Interaction Control (Human-Quadcopter)

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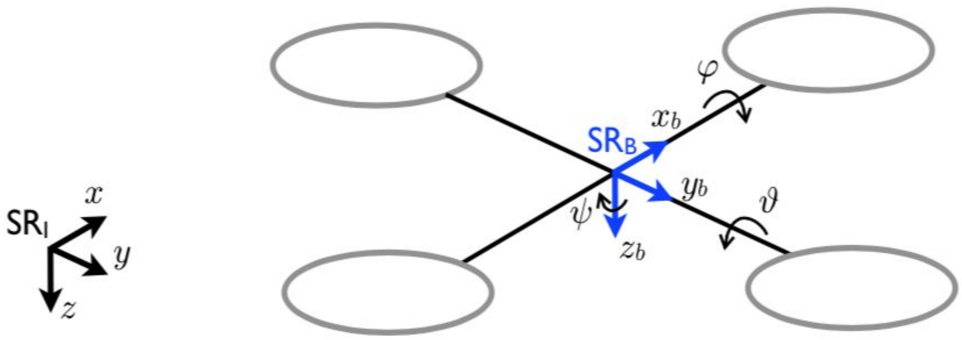
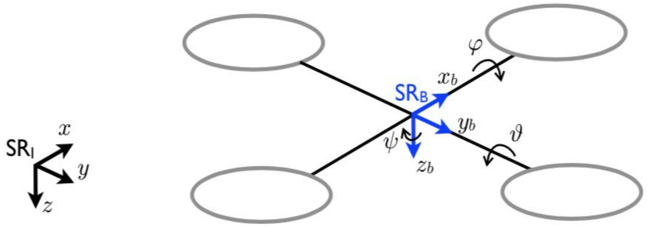
Introduction

- The proposed task is to apply the framework of Force Control to a Quadrotor, allowing an interaction between the vehicle and its environment, mainly, interaction with humans. This hybrid Position/Force control technique has been already deeply researched for manipulators and ground robots, but not for aerial robots.



Quadcopter Model

- In order to design our controller, we performed a linear approximation of the system dynamics around a desired equilibrium point considering:
- Near-hover condition: $\omega = (p, q, r) = (\dot{\phi}, \dot{\theta}, \dot{\psi})$
- State: $\xi = (x, y, z, \dot{x}, \dot{y}, \dot{z}, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, \tau_\phi, \tau_\theta, \tau_\psi)$



$X = (x, y, z)$ is the position of SR_B w.r.t SR_I;

$\dot{X} = (\dot{x}, \dot{y}, \dot{z})$ is the velocity of SR_B w.r.t SR_I;

$\Theta = (\phi, \theta, \psi)$ are the RPY angles expressing orientation of SR_B w.r.t SR_I;

$\dot{\Theta} = \omega = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ are the rotational rates;

$\tau = (\tau_\phi, \tau_\theta, \tau_\psi)$ are the torques around x_b, y_b and z_b, respectively;

SR_I is the inertial (world) frame and SR_B is the body frame fixed to the quadrotor.

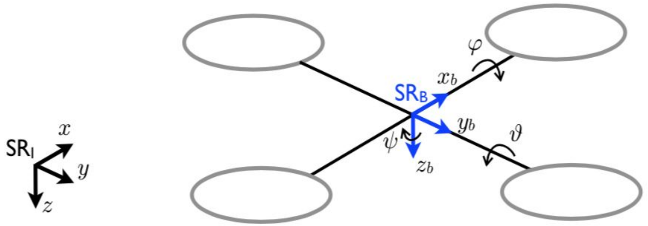
Quadcopter Model

- In case of Hovering and in the absence of wind it is possible to approximate the rotation matrix around this equilibrium, yielding the dynamic equations:

$$m\ddot{X} = -mgS(\Theta)\vec{e_3} - T\vec{e_3} \qquad (1)$$

$$\dot{\Theta} = \omega \qquad (2)$$

$$I\dot{\omega} = \tau \qquad (3)$$



Where S is the skew-symmetric matrix defined by:

$$S(\Theta) = \begin{bmatrix} 0 & -\psi & \theta \\ \psi & 0 & -\phi \\ -\theta & \phi & 0 \end{bmatrix}$$

