Connecting the Dots: The Network Nature of Shocks Propagation in Credit Markets

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November 20, 2023

Abstract

Abstract. We present a simple model of a credit market in which firms borrow from multiple banks and credit relationships are simultaneous and interdependent. In this environment, financial and real shocks induce credit reallocation across more and less affected lenders and borrowers. We show that the interdependence introduces a bias in the standard estimates of the effect of shocks on credit relationships. Moreover, we show that the use of firm fixed effects does not solve the issue, may magnify the problem and that the same bias contaminates fixed effects estimates. We propose a novel model that nests commonly used ones, uses the same information set, accounts for and quantifies spillover effects among credit relationships. We document its properties with Monte Carlo simulations and apply it to real credit register data. Evidence from the empirical application suggests that estimates not accounting for spillovers are indeed highly biased.

Keywords: Credit Markets, Shocks Propagation, Networks, Identification.

JEL Classification: C30, L14, G21.

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1 Introduction

Most research in empirical banking treats credit relationships in isolation, as determined only by firm, bank and relationship's characteristics. Recent attempts at relaxing this assumption have mostly focused on competitive interaction among firms and local spillovers due to sharing geographic locations. To the best of our knowledge, no recent work *jointly* addressed simultaneity in the borrowing decision of the same firm from multiple banks, and in the lending decision of the same bank to multiple firms.

However, credit relationships are embedded in a network of links, originated by sharing the firm or bank to which they belong. As such, shifts in banks' supply to each firm change that firm's relative cost of borrowing across its entire portfolio of relationships. Conversely, shifts in firms' demand for credit change the opportunity cost of lending for each bank, as equity and managerial resources are, after a certain point, scarce.

This paper aims at identifying the importance of spillover effects for the transmission of financial shocks over credit markets. We apply recent advancements in network econometrics to study how ignoring spillovers can affect standard estimates of the bank lending channel (e.g. Khwaja and Mian, 2008; Peek and Rosengren, 2000).[1] Moreover, we present a feasible way to correct the measurement of idiosyncratic shifts in credit demand and supply, as well as the assessment of aggregate effects in the presence of correlated demand shocks (Amiti and Weinstein, 2018; Jiménez et al., 2020), when credit-network spillovers are important.

To present our argument, we introduce an extremely simplified model of credit demand and supply joint determination. We consider the case of two banks and two firms that optimize jointly their full portfolio of relationships, nesting simultaneity in Khwaja and Mian (2008)'s model. We use this framework to argue four points. First, interdependence introduces bias in OLS estimates of the bank lending channel and any other treatment of interest. Second, in the presence of correlated demand shocks, fixed effect estimation may actually worsen the bias.[2] Third, and conversely, if spillovers are large but correlated demand shocks are not, OLS and fixed effects estimates may still differ. As a consequence, interpreting their difference as a measure of demand bias (e.g. Jiménez et al., 2020) may be misleading. Fourth and last, spillovers may contaminate the OLS estimates of firm and bank fixed effects, stopping us short of measuring pure demand and supply shifts (Amiti and Weinstein, 2018).[3]

[1] The name often given in the empirical literature to the sensitivity of credit supply to banks' shocks.

[2] Using firms with multiple bank relationships to perform a fixed effect estimate of supply movements is a standard procedure in the empirical banking literature, first popularized in Khwaja and Mian (2008). A far from complete list of influential works using this strategy in order to assess the effect of different bank shocks includes Jimenéz et al. (2012), Schnabl (2012), Jiménez et al. (2014), Behn, Haselmann, and Wachtel (2016), Bonaccorsi di Patti and Sette (2016), Jiménez et al. (2017).

[3] This point is related to the one risen by Altavilla, Boucinha, and Bouscasse (2022). We elaborate on the differences between the two contributions in the *Related Literature* part of this Introduction.

The key challenge in addressing the concerns we highlight is the reflection problem (Manski, 1993). Reflection is the type of endogeneity that arises when shocks to one agent affect all other agents, and can induce extremely complex distortion patterns. We proceed laying down an estimation framework to address such distortions, building on the literature on spatial autoregressive models (e.g. Arduini, Patacchini, and Rainone, 2015; Bramoullé, Djebbari, and Fortin, 2009; Calvó-Armengol, Patacchini, and Zenou, 2009; Lee, 2007) as extended in Rainone (2020a), to model outcomes on links (credit relationships), instead of nodes (firms or banks). We propose a new method to construct instrumental variables based on overlapping portfolios (OPIVs).

We show that the very structure of the credit market network, in which many nodes are only indirectly connected, provides instruments for spillover identification. In credit markets, firms can borrow from multiple lenders. This feature creates overlapping portfolios of relationships both at the firm and bank level. As idiosyncratic shocks to one relationship can affect the others involving the same parties, they can provide relevant instruments for the spillover effects. If these portfolios are not fully overlapping, such instruments are also exogenous. We show how we can use these OPIVs to identify spillovers and recover unbiased treatment effects, and idiosyncratic demand and supply movements.

The model we propose separately identifies two types of spillovers arising from the network nature of credit relationships, uncovering two channels that affect credit equilibrium outcomes. The first is the *bank credit reallocation effect*, it captures the effect on credit of changes in relationships sharing the same bank, for example through allocation policies of the bank. The second is the *firm credit substitution effect*, which captures the effect on credit of changes in relationships sharing the same firm, for example through substitution of bank credit by the firm. Noticeably, the model can be augmented to have bank- and firm-type specific spillovers, for example if one is interested in studying the substitution of credit from low tech banks to high tech banks by firms, or the reallocation of credit from a certain type of firms to another one by banks, and eventually combinations of the two depending on the specific empirical questions.

We explore the properties of the econometric model through Monte Carlo simulation. First, we study the bank lending channel's estimate bias if we ignore the network nature of credit relationships. We show that both the magnitude and sign of the bias depend on *observables*, as the share of relationships hit by a shock, the number of relationship sharing firms or banks (the density of the network), and *unobservables*, as the size of spillovers. Hence, such bias cannot be addressed without a strategy to estimate cross-network substitution. Second, we confirm that fixed effects can exacerbate spillover bias. What is more, we show that standard estimates of bank and firm fixed effects may pinpoint nonexistent demand and supply idiosyncratic movements, especially for banks and firms that are highly connected (have high centrality in the network). Third, we document that the network estimator performs very well in finite samples, estimating spillovers, treatment effects and idiosyncratic shocks with negligible error. We confirm the economic significance of these results calibrating the network structure of our Monte Carlo simulations to mimic real networks of credit relationships observed in the Bank of Italy's Credit Register.

Finally, we use information from 2012 to 2018 to show that spillover effects play an important role in the Italian credit market. We use complete data from the Bank of Italy's Credit Register matched with firms' financials from CERVED, the main Italian risk rating issuer, and banks' financials from Italian Supervisory Reports. We exploit changes in credit granted due to changes in the interbank rate, mediated by each relationship's exposure to other connected relationships' revolving credit fraction. Revolving credit is a natural channel of idiosyncratic shocks' transmission across firms and banks. For example, firms use revolvers as a buffer for unforeseen needs (see, e.g. Acharya and Steffen, 2020), exposing banks to shocks hitting firms. On the other hand, revolving credit contracts are renegotiated more often, as banks can easily change rates or adjust the granted limit. Hence, firms more dependent on revolvers may be more affected by shocks hitting banks.

All spillover parameters are significant, with the firms' one being particularly large. We demonstrate that spillovers lead to a significant empirical bias in treatment and idiosyncratic effects' estimates. Our exercise suggests that ignoring network spillovers would lead to overestimating the effect of interest rate changes on more revolving-intensive credit relationships by a factor of two. Furthermore, focusing on estimated fixed effects, we suggest that spillover bias may lead to an underestimation of firm fixed effects by a factor of a half to three-fourths, as well as a two-thirds to one overestimation of bank fixed effects.

1.1 Contributions To The Related Literature

We contribute to the empirical literature on financial shocks' pass-through to firms in two ways. First, we offer a methodological contribution, highlighting a possibly major issue with standard instrumental variable (e.g. Paravisini, 2008; Peek and Rosengren, 2000) and within-firm (Jiménez et al., 2014, 2017; Khwaja and Mian, 2008) estimates of bank shocks' effect on firms' credit access. We point out how this flaw may affect (i) attempts at using the difference between OLS and fixed effects estimates to track the extent of demand bias (Jiménez et al., 2020), and (ii) attempts at using estimates of bank and firm fixed effects as direct measures of pure credit demand and supply shifts (Amiti and Weinstein, 2018). We offer a solution to these problems, based on novel and classical results in network econometrics (e.g. Ballester, Calvó-Armengol, and Zenou, 2006; Calvó-Armengol, Patacchini, and Zenou, 2009).[4]

[4] See Jackson (2010), Jackson and Zenou (2015) and Jackson, Rogers, and Zenou (2017) for a complete critical survey of the theoretical literature on the economics of networks. See Bramoullé, Djebbari, and Fortin (2020), De Paula (2020) and Graham and De Paula (2020) for insightful reviews of the literature

This contribution is related to recent works deepening the identification of financial shocks' pass-through to credit and real outcomes. Paravisini, Rappoport, and Schnabl (2023) highlights that firm fixed effects control for credit demand bias only if such bias is uniformly distributed across firms' relationships. Nonetheless, credit may not be perfectly substitutable across different relationships. For example, relationship lending may create a pecking order across different credit providers. These differences may imply that demand shocks do not impact different relationships of the same firm equally. Indeed, Paravisini, Rappoport, and Schnabl (2023) document that banks specialize in funding specific projects, credit is imperfectly substitutable across relationships and this affects the impact of financial shocks.[5]

A further strand, for example Bripi (2021) and Altavilla, Boucinha, and Bouscasse (2022), builds on credit relationships models such as Herreno (2021) and Paravisini, Rappoport, and Schnabl (2023) to estimate structural credit demand systems. In particular, Bripi (2021) focuses on the estimation of credit's price elasticity to study the ease with which firms can shift their demand across different lenders within local markets. Altavilla, Boucinha, and Bouscasse (2022), instead, proposes a granular credit model to accurately measure the magnitude and impact of aggregate shocks. In doing so, it shows that two-way fixed effects models (Amiti and Weinstein, 2018) isolate demand and supply movements only under specific parametric assumptions on elasticities. Such works, though, do not directly address spillover bias.

Then, there is a growing number of recent papers in macro-finance and banking focusing on spillovers' implications for the identification of shocks' effects. For example, Mian, Sarto, and Sufi (2022) proposes a method to recover general equilibrium multipliers from differences in the regional impact of credit supply shocks. Berg, Reisinger, and Streitz (2021) studies how spillovers from firms' interactions may affect the measurement of shocks' effect on firm level outcomes. Moreover, it highlights how within group (fixed effect) identification may worsen spillover bias, in line with our evidence. Finally, Huber (2022) provides overall guidance to empirical researchers on how to deal with multiple contemporaneous spillovers (spatial, competitive, agglomeration, general equilibrium and so on) and in the presence of mechanical bias from the mismeasurement of treatment.

Whereas these works focus on the impact of peers' treatment status (being more or less hit by shocks, i.e., contextual effects) in reduced form, we uniquely focus on identifying and controlling for the effects of peers' outcomes (endogenous effects). Our approach leads us to recover parameters that can be precisely mapped to the primitives of a structural

on network econometrics.

[5] In the same thread, though not focusing on identification problems *per se*, is the ongoing working paper by De Jonghe et al. (2020) and Giometti and Pietrosanti (2022). Within-firm differences across relationships are also a standard topic of investigation in the relationship-lending literature. See, e.g., Petersen and Rajan (1994), Berger and Udell (1995), Bharath et al. (2007), Bartoli et al. (2013), Dewally and Shao (2014), Bolton et al. (2016).

model, thus having a direct behavioral interpretation. Measuring these primitives is especially interesting in the case of credit relationships. The bank-firm relationship is a key determinant of financial shocks' pass-through, but its working is a black box. Our model allows us to unpack the black box and uncover the mechanisms through which the pass-through happens. In particular, we single out the individual importance of firm-level (*firm substitution channel*) and bank-level (*bank credit allocation channel*) reallocation for shock propagation, as well as for the link between such propagation and the credit market structure.

Our investigation complements the recent works by Darmouni and Sutherland (2021) and Gupta et al. (2023), addressing spillovers in banks' credit contract design, the first in SME lending and the other in syndicated lending to large firms, both in the US. The latter paper, in particular, exploits the network structure of banks' overlapping lending portfolios through a spatial autoregressive model to estimate the degree of complementarity in banks' interest rates setting. Nonetheless, using Dealscan data which does not track credit commitments' evolution over time, Gupta et al. (2023) cannot follow changes in credit quantity and, ultimately, how shocks bolster or impair credit access. Instead, we exploit each bank's borrowers' and each firm's lenders' networks to do precisely that.

From a methodological standpoint, our work is close to other corporate finance papers that directly address Manski (1993)'s reflection problem. Recent works did so to quantify peer effects in firms' capital structure (Grieser et al., 2022; Leary and Roberts, 2014), corporate governance (Foroughi et al., 2022), and banks' liquidity choices (Silva, 2019). We cover firm access to credit and financial shocks' pass-through. From the financial shocks' pass-through perspective, our contribution is also complementary to Alfaro, García-Santana, and Moral-Benito (2021) and Huremovic et al. (2020). The latter two works study the propagation of financial shocks across the production network among firms. At the same time, we focus on the propagation of shocks through the credit network among bank-firm links. Our focus on relationships allows us to derive the identification implications of the credit-to-corporate network.

More in general, our approach is related to studies decomposing market's aggregate outcomes to derive instrumental variables, such as shift share instrumental variables (SSIVs), used initially in Bartik (1991), Blanchard et al. (1992) and recently in Borusyak, Hull, and Jaravel (2022) and Goldsmith-Pinkham, Sorkin, and Swift (2020), and granular instrumental variables (GIVs), proposed in Gabaix and Koijen (2020) and applied to banking in Galaasen et al. (2020). However, our approach differs from the GIVs and SSIVs approaches substantially. The GIVs and SSIVs are procedures designed to estimate price elasticities, while OPIVs is designed to estimate objects more similar to elasticities of substitution. OPIVs and both these approaches are actually complements, because they can be used in different types of markets. GIVs and SSIVs consider centralized markets where there is only one price, while the OPIVs consider decentralized markets where the price varies at the pair level. In Section 3.3, where we introduce the OPIVs, we compare them with SSIVs and GIVs in more detail.

In suggesting and applying this approach, we provide our second contribution: A unique quantification of spillover effects across credit relationships, and empirical evidence on the role of credit market structure for shock propagation. Such quantification is closely related to a recent stream of works measuring the link between credit market structure and the effect of financial shocks. Important examples are Andreeva and García-Posada (2021); Benetton (2021); Benetton and Fantino (2021); Corbae and D'Erasmo (2021); Giannetti and Saidi (2019). Again, our work is complementary to Huremovic et al. (2020) and Alfaro, García-Santana, and Moral-Benito (2021). Whereas they document that financial shock propagation worsens when firms' market power is greater, we show that banks' market power has a similar effect. This distinct amplification mechanism may counterbalance the stabilizing role of credit concentration documented by Giannetti and Saidi (2019).

With respect to other works studying peer effects in networks, our context allows us to achieve identification of spillover effects under lighter assumptions. Differently from social networks, credit networks are easier to observe. The detail in credit register data mitigates the concern of unobservable relevant connections, a widespread worry when studying other types of interactions (see Battaglini et al., 2020; Battaglini, Patacchini, and Rainone, 2022; De Paula, Rasul, and Souza, 2019; Miraldo, Propper, and Rose, 2021, among others). Indeed, in this paper the crucial assumption of non-overlapping bank portfolios is testable and always verified in our data.

The rest of this paper is structured as follows: In Section 2 we introduce the toy model of a credit relationships network. In Section 3 we lay down the estimation framework, and explain how network econometrics allows us to identify spillover effects and recover unbiased estimates of spillovers, treatment effects and idiosyncratic shocks. In Section 4 we explore the estimator's characteristics in finite samples, documenting bias behavior if spillovers are ignored and the unbiasedness of our estimator. In Section 5 we use Italian credit register data to show that spillover effects through firm links are highly statistically and economically significant. Section 6 presents a few extensions to the model. We take stock in Section 7.

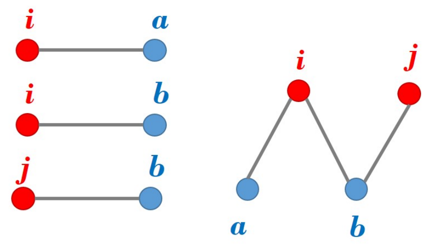
2 Model

In Section 2.1 we draft a simple example to highlight how (i) network spillovers affect the identification of financial shocks' effects; (ii) network spillovers can harm the effectiveness of fixed effects as controls for demand and supply biases; (iii) network spillovers may distort firms and banks' fixed effects estimates, lowering their correlation with actual idiosyncratic demand and supply shifts.

2.1 The Traditional Identification Challenge, With Networks

We present the logic of our results using a simple static model of bank-firm relationships, which modifies Khwaja and Mian (2008)'s to allow firms and banks to optimize simultaneously their full portfolio of relationships. Consider a network of two firms, i and j, and two banks, a and b. Bank a and b both lend to firm i, while bank b also lends to firm j. Figure 1 provides the visual counterpart. While the standard approach takes credit relationships in isolation (panel (a)), we stress how they form a network (panel (b)). Credit relationship ia links to relationship ib, as they share the same borrower (i), and to jb, as they share the same lender b.

Figure 1: A Toy Credit Network



(a) (b)

Notes: Nodes a, b are banks, i, j are firms, edges are credit relationships.

Banks (for illustrative reasons, we focus on bank b) set their credit supply maximizing:

Assumption 1. $\pi_b(c_{ib}, c_{jb}) = \big(r_{ib} - \omega(c_{ib}, x_{ib}, c_{jb}, \nu_{ib})\big)c_{ib} + \big(r_{jb} - \omega(c_{jb}, x_{jb}, c_{ib}, \nu_{jb})\big)c_{jb}$ where : $\omega(c_{ib}, x_{ib}, c_{jb}, \nu_{ib}) = \omega\frac{c_{ib}}{2} - \omega(\xi x_{ib} + \nu_{ib} - \theta c_{jb})$

The ω function captures the cost imposed on the bank by the fraction of each loan which cannot be funded with costless debt. The functional form adapts Khwaja and Mian (2008) to our setting. In particular, c_{ib}, c_{jb} are the quantity of credit supplied to firms i and j; x_{ib} is our **treatment of interest**, some relationship's characteristic that changes the marginal cost of lending to firm i for bank b by $-\xi$ dollars; ν_{ib} is an unobservable random component. Finally c_{jb} enters the function capturing the supply-side of interdependence in lending decisions due to opportunity costs. Everything else equal, if bank b already lends one more dollar to firm j, this rises the cost of lending to i by θ dollars.

We specify the cost function as linear, ω is thus a parameter that captures the baseline cost to the bank of one more dollar of commitment. The resulting supply equations follow from first order conditions:[6]

[6] We choose this specification to match as closely as possible the original by Khwaja and Mian (2008). The assumption of a common ω parameter across banks implies that banks face the same capital market. Deviations from this assumption imply that impact effects are actually heterogeneous in the system we estimate. We will empirically explore the relevance of this possibility in the further sections.

$$
\begin{aligned}
r_{ib} &= \omega c_{ib} - \omega \underbrace{(\xi x_{ib} + \nu_{ib} - \theta c_{jb})}_{u_{ib}} \\
r_{jb} &= \omega c_{jb} - \omega \underbrace{(\xi x_{jb} + \nu_{jb} - \theta c_{ib})}_{u_{jb}} \\
r_{ia} &= \omega c_{ia} - \omega \underbrace{(\xi x_{ia} + \nu_{ia})}_{u_{ia}}
\end{aligned}
\tag{1}
$$

and enter each firm's demand problem as the firm's cost function.

We assume that firms finance a single project from multiple loans, hence credit from each bank is perfectly substitutable.[7] We assume the following functional form for firms' profit function:

Assumption 2. *The firm knows the banks' pricing rules, and decides its credit demand maximizing* $\pi_f(c_{fa}, c_{fb}) = \big(e_i - \alpha(c_{ia} + c_{ib})\big)(c_{ia} + c_{ib}) - \sum_{K=a,b} c_{iK} r_{iK}(c_{iK})$.

Here, e_i is the productivity of firm f's use of funds, α tracks the quadratic decrease in returns to scale and r_{fK} is the loan's cost derived above. Firms' profit maximization results in the following structural demand system:

$$
\begin{aligned}
c_{ia} &= \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia}, \\
c_{ib} &= \rho c_{ia} + \phi c_{jb} + \beta x_{ib} + \delta_i + \epsilon_{ib}, \\
c_{jb} &= \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb}.
\end{aligned}
\tag{2}
$$

We can see that relaxing the assumption that banks and firms optimize their choices only at the single relationship level, and allowing them to more realistically maximize their profits considering all their relationships together, generate a simultaneous system of equations. In this environment, the amount of credit that firm i borrows from bank a (c_{ia}) depends on the amount that firm i borrows from bank b (c_{ib}). On the other hand, the amount of credit that bank b lends to firm j (c_{jb}) depends on the amount that bank b lends to firm i (c_{ib}). In turn, c_{ib} depends on both c_{ia} and c_{jb}, i.e. the single relationship's outcome receive impulses from both the other relationship in which the firm is involved in and the other relationship in which the bank is involved in.

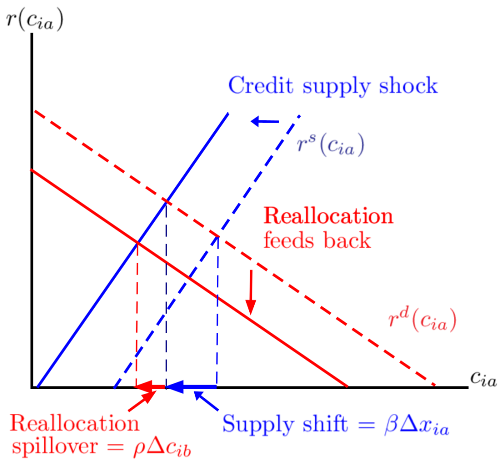
These endogenous effects induce spillovers among credit relationships, based on the structure of links in the credit network. We summarized structural parameters with β, the supply shift we want to measure; $\delta_{i,j}$, firms' demand shifters common across relationships; ρ, the spillover from credit relationships of the same firm; ϕ, the spillover from credit relationships of the same bank. In what follows, we focus on negative ρ and

[7] In this setting, firms sustain multiple relationships in equilibrium due to decreasing return to scale (increasing costs) in borrowing more and more from only one lender (for empirical evidence on the cost of credit captivity see Ioannidou and Ongena, 2010). For the moment, we focus on active credit relationships and their intensive margin. We thus ignore corner solutions in which links between a firm and a bank may not be active. We will explore matching models in future extensions.

ϕ, having in mind ρ as the spillover arising from the substitutability of credit across different relations of the same firm, and ϕ as the spillover arising from the opportunity cost of lending across different relationships of the same bank. Nevertheless, the model does not need these assumptions *a priori*; complementarities (positive ρ, or ϕ, or both) are also possible.[8]

The model illustrates how simultaneity in credit demand and supply decisions affect the final credit consumption observed in equilibrium. It can be represented graphically as a system of linear supply and demand curves that are co-determined across relationships belonging to the same firm or bank. In Figure 2, we show how this translates in a dependence of each relationship's amount of credit on both the supply shift we want to measure and reallocation spillovers, due to the change in relative cost.

Figure 2: Interdependence in a Toy Network



Notes: The Figure represents graphically the interaction between firm i and bank a and b, captured by Equation 2, in the case bank a receives a negative supply shock. The blue lines are banks' credit supply schedules to firm i, while the red lines are firm i's demand for credit from a and b respectively. Solid lines are post-shock curves, while dashed lines are pre-shock curves.

