Introduction to Bayesian linear regression with brms — Part II: Bayesian Inference

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Road map

1. Statistical inference.

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2. Frequentist inference.

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2. Frequentist inference.
3. Bayesian inference:

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3. Bayesian inference:
    3.1 Posterior distributions.

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    3.2 ROPE.

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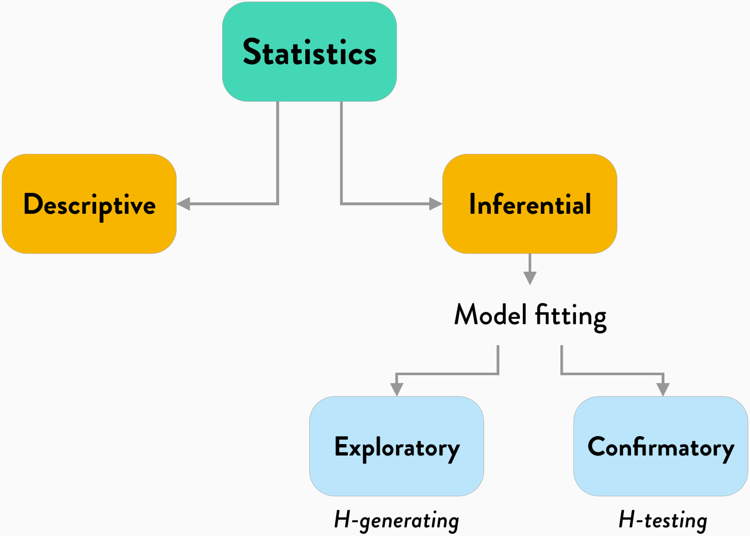
3. Bayesian inference:
    3.1 Posterior distributions.
    3.2 ROPE.
    3.3 Bayes factors.

STATISTICAL INFERENCE


Statistics
Descriptive
Inferential
Model fitting
Exploratory
Confirmatory
H-generating
H-testing

Hypothesis testing

We want to know two (interrelated) things:

- Is there **evidence** for the hypothesis H?

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- Is there **evidence** for the hypothesis H?
- What is the **strength** of the evidence?

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    - It is important to decide in advance the details of the analysis.

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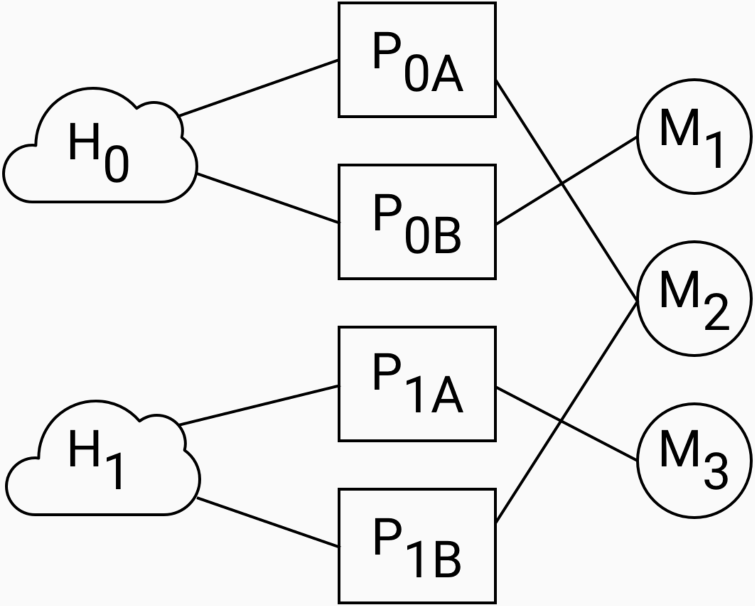
    - It is important to decide in advance the details of the analysis.
    - Even when you think you are not making decisions, the model is.
- Inference is ultimately a **long-term endeavour** (via accumulation of knowledge).


H0
H1
P0A
P0B
P1A
P1B
M1
M2
M3

FREQUENTIST INFERENCE

Frequentist statistics

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    - lme4 package.
    - **Null Hypothesis Significance Testing**.

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    - Should be as low as possible.

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- Significance is dichotomous.
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- No degrees of significance.
- “Significance” is a concept that makes sense only within frequentist statistics (NHST).

(Frequentist) confidence intervals

https://rpsychologist.com/d3/ci/

BAYESIAN INFERENCE

Bayesian inference

Bayesian statistics is based on the Bayesian interpretation of the **Bayes theorem**.

$$P(\theta \mid d) = \frac{P(d \mid \theta) \times P(\theta)}{P(d)}$$

Bayesian inference

- Evidence for any H (even the null).

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Bayesian inference

- Evidence for any H (even the null).
- Strength of evidence.
- Capitalise on previous knowledge.

Bayesian inference

Three ways of doing hypothesis testing with Bayesian statistics:

- Inference from the **posterior**.

Bayesian inference

Three ways of doing hypothesis testing with Bayesian statistics:

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- Inference using a **Region Of Practical Equivalence** (ROPE).

