Chaos detection in human EEGs: A Lyapunov exponents approach

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   - Lyapunov Exponents: Definition
   - Maximum Lyapunov Exponent: Estimation and Test
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3. EEG Signal Analysis
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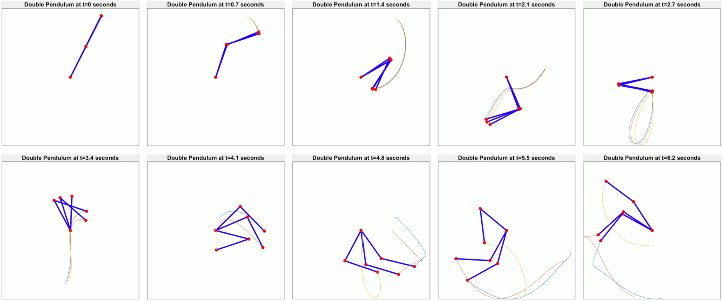
Introduction to Chaos Theory

Definition

Chaos is a peculiar dynamic, arising from either nonlinear ODEs, with at least 3 d.o.f, or certain nonlinear maps.



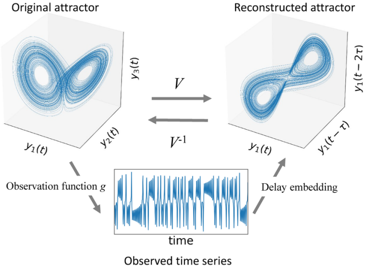
Main features of chaos:

- Random-appearing motion, even being purely deterministic
- Sensitive dependence on initial conditions (a.k.a butterfly effect)

Attractor Reconstruction

In real-world applications, the governing equations of a process are often unknown, so it is essential to reconstruct the attractor.

Let $V : \mathcal{A} \subset \mathcal{M}^m \to \mathbb{R}^d$ be a diffeomorphism with $d \geq 2d_a$, where d_a is the box-counting dimension



The **time delay embedding** $[y(t),\ y(t-\tau),\ y(t-2\tau), \ldots,\ y(t-2d\tau)]$ unfolds the squashed projection $y(t)$ into $\mathbb{R}^d$ w/o trajectory crossing.

Lyapunov Exponent

Operationally, a chaotic system is a bounded system converging to an attractor with a positive Lyapunov exponent [Eckmann and Ruelle, 1985].

The Lyapunov exponent is an attractor invariant that measures sensitivity to initial conditions.

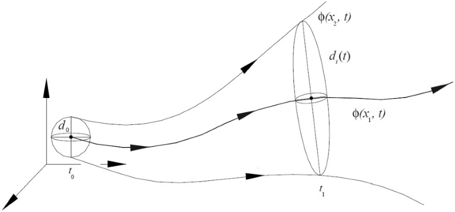
Given an initial perturbation δ, the orbits divergence after n steps is:

$$\begin{aligned}
\delta x_n &= F^n(x_0) - F^n(x_0 + \delta x_0) \\
&\approx DF(x_{n-1}) \times DF(x_{n-2}) \times \cdots \times DF(x_0)\delta x_0 \\
&\approx DF^n(x_0)\delta x_0
\end{aligned}$$

Lyapunov Spectrum

The Lyapunov spectrum λ is the *d*-dim vector of the logarithmic average rates of convergence (negative sign, stable manifolds) or divergence (positive sign, unstable manifolds).

$$\lambda = \lim_{n \to \infty} \frac{1}{n} \log |DF^n \delta x_0|$$



Maximum Lyapunov Characteristic Exponent

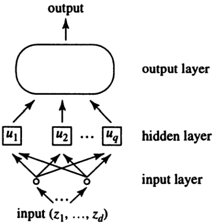
The maximum Lyapunov characteristic exponent (MLCE) is the highest exponent of the Lyapunov spectrum λ. It can be either estimated with:

- Direct methods [Kantz, 1994; Wolf et al., 1985]
- Jacobian-based methods [Eckmann and Ruelle, 1985; McCaffrey et al., 1992; Nychka et al., 1992]

Jacobian-based methods provide, under suitable conditions, a consistent estimator of the MLCE ($\hat{\lambda} \xrightarrow{\text{a.s.}} \lambda$) and they do not require a prior heuristic-based time series embedding [Chan and Tong, 2001].

