Cross-Elasticities in Credit Markets

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Abstract. We develop a methodology to estimate the cross-elasticities of credit demand and supply among firms and banks, quantifying the extent of credit reallocation across relationships following shocks. Our identification strategy leverages the fact that not all firms borrow from all banks. Partially overlapping portfolios imply that some bank-firm relationships are only indirectly connected through the credit market network, providing natural exclusion restrictions. Through theory and Monte Carlo simulations, we demonstrate the viability of our approach and show that neglecting reallocation can significantly and unpredictably bias treatment and fixed-effects estimates. Applying our estimator to twenty years of data from the Italian credit registry, we find that firms'cross-elasticities are negative, large, and vary procyclically, whereas banks' cross-elasticities are generally positive and small. Such behavior of credit cross-elasticities over the business cycle may help explain the varying impacts of credit booms and busts.

Keywords: Cross-elasticities, Credit Markets, Shocks Propagation, Networks, Identification.

JEL Classification: C30, L14, G21.

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1 Introduction

Empirical research documents how financial shocks to banks can affect firms' access to credit (Khwaja and Mian, 2008; Paravisini, 2008; Paravisini, Rappoport, and Schnabl, 2023; Peek and Rosengren, 2000) with varying real effects. Sudden contractions in credit supply are likely to restrict investment and production (Alfaro, García-Santana, and Moral-Benito, 2021; Cingano, Manaresi, and Sette, 2016; Jiménez et al., 2017), while the real effects of expansions appear mixed and context-dependent (e.g. Bottero et al., 2022; Jiménez et al., 2020). The extent to which firms can substitute credit from less to more convenient suppliers, as well as the extent of comovement in the banks' supply, probably determines an important part of this variability (Ivashina and Scharfstein, 2010). However, the present approaches only allow for indirect measurement of credit reallocation and its effects, either by comparing estimates at the firm level and within the firm (Jiménez et al., 2020; Khwaja and Mian, 2008) or by exploiting substitutions toward other sources of funding (Becker and Ivashina, 2014).

We propose *direct* measures of credit reallocation by banks and firms, show that they are important for mitigating bias due to the network nature of credit relationships, and demonstrate that their changes over time could help explain the variability in the effects of credit expansions and contractions. Such measures are estimates of the *cross-elasticity of credit*, i.e. how much the amount of credit on one relationship responds to variations of other relationships of the same firm or bank. To retrieve such cross-elasticities, we need to address a reflection problem (Manski, 1993). Reflection stems from the embeddedness of each relationship in a firm's and a bank's credit portfolio and from the fact that each player considers its entire portfolio when making decisions. This simultaneity, along with the interconnections of banks and firms in the credit network, implies that shocks to one relationship can potentially influence all others, causing complex bias in treatment and fixed effects estimates.

We use sparsity in the credit market structure to address the reflection problem, estimating directly firms' and banks' cross-elasticities. Credit relationships are the links in a network of firms and banks. As not all relationships are directly connected through a bank or a firm (partial overlap of portfolios), changes in the outcomes of some relationships will affect others only indirectly, providing natural exclusion restrictions to the econometrician. We use partially overlapping portfolio instrumental variables (OPIVs) to break the simultaneity problem, estimate cross-elasticities, and account for the reflection problem, correcting treatment and fixed effects estimators. We show evidence that our results are stable across specifications and adapt the methods proposed in Jochmans (2023) to mitigate concerns about network-endogeneity.

Importantly, the methodology we propose generalizes and nests the commonly used Khwaja and Mian (2008) model, which constrains cross-elasticities to equal zero and uses exactly the same information set; thus, any researcher using standard credit register data can employ it. Furthermore, the identification strategy requires data from only a single period and does not depend on a specific quasi-experiment, only on market sparsity. This relative context-independence allows cross-elasticities to vary over time, enabling the analysis of their dynamics without imposing restrictive assumptions.

We start this paper by introducing an extremely simplified model of joint credit demand and supply determination to argue that estimating cross-elasticities directly is not only relevant *per se*, but also key to obtaining unbiased treatment and fixed effect estimators in most cases. We describe a world with two banks and two firms that jointly optimize their full portfolio of relationships, introducing simultaneity in the credit demand system implied by the Khwaja and Mian (2008) model. We use this framework to argue four points. First, cross-elasticities introduce bias in standard estimates of the bank lending channel and any other treatment of interest. Second, whether demand and supply confounders are a legitimate concern *or not*, banks and firms fixed effects may actually worsen the bias.[1] Third, interpreting differences between treatment effects with and without the inclusion of fixed effects as a measure of demand bias (e.g. Jiménez et al., 2020) may be misleading. Fourth and last, the reflection bias may also contaminate the estimates of firm and bank fixed effects, preventing us from measuring pure demand and supply changes (Amiti and Weinstein, 2018).[2]

We proceed by establishing an estimation framework to address such distortions, based on the literature on spatial autoregressive models (e.g. Arduini, Patacchini, and Rainone, 2015; Bramoullé, Djebbari, and Fortin, 2009; Calvó-Armengol, Patacchini, and Zenou, 2009; Lee, 2007), to model outcomes on links (credit relationships) instead of nodes (firms or banks). We show that the very structure of the credit market network, in which many nodes are only indirectly connected, provides instruments for the identification of cross-elasticities, which can be seen as spillover effects from network peers connected through either banks or firms. As idiosyncratic shocks to one relationship can indirectly affect others involving the same parties, they can provide relevant instruments for spillover effects. If credit portfolios are not fully overlapping, then such instruments are exogenous. We show how to use OPIVs to identify cross-elasticities and recover unbiased treatment effects, as well as idiosyncratic firm- and bank-level credit changes, and document how we can check for robustness to network endogeneity concerns by applying the Jochmans (2023) leave-own-out approach.

[1] Using firms with multiple bank relationships to perform a fixed effect estimate of supply movements is a standard procedure in the empirical banking literature, first popularized in Khwaja and Mian (2008). A far from complete list of influential works that use this strategy to assess the effect of different bank shocks includes Jimenéz et al. (2012), Schnabl (2012), Jiménez et al. (2014), Behn, Haselmann, and Wachtel (2016), Bonaccorsi di Patti and Sette (2016), Jiménez et al. (2017).

[2] This point is related to the one raised by Altavilla, Boucinha, and Bouscasse (2022) and De Jonghe and Lewis (2025). We elaborate on the differences between contributions in the *Related Literature* part of this Introduction.

Our model separately identifies a *bank credit cross-elasticity* and a *firm credit cross-elasticity* parameter.[3] The two parameters capture the endogenous adjustment of credit across all relationships of the same bank and firm after they are hit by some shock. We do not constrain their signs ex-ante, as we consider the direct study of their signs and magnitudes a key contribution of our work. Indeed, we show that negative estimates can be interpreted as evidence of substitutability, while positive ones can be interpreted as evidence of complementarity, as long as the demand and supply curves at the relationship level are downward sloping. Well identified quantity cross-elasticity measures, interpretable as substitutability or complementarity, can inform our understanding of what drives the real effects of financial shocks. Can a firm absorb the consequences of a credit cut, or leverage the effects of an expansion, and how much so? How do these parameters change during expansions and recessions? We propose a unified framework to answer similar questions.[4]

We explore the properties of our OPIV estimation strategy through Monte Carlo simulation. First, we study the treatment estimate bias if we ignore the reallocation mechanisms (i.e., not allowing for cross-elasticities). We show that both the magnitude and sign of the bias depend on *observables*, such as the share of relationships hit by a shock and the number of firms or banks that share relationships (the density of the network), and *unobservables*, such as the magnitude of the cross-elasticities. Hence, such bias cannot be addressed without a strategy to estimate cross-elasticities. Second, we confirm that fixed effects can exacerbate spillover bias and that standard estimates of fixed effects for banks and firms may pinpoint non-existent demand and supply idiosyncratic movements, especially for banks and firms that are central to the credit network. Third, we document that the network estimator performs very well in finite samples, estimating spillovers, treatment effects, and idiosyncratic shocks with negligible error. We confirm all such findings in a network calibrated to the real data on credit relationships observed in the Bank of Italy's Credit Register, finding that in this context, the bias would still be economically significant.

We apply our method to data from 2002 to 2022 from the Bank of Italy's Credit Register, matched with firms' financials from CERVED, the main Italian risk rating issuer, and banks' financials from Italian Supervisory Reports. We present two exercises. First, we adapt to the Italian setting the study of the risk-taking channel of monetary policy

[3] In what follows, we focus on cross-elasticity between quantities, i.e., how changes in the quantities of credit for all relationships of a firm or a bank affect the quantity observed in each relationship of said firm or bank. We make this choice because quantities are the most commonly observed and studied objects in credit registers, while prices may be available only for specific subsamples. Nevertheless, our results on cross-elasticities easily extend to prices and other outcomes (see Bishop, 1952).

[4] We stress that our contribution is not about disentangling whether the effects of a shock are demand- or supply-driven. E.g., if a firm substitutes 30 percent of a credit cut using its other relationships, whether this occurs because its other banks become aware of the credit cut and pro-actively reach-out, or thanks to the firm's effort in finding new credit, does not change the ultimate result: the degree to which the firm was able to preserve its credit stock.

initially measured in Jiménez et al. (2014). Using data from 2002 to 2008, we investigate whether less capitalized banks extended more credit to riskier firms after an easing in the monetary policy stance. We estimate this channel twice; the first time within a standard Khwaja and Mian (2008) approach with firm and bank fixed effects, not accounting for cross-elasticities, and the second estimating cross-elasticities simultaneously through the use of OPIVs. We find that not accounting for cross-elasticities would have led to a gross underestimation of the risk-taking channel.

Within the same exercise, we find that the average firm cross-elasticity of demand is negative and large in the 2002-2008 time window, approximately -30 percent. That is, if a firm with three credit relationships sees granted credit growing by 50 percent on two of them, it would decrease its demand by 30 percent on the third. The average cross-elasticity of supply for banks is positive, economically significant, but smaller. In particular, if a bank having 4,403 borrowers (the 25th percentile in the number of relationships per bank over the years under study, and about a quarter of the median) extends 50 percent more credit to 4,402 of them for idiosyncratic reasons, it will further increase its supply to the remaining one by an additional 7 percent. Moreover, recovering banks and firms fixed effects estimates and averaging them over the 2002-2008 window, we find a median absolute bias of 39 (94 mean) percent for banks and 41 (96) percent for firms. The large difference between mean and median absolute bias aligns with standard network-economics intuition. More connected firms and banks will see larger errors in their naive fixed effects estimates, as the reflection bias for them is larger. Furthermore, the retrieval of average fixed effect estimates while ignoring cross-elasticities would imply misclassifications of 75 bank-level shocks (positive shocks turning negative), out of a total of 586 banks.

For our second exercise, we study how cross-elasticities evolve over the business cycle. Estimating them annually from 2002 to 2022, we find that banks' cross-elasticities are consistently positive but modest, while firms' cross-elasticities fluctuate strongly with macroeconomic conditions. In normal times, they are large and negative, indicating that firms can offset up to 70 percent of a cut in credit from one lender or reallocate nearly as much when others expand. During the four recession episodes in our sample (2008–09, 2011–13, and 2020), however, these cross-elasticities turn positive. This evidence suggests that the extent of credit reallocation after a shock varies at the firm level with the business cycle. For firms, credit is substitutable during expansions and complementary during downturns. Such heterogeneity in cross-elasticities aligns with and helps rationalize the asymmetry in the effects of credit expansions and contractions evidenced in the literature.

We proceed as follows. We discuss the literature in the rest of this introduction. In Section 2, we introduce a toy model of a credit network and derive its econometric implications. In Section 3, we lay down the estimation framework and introduce the estimators we propose. In Section 4, we explore the estimator's properties in finite samples and the bias we would incur from ignoring cross-elasticities. In Section 5, we use Italian credit data to present an empirical application. We take stock in Section 6.

1.1 Contributions to the Related Literature

We contribute to the empirical literature on the pass-through of financial shocks in two ways. First, we build a methodology to account for the endogenous reallocation of credit induced by shocks, showing that it is important to consider the reflection problem in similar settings. Second, we show that the variation of banks' and firms' credit cross-elasticities over time aligns with the intuition that firms' capacity (or incapacity) to compensate for credit cuts can explain the heterogeneity in the real effects of credit booms and busts.

The methodology we propose addresses an important limitation of standard instrumental variable (e.g. Paravisini, 2008; Peek and Rosengren, 2000) and within-firm (Jiménez et al., 2014, 2017; Khwaja and Mian, 2008) estimates of the effect of bank shocks on firms' credit access. These approaches cannot control for the endogenous reallocation of credit across portfolios of relationships. We also show how this issue may affect (i) attempts to use the difference between estimates with and without fixed effects to track the extent of demand bias (Jiménez et al., 2020), and (ii) attempts to use estimates of bank and firm fixed effects as direct measures of pure credit demand and supply changes (Amiti and Weinstein, 2018). We offer a solution to these problems based on novel and classical results in network econometrics (e.g. Ballester, Calvó-Armengol, and Zenou, 2006; Calvó-Armengol, Patacchini, and Zenou, 2009).[5]

Such contributions are complementary to works deepening the identification of financial shocks' pass-through to credit and real outcomes (Chava and Roberts, 2008; Chodorow-Reich, 2014; De Jonghe and Lewis, 2025; Paravisini, Rappoport, and Schnabl, 2023; Peek and Rosengren, 2000). While we focus on the identification challenges stemming from the endogenous reallocation of credit, a recent strand of contributions, starting with Paravisini, Rappoport, and Schnabl (2023), highlights those implied by a lack of perfect substitutability. In particular, it shows that bank specialization causes shocks to distribute unevenly across the relationships of the same firm, limiting the effectiveness of within-firm strategies to control for unobservable confounders. Research building on these premises, such as Bripi (2023) and Altavilla, Boucinha, and Bouscasse (2022), estimates credit demand systems, focusing on the price-elasticity of credit. We instead focus on measuring credit cross-elasticities within the same firm or bank.

Among such works, our results are closely related to and complementary with those

[5]See Jackson (2010), Jackson and Zenou (2015), and Jackson, Rogers, and Zenou (2017) for a complete critical survey of the theoretical literature on the economics of networks. See Bramoullé, Djebbari, and Fortin (2020a), De Paula (2020), and Graham and De Paula (2020) for insightful reviews of the literature on network econometrics.

of De Jonghe and Lewis (2025). Most papers on shock propagation in credit markets (networks) assume both homogeneity of demand and supply shocks within the same firm or bank and a lack of endogenous reallocation after a shock (lack of simultaneity bias). De Jonghe and Lewis (2025) relaxes the homogeneity assumption by disentangling relationship-level supply and demand shocks using both prices and quantities. We relax the lack of simultaneity assumption and address the reflection bias arising from endogenous reallocation across relationships, estimating credit cross-elasticities using quantity data. While our approach abstracts from recovering relationship-specific demand and supply shocks, it allows us to quantify how firms and banks substitute or complement credit across relationships—a key determinant of the real effects of financial shocks.[6] Taken together, the two approaches address distinct but equally important identification challenges, and a natural avenue for future research is to combine them.

Moreover, our work closely relates to the strand of papers studying how spillovers complicate the identification of shocks: regional variation in credit supply shocks can bias estimates of general equilibrium multipliers Mian, Sarto, and Sufi (2022); firm-to-firm interactions may distort measured effects and even be aggravated by fixed-effect strategies (Berg, Reisinger, and Streitz, 2021); and multiple channels of spillovers can interact with the mismeasurement of treatment (Huber, 2022). We shift our focus from contextual effects—peers' treatment status—to endogenous effects, the influence of peers' outcomes. In credit markets, where bank–firm relationships are crucial to the pass-through of financial shocks but remain a black box, our framework opens it by disentangling how banks and firms reallocation of credit across relationships shapes equilibrium effects.[7]

Our investigation also complements recent work by Darmouni and Sutherland (2021) and Gupta et al. (2025), addressing spillovers in bank credit contract design: the first in SME lending and the other in syndicated lending to large firms, both in the US. The latter paper, in particular, exploits the network structure of banks' overlapping lending portfolios through a spatial autoregressive model to estimate the degree of complementarity in banks' interest rate setting. Nonetheless, using Dealscan data, which does not track the evolution of credit commitments' over time, Gupta et al. (2025) does not focus on recovering credit cross-elasticities or studying their behavior over the business cycle.

From a methodological standpoint, our work is close to other corporate finance papers that directly address Manski (1993)'s reflection problem. Recent works have quantified peer effects in firms' capital structure (Grieser et al., 2022; Leary and Roberts, 2014),

[6]Our estimation procedure controls for unobservable characteristics that are constant across firms and banks (as in Abowd, Kramarz, and Margolis, 1999), recovering unbiased relationship-level complementarity/substitutability parameters, as well as the firm and bank-level idiosyncratic shock measures. The latter may include relationship-specific demand/supply components but are free of the endogenous reallocation component. Interestingly, while we build on a homogeneity assumption, our setting allows for asymmetric effects of shocks. However, this asymmetry is due to reallocation effects triggered by complementarity and substitutability through cross-elasticities.

[7]We discuss in greater detail the differences with these papers in Section A.4.4 of the Online Appendix.

corporate governance (Foroughi et al., 2022), and banks' liquidity choices (Silva, 2019). We cover firm access to credit and the pass-through of financial shocks'. From the perspective of the pass-through of financial shocks', our contribution is also complementary to Alfaro, García-Santana, and Moral-Benito (2021) and Huremovic et al. (2020). The latter two works study the propagation of financial shocks across the production network among firms. Instead, we focus on the propagation of shocks through the credit network among bank-firm links.

More generally, our approach relates to studies decomposing the market's aggregate outcomes to derive instrumental variables, such as shift share instrumental variables (SSIVs), used initially in Bartik (1991), Blanchard et al. (1992), and recently in Borusyak, Hull, and Jaravel (2022) and Goldsmith-Pinkham, Sorkin, and Swift (2020), as well as granular instrumental variables (GIVs), proposed in Gabaix and Koijen (2024) and applied to banking in Galaasen et al. (2020). However, our approach differs substantially from the GIVs and SSIVs approaches. The GIVs and SSIVs are procedures designed to estimate price elasticities, while we design OPIVs to estimate quantity cross-elasticities of substitution. OPIVs and the latter approaches are both complements because they can be used in different types of markets. GIVs and SSIVs consider centralized markets where there is only one price, while OPIVs consider decentralized markets where the price varies at the pair level. In Section A.4.3, we discuss the differences between OPIVs, SSIVs, and GIVs in more detail.

In studying the numerical properties of OPIVs in credit markets, we show how credit market structure (banks' and firms' network centrality) is key for shock propagation. From this standpoint, the paper relates to a recent stream of work studying the link between credit market structure and the effect of financial shocks. Important examples are Andreeva and García-Posada (2021); Benetton (2021); Benetton and Fantino (2021); Corbae and D'Erasmo (2021); Giannetti and Saidi (2019). Again, our work is complementary to Huremovic et al. (2020) and Alfaro, García-Santana, and Moral-Benito (2021). Whereas they document that financial shock propagation worsens when firms' market power is greater, we show that banks' market power has a similar effect. This distinct amplification mechanism may counterbalance the stabilizing role of credit concentration documented by Giannetti and Saidi (2019).

Finally, with respect to other works studying peer effects in networks, our context allows us to achieve identification of spillover effects under lighter assumptions. Unlike social networks, credit networks are easier to observe. The detail in the data from the credit register mitigates the concern of irrelevant unobservable connections, a common issue when studying other types of interaction (see Battaglini et al., 2020; Battaglini, Patacchini, and Rainone, 2022; De Paula, Rasul, and Souza, 2024; Miraldo, Propper, and Rose, 2021, among others). Indeed, the crucial assumption of non-overlapping bank portfolios is testable and is always verified in our data.

2 The Conceptual Framework

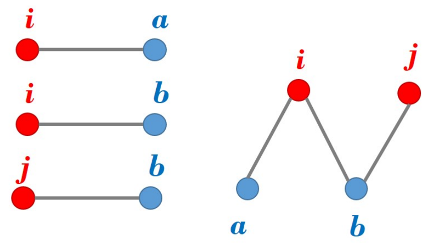
In Section 2.1 we draft a simple model to introduce the concept of cross-elasticities in credit markets and highlight how their presence can distort the estimation of (i) causal effects and (ii) firms' and banks' fixed effects.

2.1 Credit Relationships Portfolios and Identification

We present the most parsimonious necessary change to generalize the Khwaja and Mian (2008) framework and allow for reallocation across different credit relationships. The single change we introduce consists of relaxing the assumption that banks and firms optimize their choices only at the single relationship level, allowing them to more realistically maximize their profits by considering all their relationships together.

In what follows, we consider a very basic static network of two firms, i and j, and two banks, a and b, in order to keep things simple and to show the main mechanisms triggered by cross-elasticities, which hold in more complex settings as well. Bank a and b both lend to firm i, while bank b also lends to firm j. Figure 1 provides the visual counterpart. While the standard approach considers credit relationships in isolation (panel (a)), we emphasize how they form a network (panel (b)). The credit relationship ib links to the relationship ia, as they share the same borrower (i), and to jb, as they share the same lender b. In the following, we will use bank b and firm i in most derivations as examples.

Figure 1: A Toy Credit Network



(a) (b)

Notes: Nodes a, b are banks, i, j are firms, edges are credit relationships.

As in Khwaja and Mian (2008), banks fund their loans with a given mix of costless (insured) deposits and costlier resources (equity, uncovered bonds, etc). As loans cannot be fully funded with costless resources, banks face a resource allocation problem, which gives rise to the bank lending channel, i.e. the elasticity of credit to sudden changes in banks' funding.[8]

[8]In the application (Section 5), we will focus on the classic bank lending channel of monetary policy (Jiménez et al., 2014; Kashyap and Stein, 2000), i.e. the pass-through of monetary policy to banks' credit supply, possibly heterogeneous across borrowers.

Focusing on bank b for ease of exposition, let r_{ib} and r_{jb} be the interest rates that bank b earns from its credit relationships with firms i and j, and c_{ib}, c_{jb} the quantities of credit supplied to firms i and j. Let x_{ib} and x_{jb} be realizations of our treatment of interest, tracking some relationship's characteristic that changes bank b's marginal cost of lending to the specific firm, and let ν_{ib} and ν_{jb} be unobservable random shifts in supply at the relationship level. Formally, we make the following assumption about the behavior of banks.

Assumption 1. *Bank b is price-taker. Given r_{ib} and r_{jb}, bank b chooses how much credit c_{ib} and c_{ijb} to supply to firms i and j maximizing its profit $\pi_b(c_{ib}, c_{jb}) = \big[r_{ib} - \omega(c_{ib}, x_{ib}, c_{jb}, \nu_{ib})\big]c_{ib} + \big[r_{jb} - \omega(c_{jb}, x_{jb}, c_{ib}, \nu_{jb})\big]c_{jb}$. The ω cost function is linear, with $\omega(c_{ib}, x_{ib}, c_{jb}, \nu_{ib}) = \omega\frac{c_{ib}}{2} + \omega(\xi x_{ib} + \nu_{ib} + \theta c_{jb}/2)$.*[9]

The same assumptions hold for bank a, and any other bank in the market. Given the linearity of the cost function, ω is a parameter capturing the baseline cost to the bank of one more dollar in credit commitments.[10] In our setting, c_{jb} directly enters the supply choice of firm i (and viceversa), capturing the supply-side interdependence among lending decisions. Everything else being equal, if bank b already lends one more dollar to firm j, this changes the cost of lending to i by $\omega\theta$ dollars. The resulting supply equations follow from the first order conditions:

$$\begin{aligned}
r_{ib} &= \omega c_{ib} + \omega\,\underbrace{(\xi x_{ib} + \nu_{ib} + \theta c_{jb})}_{u_{ib}}, \\
r_{jb} &= \omega c_{jb} + \omega\,\underbrace{(\xi x_{jb} + \nu_{jb} + \theta c_{ib})}_{u_{jb}}, \\
r_{ia} &= \omega c_{ia} + \omega\,\underbrace{(\xi x_{ia} + \nu_{ia})}_{u_{ia}},
\end{aligned} \tag{1}$$

and enter each firm's demand problem as the firm's cost function.

