

Introduction to Bayesian linear regression with brms — Part II: Bayesian Inference

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18/01/2020

Road map

1. Statistical inference.

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2. Frequentist inference.

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3. Bayesian inference:

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3. Bayesian inference:
 - 3.1 Posterior distributions.

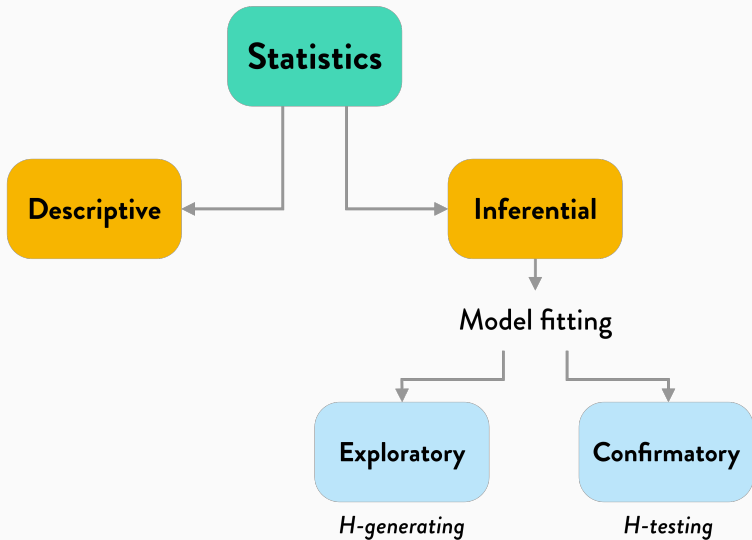
Road map

1. Statistical inference.
2. Frequentist inference.
3. Bayesian inference:
 - 3.1 Posterior distributions.
 - 3.2 ROPE.

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3. Bayesian inference:
 - 3.1 Posterior distributions.
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 - 3.3 Bayes factors.

STATISTICAL INFERENCE



Hypothesis testing

We want to know two (interrelated) things:

- Is there **evidence** for the hypothesis H ?

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- What is the **strength** of the evidence?

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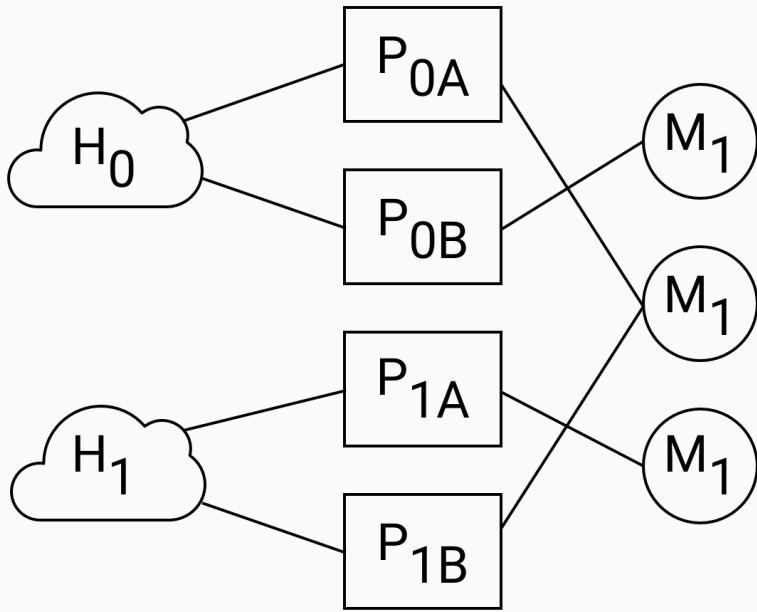
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- We test H against empirical data (**hypothesis testing**).
 - It is important to decide in advance the details of the analysis.
 - Even when you think you are not making decisions, the model is.
- Inference is ultimately a **long-term endeavour** (via accumulation of knowledge).

Hypothesis testing



FREQUENTIST INFERENCE

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 - **Null Hypothesis Significance Testing.**

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 - Should be as low as possible.

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- “Significance” is a concept that makes sense only within frequentist statistics (NHST).

(Frequentist) confidence intervals

<https://rpsychologist.com/d3/ci/>

BAYESIAN INFERENCE

Bayesian statistics is based on the Bayesian interpretation of the **Bayes theorem**.

$$P(\theta \mid d) = \frac{P(d \mid \theta) \times P(\theta)}{P(d)}$$

- Evidence for any H (even the null).

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Bayesian inference

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- Strength of evidence.
- Capitalise on previous knowledge.

Three ways of doing hypothesis testing with Bayesian statistics:

- Inference from the **posterior**.

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- Inference using the **Bayes factor**.

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5. **Inference**:
 - The posterior suggests that Condition B decreases reaction times by 15 to 80 ms at 95% confidence.
 - The posterior is **compatible with H**.

Inference from the posterior

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Inference from the posterior

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- H: Condition B decreases reaction times relative to Condition A
by 100 ms.

Inference from the posterior

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 - H: Condition B decreases reaction times relative to Condition A *by 100 ms.*
2. Choose **model specification** (including priors).

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 - The posterior suggests that Condition B decreases reaction times by 15 to 80 ms at 95% confidence.
 - The posterior is **not compatible with H**.

H0 vs H1

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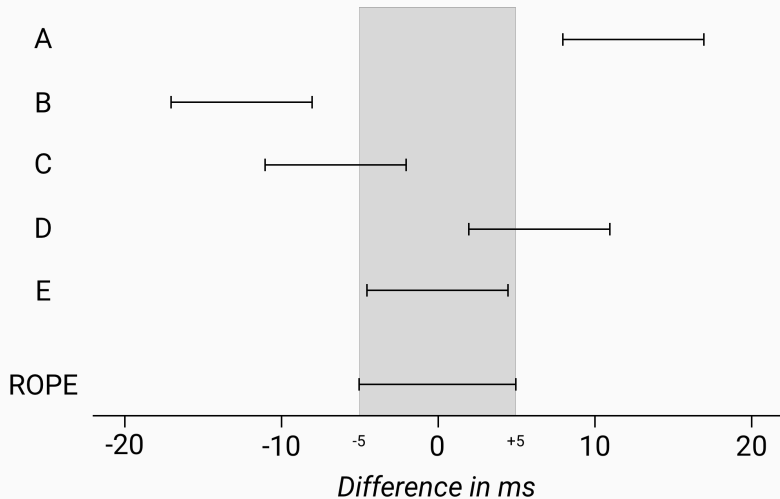
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 - This ROPE has a width of 10 ms.
- Choose a minimal sample size (ideally based on a prospective power analysis).
- Collect data until the 95% CI of β has a width equal to or smaller than the width of the ROPE.

Inference with a ROPE



- We focus on the estimate **precision** of β .

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- Higher precision means greater confidence in the estimated value of β .

The Bayes factor is the ratio of the likelihood of H_1 to the likelihood of H_2 .

$$BF_{12} = \mathcal{L}(H_1) / \mathcal{L}(H_2)$$

Bayes Factor

BF	$p(M1 D)$	evidence
1–3	0.5–0.75	weak
3–20	0.75–0.95	positive
20–150	0.95–0.99	strong
> 150	> 0.99	very strong

Bayes Factor

```
priors <- c(  
  prior(normal(40, 10), class = Intercept),  
  prior(cauchy(0, 10), class = sigma),  
  prior(normal(10, 10), class = b, coef = "c1_placecoronal"),  
  prior(normal(20, 10), class = b, coef = "c1_placevelar"),  
  prior(normal(0, 25), class = sd),  
  prior(lkj(2), class = cor)  
)
```

