}\{\mathbf{m}, \tau^a\}Q. \tag{2.108}$$

Inserting this relation in (2.106) one finds

$$\langle 0| \partial_\mu j_A^{a\mu}(x) |\pi^b(p)\rangle = -p^2 f \delta^{ab} = \langle 0| i\overline{Q}\{\mathbf{m}, \tau^a\}Q |\pi^b(p)\rangle. \tag{2.109}$$

But the last expression is an invariant quantity times

$$\text{Tr}\left(\{\mathbf{m}, \tau^a\}\tau^b\right) = \frac{1}{2}\delta^{ab}(m_u + m_d), \tag{2.110}$$

therefore, the quark and the pion masses are related by

$$m_\pi^2 = (m_u + m_d)\frac{M^2}{f}, \tag{2.111}$$

which is the notorious Goldberg-Treiman relation, where M^2 is a constant with dimensions of a mass squared.

2.8 Chiral Perturbation Theory

In light of all the previous considerations, a candidate non-decoupling effective theory for interactions of hadrons containing light quarks should exhibit the symmetries of the chiral QCD Lagrangian (2.92) and the spontaneous breaking of $SU(3)_A$. As a matter of fact, it is possible to formulate effective theories of such type. Historically, the first of these was the linear sigma model by Gell-Mann and Lévy (1960), which was formulated to describe the pion-nucleon interactions. However, the original model predicted the existence of a particle not confirmed experimentally and therefore it was generalized to the so-called non-linear sigma model, which involves only physical particles. In the following, a more general formalism due to Callan, Coleman, Wess and Zumino (CCWZ formalism, see [30] and [23]) will be shown, which will lead directly to the non-linear sigma model and to the theory for heavy mesons used in this thesis [38].

As an example, the following Lagrangian can be considered

$$\mathcal{L}_{O(N)} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \lambda \left(\boldsymbol{\phi} \cdot \boldsymbol{\phi} - v^2\right)^2, \tag{2.112}$$

where $\boldsymbol{\phi}$ is a real N-component scalar field. This Lagrangian has a global $O(N)$ symmetry. However, the potential is such that there is a continuum of configurations with minimum energy, characterized by the condition $\|\boldsymbol{\phi}\| = v$ (the so-called *vacuum manifold*, which in this case is the $N - 1$ dimensional sphere S^{N-1}). Therefore, the $O(N)$ symmetry is spontaneously broken into $O(N - 1)$ and the number of Goldstone bosons will be $N(N-1)/2 - (N-1)(N-2)/2 = N - 1$, i.e. the number of broken generators.

The Goldstone bosons will correspond to perturbations within the vacuum manifold about a standard vacuum configuration $\mathbf{v}$, which is chosen to be

$$\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ v \end{pmatrix}. \tag{2.113}$$

For this reason, one would like to introduce a set of coordinates describing the local orientation of the vacuum. The local vacua are obtained from v through $\Xi(x) \in O(N)$, hence a natural choice is

$$\boldsymbol{\phi}(x) = \Xi(x)\mathbf{v}. \tag{2.114}$$

The general CCWZ prescription is to choose a set of broken generators X^a and write $\Xi(x)$ in terms of them

$$\Xi(x) = \exp\left(i\pi^a(x)X^a\right). \tag{2.115}$$

In the considered example, such a choice could be $\{J_1, \cdots, J_{N-1}\}$, i.e. the set of generators of rotations with respect to the first $N - 1$ axes

$$\Xi(x) = \exp\left(\sum_{i=1}^{N-1} \pi^i(x)J_i\right). \tag{2.116}$$

It can be checked by an explicit calculation that the fields $\pi^i(x)$ are massless modes of the theory described by the Lagrangian (2.112) and indeed (for small perturbations about the vacuum, neglecting radial excitations)

$$\mathcal{L}_{O(N)} = \frac{1}{2}v^2 \left(\partial_\mu e^{-i\sum_{i=1}^{N-1} \pi^i(x)J^i}\partial^\mu e^{i\sum_{i=1}^{N-1} \pi^i(x)J^i}\right)_{NN}. \tag{2.117}$$

A general feature of Goldstone bosons, which is manifest in the Lagrangian (2.117), is the fact that they are derivatively coupled. Therefore, because the derivatives of the fields are proportional to the momentum of such particles, in the low energy limit the Goldstone bosons are weakly coupled. This fact has an intuitive interpretation: global transformations of the Goldstone boson fields cannot change the energy of the system and hence only their derivatives can contribute to the Lagrangian.

However, in the formulation of effective field theories the Lagrangian is not given but it is built a posteriori in a way such that it is invariant under a certain group of symmetries. Therefore, in such contexts, it is worth knowing how the Goldstone boson fields transform under the broken symmetry group.

Consider the equations (2.116) and (2.114), the same configuration $\phi(x)$ can be obtained as

$$\phi(x) = \Xi(x)h(x)\mathbf{v}, \tag{2.118}$$

where

$$h = \begin{pmatrix} h'(x) & 0 \\ 0 & 1 \end{pmatrix}, \text{ with } h'(x) \in O(N-1). \tag{2.119}$$

This observation is relevant for non-constant fields. Given a global transformation $g \in O(N)$, the transformed configuration $g\phi(x)$ will be obtained from the standard vacuum through $\Xi'(x)$ such that

$$g\Xi(x) = \Xi'(x)h(x), \tag{2.120}$$

for some (in general) non trivial $h(x)$. Therefore, the transformation law of $\Xi(x)$ is

$$\Xi(x) \mapsto g\Xi(x)h^{-1}(g, \Xi(x)) \tag{2.121}$$

This formalism can be applied to the chiral limit of QCD. In this case, neglecting $U(1)_V$, the original symmetry group of the theory is $SU(3)_R \times SU(3)_L$, while the unbroken is $SU(3)_V$. In order to define Ξ it is necessary to choose a basis of broken generators. There are two common choices in the literature, the ξ-basis and the Σ-basis. Consider, for the sake of the argument, the transformations $g \in SU(3)_R \times SU(3)_L$ and $h \in SU(3)_V$, which in the chiral representation have the form

$$g = \begin{pmatrix} R & 0 \\ 0 & L \end{pmatrix}, \tag{2.122a}$$

$$h = \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix}, \tag{2.122b}$$

where R, L, and U are $SU(3)$ transformations acting on light flavour indices.

The ξ-basis   In this case one chooses the usual basis of $SU(3)_A$, that is

$$X_\xi^a = T_R^a - T_L^a. \tag{2.123}$$

In the chiral representation this generators are block-diagonal with respect to the spinorial indices, hence

$$\Xi(x) = e^{i\pi_\xi^a X_\xi^a} = \exp\begin{pmatrix} i\pi_\xi^a T^a & 0 \\ 0 & -i\pi_\xi^a T^a \end{pmatrix} = \begin{pmatrix} \xi & 0 \\ 0 & \xi^\dagger \end{pmatrix}, \quad \xi = e^{i\pi_\xi^a T}, \tag{2.124}$$

where T^a are the $SU(3)$ generators in the fundamental representation. In this case the transformation law (2.121) has the form

$$\begin{pmatrix} \xi & 0 \\ 0 & \xi^\dagger \end{pmatrix} \mapsto \begin{pmatrix} R & 0 \\ 0 & L \end{pmatrix}\begin{pmatrix} \xi & 0 \\ 0 & \xi^\dagger \end{pmatrix}\begin{pmatrix} U^{-1} & 0 \\ 0 & U^{-1} \end{pmatrix}, \tag{2.125}$$

which implies

$$\xi \mapsto L\xi U^{-1}, \tag{2.126a}$$

$$\xi^\dagger \mapsto R\xi^\dagger U^{-1}. \tag{2.126b}$$

Therefore $U(\xi(x), R, L)$ must be the solution of

$$L\xi U^{-1} = U\xi R^\dagger. \tag{2.127}$$

The Σ-basis   In this case

$$X_\Sigma^a = T_R^a, \tag{2.128}$$

which leads to

$$\Xi(x) = e^{i\pi_\Sigma^a X_\Sigma^a} = \exp\begin{pmatrix} i\pi_\Sigma^a T^a & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \Sigma & 0 \\ 0 & \mathbb{1} \end{pmatrix}, \quad \Sigma = e^{i\pi_\Sigma^a T}. \tag{2.129}$$

The transformation law

$$\begin{pmatrix} \Sigma & 0 \\ 0 & \mathbb{1} \end{pmatrix} \mapsto \begin{pmatrix} R & 0 \\ 0 & L \end{pmatrix} \begin{pmatrix} \Sigma & 0 \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} U^{-1} & 0 \\ 0 & U^{-1} \end{pmatrix}, \tag{2.130}$$

provides the transformation law for Σ

$$\Sigma \mapsto L\Sigma U^{-1} \tag{2.131}$$

and the explicit condition $U = L$. Finally, one has

$$\Sigma \mapsto R\Sigma L^\dagger. \tag{2.132}$$

It should be noticed that the two definitions are related by

$$\Sigma(x) = \xi^2(x). \tag{2.133}$$

The Goldstone boson fields are dimensionless, as it is clear in (2.116) in which they are angular variables. When one writes the Lagrangian of an effective theory it is convenient to to use fields which have natural dimension one (i.e. of a mass). The standard choice in ChPT is to use the following definitions

$$\xi = e^{\frac{i}{f_\pi}\mathcal{M}}, \quad \Sigma = e^{\frac{2i}{f_\pi}\mathcal{M}}, \tag{2.134}$$

where $f_\pi \approx 132$ MeV is the pion decay constant and the matrix $\mathcal{M}$ is defined as

$$\mathcal{M} = \pi^a T^a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{2.135}$$

