COMPUTING INFRASTRUCTURES

EXERCISES ON QUEUING NETWORK MODEL INPUTS AND OUTPUTS

Stefano Cereda

stefano.cereda@polimi.it 

13/05/2019

Politecnico di Milano

Tomorrow’s lesson will start at 12.15

RECAP - INPUTS

- Customers are described with *workload intensity*:
  - the *arrival rate* λ for *transaction* workloads
  - the *population* N for *batch* workloads
  - N and the *think time* Z for *terminal* workloads
- Each service k is described with its type:
  - Queuing
  - Delay
- The demand at each service is described with its *service demand*: $D_k = V_k S_k$

- System Measures:
  - average system response time: R
  - system throughput: X
  - average number in system: Q
- Center Measures:
  - utilization of center k: U_k
  - average residence time at center k: R_k
  - throughput of center k: X_k
  - average queue length of center k: Q_k

EXERCISE 1

By monitoring a single class interactive system, we are able to measure the following data:

- Disk demand: 4 seconds/transaction
- CPU demand: 1 seconds/transaction
- CPU utilization: 80%
- Response time: 20 seconds/transaction
- Monitoring period: 6 minutes
- Number of users: 16

Which is the average think time of these users?

$$D_d = 4\frac{sec}{tr} \quad D_c = 1\frac{sec}{tr} \quad U_c = 0.8$$

$$R = 20\frac{sec}{tr} \quad T = 6' \quad N = 16 \quad Z = ?$$

$$Z = \frac{N}{X} - R \quad X?$$

$$X = \frac{U_c}{D_c} = \frac{0.8}{1\frac{sec}{tr}} = 0.8\frac{tr}{sec}$$

$$Z = \frac{16}{0.8\frac{tr}{sec}} - 20\frac{sec}{tr} = 0\frac{sec}{tr}$$

Consider the following measurement data for an interactive system:

- Measurement interval: 2 minutes
- number of users: 15
- average response time per transaction: 10 seconds
- Disk demand: 0.5 seconds/transaction
- CPU demand: 0.2 seconds/transaction
- Number of servers: 20
- Number of completed transactions: 60

On average, how many users are thinking?

SOLUTION

$$N = N_{think} + N_{insystem} = N' + N'' \qquad N' = ?$$

$$N' = N - N''$$

$$N'' = X \cdot R$$

$$X = \frac{C}{T} = 0.5 \frac{tr}{sec}$$

$$N'' = 0.5 \cdot 10 = 5$$

$$N' = 15 - 5 = 10$$

EXERCISE 4

In a batch system, a specific disk is performing, on average, 12 operation per second. We know that each batch transaction requires, on average, 6 accesses to this disk. Another disk in the system is handling 18 operations per second. Which is the average number of accesses to this second disk require by every batch transaction?

$$X_{D1} = 12 \quad V_{D1} = 6 \quad X_{D2} = 18$$

$$V_{D2} = \frac{X_{D2}}{X} = \frac{X_{D2}}{\frac{X_{D1}}{V_{D1}}} = \frac{18}{\frac{12}{6}} = 9$$

The storage server of an intranet consists of two groups of disks, A and B, each having service times with means S_A = 5ms and S_B = 3ms. The mean number of visits for the two components are V_A = 20 and V_B = 30. The throughput of A is 150 op./sec. The above data were collected when the system is processing a workload generated by 300 users with think time Z = 15 sec.

1. Compute the System Throughput X and the Utilization of B.
2. Compute the system response time.
3. If the number of users increases to 400, which will be the new response time?