Focusing on the two relationships belonging to firm i in System 2, say a supply shock $\Delta x_{ia} < 0$ hits bank a, contracting its supply curve, while bank b supply stays still.[9] Graphically, the shock moves a's supply from the old, dashed line, to the new, solid one by $\beta\Delta x_{ia}$. Then, for each amount of credit offered, the bank is asking an higher interest rate to i. As a consequence, i will not only comply with the restriction imposed by a, but demand less from a and more from b overall, as a's credit became relatively more

[8] If lenders are very specialized in the kind of projects they fund (Paravisini, Rappoport, and Schnabl, 2023), firm f demand for credit may increase for two different lenders that fund complementary aspects of one project. For example, we can picture a simultaneous and interdependent credit demand increase for the bank that funds the purchase of production machinery and for the bank that funds export shipping ($\rho > 0$). For simplicity, we assume here that $|\phi| < 1$ and $|\rho| < 1$. We define formally the parameter space of these parameters in the next sections.

[9] Figure 2 ignores reallocation through banks, i.e. ϕ. This is done just for clarity, as reallocation across different relationships of the same firm is enough to convey the intuition of the problem at hand.

costly with respect to credit from b. This will translate in a $\rho\Delta c_{ia}$ shift in the equilibrium consumption of c_{ib}, partly compensated by an opposite shift in $\rho\Delta c_{ib} = \rho^2\Delta c_{ia}$ in c_{ia}. In conclusion, the final change in equilibrium credit from bank a to firm i will be composed by two elements, one directly related to the supply shift and governed by parameter β, and one indirect, due to reallocation and governed by parameter ρ.

We then consider the standard identification problem in the empirical banking literature, the recovery of a treatment effect β, which can be correlated with firms' demand shocks. To further simplify the example, we assume that

Assumption 3. *Treatment only hits relationship ia ($x_{ib}, x_{jb} = 0$), $\delta_j = 0$, and $\beta < 0$ to mimic a contraction in credit.*

If the researcher ignores the simultaneity of choices across credit relationships and estimates the following system of equations instead of (2),

$$\begin{aligned} c_{ia} &= \beta x_{ia} + \varepsilon_{ia}, \\ c_{ib} &= \beta x_{ib} + \varepsilon_{ib}, \\ c_{jb} &= \beta x_{jb} + \varepsilon_{jb}. \end{aligned} \tag{3}$$

the OLS estimator for β will be biased:

Proposition 1. *Under Assumptions 1, 2, and 3 the estimator of β for the system of equations in (3) is biased. The bias can be expressed as*

$$\hat{\beta}_{OLS} = \frac{cov(c_{ia}, x_{ia})}{var(x_{ia})} = \beta + \rho \underbrace{\frac{cov(x_{ia}, c_{ib})}{var(x_{ia})}}_{\textit{spillover bias}} + \underbrace{\frac{cov(x_{ia}, \delta_i)}{var(x_{ia})}}_{\textit{demand bias}} \tag{4}$$

Proof. Looking at the first Equation in the estimated System 3 and comparing it with the real Equation in System 2, we can see that the error term ε_{ia} actually equals to:

$$\varepsilon_{ia} = \delta_i + \rho c_{ib} + \epsilon_{ia}$$

The structural demand system in 2 can be expressed in terms of its reduced form components. In particular, we can express c_{ib} as:

$$c_{ib} = \frac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \beta\frac{\rho}{1-\phi^2-\rho^2}x_{ia} + \frac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}$$

From which $\hat{\beta}_{\mathrm{OLS}} = \frac{\mathrm{cov}(c_{ia},x_{ia})}{\mathrm{var}(x_{ia})} = \beta + \rho\frac{\mathrm{cov}(x_{ia},c_{ib})}{\mathrm{var}(x_{ia})} + \frac{\mathrm{cov}(x_{ia},\delta_i)}{\mathrm{var}(x_{ia})}$ derives.

That $\frac{\mathrm{cov}(x_{ia},c_{ib})}{\mathrm{var}(x_{ia})} = \frac{1+\rho}{1-\rho^2-\phi^2}\frac{\mathrm{cov}(x_{ia},\delta_i)}{\mathrm{var}(x_{ia})} + \beta\frac{\rho}{1-\rho^2-\phi^2} \neq 0$, $\frac{\mathrm{cov}(x_{ia},\delta_i)}{\mathrm{var}(x_{ia})} \neq 0$ and that $\frac{\mathrm{cov}(x_{ia},\delta_i)}{\mathrm{var}(x_{ia})} \neq \rho\frac{\mathrm{cov}(x_{ia},c_{ib})}{\mathrm{var}(x_{ia})}$ if not for specific values of the parameters conclude the proof. Details are provided in the Appendix. ■

The presence of spillovers impacts the estimate **directly** (the *spillover bias* component in Equation (4)). A supply shift induces reallocation of credit demand, as firms demand more from the relationships that became relatively more convenient. Moreover, spillovers affect the bias **indirectly**, interacting with *demand bias*. Demand bias can be at play even in absence of spillovers, but the presence of spillovers can amplify or reduce it, depending on their magnitudes and signs. In this example, the spillover-demand bias interaction component is captured by the role of ϕ and ρ parameters in the $\frac{1+\rho}{1-\rho^2-\phi^2}$ multiplier in front of $\frac{\text{cov}(x_{ia},\delta_i)}{\text{var}(x_{ia})}$. We can see that, when the absolute value of spillovers is high, the denominator of the interaction component increases, magnifying the demand bias.

In this simplified framework, we can also see how the inclusion of **firm fixed effects** cannot control for credit relationship interdependence, unless we also address the spillovers problem directly. Say we attempt a within estimation, but ignore spillovers represented in System (2). The resulting system is:

$$\begin{aligned} c_{ia} &= \beta x_{ia} + \delta_i + \varepsilon_{ia}, \\ c_{ib} &= \beta x_{ib} + \delta_i + \varepsilon_{ib}, \\ c_{jb} &= \beta x_{jb} + \delta_j + \varepsilon_{jb}. \end{aligned} \tag{5}$$

and the resulting problem is that we will not even address demand bias.

Indicating averages with bars, so that, for example, $\bar{c}_i = \frac{c_{ia}+c_{ib}}{2}$, we can state:

Proposition 2. *Under Assumptions 1, 2 and 3, the estimator of β for the system of equations in (5), the shift in banks' supply curve, is biased and the bias can be expressed as*

$$\begin{aligned} \hat{\beta}_{FE} &= \frac{cov(c_{ia}-\bar{c}_i, x_{ia}-\bar{x}_i)}{var(x_{ia}-\bar{x}_i)} \\ &= \beta(1-\rho) + \rho(1-\rho)\frac{cov(c_{ib},x_{ia})}{var(x_{ia})} - \rho\frac{cov(\delta_i,x_{ia})}{var(x_{ia})} - \phi\frac{cov(c_{jb},x_{ia})}{var(x_{ia})}. \end{aligned} \tag{6}$$

Proof. From Assumption 3 it follows that $\hat{\delta}_i = c_{ib}$. From the structural demand system $\varepsilon_{ia} = \rho c_{ib} + \epsilon_{ia}$ and $\varepsilon_{ib} = \rho c_{ia} + \phi c_{jb} + \epsilon_{ib}$. Then we have that:

$$\begin{aligned} &\hat{\beta}_{FE} = \frac{\text{cov}(c_{ia}-\bar{c}_i, x_{ia}-\bar{x}_i)}{\text{var}(x_{ia}-\bar{x}_i)} = \frac{\text{cov}(c_{ia}-c_{ib}, x_{ia})}{\text{var}(x_{ia})} = \frac{\text{cov}(\beta x_{ia}+\varepsilon_{ia}-\varepsilon_{ib}, x_{ia})}{\text{var}(x_{ia})} = \ldots \\ &\ldots\beta + \frac{\text{cov}(\rho((1-\rho)c_{ib}-\beta x_{ia}-\delta_i)-\phi c_{jb}, x_{ia})}{\text{var}(x_{ia})} = \ldots \\ &\ldots\beta(1-\rho) + \rho(1-\rho)\frac{\text{cov}(c_{ib},x_{ia})}{\text{var}(x_{ia})} - \rho\frac{\text{cov}(\delta_i,x_{ia})}{\text{var}(x_{ia})} - \phi\frac{\text{cov}(c_{jb},x_{ia})}{\text{var}(x_{ia})} \end{aligned} \tag{7}$$

From the above, and the reduced form of System 2, it is evident that β_{FE} is biased, and that correlated demand shocks still play a role, as they are reflected back in the estimator through reallocation spillovers. β_{FE} is indeed a function of δ_i in two ways. First, through the $-\rho\text{cov}(\delta_i, x_{ia})/\text{var}(x_{ia})$ element, due to demand reallocation within the relationships of the same firm. Second, through the impact of δ_i on *all* other bias components. Details are provided in the Appendix. ■

From Proposition 2 and its proof, we can also drive another important conclusion.

Proposition 3. *Under Assumptions 1 and 2, $\hat{\beta}_{FE} \neq \hat{\beta}_{OLS}$ is possible even in the absence of demand bias ($cov(x_{ia}, \delta_i) = 0$).*

Proof. Using Assumption 3 and the absence of demand bias to simplify our calculation, we can thus express the reduced form for c_{ib}, c_{jb} in System (2):

$$c_{ib} = \beta \frac{\rho x_{ia}}{1-\phi^2-\rho^2} + \frac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}$$
$$c_{jb} = \beta \frac{\rho\phi x_{ia}}{1-\phi^2-\rho^2} + \frac{\phi\rho\epsilon_{ia}+(1-\rho^2)\epsilon_{jb}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2}$$

In the absence of demand bias, the OLS estimator equals:

$$\hat{\beta}_{\text{OLS}} = \beta \frac{1-\phi^2}{1-\rho^2-\phi^2} \tag{8}$$

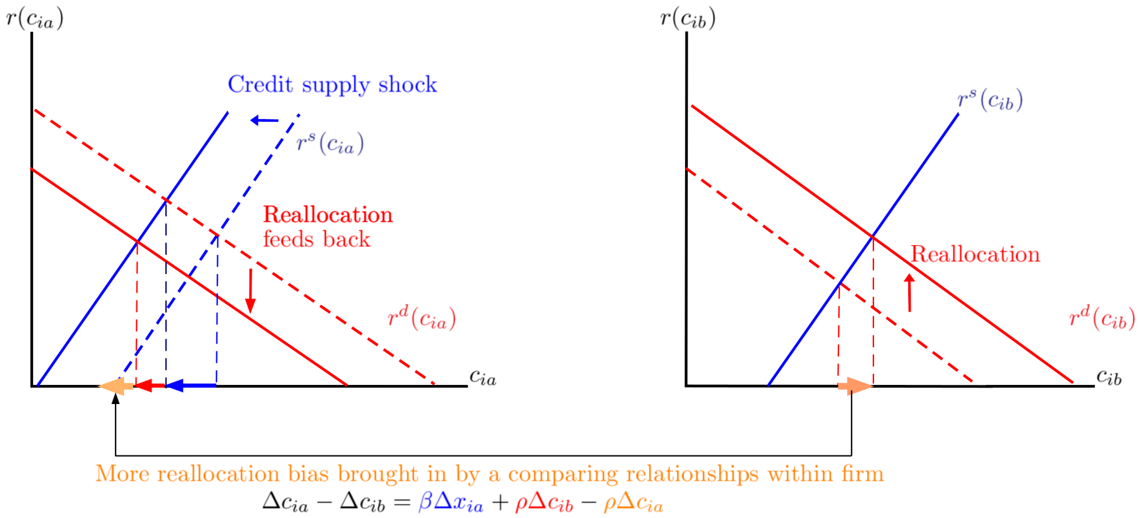
Using the bias expression in Proposition 2, the fixed effect estimator is:

$$\hat{\beta}_{FE} = \beta(1-\rho) + \beta\frac{\rho^2(1-\rho)}{1-\phi^2-\rho^2} - \beta\frac{\phi^2\rho}{1-\phi^2-\rho^2} = \beta\frac{1-\phi^2+\rho}{1-\phi^2-\rho^2}$$

which are different except for specific values of the reduced form parameters. Details are provided in the Appendix. ■

This difference implies that interpreting the distance between $\hat{\beta}_{FE}$ and $\hat{\beta}_{OLS}$ as informative on the sign of the demand bias (see, e.g. Jiménez et al., 2020) may, at least in some cases, lead to misguided conclusions. For an intuition, we shall use again our even more simplified graphical framework. In Figure 3, focusing on within-firm reallocation, we show how differentiating within firm in the absence of demand bias is tantamount to add further spillover bias back. This may lead a within-firm assessment of a supply shift even more off than a simple OLS.

Figure 3: Interdependence and Fixed Effects in the Absence of Demand Bias



Notes: The Figure represents graphically how the interaction between firm i and bank a and b, captured by Equation 2, can affect within-firm assessment of supply shifts. The blue lines are banks' credit supply schedule to firm i, while the red lines are firm i's demand for credit. Solid lines are post-shock curves, while dashed lines are pre-shock curves. Orange segments highlight how within-firm differentiation may worsen our assessment of the supply shift.

Consider again the $\beta\Delta x_{ia}$ supply shift we want to quantify, and suppose we mistakenly believe both c_{ia} and c_{ib} are affected by a demand shock specific to firm i that can bias our quantification of $\beta\Delta x_{ia}$. If we differentiate $\Delta c_{ia} - \Delta c_{ib}$ to address this nonexistent demand confounder, we add to the feedback effect of reallocation on $c_{ia} = \rho\Delta c_{ib}$, the initial reallocation from ia to ib, $\Delta c_{ib} = \rho\Delta c_{ia}$. Within comparison may actually increase the role of reallocation bias in assessing the supply shifts.

Finally, the example highlights an issue with retrieving the bank and firm fixed effects and interpreting them as credit supply and demand shifters (Amiti and Weinstein, 2018).

Proposition 4. *Under Assumptions 1 and 2, firm fixed effects' estimates contain supply shock spillovers and bank fixed effects' estimates may contain demand shock spillovers. As such, they cannot be regarded as pure measures of each firm or bank demand and supply shocks, respectively.*

Proof. We start considering an alternative version of Assumption 3, that allows for $\delta_j \neq 0$. Then, we notice that if the econometrician tries and estimate System (5), then c_{ia} is needed for the estimation of β, while:

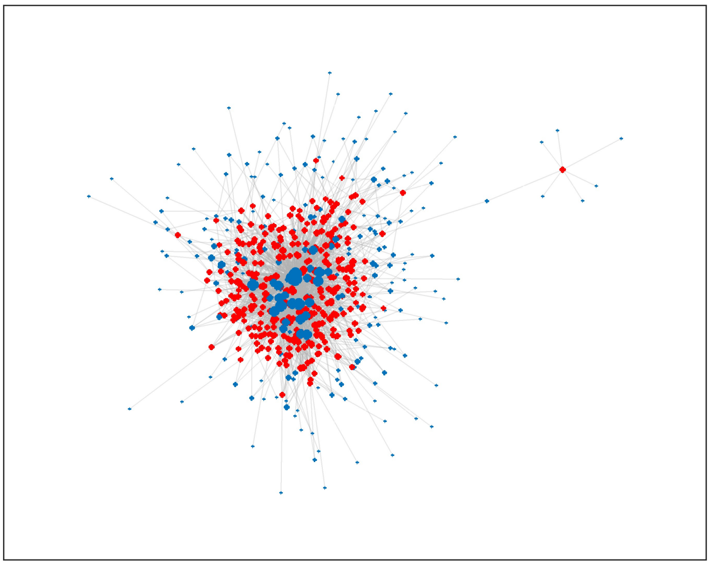
$$\begin{aligned} \hat{\delta}_i &= \tfrac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \tfrac{\phi}{1-\phi^2-\rho^2}\delta_j \\ \hat{\delta}_j &= \tfrac{\phi(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \tfrac{(1-\rho^2)}{1-\phi^2-\rho^2}\delta_j \end{aligned} \tag{9}$$

from the reduced form System (2) and clearly shows how the fixed effects are contaminated by reallocation. ■

Even in this simple setting, where we focus on firms fixed effects only, we can see that fixed effect estimate in (9) are already affected by two problems. First, fixed effects do not capture the pure demand shock exactly, but an amplified or attenuated version of it (on the base of sign and relative magnitude of ϕ, ρ). Focusing on $\hat{\delta}_i$, this is exemplified by the $\frac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i$ element. Second, other firms' shocks are reflected in the estimate through bank links, further biasing the fixed effect estimates. Focusing again on $\hat{\delta}_i$, this is exemplified by the $\frac{\phi}{1-\phi^2-\rho^2}\delta_j$ element.

This example is useful to get an intuition of the basic mechanics at play when banks and firms optimize their portfolios in an integrated way, and the simple yet powerful results that follow if these forces are ignored and standard estimation is performed. Nevertheless, actual credit networks can be much more complex, involving thousands of firms and hundreds of banks. Figure 4 provides a visual example of how complex the network formed by real credit relationships could be from a sample of only 500 real credit relationships from the Italian Credit Register data.

Figure 4: A Sampled Real Credit Network



Notes: The network is derived from a sample of 500 credit relationships observed in 2012. Banks are represented in blue and firms in red. The estimated network is represented with a force-directed layout with five iterations. It uses attractive forces between adjacent nodes and repulsive forces between distant nodes. To ease the visualization, the size of the nodes is equal to the (log) of their degree. See Fruchterman and Reingold (1991) for more details.

As a consequence, the resulting signs and magnitudes of the biases are more difficult to derive. First, as we can already see from the simple example, if the shock hit relationship ib instead of relationship ia, all bias expressions would change. This is actually an instance of a standard result in the network literature, finding that the extent of spillovers is mediated by each node's location in the network (Ballester, Calvó-Armengol, and Zenou, 2006). Furthermore, as the number of links increases, the complexity of the feedback effects increases too. A network with three links incorporates feedback loops of order two at most, i.e. a shock can affect another link and this movement can come back hitting the initial node. A network of n links can exhibit much more complicated dynamics.

Fortunately though, the intuition we built in this Section carries through to contexts of greater complexity, where existence and uniqueness of solution has been proven under mild conditions in Ballester, Calvó-Armengol, and Zenou (2006). In the rest of the document, we will introduce and estimate a generalized network model of credit relationship between F firms and B banks.

3 The Econometric Framework

In this section we introduce a econometric framework that allows for spillovers among credit relationships between banks and firms. The framework has the following advantages, that are new for this literature. First, it allows for endogenous spillovers among credit relationships, and provides consistent estimates of them. Second, it can be used to consistently estimate direct and indirect effects of treatments and shocks to firms and banks outcomes without imposing strong independence assumptions. Fourth, these estimates provide very granular, even pair-of-relationships-wise, outcome response functions which depend on the centrality of banks and firms in the credit network. Finally, as it can be derived from simple microfoundation of banks and firms behavior, discussed in Section 2.1, the estimates map with salient structural parameters that have an explicit economic interpretation.

3.1 The Credit Network Model

Suppose that there are two sets, $\mathbb{F}$ and $\mathbb{B}$, of firms and banks in the market with cardinality respectively equal to F and B. We can easily generalize the system of equations (2) in Section 2.1 to any number of banks, firms and relationships in the credit network as follows:

$$c_{ib} = \alpha + \phi \sum_{j \in \mathbb{F} \setminus i} a_{ib,jb} c_{jb} + \rho \sum_{k \in \mathbb{B} \setminus b} a_{ib,ik} c_{ik} + \delta_i + \gamma_b + x_{ib}\beta + \epsilon_{ib}, \tag{10}$$

where c_{ib} is credit from bank b to firm i. δ_i and γ_b are the firm and bank fixed effects. x_{ib} is a vector of exogenous characteristics of the loan, which may include a specific treatment administered to the relationship ib. ϵ_{ib} is the error component. The term $a_{ib,jb}$ captures the connections among credit relationships by the lender side, being equal to one if both i and j borrow from b. The term $a_{ib,ik}$ captures the connections among credit relationships by the borrower side, being equal to one if both b and k lend to i.[10] In this model, credit relationships are not i.i.d., credit granted bilaterally from banks to firms is jointly determined. On the one hand, the amount of credit from bank b to firm i depends on the credit that bank b gives to other firms j, as firms compete on the demand side to get credit from bank b, which is budget constrained. It captures the effect that a change in c_{jb} has on c_{ib}. We call this effect, captured by ϕ, the *bank credit reallocation effect* (BCR), as it captures shifts in the supply between credit relationship involving the same bank that are driven by its reallocation policies. On the other hand, the amount of credit from bank b to firm i depends on the credit that firm i takes from other banks, as banks compete on the supply side to grant credit to firm i, whose demand is not unlimited. It captures the effect that a change in c_{ik} has on c_{ib}. We call this effect, captured by ρ, the

[10] Given that we want to consider the total credit, we do not row-normalize these terms as sometimes is done in network econometrics, see Liu, Patacchini, and Zenou (2014) for a discussion.

firm credit substitution effect (FSC), as it captures shifts in the demand between credit relationship involving the same firm that are driven by its substitution choices.

Consider again Figure 1 in Section 2.1. The credit relationship ib is influenced by ia as both share the same borrower, and by jb as they share the same lender. The relationship ia is not influenced directly by jb, because it does not share any counterparty with it. Nevertheless, as we discuss in more detail below, ia is indirectly exposed to jb through adjustments in ib, as highlighted in Section 2.1. The matrix form of the *credit network model* (CNM) is:

$$\begin{aligned} C &= \alpha + \phi A_B C + \rho A_F C + X\beta + \Delta + \Gamma + \epsilon, \\ &= +\phi A_B C + \rho A_F C + Z\mu + \epsilon. \end{aligned} \tag{11}$$

where X is the matrix of loans covariates. Δ is the matrix containing the firm fixed effects. Γ is the matrix containing the banks fixed effects C is the vector containing all the N credit relationships between banks and firms in the market. A_B is the $(N \times N)$ is the adjacency matrix of the network that keeps track of connections among loans through banks whose generic element $a_{ib,jk}$ is equal to one iff $b = k$. We let $a_{ib,ib} = 0$ for all ib, following convention. A_F is the $(N \times N)$ is the adjacency matrix of the network that keeps track of connections among loans through firms whose generic element $a_{ib,jk}$ is equal to one iff $i = j$. We let $a_{ib,ib} = 0$ for all ib, following convention. The vector $A_B C$ contains for each loan the amount of credit granted by the same bank to other firms. The vector $A_F C$ contains for each loan the amount of credit obtained by the same firm from other banks. We define the *isolated-credit model* (ICM):

$$C = \alpha + X\beta + \Delta + \Gamma + \epsilon, \tag{12}$$

a model in which the credit relationships are forced to be independent, i.e. if we impose the restriction $\phi = \rho = 0$ in Equation (11). It is worth observing that (i) the CNM nests the standard ICMs commonly used in the literature, and (ii) it exploits exactly the same information set of the ICM, because the network structure (in A_B and A_F) is derived by units's ids (contained in Γ and Δ as well). From this perspective, our model can be used by every researcher working with credit register data.

The matrix form of the model makes it clearer that we deal with a simultaneous system of equations, in which the credit vector C enters the equation both on the left and the right hand side, through $A_B C$ and $A_F C$, the endogenous terms. This feature captures the more realistic assumption that credit choices are not independent, but comes at the cost of additional complexity in the econometric model and its identification. It can not be estimated by simple OLS. Nevertheless, model (11) belongs to the spatial autoregressive (SAR) models class, thus we can exploit some key results in this literature, especially the branch of the literature that extended these models to the analysis of networks (see Arduini, Patacchini, and Rainone, 2020; Bramoullé, Djebbari, and Fortin, 2009; Hsieh and Lee, 2016; Johnsson and Moon, 2021; Lee, 2007; Lee, Liu, and Lin, 2010; Patacchini, Rainone, and Zenou, 2017, among others). However, there are peculiarities and problems in our framework that deserve discussion and could need tailored solutions. For example, standard SAR models usually consider outcomes at the node level, while in our case outcomes are at the link level. In addition, nodes here belong to two different types of agents and links can be formed only between the two types, not within, and we have multiple endogenous terms and parameters.