2.9 Heavy ChPT

As a first example of application of ChPT to heavy meson spectroscopy (a theory also known as heavy ChPT), it will be reviewed the construction of the Lagrangian for the ground state doublet H at the leading order. To begin with, it should be noted that terms of the form $\mathrm{Tr}\left(\Sigma\Sigma^{\dagger}\cdots\Sigma\Sigma^{\dagger}\right)$ are invariant with respect to global transformations $\Sigma\mapsto R\Sigma L^{\dagger}$. However, because $\Sigma\Sigma^{\dagger}=\mathbb{1}$, all of them are constant. Therefore, non-constant scalar functions of Σ must contain its derivatives. This is coherent with the previous statement that Goldstone bosons are derivatively coupled. Imposing also the Lorentz and parity invariance one finds that the simplest term is

$$\mathcal{L}_{\Sigma,\,\mathrm{kin}} = \frac{1}{8}f_{\pi}^{2}\,\mathrm{Tr}\left(\partial_{\mu}\Sigma\partial^{\mu}\Sigma^{\dagger}\right), \tag{2.136}$$

where the trace is computed over the light flavour indices. The interpretation of the latter as the kinetic term for Σ is confirmed by its expansion

$$\mathcal{L}_{\Sigma,\,\mathrm{kin}} = \mathrm{Tr}\left(\partial_{\mu}\mathcal{M}\partial^{\mu}\mathcal{M}\right) + \mathcal{O}\left(\mathcal{M}^{3}\right), \tag{2.137}$$

which at the leading order in $\mathcal{M}$ reproduces the kinetic terms for the (massless limit of the) mesons of the pseudoscalar octet

$$\mathrm{Tr}\left(\partial_{\mu}\mathcal{M}\partial^{\mu}\mathcal{M}\right) = \frac{1}{2}\partial_{\mu}\pi^{0}\partial^{\mu}\pi^{0} + \partial_{\mu}\pi^{-}\partial^{\mu}\pi^{+} + \cdots. \tag{2.138}$$

In order to include H in the effective theory, one has to know how this field transforms under $SU(3)_{R}\times SU(3)_{L}$. The CCWZ prescription for H is

$$H \mapsto Hh^{\dagger}, \tag{2.139}$$

where h is defined in equation (2.122b) and U is fixed by the equation (2.127), hence is a function of R, L and $\xi(x)$. A candidate kinetic term must be quadratic in H and invariant with respect to such transformations, as well as to parity and charge conjugation, Lorentz transformations, heavy spin rotations and rotations in light and heavy flavour space.

Consider the product $\overline{H}H$, it has two light flavour indices, two heavy flavour indices and two spinorial indices. Hence, it is possible to construct a scalar by taking the trace with respect to all these indices. However, because such a trace $\mathrm{Tr}\left(\overline{H}H\right)$ is a constant, the simplest scalars that can be built using H are

$$\mathrm{Tr}\left(\overline{H}\gamma^{\mu}\partial_{\mu}H\right), \quad \mathrm{Tr}\left(\overline{H}\partial_{\mu}H\gamma^{\mu}\right). \tag{2.140}$$

However, given the property

$$\frac{1+\slashed{v}}{2}H\frac{1-\slashed{v}}{2} = H, \tag{2.141}$$

both the terms (2.140) are found to be proportional to

$$\mathcal{L}_{H,\,\mathrm{kin}} = \mathrm{Tr}\left(\overline{H}\,i\,v^{\mu}\partial_{\mu}\,H\right), \tag{2.142}$$

which has all the required symmetries but the Lorentz one. Nonetheless, the latter can be made Lorentz invariant summing covariantly over four-velocities, like the HQET Lagrangian

(2.55). Indeed, the Lorentz invariant generalization of (2.142) could be written by making explicit use of the *velocity superselection rule* in the following way

$$\mathcal{L}_{H,\text{kin}} = \int \frac{1}{2\sqrt{\|\mathbf{v}\|^2 + 1}} \frac{\mathrm{d}^3 v}{(2\pi)^3} \frac{1}{2\sqrt{\|\mathbf{v}'\|^2 + 1}} \frac{\mathrm{d}^3 v'}{(2\pi)^3} \, \text{Tr}\left(\overline{H}_v \, i \, v^\mu D_\mu \, H_{v'}\right) \delta\left(\mathbf{v}' - \mathbf{v}\right), \quad (2.143)$$

where H_v is a doublet moving with velocity v. The latter equation can be understood as the trace with respect to the *velocity index* and therefore in the following I will just write $\mathcal{L}_{H,\text{kin}}$ as in (2.142).

Finally an interaction term between π^a and H must be introduced, to this end it is convenient to introduce the definitions

$$A^\mu = \frac{i}{2}\left(\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi\right) = \frac{1}{f_\pi}\partial^\mu \pi + \cdots, \quad (2.144\text{a})$$

$$V^\mu = \frac{1}{2}\left(\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi\right) = \frac{1}{2f_\pi^2}[\pi, \partial^\mu \pi] + \cdots. \quad (2.144\text{b})$$

Using the latter and equation (2.126b), it can be shown that the transformation law of these two matrix fields are

$$A^\mu \mapsto hA^\mu h^\dagger, \quad (2.145\text{a})$$

$$V^\mu \mapsto hV^\mu h^\dagger + h\partial^\mu h^\dagger, \quad (2.145\text{b})$$

where h is the same appearing in (2.139). On one hand, this allows to define the covariant derivative

$$D^\mu H = \partial^\mu H - HV^\mu, \quad (2.146)$$

$$D^\mu H \mapsto D^\mu H h^\dagger, \quad (2.147)$$

and thus introduce an invariant interaction term with V simply using the substitution

$$\text{Tr}\left(\overline{H} \, i \, v^\mu \partial_\mu \, H\right) \mapsto \text{Tr}\left(\overline{H} \, i \, v^\mu D_\mu \, H\right). \quad (2.148)$$

On the other hand, it is possible to show that the simplest valid interaction term containing A (which is axial) is

$$\mathcal{L}_{\text{Int}} = g \, \text{Tr}\left(\overline{H} H A \gamma^5\right), \quad (2.149)$$

where the trace is over the light, heavy and *velocity* indices. Finally, it is possible to write the most general Lagrangian consistent with the required symmetries

$$\mathcal{L}_H = \text{Tr}\left(\overline{H} \, i \, v^\mu D_\mu \, H\right) + \frac{1}{8} f_\pi^2 \, \text{Tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right) + g \, \text{Tr}\left(\overline{H} H A \gamma^5\right). \quad (2.150)$$

This kind of construction can be applied to all possible doublets and their radial excitations at any order. The result is a theory with many LEC's and the S-matrix of this theory to any given order possess all the symmetries of the chiral and heavy-quark limit of the QCD (see 2.2). However, to any given process it will correspond a relevant part of the Lagrangian which will

contain only a small number of terms. Because the decays towards the ground state doublet are kinematically favoured (and experimentally more easily accessible), in the following I will focus on the modes $M \to H\pi$, where M is a member of a generic doublet, π here indicates a generic light pseudoscalar meson and H denotes a member of the fundamental doublet. At the leading order, such processes will be described by a vertex which couples directly M, H and π. Therefore, the relevant part of the heavy ChPT Lagrangian will be similar to the last term in 2.150.

The kinetic terms for M with $S_\ell^P = (k + 1/2)^{\pm}$ are of the form

$$\mathcal{L}_{M, \text{kin}} = \text{Tr} \left(\overline{M}^{\alpha_1 \cdots \alpha_k} \left(i \, v^\mu D_\mu - \Delta_M \right) M_{\alpha_1 \cdots \alpha_k} \right), \tag{2.151}$$

where the trace is performed over all free indices and $\Delta_M = \overline{m}_M - \overline{m}_H$ is the difference between the spin-averaged masses of M and H, the so-called intra-doublet mass splitting. A particle with spin J will have $2J + 1$ polarization modes, therefore, because a doublet M with $S_\ell = k + 1/2$ will contain a state P_k with spin $J = k$ and a state P_{k+1} with spin $J = k + 1$, the spin-averaged mass will be

$$\overline{m}_M = \frac{(2k + 1)m_{P_k} + (2k + 3)m_{P_{k+1}}}{4(k + 1)}. \tag{2.152}$$

There is not a general formula for the form of the interaction term(s) of M with H, which has to be derived imposing the invariance under Lorentz transformations, parity and charge conjugation, heavy-spin rotations and separate heavy and light flavour rotations. However, it is found that mesons sharing the same S_ℓ have the same interaction term form, irrespectively of their parity. It should be noticed that it is not guaranteed that there exists only one form compatible with these symmetries, as in the the case of $S_\ell = 5/2$. If there is more than one interaction term, each one will have its coupling constant and will interfere coherently with the others. I list below the interaction terms forms up to $S_\ell = 5/2$

$$\mathcal{L}_{1/2, \text{Int}} \propto \text{Tr} \left(\overline{H} M \slashed{A} \gamma^5 \right) + \text{h.c.}, \tag{2.153a}$$

$$\mathcal{L}_{3/2, \text{Int}} \propto \frac{1}{\Lambda_\chi} \text{Tr} \left(\overline{H} M^\mu \left(i \, D_\mu \slashed{A} + i \slashed{D} A_\mu \right) \gamma^5 \right) + \text{h.c.}, \tag{2.153b}$$

$$\mathcal{L}_{5/2, \text{Int}}^{(1)} \propto \frac{1}{\Lambda_\chi^2} \text{Tr} \left(\overline{H} M^{\mu\nu} \left(D_\mu D_\nu \slashed{A} + D_\nu D_\mu \slashed{A} \right) \gamma^5 \right) + \text{h.c.}, \tag{2.153c}$$

$$\mathcal{L}_{5/2, \text{Int}}^{(2)} \propto \frac{1}{\Lambda_\chi^2} \text{Tr} \left(\overline{H} M^{\mu\nu} \left(D_\mu \slashed{D} A_\nu + D_\nu \slashed{D} A_\mu \right) \gamma^5 \right) + \text{h.c.}, \tag{2.153d}$$

where $\Lambda_\chi \approx 1$ GeV is the chiral symmetry breaking scale and appears for dimensional reasons.

