

An introduction to GAM(M)s

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Outline

Part 1

- ▶ Linear models
- ▶ Introduction to GAM theory
- ▶ Comparing groups
- ▶ Significance testing

Part 2

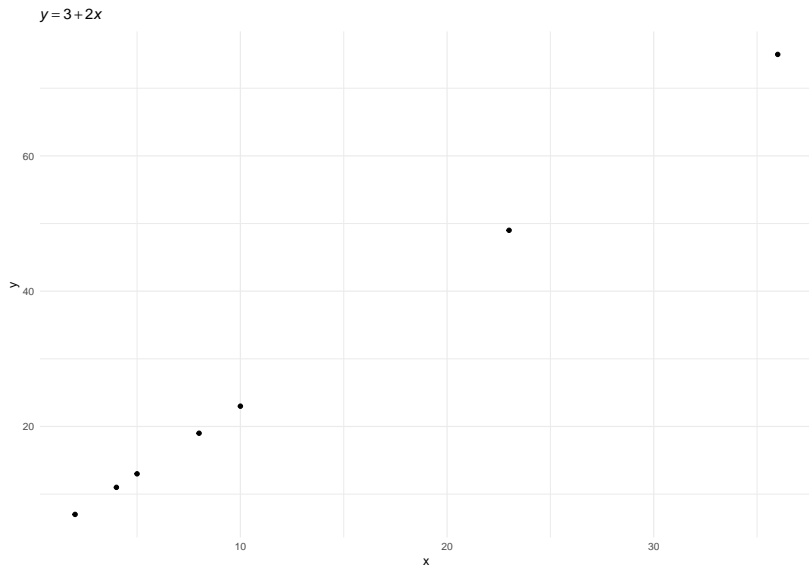
- ▶ Dynamic data
- ▶ Random effects
- ▶ Interactions

Time travel...

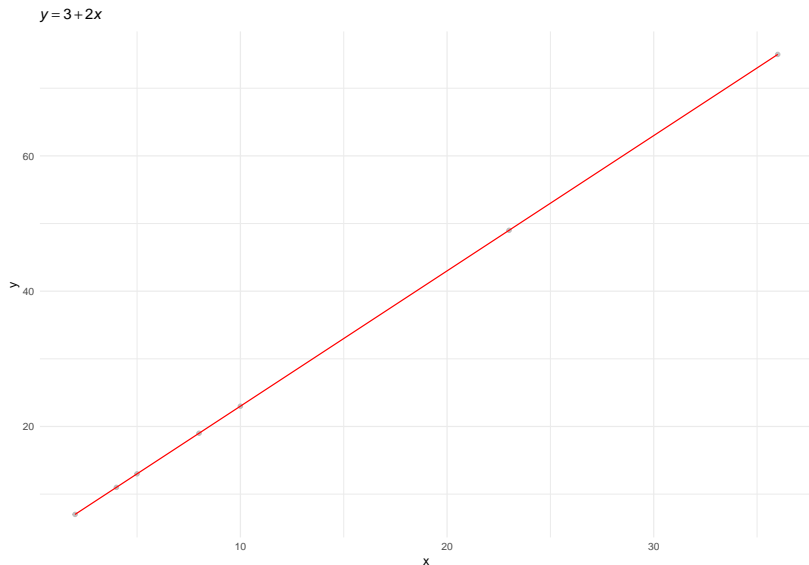
$$y = 3 + 2x$$

where $x = (2, 4, 5, 8, 10, 23, 36)$

Linear models



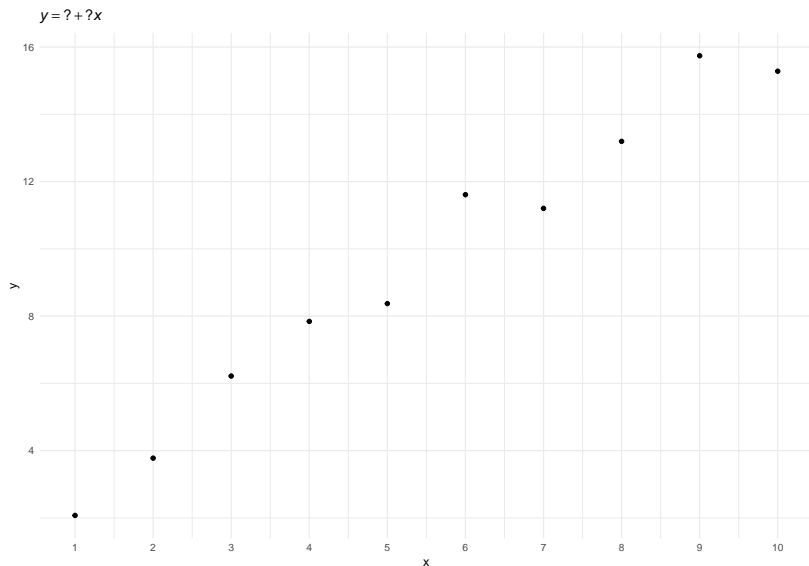
Linear models



Linear models

- ▶ In science, we have x and y ...
- ▶ for example, vowel duration and VOT, speech rate and pitch, etc...