Quadcopter Model

- It is possible to divide system (1)-(3) into 4 SISO systems. The Quadrotor will have 4 control inputs - the collective Thrust command and the 3 rotational body rates:

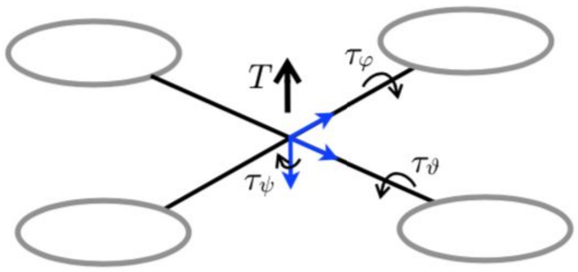
$$u = (T, \dot{\phi}_{cmd}, \dot{\theta}_{cmd}, \dot{\psi}_{cmd})$$

- The generated Torque by the vehicle reacts as a first-order system to the commanded inputs. The commanded torque is designed also to react as a first-order system to the commanded input rotational rates.

$$\tau_{cmd} = \frac{I}{T_2}(\omega_{cmd} - \omega)$$

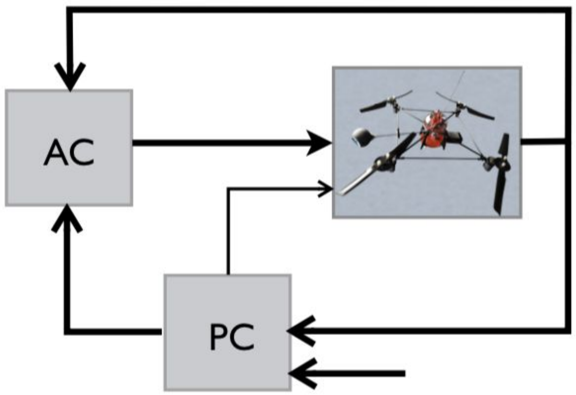
$$\dot{\tau} = \frac{1}{T_1}(\tau_{cmd} - \tau)$$

$$\dot{\tau} = \frac{I}{T_1 T_2}\omega_{cmd} - \frac{I}{T_1 T_2}\omega - \frac{1}{T_1}\tau$$



Position & Attitude Controller

Altitude and Yaw Subsystems	Control Strategy
$m\ddot{z} = -T$	$T = -m[k_{zp}(z_r - z) + k_{zd}(\dot{z}_r - \dot{z}) + \ddot{z}_r]$
$I_z\ddot{\psi} = \tau_\psi$	$\dot{\psi}_{cmd} = k_\psi(\psi_r - \psi) + \dot{\phi}_r$



Position & Attitude Controller

X + Pitch Subsystem | **Control Strategy**

$$\begin{cases} \ddot{x} = -g\theta \\ \ddot{\theta} = \frac{\tau_\theta}{I_y} \end{cases} \xrightarrow{(x_1, x_2, x_3, x_4) = (x, \dot{x}, \theta, \dot{\theta})} \begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -gx_3 \\ \dot{x_3} = x_4 \\ \dot{x_4} = \frac{\tau_\theta}{I_y} \end{cases}$$

$$\theta_r = x_{3r} = -\frac{1}{g}[k_{xp}(x_r - x) + k_{xd}(\dot{x}_r - \dot{x}) + \ddot{x}_r]$$

$$\dot{\theta}_{cmd} = k_\theta(\theta_r - \theta) + \dot{\theta}_r$$

Y + Roll Subsystem | **Control Strategy**

$$\begin{cases} \ddot{y} = g\varphi \\ \ddot{\varphi} = \frac{\tau_\varphi}{I_x} \end{cases}$$

$$\varphi_r = \frac{1}{g}[k_{yp}(y_r - y) + k_{yd}(\dot{y}_r - \dot{y}) + \ddot{y}_r]$$