Chapter 3

Taxonomy of Charmed Mesons

All'inizio, l'arte del puzzle sembra un'arte breve, di poco spessore, tutta contenuta in uno scarno insegnamento della *Gestalttheorie*: l'oggetto preso di mira–sia esso un atto percettivo, un apprendimento, un sistema fisiologico o, nel nostro caso, un puzzle di legno–non è una somma di elementi che bisognerebbe dapprima isolare e analizzare, ma un insieme, una forma cioè, una struttura: l'elemento non preesiste all'insieme, non è più immediato ne più antico, non sono gli elementi a determinare l'insieme, ma l'insieme a determinare gli elementi: la conoscenza del tutto e delle sue leggi, dell'insieme e della sua struttura, non è deducibile dalla conoscenza delle singole parti che lo compongono: la qual cosa significa che si può guardare il pezzo di un puzzle per tre giorni di seguito credendo di sapere tutto della sua configurazione e del suo colore, senza aver fatto il minimo passo avanti: conta solo la possibilità di collegare quel pezzo ad altri pezzi e in questo senso l'arte del puzzle e l'arte del go hanno qualcosa in comune; solo i pezzi ricomposti assumeranno un carattere leggibile, acquisteranno un senso: isolato, il pezzo di un puzzle non significa niente; è semplicemente una domanda impossibile, sfida opaca; ma se appena riesci, dopo molti minuti di errori e tentativi, o in un mezzo secondo prodigiosamente ispirato, a connetterlo con uno dei pezzi vicini, ecco che quello sparisce, cessa di esistere in quanto pezzo: l'intensa difficoltà che ha preceduto l'accostamento e che la parola *puzzle*—enigma—traduce così bene in inglese, non solo non ha più motivo di esistere, ma sembra non averne avuto mai, tanto si è fatta evidenza: i due pezzi miracolosamente riuniti sono diventati ormai uno, a sua volta fonte di errori, esitazioni, smarrimenti e attesa.

(George Perec, *La Vie mode d'emploi*)

In the first chapter it was seen that different open-charm meson states and some open-beauty ones have been observed and, thanks to LHCb and BelleII, one expects that as time goes by these spectra will become more and more populated. In the second chapter it was introduced a theoretical framework inspired by QCD which on one hand allows to classify the heavy-light mesons and on the other to describe their low-energy strong interactions with light pseudoscalar mesons in terms of a small number of unknown constants (which in principle should be obtained in some other way, e.g. experimentally). In this chapter, such theory is used for the classification of established and recently observed open-charm mesons according to its scheme [28]. Moreover, the decay widths of these states (with respect to selected decay modes) will be calculated within this theory and it will be shown how the result can be used to make model-independent predictions of their quantum numbers.

3.1   Strong Two-body decays to the Fundamental Doublet

I will recall some basic facts. The classification of heavy mesons is based on the quantum numbers of the light degrees of freedom (light quark plus gluons) and of the heavy quark, which decouples from the former due to its large mass. Therefore a state is characterized by the orbital angular momentum of the light degrees of freedom L, by their total angular momentum S_ℓ and by the meson spin J. Their parity will be $P = (-1)^{L+1}$ and for each value of S_ℓ there will be two possible doublets of states with opposite parity, vice versa given S_ℓ^P there is only one possible value of L. The two states of each doublet are degenerate in the exact HQ limit, they correspond to the two possible orientations of the heavy meson spin and have different J. More than one state with any given combination of these quantum numbers can exist and accounts for radial excitations identified by increasing radial quantum number n.

In the first place, the classification of states is done according to their quantum number and mass, i.e. a resonance with given J^P is a candidate for the lowest vacant states with that quantum numbers (which could have $n > 1$). However, apart for the $J = 0$ state, for each value of J^P there exist two series of doublets, with respectively $S_l = (J - 1/2)^P$ and $(J + 1/2)^P$, that contain a state with the given J^P. To distinguish between these two series it is necessary to use an additional piece of information, which is carried by the width of the state.

As reviewed in section 2.9, in heavy ChPT it is possible to write effective Lagrangian terms that describe the strong decays of a heavy meson in a given doublet to another heavy meson in a second doublet with the emission of a light pseudoscalar meson. Eqs 2.153 refer to the case in which the meson in the final state belongs to the fundamental doublet H. Each Lagrangian is weighted by a strong coupling constant that cannot be fixed within the same approach, so that it is in principle unknown. However, it should be stressed that a single parameter (i.e. the strong coupling constant) can describe (if allowed) four possible decays, corresponding to the two possible decaying mesons in the initial state doublet and two possible final state mesons in the other doublet. Therefore, such a parameter cancels in the ratio of two strong decay rates computed from the same effective Lagrangian term. As a consequence, such ratios represent model independent predictions of the method.

Exploiting this approach, in the following I focus on the decays of a meson in a generic doublet M to another one in the fundamental doublet H with emission of a light pseudoscalar meson generically denoted as π. The reason of this lies in the large phase space available for these models and in the relative simplicity of their reconstruction in experimental analysis. The kinematics of the decays $M \to H\pi$ is that of a standard two body decay and its width can be written as

$$d\Gamma = \frac{1}{32\pi^2} \left| \langle H(v), \pi(q) | T | M(v) \rangle \right|^2 \frac{\|\mathbf{q}\|}{m_M} d\Omega \, . \tag{3.1}$$

If the decaying meson has spin J the factor $1/(2J + 1)$ should be included in the equation (3.1) to average over the spin.

$\langle H(v), \pi(q) | T | M(v) \rangle$ is the transition aplitude that can be directly computed from the effective Lagrangian terms. According to the velocity superselection rule, the two heavy mesons in the initial and final state have the same velocity. It is important to notice that this

velocity superselection rule is valid only in the calculation of the matrix element and not in the calculation of the phase space that is fixed only by the kinematics.

In the frame of reference in which M is at rest (i.e. $v = (1, \mathbf{0})$) one has

$$\|\mathbf{q}\| = \frac{\sqrt{\left(m_M^2 - (m_H + m_\pi)^2\right)\left(m_M^2 - (m_H - m_\pi)^2\right)}}{2m_M}. \tag{3.2}$$

In the following I will list the allowed decays of heavy mesons belonging to one of the doublets introduced in equations (2.75), (2.76), (2.78) and write down all the corresponding Lagrangian terms. Moreover, the covariant representation and structure of the interaction Lagrangian of states with $n > 1$ are the same as the corresponding one of the state with $n = 1$ and same S_ℓ^P. However, the interaction Lagrangian terms for radial excited states have different coupling constants and, in the following, the coupling constant for the first radial excitation is denoted with the same symbol of the corresponding $n = 1$ state and marked by a tilde.

Some of the decay modes for $n = 2$ states of these mesons are explicitly listed in the following, because such decay modes towards doublets with higher L are kinematically prohibited for $n = 1$ states but may be possible for ones with $n > 1$. Finally the Clebsh-Gordan coefficients associated with the light pseudoscalar meson in the final state (and built into the definition of $\mathcal{M}$) are explicitly indicated with C_π.

It should be noticed that, the small $\|q\|$ behaviour of the decay widths reveal the dominant partial wave contribution to the decay and vice versa if the decay is known to occur in l-wave, at small $\|q\|$ will hold $\Gamma \propto \|q\|^{2l+1}$.

Decaying meson $H = (P, P^*)$ or $\tilde{H} = \left(\tilde{P}, \tilde{P}^*\right)$

$$\mathcal{L}_{H,\,\text{Int}} = g \operatorname{Tr}\left(\overline{H}_a H_b \gamma_\mu \gamma^5 A_{ba}^\mu\right). \tag{3.3}$$

Allowed decays

$$\Gamma\left(P^* \to P\pi\right) = C_\pi \frac{g^2}{6\pi f_\pi^2} \frac{m_P}{m_{P^*}} \|\mathbf{q}\|^3 \, , \tag{3.4a}$$

$$\Gamma\left(\tilde{P}^* \to P\pi\right) = C_\pi \frac{\tilde{g}^2}{6\pi f_\pi^2} \frac{m_P}{m_{\tilde{P}^*}} \|\mathbf{q}\|^3 \, , \tag{3.4b}$$

$$\Gamma\left(\tilde{P}^* \to P^*\pi\right) = C_\pi \frac{\tilde{g}^2}{3\pi f_\pi^2} \frac{m_{P^*}}{m_{\tilde{P}^*}} \|\mathbf{q}\|^3 \, , \tag{3.4c}$$

$$\Gamma\left(\tilde{P} \to P^*\pi\right) = C_\pi \frac{\tilde{g}^2}{2\pi f_\pi^2} \frac{m_{P^*}}{m_{\tilde{P}}} \|\mathbf{q}\|^3 \, . \tag{3.4d}$$

Decaying $S = (P_0^*, P_1')$

$$\mathcal{L}_{S,\,\text{Int}} = h \operatorname{Tr}\left(\overline{H}_a S_b \gamma_\mu \gamma^5 A_{ba}^\mu\right) + \text{h.c.} \, . \tag{3.5}$$

Allowed decays

$$\Gamma\left(P_0^* \to P\pi\right) = C_\pi \frac{h^2}{2\pi f_\pi^2} \frac{m_P}{m_{P_0^*}} \|\mathbf{q}\| \left(m_\pi^2 + \|\mathbf{q}\|^2\right) , \tag{3.6a}$$

$$\Gamma\left(P_1' \to P^*\pi\right) = C_\pi \frac{h^2}{2\pi f_\pi^2} \frac{m_{P^*}}{m_{P_1'}} \|\mathbf{q}\| \left(m_\pi^2 + \|\mathbf{q}\|^2\right) . \tag{3.6b}$$

Decaying $T = \left(P_1, P_2^*\right)$ or $\tilde{T} = \left(\tilde{P}_1, \tilde{P}_2^*\right)$

$$\mathcal{L}_{T,\,\mathrm{Int}} = \frac{h'}{\Lambda_\chi} \mathrm{Tr}\left(\overline{H}_a T_b^\mu \left(iD_\mu \slashed{A} + i\slashed{D}A_\mu\right)_{ba} \gamma^5\right) + \mathrm{h.c.} . \tag{3.7}$$

