

# An introduction to GAM(M)s

Stefano Coretta

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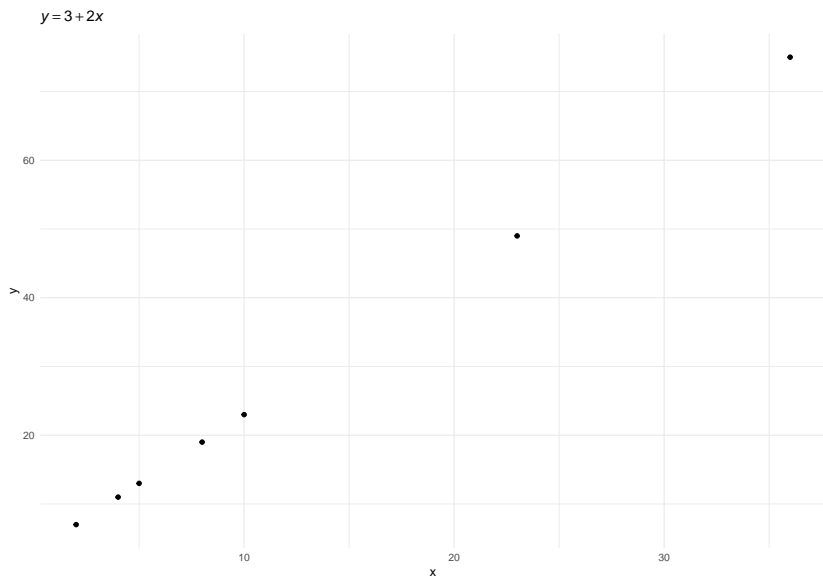
Time travel...