3.2 Identification

The main issue that arises when we want to estimate equation (11) is the endogeneity of $A_B C$ and $A_F C$, thus simple OLS estimation is not consistent. The simultaneity of equations in model (11) creates an intrinsic endogeneity problem if

$$\begin{aligned} E[(A_F C)'\epsilon] &= E[(A_F(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu + \epsilon))'\epsilon] \neq 0, \\ E[(A_B C)'\epsilon] &= E[(A_B(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu + \epsilon))'\epsilon] \neq 0. \end{aligned}$$

The last inequalities hold if

$$\begin{aligned} E[(A_F(I - \phi A_F - \rho A_B)^{-1}\epsilon)'\epsilon] &= \sigma_\epsilon^2 tr(A_F(I - \phi A_F - \rho A_B)^{-1}) \neq 0, \\ E[(A_B(I - \phi A_F - \rho A_B)^{-1}\epsilon)'\epsilon] &= \sigma_\epsilon^2 tr(A_B(I - \phi A_F - \rho A_B)^{-1}) \neq 0, \end{aligned}$$

where tr is the matrix trace operator. Endogeneity is basically determined by the structure of the observed network, represented by A_F and A_B.

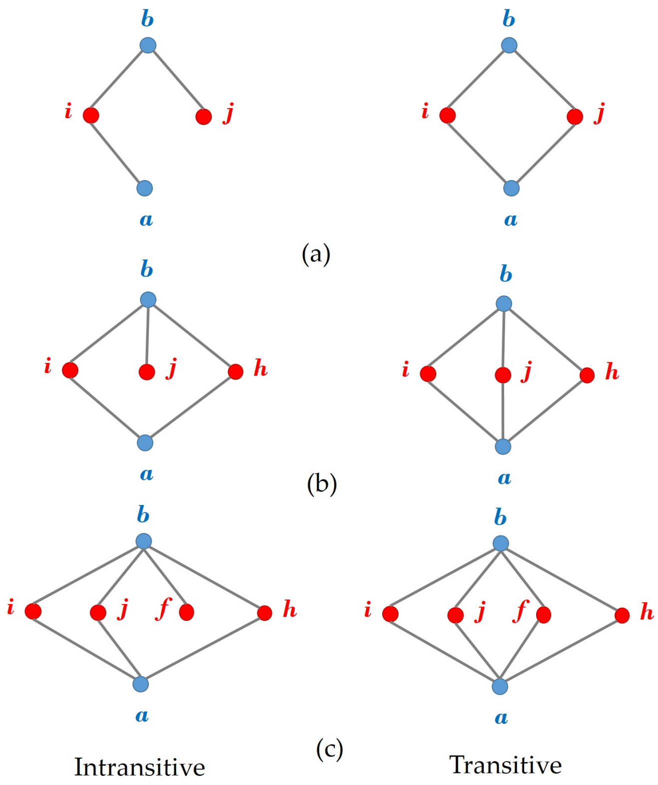
In SAR models, spatial lags of both the endogenous (here $A_B C$ and $A_F C$) and exogenous variables (here $A_B X$ and $A_F X$) can be included on the right hand side. In the social networks literature, the latter are called contextual effects, as they capture the direct influence of peers' characteristics. While endogenous effects arise quite naturally in our context, as shown in Section 2, exogenous effects are less straightforward to interpret here. We abstract from these effects, but our model can be extended to accommodate them if needed. The following proposition establishes sufficient conditions under which the parameters in model (11) are identified, even if augmented with exogenous effects $A_B X$ and $A_F X$.

Proposition 5. ***(Identification of the Credit Network Model)***. *The credit network model in (11) is identified if I_F, $A_B A_F A_B$ and A_F are linearly independent and I_B, $A_F A_B A_F$ and A_B are linearly independent -i.e. there are* intransitive quadriads *in the credit network- and $\phi\beta \neq 0$ and $\rho\beta \neq 0$.*

A nice feature of this result is that it translates into the easy-to-check requirement that banks do not have fully overlapping portfolios. In other words, it requires that not all the banks lend to the same set of firms. It follows that the credit market structure itself can provide the solution to the endogenity problem, if it meets certain conditions. Precisely, if it has a certain degree of *intransitiveness*. The level of intransitivity is the ratio of the number of intransitive quadriads over the number of quadriads. A quadriad is a set of four nodes, two banks and two firms. The quadriad is not transitive if all the between-type links are realized. The market must not be composed only by transitive quadriads. The presence of intransitive triads is a sufficient (but not necessary) condition for the identification of the model's parameters.

The intuition is that intransitivity provides exclusion restrictions that allow to identify the system of simultaneous equations in (11). Figure 5 provides examples of market structures that allow (networks on the left) and do not allow (networks on the right) for identification of spillovers among credit relationships, with two (panel (a)), three (panel (b)) and four (panel (c)) firms in the market. Let us consider the simplest networks in panel (a). In the left network, the fact that bank k does not have a relationship with firm j allows jb to be excluded from ik's equation, because j is not connected with k. It follows that jb can be used as an instrument to estimate the effect of ib on ik, as it has a direct impact on the former but not on the latter (which in turn it influences only through the former).

Figure 5: Network Structure and Identification



Notes: Nodes a, b are banks, red nodes are firms, edges are credit relationships.

Intuitively, when the number of firms grows, the number of intransitive quadriads has to grow as well. In the credit market, a transitive quadriad appears when a bank b lends to a different set of firms w.r.t. another bank k, or specularly when a firm i borrows from a different set of banks w.r.t. another firm j. If the market is composed only by transitive quadriads, we cannot identify the parameters in the system, there is no valid exclusion restriction. This situation is extremely rare credit markets.[11]

3.3 Overlapping Portfolios Instrumental Variables

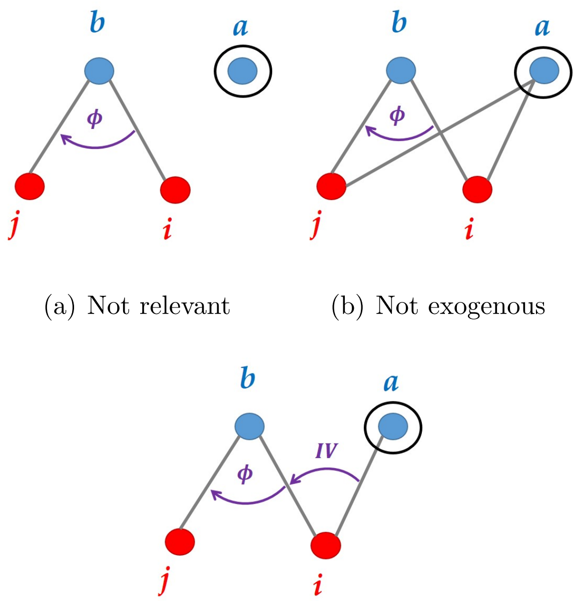
In this section, we propose a new method to construct instrumental variables based on overlapping portfolios (OPIVs). In credit markets, firms can borrow from multiple lenders. This feature creates overlapping portfolios of relationships both at the firm and bank level. As idiosyncratic shocks to one relationship can affect the others involving the

[11] Observe that in our specification we assumed that the contextual effect is equal to zero, i.e. that other credit relationships' characteristics do not affect the other loans directly. We think that this assumption is rather credible in this case, as it implies that the characteristics of counterparties that are not involved in the contract do not play a role in the loan, but only indirectly through the credit agreed with one of the counterparties. Nevertheless, model (11) can be extended including contextual effects easily.In terms of identification, Proposition 5 slightly changes, requiring $\phi\beta + \gamma \neq 0$ and $\rho\beta + \gamma \neq 0$, where γ is the contextual effect (see Proposition 1 in Bramoullé, Djebbari, and Fortin, 2009).

same parties, they can provide relevant instruments for the FSC and the BAC. If these portfolios are not fully overlapping, such instruments are also exogenous.

Intuition. Let us make a simple example in Figure 6. In panel (a) the credit network has not fully overlapping portfolios, in panel (b) the it has not overlapping portfolios, in panel (c) it has fully overlapping portfolios. For simplicity, lets focus on the capacity of a shock to bank a to identify ϕ. In panel (a), a shock to bank a is not relevant as the two banks are not connected through any firm. In panel (b), it is not exogenous because a shock to a directly influences jb through ja. In panel (c), a shock to bank a is a valid instrument because it is relevant as it directly influences bi through ai and is exogenous to jb. The same reasoning applies to shocks that are firm-specific or relationship-specific and for the identification of ρ.

Figure 6: Exogeneity and Relevance of OPIVs



(a) Not relevant

(b) Not exogenous

(c) Relvant and exogenous

Notes: Nodes a, b are banks, i, j are firms, edges are credit relationships.

Let's consider again the example we introduced in Section 2.1, in its most basic form, where there is no correlated demand confounder $\left(\frac{\text{cov}(x_{ia},\delta_i)}{\text{var}(x_{ia})} = 0\right)$ and the following assumption holds:

Assumption 4. *x_{jb} and x_{ia} do not affect directly c_{ib} and are uncorrelated with ϵ_{ia} and ϵ_{jb}.*

Then we have:

Proposition 6. *Under Assumptions 1, 2, 4, and the network structure in Section 2.1, we can identify the spillover parameters (ϕ and ρ) with a 2SLS procedure, and deliver an* *unbiased estimate of* β.

Proof. The System in 2 can be rearranged as

$$\begin{aligned} c_{ia} &= \rho\pi_\rho x_{jb} + \big(\beta + \tfrac{\beta\rho^2}{1-\phi^2-\rho^2}\big)x_{ia} + \rho\mu + \epsilon_{ia} \\ c_{ib} &= \pi_\rho x_{jb} + \pi_\phi x_{ia} + \mu \\ c_{jb} &= \phi\pi_\phi x_{ia} + \big(\tfrac{\phi^2\beta}{1-\phi^2-\rho^2} + \beta\big)x_{jb} + \phi\mu + \epsilon_{jb} \end{aligned} \tag{13}$$

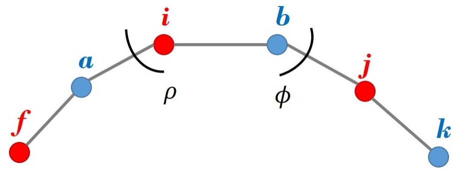
where π_ρ and π_ϕ are the reduced form parameters for the instruments' effect on c_{ib} and μ summarizes all other parameters we are not interested into. We use the c_{ib} equation of the System in 13 as first stage, recovering $\hat{\pi}_{\rho,OLS}, \hat{\pi}_{\phi,OLS}$, which in turn we use to deflate $\widehat{\pi_\rho\rho}_{OLS}, \widehat{\pi_\phi\phi}_{OLS}$ in the second stage, to finally obtain $\hat{\rho}_{IV} = \rho$, $\hat{\phi}_{IV} = \phi$. Endowed with unbiased estimates of the spillovers through bank and firms' parameters, we can correct the $\hat{\beta}_{OLS}$ and derive an unbiased $\hat{\beta}_{IV}$. ■

The key insight comes from the fact that, under Assumption 4, credit relationship jb (ia) provides exogenous variation through x_{jb} (x_{ia}) that does not affect directly c_{ib}, but does affect it indirectly through c_{jb} (c_{ia}). It then allows us to identify ϕ (ρ) with a 2SLS estimator. Identification of ρ and ϕ allows us, in turn, to retrieve an unbiased estimator of β.

Depending on the circumstances, Assumption 4 is considered to be too strong, for example because there are no credible pairwise observables (for example x_{jb}) that do not affect c_{ib}. An example could be that the econometrician does not observe other pairwise or firm specific variables at all, and only have bank specific variables at hand. If only firm-specific variables are available, j-specific variables (say x_j) can be used as an instrument for c_{jb}. If only bank-specific variables are available, b specific variables (say x_b) can not be used as an instrument, because they affect directly c_{ib}.

In such cases, variation in more distant credit relationships can be used for identification. For instance, let us add two other credit relationships to our toy example and assume that bank a supplies credit also to firm f and firm j demands credit also to bank k. Figure 7 provides the relative graph. In this highly intransitive credit network, variation induced by fa (jk) to ia (jb) can be used as an instrument to identify ρ (ϕ) -i.e. the effect of c_{ia} (c_{jb}) on c_{ib} -. In this case, even if the econometrician only observe banks' characteristics, a (k) specific variables can be used as an instrument for c_{ia} (c_{jb}) and identify ϕ (ρ). A specular strategy can be used if the econometrician only observe firms' characteristics, say f (j).

Figure 7: IV exclusion restrictions



Notes: nodes a, b, k are banks, f, i, j are firms, edges are credit relationships.

In the full model. Following Proposition 5, a natural way to proceed with the estimation of equation (11) is using OPIVs. The OPIVs are substantially "network embedded", in other words the network topology is used to create IVs that are correlated with the variables to be instrumented, being independent from the error term.[12] Indeed, the expected value of the two endogenous variables, $E(A_F C)$ and $E(A_B C)$, to be instrumented meet these two conditions. Taking advantage of the reduced form, the theoretically best OPIVs are thus derived as

$$TIV_F = E(A_F C) = E[(A_F(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu))], \quad (14)$$

$$TIV_B = E(A_B C) = E[(A_B(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu))], \quad (15)$$

since $E[(I - \phi A_F - \rho A_B)^{-1}\epsilon] = 0$. Given that the parameters in Equations (14)-(15) are unknown, TIV_B and TIV_F are unfeasible. Assuming $||\phi A_F + \rho A_B||_\infty < 1$,[13] the term $(I - \phi A_F - \rho A_B)^{-1}$ is an infinite sum of elements $\sum_{k=0}^{\infty}(\phi A_F + \rho A_B)^k$. A linear approximation, EIV_F and EIV_B, of vectors appearing in equation (14) and (15) can thus be used for the empirical IV. Exploiting only the variation of covariates, X, first order approximations of TIV_F and TIV_B are respectively:

$$EIV_F^1 = A_F X, \quad (16)$$

$$EIV_B^1 = A_B X, \quad (17)$$

second order approximations are

$$EIV_F^2 = [A_F, A_F A_B]X, \quad (18)$$

$$EIV_B^2 = [A_B, A_B A_F]X. \quad (19)$$

[12] The literature of spatial and network econometrics investigated in depth several methods to treat the endogeneity created by these simultaneous equations, Kelejian and Prucha (1999) and Liu and Lee (2010) proposed a GMM approach, and Lee (2004) used a Quasi-Maximum Likelihood Estimator. In this paper we use an IV approach in the spirit of Lee, Liu, and Lin (2010), Lee (2007) and Kelejian and Prucha (1998).

[13] This is a sufficient condition for the invertibility of $(I - \phi A_F - \rho A_B)$; it also determines the parameter space for spillover effects.

and so on and so forth.[14] Observe that here $A_F^k \equiv A_B^k \equiv \mathbf{0}$, which is the zero matrix, for $k > 1$, as we have a bipartite network with only cross types links. Let us use again our example in Figure 7, in order to consistently estimate ρ (ϕ), a first order approximation uses x_{ia} (x_{jb}) as an IV. A second order approximation would use x_{fa} (x_{jk}) in addition in the IV.[15] The 2SLS estimator is consequently

$$\hat{\theta}_{2SLS} = (W'P_Q W)^{-1}(W'P_Q C), \tag{20}$$

where $Z = [A_F C, A_B C, Z]$, $P_Q = Q(Q'Q)^{-1}Q'$, $Q = [EIV_F, EIV_B, X]$ and $\hat{\theta}_{m,t,2SLS} = [\hat{\phi}_{2SLS}, \hat{\rho}_{2SLS}, \hat{\mu}_{2SLS}]$. Given that we do not employ Γ and Δ in the construction of the OPIVs, the number of IVs does not grow with the number of banks and firms in the sample. It follows that the main asymptotic properties of the estimator are standard and follow those of the ICM. Intuitively, the estimator is consistent as long as the number of credit relationships grows faster than the number of banks and firms in the market, as it allows to include the FEs in the model.[16] Using Liu and Lee (2010)'s terminology, if we wanted to use a more efficient "many IVs" strategy instead of a "few IVs" strategy (the one we are adopting), employing also Γ and Δ in the OPIVs, we should have derived the asymptotic properties of such estimator to develop a bias-correction procedure. In this paper, our main objects of interest is the identification, the finate sample properties and the empirical analysis of this few IVs estimator, we leave the analysis of the asymptotic performance of a more efficient many IVs estimator in these contexts for future research.

In some cases, credit register or more generally bilateral credit relationship data can have a very high dimension, eventually with millions of observations. In such situation, the curse of dimensionality can severely constrain or eventually prevent the computation of the estimator in (20), because some of the matrices involved can not be manipulated with standard software. A solution for this kind of problem, is to use a within estimator. Let $F = [\Delta, \Gamma]$ and $J = I - F(F'F)^{-1}F'$. Then we have

$$JY = \phi J A_B Y + \rho J A_F Y + JX\beta + J\epsilon.$$

One can thus first estimate $\omega = (\phi, \rho, \beta')'$ by 2SLS. Let $R = [A_B Y, A_F Y, X]$ and $P_H =$

[14] The approximation is as follows. $E(A_F C) = E[(A_F(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu))] = E[A_F[\sum_{k=0}^{\infty}(\phi A_F + \rho A_B)^k](\alpha + X\beta + \Delta + \Gamma)] = E[A_F X\beta] + E[A_F A_B X\beta] + E[A_F(\alpha + \Delta + \Gamma)] + E[A_F A_B(\alpha + \Delta + \Gamma)] + E[A_F[\sum_{k=2}^{\infty}(\phi A_F + \rho A_B)^k](\alpha + X\beta + \Delta + \Gamma)]$. This is due from the fact that we have a bipartite network without within type connections an thus $A_F A_F = A_B A_B = \mathbf{0}$. A specular approximation can be derived for TIV_B.

[15] Observe that using X in the empirical IV corresponds to the 'few IV' estimator strategy in Liu and Lee (2010), we abstract from efficiency considerations and bias correction issues that would emerge from the analog of the 'many IV' estimator strategy using Z instead.

[16] We abstract here from the potential consequences of violating this assumption and selecting only firms that borrow from multiple banks in the sample, as it is a standard practice in this literature, which could affect both the ICM and CNM. Usually this operation is justified by observing that firms with only one relationship account for a very small share of the market.

$JH(H'JH)^{-1}H'J$ where H is a matrix of IVs of linearly independent columns of $[X, A_B X, A_F X]$.[17] Then,

$$\widehat{\omega}_{2SLSW} = (R'P_H Z)^{-1} R'P_H Y, \tag{21}$$

is a consistent estimator for the spillover effects. Next, one can estimate $\phi = (\gamma', \delta')'$ by OLS. Let $\widehat{U} = Y - R\widehat{\omega}_{2SLSW}$. Then,

$$\widehat{\phi}_{2SLSW} = (F'F)^{-1} F'\widehat{U}. \tag{22}$$

3.4 Bias of ICM under Interdependent Credit Relationships

After having developed an econometric framework to estimate unbiased network spillovers, treatment effects and fixed effects, in this section we derive some analytical results on the sign and magnitude of biases of treatment effects and idiosyncratic shocks when the DGP is a CNM and the econometrician estimates a ICM, i.e. when the network nature of credit relationships matters but is ignored.

3.4.1 Bias of Treatment Effects

Let us first abstract from the presence of fixed effect and the DGP be

$$C = \phi A_B C + \rho A_F C + X\beta + \epsilon, \tag{23}$$

For simplicity let $\rho = \phi$. Suppose we estimate

$$C = X\beta + U, \tag{24}$$

then the error term has the following form

$$\begin{aligned} U &= \phi A_B C + \rho A_F C + \epsilon = (\phi A_B + \rho A_F)(I - \phi A_B - \rho A_F)^{-1}[X\beta + \epsilon] + \epsilon \\ &= MX\beta + (M + I)\epsilon. \end{aligned} \tag{25}$$

[17]Observe that if pairwise exogenous X are observable in the data and exogenous effects are not considered, a first order approximation is the preferred solution if the curse of dimensionality is binding, because products of high dimensional matrices such as A_B and A_F. Unless differently specified we will use such approximation in what follows.

Suppose that X is univariate and $X \perp \epsilon$,[18] we have

$$\begin{aligned} X'U &= X'MX\beta + X'(M+I)\epsilon = X'\phi A \sum_{k=0}^{\inf} (\phi A)^k X\beta \\ &= X' \sum_{k=1}^{\inf} (\phi A)^k X\beta = \beta \sum_{k=1}^{\inf} \phi^k X' A^k X = S. \end{aligned} \tag{26}$$

If $\beta, \phi > 0$ then $B = \hat{\beta} - \beta = (X'X)^{-1}S > 0$. The positive bias is given by the amplification generated by spillovers, which is not disentangled in the reduced form estimate. If $\phi < 0$ then the sign of B depends on A, i.e. the network structure, and the intensity of the spillovers, as it contains decaying functions of ϕ. Suppose X is binary, if for example the population is divided between agents that are exposed to a treatment and those who are not. Then $X'A^kX = p_k$ is the sum of the number of $k-$ distant treated credit relationships from each treated relationship. In this case

$$S = \beta \sum_{k=1}^{\inf} \phi^k p_k = \beta(\sum_{k\ odd} \phi^k p_k + \sum_{k\ even} \phi^k p_k) = \beta(OD + EV).$$

Given that OD is negative and EV is positive, $B > 0$ if $OD > -EV$. The intuition behind the indeterminate sign is that when the spillovers are negative, they lower the outcomes of odd-distant agents (at distance 1, 3, 5, etc) but increase the outcomes of even-distant agents (at distance 2, 4, etc). Nevertheless, the negative sign is likely to prevail as the first round effects have a higher weight, because $\phi < 0$, especially in denser networks. Let us now introduce also the presence of bank and firm fixed effects. Suppose now that the DGP is

$$\begin{aligned} C &= \phi A_F C + \rho A_B C + X\beta + \Delta + \Gamma + \epsilon, \\ &= +\phi A_B C + \rho A_F C + Z\mu + \epsilon, \end{aligned} \tag{27}$$

where $\Delta = D\delta$ is a matrix of firm FEs and $\Gamma = G\gamma$ is a matrix of bank FEs. For simplicity let again $\rho = \phi$. Suppose we estimate

$$C = X\beta + \Delta + \Gamma + U, \tag{28}$$

[18] To ease the notation we assume independence here, the same conclusions can be reached assuming $E[\epsilon' X] = 0$.

then the error term has the following form

$$\begin{aligned} U &= \phi A_F C + \rho A_B C + \epsilon \\ &= (\phi A_B + \rho A_F)(I - \phi A_B - \rho A_F)^{-1}[X\beta + \Delta + \Gamma + \epsilon] + \epsilon \\ &= M[X\beta + \Delta + \Gamma] + (M + I)\epsilon \end{aligned}$$

Suppose again that X is univariate and $X \perp \epsilon$, we have

$$\begin{aligned} X'U &= X'M[X\beta + \Delta + \Gamma] + (M + I)\epsilon = X'\phi A \sum_{k=0}^{\inf}(\phi A)^k[X\beta + \Delta + \Gamma] \\ &= X' \sum_{k=1}^{\inf}(\phi A)^k[X\beta + \Delta + \Gamma] = S + \sum_{k=1}^{\inf}\phi^k X' A^k[\Delta + \Gamma] \end{aligned}$$

If $X \perp \Delta, \Gamma$, similarly to random effects models, then the bias obtained estimating (28) is equal to S. If not, $\phi > 0$ and X is positively correlated with Δ and Γ then $B = (X'X)^{-1}X'U > 0$, otherwise the sign of the bias is ambiguous. $A^k\Delta = D_k$ is a vector whose generic element contains the FEs of borrowers of credit relationships at distance k from the relative relationship. Symmetrically, $A^k\Gamma = G_k$ is a vector whose generic element contains the FEs of lenders of credit relationships at distance k from the relative relationship.