$$\dot{\varphi}_{cmd} = k_\varphi(\varphi_r - \varphi) + \dot{\varphi}_r$$

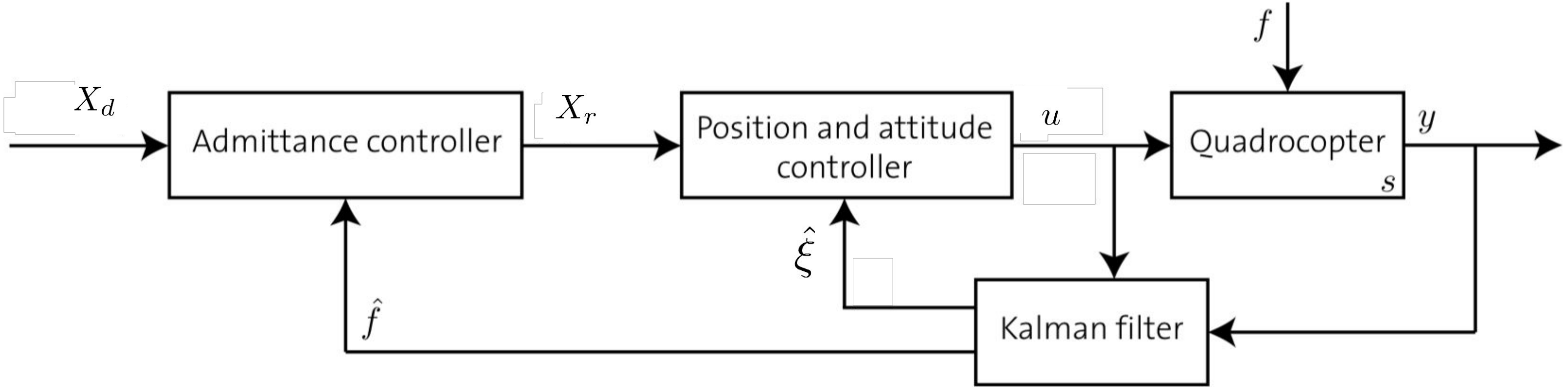
Admittance Controller

- Designed to behave like a Mass-Spring-Damper system that changes the desired reference to comply with the force

$$M(\ddot{X}_d - \ddot{X}_r) + D(\dot{X}_d - \dot{X}_r) + K(X_d - X_r) = -F$$

- X_d = Desired Trajectory (Input to Admittance Controller);
- X_r = Modified Reference Trajectory;
- M = Apparent Inertia;
- D = Desired Damping;
- K = Desired Stiffness;
- $F = (F_x, F_y, F_z)$ is the applied force to the Quadrotor.

Full Control Scheme



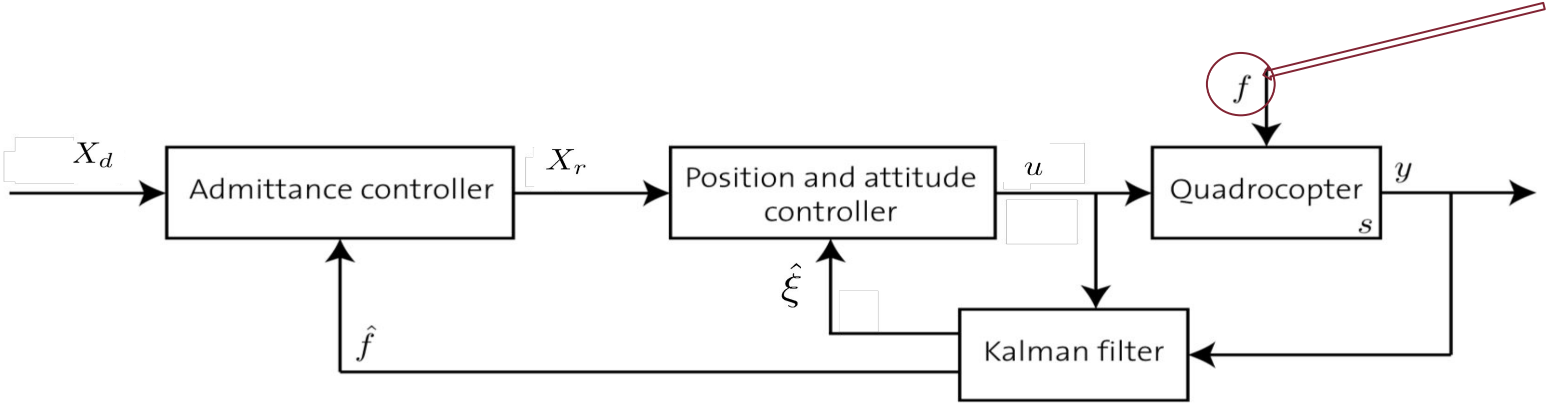
Force Estimation

Kalman Filter with extended state

- A Kalman Filter is used to determine the state of the system together with the external forces acting on it. We thus choose the augmented state:

$$\xi_k = (\xi, F)$$

- We define f = 1/m*(Fx,Fy,Fz) as the mass-normalized force, allowing to assume that the force is purely driven by noise