## Linear models

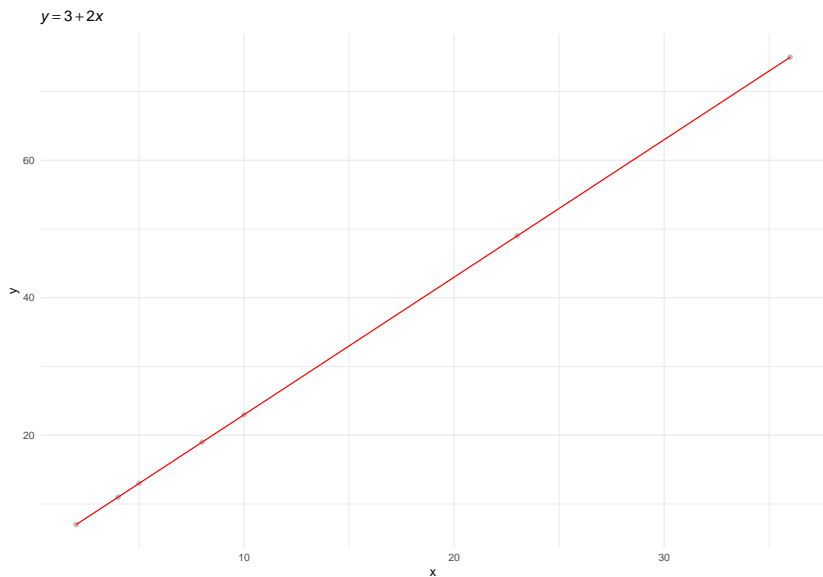
$$y = 3 + 2x$$

where  $x = (2, 4, 5, 8, 10, 23, 36)$

# Linear models



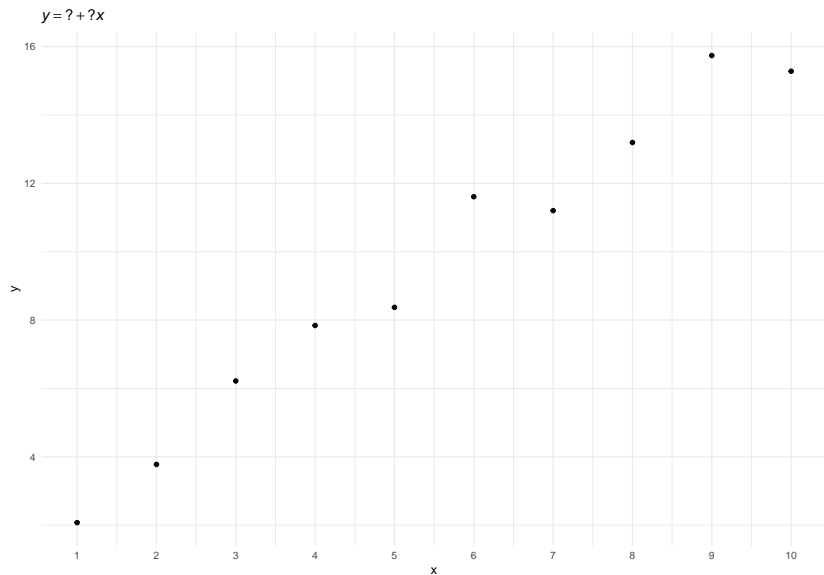
# Linear models



# Linear models

- ▶ In science, we have  $x$  and  $y$ ...
- ▶ for example, vowel duration and VOT

# Linear models



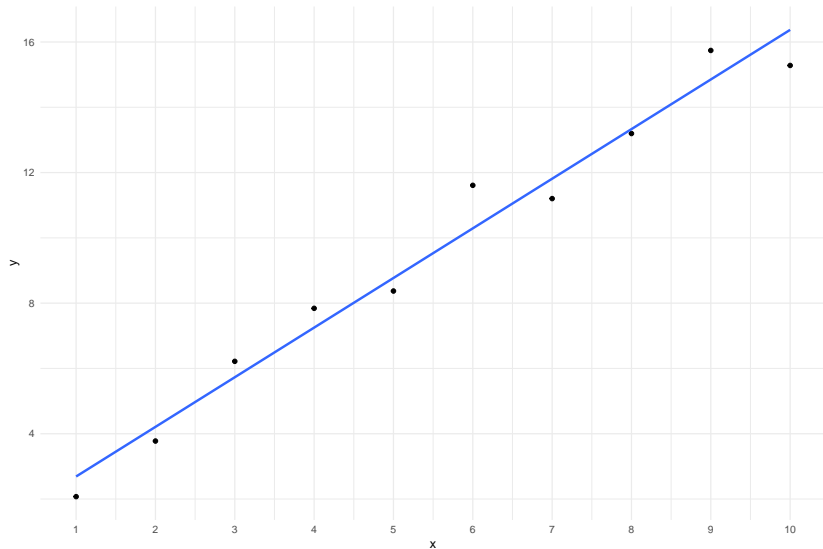
# Linear models

- ▶ The formula:  $y = \beta_0 + \beta_1 x$ 
  - ▶  $\beta_0$  is the **intercept**
  - ▶  $\beta_1$  is the **slope**
- ▶ We know  $x$  and  $y$ 
  - ▶ we need to estimate  $\beta_0, \beta_1 = \hat{\beta}_0, \hat{\beta}_1$
- ▶ We can add more predictors
  - ▶  $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$
- ▶ `lm(y ~ x, data)` ('y as a function of x')