3.4.2 Bias with Spillovers and Endogenous Treatments

In the previous analysis, we assumed that the main regressor is exogenous. Let us now allow X to be an endogenous regressor. In credit markets, as in other fields in which experiments are not possible or easy to implement, regressors are often endogenous. Endogeneity can arise because of self-selection in the extensive margin (see Jiménez et al., 2014, for example) or in the in the intensive margin (see Paravisini, Rappoport, and Schnabl, 2017, for example), or for the omission of relevant variables on the RHS. In some cases a credible instrument (a selection step) can be found (added), but in many situations this is not the case and the omission of relevant, and eventually endogenous, variables is always difficult to assess in practice. In what follows, we want to understand analytically what are the consequences for the spillover bias we are interested in. For simplicity, let us focus on the simple case used above when $\phi = \rho$ and there are no fixed effects and assume that $\epsilon = \iota X + V$, with V being an error term such that $V \perp X, \epsilon$. Suppose we estimate the parameters of the two following models,

$$C = \phi A_B C + \rho A_F C + X\beta + \epsilon, \tag{29}$$

$$C = X\beta + U, \tag{30}$$

when the DGP comes from the first equation the error term of the second equation has the following form

$$\begin{aligned} U &= \phi A_B C + \rho A_F C + \iota X + V \\ &= (\phi A_B + \rho A_F)(I - \phi A_B - \rho A_F)^{-1}[X\beta + \iota X + V] + \iota X + V, \end{aligned}$$

then, given that $V \perp X$, we have

$$\begin{aligned} X'U &= S + X'(M + I)(\iota X + V) \\ &= \underbrace{S}_{spillovers} + \underbrace{\iota X'X}_{endogeneity} + \underbrace{\iota X'MX}_{combination}, \end{aligned}$$

$$X'\epsilon = \iota X'X + \iota X'MX.$$

In this case, the estimate of β is biased for both models (29) and (30), but while the bias of (30), B_{ICM}, is affected by the pure *spillover* component, the pure *endogeneity* component and the *combination* of the two, the bias of (29), B_{CNM}, is only affected by the last two. It follows that $D = B_{ICM} - B_{CNM} \neq 0$ does not imply that $\iota \neq 0$, but it does imply that $S \neq 0$, and thus D can inform about the presence of (and the bias induced by) spillovers even if the estimate of β is biased. This is because unbiased spillovers can be recovered even in the presence of treatment endogeneity: the network lags used as instrumental variables for the identification of the spillovers, for example the EIV in (16) and (17), are still uncorrelated with the error term, i.e. $E[\epsilon' AX] = 0$. The intuition behind this result is that the treatments to other agents in the economy can still be valid instruments for their outcomes, even if they are endogenous.[19] The key requirement is that they are not endogenous to their network lags. In Section 4, we use numerical simulations to study the performance of our estimator in finite samples when the treatment is correlated with the error term and the difference in the bias when spillovers are accounted for or ignored. In Section 6, we show how our method can easily accommodate a instrumental variable strategy.

3.4.3 Bias of Idiosyncratic Firm and Bank Shocks

We can derive the bias of firm and bank FEs from the same DGP as in equation (28). Let Δ_i (Γ_b) be the i_{th} (b_{th}) column of Δ (Γ) and Δ_{-i} (Γ_{-b}) the matrix containing all the

[19]Observe that this could not be the case if the order of the EIV used to approximate the TIV is higher, because $X'A^kX$ could contain powers of the same x_i when $k \geq 2$. Using a first order approximation helps to avoid it.

columns of Δ (Γ) but the i_{th} (b_{th}). Following the same derivations shown above, we have

$$\begin{aligned} D_i'U &= D_i'\phi AM[X\beta + \Delta_i + \Delta_{-i} + \Gamma] + (M+I)\epsilon = \delta_i \sum_{k=1}^{\inf} \phi^k D_i' A^k D_i \\ &= \delta_i \sum_{k=1}^{\inf} \phi^k 1 = \delta_i (\sum_{k\ even} \phi^k l_k^i - \sum_{k\ odd} \phi^k l_k^i) = \delta_i (E_i - O_i) \end{aligned} \tag{31}$$

where l_k^i is the number of loops from (to) firm i that involve chains of length k in the credit network. A similar derivation can be done for $G_i'U$:

$$\begin{aligned} G_i'U &= D_i'\phi AM[X\beta + \Delta + \Gamma_b + \Gamma_{-b}] + (M+I)\epsilon \\ &= \gamma_b (\sum_{k\ even} \phi^k l_k^b - \sum_{k\ odd} \phi^k l_k^b) = \gamma_b (E_b - O_b) \end{aligned} \tag{32}$$

where l_k^b is the number of loops from (to) bank b that involve chains of length k in the credit network. Given that both O_b and E_b are positive, the sign of the bias depends on the credit network topology. In indirect networks, we may expect $E_b > O_b$, as loops generate paths of even length going back and forth at each step. The intuition behind this result is similar to the treatment effect's bias: competitive interactions let idiosyncratic shocks diffuse through the credit network. This propagation can bring to an over-(or under-) estimation of them, if not accounted for. Embedding market interactions requires a slightly more complex model. In the next section, we provide a model that allows to (i) avoid bias in both idiosyncratic shocks and treatment effects and (ii) estimate spillover effects both through BCR and FSC, in avery parsimonious way.

4 Monte Carlo Study

In this section, we simulate networks to illustrate our method's validity and its properties in finite samples, under different settings. Furthermore, we use these numerical experiments to study the bias of the isolated-credit network estimator. The **key take away** from our simulation is that the *sign and size of the bias from ignoring the network structure cannot be derived ex-ante.* Indeed, the bias is a convolution of observables, such as the market structure (in particular its density) and the number of treated units, and unobservables, such as the structural parameters determining the spillovers' magnitude. Thus, we can experience high non linearity of the bias, sign uncertainty, and we need to tackle spillovers directly.

4.1 Setting

We use a given set of parameters ϕ, ρ and β, randomly generated characteristics X, error terms ϵ, and a network G as inputs. We generate G as a 'circular network', ordering nodes according to natural numbers from 1 to N. If node i is odd, it is a bank, if it is even, it is a firm. We link node i to all opposite type nodes, till node $i + j$ for $j \leq z_i$, where z_i is an independent realization from a uniform distribution $U(0, m)$ for each node i.[20] For example, if z_1 (z_2) = 10, bank 1 (firm 2) links with firms 2,4,6 and 8 (banks 3,5,7 and 9). In this way, there are only links between banks and firms, none within the two types.

We label this experiment the circular network, because, when $z_i = 1$ for all i, nodes' connections are one to one (banks can share one borrower at most), with the last reconnecting to the first, describing a circle. The model we use is indeed a generalization of a simple circle that allows for a random number of connections. The variable m defines network density. When m increases, we are increasing the average number of shared borrowers, as well as the number of banks between which they are shared. This setup is useful for the comparative statics exercise, because it allows us to easily change features (size, sparsity, etc.).

In our benchmark simulation, we generate 500 networks G with $n = 200$ nodes in each, 100 banks and 100 firms. Once G is randomly generated, we then derive the links' adjacency matrix A from G. For each link we create an observable (x) and an unobservable (ϵ) variable. x is a dummy equal to 1 for a certain share s of the population and zero otherwise, tracking whether the credit relationship received the shock. We extract ϵ from a normal distribution with mean equal to zero and variance equal to σ. We generate lender and borrower fixed effects from independent normal distributions $\theta N(0, 1)$ draws, constraining them to be positive by adding the absolute value of the minimum draw. Finally, when we include fixed effects in the model, we allow the main regressor to be correlated with the fixed effects, $X = \mu(\Delta + \Gamma) + x$. We thus generate credit relationships in reduced form from: $C = (I - \phi A_B - \rho A_F)^{-1}(\beta X + \Delta + \Gamma + \epsilon)$. In particular, our pivotal setting has $(\beta, \mu, N, \sigma, \delta) = (-2, 0, 200, 1, 0.1)$ and $R = 500$ simulated samples.

We perform two classes of exercises within this framework. First, we look at the bias of $\hat{\beta}$ when we ignore the interdependence among credit relationships. Second, we study the performance of our estimators for the treatment effect β, the spillovers ϕ, ρ, and firms and banks' fixed effects in finite samples, under different settings and assumptions. Across all these exercises, on top and above negative treatment effect, we also assume negative spillovers. Doing so, we focus on a framework resembling a typical negative shock hitting the credit supply of some banks to some firms, where these firms and banks can reallocate credit freely across relationships in their portfolio.

[20] We have also repeated our simulation experiment using other distributions for z_i (different from uniform). The results are not sensitive to different specifications. Such results are available upon request.

4.2 Results in Simulated Networks

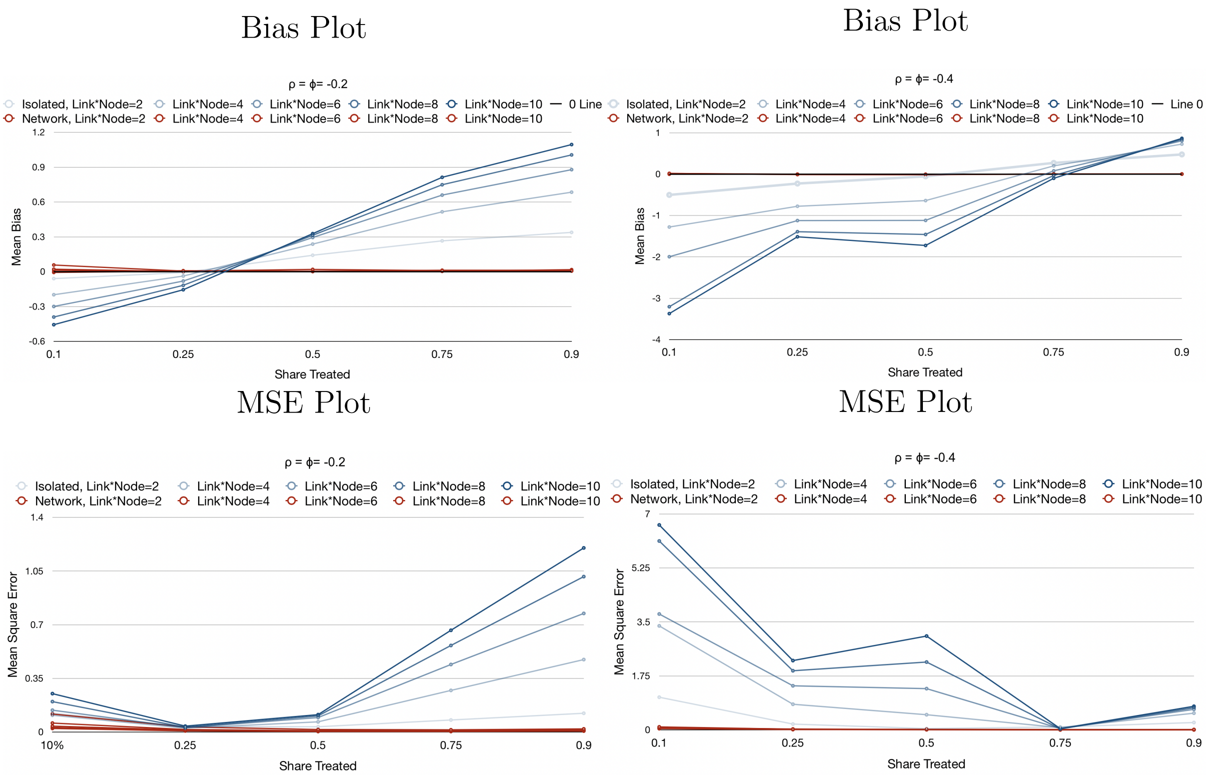
4.2.1 Treatment Effects

Bias under Interdependencies. We first study the bias of β in absence of fixed effects. We set the spillover parameters ϕ and ρ such that $\phi = \rho$ for simplicity, but our results hold in more general settings, as shown in the Appendix. Across exercises, we vary the value of spillovers, the share of treated relationships in the population, and we attach different amounts of credit relationships to each bank and firm node (i.e., we vary the network's number of connections, its density).

Figure 8 reports the mean bias (upper panel) and mean square errors (MSE) from estimating the treatment effect parameter β in the isolated and connected credit models. Blue lines represent ICM results, while red line represent CNM results. Shade intensity is darker as we increase the network density, while we report across different elements of each panel the results we obtain changing the strength of spillovers. In particular, we display results for $\rho = \phi = -0.2$ in the first element of each panel, and $\rho = \phi = -0.4$ in the second.[21]

[21] We report only two such plots per panel for expositional clarity, and we will do so through other Figures over this Section. Nonetheless, in Appendix Tables A.1 through A.7, we report all underlying numbers, also presenting results for the intermediate value of $\rho = \phi = -0.3$ and additional experiments that add to the scope of our results.

Figure 8: ICM Bias and CNM Performance



Notes: We compute the mean bias across the 500 simulated samples. For each sample, the number of nodes N in the network is 200, 100 firms and 100 banks. In black, we plot the zero line. In different shades of blue, we plot how the mean bias (upper two figures) and the mean square error (lower two figures) of the ICM's estimate change. In red, we do the same for the CNM's treatment estimates. Darker shades signal denser networks. We display the plots' underlying data in Appendix Table A.1.

In Figure 8's upper panel, we can appreciate that the sign and intensity of the bias depend on observable features of the network, i.e. the treated share and the density of relationships. Nonetheless, comparing the left and right elements of Figure 8's upper panel, we can also see that the bias depends on the magnitude of spillovers too. Such magnitudes we cannot observe directly and, as a consequence, we cannot mend the bias without directly confronting it in estimation.

Furthermore, we note that when we treat a small fraction of units, we overestimate the magnitude of β to be much more negative than what it really is, especially if the spillover is larger (right upper panel), while we underestimate it eventually, as the fraction of treated units increases and reaches a certain threshold, which depends on the magnitude of spillovers and the density of the network.

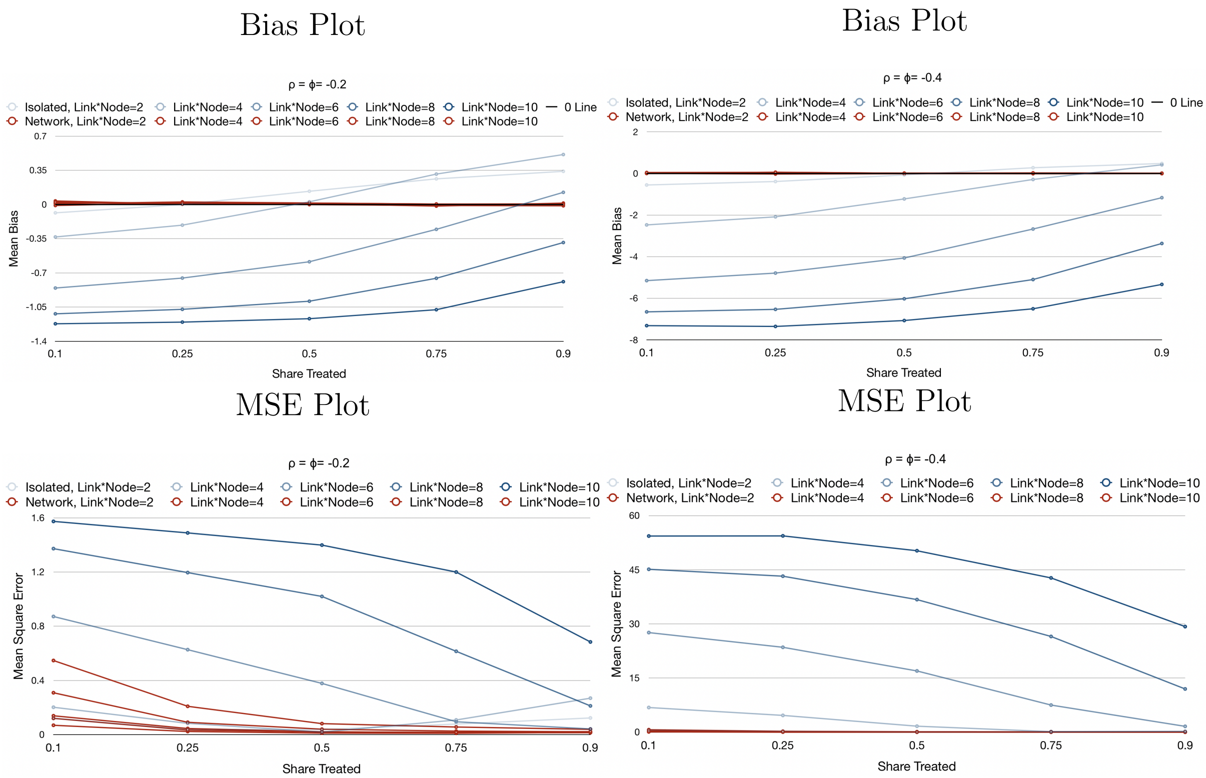
The negative (βEV) and positive (βOD) components of the bias, documented in Section 3.4, explain such a pattern of over and underestimation. When there are few treated units, βEV prevails, as most higher-order effects are feedback loops triggered by own-treatment, which amplifies the negative direct effect. To provide economic interpretation, we think of a case in which credit from shocked banks becomes scarcer, and firms with multiple lenders systematically start sourcing more credit from other providers, further decreasing what they demand from the shocked banks in equilibrium. We stress that this is the numeric counterpart of what we showcased through the theoretical example in Section 2.1, when only one credit relationship faces a supply contraction.

Instead, when the supply contraction hits more nodes, the amplifying effect of feedback loops ends up being more than offset by indirect effects from other treated units, which are primarily positive under the assumption of negative treatment and negative spillovers. Again, interpreting the results through economic logic, we can think of the case in which the majority of banks must suddenly contract credit supply, causing each firm to face a sudden increase in all other firms' demand for credit at the few spared banks. Eventually, this indirect effect can be so strong and credit from spared banks so scarce that firms source more credit from the initially hit banks than they would have done in the absence of interconnections.

Higher density, i.e. more relationships per node and thus higher share of borrowers shared by lenders, allows for more feedback loops and thus amplifies the bias' magnitude in both directions. The density also determines the point in which the bias switches its sign: the higher the number of links, the higher the number of treated units needed to switch the sign of the bias to positive, as crowd out is less striking when each firm has more options. Also the value of spillover effects (ϕ and ρ) determines the point at which the bias changes its sign. The higher they are, the stronger direct effects and own-treatment feedback loops, the more treated units we need to switch the bias from negative to positive. This conclusion is the more true, the fewer units are treated, as there is more 'space' to directly absorb the supply contraction.

The results shown do not depend on the setting chosen, which is quite simple but able to point out the main forces at work. We have chosen the density as the main metric for the network, others can be considered. Here we are not particularly interested in finding a "sufficient" statistic for the contribution of the network structure to the bias of β, because we can derive it: endowed with a credit register, we observe A_F and A_B, as well as the treatment vector, and we provide in Section 5 a consistent estimator for ϕ and ρ. With all these elements, we can precisely derive the bias *ex post*.

Figure 9: ICM Bias and CNM Performance with Correlated Fixed Effects



Notes: We compute the mean bias across the 500 simulated samples, where the data generating process includes a set of fixed effects correlated with treatment. For each sample, the number of nodes N in the network is 200, 100 firms and 100 banks. In black, we plot the zero line. In different shades of blue, we plot how the mean bias (upper two figures) and the mean square error (lower two figures) of the ICM's estimate change. In red, we do the same for the CNM's treatment estimates. Darker shades signal denser networks. We display the plots' underlying data in Appendix Table A.2.

Finally, we show that the inclusion of bank and firm fixed effects does not solve this issue. On the contrary, the inclusion of fixed effects, ignoring spillovers, can exacerbate bias. In Figure 9's upper panel, we display with blue lines how the bias' severity increases and its sign turns almost always negative, especially when spillovers are high and the network is sparse. This change in sign, again, confirms the intuition we conveyed within our theoretical example in Section 2.1, especially in Figure 3. By adding back endogenous reallocation in the treatment, fixed effects put more weight on own-treatment's feedback loops, overplaying shocks' severity.

Estimator Performance. Next, we analyze the performance of our estimator. First, we report the mean bias and the MSE of $\hat{\beta}$ for the same setting used above. We visualize the results as the red lines in Figures 8 and 9. In every simulation, the mean bias is negligible in all settings, with different shares of treated units, magnitude of spillovers and density of the credit network (Figure 8's upper panel). Moreover, our estimator is precise, as measured by the low MSE, which decreases with the number of treated units and with the density (i.e. with the increase in the number of relationships, thus the sample size; see Figure 8's lower panel). Last, we also see that such performance is not affected by the inclusion of correlated fixed effects in the data generating process (Figure 9)

Second, we analyze the performance of the estimates of the spillover parameters ϕ and ρ, relaxing the assumption that $\phi = \rho$, and also studying what happens when n grows. In Table 1 we report the mean and the standard deviation of the estimates of ϕ and ρ at different intensities (by columns) and densities (by rows). We can see that the estimates are always centered at the true values. The standard deviation decreases with the number of nodes in the network and the density of the connections among them. With a quite small sample size, 800 nodes, the dispersion of the estimates is limited even with very small density (when $m = 2$).

Table 1: Performance of the Spillovers' Estimators (OPIV $\hat{\phi}$, $\hat{\rho}$)

n	m		true		true		true		true		true	
			ϕ	ρ	ϕ	ρ	ϕ	ρ	ϕ	ρ	ϕ	ρ
200			-0.1	-0.1	-0.1	-0.2	-0.1	-0.3	-0.1	-0.4	-0.4	-0.4
	2	mean	-0.097	-0.100	-0.100	-0.209	-0.101	-0.306	-0.093	-0.414	-0.406	-0.406
		std	0.084	0.087	0.090	0.089	0.082	0.081	0.082	0.076	0.066	0.067
	6	mean	-0.098	-0.098	-0.097	-0.197	-0.097	-0.295	-0.096	-0.398	-0.402	-0.395
		std	0.029	0.030	0.029	0.030	0.029	0.033	0.028	0.032	0.039	0.040
	10	mean	-0.102	-0.096	-0.099	-0.198	0.000	-0.297	-0.098	-0.398	-0.402	-0.397
		std	0.021	0.020	0.023	0.023	0.022	0.024	0.021	0.024	0.031	0.030
800												
	2	mean	-0.102	-0.098	-0.100	-0.201	-0.097	-0.301	-0.097	-0.401	-0.398	-0.401
		std	0.041	0.043	0.044	0.042	0.042	0.042	0.040	0.037	0.034	0.033
	6	mean	-0.099	-0.100	-0.099	-0.200	-0.097	-0.300	-0.099	-0.400	-0.398	-0.401
		std	0.015	0.014	0.015	0.016	0.014	0.016	0.014	0.016	0.021	0.020
	10	mean	-0.100	-0.099	-0.100	-0.200	-0.099	-0.300	-0.099	-0.400	-0.399	-0.400
		std	0.010	0.011	0.010	0.011	0.011	0.012	0.010	0.012	0.015	0.015

Notes: Over this Table, we compute the mean and the standard deviation across 500 simulated samples. n is the number of nodes in the network, m regulates the network density as described in Section 4.1. We report further simulation results in Appendix Table A.5.