On the demand side, we consider a setting in which firms may entertain multiple relationships.[11] Reflecting the fungibility of funds in reality, multiple loans to a firm can

[9] The banks may be price-taking because of competition, or because pre-set contracts fix the cost of credit and grant each bank the right to change the credit commitment at will over the life of the credit relationship, as in the case of revolving credit. The main purpose of this model is to propose the smallest possible modification to allow simultaneity in the structural framework behind most empirical banking papers based on Khwaja and Mian (2008), thus, we take the market structure as given. This choice implies that we do not model explicitly the extensive margin of credit granting, i.e. the matching process between banks and firms, either. We focus on the intensive margin outcomes only. However, bank-firm matching underpins network formation and network-endogeneity may challenge our identification strategy. Thus, we address potential network endogeneity using the method proposed by Jochmans (2023) to prove the robustness of our empirical exercises.

[10] The assumption of a common ω parameter across banks implies that banks face the same capital market. This assumption can be relaxed to allow for heterogeneity among intermediaries.

[11] This choice is coherent with empirical facts, as the median number of credit relationships for Italian firms is three (see Table 3) and can be justified by the costs and risks of locking in with a single credit provider (for empirical evidence on the rising cost of credit captivity, see Ioannidou and Ongena, 2010).

be used to finance the same project (or multiple projects); hence, credit from one source can directly substitute or complement credit from another source in the firm's objective function, creating scope for the endogenous reallocation of demand. Formally, the above translates into:

Assumption 2. *Firm i knows the supply schedule of each bank it has a relationship with and sets its credit demand from each bank simultaneously, to maximize its profit, respectively:*

$$\pi_i(c_{ia}, c_{ib}) = \big[e_i - \alpha(c_{ia} + c_{ib})\big](c_{ia} + c_{ib}) - \sum_{k=a,b} c_{ik} r_{ik}(c_{ik}, u_{ik}).$$

The same assumptions hold for firm j and any other firm in the market. Focusing on firm i for ease of exposition, e_i is the productivity of firm f's use of funds and the source of demand shocks. α drives the marginal productivity of one more dollar borrowed, underpinning credit complementarity ($\alpha < 0$) or substitutability ($\alpha > 0$) across relationships, while r_{ik} is the cost of credit derived in Equation (1).

Allowing firms and banks to optimize their full portfolio of relationships jointly, we obtain the following simultaneous system of equations:

$$\begin{aligned} c_{ia} &= \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia}, \\ c_{ib} &= \rho c_{ia} + \phi c_{jb} + \beta x_{ib} + \delta_i + \epsilon_{ib}, \\ c_{jb} &= \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb}, \end{aligned} \tag{2}$$

where we summarize the convolution of structural parameters in each equation with $\beta = -\frac{\xi\omega}{2(\alpha+\omega)}$ for the treatment we are interested in, $\delta_i = \frac{1}{2(\alpha+\omega)} e_i$ for the firm idiosyncratic component common across relationships, $\rho = -\frac{\alpha}{\alpha+\omega}$ and $\phi = -\frac{\theta\omega}{2(\alpha+\omega)}$ for the cross-elasticities.[12] In such an environment, the amount of credit that firm i borrows from bank a (c_{ia}) depends on the amount that firm i borrows from bank b. On the other hand, the amount of credit that bank b lends to firm j depends on the amount that bank b lends to firm i (c_{ib}). In turn, c_{ib} depends on both c_{ia} and c_{jb}; i.e., the outcome of a single relationship receives impulses from all relationships sharing the same firm or bank. These endogenous effects induce reflection, i.e., spillovers among credit relationships based on the structure of links in the credit network.

The key parameters tracking spillovers over the network are ρ, which measures the *firm credit cross-elasticity* (FCC), and ϕ, which captures the *bank credit cross-elasticity* (BCC). We provide a formal discussion of the interpretation of ϕ and ρ estimates in Appendix A.1.2. At an intuitive level, as long as downward sloping demand schedules and upward sloping supply schedules generate the data we observe, we can interpret a positive ρ or ϕ as evidence of complementarity in credit at the firm or bank level, and a

[12] To solve the system, we assume here that $|\phi| < 1$ and $|\rho| < 1$. This is not strictly required in more general cases. We define the space of these parameters formally in the next section.

negative ρ or ϕ as evidence of substitutability.[13] In our simple example, firm i substituting its credit from bank a with credit from bank b implies a negative ρ. Conversely, bank b increasing its credit to firm j in complement to its higher lending to firm i would imply a positive ϕ

It is worth noting that the standard Khwaja and Mian (2008) approach, assuming non fungibility of funds, shuts any reallocation mechanism down, forcing ϕ and ρ to be equal to zero. In what follows, we show that whenever we have non-null cross-elasticities in our model, estimates of the standard approach will be biased. Consider an econometrician estimating, instead of Equations (2), the following system:

$$\begin{aligned} c_{ia} &= \beta x_{ia} + \delta_i + \varepsilon_{ia}, \\ c_{ib} &= \beta x_{ib} + \delta_i + \varepsilon_{ib}, \\ c_{jb} &= \beta x_{jb} + \delta_j + \varepsilon_{jb}. \end{aligned} \tag{3}$$

Let $\hat{\beta}_{FE}$ be the estimator of β for this model and $\hat{\beta}_{OLS}$ the estimator from a version of Equations (3) that omits the fixed effects.

Proposition 1. *Under Assumptions 1, 2, the estimator $\hat{\beta}_{FE}$ of the shift in banks' supply curve, β, for Equations (3), is biased.*

Moreover, as the estimator of β from Equations (3) will be affected by omitted variable bias, it follows that comparing $\hat{\beta}_{FE}$ with the $\hat{\beta}_{OLS}$ will not necessarily be informative of the sign of the idiosyncratic bias we are concerned about.[14]

Proposition 2. *Under Assumptions 1 and 2, $\hat{\beta}_{FE} \neq \hat{\beta}_{OLS}$ is possible even in the absence of demand bias.*

Finally, Proposition 1 easily extends to all estimates of parameters from Equations (3), including fixed effects.

Proposition 3. *Under Assumptions 1 and 2, firm fixed effects' estimates include supply shock spillovers, and bank fixed effects' estimates include demand shock spillovers. As such, they cannot be regarded as pure measures of each firm or bank demand and supply shocks, respectively.*

This last Proposition implies that retrieving bank and firm fixed effects estimates and interpreting them as credit supply and demand shifters (Amiti and Weinstein, 2018) may

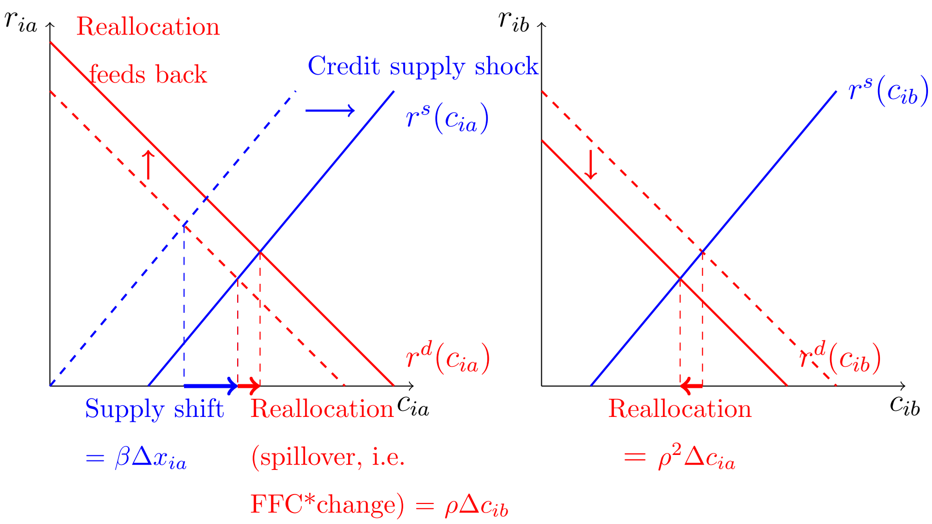
[13] Having slopes of credit demand and supply, respectively upward and downward, aligns with basic economic logic and with evidence derived from the same data we employ in our empirical exercises. For example, Burlon et al. (2016, p.22) showcases results for the estimation of a credit demand system meant to study credit rationing among firms and recovers an upward sloping supply and a downward sloping demand for credit.

[14] A very common approach to reconciling firm-level and relationship level estimates (see, e.g., Bonaccorsi di Patti and Sette, 2016; Jiménez et al., 2020).

be misleading, as they will probably incorporate the reflections of bank shocks through firm links and firm shocks through bank links.

We provide the full analytical description of the model, along with proofs of the above propositions, in Appendix A.1. Here, we focus on an intuitive exploration of the model's mechanics, starting with Figure 2, where we display a simplified graphical representation of Equations (2). In particular, we drop relationship jb and shut down the banks' reallocation channel, setting cross-elasticity ϕ to 0. Furthermore, we assume a negative firm cross-elasticity ρ; hence, firm i will substitute away from the bank whose credit becomes relatively more costly. Finally, we assume that the firm's idiosyncratic shock is null $\delta_i = 0$.

Figure 2: Cross-elasticities at Work - Reallocation in a Toy Network



Notes: The Figure represents graphically the interaction between firm i and bank a and b, captured by Equations (2), in the case bank a receives a positive supply shock. The blue lines are banks' credit supply schedules to firm i, while the red lines are firm i's demand for credit from a and b respectively. Solid lines are post-shock curves, while dashed lines are pre-shock curves.

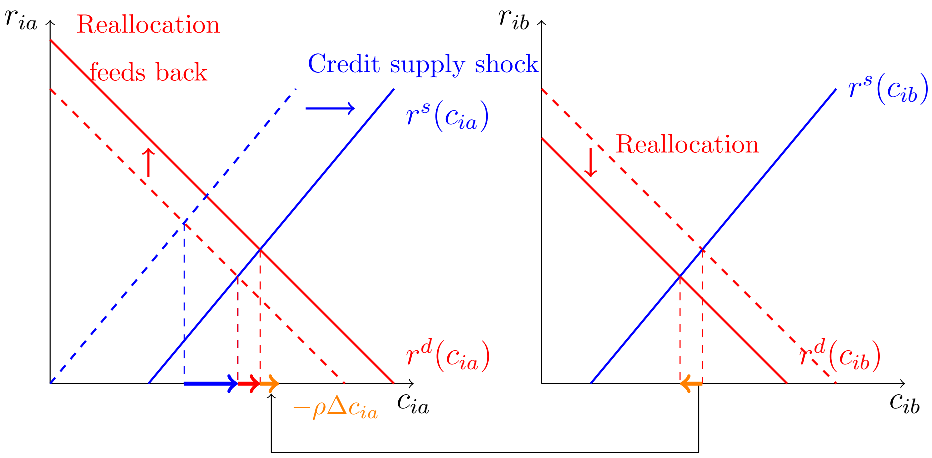
Focusing on the two relationships belonging to firm i in Equations (2), suppose a supply shock $\Delta x_{ia} > 0$ hits bank a as a consequence of monetary policy easing, to which a is especially sensitive, relaxing its supply curve (to mimic the dynamics described in Jiménez et al., 2014), while bank b's supply remains unchanged. Graphically, the shock moves a's supply from the old dashed line to the new solid one by $\beta\Delta x_{ia}$. Then, for each interest rate, bank a is happy to offer more credit to firm i. As a consequence, i will not only passively take advantage of the expansion in a's supply, sliding along the original red dotted demand curve, but will also readjust its demand to source more of its credit from a than from b, as a's credit has become relatively less costly. A $\rho\Delta c_{ia}$ shift in the equilibrium consumption of c_{ib}, partly compensated by an opposite shift by $\rho\Delta c_{ib} = \rho^2\Delta c_{ia}$ in c_{ia}, graphically describes this readjustment.[15]

[15]We limit our example to second order adjustments for simplicity; the transition to the new equilib-

In such a setting, the final change in equilibrium credit from bank a to firm i will be composed of two elements: one directly related to the supply shift and governed by parameter β, and one indirect, due to reallocation and governed also by the cross-elasticity ρ. The reallocation mechanism will bias the estimates if not properly accounted for in an empirical setting, which leads to the conclusion of Proposition 1 (and 3). In this specific example, using a typical diff-in-diff setting, where we compare the pre- and post-treatment values for treated (ia) and control (ib) units, would overestimate the effect of the treatment: the positive variation in the treated relationship c_{ia} is increased (the red arrow adds to the one in the left-hand panel in Figure 2), while the control relationship c_{ib}decreases instead of being stable (the red arrow in the right-hand panel in Figure 2).

This graphical presentation helps visualize the logic of Proposition 2 as well. In Figure 3, we show how differentiating within the firm (i.e., including a firm fixed effect on the right-hand side of an estimation equation) *even in the absence of demand bias* may lead to different estimates by changing how the bias affects the estimates.

Figure 3: Cross-elasticities and Fixed Effects in the Absence of Demand Bias



$$\Delta c_{ia} - \Delta c_{ib} = \beta \Delta x_{ia} + \rho \Delta c_{ib} - \rho \mathbf{\Delta c_{ia}}$$

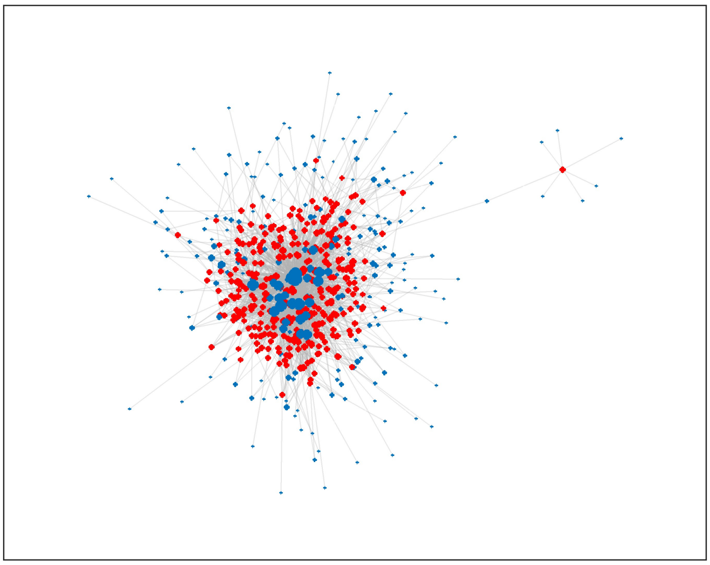
Notes: The Figure represents graphically how the interaction between firm i and bank a and b, captured by Equations (2), can affect within-firm assessment of supply shifts. The blue lines are banks' credit supply schedule to firm i, while the red lines are firm i's demand for credit. Solid lines are post-shock curves, while dashed lines are pre-shock curves. Orange segments highlight how within-firm differentiation may worsen our assessment of the supply shift.

Consider again the $\beta\Delta x_{ia}$ supply change we want to quantify, and suppose that we mistakenly believe that both c_{ia} and c_{ib} are affected by firm i-level unobservables linked to demand-shock confounders. If we differentiate $\Delta c_{ia} - \Delta c_{ib}$ to address this nonexistent demand confounder, we add to the feedback effect of reallocation on $c_{ia} = \rho\Delta c_{ib}$, which is the initial reallocation from ia to ib, $\Delta c_{ib} = \rho\Delta c_{ia}$. Within comparisons may actually increase the reallocation bias affecting the supply shifts.

rium involves an infinite sequence of adjustments whose magnitude converges to zero as $|\rho| < 1$.

The simple example presented in this section provides an intuition of the mechanics at play when banks and firms optimize their portfolios simultaneously and highlights the consequences of ignoring reallocation for the estimates of the main parameters of interest. However, actual credit networks can be even more complex, involving thousands of firms and hundreds of banks. Figure 4 provides a visual example of how complex the network formed by real credit relationships could be, graphically describing the connections in a sample of only 500 real credit relationships from the Italian Credit Register data.

Figure 4: A Sampled Real Credit Network



Notes: The network is derived from a sample of 500 credit relationships observed in 2012. Banks are represented in blue and firms in red. The estimated network is represented with a force-directed layout with five iterations. It uses attractive forces between adjacent nodes and repulsive forces between distant nodes. To ease the visualization, the size of the nodes is equal to the (log) of their degree. See Fruchterman and Reingold (1991) for more details.

As a consequence, deriving generalized expressions for the bias is more complex as well. First, as we can already see from the example, if the shock hits relationship ib instead of relationship ia, all bias expressions would change, as ib's connections are different from ia's, altering the bias. This is actually an instance of a standard result in the network literature, stating that each node's location in the network drives the extent and sign of the bias, with feedback effects increasing in complexity with the number of links (Ballester, Calvó-Armengol, and Zenou, 2006).[16] In the remainder of the document, we will introduce a general network model of credit relationships that can accommodate F

[16] We kept the model in this section as simple as possible, with the main goal of providing basic intuition about why cross-elasticities can bias standard estimates for credit market outcomes. Equilibrium conditions for a more general model, incorporating a growing number of banks and firms, would follow the results in Ballester, Calvó-Armengol, and Zenou (2006).

firms and B banks, derive unbiased estimators for β, δ, and γ, and provide expressions for the bias of commonly used estimators.

3 The Econometric Framework

In this section, we introduce an econometric framework encompassing cross-elasticities among credit relationships between banks and firms. The framework adds in two ways to the standard toolkit available in the literature. First, it allows for the identification and direct measurement of credit cross-elasticities. Second, it can be used to consistently estimate the direct and indirect effects of treatments and idiosyncratic shocks on firm and bank outcomes without imposing strong independence assumptions. We conclude this section by briefly discussing the possibility of heterogeneous cross-elasticities and unobserved credit relationships; these matters are explored in more detail in Section A.4 of the Online Appendix.

3.1 The Credit Network Model

Suppose that there are two sets, $\mathbb{F}$ and $\mathbb{B}$, of firms and banks in the market with cardinality respectively equal to F and B. We can easily generalize the system described by Equations (2) in Section 2.1 to any number of banks, firms, and relationships in the credit network as:

$$c_{ib} = \alpha + \phi \sum_{j \in \mathbb{F} \setminus i} a_{ib,jb} c_{jb} + \rho \sum_{k \in \mathbb{B} \setminus b} a_{ib,ik} c_{ik} + \delta_i + \gamma_b + x_{ib}\beta + \epsilon_{ib}, \tag{4}$$

where c_{ib} is credit from bank b to firm i. δ_i and γ_b are the firm and bank fixed effects, respectively.[17] x_{ib} is a vector of characteristics of the loan, which may include a specific treatment administered to the relationship ib.[18] ϵ_{ib} is the error component. The term $a_{ib,jb}$ captures the connections among credit relationships from the lender side, being equal to one if both firms i and j borrow from bank b, and zero otherwise. The term $a_{ib,ik}$ captures the connections from the borrower side, being equal to one if both banks b and k lend to firm i, and zero otherwise.

In this model, credit relationships are not i.i.d.; credit granted bilaterally from banks

[17]Observe that this is a two-way regression model on a bipartite graph. Given the goal of comparing our method with standard approaches followed in the literature, which overlook such features, in what follows we do not focus on how the structure of the network affects the accuracy with which these fixed effects can be estimated Jochmans and Weidner (2019).

[18]Here, the treatment is at the same level of aggregation as the outcome, the relationship level. We do not focus on treatments that are bank- or firm-specific, although the framework can be adapted. Observe that our model includes a bipartite network (Bonhomme, 2020). We do not deal with situations in which outcomes are observed for one type of node and treatments are administered to the other type (see Doudchenko et al., 2020, for example).

to firms is jointly determined. On the one hand, credit from a bank to a firm (c_{ib}) reacts to changes in lending from that bank to other firms (c_{jb}, over all js borrowing from b), as firms compete on the demand side to obtain credit from bank b, which is resource constrained. This effect is captured by the parameter ϕ, which measures the BCC (*bank credit cross-elasticity*). On the other hand, the credit to a firm by a bank (c_{ib}) reacts to changes in lending to that firm from other banks (c_{ik}, across all ks lending to i), as banks compete on the supply side to grant credit to firm i. This effect is captured by the parameter ρ, which measures the FCC (*firm credit cross-elasticity*).

The matrix form of this *credit network model* (CNM) is:

$$\begin{aligned} C &= \alpha + \phi A_B C + \rho A_F C + X\beta + \Delta + \Gamma + \epsilon, \\ &= \phi A_B C + \rho A_F C + Z\mu + \epsilon, \end{aligned} \tag{5}$$

where X is the matrix of loan covariates, Δ is the matrix containing the firm fixed effects, Γ is the matrix containing the bank fixed effects, C is the vector containing all credit granted across the N credit relationships between banks and firms in the market, A_B is the $(N \times N)$ adjacency matrix of the network that keeps track of connections among loans through banks, whose generic element $a_{ib,jk}$ is equal to one if and only if $b = k$; A_F is the $(N \times N)$ adjacency matrix of the network that keeps track of connections among loans through firms, whose generic element $a_{ib,jk}$ is equal to one if and only if $i = j$. For both adjacency matrices, we let $a_{ib,ib} = 0$ for all ib, following convention. The vector $A_B C$ (*bank-network lag*) contains, for each loan, the amount of credit granted by the same bank to other firms. The vector $A_F C$ (*firm-network lag*) contains, for each loan, the amount of credit obtained by the same firm from other banks.[19]

We define the *isolated-credit model* (ICM) as follows:

$$C = \alpha + X\beta + \Delta + \Gamma + \epsilon, \tag{6}$$

the standard model used in the literature in which credit relationships are forced to be independent, i.e., imposing the restriction $\phi = \rho = 0$ in Equation (5). It is worth observing that (i) the CNM nests the ICM, and (ii) the CNM exploits exactly the same information set necessary for estimating the ICM, because the network structure (summarized by A_B

[19]Given that our focus is on reallocation mechanisms, what matters for us is the total amount of credit; thus, we do not row-normalize these terms as is sometimes done in network econometrics. see Liu, Patacchini, and Zenou (2014) for a discussion. Observe also that the model can eventually accommodate the inclusion of relationships involving firms without multiple lenders, with the usual limitations on fixed effects. Our model can be extended to accommodate contextual effects ($A_B X$ and $A_F X$), if needed. While endogenous effects arise quite naturally as cross-elasticities in this context (see Section 2), exogenous effects are not straightforward. Other relationship' characteristics can hardly have direct effects on the equilibrium credit amounts of each relationship that do not pass through cross-elasticities of firms and banks. However, in the empirical exercise in Section 5, we also present estimates that include contextual effects as a robustness check to confirm our main findings.

and A_F) comes directly from the observable firms and banks' identities, which are used in Δ and Γ. Thus, our model can be used by every researcher working with credit register data without further addition to her dataset.

The matrix form of the model highlights how we deal with a simultaneous system of equations, in which the credit vector C enters both on the left and the right hand side, through A_BC and A_FC, the endogenous terms. This feature captures the more realistic assumption that credit choices are not independent, but it comes at the cost of additional complexity in the econometric model and its identification. Indeed, a CNM cannot be estimated by simple OLS. Nevertheless, the model described by Equation (5) belongs to the class of spatial autoregressive (SAR) models; thus, we can exploit some key results in this literature, especially from the branch that extended these models to the analysis of networks (see Arduini, Patacchini, and Rainone, 2020; Bramoullé, Djebbari, and Fortin, 2009; Hsieh and Lee, 2016; Johnsson and Moon, 2021; Lee, 2007; Lee, Liu, and Lin, 2010; Patacchini, Rainone, and Zenou, 2017, among others). However, standard SAR models usually consider outcomes at the node level, while in our case, outcomes are at the link level; thus, we proceed by describing tailored solutions designed for the features of our framework.

