An introduction to GAM(M)s

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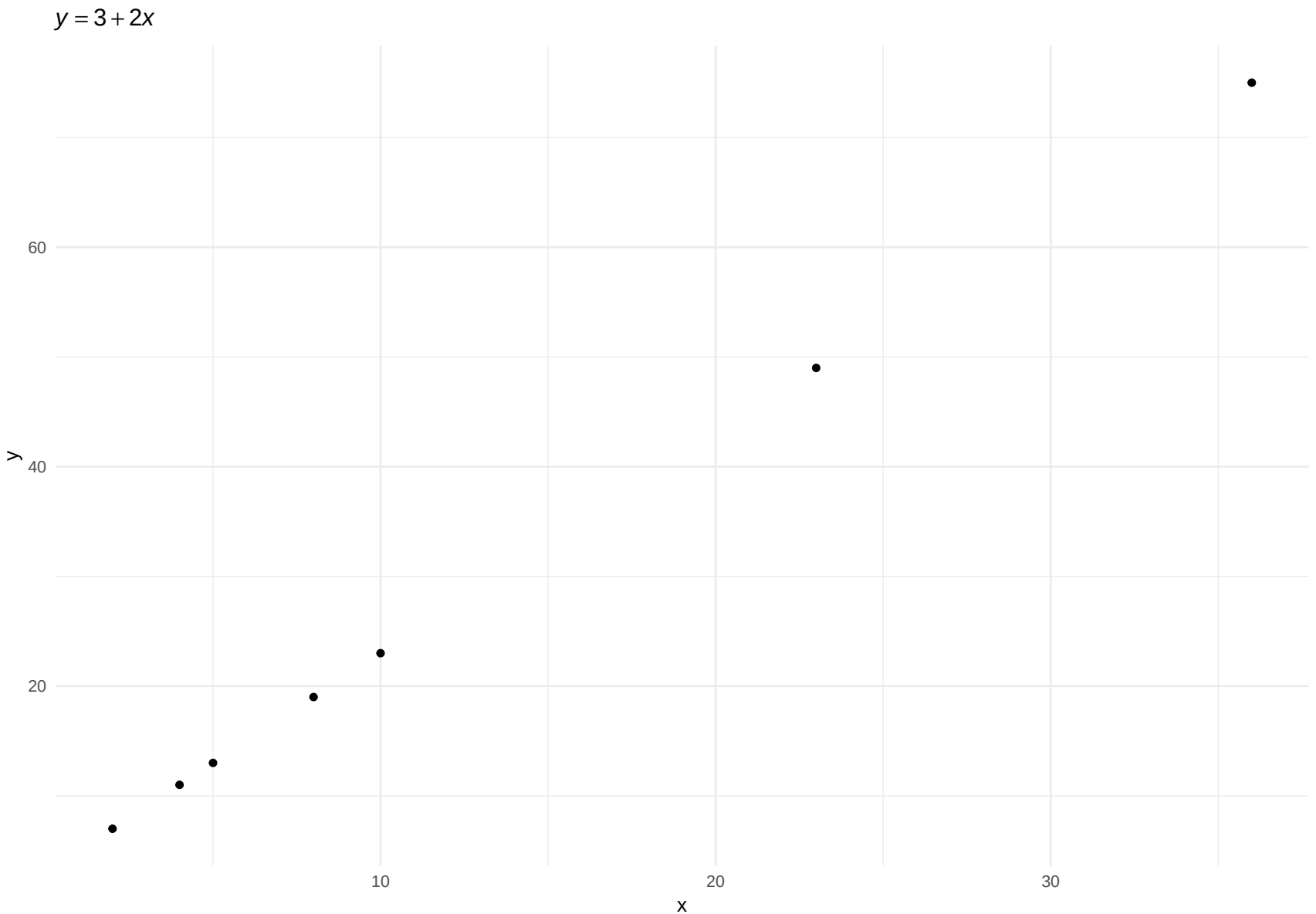
Time travel. . .

