

# Computer Systems

## Exercises on Models with One Job Class

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**SKIP THE MVA SECTIONS**

# Open models

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## Recap - Performance bounds

$$X(\lambda) \leq \frac{1}{D_{\max}} \quad D \leq R(\lambda)$$

↓

$$X(\lambda) \leq \frac{1}{D_{\max}} \quad \frac{D}{1-\lambda D_{\text{avg}}} \leq R(\lambda) \leq \frac{D}{1-\lambda D_{\max}}$$

# Open Model Solution Technique

Processing capacity:  $\lambda_{sat} = \frac{1}{D_{max}}$ . We assume  $\lambda < \lambda_{sat}$

Throughput:  $X(\lambda) = \lambda$

Utilization:  $U_k(\lambda) = \lambda D_k$

Residence time:  $R_k(\lambda) = \begin{cases} D_k, & \text{for delay centers} \\ \frac{D_k}{1-U_k(\lambda)}, & \text{for queueing centers} \end{cases}$

Queue length:  $Q_k(\lambda) = \lambda R_k(\lambda) = \begin{cases} U_k(\lambda), & \text{delay} \\ \frac{U_k(\lambda)}{1-U_k(\lambda)}, & \text{queueing} \end{cases}$

System response time:  $R(\lambda) = \sum_{k=1}^K R_k(\lambda)$

Avg num. in sys.:  $Q(\lambda) = \lambda R(\lambda) = \sum_{k=1}^K Q_k(\lambda)$

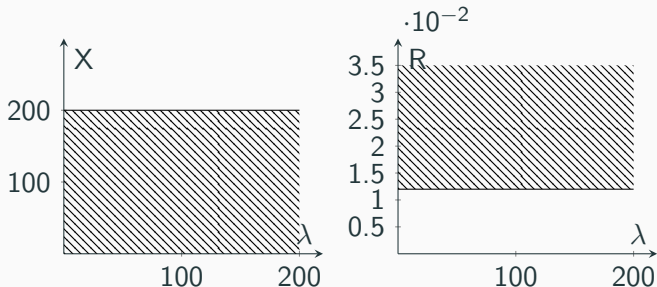
## Exercise 1

Consider an open system composed by a web server with  $D_{ws} = 5ms$ , an application server with  $D_{as} = 4ms$  and a database with  $D_{db} = 3ms$ .

Compute and draw throughput and response time bounds, then the tighter ones and compare them with the exact solution.

## Solution - performance bounds

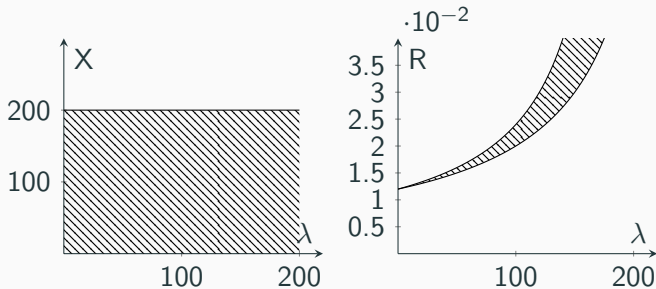
$$X(\lambda) \leq \frac{1}{D_{\max}} = \frac{1}{0.005} = 200 \quad 0.012 \leq R(\lambda)$$



Nearly useless bounds.

## Solution - Balanced system bounds

$$X(\lambda) \leq \frac{1}{D_{\max}} = \frac{1}{0.005} = 200 \quad \frac{0.012}{1-\lambda 0.004} \leq R(\lambda) \leq \frac{0.012}{1-\lambda 0.005}$$