3.2 Identification

The main issue that arises when we want to estimate Equation (5) is the endogeneity of A_BC and A_FC, which negates the consistency of OLS estimation. The simultaneity of Equations (5) creates an intrinsic endogeneity problem if

$$\begin{aligned} E[(A_FC)'\epsilon] &= E[(A_F(I-\phi A_F-\rho A_B)^{-1}(\alpha+Z\mu+\epsilon))'\epsilon] \neq 0, \\ E[(A_BC)'\epsilon] &= E[(A_B(I-\phi A_F-\rho A_B)^{-1}(\alpha+Z\mu+\epsilon))'\epsilon] \neq 0. \end{aligned}$$

The last inequalities hold if

$$\begin{aligned} E[(A_F(I-\phi A_F-\rho A_B)^{-1}\epsilon)'\epsilon] &= \sigma_\epsilon^2 tr(A_F(I-\phi A_F-\rho A_B)^{-1}) \neq 0, \\ E[(A_B(I-\phi A_F-\rho A_B)^{-1}\epsilon)'\epsilon] &= \sigma_\epsilon^2 tr(A_B(I-\phi A_F-\rho A_B)^{-1}) \neq 0, \end{aligned}$$

where tr is the matrix trace operator. Endogeneity is determined by the structure of the observed network, represented by A_F and A_B. In what follows, we achieve identification under conditions on A_B and A_F that are reminiscent of the linear independence assumptions in Bramoullé, Djebbari, and Fortin (2009).[20] Below, we establish sufficient

[20]There are also other ways of identifying spillovers among outcomes on networks. See De Paula (2017) and Bramoullé, Djebbari, and Fortin (2020b) for discussions and reviews of the literature. Focusing on covariance restrictions to identify spillovers, De Paula (2017) provides an interesting discussion on how they imply a lower bound on the correlation among observable outcomes, even when spillovers are negative (which is relevant in our framework). Restrictions on unobservables and higher moments have

conditions under which the parameters in Equation (5) are identified, even if augmented with exogenous effects $A_B X$ and $A_F X$.[21]

Proposition 4. ***(Identification of the Credit Network Model)****. The credit network model described by Equation (5) is identified if I_F, $A_B A_F A_B$, and A_F are linearly independent and I_B, $A_F A_B A_F$, and A_B are linearly independent—i.e., there are* intransitive quadriads *in the credit network—and $\phi\beta \neq 0$ and $\rho\beta \neq 0$.*

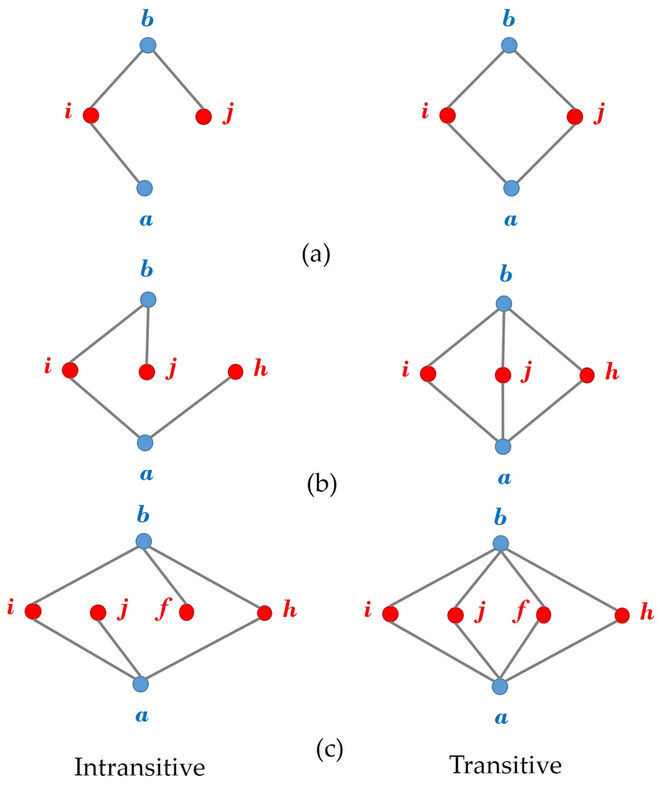
A quadriad is a set of four nodes: two banks and two firms, in this case. The quadriad is transitive if all the between-type (firm-bank) links are realized. For identification purposes, the market must not be composed of transitive quadriads only. The presence of intransitive quadriads is a sufficient (but not necessary) condition for the identification of the model's parameters. A nice feature of this result is that it translates into the easy-to-check requirement that banks do not have fully overlapping portfolios. In other words, it requires that not all the banks lend to the same set of firms. It follows that the credit market structure itself can provide the solution to the endogeneity problem if it meets certain conditions. Precisely, we need the market structure to show a certain degree of *intransitivity*. The level of intransitivity is the ratio between the number of intransitive quadriads and the total number of quadriads. In the credit market, an intransitive quadriad appears when a bank b lends to a different set of firms with respect to another bank k, or specularly when a firm i borrows from a different set of banks with respect to another firm j.

The intuition is that intransitivity provides exclusion restrictions that allow us to identify the system of simultaneous equations (5). If the market is composed only of transitive quadriads, we cannot identify the parameters in the system, as there are no valid exclusion restrictions. This situation is extremely rare, if not impossible, in credit markets. Figure 5 provides examples of market structures that allow (networks on the left) and do not allow (networks on the right) the identification of spillovers among credit relationships, with two (panel (a)), three (panel (b)), and four (panel (c)) firms in the market. Let us consider the simplest networks in panel (a). In the left network, the fact that bank k does not have a relationship with firm j allows jb to be excluded from ik's equation. It follows that X_{jb} can be used as an instrument to estimate the effect of C_{ib} on C_{ik}, as it has a direct impact on the former but not on the latter (which influences it only through the former).

been explored in the literature (see Graham, 2008, for example).

[21]In SAR models, spatial lags of both the endogenous (here $A_B C$ and $A_F C$) and exogenous (here $A_B X$ and $A_F X$) effects can be included on the right hand side. The social networks literature calls the latter contextual effects, as they capture the direct influence of peers' characteristics on outcomes. While endogenous effects arise quite naturally in our context, as shown in Section 2, exogenous effects are less straightforward in credit markets. We abstract from these effects, but our model can be extended to accommodate them if needed.

Figure 5: Network Structure and Identification



Notes: Nodes a, b are banks, red nodes are firms, edges are credit relationships.

Intuitively, when the number of firms and banks grows, the number of intransitive quadriads must also grow. Panels (b) and (c) show credit networks with and without intransitive quadriads when the number of firms increases to three and four units. Similar examples can be constructed for a growing number of banks.

3.3 Overlapping Portfolios Instrumental Variables

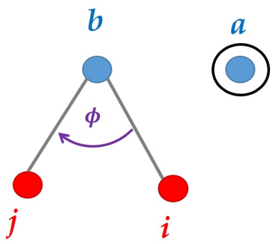
In this section, we show how to apply the result in Proposition 4 to identify treatment effects, fixed effects and cross-elasticities using instrumental variables based on overlapping portfolios (OPIVs).[22]

Intuition. Let us consider a simple example in Figure 6. In panel (a), the credit network is disconnected, as bank a does not lend at all. In panel (b), we have full overlap. Everybody lends/borrows from everybody. In panel (c), instead, the credit relationship portfolios of bank a and b are only partially overlapping (respecting the intransitivity conditions for identification). For simplicity, let's focus on how a shock to

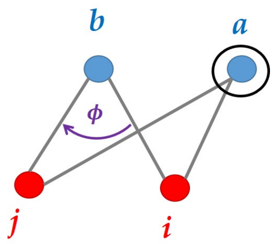
[22]The literature of spatial and network econometrics investigated in depth several methods to treat the endogeneity created by these simultaneous equations, Kelejian and Prucha (1999) and Liu and Lee (2010) proposed a GMM approach, and Lee (2004) and Lee, Liu, and Lin (2010) use a Quasi-Maximum Likelihood Estimator. In this paper we use an IV approach in the spirit of Lee (2007) and Kelejian and Prucha (1998).

bank a helps us identify ϕ. In panel (a), a shock to bank a is not relevant, as the two banks are not connected through any firm. In panel (b), the same shock is relevant but not exogenous, because it directly influences jb through ja. In panel (c), a shock to bank a is a valid instrument, instead. Indeed, it is relevant, as it directly influences bi through ai; moreover, it is exogenous to jb, as the two relationships are only indirectly connected. The same reasoning applies to shocks that are firm-specific for the identification of ρ. If we have bank and firm fixed effects in the model, variation at the relationship-level can be exploited instead, as we do in our empirical exercise.

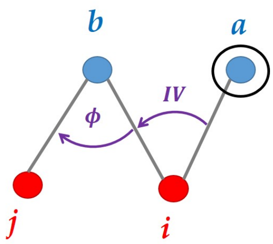
Figure 6: Exogeneity and Relevance of OPIVs



(a) Not relevant



(b) Not exogenous

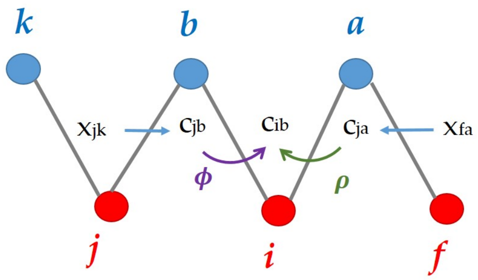


(c) Relevant, exogenous

Notes: Nodes a, b are banks, i, j are firms, edges are credit relationships.

Depending on the circumstances, there could be situations in which the econometrician observes only firm- or bank-specific variables and no pairwise variables. In such cases, we can use variation in more distant credit relationships for identification. For instance, let us add two other credit relationships to our toy example in panel (c) of Figure 6 and assume that bank a supplies credit to firm f, and firm j demands credit from bank k. Figure 7 provides the relative graph. In this highly intransitive credit network, variation induced by fa (jk) to ia (jb) can be used as an instrument to identify ρ (ϕ), i.e., the effect of c_{ia} (c_{jb}) on c_{ib}. Here, even if the econometrician only observes banks' characteristics, a's (k's) specific variables can be used as an instrument for c_{ia} (c_{jb}) to identify ϕ (ρ). A specular strategy can be used if the econometrician only observes firms' characteristics, say f (j). In Section 3.3.2, we discuss how variation far away through the network can be used to address relationships' endogeneity concerns.

Figure 7: IV exclusion restrictions



Notes: nodes k, b, a are banks, j, i, f are firms, edges are credit relationships.

In the full model. Following Proposition 5, OPIVs are a natural way to proceed with the estimation of Equation (5). The OPIVs are basically "network embedded". Following standard results in Liu and Lee (2010) the theoretically best OPIVs are just the expected values of the endogenous terms:

$$TOPIV_F = E(A_F C) = E[(A_F(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu))], \quad (7)$$
$$TOPIV_B = E(A_B C) = E[(A_B(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu))]. \quad (8)$$

Given that the parameters in Equations (7)-(8) are unknown, $TOPIV_B$ and $TOPIV_F$ are unfeasible. Assuming $||\phi A_F + \rho A_B||_\infty < 1$,[23] the term $(I - \phi A_F - \rho A_B)^{-1}$ is an infinite sum of elements $\sum_{k=0}^{\infty}(\phi A_F + \rho A_B)^k$. A linear approximation, $OPIV_F$ and $OPIV_B$, of vectors appearing in Equation (7) and (8) can thus be used for the empirical IVs. Exploiting only the variation of covariates,[24] first order approximations of $TOPIV_F$ and $TOPIV_B$ are respectively:

$$OPIV_F^1 = A_F X, \quad (9)$$
$$OPIV_B^1 = A_B X; \quad (10)$$

second order approximations are

$$OPIV_F^2 = [A_F, A_F A_B]X, \quad (11)$$
$$OPIV_B^2 = [A_B, A_B A_F]X; \quad (12)$$

and so on and so forth.[25] Observe that here $A_F^k \equiv A_B^k \equiv \mathbf{0}$, which is the zero matrix,

[23] This is a sufficient condition for the invertibility of $(I - \phi A_F - \rho A_B)$; it also determines the parameter space for spillover effects.

[24] To keep things easy we do not employ Γ and Δ in the construction of the OPIVs. In this way, we do not incur the "many IV" bias and make assumptions on how the ratio between the number of relationships and the number of firms and banks in the market grows asymptotically. We consequently abstract from efficiency considerations and the bias correction procedures strategy discussed in Liu and Lee (2010).

[25] The approximation is as follows. $E(A_F C) = E[(A_F(I - \phi A_F - \rho A_B)^{-1}(\alpha + Z\mu))] =$

for $k > 1$, as we have a bipartite network with only cross type (firm-bank) links. Let us use our example again in Figure 7 to consistently estimate ρ (ϕ), where a first order approximation uses x_{ia} (x_{jb}) as an IV. A second order approximation also includes x_{fa} (x_{jk}) in the IV. Given the high dimensionality of credit registers, which contain hundreds of banks and thousands of firms, it is preferable to use a within transformation:

$$JY = \phi J A_B Y + \rho J A_F Y + JX\beta + J\epsilon,$$

where $F = [\Delta, \Gamma]$ and $J = I - F(F'F)^{-1}F'$. One can thus first estimate $\omega = (\phi, \rho, \beta')'$ by 2SLS. Let $R = J[A_B Y, A_F Y, X]$ and $P_H = JH(H'J'JH)^{-1}H'J'$ where $H = [OPIV_F, OPIV_B, X]$. It follows that

$$\widehat{\omega}_{2SLS} = (R'P_H R)^{-1} R' P_H Y \tag{13}$$

is a consistent estimator for cross-elasticities and treatment effects. Next, one can recover the idiosyncratic components $\zeta = (\gamma', \delta')'$ by OLS. Let $\widehat{U} = Y - R\widehat{\omega}_{2SLSW}$, then

$$\widehat{\zeta}_{2SLS} = (F'F)^{-1} F' \widehat{U} \tag{14}$$

provides a consistent estimator for bank and firm fixed effects.[26]

The methodology can also accommodate an instrumental variable for the treatment if it is not exogenous. The instrument can be included in the first step in a quite straightforward way (see Anselin and Lozano-Gracia, 2008; Dall'Erba and Le Gallo, 2008, for applications of this procedure).[27] Interestingly, unbiased cross-elasticities can be recovered even in the presence of treatment endogeneity and in the absence of a valid instrument for the treatment. The intuition is that the network lags used as instrumental variables for the identification of spillovers can still be valid instruments under quite mild conditions. The formal results are shown in the supplementary online Appendix A.1.5; we present some Monte Carlo exercises in Section 4.

3.3.1 Bias Induced by Cross-Elasticities

What happens to standard estimators when we ignore cross-elasticities in credit markets? While our CNM estimator $\hat{\beta}_{2SLS}$ in (13) is consistent, what happens to the ICM estimator

$E[A_F[\sum_{k=0}^{\infty}(\phi A_F + \rho A_B)^k](\alpha + X\beta + \Delta + \Gamma)] = E[A_F X\beta] + E[A_F A_B X\beta] + E[A_F(\alpha + \Delta + \Gamma)] + E[A_F A_B(\alpha + \Delta + \Gamma)] + E[A_F[\sum_{k=2}^{\infty}(\phi A_F + \rho A_B)^k](\alpha + X\beta + \Delta + \Gamma)]$. This is due to the fact that we have a bipartite network without within type connections and thus $A_F A_F = A_B A_B = \mathbf{0}$. A specular approximation can be derived for TIV_B.

[26] The 2SLS estimator without within transformation is instead

$$\hat{\theta}_{2SLS} = (W'P_Q W)^{-1}(W'P_Q C), \tag{15}$$

where $W = [A_F C, A_B C, Z]$, $P_Q = Q(Q'Q)^{-1}Q'$, $Q = [OPIV_F, OPIV_B, X]$, and $\hat{\theta}_{m,t,2SLS} = [\hat{\phi}_{2SLS}, \hat{\rho}_{2SLS}, \hat{\mu}_{2SLS}]$.

[27] See Fingleton and Le Gallo (2008) for the finite sample properties of this type of estimators.

of β in (6)? Here we provide some analytical results; in Section 4, we also report evidence through numerical experiments.

Proposition 5. ***(Treatment Effects Bias)****. If cross-elasticities are positive, network effects imply a positive bias for treatment effects. When cross-elasticities are negative, the sign of the bias of the ICM is undetermined. The bias depends on known information, i.e., the network structure and the treatment vector, and unknown parameters, i.e., the intensity of cross-elasticities themselves.*

Given that cross-elasticities can be negative in credit markets, this result is important because it states that standard estimates cannot be taken as lower or upper bounds. The intuition behind the indeterminate sign is that when cross-elasticities are negative, they lower the outcomes of odd-distant credit relationships (at distances 1, 3, 5, etc), but increase the outcomes of even-distant relationships (at distances 2, 4, etc).[28]

Intuitively, the bias of firm and bank fixed effects has an indeterminate sign as well. The intuition behind this result is similar to that of the indeterminacy of the treatment effect bias': portfolio joint optimization allows idiosyncratic shocks to diffuse through the credit network. We provide the details in online Appendix A.1.4. In Section 4, we show that the bias is larger for nodes that are more central in the credit network when using finite samples.

3.3.2 Endogenous Credit Relationships

A common concern in the econometric analysis of spillovers over networks is the possible endogeneity of the network itself. Issues can arise if some unobserved factors drive the formation of links (here, credit relationships) and outcomes (credit quantities exchanged). Suppose, for example, that the link formation process is as follows:

$$g_{ib} = I(d(h_i, h_b) \geq u_{ib}), \tag{16}$$

where $g_{ib} = 1$ if there exists a credit relationship between firm i and bank b, h_i and h_b are unobserved node-specific (bank- and firm-specific) characteristics, u_{ib} is a relationship-specific random component, and $d(.,.)$ is a matching function. The unobserved node-specific characteristic h_i can be interpreted as a factor that increases the likelihood of forming a link. Network endogeneity may arise if the individual unobserved characteristics of the firm (bank) h_i (h_b), which affect link formation, are correlated with the firm's (bank's) unobserved characteristic determining the outcome c_{ib}. It is indeed easy to imagine that lower firm risk or higher profitability of projects, as well as advantages in monitoring or screening for banks, affect both match-likelihood and credit granted.

[28]The negative sign could prevail, as the first round effects have a higher weight, especially in denser networks. See also De Paula (2017) for a discussion on identification when spillovers are negative.

Compared to models in which the outcome is at the node level (see Arduini, Patacchini, and Rainone, 2015; Auerbach, 2022; Goldsmith-Pinkham and Imbens, 2013; Hsieh and Lee, 2016; Johnsson and Moon, 2021; Patacchini and Rainone, 2017; Qu and Lee, 2015, among the others), a key difference in our context is that we model outcomes at the link level. This feature allows us to include node fixed effects (δ_i and γ_b), which alleviates concerns related to assortative matching on unobserved agent characteristics (homophily or heterophily) as well as agent-level heterogeneity in link surplus (degree heterogeneity, see Graham, 2017). Nevertheless, there could also be link-level correlated unobservables:

$$g_{ib} = I(d(h_i, h_b, h_{ib}) \geq u_{ib}). \tag{17}$$

In model (4), a connection between two credit relationships is thus observed if

$$a_{ib,jb} = g_{ib}g_{jb} = I(d(h_i, h_b, h_{ib}) \geq u_{ib})I(d(h_j, h_b, h_{jb}) \geq u_{jb}) = 1, \tag{18}$$

or

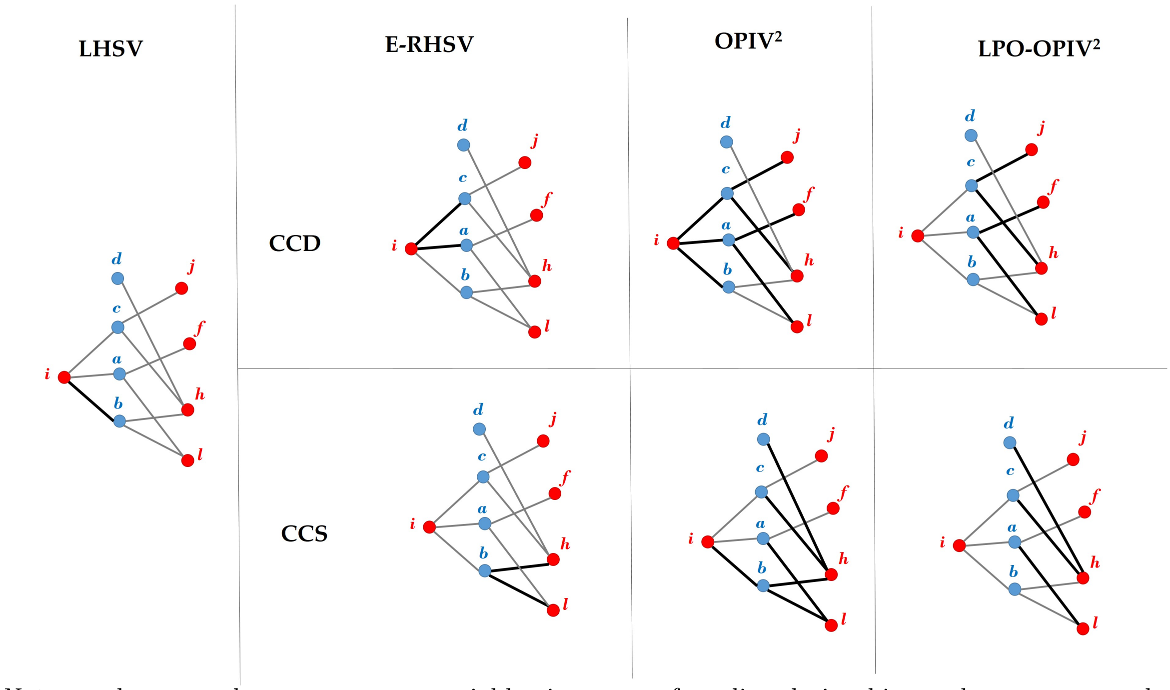
$$a_{ib,ik} = g_{ib}g_{ik} = I(d(h_i, h_b, h_{ib}) \geq u_{ib})I(d(h_i, h_k, h_{ik}) \geq u_{ik}) = 1. \tag{19}$$

It follows that if $E[a_{ib,jb}\epsilon_{ib}] \neq 0$ or $E[a_{ib,ik}\epsilon_{ib}] \neq 0$ the adjacency matrices A_B or A_F are endogenous.

To tackle this concern, we propose an IV strategy that allows for endogenous credit relationships. Following Jochmans (2023), instead of specifying a model for the matching between banks and firms building on Equations (18) and (19), we exploit two restrictions to construct conditional moment conditions: (i) link decisions that involve a given firm or bank are not independent of one another, (ii) but they are independent of the link decisions made between other pairs of banks and firms that are located sufficiently far away in the network. These conditions imply that instrumental variables can be constructed from leave-own-out networks. Given that we are modeling pairwise outcomes at the relationship level, we leave out both the firm and the bank involved in it (leave-the-pair-out, LPO henceforth).

To provide intuition about the way the instrumental variables are constructed, let us make a simple example. Consider the credit network in Figure 8 and focus on relationship ib. Suppose we use distance-2 ($OPIV^2$). In model (4), the LHS variable is c_{ib}. FCC (upper panels in the figure) comes from relationships involving the same firm, i.e. the endogenous RHS variables c_{ic} and c_{ia} in the example. Unconstrained distance-2 OPIV would exploit the variation in x of all relationships involving firm i and banks c and a, in this example including ib. In distance-2 LPO-OPIV ($LPO - OPIV^2$), instead, all relationships involving firm i or bank b are excluded. Variation in relationships involving banks a, c and firms j, f, h, l would be exogenous to ϵ_{ib}, as it involves completely different agents far away in the credit network, being relevant for c_{ic} and c_{ia}. Symmetrically, the lower panels of Figure 8 depict how the IV is constructed for the BCC.