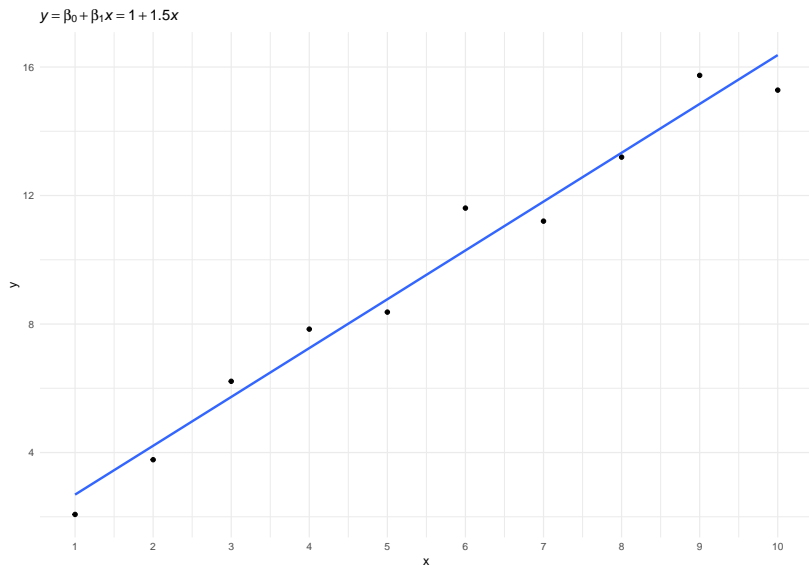
Linear models



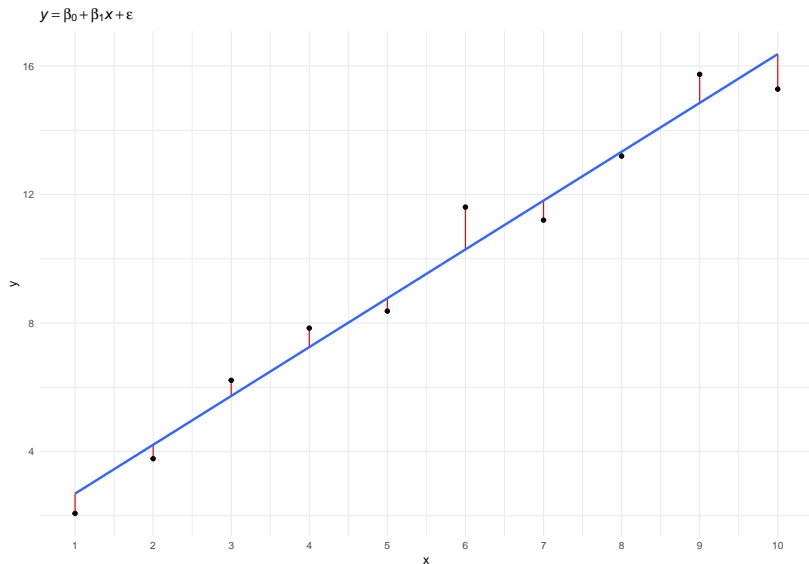
Linear models

- ▶ The formula: $y = \beta_0 + \beta_1 x$
 - ▶ β_0 is the **intercept**
 - ▶ β_1 is the **slope**
- ▶ We know x and y
 - ▶ we need to estimate $\beta_0, \beta_1 = \hat{\beta}_0, \hat{\beta}_1$
- ▶ We can add more predictors
 - ▶ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$
- ▶ `lm(y ~ x, data)` ('y as a function of x')

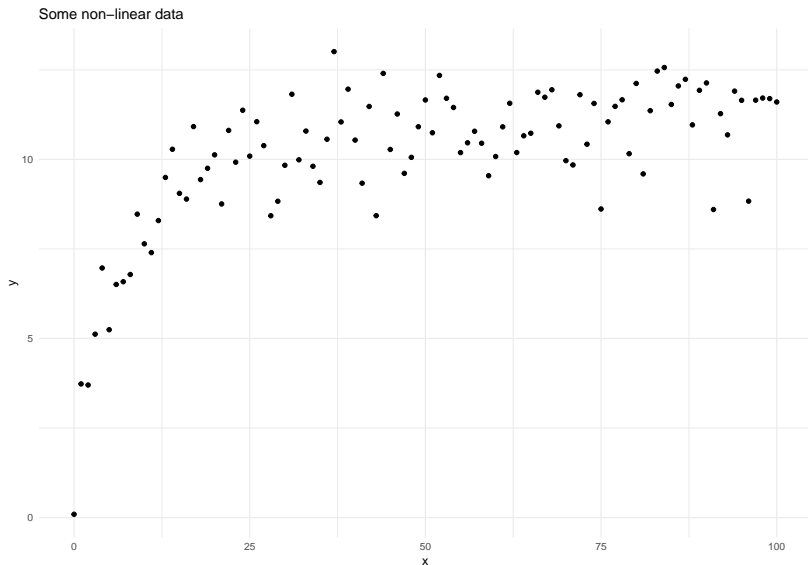
Linear models



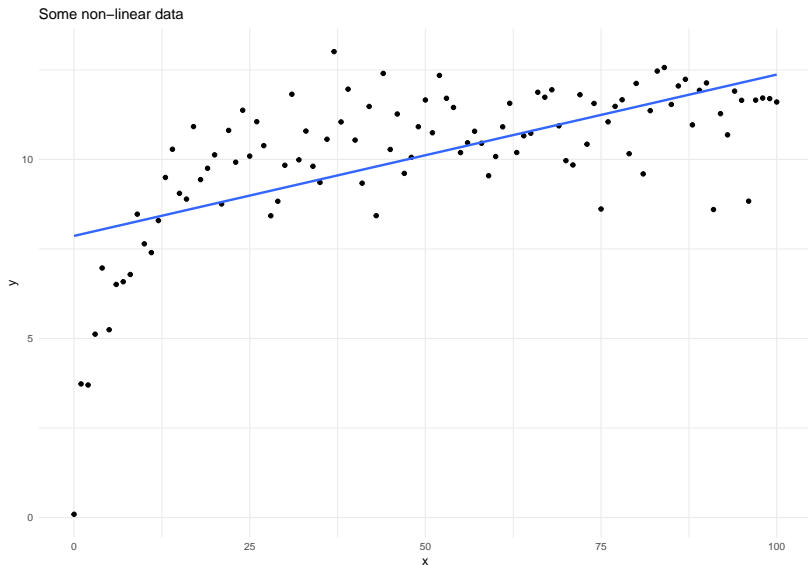
Linear models



LM with non-linear data

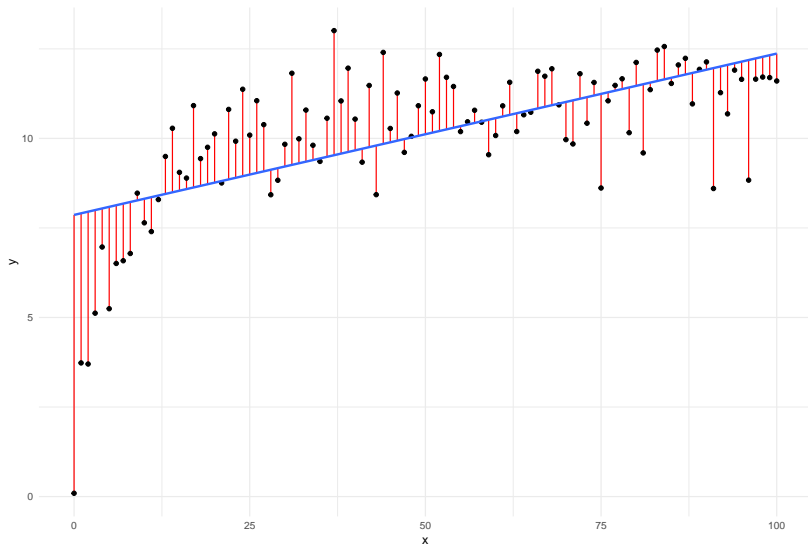


LM with non-linear data



LM with non-linear data

Some non-linear data



LM with non-linear data

How to account for non-linearity in a linear model?