Allowed decays

$$\Gamma\left(P_1 \to P^*\pi\right) = C_\pi \frac{2h'^2}{3\pi f_\pi^2} \frac{m_{P^*}}{m_{P_1}} \|\mathbf{q}\|^5 , \tag{3.8a}$$

$$\Gamma\left(P_2^* \to P\pi\right) = C_\pi \frac{4h'^2}{15\pi f_\pi^2} \frac{m_P}{m_{P_2^*}} \|\mathbf{q}\|^5 , \tag{3.8b}$$

$$\Gamma\left(P_2^* \to P^*\pi\right) = C_\pi \frac{2h'^2}{5\pi f_\pi^2} \frac{m_{P^*}}{m_{P_2^*}} \|\mathbf{q}\|^5 . \tag{3.8c}$$

Decaying $X = \left(P_1^*, P_2\right)$

$$\mathcal{L}_{X,\,\mathrm{Int}} = \frac{k'}{\Lambda_\chi} \mathrm{Tr}\left(\overline{H}_a X_b^\mu \left(iD_\mu \slashed{A} + i\slashed{D}A_\mu\right)_{ba} \gamma^5\right) + \mathrm{h.c.} . \tag{3.9}$$

Allowed decays

$$\Gamma\left(P_1^* \to P\pi\right) = C_\pi \frac{4k'^2}{9\pi f_\pi^2} \frac{m_P}{m_{P_1^*}} \|\mathbf{q}\|^3 \left(m_\pi^2 + \|\mathbf{q}\|^2\right) , \tag{3.10a}$$

$$\Gamma\left(P_1^* \to P^*\pi\right) = C_\pi \frac{2k'^2}{9\pi f_\pi^2} \frac{m_{P^*}}{m_{P_1^*}} \|\mathbf{q}\|^3 \left(m_\pi^2 + \|\mathbf{q}\|^2\right) , \tag{3.10b}$$

$$\Gamma\left(P_2 \to P^*\pi\right) = C_\pi \frac{2k'^2}{3\pi f_\pi^2} \frac{m_{P^*}}{m_{P_2}} \|\mathbf{q}\|^3 \left(m_\pi^2 + \|\mathbf{q}\|^2\right) . \tag{3.10c}$$

$$\tag{3.10d}$$

Decaying $X' = \left(P_2, P_3^*\right)$

$$\mathcal{L}_{X',\,\mathrm{Int}} = \frac{1}{\Lambda_\chi^2} \mathrm{Tr}\Big(\overline{H}_a X_b'^{\mu\nu} \big(k_1 \left(D_\mu D_\nu A_\lambda + D_\nu D_\mu A_\lambda\right)$$
$$+ \ k_2 \left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)\big)_{ba} \gamma^\lambda \gamma^5\Big) + \mathrm{h.c.} . \tag{3.11}$$

Allowed decays

$$\Gamma\left(P_2 \to P^*\pi\right) = C_\pi \frac{4k^2}{15\pi f_\pi^2} \frac{m_{P^*}}{m_{P_2'^*}} \|\mathbf{q}\|^7 \,, \tag{3.12a}$$

$$\Gamma\left(P_3^* \to P\pi\right) = C_\pi \frac{4k^2}{35} f_\pi^2 \frac{m_P}{m_{P_3^*}} \|\mathbf{q}\|^7 \,, \tag{3.12b}$$

$$\Gamma\left(P_3^* \to P^*\pi\right) = C_\pi \frac{16k^2}{105\pi f_\pi^2} \frac{m_{P^*}}{m_{P_3^*}} \|\mathbf{q}\|^7 \,, \tag{3.12c}$$

with $k = k_1 + k_2$.

Decaying $F = (P_2^{*\prime}, P_3)$

$$\mathcal{L}_{F,\,\mathrm{Int}} = \frac{1}{\Lambda_\chi^2} \mathrm{Tr}\left(\overline{H}_a F_b^{\mu\nu} \left(l_1 \left(D_\mu D_\nu A_\lambda + D_\nu D_\mu A_\lambda\right)\right.\right.$$
$$\left.\left. + l_2 \left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)\right)_{ba} \gamma^\lambda \gamma^5\right) + \mathrm{h.c.} \,. \tag{3.13}$$

Allowed decays

$$\Gamma\left(P_2^{*\prime} \to P\pi\right) = C_\pi \frac{4l^2}{25\pi f_\pi^2 \Lambda_\chi^4} \frac{m_P}{m_{P_2'^*}} \|\mathbf{q}\|^5 \left(m_\pi^2 + \|\mathbf{q}\|^2\right) \,, \tag{3.14a}$$

$$\Gamma\left(P_2^{*\prime} \to P^*\pi\right) = C_\pi \frac{8l^2}{75\pi f_\pi^2 \Lambda_\chi^4} \frac{m_P^*}{m_{P_2'^*}} \|\mathbf{q}\|^5 \left(m_\pi^2 + \|\mathbf{q}\|^2\right) \,, \tag{3.14b}$$

$$\Gamma\left(P_3 \to P^*\pi\right) = C_\pi \frac{4l^2}{105\pi f_\pi^2 \Lambda_\chi^4} \frac{m_{P^*}}{m_{P_3}} \|\mathbf{q}\|^5 \left(7m_\pi^2 + 4\|\mathbf{q}\|^2\right) \,, \tag{3.14c}$$

with $l = l_1 + l_2$.

3.2 Classification of Established States

The established states of the open-charm meson spectrum are listed in table 3.1 (together with the recently observed ones according to the classification proposed in this thesis) and, to give a bird's eye view, one could say that these states fill the H, S and T doublets. In the following I give an overview of their classification in some detail.

The fundamental doublet with $S_\ell^P = 1/2^-$ and $n = 1$ is filled by the well known states both in the strange and non-strange sector: the lightest $D_{(s)}$ meson with $J^P = 0^-$ and the $D_{(s)}^*$ meson with $J^P = 1^-$ meson; their properties are in agreement with quark model for both neutral and charges states.

If one looks at the observations of the open-charm and open-beauty states from a historical perspective, it is easy to spot a pattern in which states of the T doublet are the first to be observed after the fundamental ones. This should cause no surprise: although the states of the S doublet are lighter than those of the T doublet (as expected by the quark model),

the former have broad widths since they decay through *s*-wave and therefore they are more difficult to observe than the latter, which decay through *d*-wave and thus are rather narrow. Indeed, the $S_\ell^P = 3/2^+$ states with $J^P = 1^+$ and 2^+ are filled respectively by the $D_1(2420)$ and the $D_2^*(2460)$ in the non-strange sector and by the $D_{s1}(2536)$ and by the $D_{s2}^*(2573)$ in the strange one.

The non-strange states with $S_\ell^P = 1/2^+$ and $n = 1$ and are filled by the $D_0^*(2400)$ and by the $D_1'(2430)$, which have large widths and are the lightest known states with respectively $J^P = 0^+$ and 1^+. Notice that the masses of the four states of both the S and T doublets lie in a range of about 100 MeV, which is small compared to the splitting with respect to the H doublet (≈ 350 MeV): this can be justified in light of the quark model because the S and T doublets have both $L = 1$ while H has $L = 0$.

The case of charmed mesons with strangeness is more convoluted. As noticed in the first chapter, the mass of the $D_0^*(2317)$ and of the $D_1'(2460)$ are much smaller than suggested by the quark model for this doublet. This led to the speculation that these states may contain four quarks, either as tetraquark or as molecule formed by D and K mesons. The tetraquark picture predicts a large number of states, which were not observed by further search of the BABAR and Belle collaborations in the mass region of the D_{s0}^*, hence making this picture disfavoured. Moreover, the masses of the D_{s0}^* and the D_{s1} are just below the threshold of DK and D^*K respectively, which accounts for the small widths of these states (unexpected for S doublet states) since they are forced to decay violating isospin. This has suggested that these two states could be hadronic molecules or mixtures of hadronic molecules and ordinary $c\bar{s}$ mesons. However, theoretical predictions for their (isospin violating) hadronic and radiative decays [24, 26] strongly support their identification with the ordinary $c\bar{s}$ mesons belonging to the S doublet.

The only remaining established state without strangeness is the $D_3^*(2760)$, also known as $D^*(2760)$ or $D(2750)$ in the PDG nomenclature. This state was known to have natural parity since its observation by BABAR (it decays to $D\pi$) and its spin-parity was later fixed by LHCb. Possible states with $J^P = 3^-$ and $n = 1$ are those belonging to X' doublet, which has $L = 2$, and to a higher doublet with $L = 4$ (not considered in this thesis); however, the latter is expected to have much larger mass and so the $D_3^*(2760)$ is reasonably identified with the former.

The $D_{sJ}(2860)$ and $D_{s1}^*(2700)$ were observed both in the DK and D^*K channels by Belle and BABAR and later confirmed by LHCb. The spin-parity of $D_{s1}^*(2700)$ was established by production studies, finding $J^P = 1^-$. Measurements of branching ratios and criteria based on heavy ChPT (analogous to those object of this thesis) suggested the identification with the $\tilde{D}_{s1}^*$ [27], the first radial excitation of the $D_s^*(2112)$.

The common history of the $D_{s1}^*(2860)$ and the $D_{s3}^*(2860)$ dates back to an observation by the BABAR collaboration, indicated as the $D_{sJ}(2860)$, the identification of which constituted a puzzle [28]. In [25] the identification of the $D_{s3}^*(2860)$ with the state having $J^P = 3^-$ belonging to the X' was proposed and later confirmed by LHCb [7]. After that, $D_3^*(2760)$ was recognized as the non-strange partner of the $D_{s3}^*(2860)$. The case of the $D_{s1}^*(2860)$ is more uncertain and therefore is postponed in the following paragraph.