Figure 8: Construction of distance-2 LPO-OPIV



Notes: columns and rows represent variables in terms of credit relationship used to construct them. FCC: firm credit cross-elasticitiy; BCC: bank credit cross-elasticitiy; $LHSV$: left hand side variable; $E-RHSV$: endogenous right hand side variable; $E-RHSV$: endogenous left hand side variable. $OPIV^2$: distance 2 overlapping instrumental variable variable; $LPO-OPIV^2$: distance-2 leave-the-pair-out overlapping instrumental variable variable. Red nodes are firms, blue ones are banks. Edges represent credit relationships. Relationships involved in the construction of variables are highlighted in black.

Formally, the second order approximation LPO-OPIV are constructed as follows:

$$LPO-OPIV^2_{B,ib} = \sum_{j\in\mathbb{F}\setminus i}\sum_{d\in\mathbb{B}\setminus b} a_{ib,jb}a_{jb,jd}x_{jd}, \tag{20}$$

$$LPO-OPIV^2_{F,ib} = \sum_{k\in\mathbb{B}\setminus b}\sum_{f\in\mathbb{F}\setminus i} a_{ib,ik}a_{ik,fk}x_{fk}. \tag{21}$$

The construction of higher order OPIV follows the same logic.

3.3.3 Extensions and Further Discussion

One limitation of our model is that it imposes constant FCC and BCC across different agents. For example, such cross-elasticities may vary depending on the size, level of capitalization (for banks) or sector and riskiness (for firms). Our parameters, ϕ and ρ, provide just an average across potentially quite different agents. Furthermore, to address specific empirical questions on reallocation of credit, it may be useful to characterize these parameters depending on both the types of bank and firm involved. For instance, one may be interested in measuring the degree of reallocation of innovative firms from low to high-tech banks or whether specialized banks reallocate differently between sectors or type of borrowers they know better or not. Extending our work in these dimensions would undermine the focus of this paper. However, in Section A.4.1 of the supplementary online Appendix we present some extensions of the CNM, to allow for heterogeneous ϕ and ρ, which may be of practical relevance for future research in empirical studies addressing questions similar to those outlined above.

A residual concern stems from unobserved credit relationships. They could create an omitted variable problem as they may be correlated with the treatment and the network lags. Sampling and measurement error in links are the two main possible determinants. From this standpoint, a reassuring feature of the credit register data we use is that it is not a sample. It covers the entire population of loans above a very low size threshold: 30 thousand euros as total exposure in terms of granted credit.[29] However, even though the issues of sampling and measurement error are minor, there could be some market interactions between realized and unrealized credit relationships that may be in principle at work. For example, that would be the case if the supply of bank b affects the terms that bank a applies to firm i, even when bank b does not lend to i. This could be an issue only in very specific situations, for instance if banks have a poorly diversified but highly overlapping portfolios of borrowers and apply specific pricing rules for them. As we measure reallocation from the total amount of credit in other same-bank or same-firm relationships, such type of unobserved interactions are very unlikely to be material. We characterize in more detail the conditions under which market interactions between realized and unrealized credit relationships may threaten identification in online Appendix A.4.2.

In addition, in online Appendix A.4.3 and A.4.2 we discuss the relationship between OPIVs and other IVs based on market structure, like granular (GIV) and shift share instrumental variables (SSIV), and deepen how our paper relates to the literature on spillovers in corporate finance.

4 Monte Carlo Simulations

In this section, we simulate credit networks, both entirely fictional and calibrated to real-world characteristics, to study the finite sample properties of our methodology in different settings. Furthermore, we use these numerical experiments to assess the bias of the standard estimators (ICM) used in the literature.

[29] In several countries, like Spain, the threshold is even lower. Issues may arise if other type of data is used.

4.1 Setting

We use a given set of parameters ϕ, ρ and β, randomly generated characteristics X, error terms ϵ, and a network G as inputs. We create G as a 'circular network', ordering nodes according to natural numbers from 1 to N. Odd nodes are banks, and even nodes are firms. We link node i to all opposite type nodes, from node $i+1$ to node $i+z_i$, where z_i is an independent realization from a uniform distribution $U(0,m)$ for each node i. m regulates the network density, i.e., the average number of shared borrowers among banks.[30] For example, if z_1 $(z_2) = 10$, bank 1 (firm 2) links with firms 2,4,6,8, and 10 (banks 3,5,7,9, and 11). In this way, there are only links between banks and firms, with none within the two types. This setup is useful because it allows us to easily change salient features of the network (like size and sparsity).

In our benchmark simulation, we generate 500 networks G with l links among $n = 200$ nodes in each, comprising 100 banks and 100 firms. Once G is randomly generated, we then derive the adjacency matrix A of the links in G. For each link, we create an observable (x) and an unobservable (ϵ) variable. x is a dummy variable equal to 1 if the link belongs to a certain subsample of size s of the population and zero otherwise. We extract ϵ from a normal distribution with a mean equal to zero and a variance equal to σ. We generate strictly positive fixed effects for each lender and borrower from independent normal distributions $\delta N(0,1)$. We generate outcomes on credit relationships from the reduced form: $C = (I - \phi A_B - \rho A_F)^{-1}(\beta X + \Delta + \Gamma + \epsilon)$. In particular, our pivotal setting for sample simulation is $(\beta, N, m, \sigma, \delta) = (2, 200, 2, 1, 0.1)$, with $R = 500$. - We perform two classes of exercises within this framework. First, we look at the bias of $\hat{\beta}$ and the estimated firm and bank fixed effects (FEs) when we ignore the interdependence among credit relationships, estimating the isolated credit model (ICM). Second, we study the performance of our credit network model (CNM) estimator for the recovery of the treatment effect β, the cross-elasticities ϕ, ρ, and the firm and bank FEs under different settings and assumptions. Across all these exercises, we assume positive treatment effects. In doing so, we focus on a framework similar to the risk-taking channel of monetary policy, by which less capitalized banks relax their supply more or contract it less toward some firms after the easing of monetary policy (captured by the X dummy), which we explore in our empirical exercise in Section 5. Moreover, guided by the theory and our empirical results, we focus on negative cross-elasticities.[31]

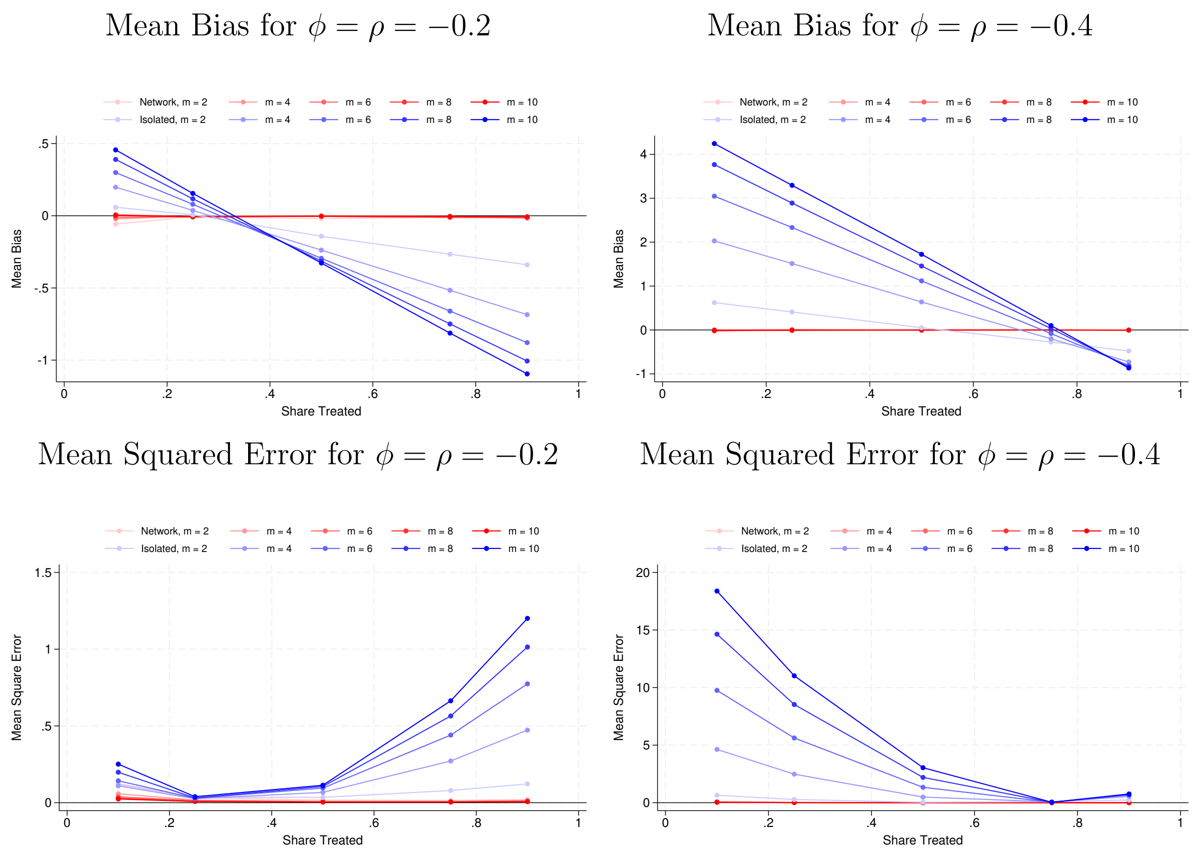
[30] When $z_i = 1$ for all i, the network is a circle. The results are not sensitive to the distribution used for z_i.

[31] In our first empirical application, $\hat{\rho}$ is negative and larger in magnitude than the positive $\hat{\phi}$. At the end of this section, we also show the good performance of our estimator when simulations are calibrated based on the empirical exercise. The sign and magnitude of the treatment effect bias would still be indeterminate when studying negative treatment effects.

4.2 Results with Simulated Networks

4.2.1 Treatment Effects

Figure 9: Treatment Effect Estimates - ICM and CNM Performance



Notes: X-axis: share of treated units (s). Y-axis: mean bias and mean squared error for treatment effect (β). We compute the mean bias across the 500 simulated samples. For each sample, the number of nodes N in the network is 200, 100 firms and 100 banks. In black, we plot the zero line. In different shades of blue, we plot how the mean bias (upper two figures) and the mean square error (lower two figures) of the ICM's estimate change. In red, we do the same for the CNM's estimates. Darker shades signal denser networks (m=2,4,6,8,10). In left panels $\phi = -0.2$, in right panels $\phi = -0.4$. We display the plots' underlying data in the supplementary online Appendix Table A.1.

ICM Bias. We first focus on the bias of β. For simplicity, we set $\phi = \rho$ and $\Delta = \Gamma = 0$; we then show that our results hold in more general settings. Across exercises, we vary the value of the cross-elasticities, the share of treated relationships in the population, and the network's density. Figure 9 reports the mean bias (upper panel) and the mean squared error (MSE) of β for ICM (blue lines) and CNM (red lines). The intensity of the shade is darker for denser networks. The right panels are generated with stronger cross-elasticities.[32] In Figure 9's upper panels, we can see how the sign and intensity of the bias depend on observable features of the network, i.e., the treated share and the density

[32]We report all the underlying numbers in the supplementary online Appendix A.2, also presenting results for $\rho = \phi = -0.3$, as well as additional experiments.

of relationships. Nevertheless, comparing the left and right elements of Figure 9's upper panel, we can also see that the bias depends on the magnitude of the cross-elasticities as well. In empirical contexts, we cannot observe these parameters directly and, as a consequence, we cannot mend the bias without directly confronting it in the estimation.

Furthermore, Figure 9 shows that when a greater fraction of the population receives the treatment, the mean bias declines faster if cross-elasticities are greater in magnitude and the network is denser. The alternating sum of positive and negative bias components, when cross-elasticities are negative (see Proposition 5 and equation A.18 in its proof), explains this pattern. When there are few treated units, the positive effect prevails, as most higher-order effects are feedback loops triggered by own-treatment, amplifying the direct positive effect of treatment. To provide economic interpretation, we think of a case in which credit from shocked banks becomes more abundant, and firms with multiple lenders systematically start sourcing more credit from relationships treated by the positive supply shock, further decreasing their use of the non-treated relationships in equilibrium. We stress that this is the numeric counterpart of what we showcased through the theoretical example in Section 2.1, when only one credit relationship faces a supply expansion.

Instead, when the supply expansion hits more nodes, the amplifying effect of feedback loops being more than offset by indirect effects from other treated units, which are primarily negative under negative cross-elasticities. Again, interpreting the results through economic logic, we can consider the case in which the majority of banks suddenly expand their credit supply. When many banks are treated, the direct positive effect of the treatment on the credit of a single relationship can be offset by the negative indirect effect from relationships that share the same borrower.

Higher density, i.e., more relationships per node and thus higher overlap across lenders' portfolios of borrowers, allows for more feedback loops and indirect effects, thereby amplifying the magnitude of bias in both directions. The density also determines the point in which the bias switches its sign: the higher the number of shared links, the smaller the number of treated units needed to switch the sign of the bias to negative. The values of cross-elasticity parameters ϕ, ρ determine the point at which the bias changes its sign as well. The higher such a value, the more treated units are needed to switch the bias from positive to negative. This conclusion is more true the fewer units are treated, as there are more banks from which to reallocate away credit demand.

The results shown do not depend on the chosen setting, which is quite simple but is able to highlight the main forces at work. We have chosen density as the main metric for the network; others can be considered. Here, we are not particularly interested in finding a "sufficient" statistic for the contribution of the network structure to the bias of β, because we can derive it: endowed with a credit register, we observe A_F and A_B, as well as the treatment vector, and we provide in Section 5 a consistent estimator for

ϕ and ρ. With all these elements, we can precisely derive the bias (equation A.18 in the online Appendix). In Figure A.1 in the online Appendix, we report the same evidence when bank and firm FEs are included.

Estimator Performance. Next, we analyze the performance of our estimator in more detail. First, we report the mean bias and the MSE of $\hat{\beta}$ for the same setting used above. We visualize the results as the red lines in Figures 9 and A.1. The mean bias is negligible in all settings, with different shares of treated units, magnitudes of spillovers, and densities of the credit network (Figure 9's upper panel). Moreover, our estimator is precise, as measured by the low MSE, which decreases with the number of treated units and with density (i.e., with the increase in the number of relationships, thus the sample size; see Figure 9's lower panel). Lastly, we also see that such performance is not affected by the inclusion of fixed effects in the data generating process (Figure A.1)

Second, we analyze the performance of the estimates of the spillover parameters ϕ and ρ, relaxing the assumption that $\phi = \rho$ and also studying what happens when n grows. In Table 1, we report the mean and the standard deviation of the estimates of ϕ and ρ at different values (by columns) and network sizes and densities (by rows). We can see that the estimates are always centered at the true values. The standard deviation decreases with the number of nodes in the network and the density of the connections among them. With a quite small sample size of 800 nodes, the dispersion of the estimates is limited even with a very small density (when $m = 2$). In the online Appendix A.2.1, we show how cross-elasticities can be recovered in finite samples even when treatments are endogenous.

Table 1: Performance of the Spillovers' Estimators ($\hat{\phi}$, $\hat{\rho}$)

n	m		true		true		true		true		true	
			ϕ	ρ	ϕ	ρ	ϕ	ρ	ϕ	ρ	ϕ	ρ
200			-0.1	-0.1	-0.1	-0.2	-0.1	-0.3	-0.1	-0.4	-0.4	-0.4
	2	mean	-0.097	-0.100	-0.100	-0.209	-0.101	-0.306	-0.093	-0.414	-0.406	-0.406
		std	0.084	0.087	0.090	0.089	0.082	0.081	0.082	0.076	0.066	0.067
	6	mean	-0.098	-0.098	-0.097	-0.197	-0.097	-0.295	-0.096	-0.398	-0.402	-0.395
		std	0.029	0.030	0.029	0.030	0.029	0.033	0.028	0.032	0.039	0.040
	10	mean	-0.102	-0.096	-0.099	-0.198	-0.099	-0.297	-0.098	-0.398	-0.402	-0.397
		std	0.021	0.020	0.023	0.023	0.022	0.024	0.021	0.024	0.031	0.030
800												
	2	mean	-0.102	-0.098	-0.100	-0.201	-0.097	-0.301	-0.097	-0.401	-0.398	-0.401
		std	0.041	0.043	0.044	0.042	0.042	0.042	0.040	0.037	0.034	0.033
	6	mean	-0.099	-0.100	-0.099	-0.200	-0.097	-0.300	-0.099	-0.400	-0.398	-0.401
		std	0.015	0.014	0.015	0.016	0.014	0.016	0.014	0.016	0.021	0.020
	10	mean	-0.100	-0.099	-0.100	-0.200	-0.099	-0.300	-0.099	-0.400	-0.399	-0.400
		std	0.010	0.011	0.010	0.011	0.011	0.012	0.010	0.012	0.015	0.015

Notes: Over this Table, we compute the mean and the standard deviation across 500 simulated samples. n is the number of nodes in the network, m regulates the network density as described in Section 4.1. We report further simulation results in the supplementary online Appendix Table A.5.

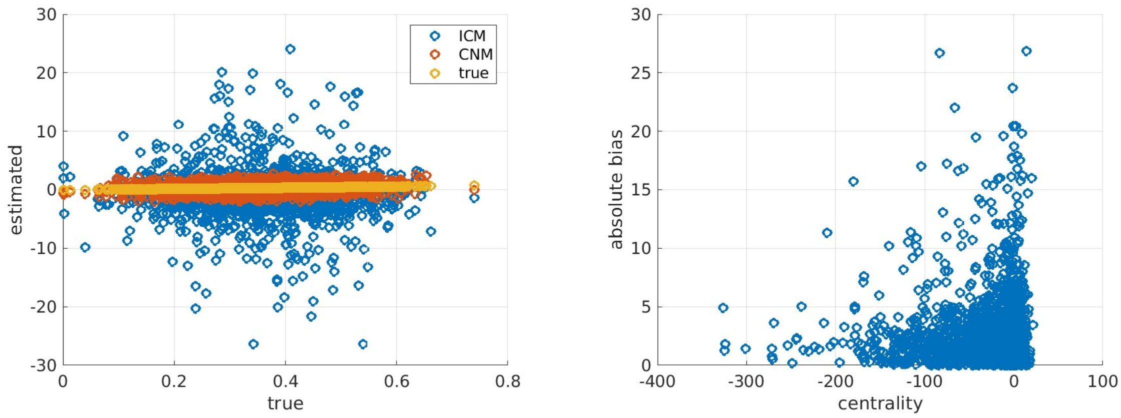
4.2.2 Idiosyncratic Firm and Bank Shocks

Thanks to our methodology, we can also improve the estimation of firms and banks fixed effects. Fixed effects are not only an important tool to mitigate the concern of unobservable demand and supply shocks that bias treatment estimates, but they are also used in the literature to measure idiosyncratic credit demand and supply shifts and quantify their real effects (Amiti and Weinstein, 2018).

For conciseness, we focus on a short graphical presentation, stressing how bias distributes across nodes in the network and its relation to centrality. We discuss the details of our fixed effects estimators' performance in the supplementary online Appendix (see Table A.6). We focus on a setting with $n = 2000$, $m = 10$, and $\phi = \rho = -0.4$ to see how idiosyncratic shock estimates perform with dense networks and high cross-elasticities.

In panel (a) of Figure 10, we plot the true value of the idiosyncratic shocks on the x-axis against themselves (in yellow), as well as the CNM fixed effects estimates (in orange) and the ICM fixed effects estimates (in blue) on the y-axis. We can see that the bias for the ICM estimates can be quite severe for some nodes, either upward or downward. Indeed, the bias can be positive or negative under negative spillovers, with the sign depending on the network topology.[33]

Figure 10: ICM and CNM Fixed Effects' Estimates and Distribution over Nodes



(a) True, CNM and ICM estimates

(b) Centrality and FE bias

Notes: We sample this network setting $n = 2000$, $m = 10$ and $\phi = \rho = -0.4$, the other parameters are the same as in the pivotal simulation described previously. In panel (a), x-axis: true value of the idyosincratic shock, y-axis: true value (in yellow), CNM FE estimates (in orange) and ICM FE estimates (in blue). In panel (b), x-axis: our node i centrality measure $D_i'MD_i$, y-axis: ICM FE estimate for node i.

Panel (b) of Figure 10 plots the nodes'centrality on the x-axis against the absolute value of the ICM-estimates' bias on the y-axis. We measure the centrality of relationships in which node i is involved with $E_i = D_i'MD_i$, where $M = (I - \phi A_B - \rho A_F)^{-1}$ and D_i are the i_{th} column of the D matrix, whose ji_{th} entry takes a value of 1 when node i is involved in the credit relationship j.

As shown in Equations (A.30)-(A.31) in the Appendix, the bias sign and magnitude depend on the number of loops in which the firm or bank is involved. In particular, spillovers distort the fixed effects estimate of more 'central' lenders and borrowers in the credit network more severely.[34] In the Figure, we can indeed see that the higher the centrality of the node, the higher the value of the ICM estimates' bias.

In other words, even if their idiosyncratic variation is negligible, lenders or borrowers that are more central in the credit network may show particularly high absolute values

[33] This is consistent with the results in Section 3.3.1.

[34] Still, we remind the reader that under negative spillovers, the bias can be close to zero even for central nodes, as positive spillovers from even loops can offset negative ones from odd loops.

of their ICM-estimated fixed effects. Nevertheless, these large values may just be the result of shocks originating from other nodes in the network. Given that central nodes are more exposed to the influence of other nodes', they may accumulate a large amount of variation that is not originated by themselves but comes from other banks and firms instead.

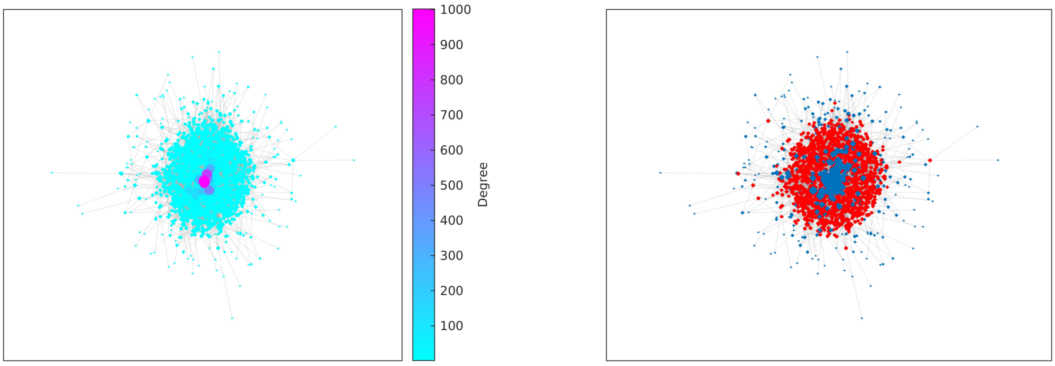
4.3 Results with Networks Sampled from the Credit Register

We conclude our simulation study with experiments based on a set of observed real credit relationships. So far, we have considered fairly simple network structures. Here, we test whether our method still works when the complexity of the topology of the credit relationships increases, resembling a real credit market structure. We thus construct G using realized credit relationships between banks and firms from the credit network used in the empirical application in Section 5. We randomly extract $n = 400, 800, 2000$ nodes from the full set of relationships observed in 2012 in the Italian credit market. The other elements of the simulation are generated in the same way described above.

Figure 11 depicts the network for the 2000 nodes sample. We highlight two features of such a credit network. First, the high interconnectedness between banks and firms; second, the high concentration of connections. In panel (a), the color of each node changes with its degree, i.e., the number of connections it has; more violet nodes represent more connected banks and firms. We can see that the real credit network features high skewness, with some nodes, especially banks, having a very high number of relationships. In panel (b) we color the nodes denoting firms in blue and banks in red. We note a tight group of banks with a prominently central position in the network, surrounded by a cloud of more peripheral banks. We also highlight a large layer of firms connected to both central and peripheral banks, whose credit relationships connect many banks indirectly.[35]

[35] This type of core-periphery structure is often observed in financial networks (see Boss et al., 2004; Craig and Von Peter, 2014; Iori et al., 2008; Soramäki et al., 2007, among others). As we are working with a sample, the position of each node is not necessarily represented accurately. Nevertheless, the sampling procedure guarantees that all the connections among sampled nodes are included. In the empirical analysis, we consider the whole credit network. Unfortunately, its huge dimension does not allow us to examine it visually with standard software.

