Computer Systems

Exercises on Models with One Job Class

Stefano Cereda
stefano.cereda@polimi.it

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Politecnico di Milano

Disclaimer

SKIP THE MVA SECTIONS

Open models

Recap - Performance bounds

$$X(\lambda) \le \frac{1}{D_{max}} \qquad D \le R(\lambda)$$

$$\downarrow$$

$$X(\lambda) \le \frac{1}{D_{max}} \qquad \frac{D}{1-\lambda D_{avg}} \le R(\lambda) \le \frac{D}{1-\lambda D_{max}}$$

Open Model Solution Technique

Processing capacity: $\lambda_{sat} = \frac{1}{D_{max}}$. We assume $\lambda < \lambda_{sat}$

Throughput: $X(\lambda) = \lambda$

Utilization: $U_k(\lambda) = \lambda D_k$

Residence time: $R_k(\lambda) = \begin{cases} D_k, & \text{for delay centers} \\ \frac{D_k}{1-U_k(\lambda)}, & \text{for queueing centers} \end{cases}$

Queue length: $Q_k(\lambda) = \lambda R_k(\lambda) = \begin{cases} U_k(\lambda), & \text{delay} \\ \frac{U_k(\lambda)}{1-U_K(\lambda)}, & \text{queueing} \end{cases}$

System response time: $R(\lambda) = \sum_{k=1}^{K} R_k(\lambda)$

Avg num. in sys.: $Q(\lambda) = \lambda R(\lambda) = \sum_{k=1}^{K} Q_k(\lambda)$

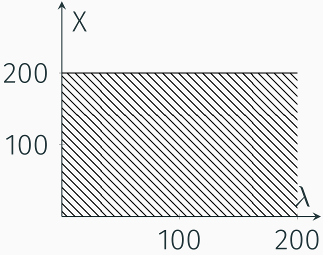
Exercise 1

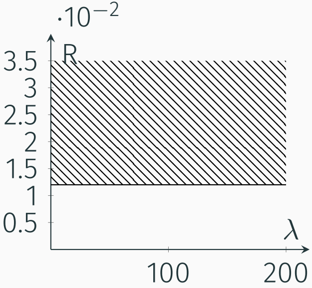
Consider an open system composed by a web server with $D_{ws} = 5ms$, an application server with $D_{as} = 4ms$ and a database with $D_{db} = 3ms$.

Compute and draw throughput and response time bounds, then the tighter ones and compare them with the exact solution.

Solution - performance bounds

$X(\lambda) \leq \frac{1}{D_{max}} = \frac{1}{0.005} = 200 \qquad 0.012 \leq R(\lambda)$

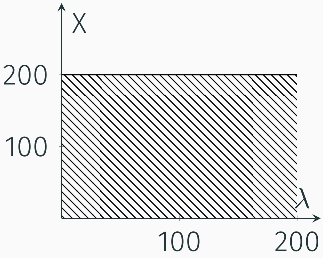


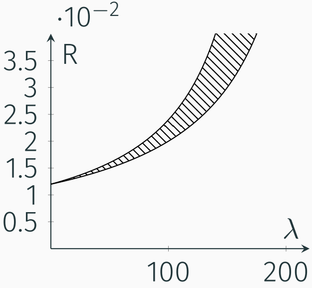


Nearly useless bounds.

Solution - Balanced system bounds

$$X(\lambda) \leq \frac{1}{D_{max}} = \frac{1}{0.005} = 200 \qquad \frac{0.012}{1-\lambda 0.004} \leq R(\lambda) \leq \frac{0.012}{1-\lambda 0.005}$$



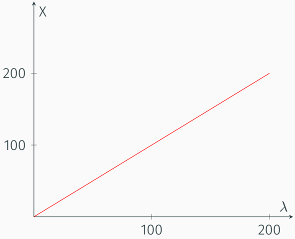


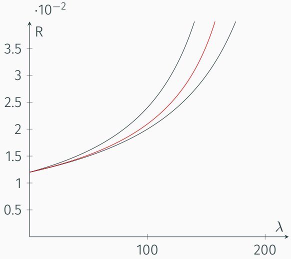
Solution - Open model solution

$\lambda < \lambda_{sat} \qquad X(\lambda) = \lambda$

$R(\lambda) = \sum R_k(\lambda) \qquad R_k = \frac{D_k}{1-U_k(\lambda)} \qquad U_k(\lambda) = \lambda D_k$

$R(\lambda) = \frac{0.005}{1-\lambda 0.005} + \frac{0.004}{1-\lambda 0.004} + \frac{0.003}{1-\lambda 0.003}$





