

An introduction to GAM(M)s

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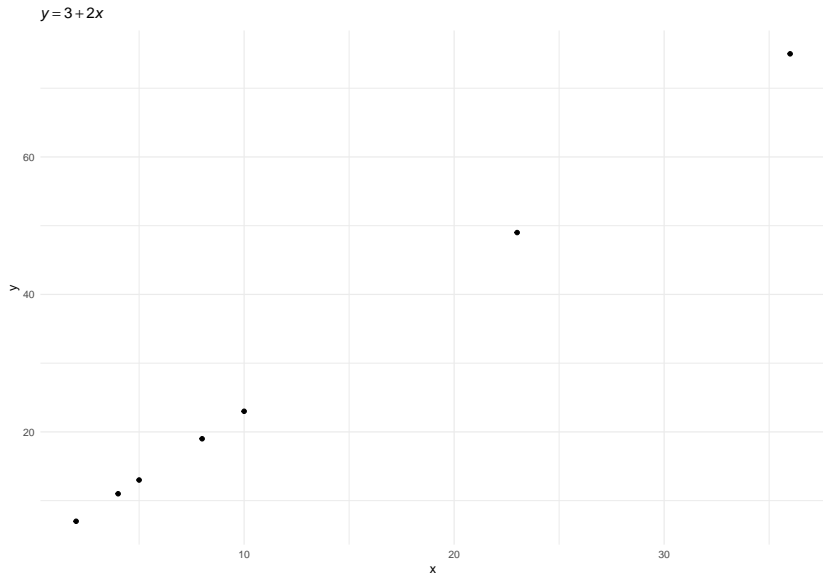
Time travel...

