Project: Interaction Control (Human-Quadcopter)

Written Report on the Final Project for EiR-UAVs Module

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***Abstract*—This report analyzes the suitability of applying an Admittance Control strategy for the interaction between a Human and a Quadcopter. This strategy allows users to define a desired behaviour to the robot when interaction happens. The forces applied to the system are estimated from the position and orientation information coming from the vehicles sensors, inputed to the Admittance Controller, that will respectively modify the reference trajectory to accommodate this force. This trajectory is then tracked by an underlying Position and Attitude Controller that is built with a Backstepping approach for the specific case of near-hover conditions. The strategy used is followed by simulations that show the method is applicable for such task.**

I. Introduction

This report is based on the work performed and presented in [1]. The proposed task is to apply the framework of Force Control to a Quadrotor, allowing an interaction between the vehicle and its environment, mainly, interaction with humans. This hybrid Position/Force control technique has been already deeply researched for manipulators and ground robots. In fact, this strategy allows to define a mechanical impedance, by regulating an apparent inertia, damping and stiffness as needed [2], [3], providing a desired dynamic behaviour to the system [4], [5].

In this work, we want to exploit the possibility of applying this strategy to a Quadrotor. In section II we present the dynamic model used to describe the Quadrotor, followed by the overall control strategy used to design the Position and Attitude Controller, the Admittance Controller and the Kalman Filter (Section III). Section IV presents the state-space model used for the simulations and finally in section V three scenarios of interaction between Human and Quadcopter are analyzed.

II. Quadcopter Model

Many examples of dynamic models for VTOL's (Vertical Take-Off and Landing vehicles) can be found, refer to [6] for an example. In particular, Quadrotors can be modeled with different approaches depending on the control objective - we refer the reader to [7] for an extensive example that takes into account different effects acting on the quadrotor. However, for the objective of this experiment, a simplified dynamic model was used for the design of the control scheme. Considering near-hover conditions, at constant height, it is reasonable to neglect aerodynamic effects, which has been proved to provide good results for low-speed purposes. [8]

The control technique that will be used is based on a linear approximation of the system dynamics around a desired equilibrium point [6]. The state will be modeled considering a near-hover condition, $\omega = (p, q, r) = (\dot{\phi}, \dot{\theta}, \dot{\psi})$, meaning:

$$\xi = (x, y, z, \dot{x}, \dot{y}, \dot{z}, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, \tau_\phi, \tau_\theta, \tau_\psi)$$

Where,

- $X = (x, y, z)$ is the position of SR_B w.r.t SR_I;
- $\dot{X} = (\dot{x}, \dot{y}, \dot{z})$ is the velocity of SR_B w.r.t SR_I;
- $\Theta = (\phi, \theta, \psi)$ are the RPY angles expressing orientation of SR_B w.r.t SR_I;
- $\dot{\Theta} = \omega = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ are the rotational rates;
- $\tau = (\tau_\phi, \tau_\theta, \tau_\psi)$ are the torques around x_b, y_b and z_b, respectively;
- SR_I is the inertial (world) frame and SR_B is the body frame fixed to the quadrotor, seen in figure 1.

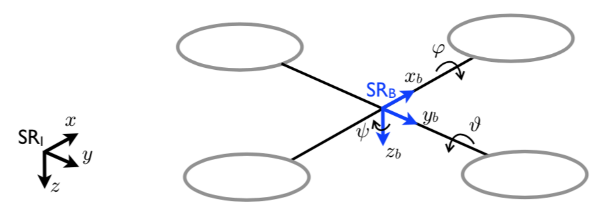

Fig. 1. Configuration of a Quadcopter

In case of hovering and in the absence of wind, it is possible to make a first-order approximation of the rotation matrix about this equilibrium [9], $R = I_{[3x3]} + S(\Theta)$, where S is the skew-symmetric matrix given by:

$$S(\Theta) = \begin{bmatrix} 0 & -\psi & \theta \\ \psi & 0 & -\phi \\ -\theta & \phi & 0 \end{bmatrix}$$

This yields the following linear approximation of the system around the equilibrium $\xi = 0$, considering zero external torque and defining $\vec{e_3}$ as the direction vector of the Thrust vector ($\vec{T}$):

$$m\ddot{X} = -mgS(\Theta)\vec{e_3} - T\vec{e_3} \tag{1}$$

$$\dot{\Theta} = \omega \tag{2}$$

$$I\dot{\omega} = \tau \quad (3)$$

With $T = \tilde{T} - mg$. Finally, if we assume that the quadrotor's inertia matrix is diagonal, $I = diag(I_x, I_y, I_z)$ we can divide the above system into 4 Single-Input-Single-Output (SISO) systems, that will be explored in the next section. [6]