Exercise 2

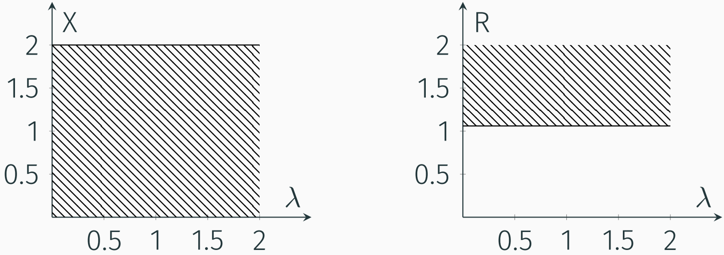
$\lambda = 1\frac{jobs}{sec}$

$$\begin{array}{ccc} V_{cpu} = 200 & V_{disk1} = 50 & V_{disk2} = 30 \\ S_{cpu} = 1ms & S_{disk1} = 10ms & S_{disk2} = 12ms \\ \hline D_{cpu} = 0.2 & D_{disk1} = 0.5 & D_{disk2} = 0.36 \end{array}$$

$D_{max} = 0.5 \qquad D = 1.06 \qquad D_{avg} = 0.35$

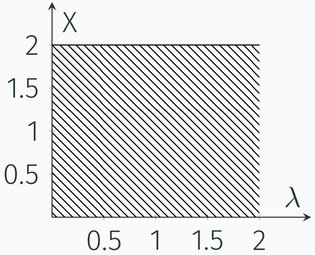
Solution - performance bounds

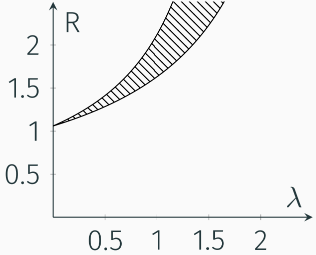
$X(\lambda) \leq \frac{1}{D_{max}} = \frac{1}{0.5} = 2 \qquad 1.06 \leq R(\lambda)$



Solution - Balanced system bounds

$$X(\lambda) \leq \frac{1}{D_{max}} = \frac{1}{0.5} = 2 \qquad \frac{1.06}{1-\lambda 0.35} \leq R(\lambda) \leq \frac{1.06}{1-\lambda 0.5}$$





Some outputs of the model

$\lambda = 1.5\frac{jobs}{sec}$

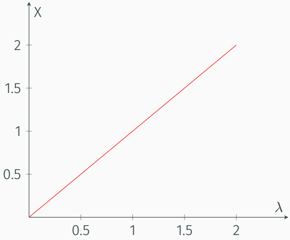
- $X_{cpu}(1.5) = \lambda V_{cpu} = 1.5200 = 300$
- $U_{cpu}(1.5) = \lambda D_{cpu} = 1.50.2 = 0.3$
- $R_{cpu}(1.5) = \frac{D_{cpu}}{1-U_{cpu(1.5)}} = \frac{0.2}{0.7} = 0.286sec$
- $Q_{cpu}(1.5) = \frac{U_{cpu}(1.5)}{1-U_{cpu(1.5)}} = \frac{0.3}{0.7} = 0.428jobs$
- $R(1.5) = R_{cpu}(1.5) + R_{disk1}(1.5) + R_{disk2}(1.5) = 3.068sec$
- $Q(1.5) = \lambda R(1.5) = 4.602jobs$

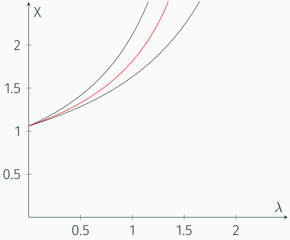
Solution - Open model solution

$\lambda < \lambda_{sat} \qquad X(\lambda) = \lambda$

$R(\lambda) = \sum R_k(\lambda) \qquad R_k = \frac{D_k}{1 - U_k(\lambda)} \qquad U_k(\lambda) = \lambda D_k$

$R(\lambda) = \frac{0.2}{1 - \lambda 0.2} + \frac{0.5}{1 - \lambda 0.5} + \frac{0.36}{1 - \lambda 0.36}$