Linear models

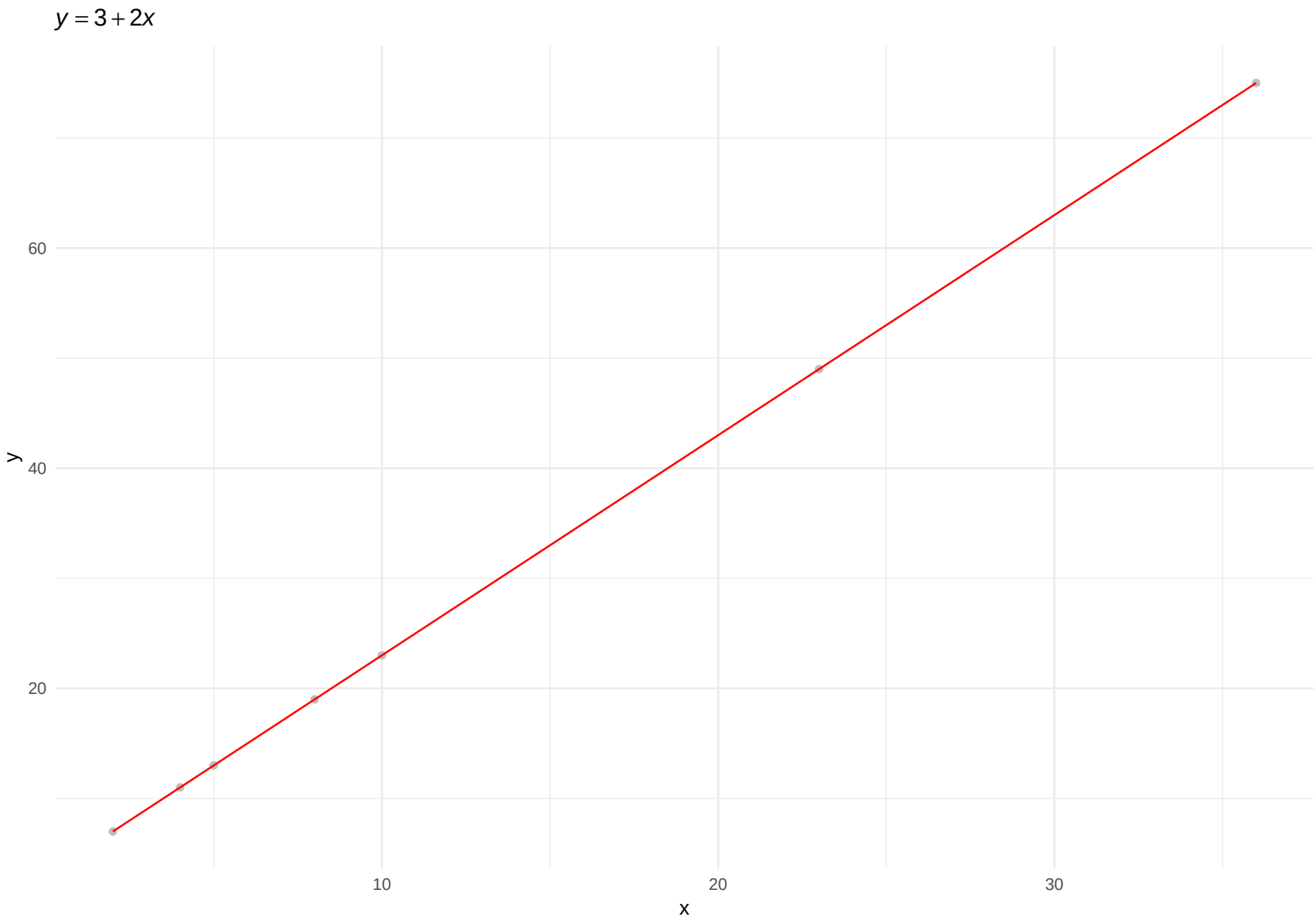
$$y = 3 + 2x$$

where $x = (2, 4, 5, 8, 10, 23, 36)$

Linear models



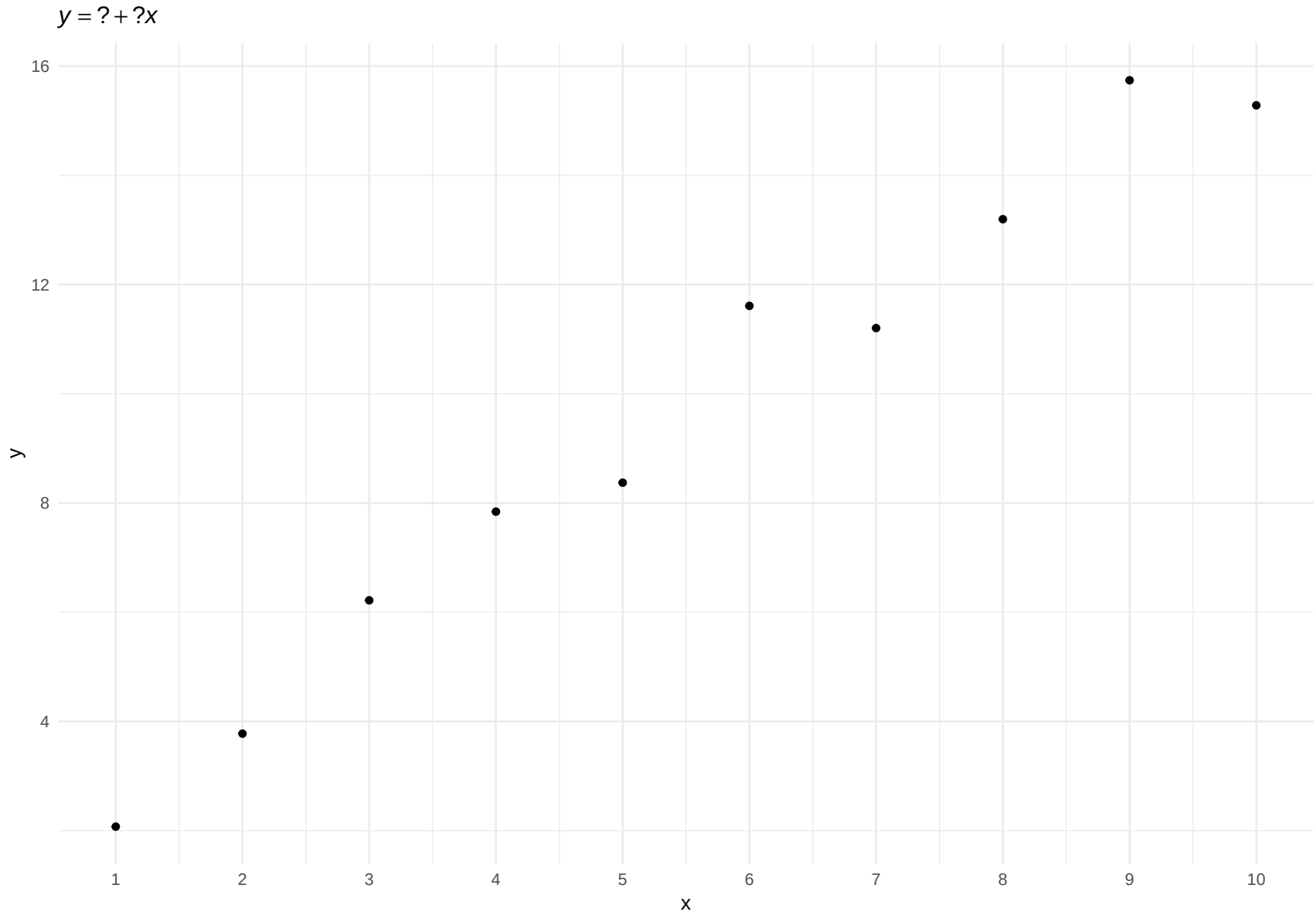
Linear models



Linear models

- In science, we have x and y...
- for example, vowel duration and VOT

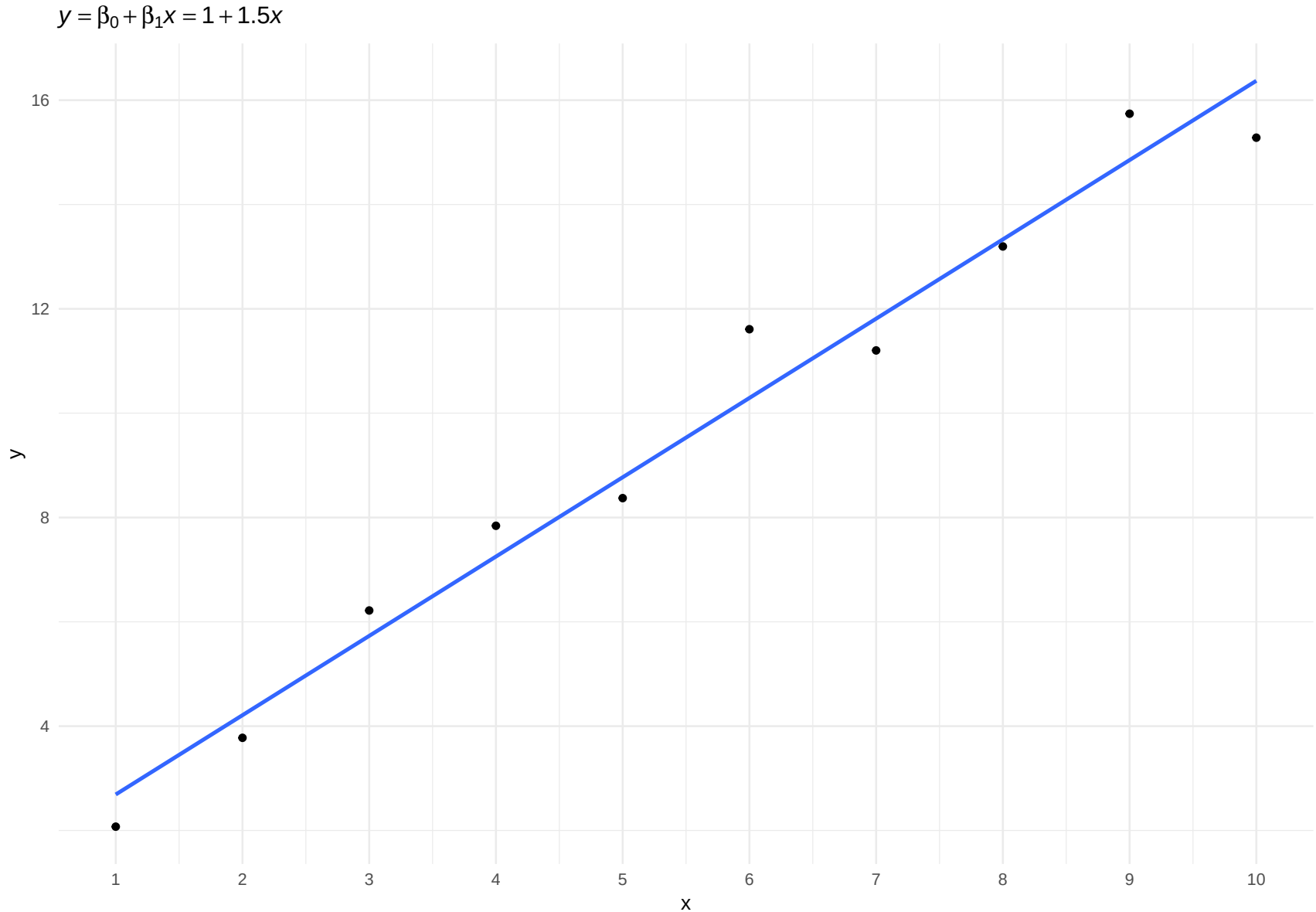
Linear models



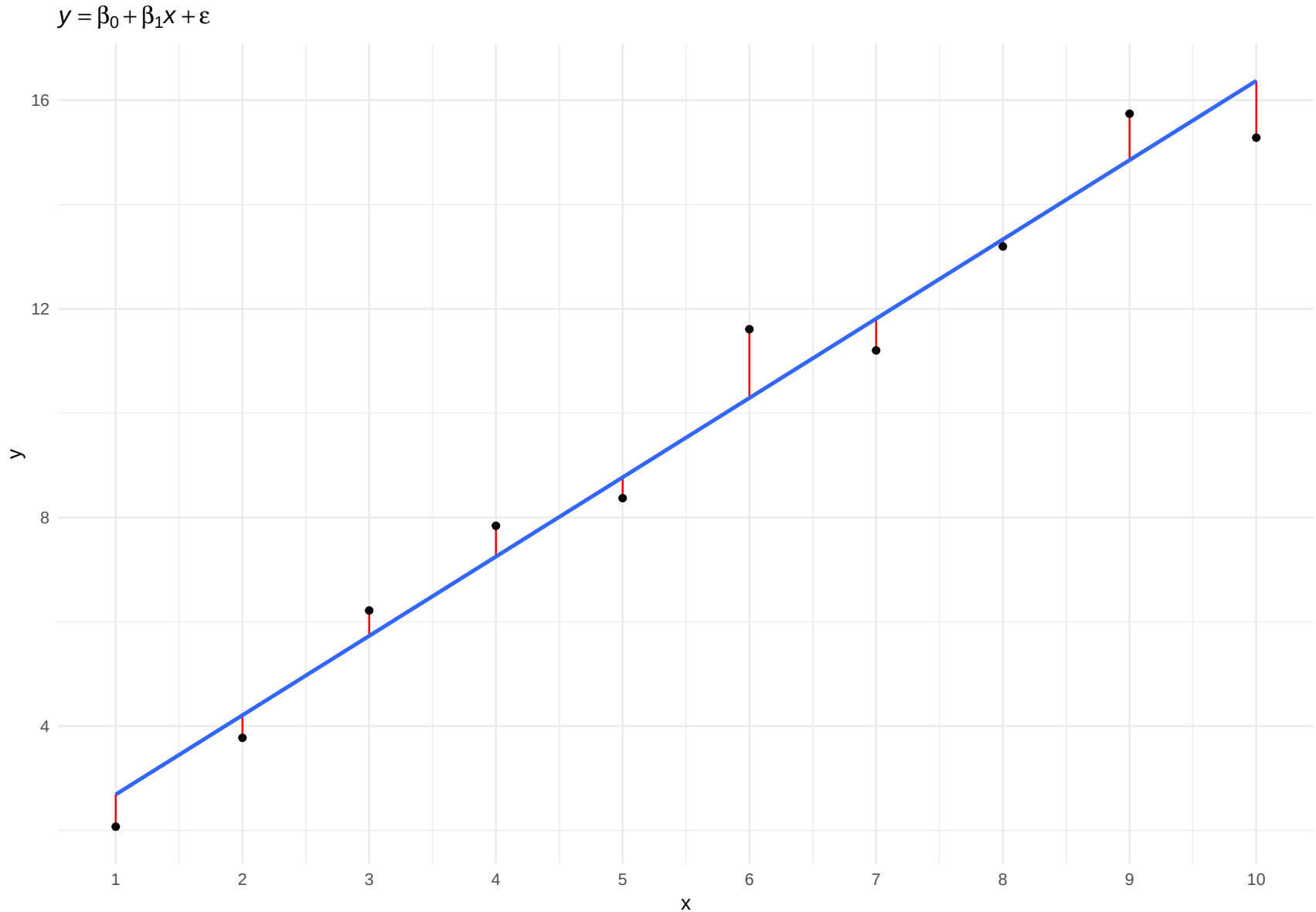
Linear models

- The formula: $y = \beta_0 + \beta_1 x$
  - β_0 is the **intercept**
  - β_1 is the **slope**
- We know x and y
  - we need to estimate β_0, $\beta_1 = \hat{\beta}_0, \hat{\beta}_1$
- We can add more predictors
  - $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n$
- `lm(y ~ x, data)` ('y as a function of x')