The quadrotors will have 4 control inputs - the collective thrust command and the three rotational body rates:

$$u = (T, \dot{\phi}_{cmd}, \dot{\theta}_{cmd}, \dot{\psi}_{cmd})$$

This follows from [1]. The commanded torque generated will depend on the on-board control loop, that is designed to react as a first-order system to the rotational rates (with time constant T_2). Consequently, by assuming that the generated torque by the vehicle also reacts as a first-order system to the commanded input torques (with time constant T_1), we can write the expressions:

$$\tau_{cmd} = \frac{I}{T_2}(\omega_{cmd} - \omega) \quad (4)$$

$$\dot{\tau} = \frac{1}{T_1}(\tau_{cmd} - \tau) \quad (5)$$

$$\dot{\tau} = \frac{I}{T_1 T_2}\omega_{cmd} - \frac{I}{T_1 T_2}\omega - \frac{1}{T_1}\tau \quad (6)$$

Lastly, it was considered that the quadrotor is equipped with the adequate sensors to capture its position and orientation, being the measurement vector equal to $y = (X, \Theta)$.

III. Control Strategy

The control strategy for the desired task was subdivided into 2 parts: first, the underlying position and attitude controller was designed (section III-A) to receive as input the modified reference trajectory, outputting the corresponding command vector u to the quadrotor; secondly, the admittance controller scheme (section III-B) reacts to an estimation of the force acting on the quadrotor and modifies the desired trajectory to comply with this force.

A. Position and Attitude Controller

Although the Position and Attitude controller for a linearized model of the quadrotor could be based on a simple PID-controller, the fact that the system can be subdivided into 4 SISO systems was exploited to create a Backstepping-like control scheme, that will have a smaller position error when compared to a PID.

Altitude+Yaw Control

Considering first the 2 subsystems concerning the vehicle's altitude and yaw angle, $(X_3, \Theta_3) = (z, \psi)$ we can write:

$$m\ddot{z} = -T \quad (7)$$

$$I_z\ddot{\psi} = \tau_\psi \quad (8)$$

Here, we use a classical PD controller to stabilize the dynamics of these 2 variables, with (z_r, ψ_r) being the input reference:

$$T = -m[k_{zp}(z_r - z) + k_{zd}(\dot{z_r} - \dot{z}) + \ddot{z}_r] \quad (9)$$

$$\dot{\psi}_{cmd} = k_\psi(\psi_r - \psi) + \dot{\phi}_r \quad (10)$$

X+Pitch Control

Considering now the subsystem $(X_1, \Theta_2) = (x, \theta)$, the linear system can be written like:

$$\begin{cases} \ddot{x} = -g\theta \\ \ddot{\theta} = \frac{\tau_\theta}{I_y} \end{cases} \quad (11)$$

Putting this system in lower-triangular form (see appendix of [10]) with $(x_1, x_2, x_3, x_4) = (x, \dot{x}, \theta, \dot{\theta})$:

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -gx_3 \\ \dot{x_3} = x_4 \\ \dot{x_4} = \frac{\tau_\theta}{I_y} \end{cases} \quad (12)$$

We are now in conditions to stabilize the subsystem with a Backstepping approach. Considering x_3 as a virtual control input (see [6]) that stabilizes (x_1, x_2) by doing:

$$\theta_r = x_{3r} = -\frac{1}{g}[k_{xp}(x_r - x) + k_{xd}(\dot{x}_r - \dot{x}) + \ddot{x}_r] \quad (13)$$

The commanded rotational rate can be generated from:

$$\dot{\theta}_{cmd} = k_\theta(\theta_r - \theta) + \dot{\theta}_r \quad (14)$$

Equation (12) represents the state-space linear model of the longitudinal dynamics for the Quadrotor, while equations (13) and (14) are the equations for the controller.

Y+Roll Control

One can find the equations for the lateral dynamics and the corresponding controllers by proceeding similarly as before, but considering now the subsystem $(X_2, \Theta_1) = (y, \phi)$. A full description of the state-space model is found in section IV.