SOLUTION 1

$$X = \frac{U_A}{D_A}$$

$$U_A = X_A \cdot S_A = 5\frac{ms}{op} \cdot 150\frac{op}{sec} = 75\%$$

$$D_A = V_A \cdot S_A = 20 \cdot 5ms = 100ms$$

$$X = \frac{0.75}{0.1} = 7.5\frac{op}{sec}$$

$$U_B = X \cdot D_B = X \cdot (S_B \cdot V_B) = 7.5 \cdot (3ms \cdot 30) = 7.5 \cdot 0.09\frac{sec}{op} = 67.5\%$$

$$R = \frac{N}{X} - Z = \frac{300op}{7.5\frac{op}{sec}} - 15sec = 25sec$$

We cannot say anything for $N = 400$

The throughput of a disk is 100 I/O operations per second. To complete a given request 20 visits to the disk are required. The number of users is 100 and the response time is 15 seconds.

Compute the users think time.

SOLUTION

$$Z = \frac{N}{X} - R$$

$$X = \frac{X_D}{V_D} = 5\frac{op}{sec}$$

$$Z = \frac{100}{5} - 15 = 5sec$$

A web server of a company is connected to an intranet and is accessed by the employees that work internally in the company resulting in a population of fixed size: N=21 users. The average think time of the users is Z=20 sec. A complete execution of a request generates a load of $V_S = 20$ operations to a specific storage device whose utilization is $U_S = 0.30$. The service time of the storage device per each visit is $S_S = 0.025$ sec.

1. Determine the average system response time R
2. Compute the average throughput and system response time with N=40 users.

$$R = \frac{N}{X} - Z$$

$$X = \frac{U_S}{D_S} = \frac{0.3}{V_S} S_S = \frac{0.3}{20 \cdot 0.025 sec} = 0.6 \frac{op}{sec}$$

$$R = \frac{21}{0.6} - 20 = 15 sec$$

We cannot say anything for $N = 40$

On a system with 3 disks, the following measurements were made:

- throughput of each disk is 40 request/seconds ($X_{disk} = 40r/s$)
- 4 customers, on average, are either in queue or in service on each disk ($N_{disk+queue} = 4$)
- average think time is 5 seconds ($Z = 5s$)
- the service time on each disk is 0.0225 seconds ($S_{disk} = 0.0225s$)

1. calculate the utilization of one disk without considering its queue.

2. calculate the average time spent by each request in each disk, both in queue and in service.
3. calculate the average number of users in service and the average number of users awaiting service.
4. calculate the number of interactions per second, considering that the system-level average response time is 15 seconds and there are 7.5 active (i.e., non-thinking) users.
5. calculate the total number of users.

D indicates just the disk, Q is the queue and $D+Q$ the disk with its queue. Notice that $X_D = X_Q$ and that when we look at the disk+queue subsystem we have $R_D = S_D$.

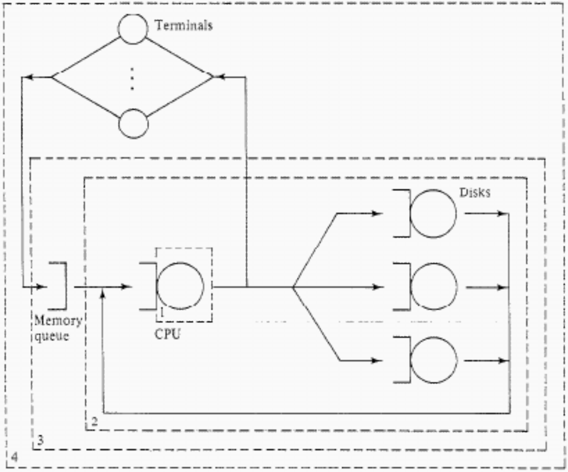
$$U_D = X_D \cdot S_D = 90\%$$
$$R_{D+Q} = \frac{N_D}{X_D} = \frac{4}{40} = 0.1sec$$

$$N_Q = X_D \cdot R_Q = X_D \cdot (R_{D+Q} - R_D) = 3.1$$
$$N_D = X_D \cdot R_D = 0.9 = U_D$$

$$X = \frac{N_{active}}{R} = \frac{7.5}{15} = 0.5\frac{int}{sec}$$
$$N = X(R+Z) = 0.5(15+5) = 10$$

On the system depicted below, the following measurements were made:

- average number of users is 23
- average response time as perceived by a user is 30 seconds
- throughput is 0.45 interactions/second
- average number of requests in memory (box 2) is 1.9
- average CPU service demand per interaction is 0.63 seconds


Terminals
Disks
Memory queue
1
CPU
2
3
4

1. What is the average think time of a user?
2. how many users are attempting to obtain service and how many of them are instead thinking?
3. how much time elapses between the acquisition of memory and the completion of an interaction?
4. what is the utilization of the CPU?

$$N = 23 \quad R = 30sec \quad X = 0.45\frac{int}{sec} \quad N_{mem} = 1.9 \quad D_{cpu} = 0.63sec$$

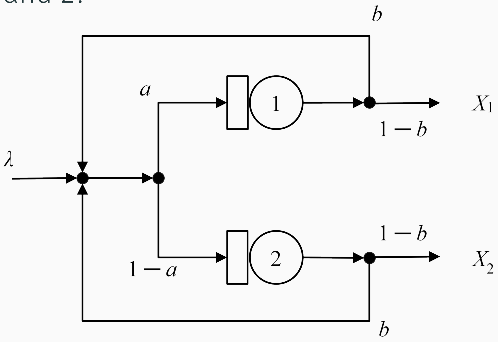
$Z =$? in box 4 $Z = \frac{N}{X} - R = \frac{23}{0.45} - 30 = 21sec$

N_{think}? $\quad N_{queue}$?
$N_q = NnotThink - N_{mem} = X \cdot R - 1.9 = 0.45 \cdot 30 - 1.9 = 11.6$
$N_{think} = N - N_{notThink} = N - 13.5 = 9.5$

$? = R_{box2} = \frac{N_{mem}}{X} = \frac{1.9}{0.45} = 4.2sec >> (R - 4.2)$ most time is spent in queue.

$U_{cpu} = X \cdot D_{cpu} = 0.45 \cdot 0.63 = 0.28 << 1$ the cpu has a very low usage as the system is limited by the memory queue.

Consider the following open network, where a and b are routing probabilities. Which is the number of visits at station 1 and 2?



$$V_1 = a(1 + bV_1 + bV2) = a(1 + b(V_1 + V_2))$$

$$V_2 = (1 - a)(1 + bV - 1 + bV_2) \ = (1 - a)(1 + b(V_1 + V_2))$$

$$\frac{V_2}{V_1} = \frac{1 - a}{a}$$

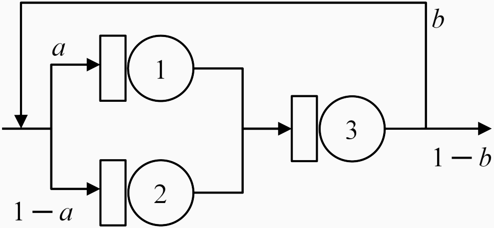
$$V_2 = V_1 \frac{1 - a}{a} = \frac{V_1}{a} - \frac{aV_1}{a}$$

$$V_1 = a(1 + b(V_1 + \frac{V1}{a} - V_1)) = a + \frac{abV_1}{a}$$

$$V_1 = \frac{a}{1 - b}$$

$$V_2 = \frac{V_1}{a} - V_1 = \frac{1}{1 - b} - \frac{a}{1 - b} = \frac{1 - a}{1 - b}$$

Consider the following open network, where a and b are routing probabilities. Which is the number of visits at station 1, 2 and 3?



SOLUTION

$$V_1 = a(bV_3 + 1)$$

$$V_2 = (1 - a)(bV_3 + 1)$$

$$V_3 = V1 + V2 = 1 + bV_3 = \frac{1}{1 - b}$$

$$V_1 = \frac{ab}{1 - b} + a = \frac{a - ab + ab}{1 - b} = \frac{a}{1 - b}$$

$$V_2 = \frac{(1 - a)b}{1 - b} + \frac{(1 - a)(1 - b)}{1 - b} = \frac{b - ab + 1 - a - b + ab}{1 - b} = \frac{1 - a}{1 - b}$$