Bias under Endogenous Treatment. The analytical results in Section 3.4.2 show that in the presence of endogenous treatments, both the CNM and ICM estimates of β can be biased. However, the difference between the two biases can inform about the presence of spillovers and the latter can be still estimated without distortion. In this section, we study the performance of both estimators when treatment endogeneity is sequentially introduced in finite samples. To study this case, we start from the settings used above and generate $\epsilon = \iota X + V$, with V being a normal error term with mean equal to 0 and variance equal to σ. On the one hand, we increase sequentially the endogeneity of the treatment with $\iota = 0, 0.2, 0.5$. On the other hand, we increase the magnitude of spillovers $\phi = \rho = 0, -0.2, -0.4$, as in the previous exercises.

Table 2 reports our results. The first interesting result is that spillovers are always correctly estimated by the CNM, even in the presence of high treatments endogeneity. This is because the spillovers among agents' outcomes can be still recovered even if the treatment is endogenous, as discussed in Section 3.4.2. Moreover, in the Table's first line, we can see how, if spillovers do not really matter, the estimator we propose does never worse than the simple OLS estimator. The second one is that CNM's estimate bias increases steadily in ι, but is not sensitive to ϕ and ρ, at difference with the ICM estimates', where biases compound. Indeed, we can see that the bias of the ICM increases in both ι and $\phi = \rho$, almost doubling the magnitude of the estimated effect when spillovers and treatment endogeneity are high.

Table 2: Performance of All Estimators When Treatment is Endogenous

			ι = 0			ι = -0.2			ι = -0.5		
			$\hat{\phi}$	$\hat{\rho}$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\rho}$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\rho}$	$\hat{\beta}$
$\phi = \rho = 0$	CNM	mean	0.001	-0.001	-2.007	0.003	0.004	-2.194	0.003	0.001	-2.495
		std	0.033	0.035	0.076	0.031	0.032	0.075	0.027	0.027	0.077
	ICM	mean			-2.008			-2.193			-2.494
		std			0.073			0.071			0.074
$\phi = \rho =$ -0.2	CNM	mean	-0.197	-0.197	-2.002	-0.199	-0.197	-2.203	-0.200	-0.199	-2.500
		std	0.033	0.034	0.073	0.029	0.031	0.072	0.027	0.026	0.073
	ICM	mean			-2.231			-2.457			-2.789
		std			0.085			0.084			0.087
$\phi = \rho =$ -0.4	CNM	mean	-0.398	-0.399	-2.003	-0.400	-0.398	-2.196	-0.399	-0.401	-2.496
		std	0.030	0.030	0.077	0.027	0.028	0.083	0.023	0.023	0.076
	ICM	mean			-2.915			-3.197			-3.649
		std			0.152			0.152			0.183

Notes: Over this Table, we compute the mean and the standard deviation across 500 simulated samples. The number of nodes is 800, $m = 2$ and $\beta = -2$.

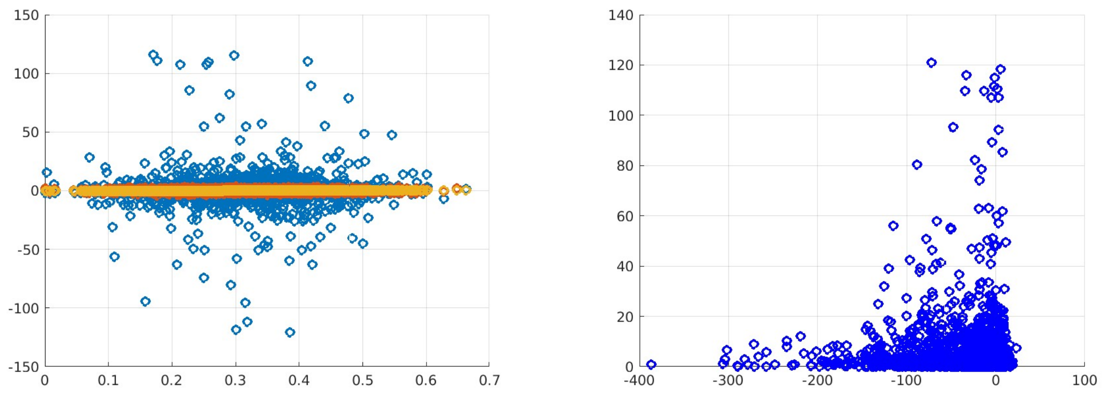
4.2.2 Idiosyncratic Firm and Bank Shocks

On top and above treatment effect's bias correction, and actual identification of the spillover parameters, our simultaneous demand system allows us to retrieve firms and banks fixed effects' estimates, while carefully separating reallocation from actual idiosyncratic credit supply and demand movements. Fixed effect are indeed not only an important tool to mitigate the concern of unobservable drivers of demand and supply biasing our estimates. They are also key tools to actually measure demand and supply shifts, allowing us to quantify the impact of finance on real variables (Amiti and Weinstein, 2018). Nonetheless, if we ignore endogenous reallocation across the network of credit relationships, we may confuse endogenous reallocation of credit demand and supply for actual idiosyncratic shocks. Moreover, idiosyncratic shocks diffuse through the credit network themselves, producing feedback looks that may unduly magnify fixed effects' estimates if not accounted for.

We discuss the details of our fixed effects estimators' good performance in the Appendix, commenting Table A.6. Here, we instead focus on a short graphical presentation, stressing how bias distributes across nodes in the network. We focus on a setting with $n = 2000$, $m = 10$ and $\phi = \rho = -0.4$, to see how idiosyncratic shocks estimates perform with dense networks and high spillovers.

In panel (a) of Figure 10 we plot the true value of the idiosyncratic shocks on the x-axis against themselves (in yellow), the CNM fixed effects estimates (in orange) and the ICM fixed effects estimates (in blue) on the y-axis. We can see that bias for the ICM estimates can be quite severe for some nodes, upward or downward. Indeed, consistently with the results in Section 3.4.3, the bias can be positive or negative under negative spillovers, with sign depending on the network topology.

Figure 10: ICM and CNM Fixed Effects' Estimate and Distribution over Nodes



(a) True, CNM and ICM estimates (b) Centrality and FE bias

Notes: We sample this network setting $n = 2000$, $m = 10$ and $\phi = \rho = -0.4$, the other parameters are the same as in the pivotal simulation described previously. In panel (a), x-axis: true value of the idyosincratic shock, y-axis: true value (in yellow), CNM FE estimates (in orange) and ICM FE estimates (in blue). In panel (b), x-axis: our node i centrality measure $D_i'MD_i$, y-axis: ICM FE estimate for node i.

Panel (b) of Figure 10 plots the nodes' centrality on the x-axis, against the absolute value of the ICM-estimates' bias on the y-axis. We measure the centrality of relationships in which node i is involved with $D_i'MD_i$. As shown in Equations (31)-(32), the bias sign and magnitude depends on the number of loops in which the firm or bank is involved in. In particular, spillover distort more the fixed effects estimate of more 'central' lender and borrowers in the credit network.[22] In the Figure, we can indeed see that the higher the centrality of the node, the higher the value of the ICM estimates' bias.

In other words, even if their idiosyncratic variation is negligible, lenders or borrowers that are more central in the credit network may show particularly high values of their ICM-estimated fixed effects. Nevertheless, these large values may just be the result of shocks originated by other nodes in the network. Given that central nodes are more subject to other nodes' feedback loops, they may accumulate a large amount of variation which is not originated by themselves, but which comes from other banks and firms instead.

4.3 Results in Networks Sampled from the Credit Register

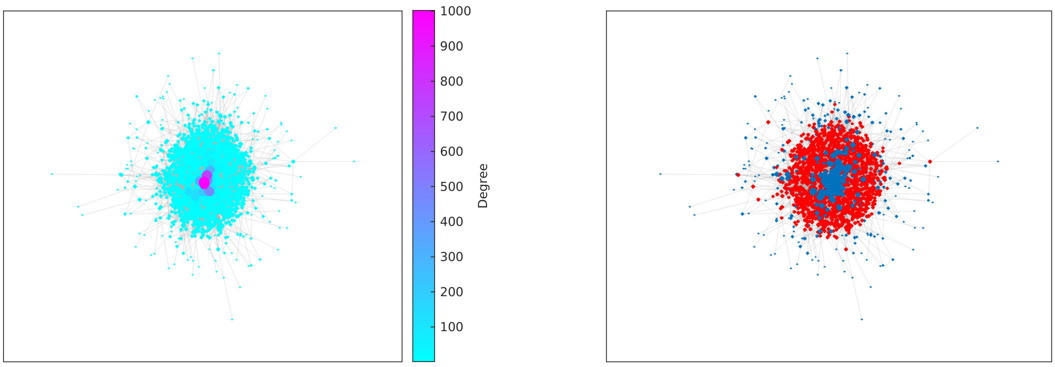
We conclude our simulation study with Monte Carlo experiments based on an observed set of credit relationships. So far we considered fairly simple network structures. Here, we test whether our method still works when the complexity of the credit relationships topology increases, resembling a real credit market structure. We thus construct G using

[22] Still, we remind that under negative spillovers the bias can be close to zero even for central nodes, as positive spillovers from even loops can offset negative ones from odd loops.

realized credit relationships between banks and firms from the credit network we use in the empirical application in Section 5. We randomly extract $n = 400, 800, 2000$ nodes from the full set of credit relationships observed in 2012 and use the links among them in the simulation exercise. The other elements of the simulation are generated in the same way described above.

Figure 11 depicts the network for the 2000 nodes sample. We highlight two features of such a credit network. First, the high interconnectedness between banks and firms; second, the high concentration of connections. In panel (a), the color of each node changes with its degree, i.e. the number of connections it has; more violet nodes represent more connected banks and firms. We can se that the real credit network features high skewness, with some nodes, especially banks, having a very high number of relationships. Some banks lend to thousands of firms in the sample, others only to few of them. This feature of the real credit network is also important for identification, as it guarantees that banks have not fully overlapping portfolios (see Section 3.2 and Proposition 5).

Figure 11: The Credit Network



(a) Degree

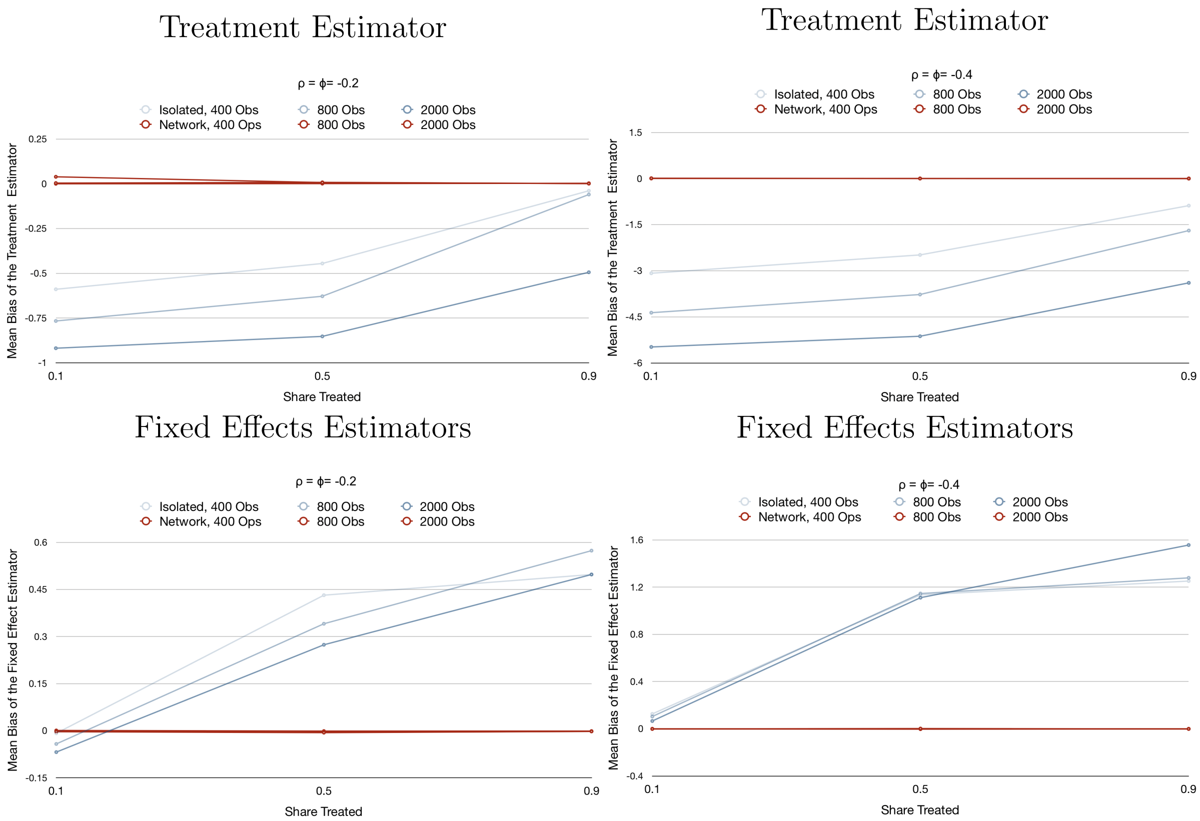
(b) Banks and firms

Notes: We derive the Figure's network sampling 2000 credit relationships observed in 2012. In panel (a), the color of each node is proportional to its degree, more violet nodes represent more connected banks and firms. In panel (b), we represent banks in red and firms in blue. We plot the estimated network with a force-directed layout with five iterations. A force-directed layout uses attractive forces between adjacent nodes and repulsive forces between distant nodes. To ease the visualization, the size of the nodes is equal to the (log) of their degree. See Fruchterman and Reingold (1991) for more details.

In panel (b) we color the nodes denoting firms in blue and banks in red. Looking at the Figure, we note a tight group of banks with a prominently central position in the network, surrounded by a cloud of more peripheral banks. We also highlight a large layer of firms connected to both central and peripheral banks, whose credit relationships connect indirectly many banks. This core-periphery structure resembles that observed in interbank networks (see Boss et al., 2004; Craig and Von Peter, 2014; Iori et al., 2008;

Soramäki et al., 2007, among others), only, in this case, connections are not directly through banks, but indirectly through firms.[23]

Figure 12: ICM Bias and CNM Performance in a Real Network



Notes: We present results from sampled networks, chosen as random subsets of 400, 800, 2000 relationships from the 2012 Italian Credit Register. In different shades of blue, we plot how the mean bias of the isolated-credit model's estimates changes as the share of treated increase. In red, how the mean bias for the credit network model does so. Darker shades signal larger real samples. We report the underlying simulation results in Table A.7.

We report mean bias for treatment effects (upper panel) and idiosyncratic shocks (lower panel) in Figure 12, varying the intensity of spillovers, the share of treated units and the sample size.[24] First, we note that the mean bias of the CNM estimator is always around zero for both treatment effects and idiosyncratic shocks. On the contrary, the ICM estimator presents a relevant bias, which on average increases with spillovers' magnitude and the size of the network. We note that latter is mainly driven by the higher density of bigger networks. Indeed, the bigger the network considered, the lower the probability of censoring links that sampled nodes have with other nodes outside the sample. Accordingly, our 400 nodes sample has a density of 1.25, while the 800 and 2000

[23] As we are working with a sample, the position of each node is not necessarily correctly represented. Nevertheless, the sampling procedure guarantees that all the connections among sampled nodes are included. In the empirical analysis we consider the whole credit network. Unfortunately, its huge dimension does not allow to examine it visually with standard software.

[24] Clearly, as we are sampling from a real network, we cannot vary the density here.

samples have respectively 1.75 and 2.4.

Second, we highlight how the treatment effect's bias decreases when the share of treated units increases, while the the idiosyncratic shocks' bias grows instead. The biases' behavior mirror each other as, when we treat a larger share of relationships, the overall spillover bias grows in importance and it loads in the fixed effects, which capture the average level of each node in the network. On the contrary, such higher share of treated units limits the scope of reallocation in the network, dampening the overestimation of the treatment effect.

In conclusion, the results of our last Monte Carlo experiment confirm that our estimator provides consistent estimates for treatment effects and idiosyncratic shocks even under a more complex structure of credit relationships, while conventional estimators are still biased.

5 Empirical Application

In our empirical exercise, we focus on the dynamics of credit to firms at the intensive margin. Following the vast majority of recent and past works in empirical banking, we employ the yearly log change of credit granted on each relationship as our dependent variable of interest.

As we want to study the bias of estimated bank and firm fixed effects (idiosyncratic shocks) as well as of treatment effects, we do not focus on treatments at the firm or bank-level (e.g., banks' interbank market exposure during a freeze, as in Bonaccorsi di Patti and Sette, 2016; Iyer et al., 2013). We instead focus on *relationship-level* exposure to macroeconomic shocks. The relationship-level effect of macroeconomic shocks has been often explored in the literature by interacting relationships' characteristics with changes in macroeconomic variables, which allows joint estimation of banks and firms' fixed effects (see Jiménez et al., 2014, among others).

Our aim here is not validating a specific strategy used in the literature, our focus is on the quantification of relationship-level endogenous spillovers, as well as their effect on treatment and fixed effects' estimates, instead. The specific setting, or the endogeneity challenge each single study aims to tackle, is really not the point here, also because we have shown analytically and numerically (see Section 3.4.2 and 4.2.1, respectively) that we can precisely quantify spillovers and their importance for other parameters even if the treatment is endogenous.

For all these reasons, we do not replicate a specific analysis. We instead focus on relationship-level effects of changes in the policy rates. Monetary policy is a consistent source of changes in banks cost of funding, allows this paper not to be time or experiment-specific, and it is especially interesting nowadays, when we want to better understand the foreseeable effects of rates increases on firms. In particular, we use as our primary independent variable the interaction between the one-year lag of relationships' revolving intensity and percentage points changes in the Italian interbank overnight rate. Indeed, revolving credit lines have either variable or relatively easy-to-re-bargain rates; hence, revolving-intensive credit relationships will likely bear the most immediate effects of changes in banks' cost of funding.

5.1 Data Description

To perform such an exercise, we use 2012 to 2018 data from (i) the Italian credit register, which tracks all credit relationships between Italian firms and banks whose total exposure in terms of granted credit is greater than 30 thousand euros. We match this data with (ii) the Company Accounts Data System (CADS), balance sheet information for the universe of Italian non-financial corporations provided by the Cerved group, and (iii) the Italian Supervisory Reports, which contain Italian banks' balance sheets and group structure.

Following the literature, we focus on firms with multiple credit relationships so that we can estimate firm fixed effects. Thus, we drop observations belonging to firms with only one relationship each year. Furthermore, we drop all observations belonging to firms with troubled credit relationships (*deteriorati* and *sofferenze*), as well as relationships belonging to foreign banks or non-bank financial intermediaries. Finally, we drop observations with missing granted credit data. We obtain seven yearly samples as a result. Each covers between five and four hundred thousand observations, belonging to about 150 thousand unique firms and five-to-four hundred banks.

Table 3 documents the basic characteristics of each yearly sample. First, the dynamic of granted credit in log changes is mostly negative across the whole study period, with the worst performance recorded in 2012, immediately after the European sovereign debt crisis. Then, we report our relationship-level explanatory variables, the ratio between revolving and total credit (F. Revolving); the ratio between credit granted on the relationship and total credit granted to the firm (F. Granted); a dummy taking value one if the firm and bank's headquarters are located in the same province (Same Prov.). These variables are mostly stable across the years we consider.

As our independent variable of interest is the interaction between lag revolving ratios and overnight rate changes, tracking the heterogeneous impact of movements in banks' refinancing rates, we also document the end-of-year change (year-on-year) in the Italian banks overnight refinancing rate in percentage points.[25] Finally, due to our interest in fixed effects estimation, we report data on the number of relationships per firm and bank, which are stable over the years. The number of banks decreases gradually, instead, mainly because of consolidation in the banking sector.

[25] 3-Month or 90-day Interbank Rates for Italy, retrieved from the ECB Statistical Data Warehouse.

Table 3: Descriptive Statistics

	2012			**2013**			**2014**		
	Mean	*Std Dev.*	*Count*	*Mean*	*Std Dev.*	*Count*	*Mean*	*Std Dev.*	*Count*
Δ Log Granted	-0.092	0.445	517,885	-0.070	0.418	484,953	-0.028	0.431	446,107
F. Revolving	0.289	0.336	517,885	0.298	0.342	484,953	0.295	0.342	446,107
F. Granted	0.084	0.068	517,885	0.084	0.067	484,953	0.085	0.066	446,107
Same Prov.	0.275	0.446	517,885	0.270	0.444	484,953	0.258	0.438	446,107
	Value			*Value*			*Value*		
Δ Overnight Rate %	-1.3			0.046			-0.16		
	Mean	*Median*	*Max*	*Mean*	*Median*	*Max*	*Mean*	*Median*	*Max*
N. Rel. Firm	3.922	2.239	27	3.907	2.238	25	3.898	2.258	23
N. Rel. Bank	30,417	20,668	70,708	27,964	19,480	66,748	25,437	18,037	60,554
	Count			*Count*			*Count*		
N. Firm	159,893			157,352		145,474			
N. Banks	546			542		525			
	2015			**2016**			**2017**		
	Mean	*Std Dev.*	*Count*	*Mean*	*Std Dev.*	*Count*	*Mean*	*Std Dev.*	*Count*
Δ Log Granted	-0.024	0.464	423,056	-0.021	0.452	423,829	-0.011	0.453	410,045
F. Revolving	0.281	0.337	423,056	0.267	0.332	423,829	0.255	0.327	410,045
F. Granted	0.086	0.069	423,056	0.086	0.071	423,829	0.086	0.071	410,045
Same Prov.	0.247	0.431	423,056	0.235	0.424	423,829	0.225	0.418	410,045
	Value			*Value*			*Value*		
Δ Overnight Rate %	-0.171			-0.223			-0.016		
	Mean	*Median*	*Max*	*Mean*	*Median*	*Max*	*Mean*	*Median*	*Max*
N. Rel. Firm	3.917	2.290	27	3.969	2.364	25	3.929	2.345	21
N. Rel. Bank	24,336	17,638	57,891	23,635	17,733	56,275	23,372	18,053	54,870
	Count			*Count*			*Count*		
N. Firm	141,297			137,321			128,953		
N. Banks	493			467			428		
	2018								
	Mean	*Std Dev.*	*Count*						
Δ Log Granted	-0.019	0.467	402,919						
F. Revolving	0.245	0.322	402,919						
F. Granted	0.085	0.070	402,919						
Same Prov.	0.206	0.404	402,919						
	Value								
Δ Overnight Rate %	0.013								
	Mean	*Median*	*Max*						
N. Rel. Firm	3.908	2.339	37						
N. Rel. Bank	25,034	27,786	59,289						
	Count								
N. Firm	138,328								
N. Banks	400								

Notes: This Table presents descriptives for the samples used in the estimation. Each panel covers one year. In each panel, the first four lines record descriptives for the dependent and main relationship-level independent variables. The fifth line reports the value of the end-of-year change in the bank overnight rate in percentage points. The sixth and seventh lines report descriptives concerning the number of relationships per firm and bank. Finally, the last two lines report the total firm and bank count.

5.2 Empirical Specification

We estimate model (10) using the estimators in (21)-(22), and data pooled from 2012 to 2018. As said, we want to measure the impact on relationships' granted credit growth of the interaction between the one-year lag of revolving intensity and percentage point changes in overnight interbank rate. Our main concern with such a specification is that revolving credit intensity can correlate with relationship lending, as revolving lines embody the actual bank-firm relationship (Berger and Udell, 1995), while other forms of credit most often come from non-recurring needs for funding.