Closed models - batch workload

Recap - Performance bounds

$$\frac{1}{D} \leq X(N) \leq \min\left(\frac{N}{D}, \frac{1}{D_{max}}\right) \qquad \max\left(D, ND_{max}\right) \leq R(N) \leq ND$$

$$\downarrow$$

$$\frac{N}{D+(N-1)D_{max}} \leq X(N) \leq \min\left(\frac{1}{D_{max}}, \frac{N}{D+(N-1)D_{avg}}\right)$$

$$\max(ND_{max}, D + (N-1)D_{avg}) \leq R(N) \leq D + (N-1)D_{max}$$

$$N^* = D/D_{max} \qquad N^+ = \frac{D-D_{avg}}{D_{max}-D_{avg}}$$

The throughput must be computed, the response time is bounded by lines passing through $(1, D)$ and $(0, D - D_{avg})(0, D - D_{max})$

Exercise 3

Consider a batch system composed by a web server with $D_{ws} = 5ms$, an application server with $D_{as} = 4ms$ and a database with $D_{db} = 3ms$.

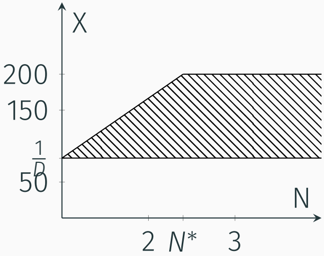
Compute and draw throughput and response time bounds, then the tighter ones and compare them with the exact solution. Then use approximate MVA to compute an estimate of X and R for N=33000.

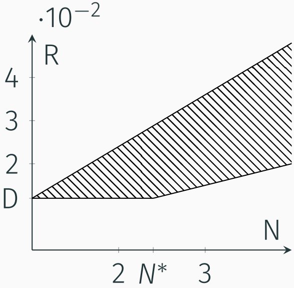
Solution - performance bounds

$\frac{1}{0.012} \leq X(n) \leq \min(\frac{N}{0.012}, \frac{1}{0.005})$

$\max(0.012, N \cdot 0.005) \leq R(N) \leq N \cdot 0.012$

$N^* = \frac{0.012}{0.005} = 2.4$



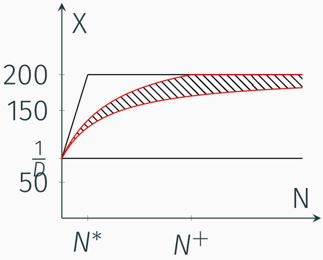


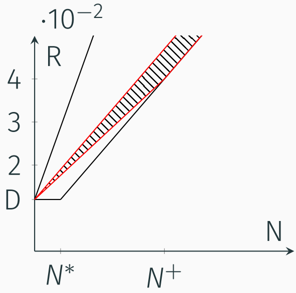
Solution - Balanced system bounds

$$\frac{N}{0.012+(N-1)0.005} \leq X(N) \leq \min(\frac{1}{0.005}, \frac{N}{D+(N-1)0.004})$$

$$\max(N0.005, D+(N-1)0.004) \leq R(N) \leq D+(N-1)0.005$$

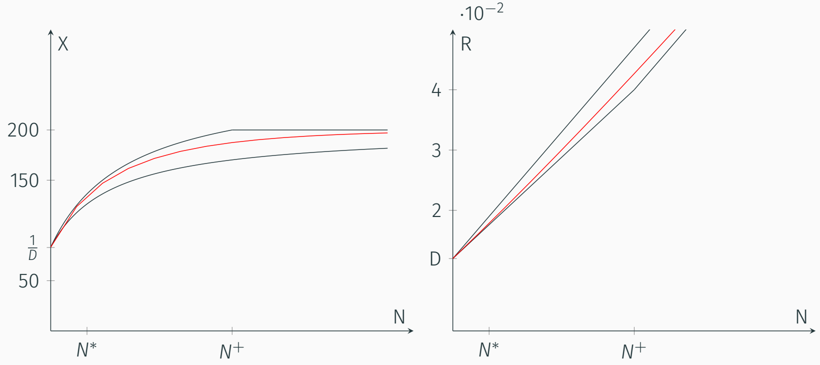
$$N^{+} = \frac{0.012-0.004}{0.005-0.004} = 8$$





Solution - Exact MVA

Use the python code to get N, X, R.



Solution - Approximate MVA

From exact MVA we know that $X(33000) = 200$ and $R(33000) = 165$

Using 20 iterations of python code for approximate MVA we get: $X(33000) = 200$ and $R(33000) = 164$

This is a good estimate and requires much less time to be computed.