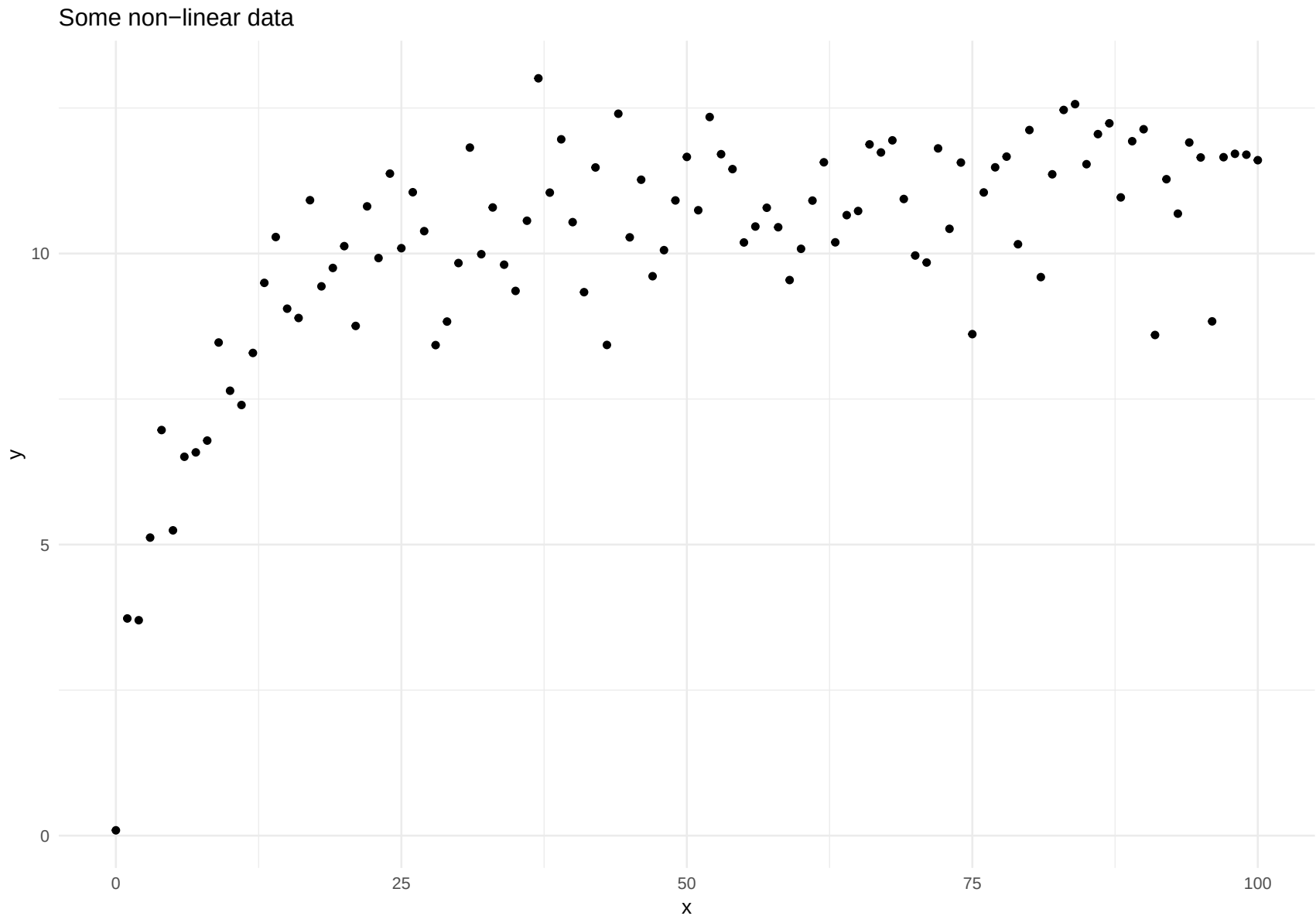
Linear models



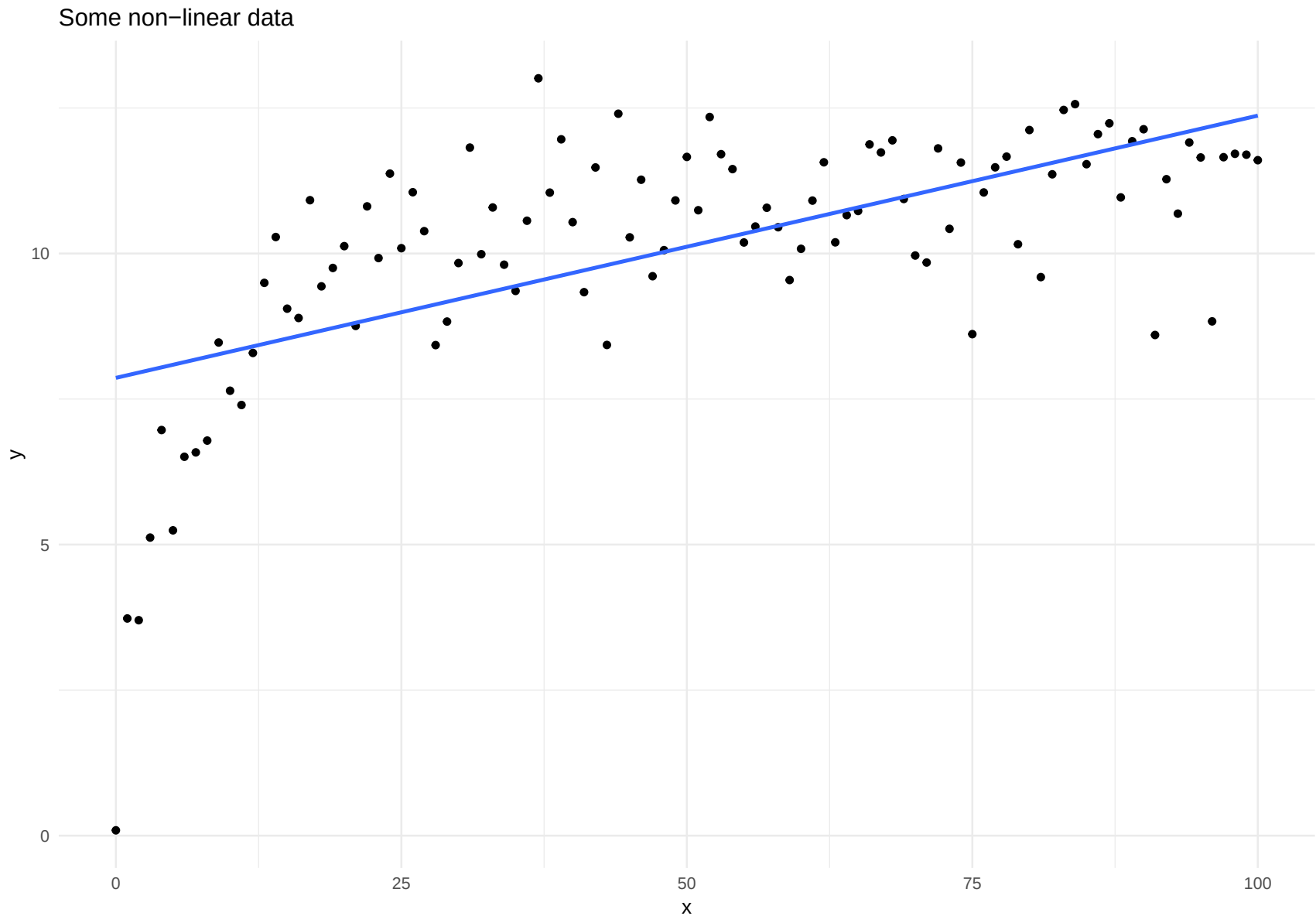
Linear models



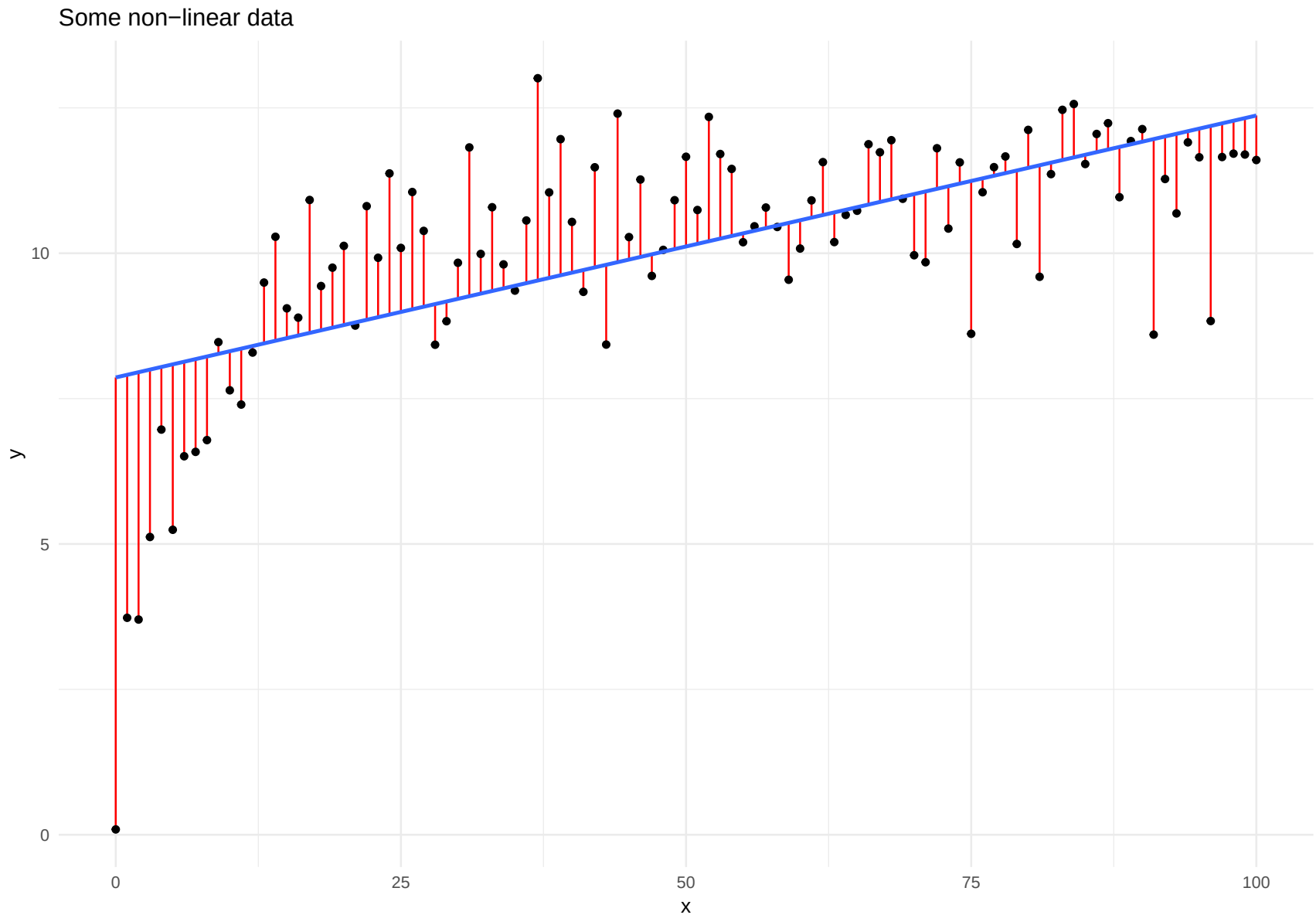
LM with non-linear data



LM with non-linear data



LM with non-linear data

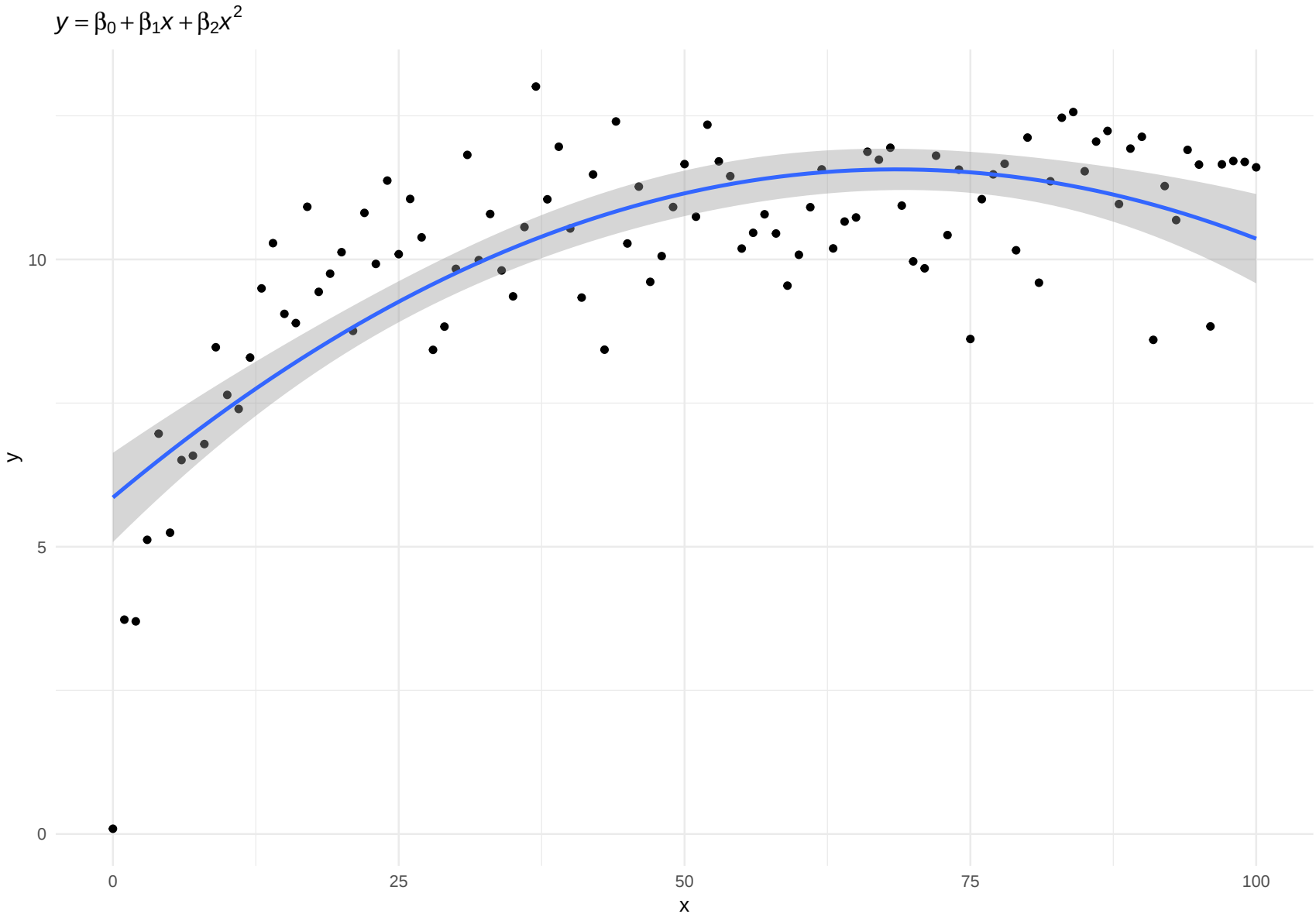


LM with non-linear data

How to account for non-linearity in a linear model?