First, we notice that this potential bias is acceptable in principle because it implies underestimating rate changes' effect. There is indeed ample evidence (e.g. Berlin and Mester, 1998; Sette and Gobbi, 2015) that relationship lenders smooth the impact of shocks for their long-term customers. Moreover, we use our rich dataset to mitigate this concern further. As relationship lending can be correlated with unobservable firm and bank characteristics, we fully control for these with bank-time and firm-time fixed effects. Then, we add two different proxies of relationship lending, i.e., the lag of the ratio between credit granted on the relationship and credit granted to the firm, measuring the relationship's importance to the firm, and a dummy taking the value of one if the firm and bank's headquarters are located in the same province.[26]

The resulting Isolated Credit Model (ICM) is as follows:

$$\begin{aligned} \Delta\text{Log Granted}_{ibt} = \quad & \delta_{it} + \gamma_{bt} + \beta\Delta\text{Overnight Rate}_{t-1} * \text{F. Revolving}_{ibt-1} + ... \\ & \mu\text{Controls}_{ibt-1} + \varepsilon_{ibt} \end{aligned} \tag{33}$$

where δ_{it} is the firm-time fixed effect; γ_{bt} the bank-time fixed effect; $\Delta\text{Overnight Rate}_t *$ $\text{F. Revolving}_{ibt}$ is our main variable of interest, which we will refer to as Treat_{ibt} when displaying results in Table 4; Controls_{ibt} is a matrix containing the lag in the relationship revolving ratio not interacted with changes in rates, the ratio between credit granted on the relationship and total granted, and the headquarter location dummy.

The ICM's β tracks differences in the growth of credit granted on bank-firm relationships that are more revolving credit intensive, after a rate change, within the *same* firm, absorbing all time-varying bank unobservables, and controlling for relationship characteristics. However, the ICM does not control for spillover effects among relationships. Such spillovers arise from firms and banks reallocating credit across their relationships' portfolios as relative cost changes. For example, following the arguments in Section 2.1, after rates drop, a firm could reallocate its credit demand towards relationships for which the discount's pass-through is greater from relationships for which it is smaller. Compar-

[26] On the relationship between distance and lending, see Agarwal and Hauswald (2010); Degryse and Ongena (2005).

ing the variation in credit growth on two such relationships without accounting for the endogenous reallocation will bias the estimated β's magnitude upward.

To account for the bias from similar direct adjustments (also described in Section 2.1), as well as from higher order indirect effects (derived and presented in Section 3.4), we estimate the following Credit Network Model (CNM):

$$\begin{aligned}\Delta\text{Log Granted}_{ibt} = \quad & \delta_{it} + \gamma_{bt} + \beta\Delta\text{Overnight Rate}_t * \text{F. Revolving}_{ibt-1} + ... \\ & \phi N_B \Delta\text{Log Granted}_{ibt} + \rho N_F \Delta\text{Log Granted}_{ibt} + ... \\ & \mu\text{Controls}_{ibt-1} + \varepsilon_{ibt}\end{aligned} \tag{34}$$

here, we introduce as further controls the bank- and firm-network lags of the dependent variable, which keep track of the connections among relationships through banks and firms. We formalize this addition with the N_B and N_F operators, such that $N_B x_{ibt} = \sum_{j\in\mathbb{F}\backslash i} a_{ib,jb} x_{jbt}$ is the bank-network lag of x_{ibt} and $N_F x_{ibt} = \sum_{k\in\mathbb{B}\backslash b} a_{ib,ik} x_{ikt}$ the firm-network lag of x_{ibt}.

5.3 Main Results

Estimated Spillovers and Treatment Effects. Table 4 reports the results, with the 2SLS' second stage estimates of the ICM and the CNM on the left panel and the CNM's first stages for the two endogenous variables on the right panel.

First, we note that the bank and firm's spillover coefficients are highly statistically significant, with ratios between coefficients and standard errors well above three. Concerning magnitudes, the estimate of ρ (FSC) is about -0.6, while the estimate of ϕ (BCR) is much smaller. We expect this size disparity, as banks have many more relationships than firms. Thus, we re-scale the ϕ estimate by 10,000, or slightly less than half the number of corporate credit relationships of the average bank (see Table 3), obtaining the final $\phi*$ estimate of -0.07.

We can perform the following two back-of-the-envelope calculations to understand the above estimates' magnitudes and their economic significance. For firms, we can think of one with three relationships (the average number we report in Table 3), seeing granted credit shrinking by 20 percent from two of its three banks. Such a firm will likely ask for an extension of the remaining line, and our ρ estimate suggests that this last will grow by about 24 percent ($0.24 = -0.6 * 2 * -0.20$). For banks, we can think of one granting 20 percent more credit to 10,000 borrowers. Our $\phi*$ estimate suggests that such action would imply a 1.5 percent crowd-out on all bank's other credit lines ($-0.0146 = 0.2 * -0.073$).

Then, we report the estimates of treatment and controls' coefficients. First, we see that credit relationships belonging to the same firm with greater revolving ratios expand more after overnight rate decreases.[27] Then, overall, credit relationships that are more revolving intensive (F. Revolving$_{ibt-1}$), as well as relationships that are more important for the total credit access of the firm (F. Granted$_{ibt-1}$), grow less, while relationships for which firm and bank's headquarters are in the same province grow slightly more. These conclusions hold in both the estimated CNM and ICM.

Focusing on the main variable of interest in the CNM (Table 4, Column (1), line 4) and re-scaling the treatment effect by the standard deviation in revolving fractions (about 0.3, see Table 3), we observe that credit relationships one standard deviation more revolving-intensive grow by 6 percent more after a one percent decrease in the overnight rate. We would have overestimated the magnitude of this coefficient by a factor of 2.5 had we not accounted for the network nature of credit relationships, as we can appreciate from the ICM panel (Table 4, Column (3), line 4). We expect such a large bias, as, looking at within-firm changes, the ICM estimator is likely to sum the endogenous demand reallocation induced by supply shifts, magnifying the estimates.

Regarding the first stage, we derive the dependent variables computing, for each relationship, the total growth in credit on other relationships by the same bank (Table 4, Columns (5-6)) or firm (Table 4, Columns (7-8)).[28] Then, we regress these quantities on all the bank and firm-network lags of controls we include in the second stage. As we employ both network lags jointly in each first stage, the coefficients we display are net of the bank-lags and firm-lags effects and are difficult to interpret meaningfully. Nonetheless, the critical insight from the first stage panel in Table 4 is that the values of F_{SW}, the F-test for weak instruments in linear IV models with multiple endogenous variables proposed by Sanderson and Windmeijer (2016), are large and do not lend support to weak IV concerns (see Table 4, Columns (5) through (8), third line from the bottom).[29] These large F_{SW}s indicate that the instruments are relevant and both endogenous variables are sharply identified.

[27] We comment on negative variations of the overnight rate as most large changes in our sample, documented in Table 3, are negative. Hence, the most empirically relevant variation is the effect of a one percent drop, as the one registered in 2012.

[28] Coefficients and errors for variables' bank-network lag in the first stage of $N_F \Delta$Log Credit$_{ibt}$ are multiplied by 10,000(Table's Columns (7-8), first four lines). Moreover, in the $N_B \Delta$Log Credit$_{ibt}$ first stage, the coefficients and errors for the variables' firm-network lags are divided by 10,000 (Table's Columns (5-6), last four lines).

[29] In greater detail, both figures are largely above the relevant minimum eigenvalue's threshold for the Cragg-Donald statistic, tabulated by Stock and Yogo (2002).

Table 4: Spillovers Estimates, First and Second Stage

	(1)	(2)	(3)	(4)		(5)	(6)	(7)	(8)
Dependent Variable:	Second Stage ΔLog Credit$_{ibt}$					First Stage $N_B\Delta$Log Credit$_{ibt}$		$N_F\Delta$Log Credit$_{ibt}$	
	CNM		*ICM*						
	coeff.	std. err.	coeff.	std. err.	IV	coeff.	std. err.	coeff.	std. err.
Firm Spillover: ρ	-0.6204	0.0032			N_BTreat.$_{ibt-1}$	0.1050	0.0002	-0.0161	0.0011
Bank Spillover: $\phi*$	-0.0703	0.0001			N_BF. Revolving$_{ibt-1}$	-0.2388	0.0004	0.0252	0.0029
					N_BSame Prov.$_{ibt-1}$	-0.1419	0.0003	0.0353	0.0019
Treat.$_{ibt-1}$: β	-0.1887	0.0016	-0.4661	0.0020	N_BF. Granted$_{ibt}$	0.5796	0.0015	-0.2028	0.0103
F. Revolving$_{ibt-1}$: μ_1	-0.1616	0.0014	-0.4148	0.0011	N_FTreat.$_{ibt-1}$	-0.0201	0.0004	-0.1760	0.0029
Same Prov.$_{ibt}$: μ_2	0.0112	0.0003	0.0258	0.0006	N_FF. Revolving$_{ibt-1}$	-0.0087	0.0002	-0.1847	0.0016
F. Granted$_{ibt-1}$: μ_3	-0.5302	0.0049	-1.4116	0.0042	N_FSame Prov.$_{ibt-1}$	-0.0023	0.0001	-0.0115	0.0007
					N_FF. Granted$_{ibt}$	-0.3282	0.0048	-0.5356	0.0322
N	3,108,758		3,108,758			3,108,758		3,108,758	
F_{SW}						154,602		2,144	
Bank-FE	Yes		Yes			Yes		Yes	
Firm-FE	Yes		Yes			Yes		Yes	

Notes: Estimated coefficients and standard errors for the second and first stages of model (11) employing the 2SLS estimator in Equation (22), in the particular case of Equation (34). The second stage dependent variable ΔLog Credit$_{ibt}$ is the yearly log growth rate of the credit relationship. N_B is the bank-network lag operator, and it equals to $\sum_{j\in\mathbb{F}\setminus i} a_{ib,jb}x_{jbt}$ for every x covariate; N_F is the firm-network lag operator, and it equals to $\sum_{k\in\mathbb{B}\setminus b} a_{ib,ik}x_{ikt}$ for every x covariate. Treat.$_{ibt-1}$ is the revolving ratio multiplied by the change in the overnight interest rate. Bank-related coefficients are re-scaled to account for the disparity in the number of relationships between banks and firms. In particular, $\phi*$ and its errors are multiplied by 10,000, as well as coefficients and errors for variables' bank-network lag in the first stage of $N_F\Delta$Log Credit$_{ibt}$ (Table's columns 7 and 8, first four lines). Moreover, in the $N_B\Delta$Log Credit$_{ibt}$ first stage, the coefficients and errors for the variables' firm-network lags are divided by 10,000 (Table's columns 5 and 6, last four lines). F_{SW} statistics are reported for the first stages. The F_{SW} is the F-test for weak instruments in linear IV models with multiple endogenous variables proposed by Sanderson and Windmeijer (2016).

Idiosyncratic Shocks' Bias. We now analyze the estimated banks' and firms' fixed effects. Following our simulation study in Section 4, we focus on the distortion that occurs when spillovers are not accounted for. Given that we found significant spillover effects, we use the estimates from the CNM as unbiased measures of γ_{bt} and δ_{it} and compare them with ICM estimates, i.e., the ones not including the endogenous terms capturing the BCR and the FSC.

In Table 5, we report the empirical Mean and Median Bias, as well as the Mean and Median Absolute Bias, separately for firms and banks' fixed effects (FEs henceforth) estimates.

Table 5: Fixed Effects, Empirical Bias Measures

	Bank	Firm
Mean Bias	1.329	-0.750
Median Bias	0.735	-0.444
Mean Absolute Bias	1.414	1.297
Median Absolute Bias	0.742	0.564

Notes: The Table reports the empirical mean bias, mean absolute bias, median bias, and median absolute bias for firm and bank estimated fixed effects. We consider the fixed effects estimated correcting for network structure as the true parameters. We compute the bias measures with the following formulas, here reported only for the bank fixed effect case: $\text{MB}_{\text{Bank}} = 1/B\left\{\sum_{k=1}^{B}(\hat{\gamma}^k_{ICM} - \hat{\gamma}^k_{CNM})/|\hat{\gamma}^k_{CNM}|\right\}$, $\text{MAB}_{\text{Bank}} = 1/B\left\{\sum_{k=1}^{B}|\hat{\gamma}^k_{ICM} - \hat{\gamma}^k_{CNM}|/|\hat{\gamma}^k_{CNM}|\right\}$, $\text{MedB}_{\text{Bank}} = Med\left\{(\hat{\gamma}_{ICM} - \hat{\gamma}_{CNM})\, Diag\left[||\hat{\gamma}_{CNM}||^{-1}\right]\right\}$, $\text{MedAB}_{\text{Bank}} = Med\left\{(||\hat{\gamma}_{ICM} - \hat{\gamma}_{CNM}||)\, Diag\left[||\hat{\gamma}_{CNM}||^{-1}\right]\right\}$. Where B is the number of banks in the market, Med is the median operator, MB stands for Mean Bias, MedB for Median Bias, MAB for Mean Absolute Bias, and MedAB for Median Absolute Bias.

In Table 5, each indicator aggregates the difference between the FE estimated by the ICM and the FE estimated by the CNM. We focus on differences divided by the absolute value of the CNM's FEs so that magnitudes are in percentage of the unbiased estimate and easier to compare. First, looking at the mean and median bias, bank FEs are overestimated on average by the ICM. On the contrary, firm FEs are underestimated. The positive bias for banks ranges between 73 (median) to 133 (mean) percent of the true parameter, while the negative bias for firms ranges between 44 (median) to 75 (mean) percent. Second, we notice that the bias's average and median absolute values are prominent for bank and firm fixed effects. The median absolute bias stands around 74 percent for banks and 56 percent for firms. The values for the mean absolute bias are even higher, pointing to the presence of highly biased FE for some banks and firms.

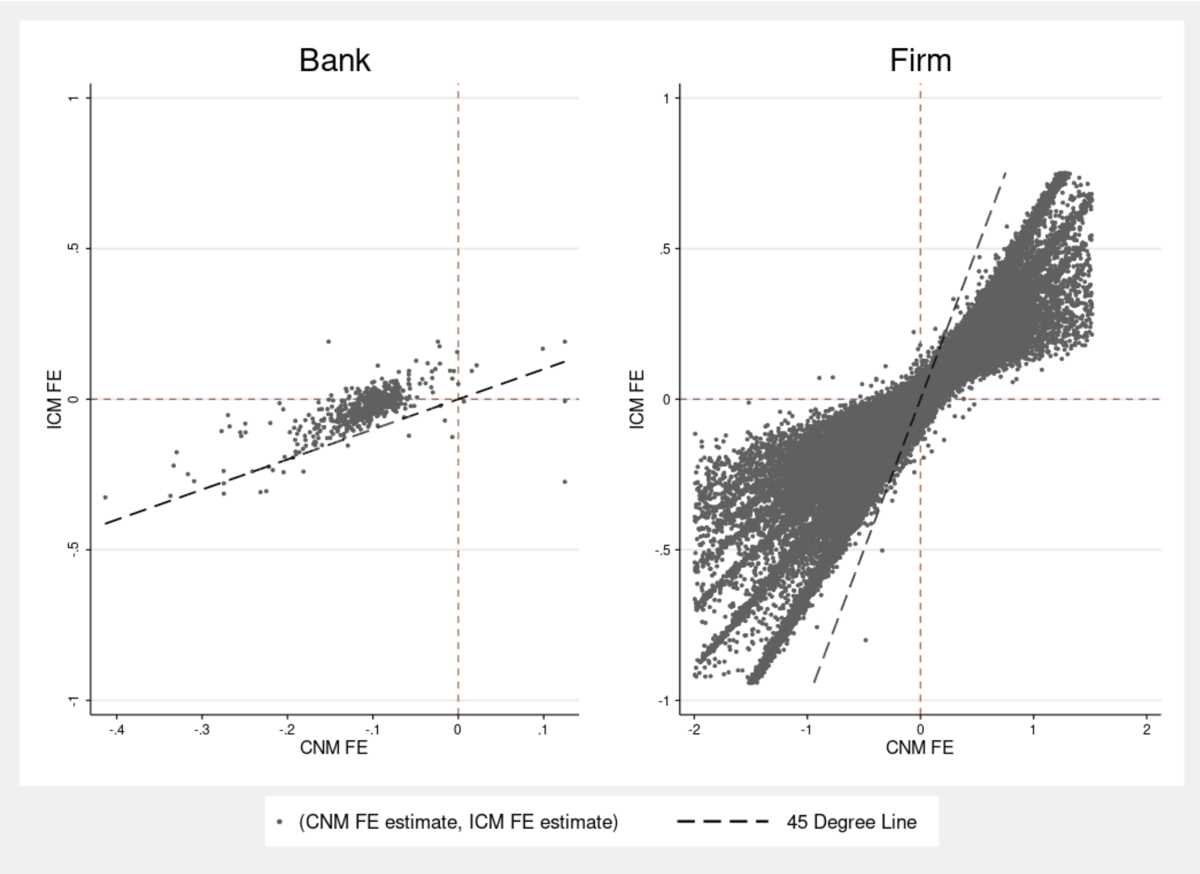
The large magnitude of the bias is easily explained by looking at the economic importance of spillover effects in Table 4's estimates and thinking of links' density over banks. At the firm level, first-order spillovers are substantial, while many relationships are connected through banks. When not accounting for the network nature of credit relationships, first-order spillovers through firm links lead to overestimating treatment effects. This overestimation happens at the expense of the idiosyncratic firm component if the latter is correlated with the treatment status. Then, as banks connect many nodes in the network, the network-wide transmission of spillovers through bank links leads us to overestimate the banks' idiosyncratic effects. When accounting for the network structure, we see that this last bias was just the result of the high centrality of banks in the credit relationships' network, as suggested before by simulation results (see Figure 10).

We can deepen the nature of the ICM-estimated fixed effects bias by plotting the

ICM estimates against the CNM unbiased estimates for each firm and bank. We do so in Figure 13, which we read as follows. First, on the left panel, we see that almost all ICM-estimated bank fixed effects are above the 45-degree. Hence, the ICM model is systematically overestimating idiosyncratic credit supply changes. In particular, it *underestimates the magnitude* of bank-level credit *contractions*. Furthermore, one-quarter of the scatter points lie in the upper-left quadrant; which is, we have numerous cases in which the CNM unbiased estimate is negative while the ICM biased estimate is positive. This sign discrepancy means that the bias in the ICM model would point to nonexistent idiosyncratic supply expansions for one in four banks.

Then, in the right panel, we can see a graphical depiction of how the ICM systematically underestimates the magnitude of idiosyncratic demand changes. Moreover, even if less often than with banks, the ICM switches the sign of firms' idiosyncratic demand changes for seven percent of firms. In conclusion, the evidence we gather points to the fact that estimating bank and firm fixed effects in an ICM may lead to a large bias in a real empirical setting. The bias's size and magnitude are node-specific, often switching the sign of the estimated idiosyncratic shock. Thus, using such estimates may be highly misleading.

Figure 13: The Empirical Distribution of Fixed Effects Estimates' Bias



Notes: The Figure displays, on the left, the scatterplot of bank fixed effects' estimates in the CNM (x-axis) and ICM (y-axis); on the right, the scatterplot of firm fixed effects' estimates in the CNM (x-axis) and ICM (y-axis). The short-dashed line is the 45-degree line.

6 Extensions and Discussions

In this section, we present some extensions of the CNM to allow for the endogeneity of credit relationships and treatments, and for heterogeneous ϕ and ρ, which may be of practical relevance in empirical studies. We then compare our model with other approaches used in the literature.

6.1 Endogenous Credit Relationships

A common concern in the econometric analysis of spillovers over networks is the possible endogeneity of the network itself. Issues can arise if for example some unobserved factors drive the formation of the links (here the credit relationships) and the outcomes (credit quantity). For the link formation, it is often considered the following type of dyadic network formation model,

$$g_{ib} = I(d(h_i, h_b) \geq u_{ib}), \tag{35}$$

where $g_{ib} = 1$ if there exist a credit relationship between firm i and bank b, h_i and h_b are unobserved individual specific characteristics, u_{ib} is a link-specific random component, and $d(.,.)$ is some function. The unobserved node-specific characteristic h_i can be interpreted as a factor that increases the likelihood of forming a link. Network endogeneity may arise if firm (bank) individual unobserved characteristic h_i (h_b), which affects link formation, is correlated with i's (b's) unobserved characteristic that affects the outcome c_{ib}. In this context, it can be interpreted as a firm or bank specific characteristic that makes it more likely to form relationships in the market and increase the credit granted. For example, it could be lower risk or higher profitability of projects for firms, higher monitoring or screening capacity for banks. Compared to models in which the outcome is at the node level (see Arduini, Patacchini, and Rainone, 2015; Auerbach, 2022; Goldsmith-Pinkham and Imbens, 2013; Hsieh and Lee, 2016; Johnsson and Moon, 2021; Patacchini and Rainone, 2017; Qu and Lee, 2015, among the others), a key difference in our context is that we model outcomes at the link level. It allows us to include node fixed effects (δ_i and γ_b), which alleviates this type of concerns. Nevertheless, one can also assume that there are link level correlated unobservables, if for example

$$g_{ib} = I(d(h_i, h_b, h_{ib}) \geq u_{ib}), \tag{36}$$

and h_{ib} is correlated with ϵ_{ib}. In this case, a connection between two credit relationships is observed if

$$a_{ib,jb} = g_{ib}g_{jb} = I(d(h_i, h_b, h_{ib}) \geq u_{ib})I(d(h_j, h_b, h_{jb}) \geq u_{jb}). \tag{37}$$

It follows that even if $E[h'_{ib}\epsilon_{ib}] \neq 0$ it does not imply that $E[a'_{ib,jb}\epsilon_{ib}] \neq 0$ because h_{ib} enters $a_{ib,jb}$ in a highly non linear form and it is multiplied by terms not necessarily correlated with ϵ_{ib}. In addition, all the terms in (37) only share b-indexed variables, but node b specific factors are absorbed by γ_b. It is, however, possible to apply a control function approach at the link level similar to those developed by Arduini, Patacchini, and Rainone (2015); Johnsson and Moon (2021),[30] or a Bayesian approach (in the spirit of Goldsmith-Pinkham and Imbens, 2013; Hsieh and Lee, 2016; Patacchini and Rainone, 2017). Therefore, controlling for h_{ib}, the network A and ϵ_{ib} become mean independent, that is,

$$E(\epsilon_{ib}|A, h_{ib}) = E(\epsilon_{ib}|h_{ib}) =: k(h_{ib}). \tag{38}$$

We can then consider the outcome equation that controls for $\hat{h}_{ib}$ nonparametrically,

$$c_{ib} = \alpha + \phi \sum_{j \in \mathbb{F} \setminus i} a_{ib,jb}c_{jb} + \rho \sum_{k \in \mathbb{B} \setminus b} a_{ib,ik}c_{ik} + \delta_i + \gamma_b + x_{ib}\beta + k(\hat{h}_{ib}) + u_{ib}, \tag{39}$$

[30]See Rainone (2020b) for an application of interbank markets. Even if his model does not include node fixed effects, the control function approach does not change the estimated parameters significantly.

where $u_{ib} := \epsilon_{ib} - k(\hat{h_{ib}})$. Once we control the endogeneity of the network, the regressor of the spillover effect becomes exogenous, and we can estimate coefficient ϕ using the conventional partially linear regression estimation method (Robinson, 1988). Observe that modeling the formation of credit relationships among hundred of thousands of firms and hundreds of banks with multiple matches could be challenging not only in terms of correct modeling but also computationally. To the best of our knowledge, the only paper that accounts for endogeneity of links in the credit market is Jiménez et al. (2014). They exploit unique data on loan applications to restrict the dimensionality problem, estimate a selection equation that involves the granting of loans in the first stage and credit outcome equations for the applications granted in the second stage. Differently from ours they focus on sample selection issues, and do not model interdependence among credit relationships. It is worth noting, that in our model the endogeneity of credit relationship may not only affect the estimate of β or other coefficients per se, it can also affect the estimates of ϕ and ρ through the potential endogeneity of A_B and A_F, as discussed above.[31] Unfortunately, loan application information is not always available in credit register data. It has to be noted that the types of models of network formation commonly used do not allow for network effects in link formation, as a link between i and b depends only on the characteristics of i and b. Indeed, especially in this context an interesting extension could incorporate a transferable matching step in the spirit of Fox (2009). As this is a non-trivial extension, we leave this work for future research. For all these reasons, we focus on conditionally exogenous networks mainly.