Bayesian inference

Three ways of doing hypothesis testing with Bayesian statistics:

- Inference from the **posterior**.
- Inference using a **Region Of Practical Equivalence** (ROPE).
- Inference using the **Bayes factor**.

Inference from the posterior

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Inference from the posterior

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3. Collect data.

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    - The posterior suggests that Condition B decreases reaction times by 15 to 80 ms at 95% confidence.
    - The posterior is **not compatible with H**.

Inference with a ROPE

H0 vs H1

- H1 states that Condition B increases segment duration (alternative hypothesis), while H0 states that Condition B does not increase segment duration (null hypothesis, null effect).

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  - $H_1 : \beta > 0$
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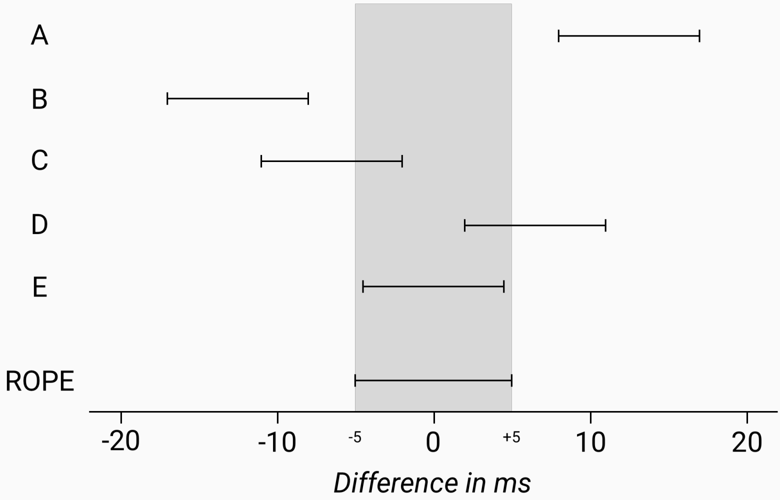
    - This ROPE has a width of 10 ms.
- Collect data until the 95% CI of β has a width equal to or smaller than the width of the ROPE.
    - Choose a minimal sample size (ideally based on a prospective power analysis).
    - Collect data and check 95% CI. If the width is greater than the ROPE, collect more data and repeat (*sequential testing*).

Inference with a ROPE



Inference with a ROPE

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- Independent from the value of β.

Inference with a ROPE

- We focus on the estimate **precision** of β.
- Independent from the value of β.
- Higher precision means greater confidence in the estimated value of β.

Bayes Factor

The Bayes factor is the ratio of the likelihood of H1 to the likelihood of H2.

$$BF_{12} = \mathcal{L}(H_1)/\mathcal{L}(H_2)$$

Bayes Factor

BF	p(M1|D)	evidence
1–3	0.5–0.75	weak
3–20	0.75–0.95	positive
20–150	0.95–0.99	strong
> 150	> 0.99	very strong

Bayes Factor

```r
priors <- c(
  prior(normal(0, 500), class = Intercept),
  prior(cauchy(0, 15), class = sigma),
  prior(normal(0, 750), class = b, coef = "vowelo"),
  prior(normal(0, 750), class = b, coef = "vowelu"),
  prior(cauchy(0, 15), class = sd),
  prior(lkj(2), class = cor)
)
```

Bayes Factor

```r
f1_3_bf <- brm(
  f1 ~ 1 + vowel + (1 + vowel | speaker),
  family = gaussian(),
  prior = priors,
  data = f_end,
  chains = 4,
  iter = 2000,
  file = "./cache/f1_3_bf",
  save_all_pars = TRUE
)
```

Bayes Factor

```
priors <- c(
  prior(normal(0, 500), class = Intercept),
  prior(cauchy(0, 15), class = sigma),
  prior(normal(0, 750), class = b, coef = "vowelo"),
  prior(normal(0, 750), class = b, coef = "vowelu"),
  prior(normal(0, 750), class = b, coef = "c2_placevelar"),
  prior(cauchy(0, 15), class = sd),
  prior(lkj(2), class = cor)
)
```

Bayes Factor

```
f1_3_place <- brm(
  f1 ~ 1 + vowel + c2_place + (1 + vowel + c2_place | sp
  family = gaussian(),
  prior = priors,
  data = f_end,
  chains = 4,
  iter = 2000,
  file = "./cache/f1_3_place",
  save_all_pars = TRUE
)
```

Bayes Factor

```r
bf <- bayes_factor(f1_3_bf, f1_3_place)
```

```
## Iteration: 1
## Iteration: 2
## Iteration: 3
## Iteration: 4
## Iteration: 5
## Iteration: 6
## Iteration: 7
## Iteration: 8
## Iteration: 9
## Iteration: 1
## Iteration: 2
## Iteration: 3
```

Bayes Factor

```
bf
```

```
## Estimated Bayes factor in favor of f1_3_bf over f1_3_place: 0.58852
```

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- BFs are ***VERY*** sensitive to priors and data.
- Always calculate and report BFs by comparing models with increasingly narrower priors (at least 3-4).
- It's important to run *sensitivity analyses* that assess the influece of the priors on the posterior (not only if you you BFs, but always).

THE END