State Space Model

Linearized Model

Quadcopter

$$\begin{cases} \dot{\xi} = A\xi + Bu + f + w \\ y = C\xi + Du + v \end{cases}$$

Position & Attitude

$$u = K_\xi \hat{\xi} + K_r X_r$$

Kalman Filter

$$\begin{cases} \dot{\xi}_k = A_k\xi_k + B_k u + w_k \\ y_k = C_k\xi_k + v \end{cases}$$

$$A_k = \begin{bmatrix} A & A_{k1[15x3]} \\ 0_{[3x3]} & 0_{[3x3]} \end{bmatrix}$$

$$B_k = \begin{bmatrix} B \\ 0_{[3x3]} \end{bmatrix}$$

$$C_k = \begin{bmatrix} C & 0_{[3x3]} \end{bmatrix}$$

Admittance Controller

$$\begin{cases} \dot{\xi}_{ad} = A_{ad}\xi_{ad} + B_{ad}u_{ad} \\ y = C_{ad}\xi_{ad} \end{cases}$$

$$A_{ad} = \begin{bmatrix} 0_{[3x3]} & I_{[3x3]} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$$

$$B_{ad} = \begin{bmatrix} 0_{[3x3]} & 0_{[3x3]} \\ M^{-1}K & -M^{-1} \end{bmatrix}$$

$$C_{ad} = \begin{bmatrix} I_{[3x3]} & 0_{[3x3]} \end{bmatrix}$$

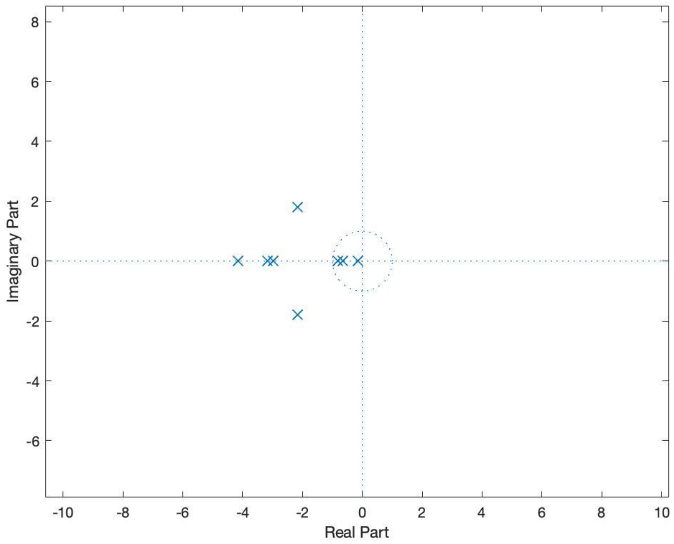
Stability

Admittance Controller

Since we are imposing the behaviour of a spring-mass-damper system:

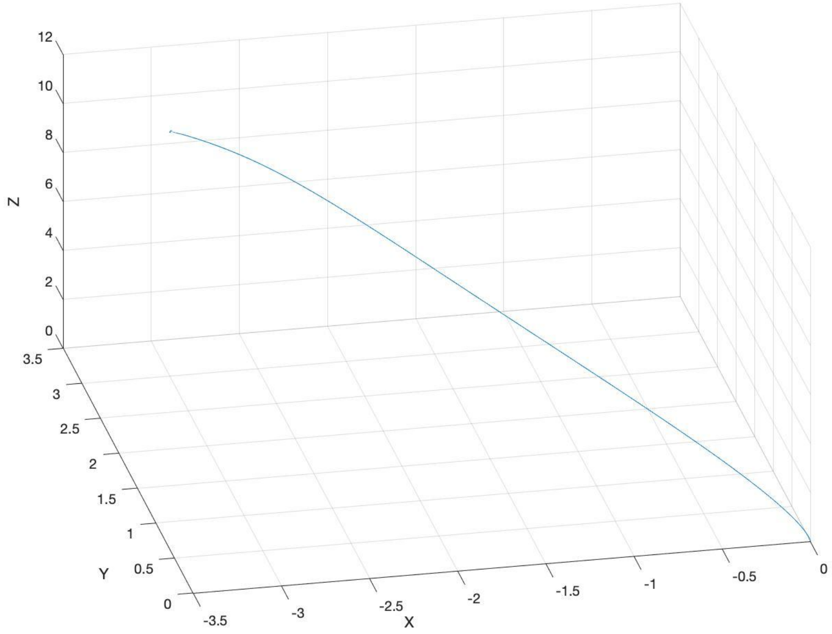
- The system will be stable for a stiffness different than zero (with 3 pairs of complex-conjugate poles)
- The system will be marginally stable if the stiffness is set to be zero (with a triple pole at the origin and another real triple pole).