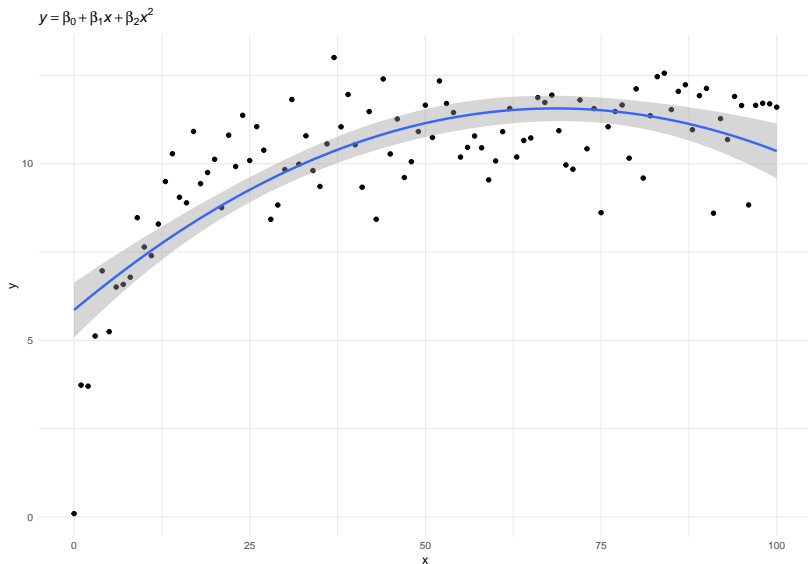
- ▶ Use **higher-degree polynomials**

- ▶ quadratic: $y = \beta_0 + \beta_1x + \beta_2x^2$

- ▶ cubic: $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3$

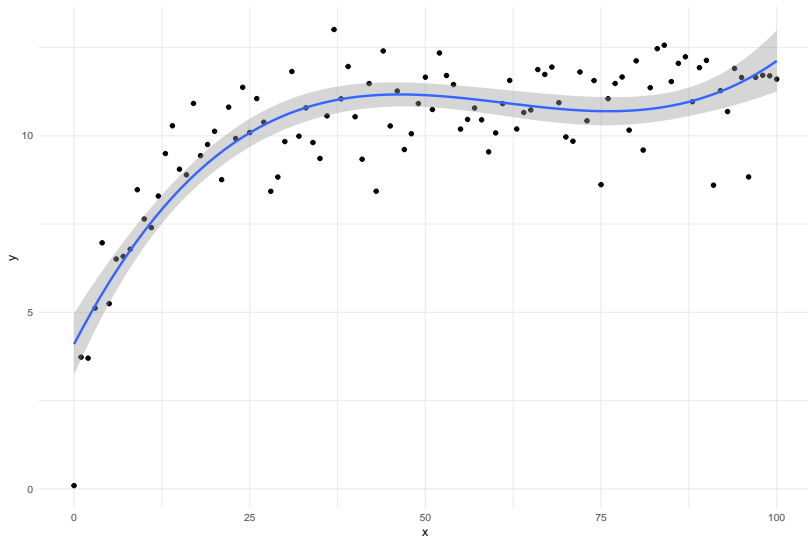
- ▶ n th: $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3 + \dots + \beta_nx^n$

LM with non-linear data



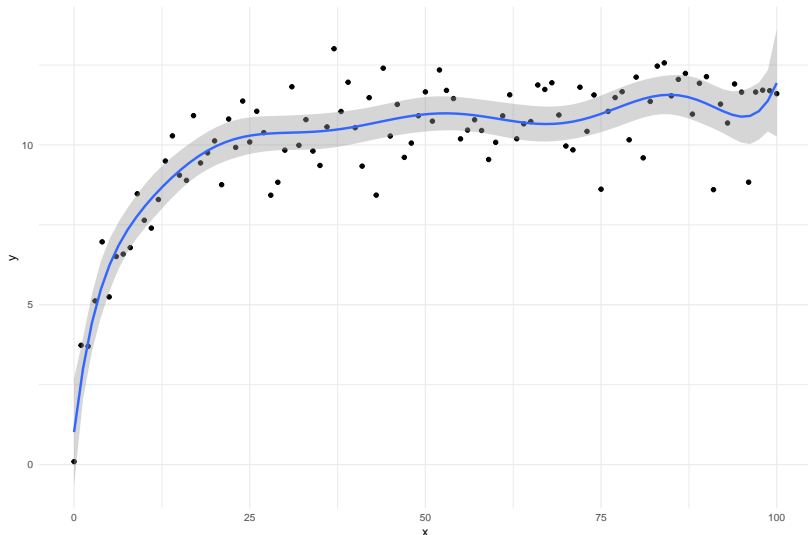
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$



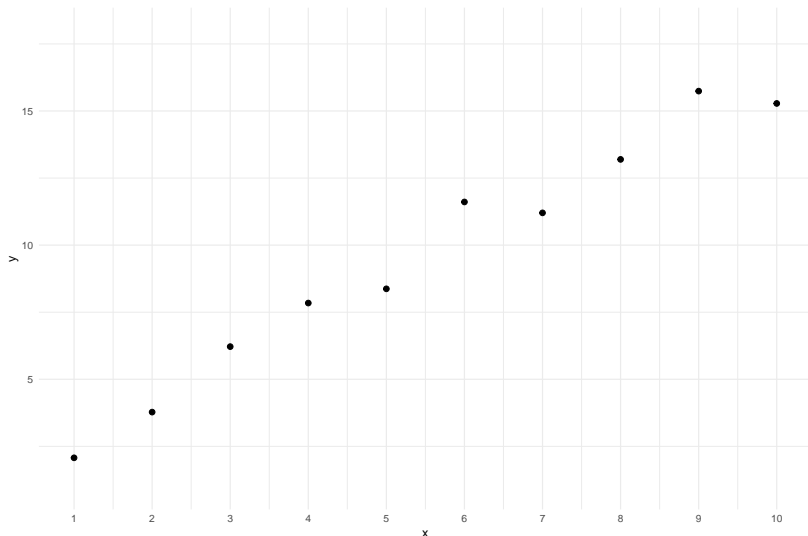
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



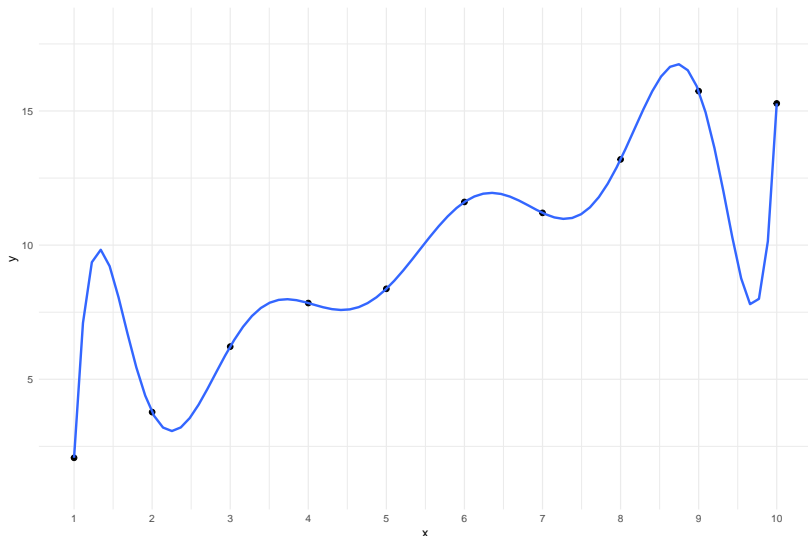
LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



LM with non-linear data

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8 + \beta_9 x^9 + \beta_{10} x^{10}$$



Generalised additive models

- ▶ **Generalised Additive Models**

- ▶ $y = f(x) + \epsilon$

- ▶ $f(x)$ = 'some function of x ' (or *smooth function*)

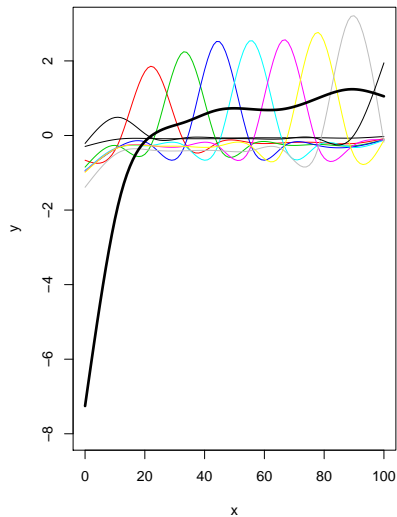
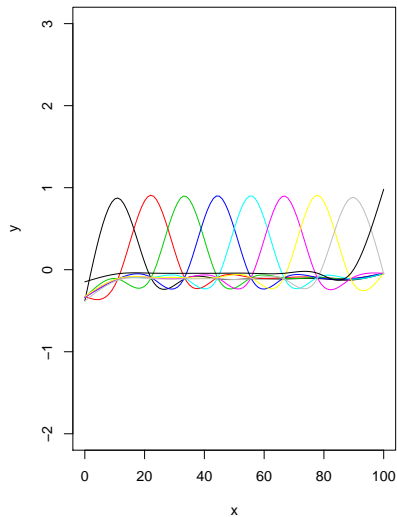
Smooth terms