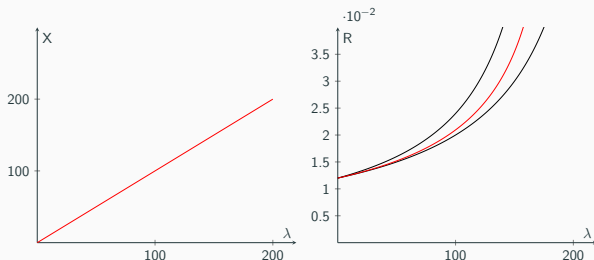


## Solution - Open model solution

$$\lambda < \lambda_{sat} \quad X(\lambda) = \lambda$$

$$R(\lambda) = \sum R_k(\lambda) \quad R_k = \frac{D_k}{1 - U_k(\lambda)} \quad U_k(\lambda) = \lambda D_k$$

$$R(\lambda) = \frac{0.005}{1 - \lambda 0.005} + \frac{0.004}{1 - \lambda 0.004} + \frac{0.003}{1 - \lambda 0.003}$$



## Exercise 2

$$\lambda = 1 \frac{\text{jobs}}{\text{sec}}$$

$$V_{cpu} = 200 \quad V_{disk1} = 50 \quad V_{disk2} = 30$$

$$S_{cpu} = 1ms \quad S_{disk1} = 10ms \quad S_{disk2} = 12ms$$

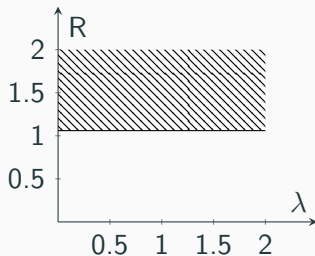
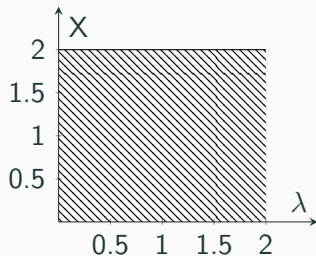
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$$D_{cpu} = 0.2 \quad D_{disk1} = 0.5 \quad D_{disk2} = 0.36$$

$$D_{max} = 0.5 \quad D = 1.06 \quad D_{avg} = 0.35$$

## Solution - performance bounds

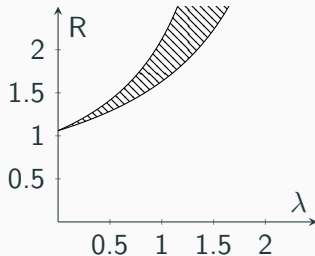
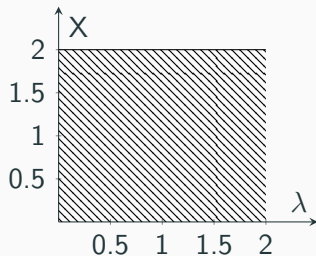
$$X(\lambda) \leq \frac{1}{D_{\max}} = \frac{1}{0.5} = 2 \quad 1.06 \leq R(\lambda)$$



## Solution - Balanced system bounds

$$X(\lambda) \leq \frac{1}{D_{\max}} = \frac{1}{0.5} = 2$$

$$\frac{1.06}{1-\lambda 0.35} \leq R(\lambda) \leq \frac{1.06}{1-\lambda 0.5}$$



## Some outputs of the model

$$\lambda = 1.5 \frac{\text{jobs}}{\text{sec}}$$

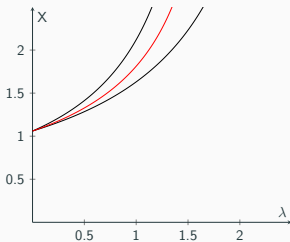
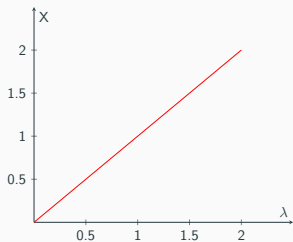
- $X_{cpu}(1.5) = \lambda V_{cpu} = 1.5200 = 300$
- $U_{cpu}(1.5) = \lambda D_{cpu} = 1.50.2 = 0.3$
- $R_{cpu}(1.5) = \frac{D_{cpu}}{1 - U_{cpu}(1.5)} = \frac{0.2}{0.7} = 0.286\text{sec}$
- $Q_{cpu}(1.5) = \frac{U_{cpu}(1.5)}{1 - U_{cpu}(1.5)} = \frac{0.3}{0.7} = 0.428\text{jobs}$
- $R(1.5) = R_{cpu}(1.5) + R_{disk1}(1.5) + R_{disk2}(1.5) = 3.068\text{sec}$
- $Q(1.5) = \lambda R(1.5) = 4.602\text{jobs}$