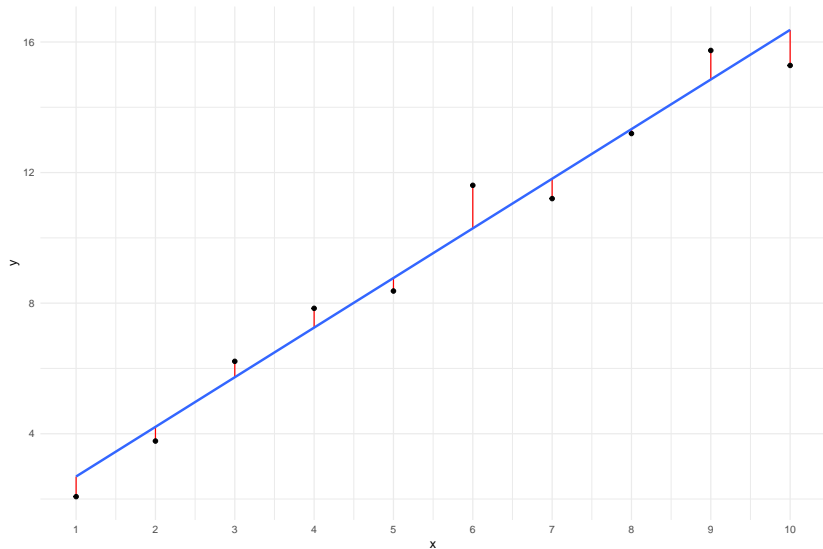
# Linear models

$$y = \beta_0 + \beta_1 x = 1 + 1.5x$$



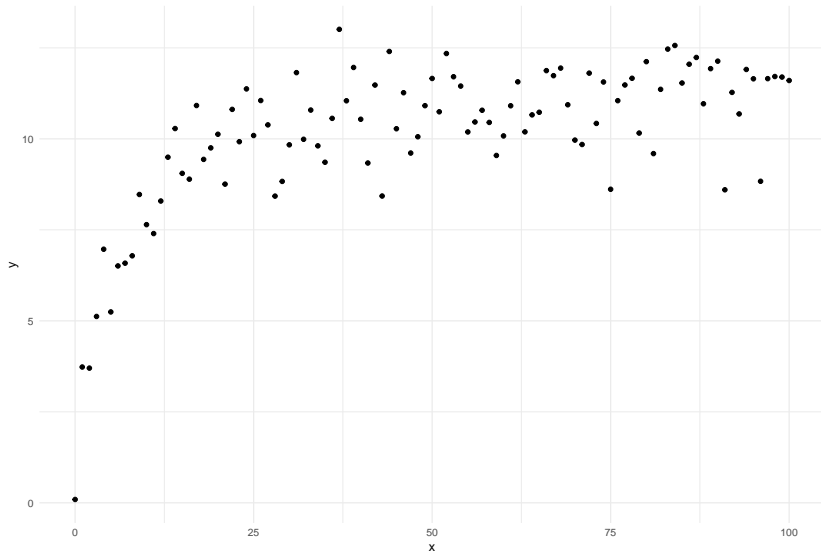
# Linear models

$$y = \beta_0 + \beta_1 x + \varepsilon$$



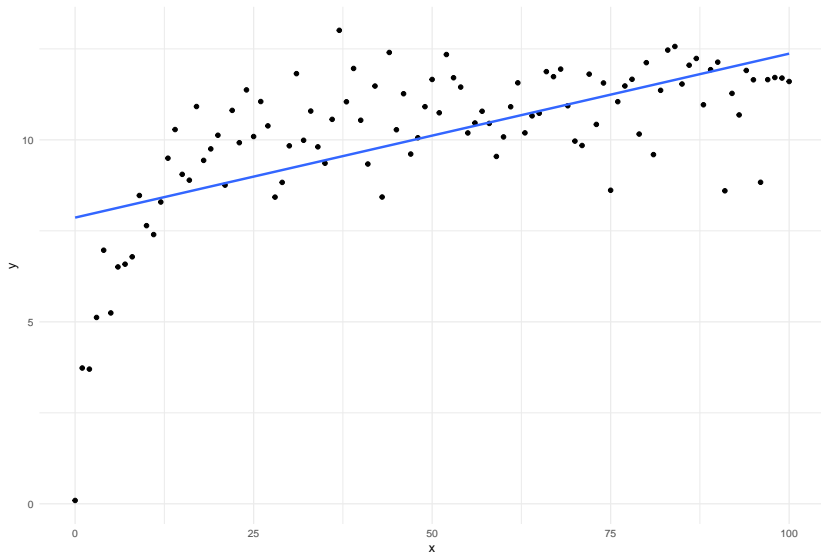
# LM with non-linear data

Some non-linear data



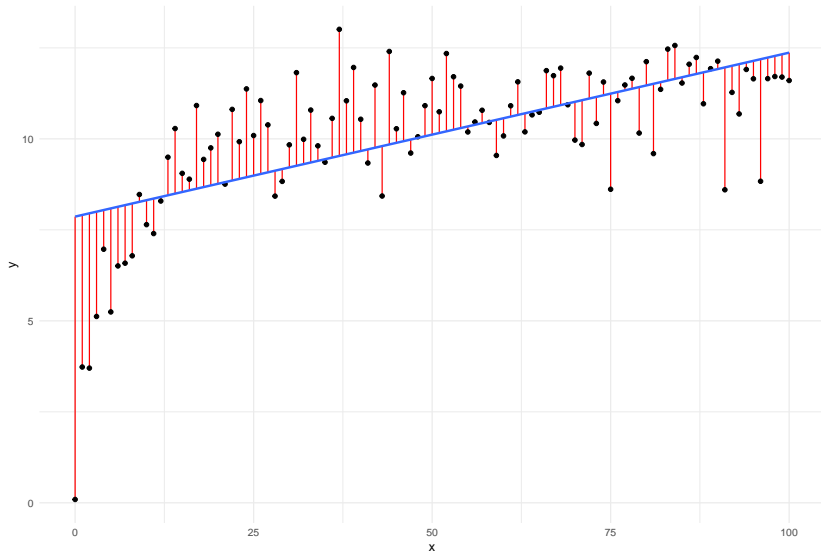
# LM with non-linear data

Some non-linear data



# LM with non-linear data

Some non-linear data