- ▶ LMs have **parametric terms**
 - ▶ $\beta_n x_n$
 - ▶ x in \mathbb{R}
 - ▶ linear effects
- ▶ GAMs add (non-parametric) **smooth terms** (or simply smooths, also smoothers)
 - ▶ $f(x)$
 - ▶ $s(x)$ in \mathbb{R}
 - ▶ non-linear effects
- ▶ `library(mgcv); gam(y ~ s(x), data), 'y as some function of x'`

Smoothing splines, basis, basis functions

- ▶ Smooths in GAMs are **smoothing splines**
 - ▶ splines are defined piecewise with a set of polynomials
- ▶ The set of polynomials is called a **basis**
 - ▶ the basis is composed of **basis functions** (the polynomials)
- ▶ A spline is the sum of the products of each basis function and its coefficient

Basis functions and knots



Smoothing parameter

- ▶ 'Wiggleness' is related to number of basis functions
 - ▶ more basis functions, more wiggleness (less smoothing)
- ▶ The **smoothing parameter** penalises wiggleness
 - ▶ high values = less wiggleness (more smoothing)
 - ▶ estimated from the data

Smoothing splines

- ▶ There are **several kinds** of splines
 - ▶ each with their own basis functions
- ▶ Most common
 - ▶ *thin plate regression splines*
 - ▶ *cubic regression splines*
- ▶ For more info, run `?smooth.terms`

A simple GAM

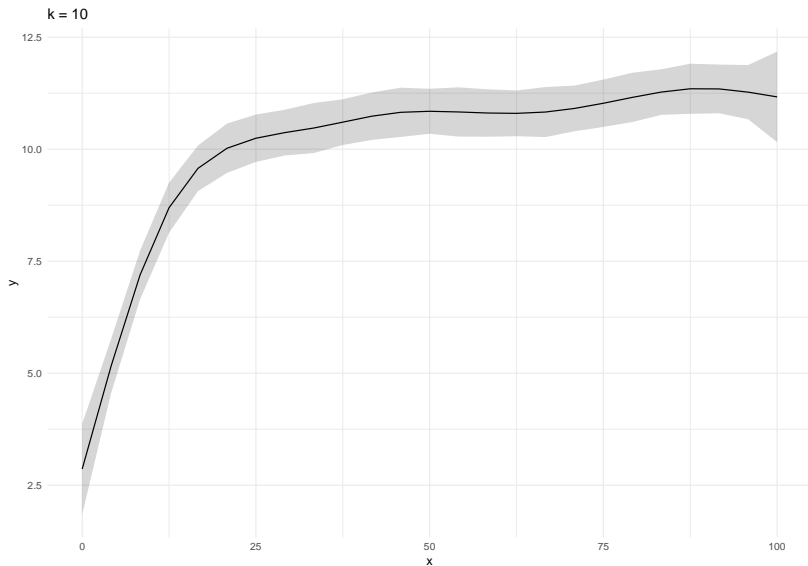
```
simple <- gam(  
  y ~  
    s(x, bs = "cr", k = 10),  
  data = sim_nl_a  
)
```

A simple GAM

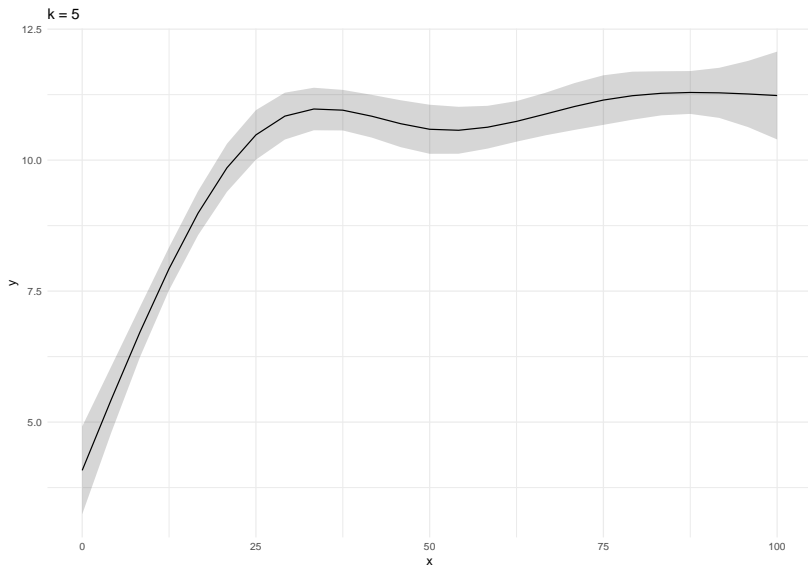
```
summary(simple)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ s(x, bs = "cr", k = 10)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.1165     0.1028   98.37  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##             edf Ref.df    F p-value
## s(x) 6.939   8.01 38.69  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.755   Deviance explained = 77.2%
## GCV = 1.1593   Scale est. = 1.0681    n = 101
```

