Assignment 1
Financial Engineering

Option price in Matlab

Price a European Call Option with the following characteristics:

Strike price:	1.1 Euro
value date:	15st of February 2008
time-to-maturity (ttm):	4 months (consider a yearfrac 1/3)
volatility:	21.2% (per year)
ttm-zero-rate:	2.5%
Underlying:	equity stock
Dividend Yield:	2%
Settlement:	physical delivery
Number of contracts:	1 Mln
Underlying price	1 Euro

Questions

a. Price the option,
considering an underlying price equal to 1 Euro (i.e a derivative Notional of 1 Mln Euro):
i) via blkprice Matlab function;
ii) with a CRR tree approach;
iii) with a Monte-Carlo (MC) approach.

b. Consider M, as the number of intervals in CRR and as the number of simulations in the MC. Focus on a call. Select M according to the criteria mentioned in the class.
[Hint1: As error for the CRR consider the difference in absolute value w.r.t. to the exact value, while as error for the MC an estimation of the unbiased standard deviation of the MC price]
[Hint2: Option bid/ask for these equity options is 1bp (0.01%).]

c. Show that the numerical errors for a call rescale with M approximately as $1/M$ for CRR and as $1/\sqrt{M}$ for MC.
[Hint3: Show, for a call, the error for CRR and MC varying M in a log-log scale.]

d. Price also a European Call Option with European barrier at €1.4 (up&out) and same parameters with the two numerical techniques (tree & MC). Does it exist a closed formula also in this case? If yes, compare the results.

e. For this barrier option, plot the Vega (possibly using both the closed formula and a numerical estimate) with the underlying price in the range 0.65 Euro and 1.45 Euro. Comment the results.

f. Consider a Call Option with "American" barrier at €1.4 (up&out) and same parameters. Compare the price, Delta and Vega with the European Call Option with European barrier at €1.4 (up&out). Discuss the results.

g. [Facultative]. Does antithetic variables technique (Hull 2009, Ch.19.7) reduce MC error of point b.?

h. [Facultative] Price also -with the Tree- a Bermudan option, where the holder has also the right to exercise the option at the end of every month, obtaining the stock at the strike price K. Does it significantly differ from a Call Option Price?

i. [Facultative] Pricing the Bermudan option, vary the dividend yield between 0% and 5% and compare with the corresponding European price. Discuss the results.

What if

What if the forward can assume negative values. Can you solve the SDE and price a Call option?

Start playing with Python

Install python (you can use the last stable release), jupyter and the IDE you prefer (e.g. PyCharm or Visual Studio). You can also consider the attached instructions if they are helpful to you.
Complete the following exercises using python [between squared parenthesis the suggested library]. Deliver a jupyter notebook (i.e. a .ipynb file)

a. Consider the matrix 5x2, using a dataframe [Pandas, Polars]

	Col0	Col1
Idx0	1	2
Idx1	3	4
Idx2	5	6
Idx3	7	8
Idx4	9	10
Idx5	11	12

   - obtain the new matrix with one unit more in the zero-based position (3,2) and two units more in the cell with column named "Col1" row named "Idx0"

b. Compute $\log\left(\frac{x\,Sinh\,(x)}{1+e^{x^2/2}}\right)$ for $x = 0.4$ [Numpy]

c. Find the max of $xe^{(x-\mu)^2/2}$ with $\mu = 1$ [Scipy]

d. Plot with Matplotlib [Matplotlib]
   - the function $y = sin(x)$ for $x \in [-2\,\pi, 2\pi]$
   - the conic $x^2 - 2\,\rho\,x\,y + y^2 = 1$ for $\rho = \frac{1}{2}, \frac{1}{\sqrt{2}}$

e. Consider the values in the matrix obtained in ex.a 5 observations with 2 values, consider a linear model and fit model parameters [Statsmodels]

f. Determine (possibly with Quantlib) [Quantlib: https://pypi.org/project/QuantLib/]
   - the yearfrac 30/360 European between the 19th of February '08 and 19th of May '08;
   - the monthly modified adjusted calendar starting on the 31st Oct. '25 (included) and ending six months later.

Function signatures

a. optionPrice=EuropeanOptionClosed(F0,K,B,T,sigma,flag), flag 1 call, -1 put.
b. optionPrice=EuropeanOptionMC(F0,K,B,T,sigma,N,flag)
c. optionPrice=EuropeanOptionCRR(F0,K,B,T,sigma,N,flag)
d. [M,errorCRR]=PlotErrorCRR(F0,K,B,T,sigma); M=2^m with m=1:10; (return row vectors)
e. [M,stdEstim]=PlotErrorMC(F0,K,B,T,sigma); M=2^m with m=1:20; (return row vectors)
f. optionPrice=EuropeanOptionKOMC(F0,K, KO,B,T,sigma,N)
g. optionPrice=EuropeanOptionKOCRR(F0,K, KO,B,T,sigma,N)
h. gamma= VegaKO(F0,K,KO,B,T,sigma,N,flagNum), flagNum=1 CRR, flagNum=2 MC, flagNum=3 Exact.

Comment codes, use explicative variable names and divide into sections. It's IMPORTANT to use **exactly the same** signatures provided above. If not discuss why.

Delivery Date: Wed 4th of March @ 18:00. Pay attention to the delivery address below.
Email Subject: Follow rules instructions