## Solution - Open model solution

$$\lambda < \lambda_{sat} \quad X(\lambda) = \lambda$$

$$R(\lambda) = \sum R_k(\lambda) \quad R_k = \frac{D_k}{1 - U_k(\lambda)} \quad U_k(\lambda) = \lambda D_k$$

$$R(\lambda) = \frac{0.2}{1 - \lambda 0.2} + \frac{0.5}{1 - \lambda 0.5} + \frac{0.36}{1 - \lambda 0.36}$$



## Closed models - batch workload

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## Recap - Performance bounds

$$\frac{1}{D} \leq X(N) \leq \min\left(\frac{N}{D}, \frac{1}{D_{\max}}\right) \quad \max(D, ND_{\max}) \leq R(N) \leq ND$$

↓

$$\frac{N}{D+(N-1)D_{\max}} \leq X(N) \leq \min\left(\frac{1}{D_{\max}}, \frac{N}{D+(N-1)D_{\text{avg}}}\right)$$

$$\max(ND_{\max}, D + (N-1)D_{\text{avg}}) \leq R(N) \leq D + (N-1)D_{\max}$$

$$N^* = D/D_{\max} \quad N^+ = \frac{D-D_{\text{avg}}}{D_{\max}-D_{\text{avg}}}$$

The throughput must be computed, the response time is bounded by lines passing through  $(1, D)$  and  $(0, D - D_{\text{avg}})$   $(0, D - D_{\max})$



## Exact MVA Solution technique

```
""" Compute exact MVA:
```

```
K (int) number of service center
```

```
N (int) maximum number of users to use
```

```
D (array of float) service demands
```

```
delay (array of bool) service_k is delay center?
```

```
Z (float) think time
```

```
"""
```

```
Q = np.zeros(K)
```

```
for n in range(1, N+1):
```

```
    R_d = D
```

```
    R_q = D * (1 + Q)
```

```
    R = np.where(delay, R_d, R_q)
```

```
    X = n / (Z + np.sum(R))
```

```
    Q = X * R
```

```
    print(n, X, np.sum(R), sep=', ')
```

## Approximate MVA Solution technique

```
""" Compute approximate MVA:
K (int) number of service center
N (int) number of users
D (array of float) service demands
delay (array of bool) service_k is delay center?
Z (float) think time
rep (int) iterations of the algorithm
"""
```

```
Q = N / K
for r in range(rep):
    A = (N-1) / N * Q
    R_d = D
    R_q = D * (1+A)
    R = np.where(delay , R_d , R_q)
    X = N / (Z + np.sum(R))
    print(r, Q, A, R, np.sum(R), X)
    Q = X * R
```

## Exercise 3

Consider a batch system composed by a web server with  $D_{ws} = 5ms$ , an application server with  $D_{as} = 4ms$  and a database with  $D_{db} = 3ms$ .

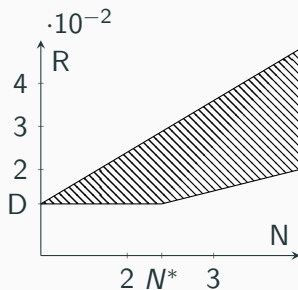
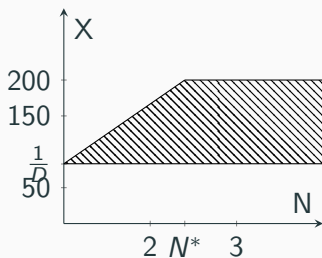
Compute and draw throughput and response time bounds, then the tighter ones and compare them with the exact solution. Then use approximate MVA to compute an estimate of  $X$  and  $R$  for  $N=33000$ .