Nonparametric ANN method

The Jacobian-based nonparametric ANN method [Nychka et al., 1992] consistently estimates the MLCE by fitting a single-hidden-layer neural network (SLFN) of the map and its derivative.



The resulting MLCE estimator supports:

- The CLT $\sqrt{m}(\hat{\lambda} - \lambda) \to N(0, \Phi)$ as $m \to \infty$ [Bailey, 1996]
- A consistent asymptotic variance estimator $\hat{\Phi}$
- A statistical chaos test $H_0 : \lambda \leq 0$ vs. $H_1 : \lambda > 0$, with $\hat{z} = \frac{\hat{\lambda}}{\sqrt{\hat{\Phi}/m}}$ as test statistic and z_α as critical value s.t $Pr(Z \geq z_\alpha) = \alpha$ and $Z \sim \mathcal{N}$ [Shintani and Linton, 2004]

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Nonlinear dynamics of the human brain

- The human brain involves more than 85 billiards of neurons.
- Nonlinearity even on a cellular level (threshold/saturation mechanisms).
- Electroencephalograms (EEGs) record the brain electrical activity.

Since the mid-1980s, chaos theory tools have been applied to EEG analysis.

- Sleep, epilepsy and eye closure [Babloyantz et al., 1985, 1986]
- Parkinson's [Müller et al., 2001]
- Schizophrenia [Lee et al., 2001]

However, the lack of statistically sound methods has led to conflicting results.

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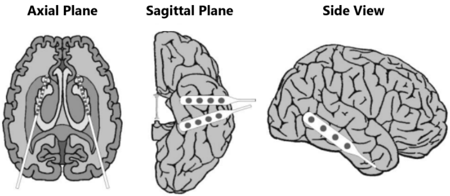
Research Question

Test the existence of chaotic dynamics and/or local instability in human EEGs by using the maximum Lyapunov exponent.



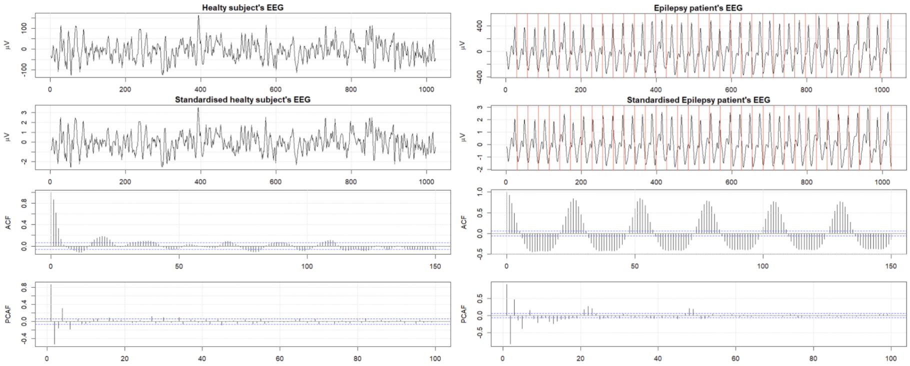
Data

The UKB dataset is composed of 5 groups of 100 single-channel EEG segments of 23.6 sec duration — 173 Hz.

Group	**A**	**B**	**C**	**D**	**E**
Subjects	Five healthy volunteers	Five healthy volunteers	Five epilepsy patients	Five epilepsy patients	Five epilepsy patients
Subject status	Awake, eyes open	Awake, eyes closed	Seizure-free interval (Interictal phase)	Seizure-free interval (Interictal phase)	Ictal phase
EEG type	Extracranial EEG (10–20 system)	Extracranial EEG (10–20 system)	Intracranial EEG, Contralateral hippocampus	Intracranial EEG, Epileptogenic hippocampus	Intracranial EEG (strip and depth electrodes)

Preliminary Analysis

- The brain region synchronisation during the ictal stage leads to an increase in the range of variation and to more regular oscillating patterns.