Linear models

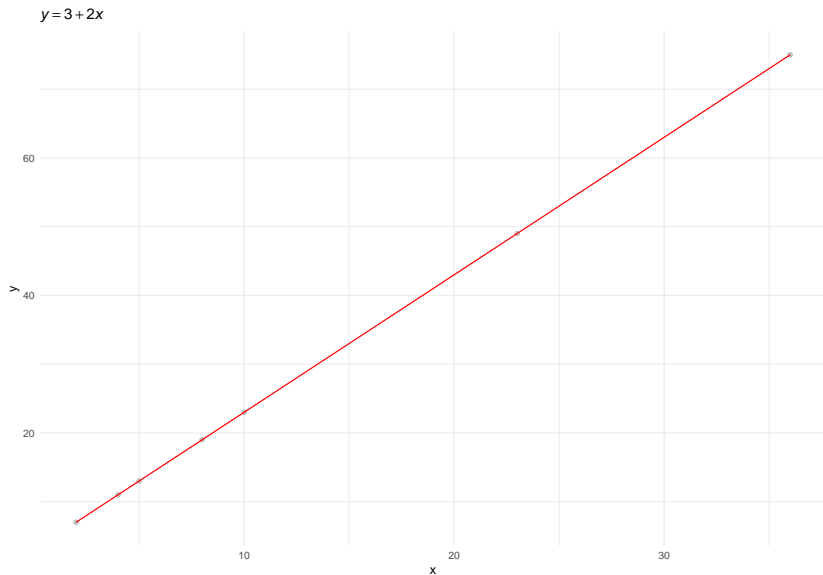
$$y = 3 + 2x$$

where $x = (2, 4, 5, 8, 10, 23, 36)$

Linear models



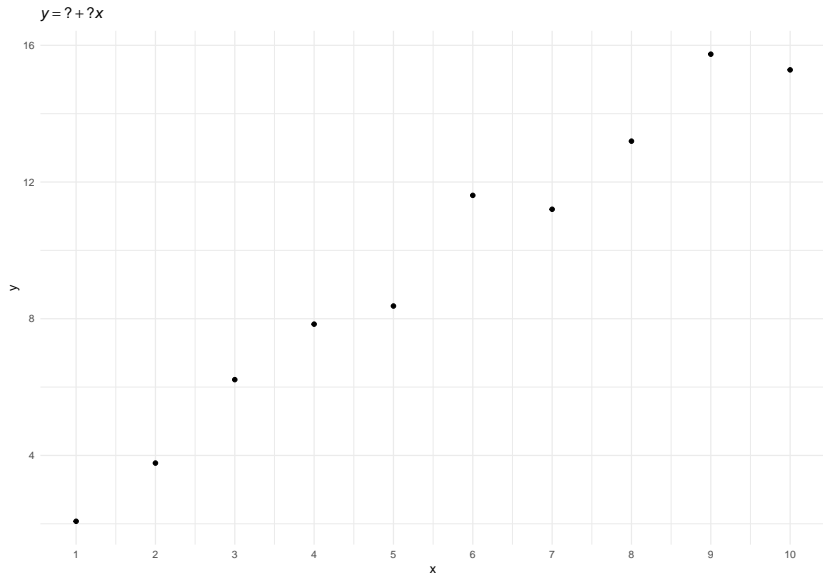
Linear models



Linear models

- ▶ In science, we have x and y ...
- ▶ for example, vowel duration and VOT

Linear models

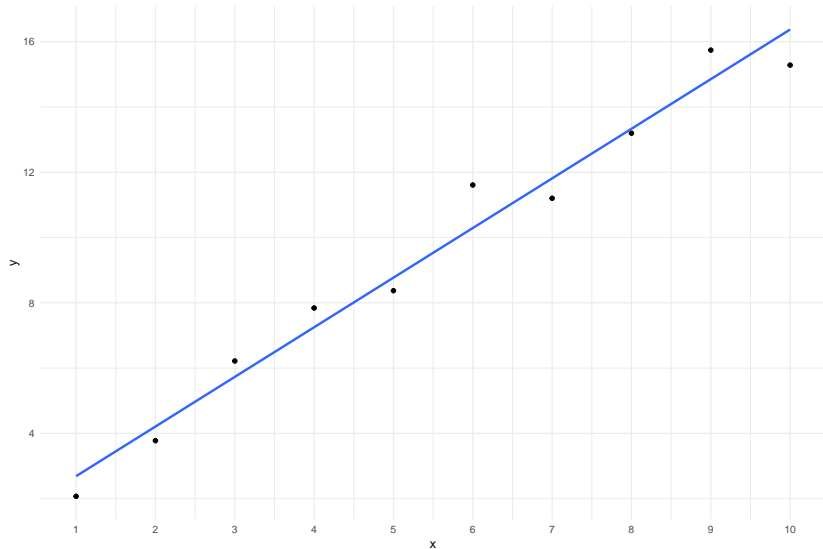


Linear models

- ▶ The formula: $y = \beta_0 + \beta_1 x$
 - ▶ β_0 is the **intercept**
 - ▶ β_1 is the **slope**
- ▶ We know x and y
 - ▶ we need to estimate $\beta_0, \beta_1 = \hat{\beta}_0, \hat{\beta}_1$
- ▶ We can add more predictors
 - ▶ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$
- ▶ `lm(y ~ x, data)` ('y as a function of x')