# LM with non-linear data

How to account for non-linearity in a linear model?

- ▶ Use **higher-degree polynomials**

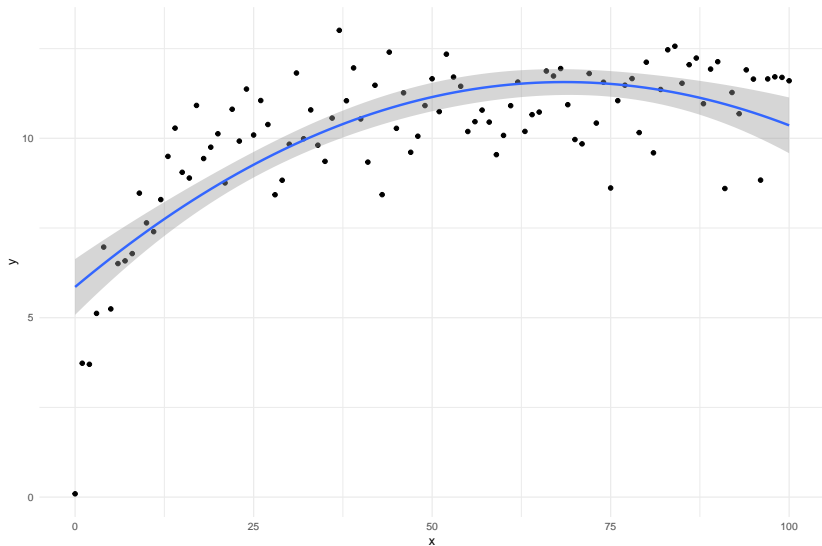
- ▶ quadratic:  $y = \beta_0 + \beta_1x + \beta_2x^2$

- ▶ cubic:  $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3$

- ▶  $n$ th:  $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3 + \dots + \beta_nx^n$

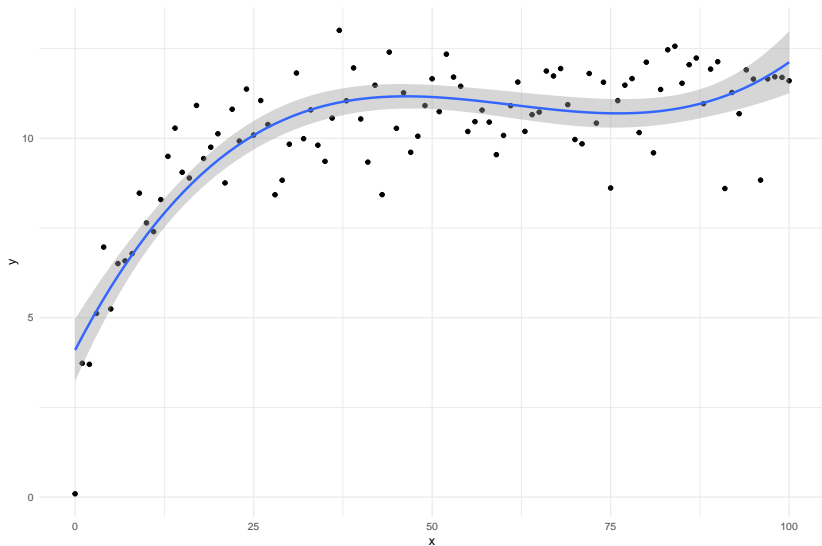
# LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2$$