B. Admittance Controller

The successful interaction between Quadrotor and human will be done by adding an outer-loop to the aforementioned control scheme, like seen in figure 2, that changes the desired trajectory by taking into account external forces imposed to the vehicle. We design the controller to impose a desired dynamic behaviour to the interaction, making it behave like a mass-spring-damper system:

$$M(\ddot{X}_d - \ddot{X}_r) + D(\dot{X}_d - \dot{X}_r) + K(X_d - X_r) = -F \quad (15)$$

Where,

- X_d = Desired Trajectory (Input to Admittance Controller);
- X_r = Modified Reference Trajectory;
- M = Apparent Inertia;
- D = Desired Damping;
- K = Desired Stiffness;
- $F = (F_x, F_y, F_z)$ is the applied force to the Quadrotor.

M, D and K are [3x3] diagonal matrices that will impose the desired behaviour to the system, thus can be seen as parameters to tune. This means that, assuming perfect trajectory tracking, when a force acts on the Quadrotor, it will behave according to (15). Since it is desired to let the force define the movement, we set $\ddot{X}_d = \dot{X}_d = 0$. If one desires that the Vehicle doesn't return to tracking the original trajectory, the stiffness can also be defined to be zero, $K = 0$. In section V-C both cases are ilustrated.

The Admittance Controller can be modeled like a linear system as well. By doing $\xi_{ad} = (X_r, v_r)$ and $u_{ad} = (X_d, F)$, with $X_r = (x_r, y_r, z_r)$ and $v_r = (\dot{x_r}, \dot{y_r}, \dot{z_r})$ we can write:

$$\begin{cases} \dot{x}_r = v_r \\ \dot{v}_r = -M^{-1}KX_r - M^{-1}Dv_r + M^{-1}KX_d + M^{-1}F \end{cases} \tag{16}$$

With the output of this system being $y_{ad} = X_r$, that will be input to the Position and Attitude Controller. A full description of the state-space model for the admittance controller is described in section IV.

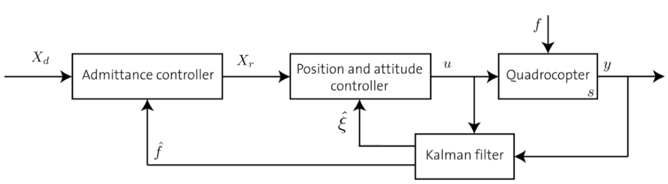

Fig. 2. Full Control Scheme

C. Force Estimation

The Admittance Controller presented in section III-B requires that the forces acting on the Quadrotor are known. A force sensor, however, cannot be used in this case, since the force can be applied anywhere on the vehicle (see [1]). Instead, a Kalman Filter will be used to estimate the state of the system together with the external forces acting on it. For this purpose we need to choose an augmented state for the Kalman Filter, $\xi_k = (\xi, F)$.

However, the linear system's state doesn't account for any external forces acting on it, meaning it is assumed that the force is purely driven by noise. Defining $f = \frac{1}{m}(F_x, F_y, F_z)$ as the mass-normalized force, it is possible to write:

$$\dot{f} = w \tag{17}$$

This allows to extend the state by assuming simply that the force acts on the center of mass of the system, meaning it can be estimated by accounting for the effect of f on the acceleration terms $\ddot{X}$ of the matrices that will define the Kalman Filter. Refer to section IV-B for a complete description of the state-space model for this filter.

IV. State-Space Model

A. Linear Model of the Quadrocopter

Like stated in section II it is possible to describe the dynamic model used in this report as a linear state-space model, by appropriately defining matrices $A_{[15x15]}$, $B_{[15x4]}$, $C_{[6x15]}$ and $D_{[6x4]}$ and using the state and input vector described before. Rewriting equations (1)-(3) and (6), including the external forces (seen as disturbances):

$$\begin{cases} \dot{\xi} = A\xi + Bu + f + w \\ y = C\xi + Du + v \end{cases} \tag{18}$$

Where $w \sim (0, W)$ is the zero-mean additive Gaussian process noise with covariance Matrix $W_{[15x15]}$ and $v \sim (0, V)$ is the zero-mean additive Gaussian measurement noise with covariance Matrix $V_{[6x6]}$.