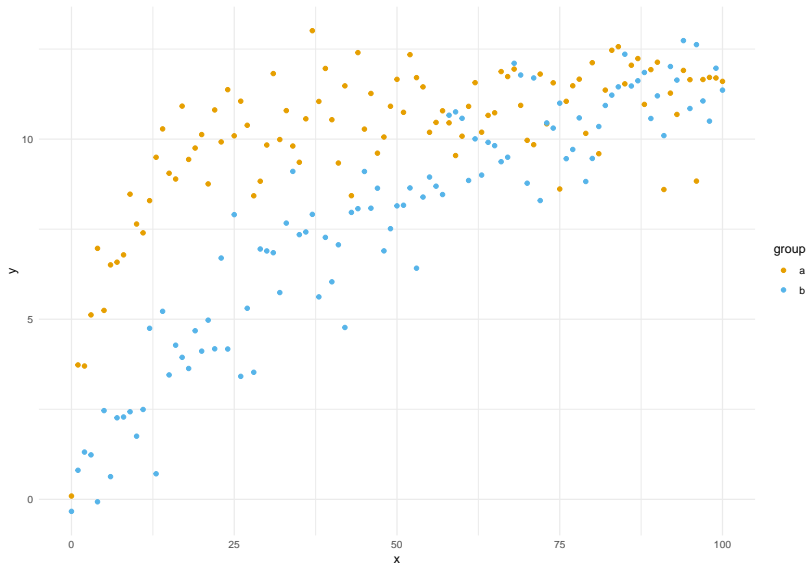
A simple GAM



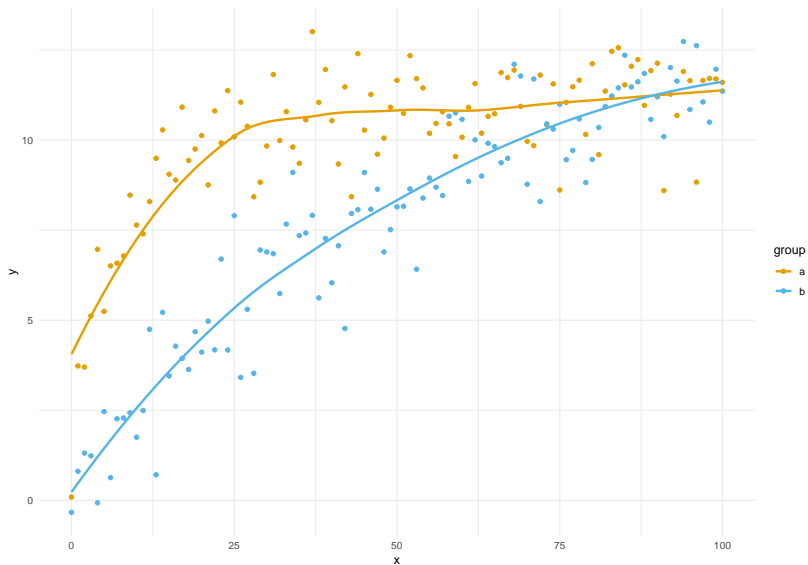
A simple GAM:



Comparing groups



Comparing groups



Comparing groups

- ▶ by-variables with ordered factors

```
compare <- gam(  
  y ~  
    group +  
    s(x, bs = "cr", k = 5) +  
    s(x, bs = "cr", k = 5, by = group),  
  data = sim_n1  
)
```

Comparing groups

- ▶ To use by-variables with ordered factors
 - ▶ change factor to **ordered factor**
 - ▶ change factor contrast to **treatment contrast** (`contr.treatment`)
 - ▶ the default in ordered factors is `contr.poly`, this won't work
 - ▶ include factor as **parametric term**
 - ▶ include a **reference smooth** and a **difference smooth** with the by-variable

Comparing groups

```
sim_n1 <- sim_n1 %>%  
  mutate(group = ordered(group, levels = c("a", "b")))  
contrasts(sim_n1$group) <- "contr.treatment"
```

Comparing groups

```
compare <- gam(  
  y ~  
    # parametric term  
    group +  
    # reference smooth  
    s(x, bs = "cr", k = 5) +  
    # difference smooth  
    s(x, bs = "cr", k = 5, by = group),  
  data = sim_n1  
)
```

Comparing groups

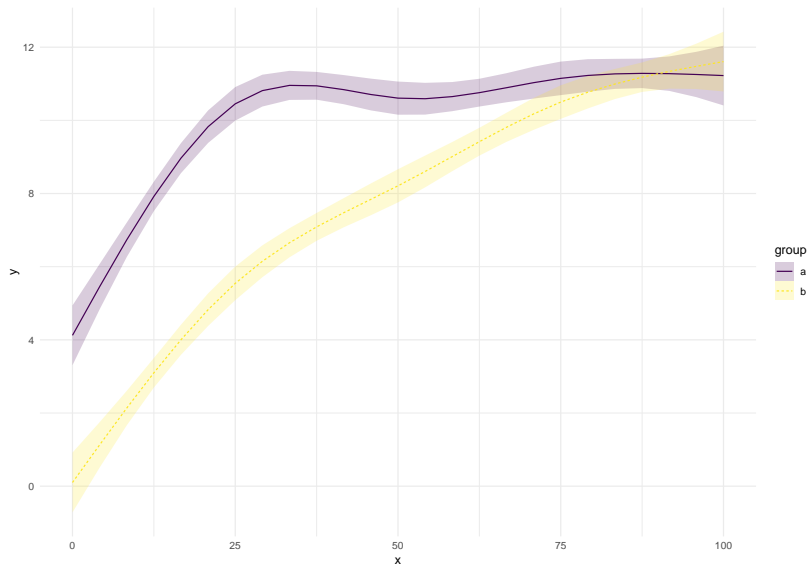
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ group + s(x, bs = "cr", k = 5) + s(x, bs = "cr", k = 5, by = group)
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.1165    0.1096   92.34  <2e-16 ***
## groupb       -2.4947    0.1549  -16.10  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##              edf Ref.df    F p-value
## s(x)          4.000  4.000 64.99  <2e-16 ***
## s(x):groupb    3.576   3.896 39.67  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.873   Deviance explained = 87.8%
## GCV = 1.2725   Scale est. = 1.2122    n = 202
```

Comparing groups

```
library(tidymv)

plot_smooths(
  model = compare,
  time_series = x,
  comparison = group
)
```

Comparing groups



Significance testing

- ▶ Several ways for testing significance of smooths
- ▶ We will use a combined method
 - ▶ model comparison with `itsadug::compareML()` of a full and a null model
 - ▶ visualisation of the difference smooth with `tidymv::plot_difference()`
 - ▶ (you can also use `itsadug::plot_diff()`)
- ▶ Caveats
 - ▶ models need to be fitted with `method = "ML"`
 - ▶ can test either fixed or random effects, not both at the same time

Significance testing

```
group_full <- gam(  
  y ~  
    group +  
    s(x, bs = "cr", k = 5) +  
    s(x, bs = "cr", k = 5, by = group),  
  data = sim_n1,  
  method = "ML"  
)
```

```
group_null <- gam(  
  y ~  
    s(x, bs = "cr", k = 5),  
  data = sim_n1,  
  method = "ML"  
)
```

Significance testing

```
compareML(group_null, group_full)
```

```
## group_null: y ~ s(x, bs = "cr", k = 5)
```

```
##
```

```
## group_full: y ~ group + s(x, bs = "cr", k = 5) + s(x, bs = "cr", k = 5, by =
```

```
##
```

```
## Chi-square test of ML scores
```

```
## -----
```

```
##           Model      Score Edf Difference    Df  p.value Sig.
```

```
## 1 group_null 422.4827    3
```

```
## 2 group_full 314.0105    6    108.472 3.000 < 2e-16 ***
```

```
##
```

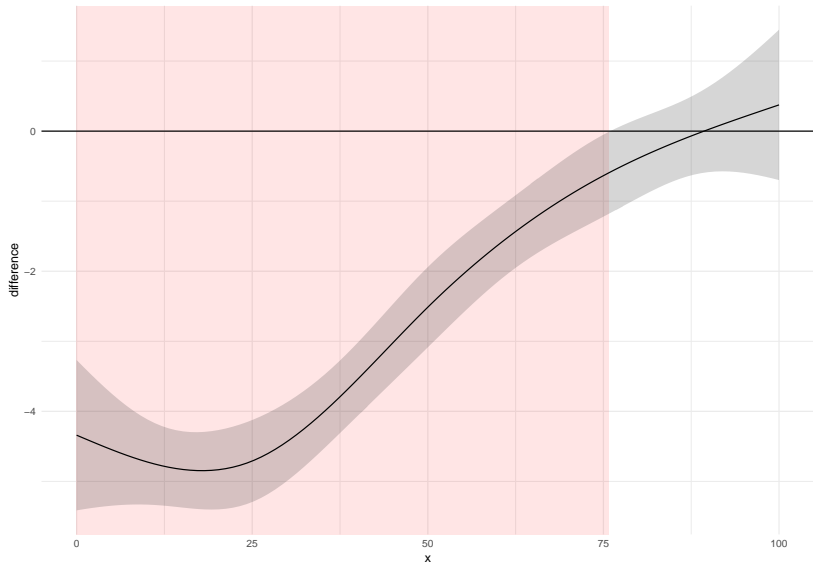
```
## AIC difference: 221.22, model group_full has lower AIC.
```

Significance testing

- ▶ Let's plot the difference smooth with `tidymv::plot_difference()`