Data Analysis

For each of the 500 time series, we estimate the MLCE and the embedding dimension, a proxy of the system complexity, through the Jacobian-based nonparametric ANN method [McCaffrey et al., 1992].

The ANN is fitted on the following parameter space and the best model is chosen by means of the Bayesian Information Criterion (BIC).

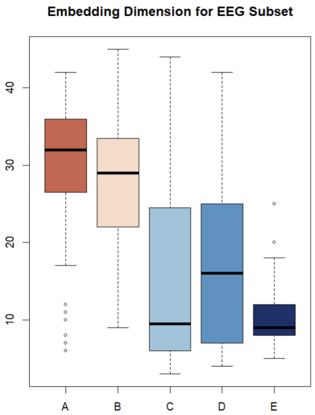
Parameter	**Grid**
Embedding dimension (m)	2–45
Time delay (d)	1–5
Number of hidden units $(size)$	1–15

Due to the computational burden, we employ the *Newton server* at the Department of Statistics, University of Bologna.

Results: System Complexity

- Ictal (E) EEGs exhibit a lower dimensionality compared to interictal phase (C and D) and healthy subjects' (A and B) EEGs.
- The sense of sight contributes to a greater complexity of open-eye brain dynamics.

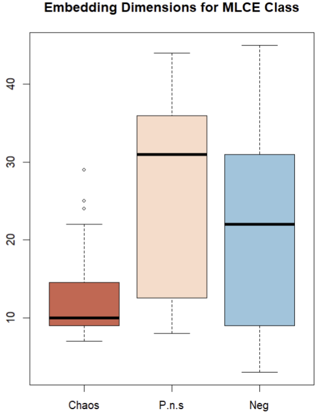
Recording Group	Mean Embedding Dim.
Healthy Open Eyes (A)	29.3
Healthy Closed Eyes (B)	27.2
Non-focal Interictal (C)	15.6
Focal Interictal (D)	17.7
Ictal (E)	10.4



Results: Chaotic Dynamics

- Ictal (E) EEGs display low-dimensional chaos.
- Interpretation uncertainties arises for interictal (C and D) and healthy subjects' (A and B) EEGs.

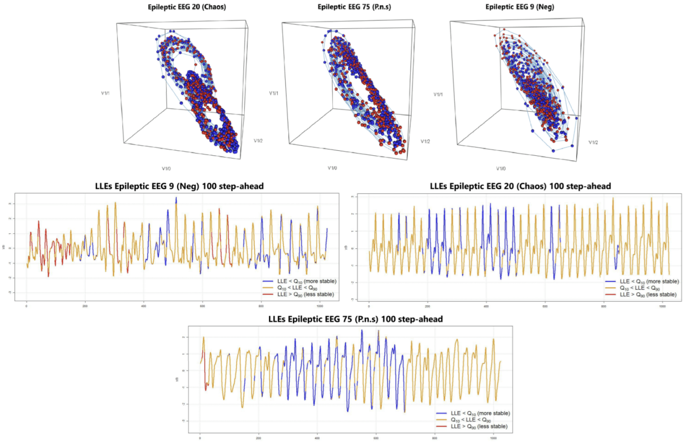
Recording Group	Chaos	P.n.s	Neg
Healthy Open Eyes (A)	0	5	95
Healthy Closed Eyes (B)	4	1	95
Non-focal Interictal (C)	0	1	99
Focal Interictal (D)	9	2	89
Ictal (E)	47	6	47



i *Chaos*, positive and significant MLCE ($\lambda > 0$ and p-value < 0.05)
ii *P.n.s*, positive and not significant MLCE ($\lambda > 0$ and p-value > 0.05)
iii *Neg*, negative and not significant MLCE ($\lambda < 0$ and p-value > 0.05)

Results: Local Instability

- Non-chaotic systems may still be characterised by regions of finite-time instability and lower predictability.
- The local Lyapunov exponent detects local instability.



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Normal brain activity does have a higher system complexity compared to both ictal and interictal phase activity.

Ictal phase EEGs do exhibit low-dimensional chaos.

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Thanks for the attention.