B. Kalman Filter

The Kalman Filter is responsible for determining the estimate of the state of the Quadrocopter that is input to the Position and Attitude Controller and to estimate the external forces for the Admittance Controller. The augmented system is given by:

$$\begin{cases} \dot{\xi_k} = A_k\xi_k + B_k u + w_k \\ y_k = C_k\xi_k + v \end{cases} \tag{19}$$

Where A_k, B_k, C_k and $w_k \sim (0, W_k)$ have to be augmented accordingly:

$$A_k = \begin{bmatrix} A & A_{k1[15x3]} \\ 0_{[3x3]} & 0_{[3x3]} \end{bmatrix} \tag{20}$$

$$B_k = \begin{bmatrix} B \\ 0_{[3x3]} \end{bmatrix} \tag{21}$$

$$C_k = \begin{bmatrix} C & 0_{[3x3]} \end{bmatrix} \tag{22}$$

And W_k is the [18x18] augmented Process Noise Covariance Matrix. The state estimate, $\hat{\xi}_k = (\hat{\xi}, \hat{F})$, will evolve as:

$$\dot{\hat{\xi}}_k = (A_k - LC_k)\hat{\xi}_k + Ly + B_k u \tag{23}$$

Where L is the Kalman Gain Matrix. Refer, for example, to [11] or [12] for a complete definition on the Kalman Filter.

C. Position and Attitude Controller

Using the state estimation from the Kalman Filter and the reference trajectory from the Admittance Controller as inputs, equations (9), (10), (13) and (14) can be rewritten in the form:

$$u = K_\xi \hat{\xi} + K_r X_r \tag{24}$$

With K_ξ and K_r being the diagonal Matrices of the controller Gains.

D. Admittance Controller

Proceeding in a similar way for the Admittance Controller, we can rewrite equation (16) as a second-order system, using the state ξ_{ad} and the input u_{ad} described in section III-B:

$$\begin{cases} \dot{\xi}_{ad} = A_{ad}\xi_{ad} + B_{ad}u_{ad} \\ y = C_{ad}\xi_{ad} \end{cases} \tag{25}$$

Where the matrices are defined as:

$$A_{ad} = \begin{bmatrix} 0_{[3x3]} & I_{[3x3]} \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \tag{26}$$

$$B_{ad} = \begin{bmatrix} 0_{[3x3]} & 0_{[3x3]} \\ M^{-1}K & -M^{-1} \end{bmatrix} \tag{27}$$

$$C_{ad} = \begin{bmatrix} I_{[3x3]} & 0_{[3x3]} \end{bmatrix} \tag{28}$$

E. Closed-Loop System

Putting together equations (18), (23), (24) and (25) to close the loop, we can observe that the closed-loop eigenvalues will be the eigenvalues of the Position and Attitude Controller with perfect state feedback plus the eigenvalues of the Admittance Controller and the Kalman Filter. [1]

Thus, it is possible to design the Admittance Controller separately without affecting the overall dynamics of the inner-loop. In the next section we take advantage of this fact to build and tune the parameters of the Position and Attitude Controller, to test the Kalman Filter with the extended state and to outline the Admittance Controller, bearing in mind that the stability of each of these blocks guarantees the stability of the entire system.

V. Analysis and Results

A. Regulation / Trajectory Tracking

Firstly, the Position and Attitude Controller was tested alone, to perform parameter tuning and to assess its correct functionality. The simulation was done recurring to a simple Kalman Filter that only estimates the state of the Quadrotor, given the control inputs and the information of position and orientation from the sensors. Minimal process and measurement noise was added to the simulation.

Two simple tasks were given to the Controller: a Regulation problem that asked the Vehicle to go from position (0,0,0) to (-3,3,10), that can be seen in figure 3; a spiral Trajectory Tracking, observable in figure 4.

As it can be seen, the tasks are achieved successfully, with small errors driven from the effect of noise. However, the controller lacks robustness on the z-direction, since the linearization was made around an equilibrium point, at hovering conditions and assuming the Quadrotor is kept at constant height. This means that, although the controller can properly track trajectories in all directions, it will behave better at constant height, given the original task of reacting to forces on the x-y direction.