Doublet	J^P	$c\bar{q}$ $n=1$	$n=2$	$n=3$	$c\bar{s}$ $n=1$	$n=2$	$n=3$
H	0^-	$D(1869)$	$D(2550)^\star$		$D_s(1968)$		
H	1^-	$D^*(2010)$	$D^*(2600)^\star$	$D_1^*(2680)^\star$	$D_s^*(2112)$	$D_{s1}^*(2700)$	$D_{s1}^*(2860)^\star$
S	0^+	$D_0^*(2400)$			$D_{s0}^*(2317)$		
S	1^+	$D_1'(2430)$	$D(3000)^\star$		$D_{s1}'(2460)$	$D_{sJ}(3040)^\star$	
T	1^+	$D_1(2420)$	$D(3000)^\star$		$D_{s1}(2536)$	$D_{sJ}(3040)^\star$	
T	2^+	$D_2^*(2460)$			$D_{s2}^*(2573)$		
X	1^-	$D_1^*(2680)^\star$			$D_{s1}^*(2860)^\star$		
X	2^-						
X'	2^-	$D(2750)^\star$					
X'	3^-	$D_3^*(2760)$			$D_{s3}^*(2860)$		
F	2^+	$D_2^*(3000)^\star$					
F	3^+						

Table 3.1: Observed open-charm mesons, classified in HQ doublets. States with uncertain identification are indicated with $\star$.

3.3 Discussion of States of Uncertain Classification

At present, the non-strange states with uncertain classification are the $D(2550)$ [11, 1], the $D^*(2600)$ [11], the $D_J^*(2650)$ [1, 3], the $D(2750)$ [11], the $D(3000)$ [1] and the $D_2^*(3000)$ [1, 3]; while the strange ones are the $D_{s1}^*(2860)$ [7] and $D_{sJ}^*(3040)$ [15].

State Classification Discussed in the Literature

As of their first observation, the BABAR collaboration suggested that the $D(2550)$ and the $D^*(2600)$ could belong to the $\tilde{H}$ doublet, i.e. with $S_\ell^P = 1/2^-$ and $n = 2$, respectively with $J^P = 0^+$ and 1^-. Hence these states are likely to be identified with the first radial excitations of (D, D^*). The $D(2750)$ is regarded as the spin partner of the $D^*(2760)$, thus having $J^P = 2^-$. Comparison with quark model results corroborates these identifications (for a discussion see [28]).

The decay of $D_{sJ}(3040)$ to DK has not been observed, therefore unnatural parity assignment is highly favoured. The lightest not yet observed states of this series are the two $J^P = 2^-$ states with $L = 2$ and $n = 1$, the 3^+ of the F doublet and the two $J^P = 1^+$ states with $L = 1$ and $n = 2$. The two 2^- states are expected to be broader, while the 3^+ to be heavier. Therefore the $D_{sJ}(3040)$ is one of the two axial mesons[1] [29].

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[1]Since this particle has such a large, several decay modes are possible. In particular to decays to DK^* and $D_s\phi$ should allow to discriminate between the $\tilde{D}_{s1}$ and the $\tilde{D}_{s1}'$, since the corresponding branching ratios are larger for the latter.

State Classification Proposed in this Thesis

The remaining states still requiring identification are those observed by LHCb in 2013 [1] and 2016 [3]. Among the latter, the $D_3^*(2760)$ can be identified with one of the previous observations, indeed it has the same J^P of the $D(2750)$ (using the PDG nomenclature) and compatible mass and width, so it is excluded from further considerations. On the other hand, the identification of the $D(3000)$, $D_1^*(2680)$ and $D_2^*(3000)$ is uncertain. In the remaining of this section I propose a classification of these mesons, which still lacks in the literature.

The $D_1^*(2680)$ could be the $D_J^*(2650)$ seen by the LHCb collaboration in 2013 [1], which has natural parity and similar mass and width. This state could be a higher radial excitation of the fundamental doublet, possibly with $n = 3$, as far as the $D^*(2600)$ is correctly identified with the $n = 2$ one. Another possibility is the identification with the 1^- state of the X doublet, both of which states are vacant.

At first sight, the mass of the $D_1^*(2680)$ does not exclude neither of these identifications. On one hand, if this state belongs to the X doublet, one would expect it to have a mass between the masses of the mesons in the T doublet (≈ 2450 MeV) and in the X' (≈ 2750 MeV). Therefore ≈ 2.68 GeV would be a plausible value. On the other hand, heuristic arguments suggest that the same mass would be compatible with a $n = 3$ state. At the present, no states with $n = 3$ have been identified and hence the mass splitting with respect to the ones with $n = 2$ is not known. However, such mass splitting is expected to be smaller than the one between corresponding pairs of $n = 2$ and $n = 1$ states. Moreover, given the mass difference between the $n = 1$ and $n = 2$ observed states, which is tipically about 500 MeV, as a crude estimate, a hydrogen-like behaviour with masses proportional to $1/n^2$ would suggest that the separation between states with $n = 2$ and 3 would be about 90 MeV. This estimate is compatible with the observed mass of the $D_1^*(2680)$. Finally, both the 1^- states of the $\tilde{H}$ and X doublets are expected to decay in p-wave and therefore neither of the alternatives can be ruled out as implausible on the basis of the width of the $D_1^*(2680)$ (which, by the way, is rather broad, about 180 MeV).

However, further conclusions can be drawn, by taking in consideration the identification of the $D_{s1}^*(2860)$. The case for this state mirrors the one of the $D_1^*(2680)$: possible identifications are with the vector meson of the X doublet with $n = 1$ and with the one of the H doublet with $n = 3$. Leaving alone the anomalous case of the $D_{s0}^*(2317)$ and $D_{s1}'(2460)$, as already noticed, the mass difference between corresponding strange and non-strange states is about 100 MeV. But the mass difference between the $D_1^*(2680)$ and the $D_{s1}^*(2860)$ is about 200 MeV, therefore is likely that the assignment to doublets with the same S_ℓ^P are mutually exclusive. The possible scenarios are: $D_1^*(2680)$ belongs to the X doublet with $n = 1$ and $D_{s1}^*(2860)$ to the H doublet with $n = 3$ or the other way around. It is remarkable that in every case one of the two states has $n = 3$, which if confirmed would be the first of such states.

The identification of the $D(3000)$ is more complicated, being known only that it has unnatural parity. However, due to its large mass, several identifications are unlikely. As will be shown in the following, the $D_2^*(3000)$ is likely to be the 2^+ state of the F doublet and its mass is larger than the one of the $D(3000)$ by more than 200 MeV, hence, if the $D(3000)$ is the 3^+ state of the F doublet there would be a mass inversion and therefore this identification is precluded. Instead, if it is the 2^- of the X doublet it would be expected to have mass between

the ones of the states of the T and X' doublet, which are sensibly less than 3 GeV. Therefore also this identification is unlikely. The remaining possibilities are the $n = 3$ and $J^P = 0^-$ state of the H doublet[2] and the following three $n = 2$ states: the 2^- state of the X' doublet, and the two 1^+ states. Notice that, in the last scenario, the $D(3000)$ would be the non-strange partner of the $D_{sJ}(3040)$ and indeed it has a mass about 100 MeV smaller than such state. Moreover, always in this scenario, its mass would be about 500 MeV greater than the corresponding $n = 1$ state. Overall, this hypothesis seems to fit nicely with the available data and thus is reported in table 3.1.

3.4 The Classification of the $D_2^*(3000)$

In this section I present my original contribution aiming at providing all possible ingredients to properly identify the $D_2^*(3000)$, as well as its yet unobserved strange partner.

The $D_2^*(3000)$ could be the $D^*(3000)$ observed by LHCb in 2013 [1], however, as noticed in the 2016 LHCb analysis, the unquoted systematic errors of the 2013 study makes uncertain this identification. Moreover, in the study [1] the LHCb collaboration had already put forward the possibility that the observed $D^*(3000)$ could be a superposition of several states in that mass region. Overall, although both states have the same parity and lie in the same mass region (at the edge of the spectrum of observed states), the $D_2^*(3000)$ could be considered a newly observed D meson.

Having $J^P = 2^+$, the $D_2^*(3000)$ could belong to the F or $\tilde{T}$ doublets, which at the moment are both empty. Moreover, at present, its strange partner is unobserved in both cases. In the following an analysis of the relative branching ratios is proposed to discriminate between the two alternatives, focusing on the neutral state $D_2^*(3000)^0$ which was effectively observed.

The decay widths of the relevant doublets have been already computed and appear in the equations (3.8b), (3.8b), (3.14a) and (3.14a). On the basis of charge conservation alone, possible decay modes of $D_2^*(3000)^0$ are $D_2^*(3000)^0 \to D^0 \pi^0$, $D_2^*(3000)^0 \to D^+ \pi^-$, $D_2^*(3000)^0 \to D^{*0} \pi^0$ and $D_2^*(3000)^0 \to D^+ \pi^-$ (charge conjugation is understood). I introduce the following definition:

$$R_\pi = \frac{\Gamma\left(D_2^*(3000)^0 \to D^{*+} \pi^-\right) + \Gamma\left(D_2^*(3000)^0 \to D^{*0} \pi^0\right)}{\Gamma\left(D_2^*(3000)^0 \to D^+ \pi^-\right) + \Gamma\left(D_2^*(3000)^0 \to D^0 \pi^0\right)}. \tag{3.15}$$

This quantity is evaluated numerically using the known masses for the pions and the $D^{(*)}$ mesons including their uncertainty and the mass measurement of the $D_2^*(3000)$ reported in [3], finding

$$R_\pi = \begin{cases} 0.40 \pm 0.01 & F \\ 1.06 \pm 0.03 & \tilde{T} \end{cases}. \tag{3.16}$$

It is worth observing that these ratios assume very different values in the two cases: hence they are a powerful piece of information to discriminate between the two possibilities. As

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[2]Notice that, if the $D_1^*(2680)$ is the $n = 3$, $J^P = 1^-$ state, in this scenario there would be a mass inversion within this doublet, therefore this assignment would be ruled out.

soon as experimental measurements will be available for these ratios, it will be possible to finalize the identification of $D_2^*(3000)$.