Closed models - terminal workloads

Recap - Performance bounds

$$\frac{N}{ND+Z} \le X(N) \le \min\left(\frac{N}{D+Z}, \frac{1}{D_{max}}\right)$$
$$\max\left(D, ND_{max} - Z\right) \le R(N) \le ND$$

$$\downarrow$$

$$\frac{N}{D+Z+\frac{(N-1)D_{max}}{1+Z/(ND)}} \le X(N) \le \min\left(\frac{1}{D_{max}}, \frac{N}{D+Z+\frac{(N-1)D_{avg}}{1+Z/D}}\right)$$

$$\max\left(ND_{max} - Z, D + \frac{(N-1)D_{avg}}{1+Z/D}\right) \le R(N) \le D + \frac{(N-1)D_{max}}{1+Z/(ND)}$$

$$N^* = \frac{D+Z}{D_{max}} \qquad N^+ = \frac{(D+Z)^2 - D \cdot D_{avg}}{(D+Z)D_{max} - D \cdot D_{avg}}$$

The throughput and the pessimistic bound on response time must be computed.The optimistic bound on response time is a line through the points: $(0, D - \frac{D_{avg}}{1+Z/D})$ and $(1, D)$.

The exact and approximate solutions can be computed as in the previous case.

Exercise 4

Consider a terminal system composed by a web server with $D_{ws} = 5ms$, an application server with $D_{as} = 4ms$ and a database with $D_{db} = 3ms$. The sleep time is $Z = 5s$.

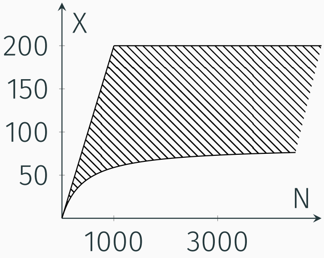
Compute and draw throughput and response time bounds, then the tighter ones and compare them with the exact solution. Then use approximate MVA to compute an estimate of X and R for N=750.

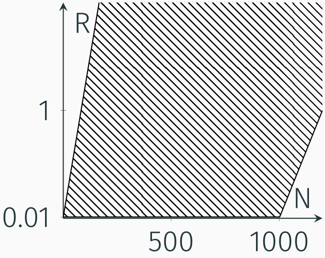
Solution - performance bounds

$\frac{N}{0.012 \cdot N+5} \leq X(n) \leq \min(\frac{N}{5.012}, \frac{1}{0.005})$

$\max(0.012, N \cdot 0.005 - 5) \leq R(N) \leq N \cdot 0.012$

$N^* = \frac{5.012}{0.005} = 1002.4$



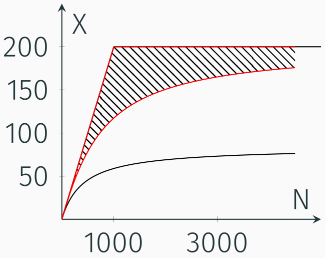


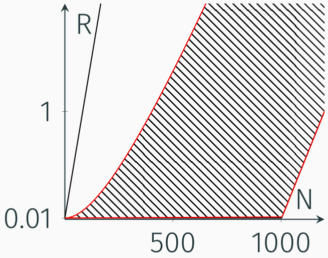
Solution - Balanced system bounds

$$\frac{N}{5.012+\frac{(N-1)0.005}{1+5/0.012N}} \leq X(N) \leq \min(\frac{1}{0.005}, \frac{N}{5.012+\frac{(N-1)0.004}{1+5/0.012}})$$

$$\max(N0.005-5, D+\frac{(N-1)0.004}{1+5/0.012}) \leq R(N) \leq D+\frac{(N-1)0.005}{1+5/0.012N}$$

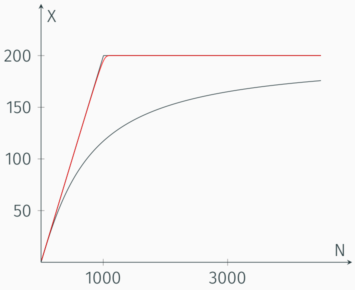
$$N^{+} = \frac{5.012^2-0.012*0.004}{5.012*0.005-0.012*0.004} = \frac{25.12}{0.025} = 1004.8$$

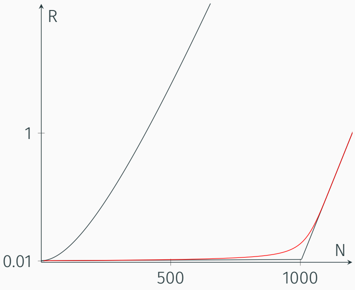




Solution - Exact MVA

Use the python code to get N, X, R.





Solution - Approximate MVA

From exact MVA we know that $X(750) = 148.97$ and $R(750) = 0.035$

Using 15 iterations of python code for approximate MVA we get: $X(750) = 148.75$ and $R(750) = 0.042$