```
plot_difference(  
  model = group_full,  
  time_series = x,  
  comparison = list(group = c("b", "a"))  
)
```

Significance testing



Practical 1

Dynamic data

- ▶ “Dynamic speech analysis is a term used to refer to analyses that look at measureable quantities of speech that **vary in space and/or time**” (Sóskuthy, 2017)
- ▶ Examples
 - ▶ formant trajectories
 - ▶ pitch contours
 - ▶ geographic (*diatopic*) variation
 - ▶ tongue contours

Dynamic data

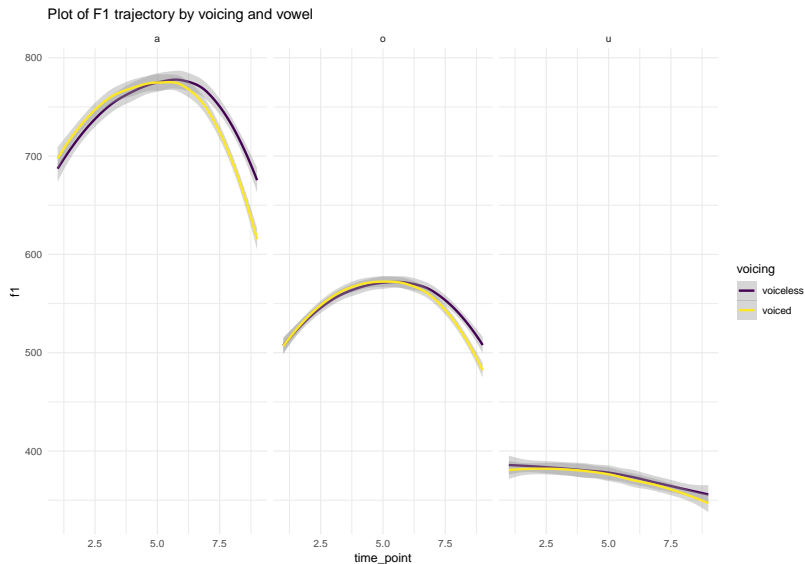
- ▶ Two main types
 - ▶ **time series data**
 - ▶ **spatial data**
- ▶ More data ($n > 1000$)
 - ▶ use `bam()` (big GAM) instead of `gam()`

Dynamic data

► formant trajectories (time series data)

```
## # A tibble: 10,705 x 13
##   speaker index word  time_point    f1    f2    f3    f0 duration vowel
##   <fct>   <chr> <fct>      <int> <dbl> <dbl> <dbl> <dbl>   <dbl> <ord>
## 1 it01   it01~  pugu         1  308.  797. 2280.  137.    95.2 u
## 2 it01   it01~  pugu         2  315.  779. 2124.  134.    95.2 u
## 3 it01   it01~  pugu         3  316.  786. 2314.  134.    95.2 u
## 4 it01   it01~  pugu         4  314.  789. 2374.  135.    95.2 u
## 5 it01   it01~  pugu         5  313.  737. 2307.  137.    95.2 u
## 6 it01   it01~  pugu         6  305.  717. 2315.  138.    95.2 u
## 7 it01   it01~  pugu         7  291.  713. 2318.  138.    95.2 u
## 8 it01   it01~  pugu         8  280.  733. 2308.  137.    95.2 u
## 9 it01   it01~  pugu         9  287.  784. 2329.  136.    95.2 u
## 10 it01  it01~  pada         1  651. 1119. 2155.  120.   139. a
## # ... with 10,695 more rows, and 3 more variables: voicing <ord>,
## #   place <ord>, vow_voi <ord>
```


Dynamic data



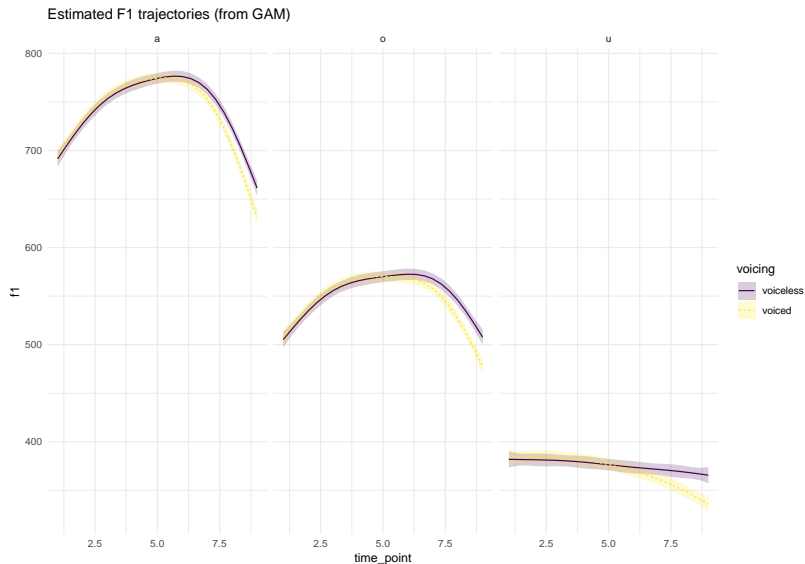
Dynamic data