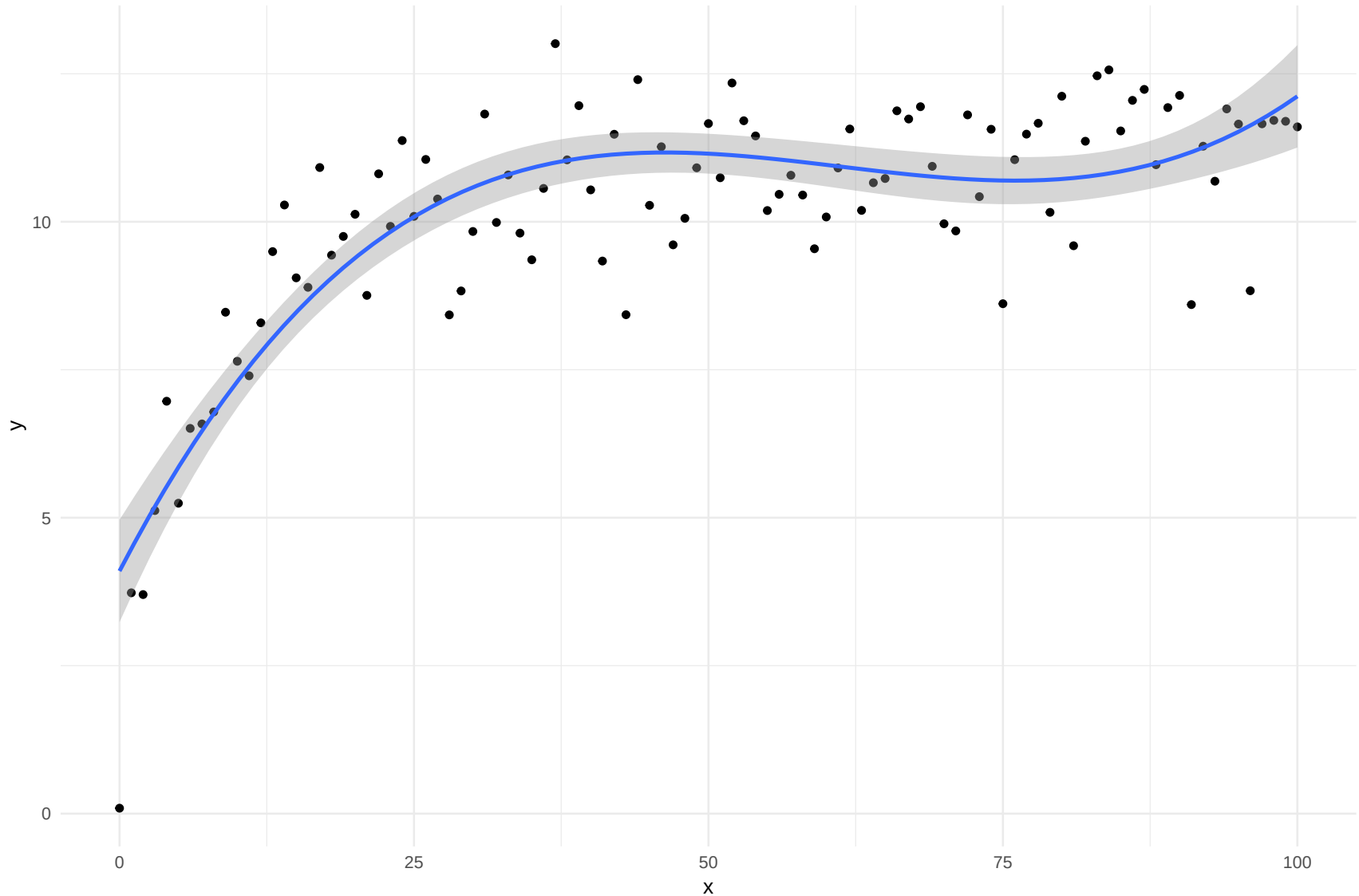
- Use **higher-degree polynomials**
  - quadratic: $y = \beta_0 + \beta_1 x + \beta_2 x^2$
  - cubic: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$
  - nth: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + ... + \beta_n x^n$

LM with non-linear data



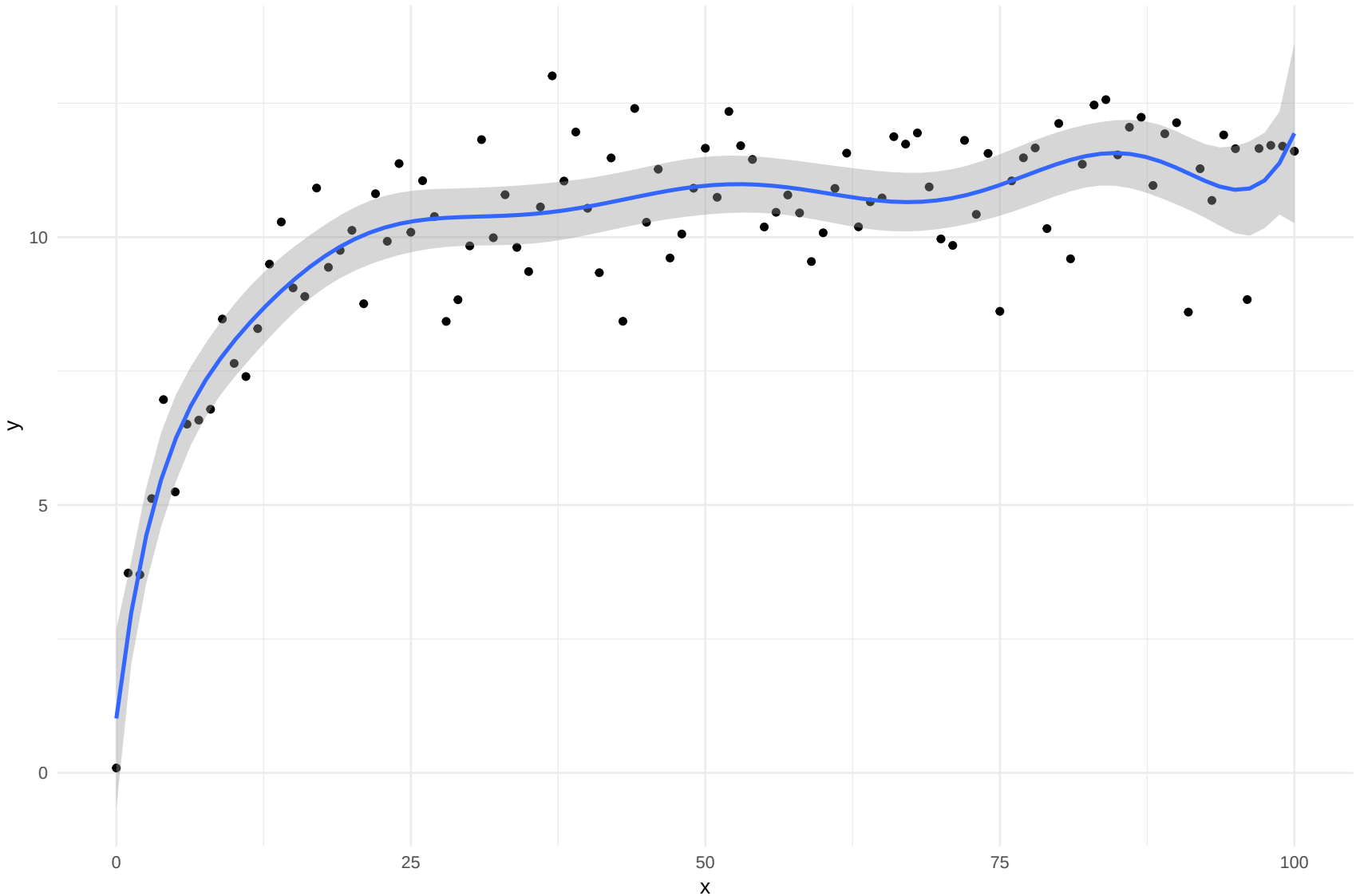
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$



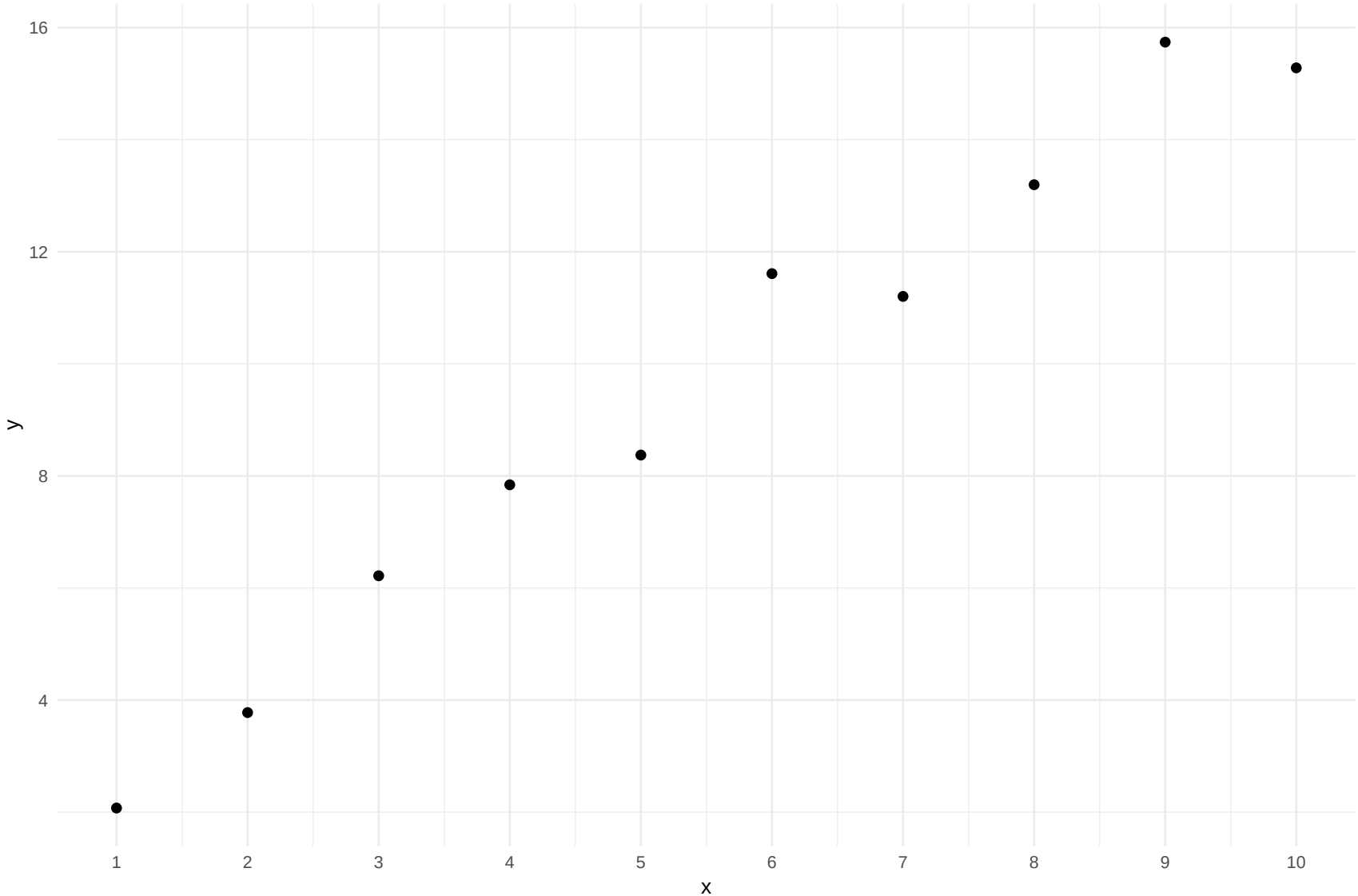
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