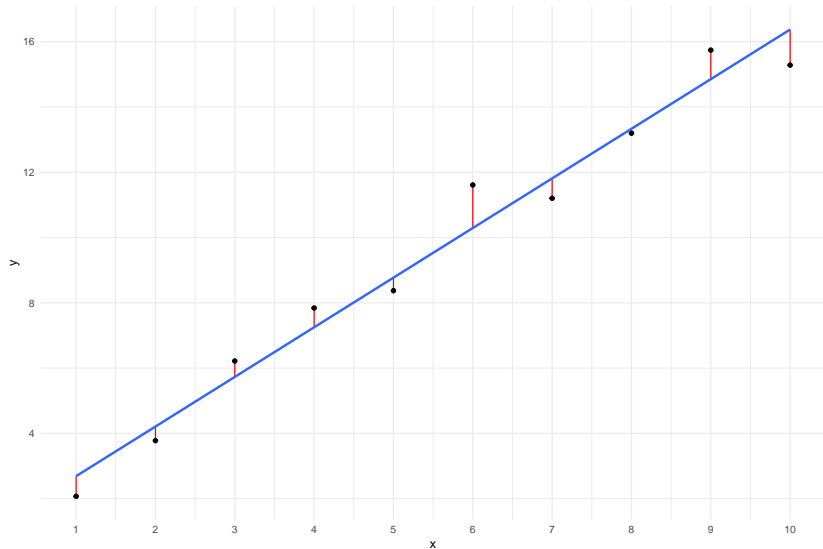
Linear models

$$y = \beta_0 + \beta_1 x = 1 + 1.5x$$



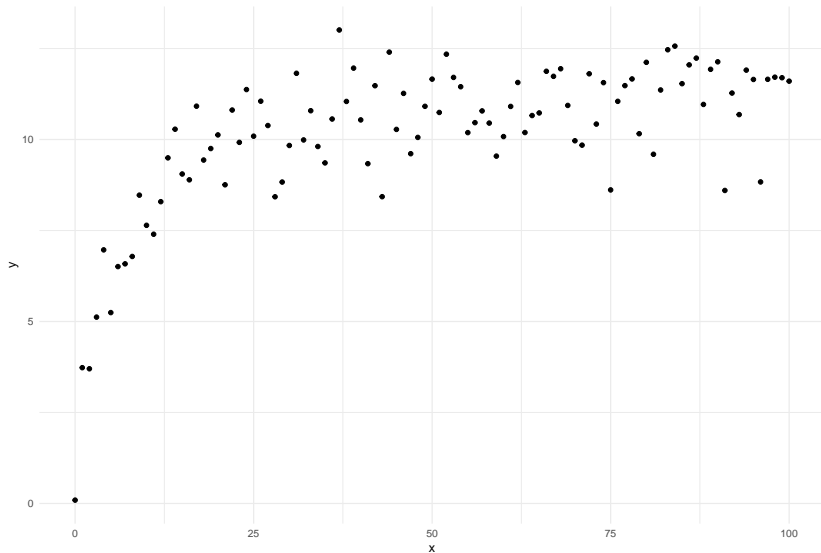
Linear models

$$y = \beta_0 + \beta_1 x + \varepsilon$$



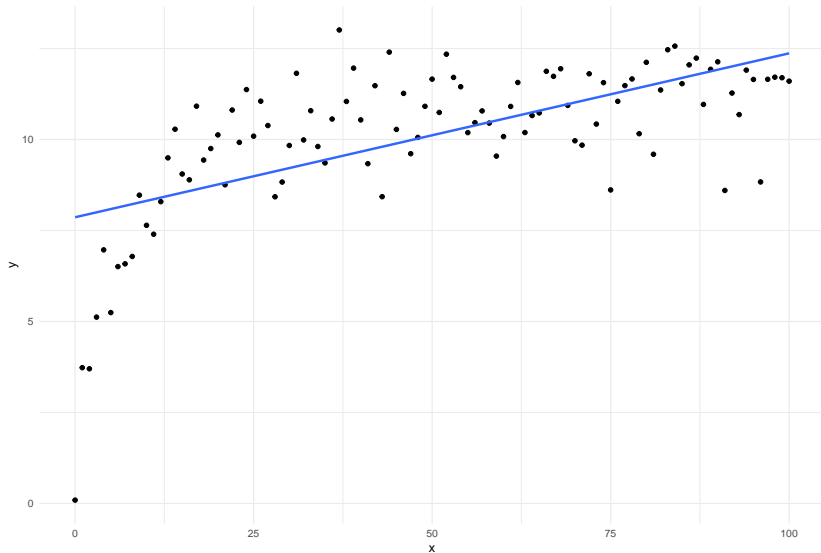
LM with non-linear data

Some non-linear data



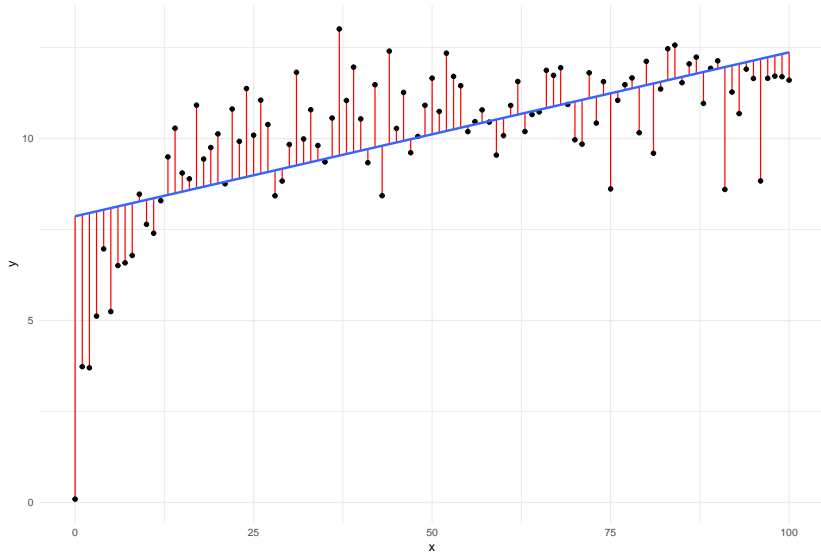
LM with non-linear data

Some non-linear data



LM with non-linear data

Some non-linear data



LM with non-linear data