6.2 Endogenous Treatments

In the previous analysis, we assumed that the main regressor is exogenous. Our method can accommodate a instrumental variable strategy. For example, let us now allow X to be an endogenous regressor and assume that a valid instrument W is available, $X = W\kappa + \omega$, and $E[\epsilon'\omega] \neq 0$, then we can include the instrument and its network-lags in the first step in a quite straightforward way (see Anselin and Lozano-Gracia, 2008; Dall'Erba and Le Gallo, 2008, for applications of this procedure).[32] In practice, the empirical IV in (16) and (17) can be augmented in the following way

$$EIV_F^{1W} = [A_F X, A_F W, W], \tag{40}$$

$$EIV_B^{1W} = [A_B X, A_B W, W]. \tag{41}$$

[31] Jiménez et al. (2014) use the method proposed by Kyriazidou (1997), (see also Arellano and Honoré, 2001; Honore, Kyriazidou, and Powell, 2000), which does not require distributional assumptions (like normality of the errors in the selection equation) and differences out both the sample selection effect and the unobservable individual effect from the equation of interest, under the conditional exchangeability assumption. This assumption could be stronger for our model, given the presence of interdependence.

[32] See Fingleton and Le Gallo (2008) for the finite sample properties of this type of estimators.

6.3 Heterogeneous BCR and FCS

The CNM can be augmented to have bank- and firm-type specific spillovers, for example if one is interested in studying the substitution of credit from a type of banks to another type of banks by firms, or the reallocation of credit from a certain type of firms to another one by banks, and eventually combinations of the two depending on the specific empirical questions. A typical example for the FSC is the substitution of credit from low technology banks to high technology ones by firms demanding new and more modern financial services.[33] Another one, focusing this time on the BCR, is the reallocation of credit by banks from sectors hit by specific shocks to other unaffected sectors in the economy.[34] Let us focus on the first case and suppose that there are H and L type banks. If we have two types, we will have four types of ρs. ρ^H captures the spillovers among relationships involving type H banks, ρ^L captures the spillovers among relationships involving type L banks, ρ^{LH} captures the spillovers from relationships with type H bank to those with type L, ρ^{HL} captures the spillovers from relationships with type L bank to those with type H. If, for example, one expects high substitution of credit from low tech banks to high tech banks, ρ^{HL} should be higher than the others. Model (11) will then become:

$$C = (\rho^H A_F^H + \rho^L A_F^L + \rho^{HL} A_F^{HL} + \rho^{LH} A_F^{LH})C + \phi A_B C + Z\mu + \epsilon. \tag{42}$$

where the matrix A_F^H keeps track of connections among relationships involving the same firm and banks of type H, the matrix A_F^L keeps track of connections among relationships involving the same firm and banks of type L, A_F^{HL} and A_F^{LH} are symmetrically equal matrices that keep track of connections among relationships involving the same firm and banks of both types. In such extension of the baseline model, the instrumental variables change accordingly to the different specification. First order approximations of the best IV for the five endogenous variable are respectively:

$$\begin{aligned}
EIV^1_{FH} &= A_F^H X, \\
EIV^1_{FL} &= A_F^L X, \\
EIV^1_{FLH} &= A_F^{LH} X, \\
EIV^1_{FHL} &= A_F^{HL} X, \\
EIV^1_{B} &= A_B X.
\end{aligned}$$

[33]See Fuster et al. (2019), Fuster et al. (2018), Branzoli, Rainone, and Supino (2023), Core and De Marco (2021) and Kwan et al. (2021), among the others, for studies on the effects of bank technological adoption and credit.

[34]See Paravisini, Rappoport, and Schnabl (2017), Federico, Marinelli, and Palazzo (2023) and Federico, Hassan, and Rappoport (2023), among the others, for studies on lending behavior by banks more specialized or with loan portfolios concentrated in sectors more exposed to shocks.

Second order approximations of the best IV for the five endogenous variable are respectively:

$$\begin{aligned} EIV^2_{FH} &= A^H_F[I, A_B]X, \\ EIV^2_{FL} &= A^L_F[I, A_B]X, \\ EIV^2_{FLH} &= A^{LH}_F[I, A_B]X, \\ EIV^2_{FHL} &= A^{HL}_F[I, A_B]X, \\ EIV^2_B &= A_B[I, A^H_F, A^L_F, A^{LH}_F, A^{HL}_F]X. \end{aligned}$$

Similar derivations can be computed for higher order approximations. If we are interested in heterogeneous reallocation policies by the banks, suppose within and between two sectors (say S and P), we will have four types of ϕs. ϕ^S captures the spillovers among relationships involving firms in sector S, ϕ^T captures the spillovers among relationships involving firms in sector T, ϕ^{TS} captures the spillovers from relationships with firms in sector S to those in sector T, ϕ^{ST} captures the spillovers from relationships with firms in sector T to those in sector S. If, for example, one expects high substitution of credit from low tech banks to high tech banks, ρ^{LH} should be higher than the others. Model (11) will then become:

$$C = (\phi^S A^S_B + \phi^T A^T_B + \phi^{ST} A^{ST}_B + \phi^{TS} A^{TS}_B)C + \rho A_F C + Z\mu + \epsilon. \quad (43)$$

where the matrix A^S_B keeps track of connections among relationships involving the same bank and firms of sector S, the matrix A^T_B keeps track of connections among relationships involving the same bank and firms of sector T, A^{ST}_B and A^{TS}_B are symmetrically equal matrices that keep track of connections among relationships involving the same bank and firms of both sectors. In such extension of the baseline model, the instrumental variables change accordingly to the different specification. First order approximations of the best IV for the five endogenous variable are respectively:

$$\begin{aligned} EIV^1_{FH} &= A^S_B X, \\ EIV^1_{FL} &= A^T_B X, \\ EIV^1_{FLH} &= A^{TS}_B X, \\ EIV^1_{FHL} &= A^{ST}_B X, \\ EIV^1_B &= A_F X. \end{aligned}$$

Second order approximations of the best IV for the five endogenous variable are respectively:

$$\begin{aligned}
EIV_{FH}^2 &= A_B^S[I, A_F]X, \\
EIV_{FL}^2 &= A_B^T[I, A_F]X, \\
EIV_{FLH}^2 &= A_B^{TS}[I, A_F]X, \\
EIV_{FHL}^2 &= A_B^{ST}[I, A_F]X, \\
EIV_B^2 &= A_F[I, A_B^S, A_B^T, A_B^{TS}, A_B^{ST}]X.
\end{aligned}$$

Similar derivations can be computed for higher order approximations. In general terms, when we are interested in heterogeneous FSC and BCR, instrumental variables depend on the final specification and thus on the specific research question. We thus do not provide sufficient identification conditions as those in Proposition 5 for any possible configuration. Nevertheless, a more general condition for identification is that the matrix including the expected value of the endogenous variables and the other covariates in the model has full rank. In this example, we need $[E(A_B^S C), E(A_B^T C), E(A_B^{ST} C), E(A_B^{TS} C), E(A_F C), Z]$ to have full rank. The intuition is that as long as there are intransitive quadriads for any combination of heterogeneous BCR and FCS resulting from the selected choices, this condition is always respected, the parameters are identified and the IVs can be constructed as linear combinations of the vectors appearing in the expected value of each endogenous terms.

6.4 Comparison between OPIVs and SSIVs and GIVs

In general terms, OPIVs are related to other approaches decomposing market's aggregate outcomes to derive instrumental variables, such as shift share instrumental variables (Bartik, 1991; Blanchard et al., 1992; Borusyak, Hull, and Jaravel, 2022; Goldsmith-Pinkham, Sorkin, and Swift, 2020, SSIVs, see), and granular instrumental variables (GIVs, Gabaix and Koijen, 2020). However, our approach differs from the GIVs and SSIVs approaches substantially. OPIVs and both these approaches are actually complements, because they can be used in different types of markets. The GIVs and SSIVs are procedures designed to estimate price elasticities in centralized markets, where there is only one price, while OPIVs is designed to estimate objects more similar to elasticities of substitution in decentralized markets, where the price varies at the pair level and the identity of counterparties matters. For example, the GIVs exploits the fact that in centralized markets single agents demand (or supply) depends on the aggregate price, but the aggregate price does not depend on the demand of the single agent, but rather the aggregate demand. The instrumental variable is obtained when few large actors account for a substantial fraction of aggregate demand (supply) and idiosyncratic shocks are volatile relative to the volatility of aggregate shocks. Although being deeply different from each other, both GIVs and SSIVs derive instruments by decomposing aggregate quantities (like the aggregate demand, for example) and using exogenous components, under different assumptions. The OPIV does not decompose any aggregate quantity, it instead derives instrumental variables for endogenous disaggregated outcomes, exploiting intransitivity in decentralized markets. The OPIVs exploits the fact that in decentralized markets agents demand (or supply) in a single contract depends on what happens in other contracts involving the same parties. The instrumental variable is obtained under the condition that agents have not fully overlapping portfolios of counterparties. Whereas GIVs uses players' size disparities to derive exclusion restrictions, OPIVs use intransitivity. GIV needs that idiosyncratic shocks to large players can be separated from systemic ones (granularity). What OPIVs need is that not all banks lend to all firms.

6.5 Comparison with other Studies on Spillovers in Corporate Finance

In this section we discuss the main differences between our model and those of the family which Huber (2022), Berg, Reisinger, and Streitz (2021) belong to. The first difference is about the main outcome variables and the source of spillovers considered. We study the formation of outcomes in the credit market, and specifically we focus on quantities of loans. They study the effects of shocks to banks to firms' outcomes, such as employment. They assume that spillovers come from firms operating in the same region or sector. Our spillovers come from relationships that share the very same counterparty. In other words, they provide tools to account for spillovers among firms when the effects of financial shocks on real outcomes are analyzed. We provide a tool to model spillovers among credit relationships and account for them when the effects of financial shocks on credit outcomes are analyzed. From this perspective the two approaches can be useful complements to study the effects of financial shocks.

This aspect implies another difference: we study outcomes at the bilateral level, because we look at credit market outcomes that are bilateral by construction, the other papers look at individual outcomes. It allows us to exploit the micro structure of the market for identification and infer on counterparties specific behaviors. It comes with non trivial differences in the complexity of the model and in the interpretation of the results. Indeed, we can provide a structural model that not only controls for spillovers, but also allows to recover parameters that have a direct behavioral interpretation in terms of credit substitution and reallocation, as discussed above. Implied by this different approach is that we focus on endogenous effects and they focus on exogenous effects. The endogenous effects capture agents' choices/outcomes that depend on that of others, and can be interpreted through assumed response functions of agents, while exogenous (or contextual) effects control for others' exogenous characteristics or treatment status.

They use linear-in-means models, which do not allow for disentangling endogenous from exogenous effects (Manski, 2013). On the contrary, exploiting the network nature of credit markets, we are able to account for and potentially estimate both (Bramoullé, Djebbari, and Fortin, 2009). Our focus is on endogenous effects because they can be easily derived from simple models, like the one proposed in Section 2, however the identification conditions and the instrumental variables proposed hold even if we want to include exogenous effects.

7 Conclusion

We present a network model of interdependent credit relationships. We use it to show that standard bank lending channel estimators, as well as bank and firm fixed effect estimators, suffer from spillovers bias due to firms and banks' endogenous reallocation over the credit relationships' network. We show numerically and empirically that such bias can be large, and depends in sign and magnitude on the credit market structure.

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Appendix

A.1 Monte Carlo Tables

Table A.1: Simulation study - ICM treatment effect bias under different share of treated, spillovers and density

$\phi = \rho$	% of treated	m=2		m=4		m=6		m=8		m=10	
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	-0.059	0.124	-0.198	0.110	-0.299	0.143	-0.390	0.199	-0.456	0.251
	0.25	-0.007	0.036	-0.037	0.026	-0.080	0.029	-0.117	0.031	-0.155	0.039
-0.2	0.50	0.142	0.035	0.238	0.067	0.295	0.095	0.314	0.106	0.328	0.114
	0.75	0.266	0.079	0.516	0.272	0.660	0.441	0.749	0.565	0.813	0.664
	0.90	0.339	0.123	0.685	0.473	0.879	0.774	1.006	1.014	1.095	1.201
	0.10	-0.253	0.213	-0.660	0.574	-0.958	1.016	-1.206	1.546	-1.352	1.909
	0.25	-0.105	0.063	-0.397	0.208	-0.588	0.388	-0.766	0.629	-0.901	0.848
-0.3	0.50	0.093	0.031	0.067	0.026	0.013	0.020	-0.064	0.022	-0.113	0.029
	0.75	0.313	0.110	0.517	0.277	0.601	0.369	0.642	0.421	0.661	0.444
	0.90	0.436	0.197	0.791	0.630	0.961	0.929	1.056	1.118	1.133	1.288
	0.10	-0.622	0.653	-2.029	4.629	-3.048	9.756	-3.766	14.635	-4.245	18.382
	0.25	-0.410	0.280	-1.512	2.479	-2.334	5.623	-2.890	8.530	-3.295	11.025
-0.4	0.50	-0.052	0.043	-0.638	0.491	-1.117	1.337	-1.457	2.199	-1.723	3.042
	0.75	0.273	0.095	0.201	0.073	0.081	0.042	-0.032	0.032	-0.099	0.040
	0.90	0.478	0.238	0.728	0.541	0.807	0.665	0.840	0.718	0.867	0.765

Notes. The mean bias and the MSE are copmuted across the 500 simulated samples. The number of nodes N in the network is 200, 100 firms and 100 banks, in each sample. The column spillovers intensity reports the value of $\phi = \rho$ in the simulations.

Table A.2: Simulation study - ICM treatment effect bias with FEs under different share of treated, spillovers and density

$\phi = \rho$	% of treated	m=2		m=4		m=6		m=8		m=10	
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	-0.085	0.121	-0.332	0.202	-0.855	0.872	-1.119	1.373	-1.221	1.574
	0.25	0.000	0.038	-0.211	0.083	-0.753	0.627	-1.073	1.196	-1.204	1.489
-0.2	0.50	0.136	0.037	0.028	0.019	-0.586	0.378	-0.990	1.020	-1.169	1.399
	0.75	0.263	0.078	0.312	0.108	-0.254	0.095	-0.755	0.615	-1.077	1.200
	0.90	0.340	0.123	0.512	0.269	0.125	0.042	-0.389	0.212	-0.790	0.684
	0.10	-0.284	0.217	-0.896	0.994	-1.936	4.023	-2.529	6.666	-2.785	7.962
	0.25	-0.116	0.064	-0.663	0.522	-1.776	3.282	-2.433	6.035	-2.773	7.776
-0.3	0.50	0.112	0.035	-0.271	0.114	-1.443	2.182	-2.255	5.170	-2.663	7.165
	0.75	0.306	0.104	0.235	0.075	-0.816	0.748	-1.833	3.480	-2.430	6.000
	0.90	0.439	0.201	0.564	0.328	-0.142	0.084	-1.099	1.347	-1.933	3.900
	0.10	-0.552	0.564	-2.471	6.830	-5.149	27.597	-6.650	45.138	-7.313	54.346
	0.25	-0.385	0.245	-2.082	4.646	-4.786	23.521	-6.530	43.221	-7.351	54.391
-0.4	0.50	-0.069	0.043	-1.222	1.657	-4.062	16.962	-6.022	36.735	-7.067	50.276
	0.75	0.277	0.094	-0.287	0.146	-2.669	7.507	-5.097	26.517	-6.503	42.740
	0.90	0.480	0.240	0.418	0.204	-1.162	1.610	-3.363	11.971	-5.331	29.257

Notes. The mean bias and the MSE are computed across the 500 simulated samples. The number of nodes N in the network is 200, 100 firms and 100 banks, in each sample. The column spillovers intensity reports the value of $\phi = \rho$ in the simulations.

Table A.3: Simulation study - CNM estimator performance under different share of treated, spillovers and density

		m=2		m=4		m=6		m=8		m=10	
$\phi = \rho$	% of treated										
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	0.058	0.118	0.022	0.059	0.013	0.039	0.000	0.030	-0.006	0.024
	0.25	0.006	0.034	0.002	0.017	0.005	0.013	0.009	0.009	0.006	0.008
-0.2	0.50	0.020	0.017	0.002	0.009	0.005	0.007	0.003	0.005	0.002	0.004
	0.75	0.010	0.015	0.004	0.009	0.013	0.007	0.006	0.005	0.005	0.004
	0.90	0.007	0.021	0.018	0.015	0.013	0.011	0.007	0.008	0.007	0.007
	0.10	0.009	0.098	0.016	0.058	0.000	0.045	0.000	0.032	0.011	0.033
	0.25	0.012	0.035	0.005	0.020	0.009	0.013	0.007	0.011	0.004	0.009
-0.3	0.50	0.010	0.016	-0.004	0.008	-0.001	0.006	-0.002	0.005	0.002	0.004
	0.75	0.009	0.015	0.003	0.007	0.004	0.005	0.008	0.003	-0.002	0.003
	0.90	0.012	0.014	0.011	0.010	0.012	0.007	0.000	0.005	0.008	0.005
	0.10	0.019	0.096	0.014	0.070	-0.001	0.058	0.019	0.045	0.007	0.040
	0.25	-0.009	0.034	0.001	0.022	-0.006	0.017	-0.001	0.011	0.009	0.012
-0.4	0.50	0.013	0.017	0.004	0.009	0.004	0.006	-0.001	0.005	0.000	0.004
	0.75	0.008	0.011	0.004	0.005	0.001	0.004	-0.001	0.000	0.000	0.002
	0.90	0.003	0.011	0.005	0.006	0.004	0.004	0.003	0.003	0.003	0.002

Notes. The mean bias and the MSE are computed across the 500 simulated samples. The number of nodes n in the network is 200, 100 firms and 100 banks, in each sample. The first column reports the spillovers intensity, i.e. the value of $\phi = \rho$ in the simulations.

Table A.4: Simulation study - CNM estimator performance with FEs under different share of treated, spillovers and density

		m=2		m=4		m=6		m=8		m=10	
$\phi = \rho$	% of treated										
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	0.025	0.120	0.037	0.069	0.008	0.139	-0.010	0.309	0.018	0.547
	0.25	0.014	0.034	0.005	0.023	0.027	0.045	0.018	0.091	0.012	0.209
-0.2	0.50	0.015	0.020	0.005	0.011	0.016	0.021	-0.001	0.040	0.009	0.081
	0.75	0.005	0.017	0.003	0.010	0.005	0.013	-0.010	0.026	-0.016	0.056
	0.90	0.003	0.022	0.007	0.016	0.007	0.014	-0.013	0.021	0.015	0.039
	0.10	0.002	0.098	0.011	0.072	0.015	0.146	0.035	0.309	0.033	0.835
	0.25	0.011	0.031	0.016	0.025	0.007	0.057	-0.003	0.116	-0.031	0.234
-0.3	0.50	0.016	0.018	-0.002	0.011	0.004	0.024	-0.009	0.052	-0.012	0.107
	0.75	0.001	0.012	0.014	0.008	0.006	0.015	0.002	0.032	-0.019	0.058
	0.90	0.016	0.018	0.005	0.009	0.003	0.013	-0.008	0.026	-0.018	0.049
	0.10	0.048	0.130	0.007	0.081	0.008	0.164	0.004	0.333	0.007	0.675
	0.25	0.017	0.034	-0.006	0.029	0.006	0.053	0.001	0.126	0.030	0.220
-0.4	0.50	0.004	0.016	0.011	0.012	-0.006	0.029	0.016	0.049	-0.029	0.102
	0.75	0.008	0.011	0.004	0.008	-0.001	0.018	-0.004	0.035	0.006	0.087
	0.90	0.015	0.014	0.003	0.003	0.012	0.015	-0.005	0.034	-0.005	0.065

Notes. The mean bias and the MSE are computed across the 500 simulated samples. The number of nodes n in the network is 200, 100 firms and 100 banks, in each sample. The first column reports the spillovers intensity, i.e. the value of $\phi = \rho$ in the simulations.

Table A.5: Simulation study - estimator performance under different spillovers, size and density

n	m		true		true		true		true		true	
			ϕ	ρ	ϕ	ρ	ϕ	ρ	ϕ	ρ	ϕ	ρ
200			-0.1	-0.1	-0.1	-0.2	-0.1	-0.3	-0.1	-0.4	-0.4	-0.4
	2	mean	-0.097	-0.100	-0.100	-0.209	-0.101	-0.306	-0.093	-0.414	-0.406	-0.406
		std	0.084	0.087	0.090	0.089	0.082	0.081	0.082	0.076	0.066	0.067
	4	mean	-0.099	-0.096	-0.095	-0.194	-0.094	-0.299	-0.096	-0.395	-0.402	-0.395
		std	0.040	0.041	0.040	0.040	0.040	0.044	0.040	0.042	0.039	0.040
	6	mean	-0.098	-0.098	-0.097	-0.197	-0.097	-0.295	-0.096	-0.398	-0.402	-0.395
		std	0.029	0.030	0.029	0.030	0.029	0.033	0.028	0.032	0.039	0.040
	8	mean	-0.101	-0.097	-0.101	-0.197	-0.100	-0.296	-0.098	-0.398	-0.403	-0.396
		std	0.024	0.022	0.025	0.026	0.024	0.026	0.024	0.028	0.033	0.032
	10	mean	-0.102	-0.096	-0.099	-0.198	0.000	-0.297	-0.098	-0.398	-0.402	-0.397
		std	0.021	0.020	0.023	0.023	0.022	0.024	0.021	0.024	0.031	0.030
800												
	2	mean	-0.102	-0.098	-0.100	-0.201	-0.097	-0.301	-0.097	-0.401	-0.398	-0.401
		std	0.041	0.043	0.044	0.042	0.042	0.042	0.040	0.037	0.034	0.033
	4	mean	-0.098	-0.098	-0.099	-0.198	-0.098	-0.300	-0.097	-0.400	-0.398	-0.401
		std	0.022	0.021	0.021	0.022	0.021	0.022	0.018	0.020	0.021	0.020
	6	mean	-0.099	-0.100	-0.099	-0.200	-0.097	-0.300	-0.099	-0.400	-0.398	-0.401
		std	0.015	0.014	0.015	0.016	0.014	0.016	0.014	0.016	0.021	0.020
	8	mean	-0.100	-0.099	-0.100	-0.200	-0.099	-0.300	-0.099	-0.399	-0.401	-0.399
		std	0.012	0.012	0.012	0.014	0.013	0.014	0.012	0.014	0.015	0.016
	10	mean	-0.100	-0.099	-0.100	-0.200	-0.099	-0.300	-0.099	-0.400	-0.399	-0.400
		std	0.010	0.011	0.010	0.011	0.011	0.012	0.010	0.012	0.015	0.015

Notes. The mean and the std are computed across the 500 simulated samples. n is the number of nodes in the network, m regulates the network density as described in Section 4.1.