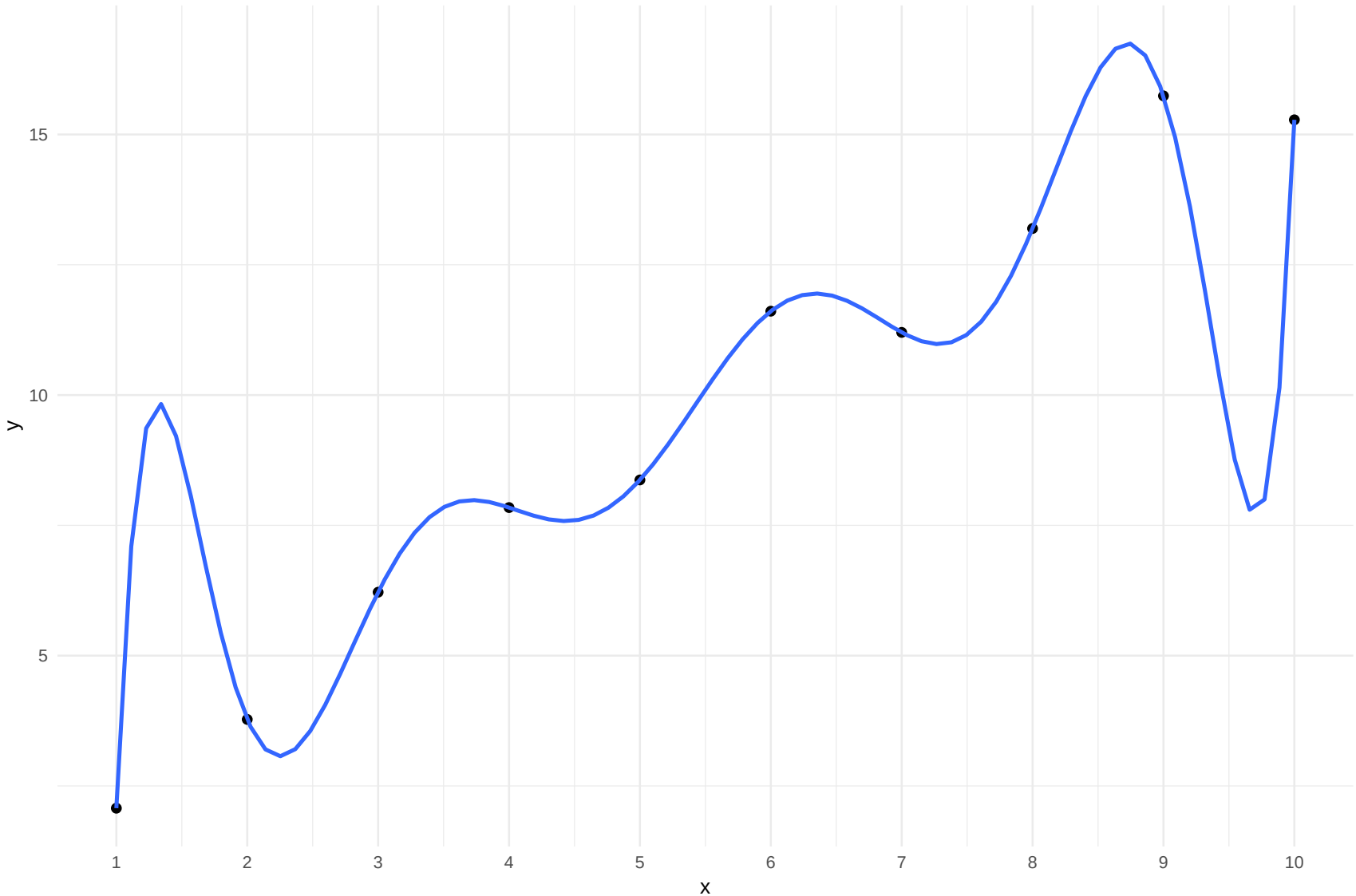
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



Generalised additive models

- **G**enrealised **A**dditive **M**odel**s**
- $y = f(x) + \epsilon$
  - $f(x) =$ 'some function of x' (or *smooth function*)

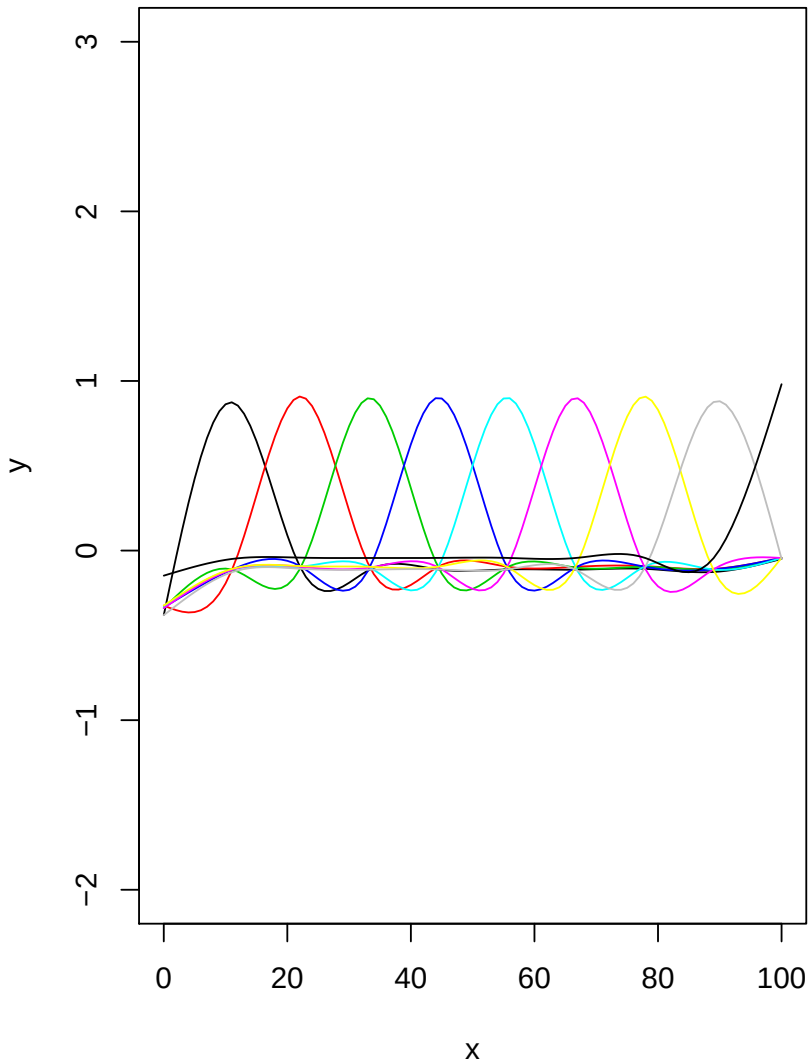
Smooth terms

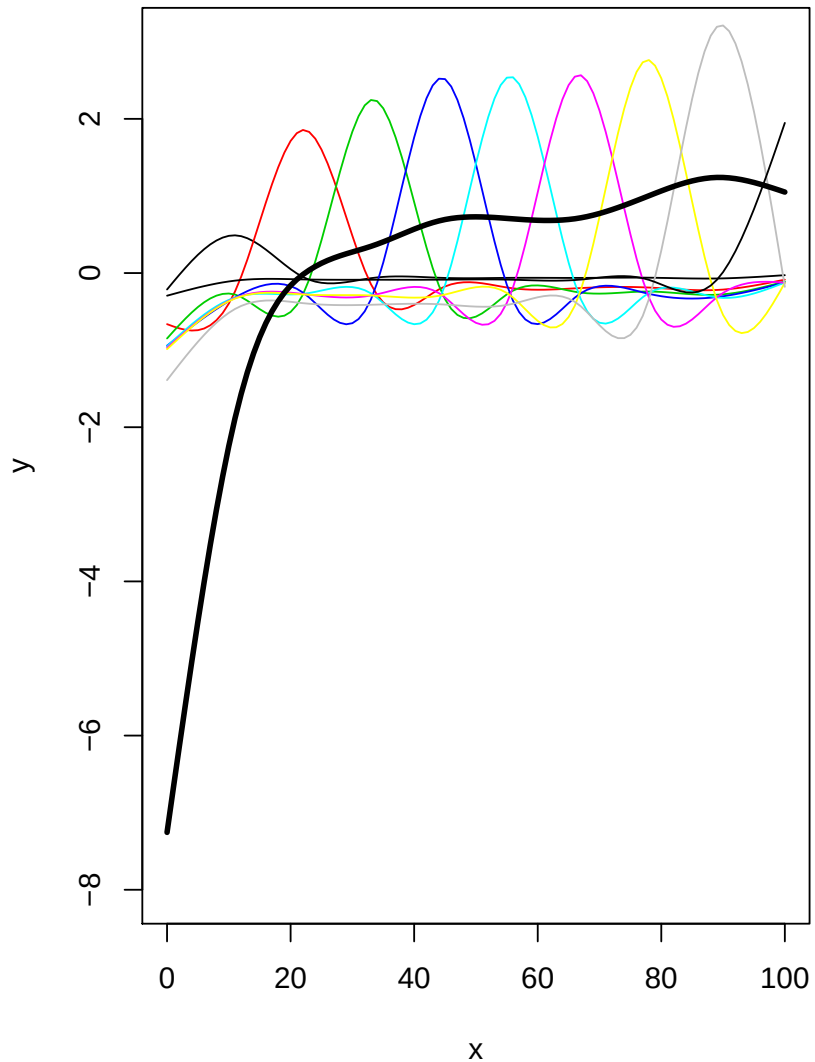
- LMs have **parametric terms**
  - $\beta_n x_n$ (`x` in `R`)
  - linear effects
- GAMs add (non-parametric) **smooth terms** (or simply smooths, also smoothers)
  - $f(x)$, `s(x)` in `R`
  - non-linear effects
- `gam(y ~ s(x), data)`, 'y as *some* function of x'

