Chaos detection in human EEGs: A Lyapunov exponents approach

Stefano Cartabia
Supervisor: Alessandra Luati, Simone Giannerini

Long Abstract

The high complexity of human brain dynamics raise the question of whether brain electrical activity recordings — such as the electroencephalograms (EEGs) — can exhibit chaos. Chaotic dynamics arises from either nonlinear ordinary differential equations (ODEs) with at least three degrees of freedom or specific nonlinear maps, and it is characterised by peculiar properties such as the so-called "sensitivity to initial conditions", which causes initially nearby trajectories to diverge exponentially fast and prevents any long-term prediction. Since the mid-1980s, there has been a growing interest in applying chaos theory methods to analyse EEG signals. The results revealed a considerable reduction of the system complexity for various mental states and neurological conditions, such as eye closure and epilepsy (Babloyantz & Destexhe, 1986; Babloyantz et al., 1985), schizophrenia (Lee et al., 2001) or Parkinson's (Müller et al., 2001). However, the absence of long, stationary, and noise-free EEGs, together with a lack of statistically sound methods, led to conflicting findings (Palus, 1996; Prichard & Theiler, 1994), especially concerning the presence of global chaotic dynamics, as measured by the maximum Lyapunov exponent (MLCE) — an attractor invariant that measures sensitivity to initial conditions. The main objective of this dissertation is to assess rigourously the existence of global chaotic dynamics and/or local instability in human EEGs by using state of the art statistical results, implemented in an experimental version of the R package tseriesChaos (Narzo, 2019). We use a consistent estimator of the MLCE through a Jacobian-based nonparametric neural network approach (Giannerini & Rosa, 2004; McCaffrey et al., 1992). The method uses a single-hidden-layer neural network (SLFN) of the system map. Next, we use a one-sided asymptotic chaos test based on the results of (Shintani & Linton, 2004).

The data used in this thesis comes from the UKB dataset provided by the Epilepsy Center at the University of Bonn, and it includes 100 single-channel EEGs for each of the following five recording groups: healthy subject with open eyes (A), healthy subject with closed eyes (B), interictal phase (C and D), and ictal phase (E), for an overall 500 series analysed.

We obtain a preliminary result based on the embedding dimension of the reconstructed attractor, used as a proxy of the true dimensionality of the system. We find that ictal phase EEGs exhibit a lower dimensionality compared to both interictal phase (C and D) and healthy subjects recordings (A and B). Physiologically, this happens because of a brain region synchronisation taking place during the ictal phase, which causes the neuronal peaks to align, reducing the overall system complexity. Moreover, we observe that the sense of sight contributes to the higher complexity of open-eye brain (A) dynamics compared to closed-eye brain (B).

The main results based on the estimate of MLCE and the associated p-values from the (Shintani & Linton, 2004) chaos test, show that the ictal stage recordings can display low-dimensional chaos, whereas there is a greater uncertainty in the case of interictal (A and B) and healthy subjects' EEGs (C and D). Indeed, if, on the one hand, these signals are mostly associated with a negative estimate of the MLCE, on the other hand, they display a much higher degree of randomness and variability. This behaviour could be explained by the different dimensionality of the series in the various stages. Indeed, chaotic dynamics may also exist in these systems but it would be a high-dimensional chaos, which is much more difficult to estimate, due to the curse of dimensionality. Finally, we investigated the local instability of human brain dynamics through Local Lyapunov exponent (LLE). The results revealed the extremely high complexity and heterogeneity of human brain dynamics.

References

Babloyantz, A., & Destexhe, A. (1986). Low-dimensional Chaos in an Instance of Epilepsy. *Proceedings of the National Academy of Sciences*, *83*(10), 3513–3517. https://doi.org/10.1073/pnas.83.10.3513

Babloyantz, A., Salazar, J., & Nicolis, C. (1985). Evidence of Chaotic Dynamics of Brain Activity During the Sleep Cycle. *Physics Letters A*, *111*(3), 152–156. https://doi.org/10.1016/0375-9601(85)90444-x

Giannerini, S., & Rosa, R. (2004). Assessing chaos in time series: Statistical aspects and perspectives. *Studies in Nonlinear Dynamics and Econometrics*, *8*(2). https://doi.org/10.2202/1558-3708.1215

Lee, Y.-J., Zhu, Y.-S., Xu, Y.-H., Shen, M.-F., Zhang, H.-X., & Thakor, N. (2001). Detection of Nonlinearity in the EEG of Schizophrenic Patients. *Clinical Neurophysiology*, *112*(7), 1288–1294. https://doi.org/10.1016/s1388-2457(01)00544-2

McCaffrey, D. F., Ellner, S., Gallant, A. R., & Nychka, D. W. (1992). Estimating the Lyapunov Exponent of a Chaotic System With Nonparametric Regression. *Journal of the American Statistical Association*, *87*(419), 682. https://doi.org/10.2307/2290206

Müller, V., Lutzenberger, W., Pulvermüller, F., Mohr, B., & Birbaumer, N. (2001). Investigation of Brain Dynamics in Parkinson's Disease by Methods Derived From Nonlinear Dynamics. *Experimental Brain Research*, *137*(1), 103–110. https://doi.org/10.1007/s002210000638

Narzo, A. F. D. (2019). *Tserieschaos: Analysis of nonlinear time series* [R package version 0.1-13.1]. https://doi.org/10.32614/CRAN.package.tseriesChaos

Palus, M. (1996). Nonlinearity in normal human eeg: Cycles, temporal asymmetry, nonstationarity and randomness, not chaos. *Biological cybernetics*, *75*, 389–96. https://doi.org/10.1007/s004220050304

Prichard, D., & Theiler, J. (1994). Generating Surrogate Data for Time Series With Several Simultaneously Measured Variables. *Physical Review Letters*, *73*(7), 951–954. https://doi.org/10.1103/physrevlett.73.951

Shintani, M., & Linton, O. (2004). Nonparametric Neural Network Estimation of Lyapunov Exponents and a Direct Test for Chaos. *Journal of Econometrics*, *120*(1), 1–33. https://doi.org/10.1016/s0304-4076(03)00205-7