How to account for non-linearity in a linear model?

- ▶ Use **higher-degree polynomials**

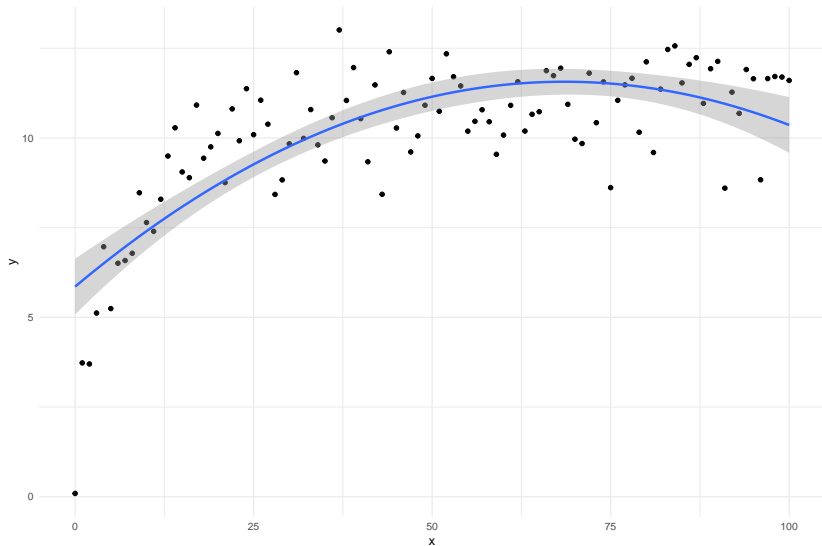
- ▶ quadratic: $y = \beta_0 + \beta_1x + \beta_2x^2$

- ▶ cubic: $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3$

- ▶ n th: $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3 + \dots + \beta_nx^n$

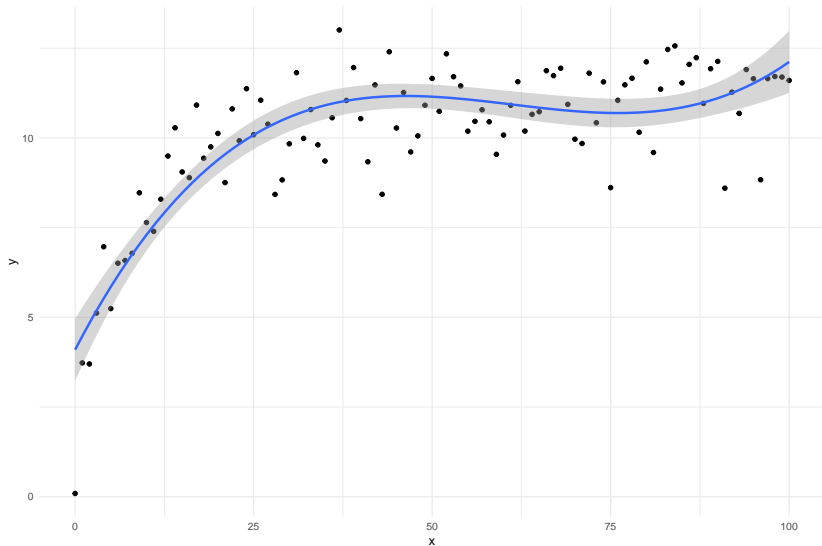
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2$$