Kalman Filter

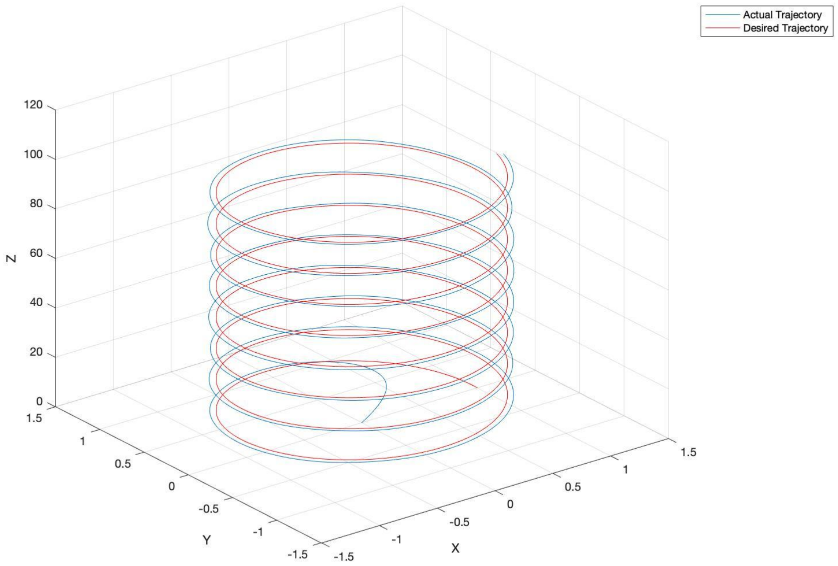


Analysis & Results

Regulation

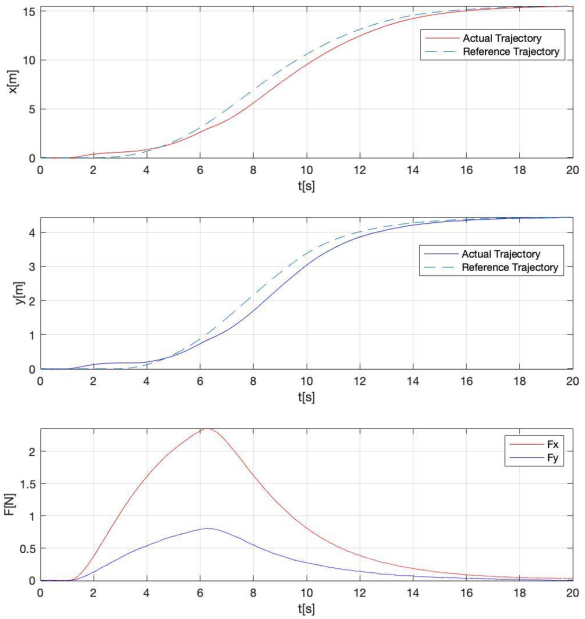


Trajectory Tracking

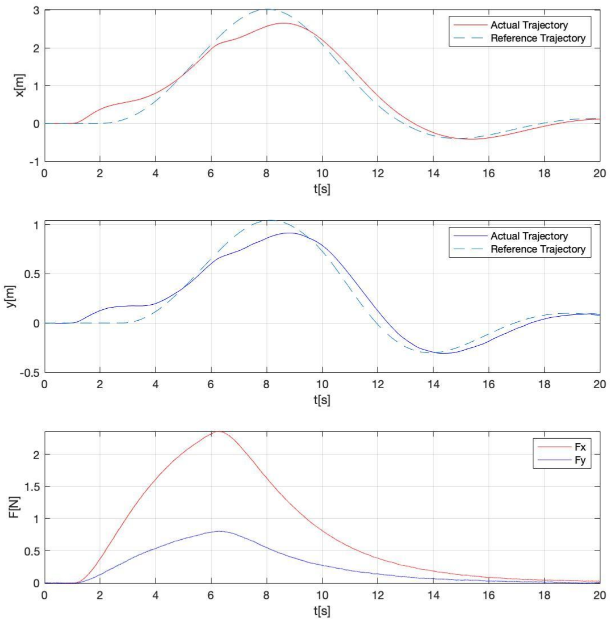


Experiments

1. Constant Force with No stiffness

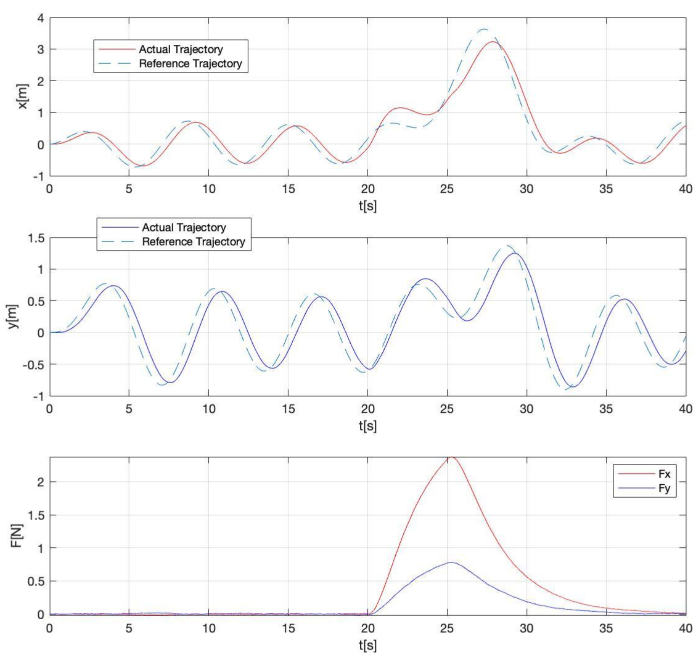


2. Constant Force with Stiffness