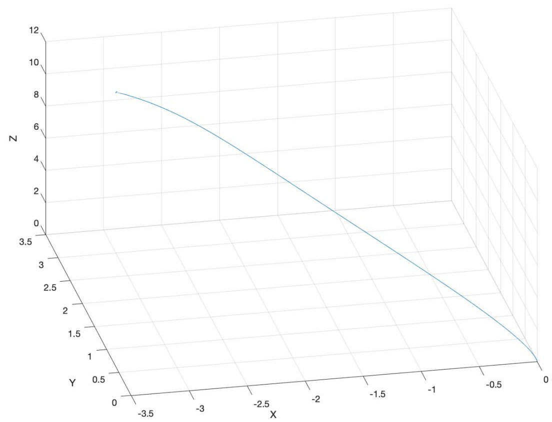

Fig. 3. Test of the Position and Attitude Controller: Regulation Problem

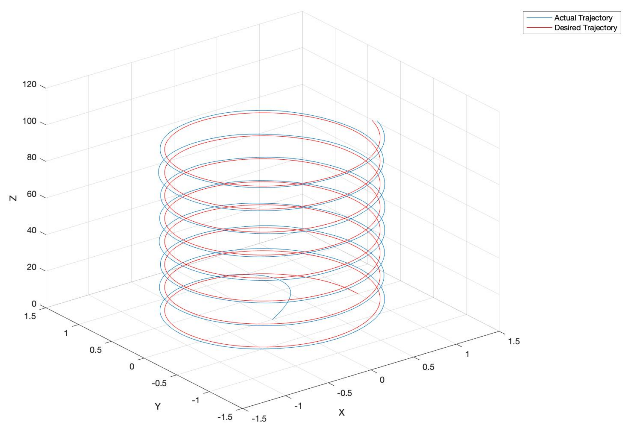

Fig. 4. Test of the Position and Attitude Controller: Trajectory Tracking Problem

B. Stability of the System

Like stated in section IV-E, it is possible to design the Kalman Filter and the Admittance Controller separately, guaranteeing only the stability of all blocks isolated.

For the Position and Attitude Controller, the gains were chosen to guarantee convergence towards the desired trajectory, seen in figure 3 and 4. For the Admittance Controller, since we are imposing the behaviour of a spring-mass-damper system, it is known that the system will be stable for a stiffness different than zero (with 3 pairs of complex-conjugate poles) and marginally stable if the stiffness is set to be zero (with a triple pole at the origin and another real triple pole). As for the Kalman Filter, the poles of the system were determined and plotted in figure 5, where it is observable that all the poles lie on the left-half plane, thus the filter is stable.

C. Quadcopter-Human Interaction

Having guaranteed the stability of the overall system, we can finally proceed to the experiment regarding Human-

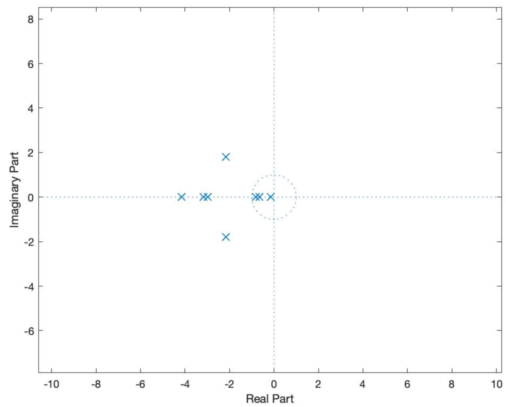

Fig. 5. Poles of the Kalman Filter

Quadcopter interaction. This will be simulated by adding an external force, assumed to be applied in the Center of Mass of the vehicle.

If no force is acting, the control scheme showed above is reduced to the underlying Position and Attitude Controller. Nonetheless, the effect of noise can result in the estimated forces not being zero, even when there are no forces applied. For this reason, a filter is added before the Admittance Controller, that activates it only when the perceived forces are higher than a threshold value (0.3 N). For smaller estimated forces, the admittance controller is ”switched off”.

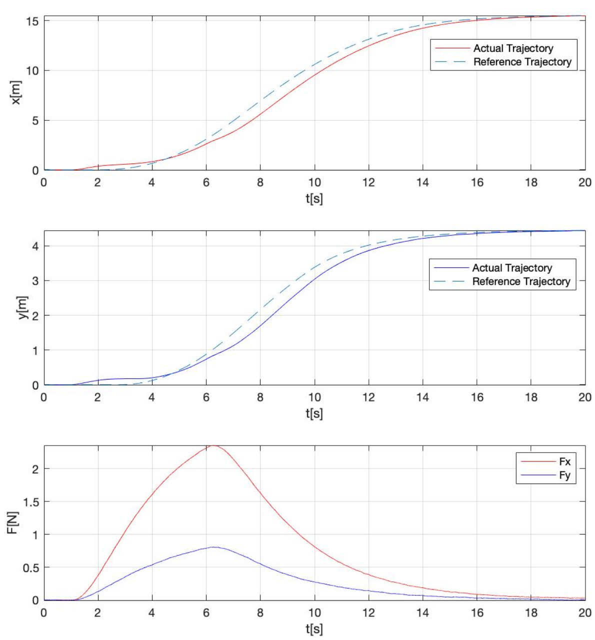

Fig. 6. Application of a constant force to the Quadrotor with stiffness set to zero

The **first experiment** (figure 6) done was to apply a constant force for a small duration of time, with the stiffness set to zero, $K = 0$, meaning the Quadrotor should react to the force but not return to the original position.