```
big_gam <- bam(  
  f1 ~  
    voicing +  
    vowel +  
    s(time_point, k = 6) +  
    s(time_point, k = 6, by = voicing) +  
    s(time_point, k = 6, by = vowel),  
  data = vowels  
)
```

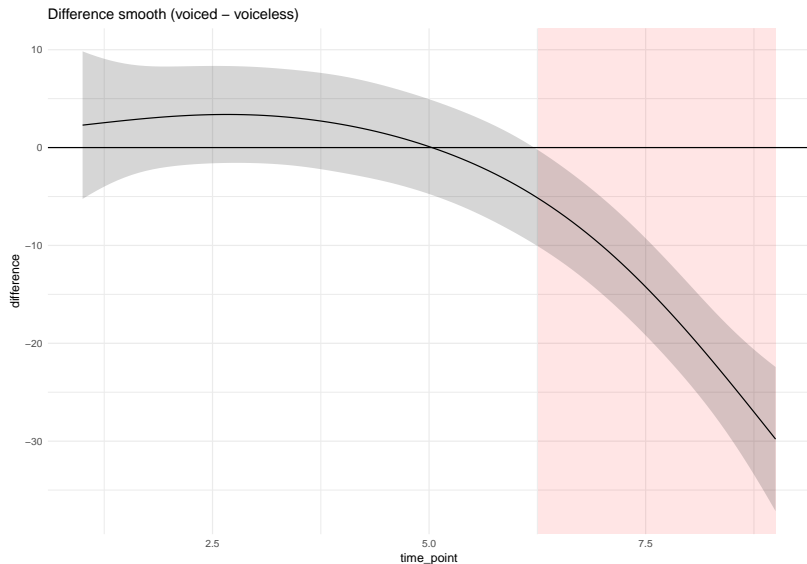
Dynamic data

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## f1 ~ voicing + vowel + s(time_point, k = 6) + s(time_point, k = 6,
##    by = voicing) + s(time_point, k = 6, by = vowel)
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   737.722     1.476  499.932 < 2e-16 ***
## voicingvoiced  -5.788     1.480   -3.911 9.25e-05 ***
## vowelo        -190.218     1.799 -105.734 < 2e-16 ***
## vowelu        -362.253     1.822 -198.779 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##              edf Ref.df      F p-value
## s(time_point)      4.816  4.944 140.16 < 2e-16 ***
## s(time_point):voicingvoiced 2.737  3.331  17.28 7.44e-12 ***
## s(time_point):vowelo      3.663  4.266  17.69 5.40e-15 ***
## s(time_point):vowelu      4.610  4.903  83.36 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Dynamic data



Dynamic data



Random effects

- ▶ Only **fixed effects** so far...
- ▶ **Generalised Additive Mixed Model (GAMM)**
 - ▶ fixed + random effects
- ▶ Include a **random smooth** term with the **factor smooth interaction** as basis

Random effects

- ▶ Factor smooth interaction
 - ▶ `bs = "fs"`
 - ▶ a smooth is fitted at each level of a factor
- ▶ the random effect variable *needs to be a factor*
- ▶ `s(time, speaker, bs = "f")`

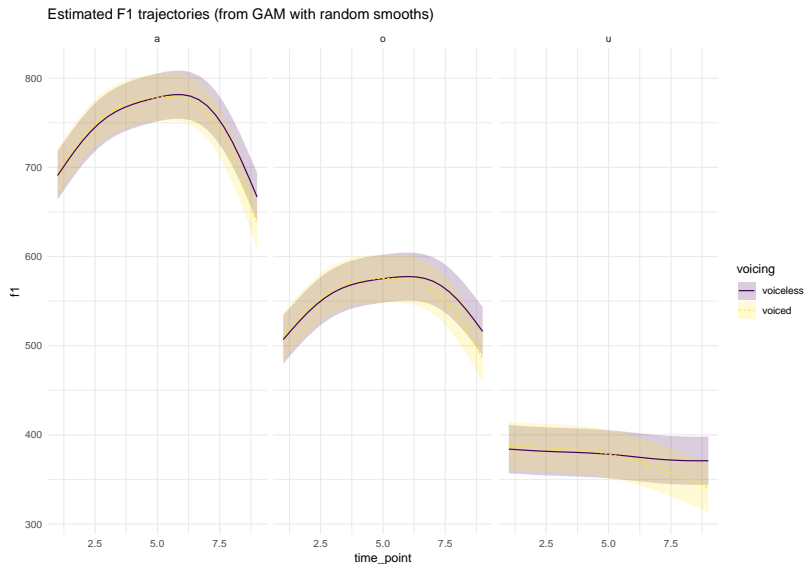
Random effects

```
random_gam <- bam(  
  f1 ~  
    voicing +  
    vowel +  
    s(time_point, k = 6) +  
    s(time_point, k = 6, by = voicing) +  
    s(time_point, k = 6, by = vowel) +  
    # random smooth  
    s(time_point, speaker, bs = "fs", m = 1),  
  data = vowels  
)
```

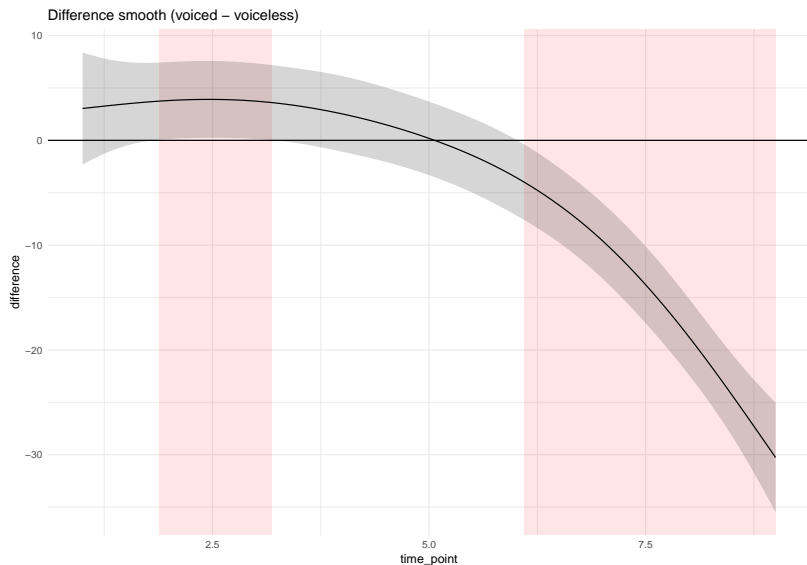

Random effects

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## f1 ~ voicing + vowel + s(time_point, k = 6) + s(time_point, k = 6,
##      by = voicing) + s(time_point, k = 6, by = vowel) + s(time_point,
##      speaker, bs = "fs", m = 1)
##
## Parametric coefficients:
##              Estimate Std. Error  t value Pr(>|t|)
## (Intercept)    741.732    13.384   55.418 < 2e-16 ***
## voicingvoiced   -5.489     1.008   -5.444 5.32e-08 ***
## vowel0         -189.579     1.226 -154.652 < 2e-16 ***
## vowel1         -364.484     1.251 -291.430 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##              edf  Ref.df    F p-value
## s(time_point)    4.853    4.927 152.71 <2e-16 ***
## s(time_point):voicingvoiced 3.174    3.808  33.58 <2e-16 ***
## s(time_point):vowel0    4.194    4.693  37.20 <2e-16 ***
## s(time_point):vowel1    4.811    4.969 189.35 <2e-16 ***
## s(time_point,speaker) 84.021 142.000  86.87 <2e-16 ***
```

Random effects



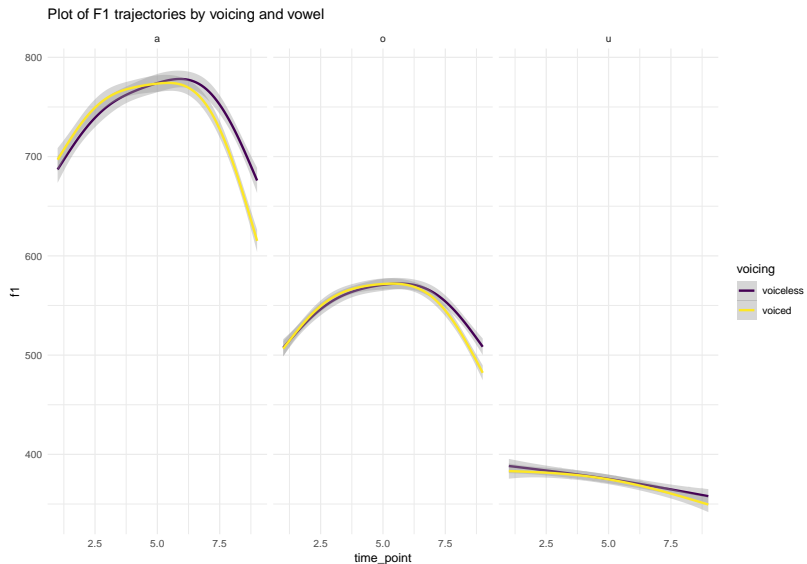
Random effects



Random effects

- ▶ You can also include classical random intercepts and slopes
- ▶ random intercept
 - ▶ `s(speaker, bs = "re")`
- ▶ random slope
 - ▶ `s(speaker, time_point, "re")`