# LM with non-linear data

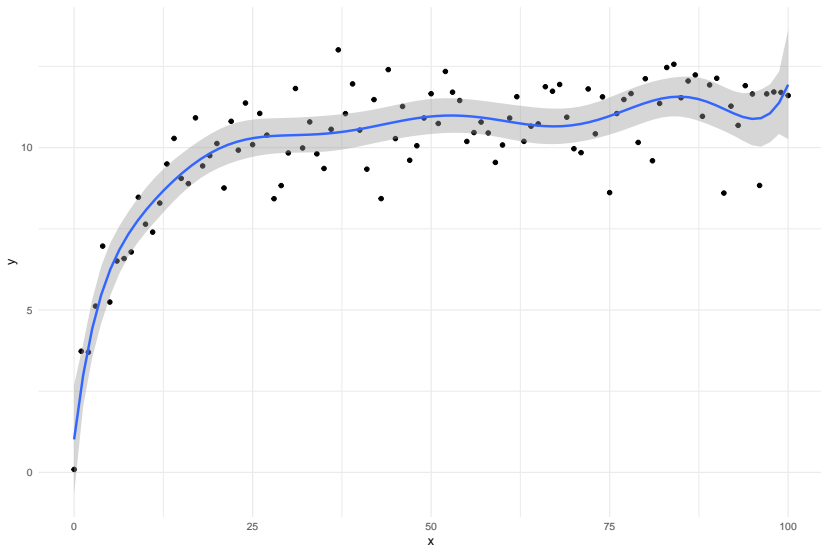
$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$





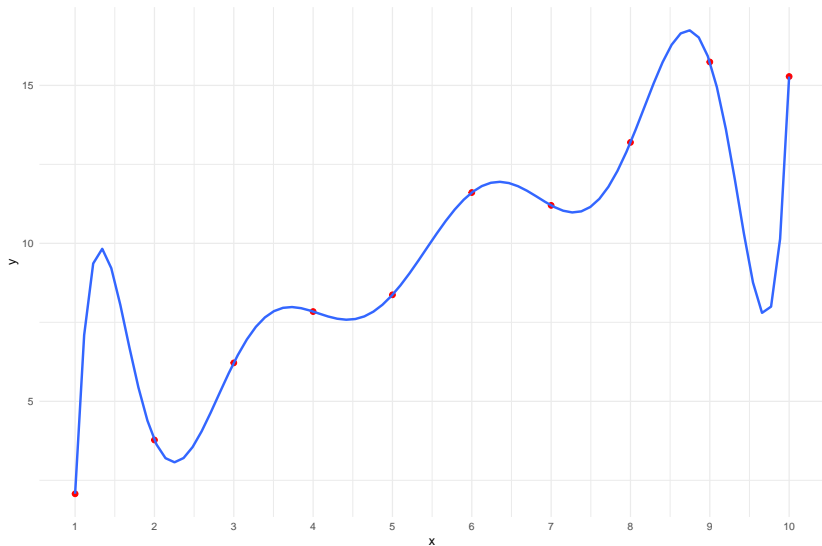
# LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



# LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



# Generalised additive models

- ▶ **Generalised Additive Models**
- ▶  $y = f(x) + \epsilon$ 
  - ▶  $f(x)$  = 'some function of  $x$ ' (or *smooth function*)

# Smooth terms

- ▶ LMs have **parametric terms**
  - ▶  $\beta_n x_n$  ( $x$  in  $\mathbb{R}$ )
  - ▶ linear effects
- ▶ GAMs add (non-parametric) **smooth terms** (or simply smooths)
  - ▶  $f(x)$ ,  $s(x)$  in  $\mathbb{R}$
  - ▶ non-linear effects
- ▶ `gam(y ~ s(x), data)`, ' $y$  as *some* function of  $x$ '

# Basis functions

- ▶ polynomials are a type of basis function
  - ▶ linear regression is the simplest polynomial (degree 1)
- ▶ **smoothing splines** are another type
  - ▶ there are several kinds of splines
  - ▶ each with their own basis functions