Further informations can be gained by exploiting the strange sector. If the strange $J^P = 2^+$ state with $n = 2$ were known, it would have been possible to use an argument similar to the case of the $D_1^*(2680)$ to estimate the mass of its non-strange partner. Unfortunately, at the moment, such state is unobserved. Nonetheless, if the $D_{sJ}(3040)$ is the 1^+ state of the $\tilde{T}$ doublet then the mass of this yet unobserved state would differ by the $D_{sJ}(3040)$ by a small quantity. By comparison with other doublets one expects this splitting to be less than about 50 MeV. Therefore, in this scenario, the strange partner of the $D_2^*(3000)$ would have mass near 3.1 GeV, which is about 100 MeV *smaller* than the mass of the $D_2^*(3000)$. But the latter is expected to be about 100 MeV *larger* than the one of the $D_2^*(3000)$. Therefore, it is more likely that the $D_2^*(3000)$ belongs to the F doublet (see table 3.1). Notice that, if the $D_{sJ}(3040)$ is the 1^+ state of the S doublet, the argument still holds, because the two 1^+ states are expected to have similar masses.

As already remarked, the quantity defined in equation (3.15) is a model independent prediction which can be used to discriminate between the identifications of the $D_2^*(3000)$ as a member of the F or $\tilde{T}$ doublet. However, it is possible to define other ratios of widths which can be used to identify the $D_2^*(3000)$ on the basis of the expected properties of the would-be spin partner in the two cases. I denote this partner by D^{**}: it would be a $J^P = 1^+$ state in the case of D_2^* belonging to the $\tilde{T}$ doublet, or the $J^P = 3^+$ state, if D_2^* belongs to the F doublet. I define:

$$R_\pi' = \frac{\Gamma\left(D^{**0} \to D^{*+}\pi^-\right) + \Gamma\left(D^{**0} \to D^{*0}\pi^0\right)}{\Gamma\left(D_2^*(3000)^0 \to D^+\pi^-\right) + \Gamma\left(D_2^*(3000)^0 \to D^0\pi^0\right)} \, . \tag{3.17}$$

Indeed, this quantity can be calculated with respect to the two hypotheses under consideration within the heavy ChPT. However, since the mass of the spin partner D^{**} is unknown, the only statement which can be made at the present is in the form of a parametric study, as shown in figure 3.1.

On the basis of all the observed cases, one can argue that in each doublet the mass splitting between the two spin partners ranges between 50 Mev and 100 MeV and is smaller for doublets with higher L or n. Moreover, usually the state with higher spin in a given doublet has larger mass than the other (the other way around is referred to as *spin inversion*). Hence, under the assumption that $D_2^*(3000)$ belongs to the $\tilde{T}$ doublet, one can compute the ratio R_π' assuming for the spin partner a mass in the range 3.1–3.214 GeV. On the other hand, if $D_2^*(3000)$ belongs to the F doublet, the same ratio is computed for masses of the spin partner in the range 3.214–3.3 GeV. Notice that in the first case, one should exploit equation (3.8a), while in the second case equation (3.14c) should be used to compute the relevant decay widths. The shaded area in figure 3.1 represents the prediction, including the uncertainty, for R_π' as a function of the mass of D^{**}: mass values smaller than 3214 MeV refer to the case of the $\tilde{T}$ doublet, larger ones to the case of the F one.

Finally, the observation of the $D_2^*(3000)$ allows to draw further conclusions. Indeed, the strange partner of the $D_2^*(3000)$, namely the $D_{s2}^{*\prime}$, should have mass of about 3.3 GeV. This information can be used to calculate the analogous of equation (3.16) for decays of the $D_{s2}^{*\prime}$ to

52
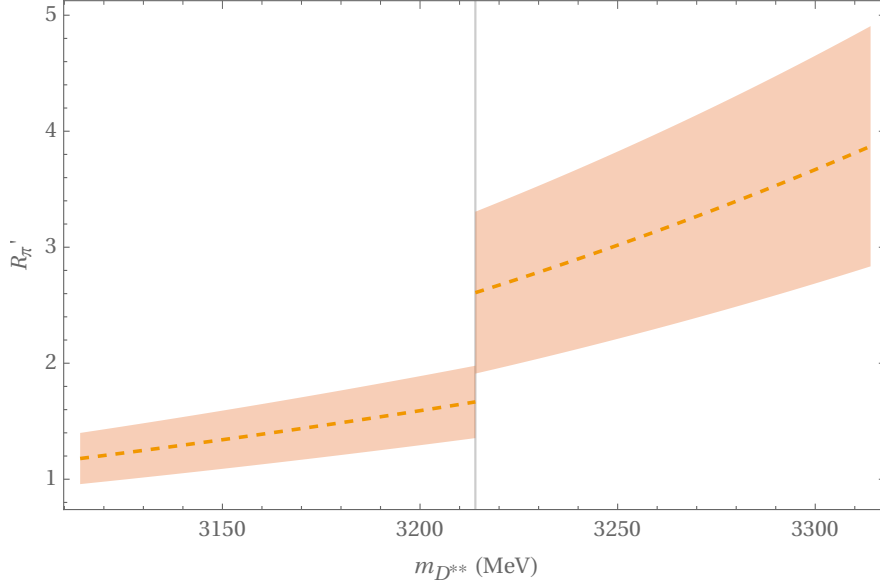
Figure 3.1: Possible values of R'_π against the mass of $D^{*\prime}_{s2}$. The left-hand side refers to the case in which $D^*_2(3000)$ belongs to the $\tilde{T}$ doublet and D^{**} is its spin partner with $J^P = 1^+$. The right-hand side refers to the case in which $D^*_2(3000)$ belongs to the F doublet and D^{**} is its spin partner with $J^P = 3^+$.

a strange meson of the H doublet plus K or η. Such quantities are listed below, where I have assumed[3] the value 3313 ± 62 MeV for the mass of the $D^{*\prime}_{s2}$

$$R_K = \frac{\Gamma\left(D^{*+}_{s2} \to D^{*0}K^+\right) + \Gamma\left(D^{*+}_{s2} \to D^{*+}K_S\right)}{\Gamma\left(D^{*+}_{s2} \to D^0 K^+\right) + \Gamma\left(D^{*+}_{s2} \to D^+ K_S\right)} , \quad R_K = \begin{cases} 0.39 \pm 0.02 & F \\ 1.02 \pm 0.03 & \tilde{T} \end{cases} , \quad (3.18\text{a})$$

$$R_\eta = \frac{\Gamma\left(D^{*+}_{s2} \to D^+_s \eta\right)}{\Gamma\left(D^{*+}_{s2} \to D^0 K^+\right) + \Gamma\left(D^{*+}_{s2} \to D^+ K_S\right)} , \quad R_\eta = \begin{cases} 0.29 \pm 0.01 & F \\ 0.31 \pm 0.01 & \tilde{T} \end{cases} , \quad (3.18\text{b})$$

$$R^*_\eta = \frac{\Gamma\left(D^{*+}_{s2} \to D^{*+}_s \eta\right)}{\Gamma\left(D^{*+}_{s2} \to D^0 K^+\right) + \Gamma\left(D^{*+}_{s2} \to D^+ K_S\right)} , \quad R^*_\eta = \begin{cases} 0.10 \pm 0.01 & F \\ 0.29 \pm 0.02 & \tilde{T} \end{cases} . \quad (3.18\text{c})$$

---

[3]It was used the following estimate

$$m_{D^{*\prime}_{s2}} = m_{D^*_{s2}(3000)} + \overline{\Delta m_s}$$

$$\sigma\left(m_{D^{*\prime}_{s2}}\right) = \sqrt{\sigma\left(m_{D^{*\prime}_{s2}(3000)}\right)^2 + \sigma\left(\Delta m_s\right)^2}$$

where $\overline{\Delta m_s}$ and $\sigma\left(\Delta m_s\right)$ are the weighted mean and standard deviation of the difference between the masses of D mesons and their strange partner $m_s = m\left(D^{**}_s\right) - m\left(D^{**}\right)$, where D^{**} are the $D(1869)$, the $D^*(2010)$, the $D^*(2600)$, the $D_1(2420)$, the $D^*_2(2460)$ and the $D^*_3(2760)$ (see table 3.1).

It can be noticed that the most significant difference is found in the ratio R_K. Moreover, in the case $D_2^*(3000)$ belongs to the F doublet the ratio R_η^* is much smaller than R_K and R_η, while in the event it belongs to the $\tilde{T}$ doublet the two ratios R_η and R_η^* are almost equal size and R_K is much larger than the other two. These findings are summarized at the end of the chapter.

3.5 Final remarks about $D_2^*(3000)$ decay modes

The mass of $D_2^*(3000)$ is large enough to allow other decay modes besides those discussed in the previous section. For example, it can decay to members of doublets other than the H doublet plus a light pseudoscalar meson. The features of such decay modes are different for the two assignments discussed in this thesis. One could argue that these modes are less important than the ones already considered, on the basis of the reduced phase space available for them. Moreover, their theoretical estimate would require the introduction of further coupling constants, at present unknown.

Other possible decay channels are those with the emission of a light vector meson, such as $D\rho$ or $D^*\rho$. In this case, the phase space would not be suppressed as in the case of decays to higher doublets and these modes cannot be neglected. Their estimate is possible using an approach similar to the one already exploited here. Indeed, an effective Lagrangian approach has been developed using the hidden gauge symmetry idea [20, 19]. The description of this method is beyond the scope of this thesis.