Figure 11: The Credit Network



(a) Degree (b) Banks and firms

Notes: We derive the Figure's network sampling 2000 credit relationships observed in 2012. In panel (a), the color of each node is proportional to its degree, more violet nodes represent more connected banks and firms. In panel (b), we represent banks in red and firms in blue. We plot the estimated network with a force-directed layout with five iterations. A force-directed layout uses attractive forces between adjacent nodes and repulsive forces between distant nodes. To ease the visualization, the size of the nodes is equal to the (log) of their degree. See Fruchterman and Reingold (1991) for more details.

We report mean bias for idiosyncratic shocks and treatment effects in Table 2, varying the intensity of spillovers, the share of treated units, and the sample size.[36] In the last panel of the table, we set ϕ and ρ equal to the values estimated in the empirical exercise (see Table 4). We note that the mean bias of the CNM estimator is always around zero for both treatment effects and idiosyncratic shocks. On the contrary, the biases of the ICM treatment and fixed effects estimates' are large and increase on average with the magnitude of cross-elasticities, and for the treatment estimator, with the number of nodes sampled.[37]

In conclusion, the results of our last experiment confirm that our estimator provides consistent estimates for treatment effects and idiosyncratic shocks, even under a more complex structure of credit relationships and highly heterogeneous values of ϕ and ρ estimated from our empirical exercise, while conventional estimators are still severely biased.

[36] We cannot vary the density in this exercise as we are sampling from a real network.

[37] This is due to the higher density of bigger networks, which is linked to larger bias, as shown, e.g., in the Figures 9 and A.1. Indeed, the bigger the network sampled, the less we censor links between sampled and left-out nodes. Accordingly, our sample of 400 nodes has a density of 1.25, while the samples of 800 and 2000 nodes have respective densities of 1.75 and 2.4.

Table 2: Simulation study - ICM and CNM estimators performance with real credit networks and ϕ and ρ as estimated in the empirical application

			mean bias					
		n =	400		800		2000	
			treatment effect	idiosyncratic shocks	treatment effect	idiosyncratic shocks	treatment effect	idiosyncratic shocks
$\phi = \rho =$ -0.2	% of treated							
	0.10	ICM	0.587	-0.224	0.754	-0.228	0.919	-0.228
		CNM	0.000	0.008	0.000	0.002	0.000	0.001
	0.25	ICM	0.546	-0.398	0.715	-0.379	0.896	-0.359
		CNM	0.000	0.007	-0.002	0.001	0.000	0.000
	0.50	ICM	0.440	-0.660	0.614	-0.607	0.847	-0.571
		CNM	0.000	0.004	-0.003	-0.001	0.000	0.001
	0.75	ICM	0.235	-0.840	0.409	-0.770	0.716	-0.749
		CNM	0.000	0.005	0.000	0.002	0.000	0.000
	0.90	ICM	0.017	-0.862	0.141	-0.780	0.480	-0.788
		CNM	0.000	-0.004	0.000	0.006	0.000	0.002
$\phi = \rho =$ -0.4								
	0.10	ICM	3.043	-0.409	4.363	-0.433	5.480	-0.434
		CNM	0.000	0.000	0.000	0.001	0.000	0.000
	0.25	ICM	2.857	-0.834	4.193	-0.862	5.379	-0.841
		CNM	0.000	0.001	0.000	0.000	0.000	-0.001
	0.50	ICM	2.448	-1.412	3.745	-1.466	5.134	-1.482
		CNM	0.000	-0.005	0.000	-0.001	0.000	0.000
	0.75	ICM	1.701	-1.696	2.836	-1.789	4.517	-1.959
		CNM	0.000	0.009	0.000	0.003	0.000	0.000
	0.90	ICM	0.846	-1.526	1.660	-1.597	3.376	-1.916
		CNM	0.000	0.003	0.000	0.004	0.000	0.002
ϕ and ρ								
from the empirical	0.10	ICM	2.359	-0.276	2.526	-0.142	2.825	-0.055
application		CNM	-0.039	0.002	-0.009	0.000	-0.005	0.002
	0.25	ICM	2.209	-0.613	2.418	-0.435	2.772	-0.302
		CNM	0.003	-0.002	-0.001	-0.005	0.016	0.001
	0.50	ICM	1.860	-1.102	2.106	-0.831	2.600	-0.689
		CNM	0.000	-0.001	0.000	0.003	0.000	0.004
	0.75	ICM	1.185	-1.311	1.479	-1.038	2.197	-0.972
		CNM	-0.001	-0.006	-0.001	-0.001	-0.006	0.004
	0.90	ICM	0.444	-0.945	0.637	-0.881	1.469	-0.945
		CNM	-0.005	0.010	-0.005	0.010	0.004	-0.002

Notes: ICM and CNM stand respectively for isolated credit model and credit network model. n is the number of nodes in the network. The links are extracted from realized credit relationships between a random sample of firms and banks from all credit relationships observed in 2016. Estimates of ϕ and ρ are from Table 4. The bias of the treatment effect is computed as in Table A.1, A.2, A.4 and 1. The bias of idiosyncratic shocks is computed as in Table A.6.

5 Empirical Application

In the following, we re-evaluate the risk-taking channel of monetary policy using our methodology in an empirical exercise inspired by Jiménez et al. (2014). We test on Italian data whether the easing of monetary policy rates encouraged less capitalized banks to extend more credit to risky firms before the Great Financial Crisis (2002-2008). Employing our credit network model, we assess the importance of taking into account cross-elasticities across the different relationships of firms and banks in the risk-taking channel regressions.

The exercise has two purposes. First, we aim to study how accounting for endogenous reallocation affects a standard result in the empirical banking literature. Second, and more importantly, we want to quantify the cross-elasticities of credit at the firm and bank level, and study their behavior over the business cycle. To do so, we estimate our model year-by-year from 2002 to 2022, documenting the heterogeneity in the sign and magnitude of the cross-elasticity estimates over time.

5.1 Data Description

We assemble 20 years of data from (i) the Italian credit register, which tracks all credit relationships between Italian firms and banks whose total exposure in terms of granted credit is greater than 30,000 euros. We match this data with (ii) the Company Accounts Data System (CADS), balance sheet information for the universe of Italian non-financial corporations provided by the Cerved group, and (iii) the Italian Supervisory Reports, which contain Italian banks' balance sheets and group structure.[38]

From the population of credit relationships, we drop all observations belonging to firms with troubled credit relationships (*deteriorati* and *sofferenze*), as they are subject to different reporting thresholds on their size,[39] as well as relationships belonging to foreign banks or non-bank financial intermediaries, as in such cases we could not track the lenders' balance sheet characteristics. Then, following the literature, we focus on firms with multiple credit relationships so that we can estimate firm fixed effects. Thus, we drop observations belonging to firms with only one relationship per year, as well as observations whose balance sheet data are not dependable.[40]

[38] As our main dependent variable is the log growth in credit, each year we project the group structure as of the 31st of December from the previous year, so that relationships entering a new banking group due to a change in bank identity (e.g., due to a merger) do not appear as new relationships with missing credit growth.

[39] Whereas healthy credit relationships are included in the register whenever they cross the 30,000 euro threshold for at least a quarter, a 250 euro reporting threshold applies to severely deteriorated credit (*sofferenze*). Moreover, banks most often set the amount granted to such customers to zero, while the drawn credit grows. The combination of these two conditions implies that the amount granted to firms with severely deteriorated credit outstanding is not a reliable statistic for our purposes. We filter out all deteriorated credit to be conservative regarding the changes in credit quantities we consider. For more details on the credit register reporting standards, we refer to Di Noia and Moretti (2020) (in Italian).

[40] The non-dependable data firms are either firms with negative leverage or those that report more than 10 credit relationships while not classified as large by the Banca d'Italia reporting system. Such firms are, respectively, either in distress, with negative equity and, thus, spotty reporting, or the financial arms of large corporations, which report separately, but whose data in isolation are not meaningful.

Table 3: Descriptive Statistics

	Mean	*S.D.*	*P.25*	*Median*	*P.75*	*N.*
			Credit Relationships Characteristics			
Δ ln(C)	-0.014	0.431	-0.108	0.000	0.000	9,008,642
Revolving on Total	0.268	0.327	0.034	0.125	0.349	9,008,642
Granted on Total	0.310	0.219	0.135	0.257	0.447	9,008,642
Same Province	0.261	0.439	0.000	0.000	1.000	9,008,642
Rel. Age (1999)	6.145	5.277	2.000	4.000	9.000	9,008,642
			Firm Characteristics across Relationships			
N. Rel. Firm	3.924	2.157	2.000	3.000	5.000	9,008,642
I(Risk)	0.276	0.447	0.000	0.000	1.000	8,968,260
Risk Rating	5.232	1.826	4.000	5.000	7.000	8,968,260
			Bank Characteristics across Relationships			
N. Rel. Bank	25,824	23,277	4,402	18,732	42,383	9,008,642
ln(Bank Eq./Asset)	1.966	0.287	1.810	1.959	2.135	8,505,613
I(Bank Id. Change)	0.044	0.206	0.000	0.000	0.000	9,008,642
			Aggregates			
Δ Overnight Rate (pp)	-0.077	1.075	-0.223	-0.044	0.299	9,008,674
			Count			
N. Firm	447,918					
N. Banks	771					
Years	2002-2022					

Notes: This Table presents descriptives for the estimation sample. The first panel, *Credit Relationships Characteristics*, describes the distribution of changes in granted credit, as well as that of the revolving intensity of credit relationships, the ratio between credit granted on the relationship and total credit granted to the firm, the dummy tracking whether firm and bank share the same headquarter province, and the age of the credit relationship since year 1999. The second panel, *Firm Characteristics across Relationships*, summarizes the distribution of number of credit relationships per firm, of the risk dummy (risk rating greater or equal to seven), and that of the underlying firm risk rating (there are 8 levels in increasing risk order) across credit relationships. The third panel, *Bank Characteristics across Relationships*, displays the distribution of the number of the credit relationships per bank, the log of the bank equity to assets ratio, and of the dummy tracking whether the id of the bank in the dataset changes (e.g., due to a merge) across credit relationships. Finally, we report descriptives for the Italian overnight rate in percentage points and the number of firms, banks and years covered in our sample.

Table 3 documents the characteristics of our sample, covering about 9 million observations belonging to approximately half a million firms and 800 banks.[41] First, we document the negative average dynamic of granted credit (in log-growth) across the entire study period, a negative performance that is strongly influenced by the two large crises included in the time-span (the GFC and the European Debt Crisis). Then, we re-

[41] In each year after 2006, we have approximately 150 thousand unique firms and between 400 and 500 banks, due to the progressive consolidation in the Italian banking industry. Before, we only have information on the 72 major banking groups, which still covers the majority of credit granted in Italy.

port our relationship-level control variables: the ratio between revolving and total credit (Revolving on Total); the ratio between credit granted on the relationship and total credit granted to the firm (Granted on Total); a dummy variable taking the value of one if the firm and bank's headquarters are located in the same province (Same Prov.); and the age of the relationship since 1999 (Rel. Age (1999)).[42] Their distributions are mostly stable across the years we consider.

As our treatment variable is the interaction between bank capitalization, firm risk, and the change in banks' overnight refinancing rates (Jiménez et al., 2014), we also document the end-of-year change (year-on-year) in the Italian banks overnight refinancing rate in percentage points;[43] a risk dummy based on the Cerved risk rating score (a firm is considered risky if the score is equal to or above seven); the logarithm of bank equity over assets, providing a regulation-independent assessment of each bank equity buffers; a dummy tracking whether the bank id in the dataset changes due to events such as changes in incorporation status or mergers. Finally, due to our interest in fixed effects estimation, we report data on the number of relationships per firm and bank within the estimation sample.

5.2 Empirical Specification

We estimate an instance of Equation (4), inspired by Jiménez et al. (2014), using the estimators in Equations (13)-(14). Our objective is to assess whether credit grew more for risky borrowers of under-capitalized banks when monetary policy rates were lowered between 2002 and 2008 (following Jiménez et al., 2014). Thus, our treatment of interest (x_{ibt} in Equation (4)) is the triple interaction between the change in the overnight interest rate (Δ Overnight Rate), a dummy variable taking the value of one if the firm is rated risky by Cerved (I(Risk)), and the logarithm of bank equity over total assets from each banking group's books (ln(Bank Eq./Asset)).[44]

A large positive value represents a relationship involving a risky firm matched with an under-capitalized bank during a rate decrease, or a risky firm matched with a highly-capitalized bank during a rate increase. As a consequence, we would find evidence of

[42] To compute the age variable, in each year we project that year's group structure back to 1999 to avoid interrupting the life of relationships involving banks that have been incorporated into new lending institutions.

[43] The 90-day Interbank Rates for Italy, which we retrieve from the ECB Statistical Data Warehouse.

[44] We choose the latter measure of bank capitalization as it is the one selected in Jiménez et al. (2014) and is independent of changes in the definitions of regulatory capital and risk weights, allowing for inter-temporal comparisons. For the risk measure, we make a slightly different choice, opting for a rating-based definition. We use the risk-rating instead of the presence of deteriorated credit, as employed by Jiménez et al. (2014). Indeed, the Italian credit register has different rules for the reporting of deteriorated and healthy credit relationships, as documented in Footnote 39. As the reporting threshold changes when a loan becomes deteriorated, the data reporting becomes noisier. Comparing firms with and without deteriorated credit would mean comparing firms with inherently different data quality, thereby obtaining unreliable estimates.

the risk-taking channel of monetary policy whenever estimating a positive impact of this triple interaction. Such a result would imply that the credit supply of under-capitalized banks is more pro-cyclical for risky customers, with greater accumulation of risky exposures during easing and greater restrictions during tightening episodes, thereby increasing overall fragility.

The informativeness of estimates from such an empirical model is threatened by at least three types of confounders. First and foremost, the matching between risky firms and under-capitalized lenders is likely to be non-random. For example, (Schwert, 2018) documents how risky borrowers match with safer lenders who have a greater buffer and can provide better liquidity insurance. Hence, when we observe a risky borrower matched to an under-capitalized lender, we are likely looking at firms that could not find credit otherwise. Such firms would probably face worse credit conditions and smaller loan sizes, biasing our estimates downward.

Jiménez et al. (2014) deals with this threat by estimating a static selection model that explicitly accounts for the matching step. To do so, they use access requests to the credit register data by banks, which can signal the evaluation of a loan application. Instead, we estimate our models on credit log-growth. First-differencing our data removes all static, match-specific confounders. As the bias we accept in implementing such a less demanding procedure is against finding significant results, it bolsters the economic significance of our findings.

Second, even after addressing pair-specific, static confounders, firms that match with banks having smaller buffers may be those expecting overall smaller idiosyncratic increases in their credit demand. With more conservative expectations, these firms would value the liquidity service less, and banks may be more conservative in extending credit due to the lower balance sheet space at their disposal. To limit the extent of these concerns about correlated demand and supply shocks, and in line with Jiménez et al. (2014), we fully saturate our model with bank-time and firm-time fixed effects. Thus, we will compare the credit growth of relationships with the same risky firm borrowing from more and less capitalized banks after demeaning all common bank-specific shocks.

Third and last, recent research on bank specialization suggests that credit in different relationships is not always perfectly substitutable (Paravisini, Rappoport, and Schnabl, 2023), and researchers should account for differences at the relationship level. We use our rich dataset to do so, controlling for the lag of the ratio between credit granted in the relationship and credit granted to the firm, measuring the relationship's importance to the firm; a dummy variable taking the value of one if the headquarters of the firm and the bank are located in the same province (Agarwal and Hauswald, 2010; Degryse and Ongena, 2005); the relationship's age expressed in years (capturing the degree of lock-in between banks and firms, Ioannidou and Ongena, 2010); the ratio between revolving and total credit granted on each relationship (proxying for the intensity of information collection, Berger and Udell, 1995); and a dummy variable taking the value of one if a lender b has changed ID due to, for example, an M&A operation at time t, accounting for possible changes in the banking group structure over time.

The resulting Isolated Credit Model (ICM) is as follows:

$$\begin{aligned}\Delta \log \left(\mathrm{C}_{ibt}\right) = \quad & \beta \Delta \text{Overnight Rate}_t * I(\text{Risk})_{it-1} * ln(\text{Bank Eq./Asset})_{bt-1} \\ & +\delta_{it} + \gamma_{bt} + \mu \text{Controls}_{ibt-1} + \varepsilon_{ibt},\end{aligned} \tag{22}$$

where $\Delta\text{Overnight Rate}_t * I(\text{Risk})_{it-1} * ln(\text{Bank Eq./Asset})_{bt-1}$ is our main variable of interest; δ_{it} is the firm-time fixed effect; γ_{bt} is the bank-time fixed effect; Controls_{ibt} is a vector collecting the five relationship characteristics described above; ε_{ibt} is the heteroskedasticity robust error term.

As we discussed in Sections 2, 3, and 4, the ICM above cannot account for endogenous reallocation among firm and bank relationships. This limitation implies that we cannot obtain direct estimates of banks and firms' credit cross-elasticities, and we may encounter difficulties identifying our main parameters of interest β, δ_{it}, and γ_{bt}.

To illustrate the problem, let us focus on the case of a drop in interest rates and a negative firm cross-elasticity of credit. If under-capitalized banks expand their supply to risky firms (say our treatment group), these firms may further reallocate their demand away from less convenient banks in their portfolio. This endogenous reshuffling will directly impact the credit amounts we observe in relationships with other banks (say our control group). We may also observe indirect effects: first, the crowding-out of other borrowers from their more convenient suppliers (see Section 3.3.1); second, reallocation by banks may kick in. On the basis of whether the different relationships of the same bank are substitutes or complements, this channel will also determine the credit granted that we observe in equilibrium.

In other words, the standard approach does not take into account that the control group's outcome also changes when the treatment status of treated units changes, improperly assuming the stable unit treatment value assumption (SUTVA, Rubin, 1986). Estimating β with the ICM will result in a bias of unforeseeable magnitude and sign, driven by the signs of bank and firm credit cross-elasticities, the topology of the credit network, and the exposure of all credit relationships to the treatment variable, as illustrated in Section 4.

To overcome these limitations, we estimate the following CNM:

$$\Delta \log\left(\mathrm{C}_{ibt}\right) = \quad \beta\Delta \text{Overnight Rate}_t * I(\text{Risk})_{it-1} * ln(\text{Bank Eq./Asset})_{bt-1}$$

$$+\phi \mathrm{N}_B \Delta \log\left(\text{granted}_{ibt}\right) + \rho \mathrm{N}_F \Delta \log\left(\text{granted}_{ibt}\right) \tag{23}$$

$$+\delta_{it} + \gamma_{bt} + \mu \text{Controls}_{ibt} + \varepsilon_{ibt},$$

which includes the bank- and firm-network lags of the dependent variable to identify the bank (ϕ) and firm (ρ) cross-elasticities. We formalize this addition with the N_B and N_F operators, such that $N_B\Delta \log\left(\text{granted}_{ibt}\right)= \sum_{j\in\mathbb{F}\setminus i} a_{ib,jb}\Delta \log\left(\text{granted}_{jbt}\right)$ is the bank-network lag of $\Delta\log\left(\text{granted}_{ibt}\right)$ and $N_F \log\left(\text{granted}_{ibt}\right) = \sum_{k\in\mathbb{B}\setminus b} a_{ib,fk}\Delta \log(\text{granted}_{ikt})$ is the firm-network lag of $\Delta \log(\text{granted}_{ibt})$. These two variables capture the dependence of one relationship's credit growth on the credit growth of other relationships by the same firm and bank. We estimate this CNM equation using the OPIV strategy described in Section 3.3.

5.3 Main Results

Estimated Risk Taking Channel and Cross-Elasticities. Table (4) reports the results, with the risk-taking channel (β) estimate for the ICM displayed in the left panel and the 2SLS second stage estimates for the risk-taking channel, as well as the cross-elasticities of demand (ρ) and supply (ϕ) for the CNM shown in the right panel.

First, we note that we are confirming the findings in Jiménez et al. (2014). Looking at $\hat{\beta}$ entries in Table 4, we observe a positive estimate for both the ICM and the CNM. Focusing on the CNM estimate displayed in the right-hand panel, we see that after a one percent drop in the overnight rate, a risky firm sourcing its credit from a bank with one standard deviation smaller capitalization than average *and* from a bank with average capitalization would see 43 basis points more credit growth[45] This is an economically (as well as statistically) significant result, as it amounts to 13 percent of the average log growth in credit granted from 2002 to 2008 (three percent). The ICM result displayed in the Table's left-hand panel is one tenth in magnitude and non-significant due to unaccounted endogenous credit reallocation.

Second, we report in the right-hand panel the results for the bank and firm's cross-elasticity estimates. ρ is approximately -0.32, while the estimate of ϕ, rescaled by 4,402 ($\phi*$) to account for the disparity in the number of relationships per firm and bank, is positive and smaller.[46] Both estimates are highly statistically significant. Before commenting

[45] We can see that $0.0043 = 0.015 * (-0.284) * (-1)$, where 0.015 is $\hat{\beta}$, -0.284 is one standard deviation smaller capitalization measure (also reported in the Table for ease), while -1 is the one percentage point drop in overnight rates.

[46] 4,402 is the 25th percentile and about a quarter of the median of the distribution of the number of

further, we stress that the data do not support concerns regarding weak IVs. We display the values of the F_{SW} test statistics at the bottom of the panel, summarizing the results of the weak IV tests for linear models with multiple endogenous variables (Sanderson and Windmeijer, 2016) for the two first stage equations. Both F_{SW}s are large against any possible benchmark.[47]

We perform two back-of-the-envelope calculations to convey the economic significance of the cross-elasticity estimates'. For firms, we can think of one with three relationships (the median number of relationships per firm we report in Table 3), observing a 50 percent increase in granted credit from two of its three banks. Our estimated cross-elasticities suggest that, on average and within the 2002-2008 time window of interest, such a firm would shrink its demand toward the third bank, satisfying its needs more conveniently. Our ρ estimate suggests that the amount granted in the third credit relationship will shrink by approximately 32 percent ($-0.32 = -0.32 * 2 * 0.5$). For banks, we can think of one increasing its credit supply by 50 percent to 4,402 borrowers (about a quarter of the median of the distribution of the number of relationships per bank). Our $\phi*$ estimate implies that such growth would entail a 3.5 percent additional crowding-in of credit for the firms borrowing from that bank ($0.07 * 0.5$), possibly due to how easing many borrowers' credit constraints at once makes all of them more profitable to lend to.

relationships per bank shown in Table 3. Thus, $\phi*$ tracks the impact from the simultaneous growth of about a quarter of relationships for the median bank. We make this normalization choice to simulate a large but realistic comovement in a bank's relationships, to better interpret the economic significance of our result.

[47] Stock and Yogo (2002) critical values may be considered excessively permissive in the case of multiple endogenous regression, such as the one presented in this paper. Nonetheless, the magnitude of the F-statistics in any of the first stages exploited to compute the results displayed in this paper limits concerns about the joint significance of the first stage equations. We report the results of the estimation of both first stage equations in Appendix Table A.9.