## Solution - performance bounds

$$\frac{1}{0.012} \leq X(n) \leq \min\left(\frac{N}{0.012}, \frac{1}{0.005}\right)$$

$$\max(0.012, N \cdot 0.005) \leq R(N) \leq N \cdot 0.012$$

$$N^* = \frac{0.012}{0.005} = 2.4$$

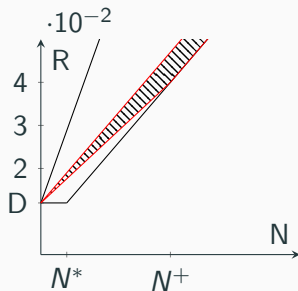
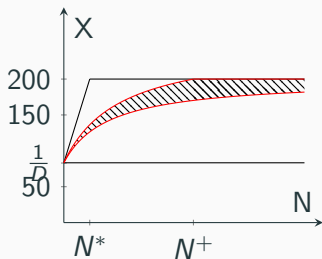


## Solution - Balanced system bounds

$$\frac{N}{0.012 + (N-1)0.005} \leq X(N) \leq \min\left(\frac{1}{0.005}, \frac{N}{D + (N-1)0.004}\right)$$

$$\max(N0.005, D + (N-1)0.004) \leq R(N) \leq D + (N-1)0.005$$

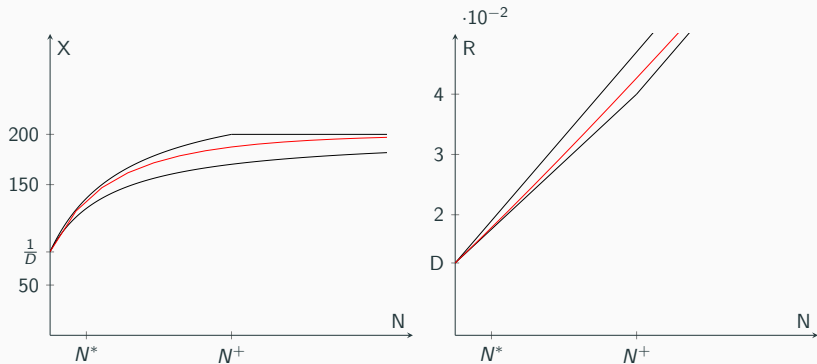
$$N^+ = \frac{0.012 - 0.004}{0.005 - 0.004} = 8$$



Easy to draw.

## Solution - Exact MVA

Use the python code to get  $N$ ,  $X$ ,  $R$ .



## Solution - Approximate MVA

From exact MVA we know that  $X(33000) = 200$  and  $R(33000) = 165$

Using 20 iterations of python code for approximate MVA we get:  
 $X(33000) = 200$  and  $R(33000) = 164$

This is a good estimate and requires much less time to be computed.

## Closed models - terminal workloads

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## Recap - Performance bounds

$$\frac{N}{ND+Z} \leq X(N) \leq \min\left(\frac{N}{D+Z}, \frac{1}{D_{\max}}\right)$$
$$\max(D, ND_{\max} - Z) \leq R(N) \leq ND$$

↓

$$\frac{N}{D+Z+\frac{(N-1)D_{\max}}{1+Z/(ND)}} \leq X(N) \leq \min\left(\frac{1}{D_{\max}}, \frac{N}{D+Z+\frac{(N-1)D_{\max}}{1+Z/D}}\right)$$
$$\max(ND_{\max} - Z, D + \frac{(N-1)D_{\max}}{1+Z/D}) \leq R(N) \leq D + \frac{(N-1)D_{\max}}{1+Z/(ND)}$$
$$N^* = \frac{D+Z}{D_{\max}} \quad N^+ = \frac{(D+Z)^2 - D \cdot D_{\max}}{(D+Z)D_{\max} - D \cdot D_{\max}}$$

The throughput and the pessimistic bound on response time must be computed. The optimistic bound on response time is a line through the points:  $(0, D - \frac{D_{\max}}{1+Z/D})$  and  $(1, D)$ .