As a final remark, I would like to mention that, if the decay modes to $D\pi$ and $D^*\pi$ were the only relevant ones, it would have been possible to estimate the strong coupling constant appearing in the effective Lagrangian (3.13) in the two cases: by equating the theoretical expression of the decay widths to the experimentally measured total width one could have easily derived such a parameter and, consequently, it would have been possible to estimate the width of the spin partner. Due to the existence of the other decay modes mentioned above, this procedure could only lead to an upper bound on the strong coupling constant.

Summary of the Results for the Classification of the $D_2^*(3000)$

Case $D_2^*(3000)$ belongs to the F doublet

- Predicted ratio R_π (defined in equation (3.15)):

$$R_\pi = 0.40 \pm 0.01$$

- Spin partner: D_3^* with $J^P = 3^+$. Should be looked for in the mass range $\approx$ 3.2–3.3 GeV.

- Predicted ratio R'_π defined in eq. (3.17):

$$R'_\pi = 3.60 \pm 1.60$$

- Predicted hierarchy for ratios R_K, R_η, R_η^* for the strange partner (see equations (3.18)):

$$R_K > R_\eta \gg R_\eta^*$$

and, in particular
$$R_K = 0.39 \pm 0.02.$$

Case $D_2^*(3000)$ belongs to the $\tilde{T}$ doublet

- Predicted ratio R_π (defined in eq. (3.15)):

$$R_\pi = 1.06 \pm 0.03$$

- Spin partner: $\tilde{D}_1$ with $J^P = 1^+$. Should be looked for in the mass range $\approx$ 3.1–3.2 GeV.

- Predicted ratio R'_π defined in eq. (3.17):

$$R'_\pi = 1.50 \pm 0.60$$

- Predicted hierarchy for ratios R_K, R_η, R_η^* for the strange partner (see equations (3.18)):

$$R_K \gg R_\eta \approx R_\eta^*$$

and, in particular
$$R_K = 1.02 \pm 0.03.$$

Chapter 4

Conclusions

Thanks to the *B*-factories and the LHCb experiment, there exist an ever growing availability of new data and hadronic spectroscopy is becoming a field of precision physics. The theoretical analysis of this data requires an approach deeply rooted in QCD. In the case of heavy-light hadrons, at present, one of the best options is the heavy chiral perturbation theory, which, unlike potential models, allows a model independent description of such states in the limit in which heavy quarks have infinite mass and light ones are massless. This theory is an established research tool and is today widely used in the discussion heavy-light hadronic systems.

At present, the states of the open-charm meson spectrum fit rather nicely the predictions of the heavy chiral perturbation theory, although the puzzle of the low mass of the strange *S* doublet with $n = 1$ still needs to be clarified.

Classification is the first step in each science, because it allows to find patterns and indicates where to look for missing pieces. Prominent examples are the Mendeleev's table and the early quark model. In the case of heavy meson spectroscopy and in particular of open-charm spectroscopy, there are many vacancies in the classification of states and future observations will surely enrich our understanding. In particular, it would be interesting to observe the spin partner of the $D^*_{s1}(2700)$ and in general other states with $n = 2$, indeed a glance at the table 3.1 reveals that practically all radial excited states have uncertain classification. These and other considerations show how the heavy-meson spectroscopy is still a young and promising field of research.

The case of $D^*_2(3000)$ was considered in some detail and original results, based on the formalism treated in this thesis, have been presented. This meson has been observed by LHCb Collaboration in 2016, and its spin parity has been fixed to $J^P = 2^+$. Considering the classification scheme discussed in this thesis, I have found that there are two possible identifications for it. In order to distinguish between the two, several strategies have been proposed in this thesis.

In particular, the first one relies on the calculation of the ratio of the branching fractions of the $D^*_2(3000)$ in $D\pi$ and $D^*\pi$. This quantity is sensitive to the quantum numbers of $D^*_2(3000)$ and hence suitable to classify it. This result is interesting in light of the upcoming measurements of the LHCb collaboration.

Other original results presented in this thesis are the predictions concerning the spin and the strange partners of the $D_2^*(3000)$. In particular, I have shown that exist a hierarchy among the possible decay modes of the latter that depends on the identification of the $D_2^*(3000)$. Hence, a measurement of such hierarchy could again provide a useful tool for the identification of this meson.

Finally, I would like to mention possible future investigations stemming from this work or connected to its theoretical framework. To begin with, within the heavy mesons spectroscopy, an interesting possibility is the determination of the low energy coupling constants of the heavy chiral perturbation theory from experimental data. This requires the evaluation of decay widths to final states with a light vector meson and involves extensions of the methods presented in this thesis. On the other hand, within the latter, it is possible to calculate decays to final states with an excited meson belonging to other doublets than the fundamental one plus a light pseudoscalar meson.

The spectroscopy of baryons containing a heavy quark, which as noticed in the first chapter is collecting new experimental achievements as well, can be studied using the same methods presented in this thesis. Indeed, once the covariant representation of baryon doublets is introduced, the decay widths of excited heavy baryons to a final state with a light pseudoscalar meson can be obtained from effective Lagrangians in a similar way.

Appendix A

Spinor Degrees of Freedom and Projectors

Throughout this thesis several algebraic properties of spinors have been exploited. Given their relevance, some general remarks are recollected and presented in a systematic way in this appendix (the approach of which is taken from [43]).

A rank 1 spinor (or Weyl spinor or simply spinor) is a two component complex vector with defined Lorentz transformation law. Spinors can belong to two different representations of the Lorentz group $SO(3,1)$, which is homomorphic to $SL(2, \mathbb{C})$, i.e. the group of 2×2 unimodular matrices, which are indicated in the literature as $(1/2, 0)$ and $(0, 1/2)$. The members of the first representation are called contraspinors or right-handed while the member of the second are called cospinors or left-handed.

The transformations of these two representations are the following

$$\mathrm{D}_R : \ u \mapsto \exp\left(\frac{i}{2}(\boldsymbol{\theta} + i\boldsymbol{\rho}) \cdot \boldsymbol{\sigma}\right) u \qquad \text{for contraspinors} \tag{A.1}$$

$$\mathrm{D}_L : \ v \mapsto \exp\left(\frac{i}{2}(\boldsymbol{\theta} - i\boldsymbol{\rho}) \cdot \boldsymbol{\sigma}\right) v \qquad \text{for cospinors,} \tag{A.2}$$

where (θ^i, ρ^i) are the parameters of the transformation (respectively the rotation angles and the rapidity vector components) and σ^i are the Pauli matrices. Therefore, *spinors of opposite chirality transform in the same way under pure rotations and in opposite way under pure boosts*. A Lorentz transformation for contraspinors is related to the transformation for cospinors with same parameters $\left(\mathrm{D}_R^\dagger\right)^{-1}$, in this way the contraction $v^\dagger u$ is Lorentz invariant (like the contractions of contravariant and covariant four-vectors).

The tensorial product of two rank 1 spinor is a rank 2 spinor, and so go on. Tensors of rank k can be represented as spinors of rank $2k$. However, some tensors can be represented with spinors of the rame rank. This is shown explicitly for four-vectors in the following.

To begin with, notice that there is a linear isomorphism between four-vectors and Hermitian 2×2 matrices. The latter form a four-dimensional real vector space and a particularly

convenient choice of a basis is $\{\mathbb{1}, \sigma^1, \sigma^2, \sigma^3\}$

$$X = \begin{pmatrix} t+z & x-iy \\ x+iy & t-z \end{pmatrix} = t\mathbb{1} + x\sigma^1 + y\sigma^2 + z\sigma^3 = \sum_\mu x^\mu \sigma^\mu, \tag{A.3}$$

with $\sigma^\mu = (\mathbb{1}, \boldsymbol{\sigma})$. These matrices belong to a representation of the (complex two-dimensional) special linear group, if M is Hermitian and $\mathrm{D}_{R/L}$ belongs to $SL(2, \mathbb{C})$ then $\mathrm{D}_{R/L} M \mathrm{D}_{R/L}^\dagger$ is Hermitian, which is homomorphic to $SO(3,1)$ and indeed

$$\det X = t^2 - \left(x^2 + y^2 + z^2\right) = x_\mu x^\mu, \tag{A.4}$$

$$\det\left(\mathrm{D}_{R/L} X \mathrm{D}_{R/L}^\dagger\right) = \det X. \tag{A.5}$$

The connection between the two groups is made apparent by the relations

$$\mathrm{D}_R^\dagger \sigma^\mu \mathrm{D}_R = \sum_\nu \Lambda^\mu{}_\nu \sigma^\nu \tag{A.6a}$$

$$\mathrm{D}_L^\dagger \sigma^\mu \mathrm{D}_L = \sum_\nu \Lambda_\mu{}^\nu \sigma^\nu \tag{A.6b}$$

where Λ is the four-vector Lorentz transformations with same parameters.

At this point, it is sufficient to observe that the external product

$$uu^\dagger = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}, \text{ with } u = \begin{pmatrix} a \\ b \end{pmatrix}, \tag{A.7}$$

is always a Hermitian null matrix which transforms covariatly with respect to Lorentz transformations, to conclude that a spinor is always associated to a null (or light-like) four-vector

$$V = \frac{1}{2} \begin{pmatrix} |a|^2 + |b|^2 \\ ab^* + ba^* \\ i(ab^* - ba^*) \\ |a|^2 - |b|^2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} u^\dagger u \\ u^\dagger \boldsymbol{\sigma} u \end{pmatrix}. \tag{A.8}$$

Moreover, it can be shown that any spinor can be uniquely represented by a tuple consisting of a light-like four-vector, a phase and a sign.

Given a spinor, the associated four-vector can be extracted using the following relations

$$V^\mu = u^\dagger \sigma^\mu u \qquad \text{for contraspinors} \tag{A.9}$$

$$V_\mu = v^\dagger \sigma^\mu v \qquad \text{for cospinors.} \tag{A.10}$$

These light-like four-vectors can be interpreted as the four-velocity of massless particles. The number of available DoF's confirm this idea: in a given reference frame the four-velocity of a massless particle is determined by its direction (2 DoF's) while from a normalized Weyl spinor can be extracted a null four-vector with unitary time component (2 DoF's).