Table 4: Spillover and Treatment Estimates for the Risk Taking Channel

Dep. Var.:		$\Delta \log(C)$		Mean Dep.: 0.032		SD log(Bank Eq./Asset): 0.284		
		ICM		CNM Second Stage				
							Cross-Elasticities	
Years		Coeff.	S.E.	Coeff.	S.E.		Coeff.	S.E.
2002-2008	$\hat{\beta}$	0.002	0.002	0.015	0.001	$\hat{\phi}*$	0.070	0.002
						$\hat{\rho}$	-0.321	0.010
	N	2,107,094					2,107,094	
							First Stage F_{SW}	
							BCC	FCC
							39,162	453

Notes: The table reports estimated coefficients and standard errors for the second stage of model (23), obtained applying the 2SLS estimator in Equation (14), with $OPIV_F^1$ and $OPIV_B^1$ in equations (11) and (12) as instruments . The dependent variable $\Delta \log(\text{granted}_{ibt})$ is the yearly log growth of credit granted on each relationship. The treatment associated with $\hat{\beta}$ equals to the change in the overnight rate*the lag of log(100*equity/asset)*the risky firm indicator. $\hat{\rho}$ is the firm-level cross-elasticity (FCC) estimate, measuring the reaction of credit granted on a certain relationship to increases in credit granted on other relationships of the same firm. $\hat{\phi}*$ is the bank-level cross-elasticity (BCC) estimate, measuring the reaction of credit granted on a certain relationship to increases in credit granted on other relationships of the same bank. ϕ estimate and its errors are multiplied by 4,402, the 25th percentile of the distribution of the number of relationships per bank ($\phi*$). The F_{SW} statistics on the bottom right of the table summarizes the results of F-test for weak instruments in linear IV models with multiple endogenous variables proposed by Sanderson and Windmeijer (2016). We display full first stage estimation output in Appendix Table A.9. BCC reports the F-test for the regressor of ϕ, FCC reports the F-test for the regressor of ρ. We report as N the effective number of observations used for our estimates, this is slightly smaller than the number of observations without missing controls (2,228,595) due to the iterative drop of singletons, necessary to identify bank and firm fixed effects. We report full descriptives for the estimation sample in Appendix Table A.8.

Idiosyncratic Shocks' Bias. Following our simulation study in Section 4, we focus on the distortion caused by ignoring endogenous reallocation on fixed effects estimates. Given that we estimated economically significant cross-elasticities, which imply widespread spillovers, we want to document their effect on the estimates of idiosyncratic credit movements at the firm and bank level by comparing $\hat{\gamma}_{bt}$ and $\hat{\delta}_{it}$ from the CNM and the ICM. The fixed effects we estimated within the CNM will proxy for the unbiased parameters. We will measure the bias through the differences between each fixed effect estimated within the ICM (Equation (22)) and the CNM (Equation (23)).

Table 5: Fixed Effects, Empirical Bias Summary Measures

	Bank	Firm
Mean Absolute Bias	0.938	0.960
Median Absolute Bias	0.387	0.409

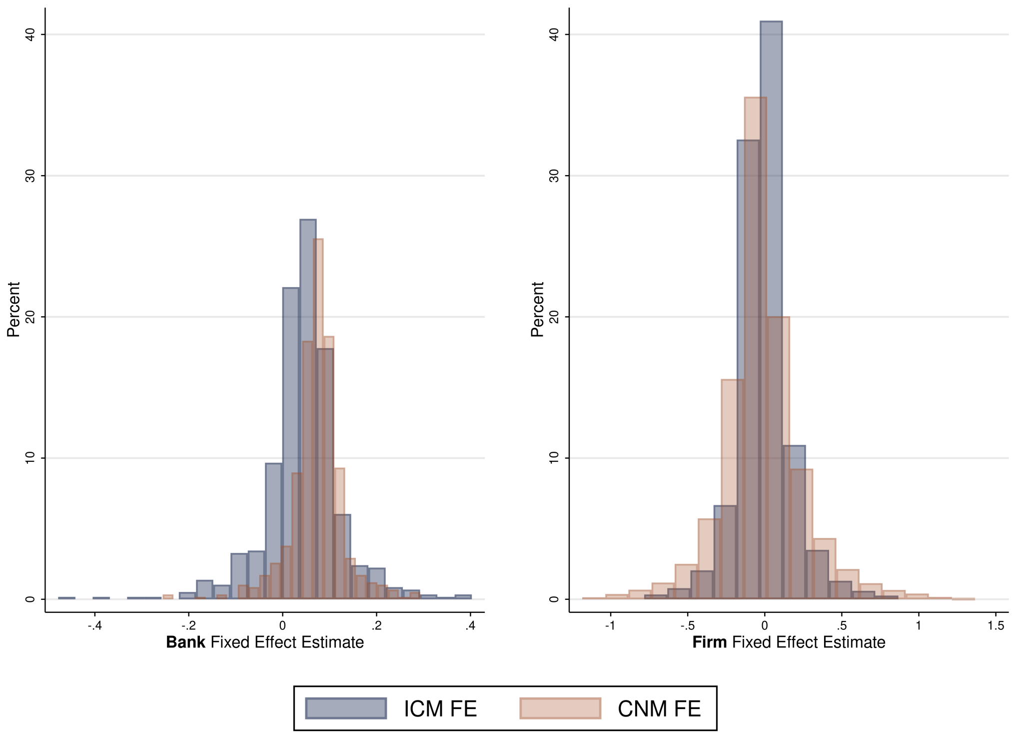
Notes: The Table reports the empirical mean absolute bias and median absolute bias for firm and bank estimated fixed effects. We consider the fixed effects estimated correcting for network structure as the true parameters. We compute the bias measures with the following formulas, here reported only for the bank fixed effect case: $\text{MAB}_{\text{Bank}} = 1/B\{\sum_{k=1}^{B} |\hat{\gamma}_{ICM}^{k} - \hat{\gamma}_{CNM}^{k}|/|\hat{\gamma}_{CNM}^{k}|\}$, $\text{MedAB}_{\text{Bank}} = Med\{(||\hat{\gamma}_{ICM} - \hat{\gamma}_{CNM}||)/Diag[||\hat{\gamma}_{CNM}||]\}$. Where B is the number of banks in the market, Med is the median operator, MAB stands for Mean Absolute Bias, and MedAB for Median Absolute Bias.

In Table 5, we focus on differences divided by the absolute value of the CNM's fixed effects, so that the magnitudes are expressed as a percentage of the unbiased estimate, thereby making them easier to compare. We summarize these quantities by reporting the mean and median absolute bias. We notice three facts: first, the bias is of first order economic importance, with the median absolute bias amounting to 39 (banks) and 41 percent (firms) of the magnitude of the CNM idiosyncratic effect estimate. Second, the mean absolute bias is orders of magnitudes larger than the median, being about twice as large as the median, implying not only large biases overall but also cases of extremely high distortion.

Figure 12 describes the above conclusions visually, displaying the relationship between the distribution of fixed effects estimates for firms and banks through the ICM (darker shade) and the CNM (lighter shade). On average, and in the context of evaluating the risk taking channel of monetary policy before the GFC, we can see how the ICM model returns a left-ward-shifted distribution of banks' idiosyncratic components (left panel). In the case of firm fixed effects (right panel), we can detect a slight right-ward shift in the mean, while the ICM performs suppression of the distribution tails.

Such behavior aligns well with the logic of reallocation effects in a network of borrowers and lenders, where credit is highly substitutable for firms, as displayed in Table 4 (a large and negative $\hat{\rho}$). In such a setting, it may be the case that banks whose credit supply eased less during rate cuts experienced widespread drops in their borrowers demand. Ignoring the network component in the model, the econometrician may end up confusing such reallocation away from a bank with an idiosyncratic cut by the bank. Conversely, for firms, the ICM cannot separate borrowers idiosyncratic shocks from the effect of lender-level complementarity in credit allocation, underestimating the magnitudes of many firm fixed effects.

Figure 12: The Empirical Distribution of Fixed Effects Estimates

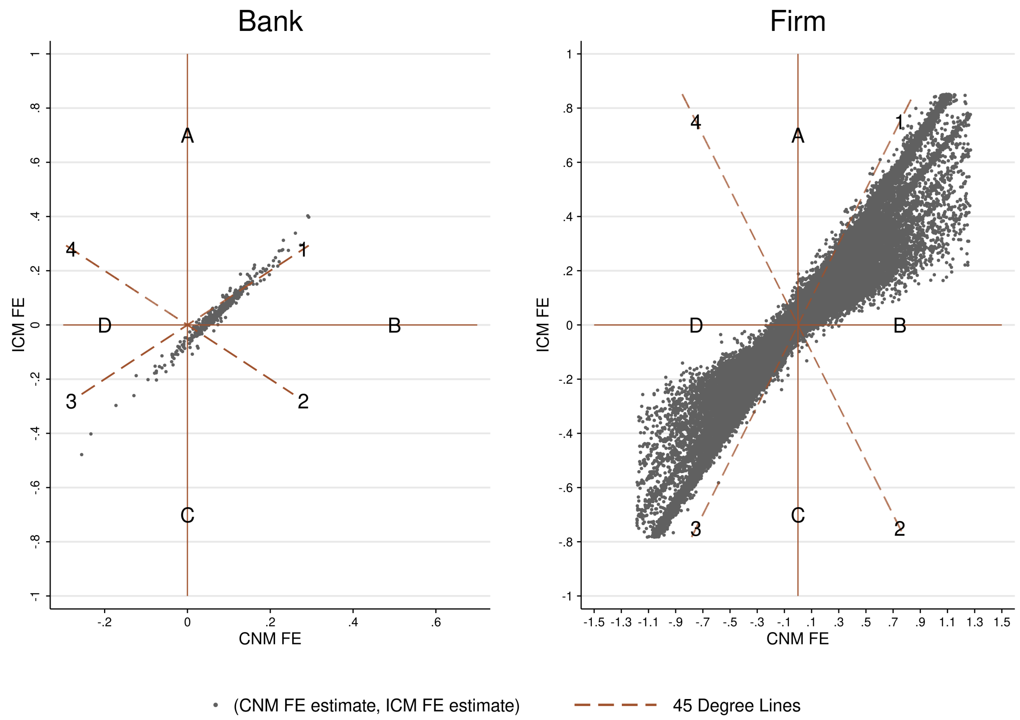


Notes: The Figures compare the distribution of estimated banks (left) and firms (right) fixed effects, obtained from the estimation of the ICM (blue shade) and CNM (red shade) from Equations (22) and (23). Estimates based on pooled data from 2002 to 2008.

We can further deepen our understanding of the ICM-estimated fixed effects bias by plotting, for each firm and bank, the mean over the years of the ICM and CNM estimates. We do so in Figure 13, where we report banks' idiosyncratic shock estimates on the left and firms' idiosyncratic shock estimates on the right. We display CNM estimates on the x-axis and ICM estimates on the y-axis, and divide each plot into eight areas, obtained by combining the regions above and below the two 45-degree lines (indexed clock-wise by capital letters from A to D) and the four quadrants of the plane (numbered clock-wise from one to four). Points in regions 2 and 4 will be misclassified, respectively, with positive shocks turning negative and negative shocks turning positive. Points in areas A1 and C3 (B1 and D3) will be overestimated (underestimated) in magnitude.

On the left, we see the depiction of 586 bank fixed effect estimates. Among these, the ICM would, on average, introduce a negative bias in the estimation of the bank-level idiosyncratic credit shifts. However, there is more. Looking at the density of observations in the eight sections of the plane, we see that the ICM would indeed underestimate most positive supply shifts (B1, 451 of 541 cases); however, it crucially turns 75 positive supply shocks negative, and in a large number of such cases, it overestimates magnitudes – see C2 density, as well as region C3, indicating the overestimation on average of all negative shocks. The misclassified estimates are 13 percent of the total. On the right, we display the 205,799 firm fixed effect estimates. The ICM underestimates the magnitude of most of them (186,233 points lie in B1, B2, D3, D4), switching signs for approximately 10,000 estimates. Although this occurs less frequently than in the case of banks, it still accounts for five percent of the total.

Figure 13: The Empirical Distribution of Fixed Effects Estimates' Bias



Notes: The Figure displays the scatterplots of the fixed effects' estimates in the CNM (x-axis) and ICM (y-axis) space, averaging estimates over the years 2002-2008 for each firm and bank. The dashed lines are the 45-degree lines in the plane, the solid ones the axes crossing zero. Quadrants between the 45-degree lines are identified by letters, quadrants between the axes by numbers, both in clockwise order. Points in the areas A1, A4 and C2, C3 indicate overestimation of the magnitude of the true parameter by the ICM; points in the areas B1, B2 and D3, D4 an underestimation of the magnitude. Points in quadrant 2 and 4 indicate cases in which the bias induces a sign error in the estimation of the idiosyncratic demand or supply component. The plot omits the top and bottom 0,5 percent of estimates for ease of display.

In conclusion, the evidence we gather points to the fact that ICM fixed effects estimates cannot be interpreted as primitive heterogeneity but rather as weighted averages of equilibrium responses to shocks.[48] Therefore, to mitigate general equilibrium contamination from spillover effects and cleanly identify the impact of monetary policy and other shocks, it is essential to account for cross-elasticities.

[48]Node-level analysis of the centrality-bias relationship was precluded by the computational complexity of measuring centrality in our large-scale credit network. However, it is reasonable to expect an outcome similar to those demonstrated by our analytical derivations and numerical exercises above.

How Credit Cross-Elasticities Vary Through the Business Cycle. Pooling the years 2002-2008 to estimate the parameters in Table 4 may mask significant heterogeneity in credit cross-elasticities over time. Analyzing this heterogeneity can reveal the conditions under which banks cut or expand credit, rather than simply reallocating it, and the circumstances that allow firms to absorb credit shocks through their relationship networks.

In Figure 14, we display point-in-time estimates and confidence intervals for the rescaled $\phi*$ (left)and for the ρ (right) estimates. We obtain each point in the plots by estimating the model in Equation (23) in repeated cross-sections over a wide time span from 2002 to 2022. In each plot, we underlay shaded areas highlighting Italian recessions based on OECD indicators (sourced from Federal Reserve Economic Data, 2024).[49]

Two facts stand out. First, the different credit relationships of the same bank are most often complementary, suggesting that the doubling of amounts granted on 4,400 relationships of the same bank would imply a ten percent to null crowd-in for all others, except for 2014, when significant substitution across customers appears to be at play. Interestingly, this coincides with the introduction of Basel III, causing system-wide changes in the risk-weighting rules determining how much regulatory capital buffers banks must hold for each exposure.[50]

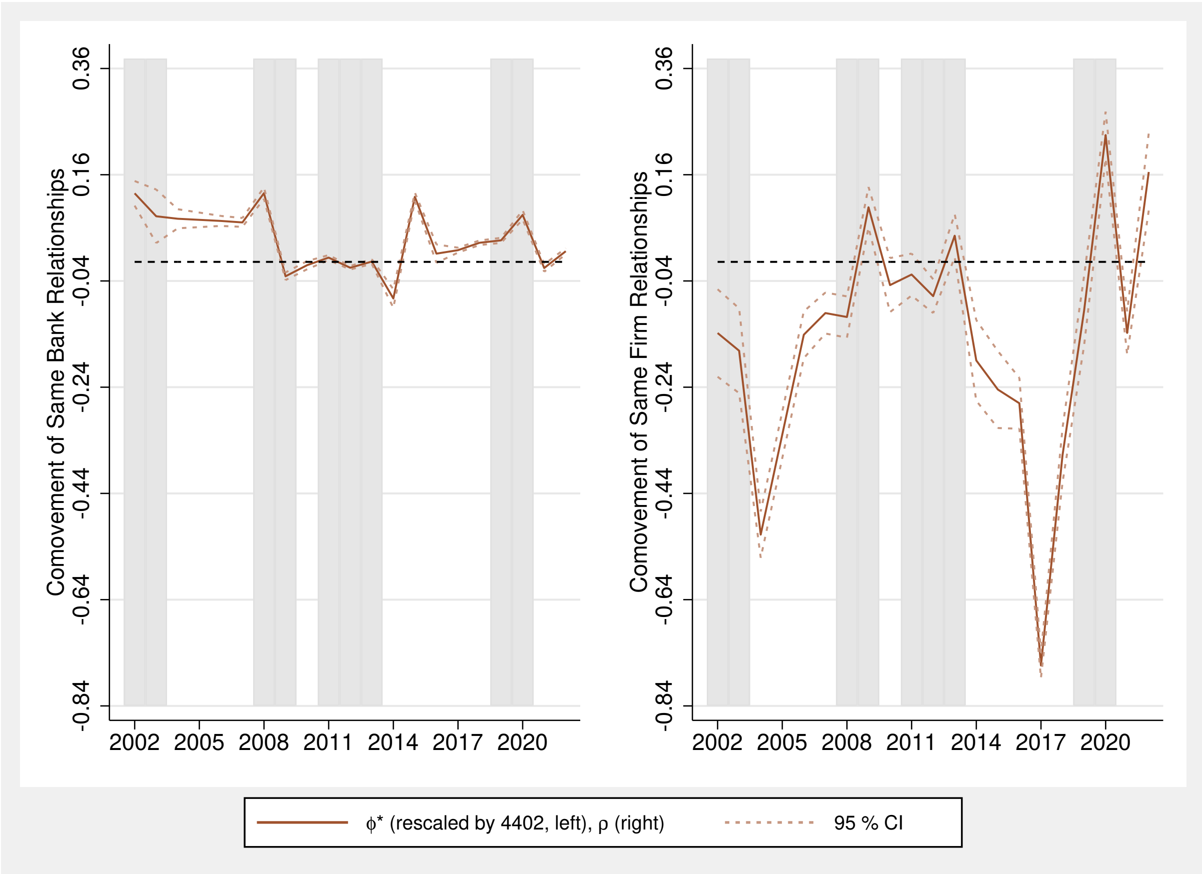
Second, the cross-elasticity between different credit relationships of the same firm shifts in a significant and intuitive way with the business cycle. In normal times, credit from different banks is substitutable, albeit imperfectly, to a high-degree. Our figures suggest that the average firm is normally able to substitute between 20 and 70 percent of a credit cut, if not more. Nonetheless, when the conjuncture worsens, such ability quickly fades. The two most severe crises we cover (2007-09, 2011-13), as well as the COVID-19 crisis, are characterized by emerging credit complementarities at the firm level, possibly reflecting simultaneous contractions across credit relationships by most affected firms and a dash for liquidity from the firms still able to access funding (Acharya and Steffen, 2020).[51]

[49]To support the conclusion that our estimates of ϕ and ρ capture the underlying banks and firms' cross-elasticities of credit across the business cycle, we document in Appendix Figures **??** and **??** how our results are practically unchanged if we change risk proxies, instrument for the overnight rate change with the Altavilla et al. (2019)'s monetary policy surprises. The results are remarkably stable even when we change the treatment of interest altogether. This last piece of evidence is of special importance. Indeed, if we can actually interpret ϕ and ρ estimates structurally, they must not be excessively context dependent. Changing the main treatment variable alters the set of relationships one substitutes from or complements to, but not whether credit is substitutable or complementary at the firm and bank level.

[50]Changing risk-weights implied sharp changes in the relative cost of lending to different customers (Bonaccorsi di Patti, Moscatelli, and Pietrosanti, 2023).

[51]The magnitude of COVID-era complementarities being the largest aligns with the fact that firms had access to unprecedented government guaranties to sustain credit access during the crisis; see, e.g., https://www.mef.gov.it/en/inevidenza/Liquidity-Decree-over-400-billion-in-guarantees/.

Figure 14: How Credit Cross-Elasticities Vary Through the Business Cycle



Notes: The Figure displays, on the left, point estimates (solid line) and 95 % confidence intervals (dotted lines) for the bank credit cross-elasticity ϕ, re-scaled by 4,402, equal to the 25th percentile, and about a quarter of the median of the distribution of the number of relationships shown in Table 3. On the right, point estimates and 95 % confidence intervals for the firm-level credit cross-elasticity ρ. Shaded areas indicate Italian economy recessions (OECD based recessions, Federal Reserve Economic Data, 2024). We obtain each point displayed estimating a year by year version of Equation (23).

Such evidence informs our view of the variability of the effects of credit booms and busts, as highlighted by the empirical banking literature. While findings of negative real effects of credit busts are common across different settings and credit contraction episodes (Alfaro, García-Santana, and Moral-Benito, 2021; Cingano, Manaresi, and Sette, 2016; Jiménez et al., 2017), the real effects of expansions are mixed (Bottero et al., 2022; Jiménez et al., 2020). In Figure 14, we see that, while the cross-elasticity of credit supply is relatively stable over twenty years of data, the cross-elasticity of demand varies widely in normal times but consistently exhibits an upward trend during recessions. As credit booms occur in normal times, their effects are determined by the varying contingencies of the credit market, dictating the extent to which firms can profit from the initial positive shock and how this shock will ripple through the market. As credit busts often occur close to or during a recession, their effects are amplified by the sharp rise in the difficulties of substituting the contracting credit supply for firms.

5.4 Robustness Checks

In our first robustness exercise, we verify the stability of our estimates using different sets of controls. The results in the upper panel of Table 6 confirm that our baseline results are not driven by specification choices. Estimated cross-elasticities and treatment effects remain broadly stable when the set of controls is progressively expanded to include the full set of covariates, as well as the lag fraction between credit drawn and granted, which proxies for the degree of credit constraints a firm faces. In particular, both firm- and bank-level cross-elasticities retain their statistical and economic significance, indicating that the interplay between credit reallocation and balance sheet characteristics is not confounded by compositional effects in the underlying data.

In our second exercise, We further address the potential endogeneity of the credit network structure, a concern discussed in Section 3.3.2, by implementing the leave-one-out estimator of Jochmans (2023). This approach mitigates the bias that could arise if unobserved factors jointly influence the formation of credit relationships and the allocation of credit among them. The results obtained with this estimator (lower panel of Table 6) are consistent with the baseline, confirming that network endogeneity does not drive the measured spillovers. As a result, $\hat{\phi}*$ is almost unchanged, while $\hat{\rho}$ attenuation is stronger; however, the statistical and economic significance of the estimate remains unaltered. In conclusion, we can support the interpretation of our parameters as genuine equilibrium cross-elasticities rather than artifacts of correlated matching between firms and banks.

Finally, in our third exercise, we assess the stability of the business-cycle patterns reported in Figure 14 when cross-elasticities are estimated (i) by instrumenting the overnight rate changes with monetary policy surprises from Altavilla et al. (2019) in panel (b); (ii) by substituting the dummy tracking firm riskiness with the lag in the Drawn on Granted fraction on the specific credit line in panel (c) and combining the two in panel (d); (iii) by using a totally different treatment variable, the interaction between the lag of the Revolving on Granted fraction in the credit line and overnight rate changes in panel (e); and (iv) by employing the leaving-own-connections-out procedure, inspired by Jochmans (2023) in panel (f).

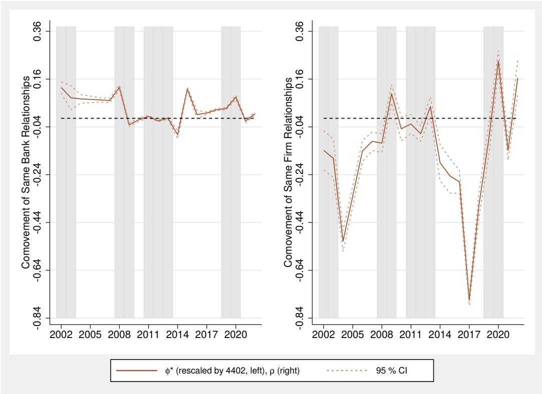
Figure 15 remains virtually unchanged when applying the leave-one-out correction, underscoring the structural nature of the estimated propagation mechanism. The persistence of stronger cross-elasticities of substitution among firms' credit relationships during expansions suggests that the cyclical behavior of credit interdependence reflects genuine shifts in market constraints rather than changes in network composition, subjective choice of the treatment variable, or their potential endogeneity to the business cycle. Overall, these results lend strong credibility to our identification strategy and confirm that the estimated reallocation dynamics capture fundamental features of credit markets rather than econometric artifacts.