The exact and approximate solutions can be computed as in the previous case.

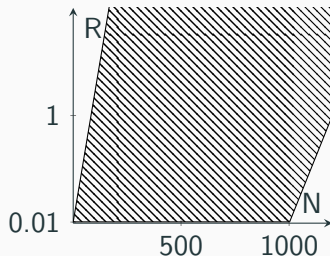
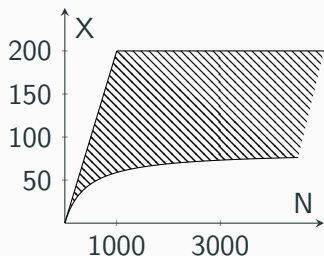
## Exercise 4

Consider a terminal system composed by a web server with  $D_{ws} = 5ms$ , an application server with  $D_{as} = 4ms$  and a database with  $D_{db} = 3ms$ . The sleep time is  $Z = 5s$ .

Compute and draw throughput and response time bounds, then the tighter ones and compare them with the exact solution. Then use approximate MVA to compute an estimate of  $X$  and  $R$  for  $N=750$ .

## Solution - performance bounds

$$\frac{N}{0.012 \cdot N + 5} \leq X(n) \leq \min\left(\frac{N}{5.012}, \frac{1}{0.005}\right)$$
$$\max(0.012, N \cdot 0.005 - 5) \leq R(N) \leq N \cdot 0.012$$
$$N^* = \frac{5.012}{0.005} = 1002.4$$

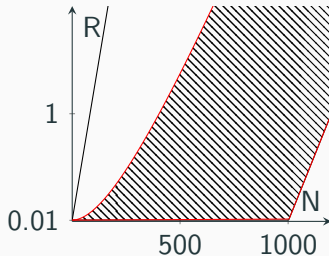
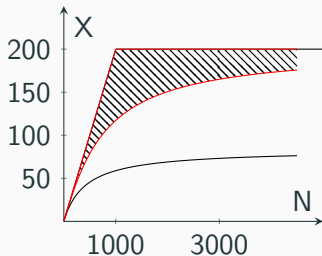


## Solution - Balanced system bounds

$$\frac{N}{5.012 + \frac{(N-1)0.005}{1+5/0.012N}} \leq X(N) \leq \min\left(\frac{1}{0.005}, \frac{N}{5.012 + \frac{(N-1)0.004}{1+5/0.012}}\right)$$

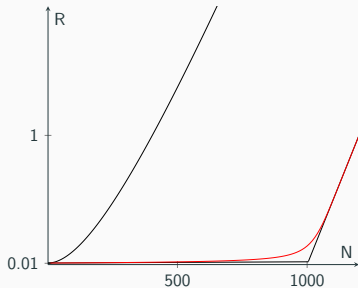
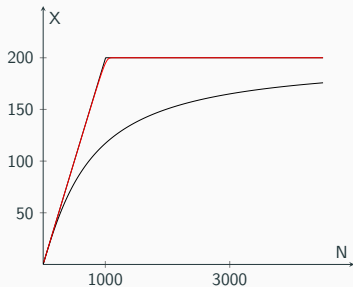
$$\max(N0.005 - 5, D + \frac{(N-1)0.004}{1+5/0.012}) \leq R(N) \leq D + \frac{(N-1)0.005}{1+5/0.012N}$$

$$N^+ = \frac{5.012^2 - 0.012 \cdot 0.004}{5.012 \cdot 0.005 - 0.012 \cdot 0.004} = \frac{25.12}{0.025} = 1004.8$$



## Solution - Exact MVA

Use the python code to get N, X, R.



## Solution - Approximate MVA

From exact MVA we know that  $X(750) = 148.97$  and  $R(750) = 0.035$

Using 15 iterations of python code for approximate MVA we get:  
 $X(750) = 148.75$  and  $R(750) = 0.042$