Indeed, if u is a spinor and P is its four-vector then

$$\left(P^0 \mathbb{1} - \mathbf{P} \cdot \boldsymbol{\sigma}\right) u = 0 \qquad \text{for contraspinors} \tag{A.11}$$

$$\left(P^0 \mathbb{1} + \mathbf{P} \cdot \boldsymbol{\sigma}\right) v = 0 \qquad \text{for cospinors.} \tag{A.12}$$

These equations can be interpreted as the *Weyl equations* in the momentum space and describe massless spin-1/2 particles. For these particles the Pauli-Lubanski four-spin is proportional to the four-momentum: parallel in the case of right-handed solutions and anti-parallel in the case of left-handed solutions. For this reason, for Weyl particles, the helicity coincides with chirality.

The Pauli-Lubanski four-spin is defined by

$$W_\mu = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}P^\nu J^{\alpha\beta}, \tag{A.13}$$

where $J^{\alpha\beta}$ is the relativistic angular momentum tensor (the charge of the Noether current of the Lorentz symmetry). This four-vector is the covariant generalization of the non relativistic spin, indeed it is proportional to the usual spin vector in the rest frame (which does not exist for Weyl particles, being massless)

$$W|_{\text{rest frame}} = (0, m\mathbf{S}) \tag{A.14}$$

and, as such, is the generator of spin rotations. Moreover, as can be seen in the rest frame, it satisfies

$$W_\mu W^\mu = -m^2 s(s+1) \qquad W_\mu P^\mu = 0, \tag{A.15}$$

where s is the spin of the field in exam.

It is finally possible to transform a contraspinor in a cospinor and vice-versa, that is to find P such that

$$\text{if } u \mapsto \mathrm{D}_R\, u \text{ then } \mathrm{P}\, u \mapsto \left(\mathrm{D}_R^\dagger\right)^{-1} \mathrm{P}\, u. \tag{A.16}$$

The interpretation of contraspinors and cospinors as solutions of the Weyl equation suggests that P is a spatial inversion. Indeed, the momentum P^μ is a polar vector while the Pauli-Lubanski four-spin W^μ is an axial one, therefore a spatial inversion transforms right-handed spinors into left-handed one and vice-versa. However, left-handed solutions of the Weyl equation correspond to negative frequency modes, that is to antiparticles. Therefore, in the case of Weyl particles, charge conjugation coincide with parity inversion. An explicit calculation shows that the form of P is

$$\mathrm{P}: u \mapsto \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} u^*. \tag{A.17}$$

A bispinor (or Dirac spinor) is a vector with four complex components, thus eight real DoF's, transforming as a pair of spinors of opposite chirality. Conventionally, in the so-called chiral representation, the upper component is the right handed one, hence

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix}, \qquad \mathrm{D}: \psi \mapsto \begin{pmatrix} \mathrm{D}_R & 0 \\ 0 & \left(\mathrm{D}_R^\dagger\right)^{-1} \end{pmatrix} \psi. \tag{A.18}$$

The right and left-handed components of a bispinor can be projected by means of the *chirality projectors*

$$P_R = \frac{1+\gamma^5}{2} \qquad P_L = \frac{1-\gamma^5}{2}, \tag{A.19}$$

where γ^5 is the fifth gamma matrix, which in the chiral representation is

$$\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.20}$$

A naive counting of the DoF's indicates that a normalized bispinor could represent an object with linearly independent four-momentum and four-spin. Because a bispinor transforms as a pair of spinors it is possible to extract two proper light-like four-vectors from them, in this way the available DoF's are reduced to five (where the normalization condition was taken into account). Notice that the two phases left out could be used to transform non trivially a bispinor without affecting its momentum or spin

$$U_R : \psi \mapsto \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \psi \tag{A.21a}$$

$$U_L : \psi \mapsto \begin{pmatrix} \mathbb{1} & 0 \\ 0 & e^{i\alpha} \end{pmatrix} \psi. \tag{A.21b}$$

To specify the state of a particle with spin, five parameters are sufficient indeed. This can be seen in two way. First: the four-momentum and the four-spin have 8 real parameters combined, 3 of which are fixed by the respective on-shell conditions and by the orthogonality condition. Second: to specify the state of such a particle is sufficient assign its velocity (3 DoF's) and the direction (and sign) of its spin (2 DoF's).

Let be A^μ and B_μ the two light-like four-vector associated with the right-handed and left-handed components of ψ

$$A^\mu = u^\dagger \sigma^\mu u \qquad B_\mu = v^\dagger \sigma^\mu v, \tag{A.22}$$

then two orthogonal linear combinations can be formed

$$V^\mu = A^\mu + B^\mu \qquad S_\mu = \frac{1}{2} \left(A_\mu - B_\mu \right), \tag{A.23}$$

which can be written as

$$V^\mu = u^\dagger \sigma^\mu u + \eta^{\mu\nu} v^\dagger \sigma^\nu v = \psi^\dagger \gamma^0 \gamma^\mu \psi \tag{A.24}$$

$$S_\mu = \frac{1}{2} \left(\eta_{\mu\nu} u^\dagger \sigma^\nu u - v^\dagger \sigma^\mu v \right) = \frac{1}{2} \psi^\dagger \gamma^0 \gamma_\mu \gamma^5 \psi, \tag{A.25}$$

where all gamma matrices are in the chiral representation

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}. \tag{A.26}$$

These two four-vectors are **not necessarily** non-null, however they have opposite invariant norm and hence if one is space-like the other is time-like and vice-versa. If in some particular frame of reference holds

$$u = v \quad \text{or} \quad u = -v \tag{A.27}$$

then $A^0 = B^0$ and $\mathbf{A} = -\mathbf{B}$. Therefore in this frame of reference V^μ and S_μ are suitable to be interpreted as the four-velocity and the four-spin of a particle at rest and the explicit expression of $\mathbf{S}$ confirms this interpretation

$$\mathbf{S} = u^\dagger \boldsymbol{\sigma} u. \tag{A.28}$$

Once the equations for bispinorial fields are given, the two mutually exclusive possibilities (A.27) are interpreted respectively as particles and antiparticles.

However, it is important to note that the conditions (A.27) are **not** Lorentz covariant and so are referred to a particular frame of reference (if it exists). Nonetheless, in any given frame of reference, any bispinor can always be written as a sum of bispinors each satisfying respectively the first and the second of the equations (A.27)

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix} \equiv \begin{pmatrix} (u+v)/2 \\ (u+v)/2 \end{pmatrix} + \begin{pmatrix} (u-v)/2 \\ (v-u)/2 \end{pmatrix}. \tag{A.29}$$

This decomposition can be obtained using the energy projectors, which in the given frame of reference have the form

$$P_+ = \frac{1+\gamma^0}{2} \qquad P_- = \frac{1-\gamma^0}{2}. \tag{A.30}$$

Moreover, if ψ represent a particle (or an antiparticle) at rest in some other frame of reference, moving with speed $\mathbf{v}$ with respect to the observer, then the decomposition (A.29) can be used in that frame and the result can be boosted with velocity $-\mathbf{v}$

$$\psi'_\pm = D(-\mathbf{v})\frac{1 \pm \gamma^0}{2}\psi = \frac{1 \pm D(-\mathbf{v})\gamma^0 D(\mathbf{v})}{2}\psi' = \frac{1 \pm v_{0\mu} D(-\mathbf{v})\gamma^\mu D(\mathbf{v})}{2}\psi' = \frac{1 \pm \slashed{v}}{2}\psi' \tag{A.31}$$

where $v_0 = (1, \mathbf{0})$, $\psi' = D(-\mathbf{v})\psi$ and were taken into account the bispinorial analogous of the relations (A.6)

$$D(-\mathbf{v})\gamma^\mu D(\mathbf{v}) = \Lambda^\mu{}_\nu(\mathbf{v})\,\gamma^\nu. \tag{A.32}$$

In this way the generalized energy projectors are introduced

$$P_\pm = \frac{1 \pm \slashed{v}}{2} \tag{A.33}$$

and the bispinors projected using these operators have indeed the right velocity

$$V^\mu = \psi_\pm \gamma^\mu \psi_\pm = \pm v^\mu. \tag{A.34}$$

The same approach can be used to obtain the projectors for bispinor with spin in their rest frame parallel or antiparallel to $\mathbf{s}$ in an arbitrary frame of reference

$$P_{\uparrow\downarrow} = \frac{1 \pm \slashed{v}\slashed{s}\gamma^5}{2}, \quad \text{with } s^\mu = \Lambda^\mu{}_\nu\, s_0^\nu \text{ and } s_0^\mu = (0, \mathbf{s}) \tag{A.35}$$

and verify that bispinors projected using the latter have the right spin

$$S^\mu = \frac{1}{2}\psi_{\uparrow\downarrow}\gamma^\mu\gamma^5\psi_{\uparrow\downarrow} = \pm s^\mu. \tag{A.36}$$

The interpretation of V and S as four-velocity and four-spin of the bispinor can be checked against another argument. If the operator which exchange the left and the right component

$$\mathrm{P} : \psi \mapsto \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \psi = \gamma^0 \psi \qquad (A.37)$$

is interpreted as the spatial inversion (or parity transformation) operator then an explicit calculation show that $\mathbf{V}$ change sign, i.e. is polar, while $\mathbf{S}$ remain the same, i.e. is axial. This is the expected behaviour of the velocity and spin vectors. Moreover, as can be checked in the standard representation where P is diagonal, particle and antiparticle bispinors are eigenstates of the parity operator respectively with eigenvalues $+1$ and -1.

Finally, the bispinor charge conjugation operator can be introduced

$$\mathrm{C} : \psi \mapsto \begin{pmatrix} 0 & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} & 0 \end{pmatrix} \psi^* \qquad (A.38)$$

which simultaneously inverts the handedness of the two components and swap them. Its action is manifest in the rest frame, where it changes a particle bispinor in an antiparticle one and vice-versa.

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