Table 6: Robustness for Spillover and Treatment Estimates for the Risk Taking Channel

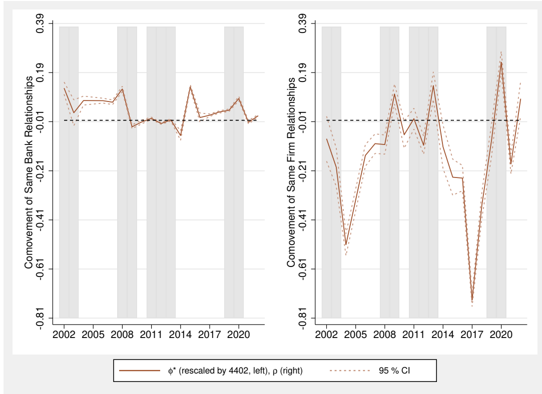
Dep. Var.:					$\Delta \log(C)$					
					CNM Second Stage, Baseline					
	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.
$\hat{\beta}$	0.013	0.0018	0.014	0.0014	0.015	0.0012	0.014	0.0012	0.014	0.0013
$\hat{\phi}*$	0.050	0.0014	0.055	0.011	0.053	0.009	0.055	0.009	0.07	0.0091
$\hat{\rho}$	-0.102	0.019	-0.280	0.011	-0.381	0.009	-0.374	0.008	-0.360	0.009
					First Stage F_{SW}					
	BCC	FCC	BCC	FCC	BCC	FCC	BCC	FCC	BCC	FCC
	79,924	435	240,198	602	193,995	542	141,329	495	25,524	409
					CNM Second Stage, Lag 2, Leave-One-Out					
	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.
$\hat{\beta}$	0.013	0.0017	0.013	0.0016	0.012	0.0015	0.012	0.0015	0.012	0.0015
$\hat{\phi}*$	0.046	0.0031	0.053	0.0029	0.051	0.0026	0.053	0.0026	0.068	0.0026
$\hat{\rho}$	-0.154	0.017	-0.182	0.013	-0.239	0.011	-0.255	0.01	-0.237	0.011
					First Stage F_{SW}					
	BCC	FCC	BCC	FCC	BCC	FCC	BCC	FCC	BCC	FCC
	20,007	482	14,601	521	45,559	495	42,661	461	11,914	375
					Controls					
Granted on Total	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
I(Bank Id. Change)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Revolving on Total			✓	✓	✓	✓	✓	✓	✓	✓
Drawn on Granted					✓	✓	✓	✓	✓	✓
Rel. Age (1999)							✓	✓	✓	✓
Same Province									✓	✓

Notes: The Table reports robustness tests for the results displayed in Table 4. The Table reports estimated coefficients and standard errors for the second stage of model (23) under an array of specifications, increasing the set of controls incrementally. The upper panel of the Table reports results for changing specifications, while still estimating model (23) with the 2SLS estimator introduced in Equation (14). The instrumental variables employed are the $OPIV^1_F$ and $OPIV^1_B$ in equations (11) and (12). The lower panel reports estimates obtained using the second network lag and leaving-own-connections-out procedure, inspired to Jochmans (2023) and outlined in Section 3.3.2, which mitigates concerns due to network endogeneity. The instrumental variables employed are the $LPO - OPIV^2_B$ and $LPO - OPIV^2_F$ in equations (20) and (21). The dependent variable $\Delta \log(C)$ is the yearly log growth of credit granted on each relationship. The treatment associated with $\hat{\beta}$ is the product between the change in the banks' overnight rate, the lag of the log-transformed equity over asset ratio in percentage points and the risky firm indicator variable. $\hat{\rho}$ is the firm-level cross-elasticity estimate, measuring the reaction of credit granted on a certain relationship whenever credit granted increases on other relationships of the same firm. $\hat{\phi}*$ is the bank-level cross-elasticity estimate, measuring the reaction of credit granted on a certain relationship whenever credit granted increases on other relationships of the same bank. $\phi*$ estimate and its errors are multiplied by 4,402. BCC reports the F-test for the regressor of ϕ, FCC reports the F-test for the regressor of ρ.

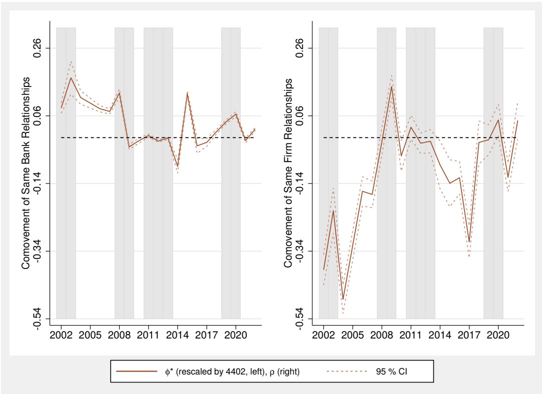
Figure 15: Cross-Elastiticities over the Business Cycle, Robustness.



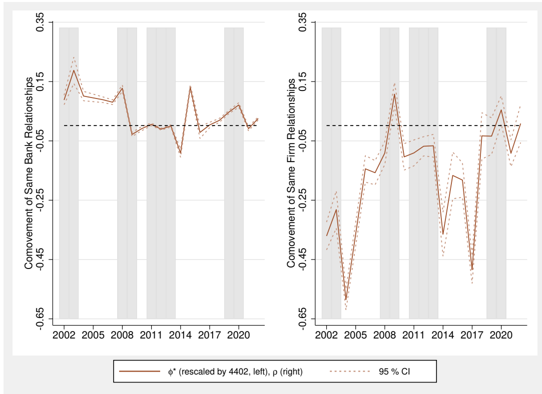
(a) Baseline



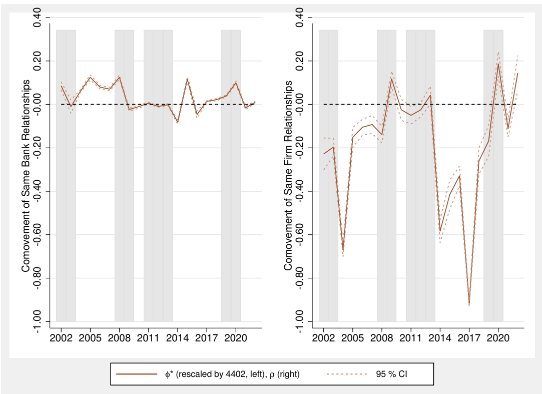
(b) Baseline with Mon. Pol. Surprises



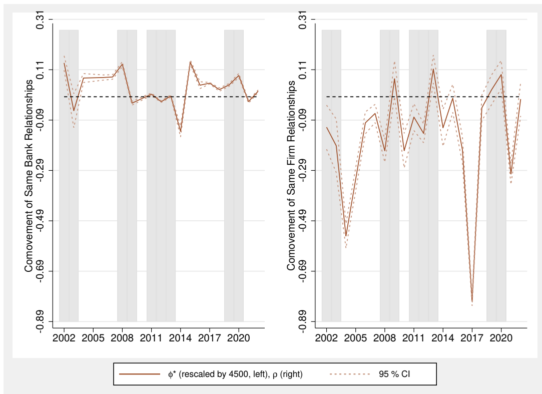
(c) Lag Drawn on Granted in Treatment



(d) Lag Drawn on Granted in Treatment and Mon. Pol. Surprises



(e) Revolving on Granted



(f) Baseline, Lag 2, Leave-One-Out

Notes: The Figure displays robustness for the results displayed in Figure 14. ϕ estimate and its errors are multiplied by 4,402, the 25th percentile and about a quarter of the median of the distribution of the number of relationships per bank ($\phi*$). The first subplot reports the main Figure, for reference purposes; the second plot results that instrument overnight rate changes with monetary policy surprises from Altavilla et al. (2019); the third figure substitutes the dummy tracking firm riskiness with the lag in the Drawn on Granted fraction on the specific credit line; the fourth figure does the same, plus employing Altavilla et al. (2019) instruments for overnight rate changes; the fifth figure uses a totally different treatment, the interaction between the lag of the Revolving on Granted fraction in the credit line and overnight rate changes; the last subplot estimates coefficient in the basic figure with the second network lag and leaving-own-connections-out procedure, inspired to Jochmans (2023) and outlined in Section 3.3.2. Shaded areas indicate Italian economy recessions (OECD based recessions, Federal Reserve Economic Data, 2024). We obtain each point displayed estimating a year by year version of Equation (23).

6 Conclusion

In the last three decades, empirical research has consistently demonstrated that shocks to banks significantly impact firms' credit availability (e.g., Behn, Haselmann, and Wachtel, 2016; Jiménez et al., 2017; Khwaja and Mian, 2008; Paravisini, 2008; Paravisini et al., 2014; Peek and Rosengren, 1995, 2000), subsequently influencing employment and investment decisions (e.g., Amiti and Weinstein, 2018; Chodorow-Reich, 2014; Chodorow-Reich and Falato, 2022; Cingano, Manaresi, and Sette, 2016; Jiménez et al., 2020).

A pivotal tool in achieving these findings has been the within-firm (and often within-bank) estimation strategy, popularized initially by Khwaja and Mian (2008). This tool is crucial due to the non-random matching between banks and firms (see, e.g., Schwert, 2018), necessitating the separation of shocks to banks from the unobservable idiosyncratic shocks affecting their partner firms. The ability to disentangle these two types of shocks is a key initial step in assessing the impact of credit supply contractions and expansions on the real economy.

However, fixed effects have inherent limitations. First, they assume a uniform distribution of idiosyncratic shocks across different credit relationships within the same firm or bank. If the data violates this assumption, residual bias may persist, affecting estimates (as noted in Paravisini, Rappoport, and Schnabl, 2023). In addition, fixed effects cannot address the endogenous reallocation of the demand and supply of credit across relationships after a shock.

In this paper, we demonstrated analytically, through Monte Carlo simulations, and empirically, by replicating standard results in the literature (Jiménez et al., 2014), the relevance of the bias affecting conventional treatment and fixed effects estimates, which are often used to retrieve proxies for idiosyncratic shifts in demand and supply. We introduced a methodology for measuring such reallocation directly, without relying on additional information, such as price data. To do so, we adapted frontier results in network theory and econometrics (Ballester, Calvó-Armengol, and Zenou, 2006; Bramoullé, Djebbari, and Fortin, 2009) to the credit market. Introducing a new overlapping portfolio instrumental variables strategy, we demonstrated how to estimate firm and bank cross-elasticities of credit — i.e., the extent to which firms and banks reallocate credit across their relationships' portfolios. By controlling for such reallocation mechanisms, we provided unbiased treatment and fixed effects estimates, generalizing the standard methodology, having the Khwaja and Mian (2008) approach as a special case when cross-elasticities are forced to be zero. Finally, adapting results from Jochmans (2023), we show that our findings are robust to network endogeneity.

The ability to directly quantify reallocation across relationship portfolios is essential for robustly assessing the effects of shocks and policies that affect credit supply and demand, translating into real effects. For example, how does a low monetary policy rate spur risk-taking by banks, and how much does reallocation across credit relationships alter its effects? How does reallocation intensity vary over the business cycle? Can such variation help explain the heterogeneity in the effects of credit booms and busts documented by the literature? We made progress in answering these questions, showing that cross-elasticities of demand and supply by firms and banks are relevant, could bias standard estimates of the risk-taking channel downward, and can amplify expansions and contractions of credit over the business cycle. Our methodology should aid researchers in addressing similar questions, even in the absence of extremely rich datasets or quasi-experimental settings to account for reallocation. Moreover, the tool we propose could be especially useful for studying the effects of regulatory changes and monetary policy, which are often enacted when reallocation is possible and easy.

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Online Appendix

A.1 Proofs

A.1.1 Model

With the toy model in Section 2.1, we describe the estimation consequences of dealing with the two-firm, two-bank network while ignoring credit relationship interdependence and incorrectly restricting the cross-elasticities in Equations (2) to zero. Here, we present the full analytical details of the toy model, complementing the presentation in Section 2.1.

Bank Problem:

$$
\begin{aligned}
&\text{Bank } b\text{: } \max_{c_{ib}, c_{jb}} \Big(r_{ib} - \omega\big(\tfrac{c_{ib}}{2} + \xi x_{ib} + \theta \tfrac{c_{jb}}{2} + \nu_{ib}\big)\Big) c_{ib} + \Big(r_{jb} - \omega\big(\tfrac{c_{jb}}{2} + \xi x_{jb} + \theta \tfrac{c_{ib}}{2} + \nu_{jb}\big)\Big) c_{jb} \\
&\text{Bank } a\text{: } \max_{c_{ia}} \Big(r_{ia} - \omega\big(\tfrac{c_{ia}}{2} + \xi x_{ia} + \nu_{ia}\big)\Big) c_{ia} \\
&\text{FOC deliver:} \\
&r_{ib} = \omega c_{ib} + \omega \underbrace{(\xi x_{ib} + \nu_{ib} + \theta c_{jb})}_{u_{ib}} \\
&r_{jb} = \omega c_{jb} + \omega \underbrace{(\xi x_{jb} + \nu_{jb} + \theta c_{ib})}_{u_{jb}} \\
&r_{ia} = \omega c_{ia} + \omega \underbrace{(\xi x_{ia} + \nu_{ia})}_{u_{ia}}
\end{aligned}
\tag{A.1}
$$

Firm problem:

Firm i: $\max_{c_{ia},c_{ib}} \big(e_i - \alpha(c_{ia}+c_{ib})\big)(c_{ia}+c_{ib}) - \sum_{K=a,b} c_{iK}\omega(c_{iK}+u_{iK})$
Firm j: $\max_{c_{jb}} \big(e_j - \alpha c_{jb}\big)c_{jb} - c_{jb}\omega(c_{jb}+u_{jb})$

FOC deliver:
$e_i - 2\alpha c_{ia} - 2\alpha c_{ib} - 2\omega c_{ia} - \omega(\xi x_{ia} + \nu_{ia}) = 0$
$e_i - 2\alpha c_{ib} - 2\alpha c_{ia} - 2\omega c_{ib} - \omega(\xi x_{ib} + \nu_{ib} + \theta c_{jb}) = 0$
$e_j - 2\alpha c_{jb} - 2\omega c_{jb} - \omega(\xi x_{jb} + \nu_{jb} + \theta c_{ib}) = 0$

Which simplifies to:
$c_{ia} = -\frac{\alpha}{\alpha+\omega}c_{ib} + \frac{1}{2(\alpha+\omega)}e_i - \frac{\omega}{2(\alpha+\omega)}(\xi x_{ia} + \nu_{ia})$
$c_{ib} = -\frac{\alpha}{\alpha+\omega}c_{ia} + \frac{1}{2(\alpha+\omega)}e_i - \frac{\omega}{2(\alpha+\omega)}(\xi x_{ib} + \nu_{ib} + \theta c_{jb})$
$c_{jb} = \frac{1}{2(\alpha+\omega)}e_j - \frac{\omega}{2(\alpha+\omega)}(\xi x_{jb} + \nu_{jb} + \theta c_{ib})$

And delivers the following structural demand system:
$c_{ia} = \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia}$
$c_{ib} = \rho c_{ia} + \phi c_{jb} + \beta x_{ib} + \delta_i + \epsilon_{ib}$
$c_{jb} = \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb}$

With:
$\rho = -\frac{\alpha}{\alpha+\omega}$
$\phi = -\frac{\theta\omega}{2(\alpha+\omega)}$
$\beta = -\frac{\xi\omega}{2(\alpha+\omega)}$
$\delta_{i,j} = \frac{1}{2(\alpha+\omega)}e_{i,j}$
$\epsilon_{ia,ib,jb} = -\frac{\omega\nu_{ia,ib,jb}}{2(\alpha+\omega)}$

From the above, we can see that the OLS estimator, when ignoring the network structure, will be biased. To ease the derivations further, we add Assumption 3.

Assumption 3. *Treatment only hits relationship ia ($x_{ib}, x_{jb} = 0$), firm j experiences no correlated idiosyncratic demand shock ($\delta_j = 0$).*

Proposition A.6. *Under Assumptions 1, 2, and 3, the estimator of β obtained from applying OLS to the following system*

$$\begin{aligned} c_{ia} &= \beta x_{ia} + \ddot{\varepsilon}_{ia}, \\ c_{ib} &= \beta x_{ib} + \ddot{\varepsilon}_{ib}, \\ c_{jb} &= \beta x_{jb} + \ddot{\varepsilon}_{jb}, \end{aligned} \tag{A.2}$$

will be biased. The bias can be expressed as

$$\hat{\beta}_{OLS} = \beta + \rho\frac{cov(x_{ia}, c_{ib})}{var(x_{ia})} + \frac{cov(x_{ia}, \delta_i)}{var(x_{ia})}. \tag{A.3}$$

Proof of Proposition A.6.

From structural System A.1.1, we can derive the reduced form system as follows:

$$\begin{aligned}
&c_{ia} = \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia} \\
&c_{ib} = \rho(\rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia})... \\
&+\phi(\phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb}) + \beta x_{ib} + \delta_i + \epsilon_{ib} \\
&c_{jb} = \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb} \\
\\
&c_{ia} = \rho c_{ib} + \beta x_{ia} + \delta_i + \epsilon_{ia} \\
&(1-\rho^2-\phi^2)c_{ib} = \beta(\rho x_{ia} + x_{ib} + \phi x_{jb})... \\
&+(1+\rho)\delta_i + \phi\delta_j + \rho\epsilon_{ia} + \phi\epsilon_{jb} + \epsilon_{ib} \\
&c_{jb} = \phi c_{ib} + \beta x_{jb} + \delta_j + \epsilon_{jb} \\
\\
&c_{ia} = \rho\big(\tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}\big)... \\
&+\beta x_{ia} + \delta_i + \epsilon_{ia} \\
&c_{ib} = \tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\
&c_{jb} = \phi\big(\tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}\big)... \\
&+\beta x_{jb} + \delta_j + \epsilon_{jb} \\
\\
&c_{ia} = \tfrac{\rho(1+\rho-\phi^2)\delta_i+\rho\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{(1-\phi^2)x_{ia}+\rho\phi x_{jb}+\rho x_{ib}}{1-\phi^2-\rho^2} + \tfrac{(1-\phi^2)\epsilon_{ia}+\rho\phi\epsilon_{jb}+\rho\epsilon_{ib}}{1-\phi^2-\rho^2} \\
&c_{ib} = \tfrac{(1+\rho)\delta_i+\phi\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho x_{ia}+\phi x_{jb}+x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\
&c_{jb} = \tfrac{\phi(1+\rho)\delta_i+(1-\rho^2)\delta_j}{1-\phi^2-\rho^2} + \beta\tfrac{\rho\phi x_{ia}+(1-\rho^2)x_{jb}+\phi x_{ib}}{1-\phi^2-\rho^2} + \tfrac{\phi\rho\epsilon_{ia}+(1-\rho^2)\epsilon_{jb}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2}
\end{aligned} \tag{A.4}$$

exploiting Assumption 3, we obtain:

$$\begin{aligned}
&c_{ia} = \beta x_{ia} + \ddot{\varepsilon}_{ia} \\
&\ddot{\varepsilon}_{ia} = \delta_i + \rho c_{ib} + \epsilon_{ia} \\
&c_{ib} = \tfrac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \beta\tfrac{\rho}{1-\phi^2-\rho^2}x_{ia} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}
\end{aligned} \tag{A.5}$$

which results in

$$\hat{\beta}_{\text{OLS}} = \frac{\text{cov}(c_{ia}, x_{ia})}{\text{var}(x_{ia})} = \beta + \rho\underbrace{\frac{\text{cov}(x_{ia}, c_{ib})}{\text{var}(x_{ia})}}_{\text{spillover bias}} + \underbrace{\frac{\text{cov}(x_{ia}, \delta_i)}{\text{var}(x_{ia})}}_{\text{demand bias}} \tag{A.6}$$

The fact that $\frac{\text{cov}(x_{ia}, c_{ib})}{\text{var}(x_{ia})} = \frac{1+\rho}{1-\rho^2-\phi^2}\frac{\text{cov}(x_{ia}, \delta_i)}{\text{var}(x_{ia})} + \beta\frac{\rho}{1-\rho^2-\phi^2} \neq 0$ concludes the proof.

■

Below, we re-state Proposition 1 with further details, adding Assumption 3 to ease calculations, and provide its proof.

Proposition A.7. *Under Assumptions 1, 2 and 3, the estimator of the shift in banks' supply curve, β, for Equations (3), is biased*

$$\begin{aligned}\hat{\beta}_{FE} &= \frac{cov(c_{ia}-\bar{c}_i, x_{ia}-\bar{x}_i)}{var(x_{ia}-\bar{x}_i)} \\ &= \beta + \rho(1-\rho)\frac{cov(c_{ib}, x_{ia})}{var(x_{ia})} - \rho\left(\beta + \frac{cov(\delta_i, x_{ia})}{var(x_{ia})}\right) - \phi\frac{cov(c_{jb}, x_{ia})}{var(x_{ia})}.\end{aligned} \tag{A.7}$$

Proof of Proposition 1.

Indicating averages with bars, so that, for example, $\bar{c}_i = \frac{c_{ia}+c_{ib}}{2}$, we have that:

$$\begin{aligned}&c_{ia} = \beta x_{ia} + \delta_i + \varepsilon_{ia}, \\ &c_{ib} = \delta_i + \varepsilon_{ib}. \\ \\ &\Rightarrow \hat{\delta}_i = c_{ib} \\ &\hat{\beta}_{FE} = \frac{\text{cov}(c_{ia}-\bar{c}_i, x_{ia}-\bar{x}_i)}{\text{var}(x_{ia}-\bar{x}_i)} = \frac{\text{cov}\left(\frac{c_{ia}-c_{ib}}{2}, \frac{x_{ia}}{2}\right)}{\text{var}\left(\frac{x_{ia}}{2}\right)} = \frac{\text{cov}(c_{ia}-c_{ib}, x_{ia})}{\text{var}(x_{ia})} = ... \\ &... \frac{\text{cov}(\beta x_{ia}+\varepsilon_{ia}-\varepsilon_{ib}, x_{ia})}{\text{var}(x_{ia})} \\ \\ &\text{From the structural demand system (Equations 2):} \\ &\varepsilon_{ia} = \rho c_{ib} + \epsilon_{ia} \\ &\varepsilon_{ib} = \rho c_{ia} + \phi c_{jb} + \epsilon_{ib} \\ \\ &\Rightarrow \frac{\text{cov}(\beta x_{ia}+\varepsilon_{ia}-\varepsilon_{ib}, x_{ia})}{\text{var}(x_{ia})} = \beta + \frac{\text{cov}(\rho(c_{ib}-c_{ia})-\phi c_{jb}, x_{ia})}{\text{var}(x_{ia})} = ... \\ &...\beta + \frac{\text{cov}(\rho((1-\rho)c_{ib}-\beta x_{ia}-\delta_i)-\phi c_{jb}, x_{ia})}{\text{var}(x_{ia})} = ... \\ &...\beta(1-\rho) + \rho(1-\rho)\frac{\text{cov}(c_{ib}, x_{ia})}{\text{var}(x_{ia})} - \rho\frac{\text{cov}(\delta_i, x_{ia})}{\text{var}(x_{ia})} - \phi\frac{\text{cov}(c_{jb}, x_{ia})}{\text{var}(x_{ia})} \\ \\ &\text{From the reduced form system, simplified thanks to Assumption 3:} \\ &c_{ib} = \frac{(1+\rho)\delta_i}{1-\phi^2-\rho^2} + \beta\frac{\rho x_{ia}}{1-\phi^2-\rho^2} + \frac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\ &c_{jb} = \frac{\phi(1+\rho)\delta_i}{1-\phi^2-\rho^2} + \beta\frac{\rho\phi x_{ia}}{1-\phi^2-\rho^2} + \frac{\phi\rho\epsilon_{ia}+(1-\rho^2)\epsilon_{jb}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2} \\ &\hat{\beta}_{FE} = \frac{\text{cov}(c_{ia}-\bar{c}_i, x_{ia}-\bar{x}_i)}{\text{var}(x_{ia}-\bar{x}_i)} = \frac{\text{cov}(c_{ia}-c_{ib}, x_{ia})}{\text{var}(x_{ia})} = \frac{\text{cov}(\beta x_{ia}+\varepsilon_{ia}-\varepsilon_{ib}, x_{ia})}{\text{var}(x_{ia})} = ... \\ &...\beta + \frac{\text{cov}(\rho((1-\rho)c_{ib}-\beta x_{ia}-\delta_i)-\phi c_{jb}, x_{ia})}{\text{var}(x_{ia})} = ... \\ &...\beta - \rho\beta + \rho(1-\rho)\frac{\text{cov}(c_{ib}, x_{ia})}{\text{var}(x_{ia})} - \phi\frac{\text{cov}(c_{jb}, x_{ia})}{\text{var}(x_{ia})} - \rho\frac{\text{cov}(\delta_i, x_{ia})}{\text{var}(x_{ia})}\end{aligned} \tag{A.8}$$

From the above, and the reduced form of System 2 displayed in the last passage, it is evident that β_{FE} is biased, and that correlated demand shocks still play a role, as they are reflected back in the estimator through reallocation spillovers. β_{FE} is indeed a function of δ_i through the $-\rho\text{cov}(\delta_i, x_{ia})/\text{var}(x_{ia})$ element, from demand reallocation within the relationships of the same firm, and through the impact of δ_i on *all* other bias components.