Analyzing the graph, where it is illustrated the force estimation on the x-y direction, the reference trajectory determined by the admittance controller (dashed line) and the corresponding response of the Quadrotor, it is possible to see that the vehicle complies with the force, with about 2 seconds of delay. This is the result of the time the Kalman Filters force estimation needs to converge. Evaluating the difference between the Actual and the Reference Trajectory, one can realize that the system starts moving before the Admittance Controller reacts to the force: the vehicle suffers an acceleration on the x-y direction due to the actuating force, but considers it first as a disturbance, therefore trying to keep the initial position until the admittance controller is ”switched on”, about 2 seconds after, changing the reference trajectory to accommodate the force. We can see that the Quadrotor moves from (0,0) to position (15,4) and remains there once it perceives that there is no longer a force acting on it.

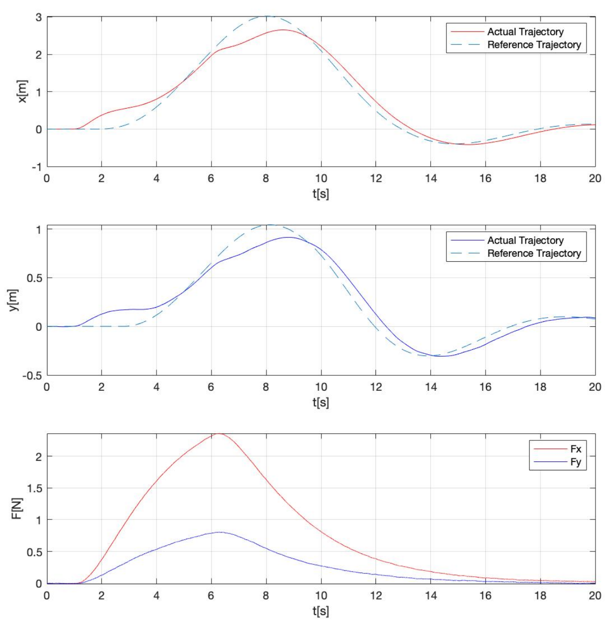

Fig. 7. Application of a constant force to the Quadrotor with stiffness set to 0.9

The **second experiment** is reported in figure 7, where the same force as before was applied with the difference that now the stiffness was set to be 0.9, meaning that the Admittance Controller determines a reference trajectory that takes into account the external forces but also the desired position (that is set to be the origin), i.e, the vehicle will have a (virtual) ”elastic force” trying to make it return to its original configuration. It is possible to observe this: the Quadrotor will only move towards (3,1) and once the force estimation peaks it returns to its original position of (0,0).

The **third experiment**, that can be analyzed in figure 8, was done by applying a force while the Quadrotor was tracking a circular trajectory on the x-y plane. As it can be observed, the system perceives that there is a force applied (t = 20sec) and the Admittance Controller is "switched on" to fit the force into the motion, returning to the original circular trajectory once the force is no longer applied.

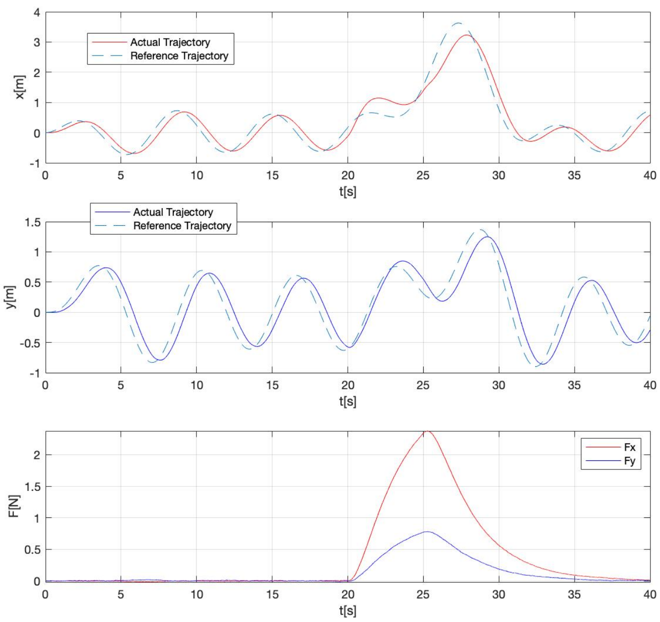

Fig. 8. Application of a constant force to the Quadrotor while it is performing a circular motion