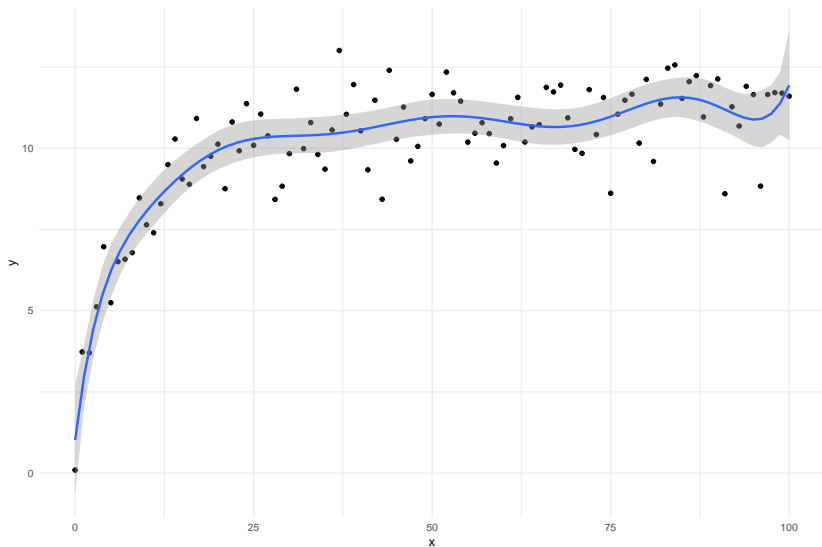
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$



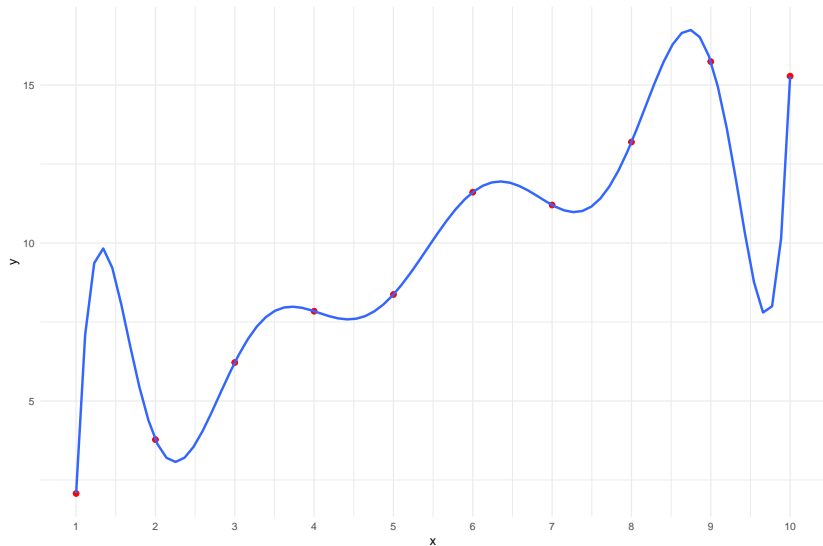
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



Generalised additive models

- ▶ **Generalised Additive Models**

- ▶ $y = f(x) + \epsilon$

- ▶ $f(x)$ = 'some function of x ' (or *smooth function*)

Smooth terms

- ▶ LMs have **parametric terms**
 - ▶ $\beta_n x_n$ (x in \mathbb{R})
 - ▶ linear effects
- ▶ GAMs add (non-parametric) **smooth terms** (or simply smooths, also smoothers)
 - ▶ $f(x)$, $s(x)$ in \mathbb{R}
 - ▶ non-linear effects
- ▶ `gam(y ~ s(x), data)`, 'y as *some* function of x'

Smoothing splines, basis, basis functions

- ▶ smooths in GAMs are **smoothing splines**
 - ▶ splines are defined piecewise with polynomials
 - ▶ the piecewise combination of polynomials is called the **basis**
 - ▶ the basis is composed of **basis functions** (the polynomials)
- ▶ there are **several kinds** of splines
 - ▶ each with their own basis functions
 - ▶ thin plate regression splines
 - ▶ cubic regression splines