Idiosyncratic Firm and Bank Shocks, Detailed Exposition. Here we deepen the properties firm and bank FEs' estimates, which account for idiosyncratic shocks, when using the isolated credit model (ICM) and the credit network model (CNM). To better assess the magnitude of the bias in finite samples, we let all FEs be positive by adding the minimum draw for each replication. To aggregate all the firms' and banks' specific parameters, we use the mean bias, the mean absolute bias and the root mean squared bias: $MB = 1/R\sum_{r=1}^{R}\{1/N[\sum_{j=1}^{F}(\hat{\delta}_j^r - \delta) + \sum_{k=1}^{B}(\hat{\gamma}_k^r - \gamma)]\}$, $MAB = 1/R\sum_{r=1}^{R}[1/N(\sum_{j=1}^{F}|\hat{\delta}_j^r - \delta| + \sum_{k=1}^{B}|\hat{\gamma}_k^r - \gamma|)]$, $RMSE = 1/R\sum_{r=1}^{R}[1/N\sqrt{\sum_{j=1}^{F}(\hat{\delta}_j^r - \delta)^2 + \sum_{k=1}^{B}(\hat{\gamma}_k^r - \gamma)^2}]$.

Table A.6 reports these indicators for different network sizes (n = 200, 800 and 2000), network densities (m = 4,6,8 and 10) and magnitude of spillovers (ϕ and ρ). The bias for the ICM is increasing in both the density of the network and the magnitude of spillovers, while the bias of the CNM is always close to zero and converges to it as n tends to infinity.

Given that $\theta = 0.1$ in our pivotal setting and we constrained the FEs to be positive, the average FE is greater than 0.3 with a probability lower than 0.001. The bias of the ICM ranges from about 0.3 (when $m = 4$ and $\phi = \rho = -0.2$) and 3 (when $m = 10$ and $\phi = \rho = -0.4$), which means that with low (high) density and small (large) spillovers the ICM estimate is on average about the double (ten times) the real idiosyncratic shock with very high probability. The intuition behind this result is that competitive interactions let positive idiosyncratic shocks diffuse through the credit network and amplify them, overestimating them, if such feedback loops are not accounted for.

Table A.6: Simulation study - ICM and CNM estimators performance under different spillovers, size and density

			$\phi = \rho = -0.2$			$\phi = \rho = -0.3$			$\phi = \rho = -0.4$		
n	m		MB	MAB	RMSE	MB	MAB	RMSE	MB	MAB	RMSE
200											
	4	ICM	0.303	0.394	61	0.412	0.460	73	0.628	0.648	98
		CNM	0.002	0.418	67	0.002	0.424	71	-0.005	0.420	69
	6	ICM	0.435	0.532	627	0.676	0.739	843	1.283	1.330	1,446
		CNM	-0.005	0.447	556	-0.009	0.448	556	-0.009	0.447	547
	8	ICM	0.650	0.732	1,472	1.066	1.125	2,163	2.230	2.281	4,316
		CNM	-0.013	0.472	1,025	0.001	0.466	1,002	-0.014	0.462	995
	10	ICM	0.820	0.891	2,234	1.414	1.462	3,599	3.074	3.138	7,628
		CNM	-0.027	0.498	1,376	-0.021	0.494	1,376	-0.018	0.474	1,312
800											
	4	ICM	0.276	0.377	484	0.376	0.429	527	0.594	0.620	729
		CNM	0.000	0.414	543	-0.002	0.415	535	0.001	0.417	539
	6	ICM	0.412	0.516	4,851	0.642	0.710	6,427	1.253	1.305	11,317
		CNM	-0.002	0.437	4,299	-0.002	0.436	4,284	-0.002	0.435	4,257
	8	ICM	0.615	0.701	11,235	1.039	1.100	16,944	2.195	2.256	34,081
		CNM	-0.004	0.448	7,707	0.000	0.448	7,664	-0.002	0.449	7,710
	10	ICM	0.789	0.862	17,467	1.374	1.430	28,187	3.015	3.081	59,562
		CNM	-0.007	0.458	10,089	-0.002	0.454	10,007	-0.009	0.457	10,041
2000											
	4	ICM	0.264	0.371	1,882	0.360	0.419	2,052	0.577	0.604	2,821
		CNM	0.000	0.415	2,144	-0.002	0.413	2,118	0.002	0.414	2,127
	6	ICM	0.399	0.507	18,845	0.632	0.703	25,099	1.238	1.295	44,536
		CNM	0.001	0.434	16,831	0.000	0.434	16,802	-0.001	0.435	16,858
	8	ICM	0.602	0.691	43,662	1.023	1.086	66,298	2.171	2.231	133,100
		CNM	-0.001	0.444	30,017	0.000	0.443	30,005	0.000	0.444	30,024
	10	ICM	0.775	0.851	68,219	1.354	1.412	109,951	3.001	3.069	234,902
		CNM	0.001	0.450	39,115	-0.005	0.450	39,012	-0.004	0.448	38,915

Notes. ICM and CNM stand respectively for isolated credit model and credit network model. n is the number of nodes in the network, m regulates the network density as described in Section 4.1. MB, MAB and RMSE stand respectively for mean bias, mean absolute bias and root mean square error. All are averaged across 500 replications and computed with the following formulas: $MB = 1/R\sum_{r=1}^{R}\{1/N[\sum_{j=1}^{F}(\hat{\delta}_j^r - \delta) + \sum_{k=1}^{B}(\hat{\gamma}_k^r - \gamma)]\}$, $MAB = 1/R\sum_{r=1}^{R}[1/N(\sum_{j=1}^{F}|\hat{\delta}_j^r - \delta| + \sum_{k=1}^{B}|\hat{\gamma}_k^r - \gamma|)]$, $RMSE = 1/R\sum_{r=1}^{R}[1/N\sqrt{\sum_{j=1}^{F}(\hat{\delta}_j^r - \delta)^2 + \sum_{k=1}^{B}(\hat{\gamma}_k^r - \gamma)^2}]$. $RMSE$ is reported without decimals.

Table A.7: Simulation study - ICM and CNM estimators performance with real credit networks

				mean bias					
			n =	400		800		2000	
$\phi = \rho$				treatment effect	idiosyncratic shocks	treatment effect	idiosyncratic shocks	treatment effect	idiosyncratic shocks
-0.2	% of treated								
	0.10								
		ICN		-0.589	-0.008	-0.766	-0.042	-0.918	-0.068
		CNM		0.039	-0.003	0.004	-0.001	0.000	0.001
	0.50								
		ICN		-0.445	0.432	-0.628	0.341	-0.852	0.274
		CNM		0.007	-0.006	0.008	-0.005	0.001	-0.001
	0.90								
		ICN		-0.039	0.498	-0.060	0.574	-0.494	0.498
		CNM		0.001	-0.002	0.001	-0.002	0.003	-0.001
-0.3	0.10								
		ICN		-1.244	0.025	-1.695	-0.012	-2.076	-0.045
		CNM		0.024	-0.002	0.017	0.000	0.011	-0.002
	0.50								
		ICN		-1.002	0.642	-1.438	0.556	-1.930	0.491
		CNM		0.004	-0.002	0.005	-0.002	0.006	-0.002
	0.90								
		ICN		-0.242	0.836	-0.266	0.764	-1.219	0.780
		CNM		0.006	-0.005	0.005	-0.004	0.002	0.001
	0.10								
-0.4		ICN		-3.080	0.126	-4.364	0.104	-5.479	0.066
		CNM		0.012	-0.001	0.004	0.000	0.006	-0.001
	0.50								
		ICN		-2.484	1.135	-3.772	1.146	-5.130	1.111
		CNM		0.001	0.002	0.004	-0.001	0.005	-0.003
	0.90								
		ICN		-0.880	1.252	-1.691	1.279	-3.393	1.557
		CNM		0.004	-0.001	0.002	0.000	0.000	-0.001

Notes. ICM and CNM stand respectively for isolated credit model and credit network model. n is the number of nodes in the network. The links are extracted from realized credit relationships between a random sample of firms and banks from all credit relationships observed in 2016. The bias of the treatment effect is computed as in Table A.1, A.2, A.4 and 1. The bias of idiosyncratic shocks is computed as in Table A.6.

A.2 Proofs

A.2.1 Model

Full derivation of the toy model, proving Proposition 1.

Bank Problem:

$$\begin{aligned}
&\text{Bank } b\text{: } \max_{c_{ib},\, c_{jb}} \big(r_{ib} - \omega(c_{ib} - \xi x_{ib} - \theta c_{jb} - \nu_{ib})\big)c_{ib} + \big(r_{jb} - \omega(c_{jb} - \xi x_{jb} - \theta c_{ib} - \nu_{jb})\big)c_{jb} \\
&\text{Bank } a\text{: } \max_{c_{ia}} \big(r_{ia} - \omega(c_{ia} - \xi x_{ia} - \nu_{ia})\big)c_{ia} \\
&\text{FOC deliver:} \\
&r_{ib} = \omega c_{ib} - \omega \underbrace{(\xi x_{ib} + \nu_{ib} - \theta c_{jb})}_{u_{ib}} \\
&r_{jb} = \omega c_{jb} - \omega \underbrace{(\xi x_{jb} + \nu_{jb} - \theta c_{ib})}_{u_{jb}} \\
&r_{ia} = \omega c_{ia} - \omega \underbrace{(\xi x_{ia} + \nu_{ia})}_{u_{ia}}
\end{aligned} \tag{A.1}$$

Firm problem:

$$\begin{aligned}
&\text{Firm i: } \max_{c_{ia},\, c_{ib}} \big(e_i - \alpha(c_{ia} + c_{ib})\big)(c_{ia} + c_{ib}) - \textstyle\sum_{K=a,b} c_{iK}\omega(c_{iK} - u_{iK}) \\
&\text{Firm j: } \max_{c_{jb}} \big(e_j - \alpha c_{jb}\big)c_{jb} - c_{jb}\omega(c_{jb} - u_{jb})
\end{aligned}$$

FOC deliver:

$$\begin{aligned}
&e_i - 2\alpha c_{ia} - 2\alpha c_{ib} - 2\omega c_{ia} + \omega(\xi x_{ia} + \nu_{ia}) = 0 \\
&e_i - 2\alpha c_{ib} - 2\alpha c_{ia} - 2\omega c_{ib} + \omega(\xi x_{ib} + \nu_{ib} - \theta x_{jb}) = 0 \\
&e_j - 2\alpha c_{jb} - 2\omega c_{jb} + \omega(\xi x_{jb} + \nu_{jb} - \theta x_{ib}) = 0
\end{aligned}$$

Which simplifies to:

$$\begin{aligned}
&c_{ia} = -\tfrac{\alpha}{\alpha+\omega} c_{ib} + \tfrac{1}{2(\alpha+\omega)} e_i + \tfrac{\omega}{2(\alpha+\omega)}(\xi x_{ia} + \nu_{ia}) \\
&c_{ib} = -\tfrac{\alpha}{\alpha+\omega} c_{ia} + \tfrac{1}{2(\alpha+\omega)} e_i + \tfrac{\omega}{2(\alpha+\omega)}(\xi x_{ib} + \nu_{ib} - \theta c_{jb}) \\
&c_{jb} = \tfrac{1}{2(\alpha+\omega)} e_j + \tfrac{\omega}{2(\alpha+\omega)}(\xi x_{jb} + \nu_{jb} - \theta c_{ib})
\end{aligned}$$

And delivers the following structural demand system:

$$\begin{aligned}
&c_{ia} = \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia} \\
&c_{ib} = \rho c_{ia} + \phi c_{jb} + \beta x_{ib} + \delta_i + \epsilon_{ib} \\
&c_{jb} = \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb}
\end{aligned}$$

Calling:

$$\begin{aligned}
&\rho = -\tfrac{\alpha}{\alpha+\omega} \\
&\phi = -\tfrac{\theta\omega}{2(\alpha+\omega)} \\
&\beta = \tfrac{\xi\omega}{2(\alpha+\omega)} \\
&\delta_{i,j} = \tfrac{1}{2(\alpha+\omega)} e_{i,j} \\
&\epsilon_{ia,ib,jb} = \tfrac{\omega\nu_{ia,ib,jb}}{2(\alpha+\omega)}
\end{aligned}$$

From the above, we can derive the following reduced form system:

$$
\begin{aligned}
&c_{ia} = \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia} \\
&c_{ib} = \rho(\rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia})... \\
&+\phi(\phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb}) + \beta x_{ib} + \delta_i + \epsilon_{ib} \\
&c_{jb} = \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb} \\
\\
&c_{ia} = \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia} \\
&(1-\rho^2-\phi^2)c_{ib} = \beta(\rho x_{ia} + x_{ib} + \phi x_{jb})... \\
&+(1+\rho)\delta_i + \phi\delta_j + \rho\epsilon_{ia} + \phi\epsilon_{jb} + \epsilon_{ib} \\
&c_{jb} = \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb} \\
\\
&c_{ia} = \rho\big(\tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}\big)... \\
&+\beta x_{ia} + \delta_i + \epsilon_{ia} \\
&c_{ib} = \tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\
&c_{jb} = \phi\big(\tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}\big)... \\
&+\beta x_{jb} + \delta_j + \epsilon_{jb} \\
\\
&c_{ia} = \tfrac{\rho(1+\rho-\phi^2)\delta_i+\rho\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{(1-\phi^2)x_{ia}+\rho\phi x_{jb}+\rho x_{ib}}{1-\phi^2-\rho^2} + \tfrac{(1-\phi^2)\epsilon_{ia}+\rho\phi\epsilon_{jb}+\rho\epsilon_{ib}}{1-\phi^2-\rho^2} \\
&c_{ib} = \tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\
&c_{jb} = \tfrac{\phi(1+\rho)\delta_i+(1-\rho^2)\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho\phi x_{ia}+(1-\rho^2)x_{jb}+\phi x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\phi\rho\epsilon_{ia}+(1-\rho^2)\epsilon_{jb}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2}
\end{aligned}
\tag{A.2}
$$

Adding Assumption 3 and considering the case in which the econometrician ignores spillovers and correlated demand shocks both, we obtain:

$$
\begin{aligned}
c_{ia} &= \beta x_{ia} + \varepsilon_{ia} \\
\varepsilon_{ia} &= \delta_i + \rho c_{ib} + \epsilon_{ia} \\
c_{ib} &= \tfrac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \beta\tfrac{\rho}{1-\phi^2-\rho^2}x_{ia} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}
\end{aligned}
\tag{A.3}
$$

which results in

$$
\hat{\beta}_{\text{OLS}} = \tfrac{\text{cov}(c_{ia},x_{ia})}{\text{var}(x_{ia})} = \beta + \underbrace{\rho\frac{\text{cov}(x_{ia},c_{ib})}{\text{var}(x_{ia})}}_{\text{spillover bias}} + \underbrace{\frac{\text{cov}(x_{ia},\delta_i)}{\text{var}(x_{ia})}}_{\text{demand bias}} \tag{A.4}
$$

The fact that $\frac{\text{cov}(x_{ia},c_{ib})}{\text{var}(x_{ia})} = \frac{1+\rho}{1-\rho^2-\phi^2}\frac{\text{cov}(x_{ia},\delta_i)}{\text{var}(x_{ia})} + \beta\frac{\rho}{1-\rho^2-\phi^2} \neq 0$ concludes the proof.

■

Proof of Proposition 2.

Indicating averages with bars, so that, for example, $\bar{c}_i = \frac{c_{ia}+c_{ib}}{2}$, we have that:

$$c_{ia} = \beta x_{ia} + \delta_i + \varepsilon_{ia},$$
$$c_{ib} = \delta_i + \varepsilon_{ib}.$$

$$\Rightarrow \hat{\delta}_i = c_{ib}$$
$$\hat{\beta}_{FE} = \frac{\text{cov}(c_{ia}-\bar{c}_i, x_{ia}-\bar{x}_i)}{\text{var}(x_{ia}-\bar{x}_i)} = \frac{\text{cov}\left(\frac{c_{ia}-c_{ib}}{2}, \frac{x_{ia}}{2}\right)}{\text{var}\left(\frac{x_{ia}}{2}\right)} = \frac{\text{cov}(c_{ia}-c_{ib}, x_{ia})}{\text{var}(x_{ia})} = ...$$
$$...\frac{\text{cov}(\beta x_{ia}+\varepsilon_{ia}-\varepsilon_{ib}, x_{ia})}{\text{var}(x_{ia})}$$

From the structural demand system:

$$\begin{aligned} \varepsilon_{ia} &= \rho c_{ib} + \epsilon_{ia} \\ \varepsilon_{ib} &= \rho c_{ia} + \phi c_{jb} + \epsilon_{ib} \end{aligned} \tag{A.5}$$

$$\Rightarrow \frac{\text{cov}(\beta x_{ia}+\varepsilon_{ia}-\varepsilon_{ib}, x_{ia})}{\text{var}(x_{ia})} = \beta + \frac{\text{cov}(\rho(c_{ib}-c_{ia})-\phi c_{jb}, x_{ia})}{\text{var}(x_{ia})} = ...$$
$$...\beta + \frac{\text{cov}(\rho((1-\rho)c_{ib}-\beta x_{ia}-\delta_i)-\phi c_{jb}, x_{ia})}{\text{var}(x_{ia})} = ...$$
$$...\beta(1-\rho) + \rho(1-\rho)\frac{\text{cov}(c_{ib}, x_{ia})}{\text{var}(x_{ia})} - \rho\frac{\text{cov}(\delta_i, x_{ia})}{\text{var}(x_{ia})} - \phi\frac{\text{cov}(c_{jb}, x_{ia})}{\text{var}(x_{ia})}$$

From the reduced form system, simplified thanks to Assumption 3:

$$c_{ib} = \frac{(1+\rho)\delta_i}{1-\phi^2-\rho^2} + \beta\frac{\rho x_{ia}}{1-\phi^2-\rho^2} + \frac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}$$
$$c_{jb} = \frac{\phi(1+\rho)\delta_i}{1-\phi^2-\rho^2} + \beta\frac{\rho\phi x_{ia}}{1-\phi^2-\rho^2} + \frac{\phi\rho\epsilon_{ia}+(1-\rho^2)\epsilon_{jb}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2}$$

From the above, and the reduced form of System 2 displayed in the last passage, it is evident that β_{FE} is biased, and that correlated demand shocks still play a role, as they are reflected back in the estimator through reallocation spillovers. β_{FE} is indeed a function of δ_i through the $-\rho\text{cov}(\delta_i, x_{ia})/\text{var}(x_{ia})$ element, from demand reallocation within the relationships of the same firm, and through the impact of δ_i on *all* other bias components.

■

Proof of Proposition 3. From the end of Proposition 1's Proof and the absence of demand bias it follows that:

$$\hat{\beta}_{OLS} = \beta\left(1 + \frac{\rho^2}{1-\phi^2-\rho^2}\right) = \beta\frac{1-\phi^2}{1-\phi^2-\rho^2}$$

From the reduced form demand system, Proposition 2's and the absence of demand bias:

$$\begin{aligned} &\hat{\beta}_{FE} = \beta(1-\rho) + \rho(1-\rho)\frac{\text{cov}(c_{ib}, x_{ia})}{\text{var}(x_{ia})} - \phi\frac{\text{cov}(c_{jb}, x_{ia})}{\text{var}(x_{ia})} = ... \\ &\beta(1-\rho) + \beta(1-\rho)\frac{\rho^2}{1-\phi^2-\rho^2} - \beta\frac{\phi^2\rho}{1-\phi^2-\rho^2} = ... \\ &\beta\frac{(1-\rho)(1-\phi^2-\rho^2)+\rho^2(1-\rho)-\phi^2\rho}{1-\phi^2-\rho^2} = \beta\frac{1-\rho-\phi^2+\rho\phi^2-\rho^2+\rho^3+\rho^2-\rho^3-\phi^2\rho}{1-\phi^2-\rho^2} = ... \\ &\beta\frac{1-\phi^2+\rho}{1-\phi^2-\rho^2} \end{aligned} \tag{A.6}$$

■

A.2.2 The econometric framework

Proof of Proposition 5. It follows from the proof of Proposition 1 in Arduini, Patacchini, and Rainone (2020) when G, the network among nodes, and its sub-matrices are replaced by A and its sub-matrices, the network among links. Moving from nodes to links implies that quadriads instead of triads intransitivity is needed. Quadriads intransitivity is implied by linear independence of I_F, $A_B A_F A_B$ and A_F and I_B, $A_F A_B A_F$ and A_B. See condition 1 of the proposition. Alternatively, also the proof of Proposition 1 in Rainone (2020a) brings to the same result if multiple endogenous terms are considered.
■

Details for the identification example in Section 3.3.

$$\begin{aligned} c_{ia} &= \rho c_{ib} + \beta x_{ia} + \epsilon_{ia} \\ c_{ib} &= \rho c_{ia} + \phi c_{jb} + \epsilon_{ib} \\ c_{jb} &= \phi c_{ib} + \beta x_{jb} + \epsilon_{jb} \end{aligned} \tag{A.7}$$

$$
\begin{gathered}
c_{ia} = \rho c_{ib} + \beta x_{ia} + \epsilon_{ia} \\
c_{ib} = \rho(\rho c_{ib} + \beta x_{ia} + \epsilon_{ia}) + \phi(\phi c_{ib} + \beta x_{jb} + \epsilon_{jb}) + \epsilon_{ib} \\
c_{jb} = \phi c_{ib} + \beta x_{jb} + \epsilon_{jb}
\end{gathered}
$$

$$
\begin{gathered}
c_{ia} = \big(\tfrac{\rho^2\beta}{1-\phi^2-\rho^2} + \beta\big)x_{ia} + \rho\tfrac{\phi\beta}{1-\phi^2-\rho^2}x_{jb} + \tfrac{(1-\phi^2)\epsilon_{ia}+\rho\phi\epsilon_{jb}+\rho\epsilon_{ib}}{1-\phi^2-\rho^2} \\
c_{ib} = \tfrac{\rho\beta}{1-\phi^2-\rho^2}x_{ia} + \tfrac{\phi\beta}{1-\phi^2-\rho^2}x_{jb} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\
c_{jb} = \big(\tfrac{\phi^2\beta}{1-\phi^2-\rho^2} + \beta\big)x_{jb} + \tfrac{\phi\rho\beta}{1-\phi^2-\rho^2}x_{ia} + \tfrac{(1-\rho^2)\epsilon_{jb}+\phi\rho\epsilon_{ia}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2}
\end{gathered}
$$

$$
\begin{gathered}
\text{call } \mu = \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\
\text{and } \pi_\rho = \tfrac{\beta\phi}{1-\phi^2-\rho^2} \\
\text{and } \pi_\phi = \tfrac{\beta\rho}{1-\phi^2-\rho^2} \\
\Rightarrow c_{ia} = \rho\pi_\rho x_{jb} + \big(\beta + \tfrac{\beta\rho^2}{1-\phi^2-\rho^2}\big)x_{ia} + \rho\mu + \epsilon_{ia} \\
c_{ib} = \pi_\rho x_{jb} + \pi_\phi x_{ia} + \mu \\
c_{jb} = \phi\pi_\phi x_{ia} + \big(\tfrac{\phi^2\beta}{1-\phi^2-\rho^2} + \beta\big)x_{jb} + \phi\mu + \epsilon_{jb}
\end{gathered} \tag{A.8}
$$

$$
\begin{gathered}
\hat{\pi}_{\rho,OLS} = \pi_\rho \\
\hat{\pi}_{\phi,OLS} = \pi_\phi \\
\widehat{\pi_\rho\rho}_{OLS} = \pi_\rho\rho \\
\widehat{\pi_\phi\phi}_{OLS} = \pi_\phi\phi \\
\hat{\rho}_{IV} = \tfrac{\widehat{\pi_\rho\rho}_{OLS}}{\hat{\pi}_{\rho,OLS}} = \rho \\
\hat{\phi}_{IV} = \tfrac{\widehat{\pi_\phi\phi}_{OLS}}{\hat{\pi}_{\phi,OLS}} = \phi \\
\hat{\beta}_{OLS} = \beta\big(1 + \tfrac{\rho^2}{1-\phi^2-\rho^2}\big) \\
\hat{\beta}_{IV} = \hat{\beta}_{OLS} - \tfrac{\hat{\rho}^2_{IV}}{1-\hat{\phi}^2_{IV}-\hat{\rho}^2_{IV}}
\end{gathered}
$$

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