Interactions



Interactions

- ▶ Use factor by-variable with the interaction of the terms
- ▶ Create the interaction with `interaction()`
 - ▶ be sure it is an ordered factor

Interactions

```
vowels <- vowels %>%  
  mutate(  
    vow_voi = interaction(vowel, voicing),  
    vow_voi = as.ordered(vow_voi)  
  )
```

Interactions

```
vowel_gam <- bam(  
  f1 ~  
    vow_voi +  
    s(time_point, k = 6) +  
    s(time_point, by = vow_voi, k = 6) +  
    s(time_point, speaker, bs = "fs", m = 1),  
  data = vowels  
)
```


Interactions

```
## Family: gaussian
## Link function: identity
##
## Formula:
## f1 ~ vow_voi + s(time_point, k = 6) + s(time_point, by = vow_voi,
##      k = 6) + s(time_point, speaker, bs = "fs", m = 1)
##
## Parametric coefficients:
##              Estimate Std. Error  t value Pr(>|t|)
## (Intercept)      743.477    13.408   55.448 < 2e-16 ***
## vow_voio.voiceless -192.215     1.729 -111.157 < 2e-16 ***
## vow_voiu.voiceless -367.254     1.757 -209.026 < 2e-16 ***
## vow_voia.voiced      -9.052     1.732   -5.225 1.77e-07 ***
## vow_voio.voiced    -195.955     1.728 -113.399 < 2e-16 ***
## vow_voiu.voiced    -370.751     1.758 -210.867 < 2e-16 ***
## ---
```

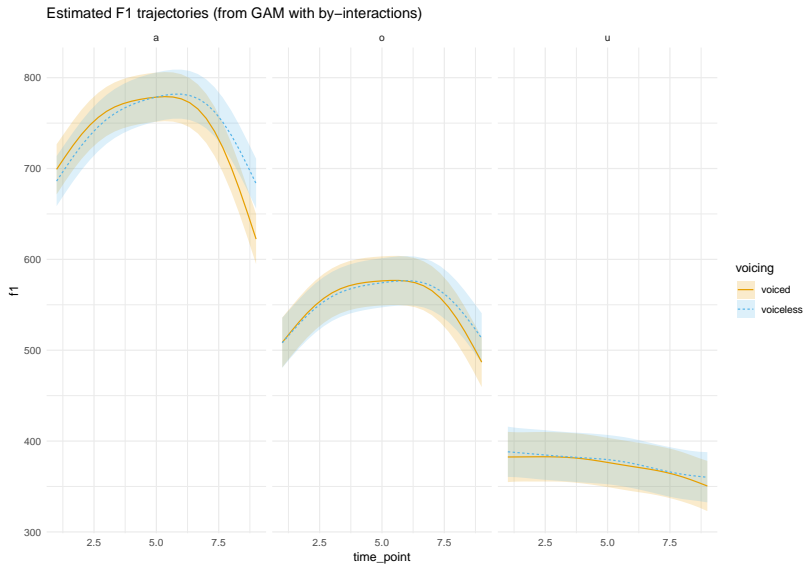
Interactions

```
## Approximate significance of smooth terms:
##
```

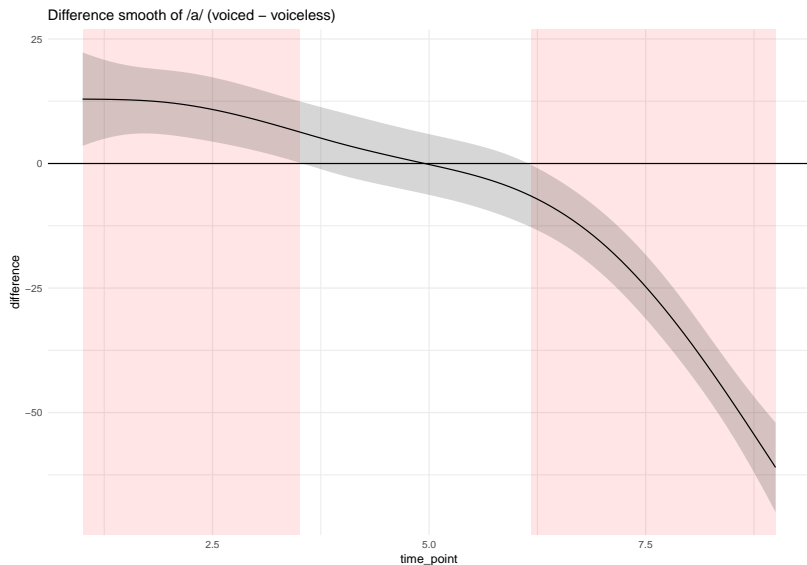
	edf	Ref.df	F	p-value
## s(time_point)	4.838	4.908	113.935	< 2e-16 ***
## s(time_point):vow_voio.voiceless	3.612	4.215	11.604	1.22e-09 ***
## s(time_point):vow_voiu.voiceless	4.668	4.926	89.411	< 2e-16 ***
## s(time_point):vow_voia.voiced	3.715	4.307	43.703	< 2e-16 ***
## s(time_point):vow_voio.voiced	2.863	3.455	8.455	4.79e-06 ***
## s(time_point):vow_voiu.voiced	4.584	4.894	81.688	< 2e-16 ***
## s(time_point,speaker)	84.132	142.000	87.515	< 2e-16 ***

```
## ---
##
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.905   Deviance explained = 90.6%
## fREML = 57607   Scale est. = 2700.4    n = 10705
```

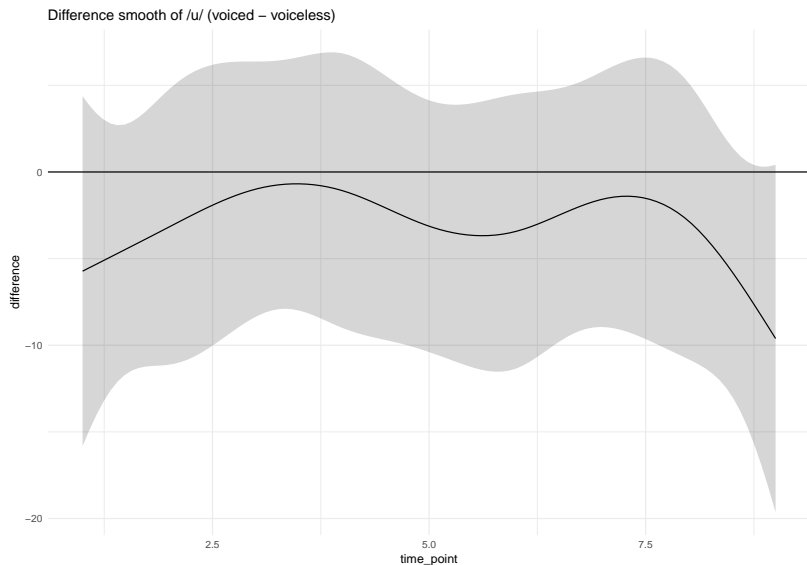
Interactions



Interactions



Interactions



Practical 2

Sóskuthy, Márton. 2017. Generalised additive mixed models for dynamic analysis in linguistics: a practical introduction. arXiv preprint arXiv:1703.05339.