The errors observable in the three experiments, between the reference trajectory and the actual trajectory performed by the Quadrotor, are a consequence of the linearization performed to the dynamics of the system, which has some unmodeled effects. Nevertheless, the results are sufficiently accurate for this kind of task, corroborrating the assumptions made in equations (1)-(3). The effect of measurement and process noise, however small, can also affect the final results.

VI. Conclusion

The Control Scheme presented above provides a good first approach to the problem introduced in section I. Interaction between humans and robots is seemingly increasing and aerial robots can help humans in the future with several tasks that require interaction between them. This framework provides an intuitive way to model the problem, by using a Kalman Filter to estimate the forces applied to the vehicle and an Admittance Controller to accommodate this forces onto a desired trajectory/position, built on top of a Position and Attitude Controller.

Yet, this approach has some limitations, especially due to the delay that exists between the application of the force and its estimation. In the future, this can be improved by equipping the Quadrotor with suitable sensors, that can estimate the force more accurately and more quickly. The hybrid position/force control strategy can also be further exploited, by using a more robust controller that is based on a full dynamic model of the Quadrotor, making it possible to perform harder tasks.

References

[1] Federico Augugliaro, Raffaello D'Andrea, "Admittance Control for Physical Human-Quadrocopter Interaction," *Proceedings of the 12th European Control Conference (ECC)*, pp. 1805–1810, 2013.

[2] B. Siciliano and O. Khatib, Eds., *Springer handbook of robotics*. Berlin: Springer, 2008, oCLC: ocn153562054.

[3] B. Siciliano, Ed., *Robotics: modelling, planning and control*, ser. Advanced textbooks in control and signal processing. London: Springer, 2009, oCLC: ocn144222188.

[4] J. J. Craig, *Introduction to robotics: mechanics and control*, 3rd ed., ser. Pearson education international. Upper Saddle River, NJ: Pearson, Prentice Hall, 2005, oCLC: 249488662.

[5] P. I. Corke, *Robotics, vision and control: fundamental algorithms in MATLAB*, ser. Springer tracts in advanced robotics. Berlin: Springer, 2011, no. v. 73, oCLC: ocn754988355.

[6] Minh-Duc Hua, Tarek Hamel (Member, IEEE), Pascal Morin, Claude Samson, "Introduction to Feedback Control of Underactuated VTOL Vehicles."

[7] R. Mahony, V. Kumar, and P. Corke, "Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor," *IEEE Robotics & Automation Magazine*, vol. 19, no. 3, pp. 20–32, Sep. 2012. [Online]. Available: http://ieeexplore.ieee.org/document/6289431/

[8] Markus Hehn, Rafaello D'Andrea, "Quadrocopter Trajectory Generation and Control," *IFAC World Congress*, pp. 1485–1491, 2011.

[9] A. P. S. Clemens Wiltsche, Raffaello D'Andrea, "Feed-Forward Parameter Identification for Precise Periodic Quadrocopter Motions," *2012 American Control Conference (ACC 2012)*, pp. 4313–4318, 2012.

[10] I. A. Raptis and K. P. Valavanis, *Linear and Nonlinear Control of Small-Scale Unmanned Helicopters*, ser. Intelligent Systems, Control and Automation: Science and Engineering. Dordrecht: Springer Netherlands, 2011, vol. 45. [Online]. Available: http://link.springer.com/10.1007/978-94-007-0023-9

[11] H. M. Choset, Ed., *Principles of robot motion: theory, algorithms, and implementation*, ser. Intelligent robotics and autonomous agents. Cambridge, Mass: MIT Press, 2005, oCLC: ocm54460979.

[12] J. K. U. Simon J. Julier, "Unscented Filtering and Nonlinear Estimation," *Proceedings of the IEEE*, vol. 92, no. 3, pp. 401–422, Mar. 2004.