Smoothing splines, basis, basis functions

- smooths in GAMs are **smoothing splines**
  - splines are defined piecewise with a set of polynomials
- the set of polynomials is called a **basis**
  - the basis is composed of **basis functions** (the polynomials)
- a spline is the sum of the products of each basis function and its coefficient

Basis functions





Smoothing parameter

- 'wiggliness' is related to number of basis functions
  - more basis functions, more wiggliness (less smoothing)
- the **smoothing parameter** penalises wiggliness
  - high values = less wiggliness (more smoothing)
  - estimated from the data

Smoothing splines

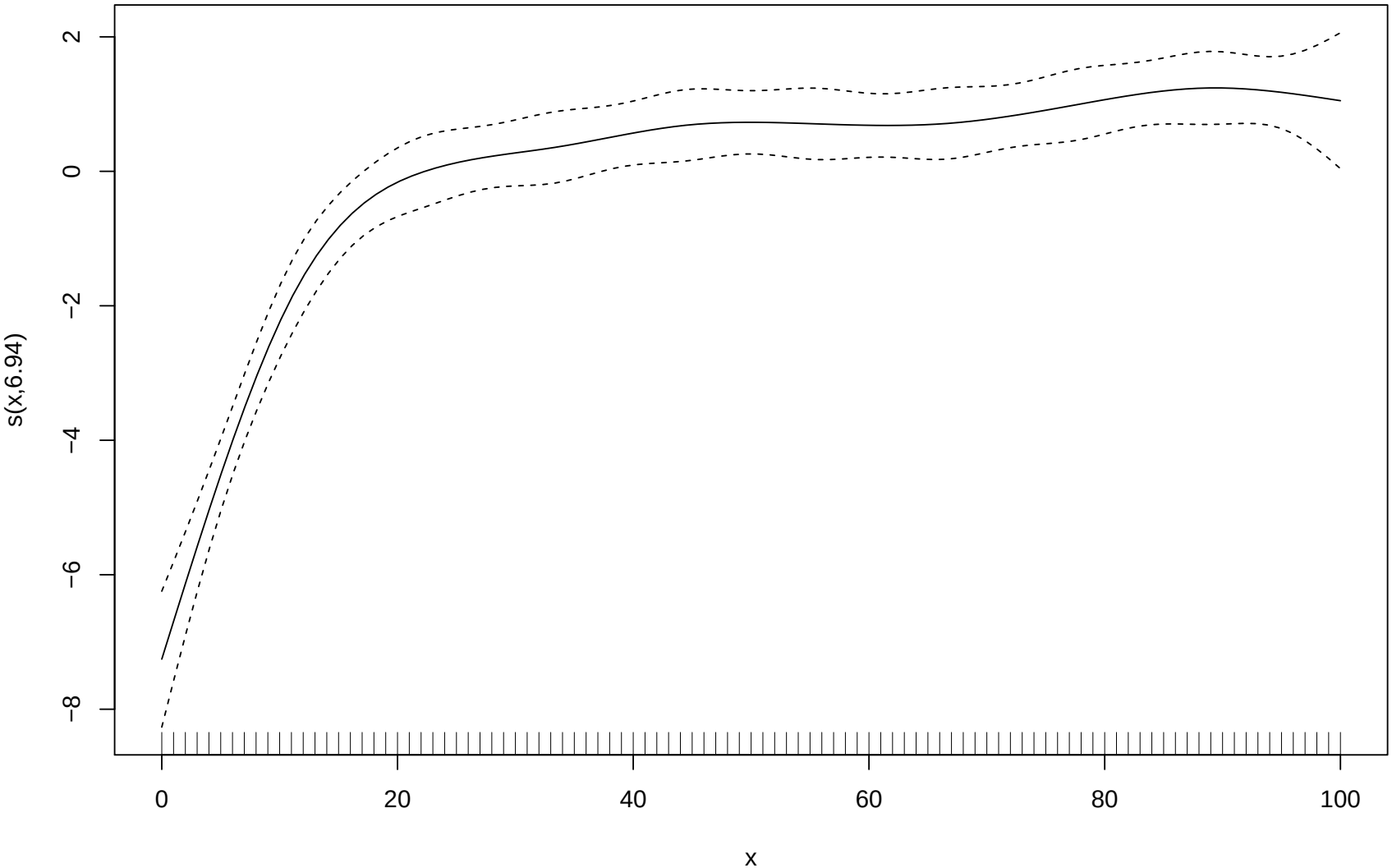
- there are **several kinds** of splines
  - each with their own basis functions
- most common
  - *thin plate regression splines*
  - *cubic regression splines*
- for more info, run `?smooth.terms`