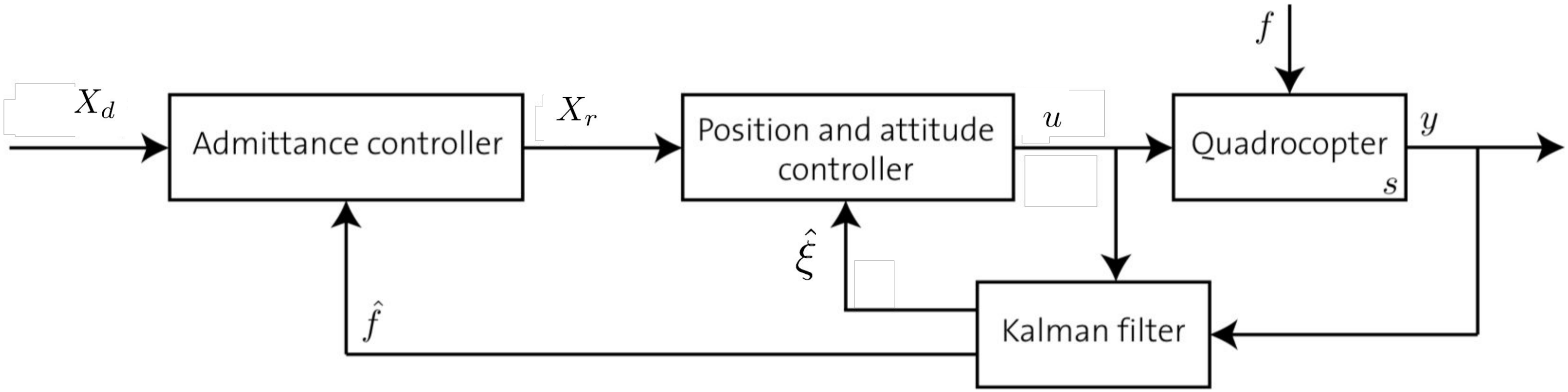
Experiments

3. Constant Force with stiffness while performing circular motion



Conclusion

- Good first approach to the problem with sufficiently accurate results
- Main limitation lies on the delay between the application of the force and its estimation



THANK YOU