A simple GAM

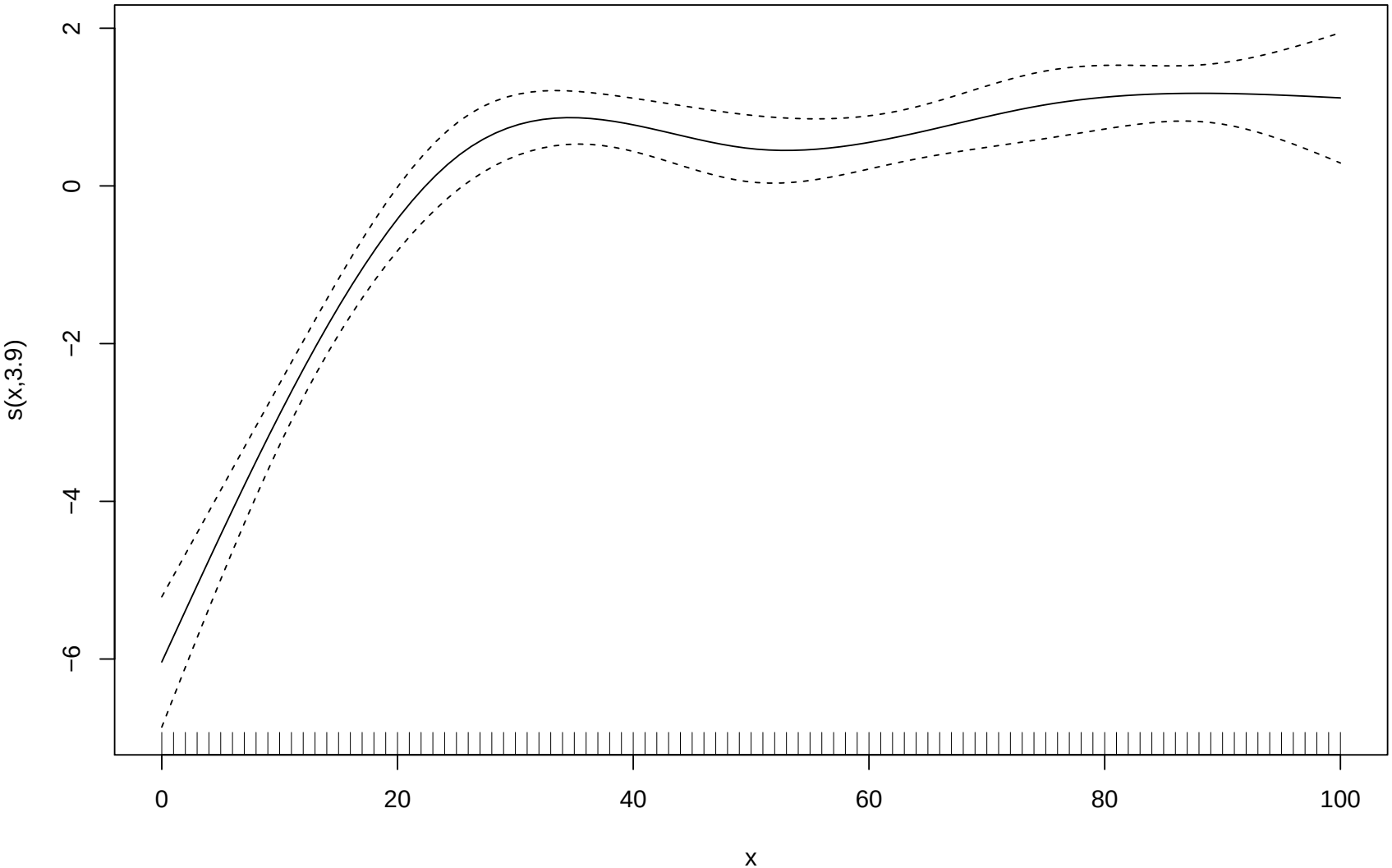
```
simple <- gam(y ~ s(x, bs = "cr", k = 10), data = sim_nl_a)
summary(simple)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ s(x, bs = "cr", k = 10)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

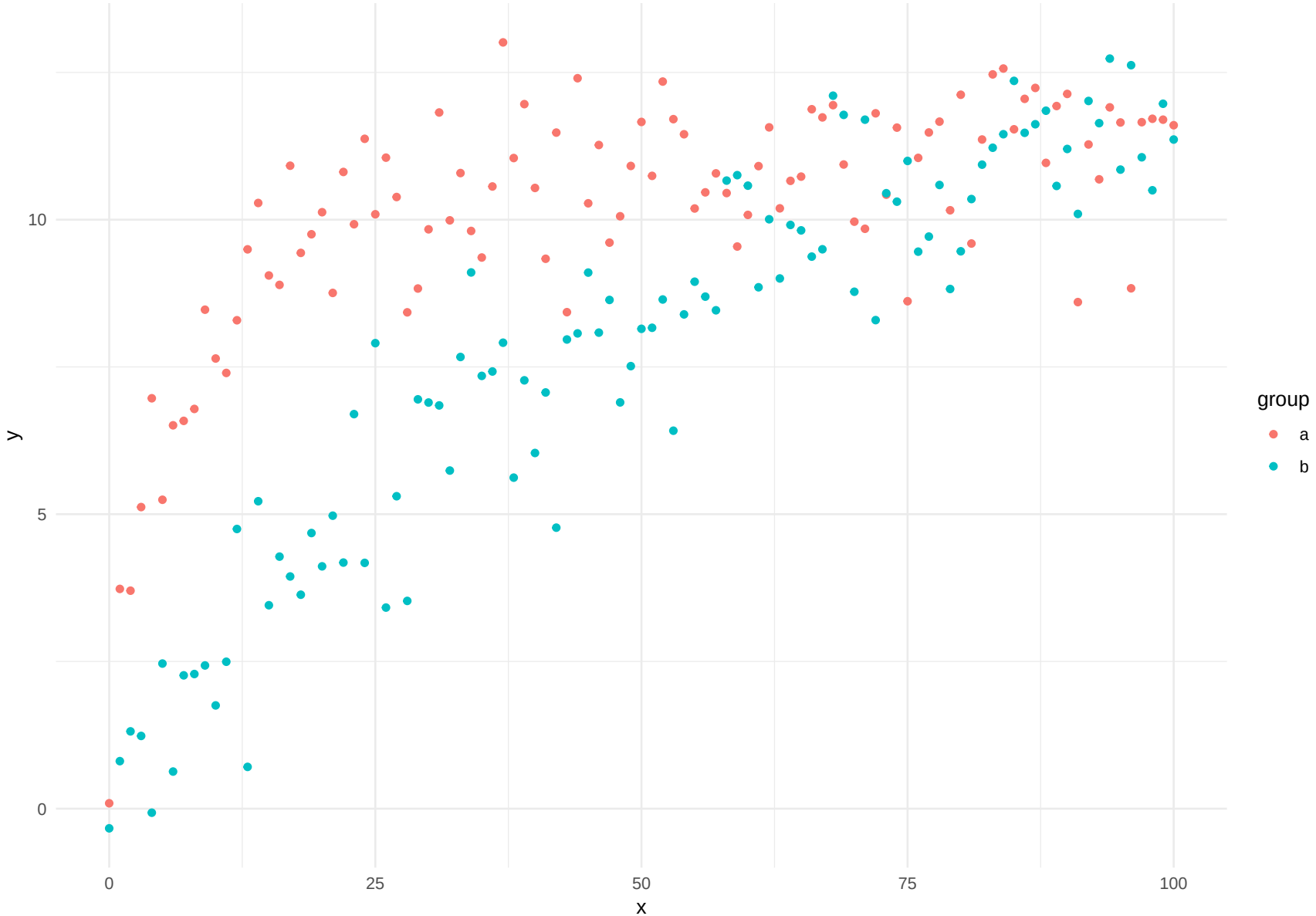
A simple GAM



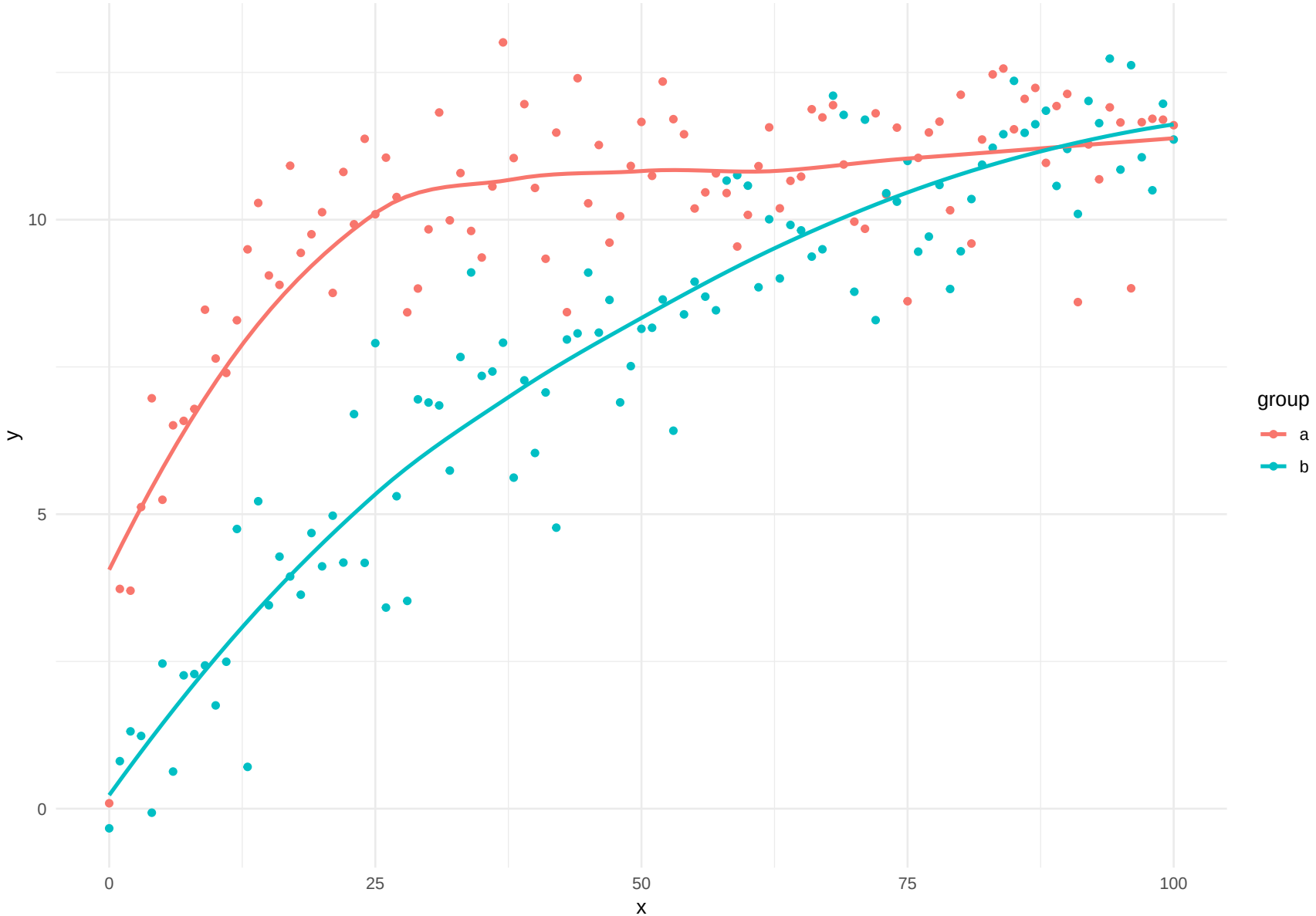
A simple GAM



Comparing levels



Comparing levels



Comparing levels

- ordered by-variables

```
compare <- gam(
  y ~
    group +
    s(x, bs = "cr", k = 5) +
    s(x, bs = "cr", k = 5, by = group),
  data = sim_nl
)
```

Comparing levels

- to use ordered `by`-variables
  - change factor to **ordered factor**
  - change factor contrast to **treatment contrast** (`contr.treatment`)
    - the default in ordered factors is `contr.poly`, this won't work
  - include factor as **parametric term**
  - include a **reference smooth** and a **difference smooth** with the `by`-variable

Comparing levels

```
sim_nl <- sim_nl %>%
  mutate(group = ordered(group, levels = c("a", "b")))
contrasts(sim_nl$group) <- "contr.treatment"
```

Comparing levels

```
compare <- gam(
  y ~
    group +
    s(x, bs = "cr", k = 5) +
    s(x, bs = "cr", k = 5, by = group),
  data = sim_nl
)
```

Comparing levels

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ group + s(x, bs = "cr", k = 5) + s(x, bs = "cr", k =
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.1165     0.1096   92.34   <2e-16 ***
## groupb       -2.4947     0.1549  -16.10   <2e-16 ***
```

Compare levels