■

Combining Propositions 1 and A.6, we can see that

Proposition A.8. *Under Assumptions 1 and 2, $\hat{\beta}_{FE} \neq \hat{\beta}_{OLS}$ is possible even in the absence of demand bias.*

Proof of Proposition 2. Using Assumption 3 and the absence of demand bias to simplify our calculation, we can thus express the reduced form for c_{ib}, c_{jb} in Equations (2):

$$
\begin{aligned}
c_{ib} &= \beta\tfrac{\rho x_{ia}}{1-\phi^2-\rho^2} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2}\\
c_{jb} &= \beta\tfrac{\rho\phi x_{ia}}{1-\phi^2-\rho^2} + \tfrac{\phi\rho\epsilon_{ia}+(1-\rho^2)\epsilon_{jb}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2}
\end{aligned}
$$

From the end of Proposition A.6's Proof and the absence of demand bias it follows that:

$$\hat{\beta}_{\mathrm{OLS}} = \beta\Big(1 + \tfrac{\rho^2}{1-\phi^2-\rho^2}\Big) = \beta\tfrac{1-\phi^2}{1-\rho^2-\phi^2}$$

Using the bias expression in Proposition 1, the fixed effect estimator is:

$$
\begin{aligned}
&\hat{\beta}_{FE} = \beta(1-\rho) + \rho(1-\rho)\tfrac{\mathrm{cov}(c_{ib},x_{ia})}{\mathrm{var}(x_{ia})} - \phi\tfrac{\mathrm{cov}(c_{jb},x_{ia})}{\mathrm{var}(x_{ia})} = \ldots\\
&\beta(1-\rho) + \beta(1-\rho)\tfrac{\rho^2}{1-\phi^2-\rho^2} - \beta\tfrac{\phi^2\rho}{1-\phi^2-\rho^2} = \ldots\\
&\beta\tfrac{(1-\rho)(1-\phi^2-\rho^2)+\rho^2(1-\rho)-\phi^2\rho}{1-\phi^2-\rho^2} = \beta\tfrac{1-\rho-\phi^2+\rho\phi^2-\rho^2+\rho^3+\rho^2-\rho^3-\phi^2\rho}{1-\phi^2-\rho^2} = \beta\tfrac{1-\phi^2+\rho}{1-\phi^2-\rho^2}
\end{aligned}
\tag{A.9}
$$

the two estimators are different except for specific values of the reduced form parameters.

■

Proposition A.9. *Under Assumptions 1 and 2, firm fixed effects'estimates contain supply shock spillovers, and bank fixed effects' estimates may contain demand shock spillovers. As such, they cannot be regarded as pure measures of each firm or bank demand and supply shocks, respectively.*

Proof of Proposition 3. We start by considering an alternative version of Assumption 3, which allows for $\delta_j \neq 0$. Then, we notice that if the econometrician tries to estimate Equations 3, she obtains:

$$
\begin{aligned}
\hat{\delta}_i &= \tfrac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \tfrac{\phi}{1-\phi^2-\rho^2}\delta_j\\
\hat{\delta}_j &= \tfrac{\phi(1+\rho)}{1-\phi^2-\rho^2}\delta_i + \tfrac{(1-\rho^2)}{1-\phi^2-\rho^2}\delta_j
\end{aligned}
\tag{A.10}
$$

from the reduced form of Equations 2 displayed as the last derivation in Equations A.4.

■

Even in this simple setting, where we focus on firm fixed effects only, we can see that the fixed effect estimates in Equations A.10 are already affected by two problems. First, fixed effects do not exactly capture the pure demand shock but rather an amplified or attenuated version of it (based on the signs and relative magnitudes of ϕ, ρ). Focusing on $\hat{\delta}_i$, this is exemplified by the $\frac{(1+\rho)}{1-\phi^2-\rho^2}\delta_i$ element. Second, bank links reflect other firms' shocks in the estimate, further biasing the fixed-effect estimates. Focusing again on $\hat{\delta}_i$, this is exemplified by the $\frac{\phi}{1-\phi^2-\rho^2}\delta_j$ element.

A.1.2 On the economic interpretation of the ϕ and ρ estimates

To interpret positive ϕ or ρ estimates as complementarity and negative ones as substitutability in our framework, it is sufficient (but not necessary) that the following two assumptions hold—conditionally on the observed relationships and their observable characteristics:

Assumption 4. *The demand for credit of each firm from each bank is all-else-equal downward sloping in the interest rate.*

Assumption 5. *The supply of credit of each bank towards each firm is all-else-equal upward sloping in the interest rate.*

Before formally discussing their implications, we stress that these assumptions are reasonable for the data at hand (see, e.g. Bripi, 2023; Burlon et al., 2016).[52]

Starting from the demand for credit, the parameter α describes the nature of the return to scale in credit in our model and determines whether different sources of credit are complements or substitutes for the firm. Consider the margin of the project for firm i, $\Big(e_i - \alpha(c_{ia}+c_{ib})\Big)(c_{ia}+c_{ib})$, where the product element between c_{ia}, c_{ib} is multiplied by $-\alpha$. A positive α implies substitutability, while a negative one indicates complementarity.[53]

From the model, it follows that

$$\rho = -\frac{\alpha}{\alpha+\omega}, \tag{A.11}$$

To interpret the sign of ρ structurally as complementarity or substitutability in the demand for credit, we need a positive denominator that does not confound the sign of α. In the case of substitutability ($\alpha > 0$), $\omega > 0$ is a sufficient condition. In the case of complementarity ($\alpha < 0$), it is necessary that $\omega > 0$ and $|\omega| > |\alpha|$.

Moving to the supply of credit, as long as $\omega > 0$ the parameter θ directly captures complementarity or substitutability in supplying credit to different borrowers. Consider the margin on the loan to firm i; $r_{ib} - \omega\Big(\frac{c_{ib}}{2} + \xi x_{ib} + \theta\frac{c_{jb}}{2} + \nu_{ib}\Big)c_{ib}$, the product element between c_{ib}, c_{jb} enters multiplied by $-\omega\theta$. A positive θ implies substitutability, while a negative one implies complementarity.

[52]The paper by Burlon and coauthors estimates a credit demand system to study credit rationing, while the one by Bripi studies the interest rate elasticity of credit demand using the same data sources we access. The latter work finds evidence of negative elasticity of substitution across sectors and local credit markets, while the former directly estimates an upward sloping supply and a downward sloping demand for credit.

[53]In terms of economic logic, $\alpha < 0$ implies either scenarios in which a firm has something extremely profitable to undertake or cases in which, to keep afloat, every source of funding is strictly necessary.

From the model, it follows that

$$\phi = -\frac{\omega\theta}{2(\alpha+\omega)}, \tag{A.12}$$

and to interpret the sign of ϕ structurally as complementarity or substitutability in the supply of credit, we again need a positive denominator. In this case, though, it is enough to have $\alpha + \omega > 0$, which implies $\alpha > 0$, $\omega > 0$, $\alpha < 0$, and $|\omega| > |\alpha|$.

We can see that, as long as we believe that the crossing of downward sloping demand and an upward sloping supply of credit generates the data we observe, our estimates respect the interpretability conditions derived above. Upward sloping supply is possible if and only if offering more credit raises costs for the bank, which maps to $\omega > 0$. To see that downward sloping demand $\Rightarrow |\alpha| < \omega$, let's derive again the firm's i demand for c_{ib}, now keeping the dependence on rates explicit:

Firm i: $\max\limits_{c_{ia},\, c_{ib}} \big(e_i - \alpha(c_{ia} + c_{ib})\big)(c_{ia} + c_{ib}) - \sum_{K=a,b} c_{iK} r(c_{iK}, u_{iK})$

FOC for ib deliver:
$e_i - 2\alpha c_{ib} - 2\alpha c_{ia} - r(c_{ib}, u_{ib}) - c_{ib} r'(c_{ib}, u_{ib}) = 0$

Which, given that the FOC in the bank's problem A.1 implies that $\partial r(c_{ib}, u_{ib})/\partial c_{ib} = \omega$, simplifies to:

$$e_i - 2\alpha c_{ib} - 2\alpha c_{ia} - r(c_{ib}, u_{ib}) - \omega c_{ib} = 0 \Rightarrow c_{ib} = \frac{e_i}{2\alpha+\omega} - \frac{2\alpha}{2\alpha+\omega} c_{ia} - \frac{r(c_{ib},u_{ib})}{2\alpha+\omega} \tag{A.13}$$

and the demand for credit is downward sloping in rates iff $2\alpha + \omega > 0$, which is true when $\alpha < 0$ iff $|\alpha| < \frac{\omega}{2} \Rightarrow |\alpha| < \omega$.

A.1.3 The econometric framework

Proof of Proposition 4. It follows from the proof of Proposition 1 in Arduini, Patacchini, and Rainone (2020) when G, the network among nodes, and its sub-matrices are replaced by A and its sub-matrices, the network among links. Moving from nodes to links implies that quadriads instead of triads intransitivity is needed. Quadriads intransitivity is implied by linear independence of I_F, $A_B A_F A_B$ and A_F and I_B, $A_F A_B A_F$ and A_B. See condition 1 of the proposition. Alternatively, also the proof of Proposition 1 in Rainone (2020) brings to the same result if multiple endogenous terms are considered.
■

Proof of Proposition 5.

Let us abstract from the presence of fixed effect and let the DGP be

$$C \;=\; \phi A_B C + \rho A_F C + X\beta + \epsilon, \tag{A.14}$$

For simplicity, let also $\rho = \phi$. Then the ICM is

$$C = X\beta + U, \tag{A.15}$$

and the error term has the following form

$$\begin{aligned} U &= \phi A_B C + \rho A_F C + \epsilon = (\phi A_B + \rho A_F)(I - \phi A_B - \rho A_F)^{-1}[X\beta + \epsilon] + \epsilon \\ &= MX\beta + (M+I)\epsilon. \end{aligned} \tag{A.16}$$

Suppose that X is univariate and $X \perp \epsilon$,[54] then we have

$$\begin{aligned} X'U &= X'MX\beta + X'(M+I)\epsilon = X'\phi A \sum_{k=0}^{\inf} (\phi A)^k X\beta \\ &= X' \sum_{k=1}^{\inf} (\phi A)^k X\beta = \beta \sum_{k=1}^{\inf} \phi^k X' A^k X = S. \end{aligned} \tag{A.17}$$

If $\beta, \phi > 0$ then $B = \hat{\beta} - \beta = (X'X)^{-1}S > 0$. The positive bias is given by the amplification generated by spillovers, which is not disentangled in the reduced form estimate. If $\phi < 0$ then the sign of B depends on A, i.e. the network structure, and the intensity of the spillovers, as it contains decaying functions of ϕ.

Suppose X is binary. For example, a stark change in regulation hits only part of our credit relationships population. Then, for each treated relationship, we call $X'A^kX = p_k$ the sum of the number of $k-$ distant treated credit relationships. It follows that

$$S = \beta \sum_{k=1}^{\inf} \phi^k p_k = \beta\left(\sum_{k\ odd} \phi^k p_k + \sum_{k\ even} \phi^k p_k\right) = \beta(OD + EV). \tag{A.18}$$

Given that OD is negative and EV is positive, $B > 0$ if $OD > -EV$.

■

Here we include, for completeness, a display of how our identification argument works in the example we introduced in Section 2.1. We work on the most basic form of the toy model, where there is no correlated demand confounder $\left(\frac{\text{cov}(x_{ia},\delta_i)}{\text{var}(x_{ia})} = 0\right)$, and the following assumption holds.

Assumption 6. *x_{jb} and x_{ia} do not affect directly c_{ib} and are uncorrelated with ϵ_{ia} and ϵ_{jb}.*

If Assumption 6 holds, then the proposition below follows.

[54] To ease the notation we assume independence here, the same conclusions can be reached assuming $E[\epsilon' X] = 0$.

Proposition A.10. *Under Assumptions 1, 2, 6, and the network structure in Section 2.1, we can identify the spillover parameters (ϕ and ρ) using a 2SLS procedure and deliver an unbiased estimate of β.*

Proof. We rewrite the Equations in (2) in light of Assumptions 1, 2, and 6

$$\begin{aligned} c_{ia} &= \rho c_{ib} + \beta x_{ia} + \epsilon_{ia} \\ c_{ib} &= \rho c_{ia} + \phi c_{jb} + \epsilon_{ib} \\ c_{jb} &= \phi c_{ib} + \beta x_{jb} + \epsilon_{jb} \end{aligned} \tag{A.19}$$

These can be rearranged as

$$\begin{aligned} c_{ia} &= \rho c_{ib} + \beta x_{ia} + \epsilon_{ia} \\ c_{ib} &= \rho(\rho c_{ib} + \beta x_{ia} + \epsilon_{ia}) + \phi(\phi c_{ib} + \beta x_{jb} + \epsilon_{jb}) + \epsilon_{ib} \\ c_{jb} &= \phi c_{ib} + \beta x_{jb} + \epsilon_{jb} \end{aligned}$$

$$\begin{aligned} c_{ia} &= \left(\tfrac{\rho^2\beta}{1-\phi^2-\rho^2} + \beta\right)x_{ia} + \rho\tfrac{\phi\beta}{1-\phi^2-\rho^2}x_{jb} + \tfrac{(1-\phi^2)\epsilon_{ia}+\rho\phi\epsilon_{jb}+\rho\epsilon_{ib}}{1-\phi^2-\rho^2} \\ c_{ib} &= \tfrac{\rho\beta}{1-\phi^2-\rho^2}x_{ia} + \tfrac{\phi\beta}{1-\phi^2-\rho^2}x_{jb} + \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\ c_{jb} &= \left(\tfrac{\phi^2\beta}{1-\phi^2-\rho^2} + \beta\right)x_{jb} + \tfrac{\phi\rho\beta}{1-\phi^2-\rho^2}x_{ia} + \tfrac{(1-\rho^2)\epsilon_{jb}+\phi\rho\epsilon_{ia}+\phi\epsilon_{ib}}{1-\phi^2-\rho^2} \end{aligned} \tag{A.20}$$

$$\begin{aligned} \text{call } \mu &= \tfrac{\rho\epsilon_{ia}+\phi\epsilon_{jb}+\epsilon_{ib}}{1-\phi^2-\rho^2} \\ \text{and } \pi_\rho &= \tfrac{\beta\phi}{1-\phi^2-\rho^2} \\ \text{and } \pi_\phi &= \tfrac{\beta\rho}{1-\phi^2-\rho^2} \end{aligned}$$

The result of the above rearrangement is

$$\begin{aligned} c_{ia} &= \rho\pi_\rho x_{jb} + \left(\beta + \tfrac{\beta\rho^2}{1-\phi^2-\rho^2}\right)x_{ia} + \rho\mu + \epsilon_{ia} \\ c_{ib} &= \pi_\rho x_{jb} + \pi_\phi x_{ia} + \mu + \epsilon_{ib} \\ c_{jb} &= \phi\pi_\phi x_{ia} + \left(\tfrac{\phi^2\beta}{1-\phi^2-\rho^2} + \beta\right)x_{jb} + \phi\mu + \epsilon_{jb} \end{aligned} \tag{A.21}$$

where π_ρ and π_ϕ are the reduced form parameters for the instruments' effect on c_{ib}. μ summarizes all other parameters and variables we are not interested in. We use the c_{ib} equation from the Equations in (A.21) as the first stage, recovering $\hat{\pi}_{\rho,OLS}, \hat{\pi}_{\phi,OLS}$, which we then use to deflate $\widehat{\pi_\rho\rho}_{OLS}, \widehat{\pi_\phi\phi}_{OLS}$ in the second stage to finally obtain $\hat{\rho}_{IV} = \rho$, $\hat{\phi}_{IV} = \phi$. Endowed with unbiased estimates of the spillovers through bank and firms' parameters, we can correct the $\hat{\beta}_{OLS}$ and derive an unbiased $\hat{\beta}_{IV}$. In particular

$$
\begin{aligned}
\hat{\pi}_{\rho,OLS} &= \pi_\rho \\
\hat{\pi}_{\phi,OLS} &= \pi_\phi \\
\widehat{\pi_\rho\rho}_{OLS} &= \pi_\rho\rho \\
\widehat{\pi_\phi\phi}_{OLS} &= \pi_\phi\phi \\
\hat{\rho}_{IV} &= \frac{\widehat{\pi_\rho\rho}_{OLS}}{\hat{\pi}_{\rho,OLS}} = \rho \\
\hat{\phi}_{IV} &= \frac{\widehat{\pi_\phi\phi}_{OLS}}{\hat{\pi}_{\phi,OLS}} = \phi \\
\hat{\beta}_{OLS} &= \beta\big(1 + \frac{\rho^2}{1-\phi^2-\rho^2}\big) \\
\hat{\beta}_{IV} &= \hat{\beta}_{OLS} - \frac{\hat{\rho}^2_{IV}}{1-\hat{\phi}^2_{IV}-\hat{\rho}^2_{IV}}
\end{aligned}
\tag{A.22}
$$

■

The key insight comes from the fact that, under Assumption 6, the credit relationship jb (ia) provides exogenous variation through x_{jb} (x_{ia}) that does not directly affect c_{ib}, but does affect it indirectly through c_{jb} (c_{ia}). It then allows us to identify ϕ (ρ) with a 2SLS estimator. Identification of ρ and ϕ allows us, in turn, to retrieve an unbiased estimator of β.

A.1.4 Bias of Treatment Effects with Fixed Effects

Let us now augment the model with bank and firm fixed effects:

$$
\begin{aligned}
C &= \phi A_F C + \rho A_B C + X\beta + \Delta + \Gamma + \epsilon, \\
&= +\phi A_B C + \rho A_F C + Z\mu + \epsilon,
\end{aligned}
\tag{A.23}
$$

where $\Delta = D\delta$ is a matrix of firm FEs and $\Gamma = G\gamma$ is a matrix of bank FEs. For simplicity let again $\rho = \phi$. Suppose we estimate

$$
C = X\beta + \Delta + \Gamma + U, \tag{A.24}
$$

then the error term has the following form

$$
\begin{aligned}
U &= \phi A_F C + \rho A_B C + \epsilon \\
&= (\phi A_B + \rho A_F)(I - \phi A_B - \rho A_F)^{-1}[X\beta + \Delta + \Gamma + \epsilon] + \epsilon \\
&= M[X\beta + \Delta + \Gamma] + (M + I)\epsilon
\end{aligned}
$$

Suppose again that X is univariate and $X \perp \epsilon$, then we have

$$
\begin{aligned}
X'U &= X'M[X\beta + \Delta + \Gamma] + (M + I)\epsilon = X'\phi A \sum_{k=0}^{\inf} (\phi A)^k [X\beta + \Delta + \Gamma] \\
&= X' \sum_{k=1}^{\inf} (\phi A)^k [X\beta + \Delta + \Gamma] = S + \sum_{k=1}^{\inf} \phi^k X' A^k [\Delta + \Gamma].
\end{aligned}
$$

Here, $A^k\Delta = D_k$ is a vector whose generic element contains the FEs of borrowers of credit relationships at a distance k from the relative relationship. Symmetrically, $A^k\Gamma = G_k$ is a vector whose generic element contains the FEs of lenders of credit relationships at a distance k from the relative relationship. If $X \perp \Delta, \Gamma$, then the bias obtained from estimating (A.24) is equal to S, similar to random effects models. If not, $\phi > 0$ and X are positively correlated with Δ and Γ, then $B = (X'X)^{-1}X'U > 0$. Otherwise, the sign of the bias is again ambiguous.

A.1.5 Bias with Endogenous Treatments

In the previous analysis, we assumed that the main regressor is exogenous. Let us now allow X to be an endogenous regressor. In credit markets, as in other fields where experiments are not possible or easy to implement, regressors are often endogenous. Endogeneity can arise because of self-selection in the extensive (see Jiménez et al., 2014) or in the intensive margin (see Paravisini, Rappoport, and Schnabl, 2023). It can also arise due to the omission of relevant variables on the right-hand side (RHS). In some cases, a credible instrument (a selection step or a clever IV) can be found, but in many situations, this is not a possibility, and the omission of relevant and potentially endogenous variables is always difficult to assess in practice.

In what follows, we want to understand analytically the consequences of endogeneity for the bias due to spillovers. For simplicity, we still focus on the case in which $\phi = \rho$ and without fixed effects. In contrast to the previous sections, we assume that $\epsilon = \iota X + V$, with V being an error term such that $V \perp X, \epsilon$. Suppose we estimate the following CNM's and ICM's parameters

$$C = \phi A_B C + \rho A_F C + X\beta + \epsilon, \tag{A.25}$$

$$C = X\beta + U, \tag{A.26}$$

When the DGP comes from Equation A.25, the error term of Equation A.26 takes the following form

$$\begin{aligned} U &= \phi A_B C + \rho A_F C + \iota X + V \qquad \text{(A.27)} \\ &= (\phi A_B + \rho A_F)(I - \phi A_B - \rho A_F)^{-1}[X\beta + \iota X + V] + \iota X + V, \end{aligned}$$

then, given that $V \perp X$, we have

$$\begin{aligned} X'U &= S + X'(M + I)(\iota X + V) \qquad \text{(A.28)} \\ &= \underbrace{S}_{spillovers} + \underbrace{\iota X'X}_{endogeneity} + \underbrace{\iota X'MX}_{combination}, \end{aligned}$$

$$X'\epsilon \quad = \quad \iota X'X + \iota X'MX. \tag{A.29}$$

In this case, the estimate of β is biased for *both* Equations A.25 and A.26. Nonetheless, the bias of A.26, B_{ICM}, has three drivers: The pure *spillover* component; the pure *endogeneity* component; the *combination* of the two. On the other hand, the bias of A.25, B_{CNM}, is only affected by the last two. It follows that even in a context with endogeneity, we can use our model to study firms and banks credit reallocation system-wide. More formally, $D = B_{ICM} - B_{CNM} \neq 0$ does not imply that $\iota \neq 0$, but it does imply that $S \neq 0$, and thus D can inform us about the presence of (and the bias induced by) spillovers, even if the estimate of β is biased.

We obtain this result because unbiased spillovers can be recovered even in the presence of treatment endogeneity: the network lags used as instrumental variables for the identification of the spillovers, for example, the OPIV in Equations 9 and 10, are still uncorrelated with the error term, i.e., $E[\epsilon' AX] = 0$. The intuition is that the treatments to other agents in the economy can still be valid instruments for their outcomes, even if they are endogenous.[55] The key requirement is that they are not endogenous to their network lags.[56]

A.1.6 Bias of Idiosyncratic Firm and Bank Shocks

We can now derive the bias of firm and bank FEs defining the DGP as $C = X\beta + \Delta + \Gamma + U$. Let Δ_i (Γ_b) be the i_{th} (b_{th}) column of Δ (Γ) and Δ_{-i} (Γ_{-b}) the matrix containing all the columns of Δ (Γ) but the i_{th} (b_{th}). Following the same derivations as above, we have

$$\begin{aligned} D_i'U &= D_i'\phi AM[X\beta + \Delta_i + \Delta_{-i} + \Gamma] + (M+I)\epsilon = \delta_i \sum_{k=1}^{\inf} \phi^k D_i' A^k D_i \\ &= \delta_i \sum_{k=1}^{\inf} \phi^k 1 = \delta_i (\sum_{k\ even} \phi^k l_k^i - \sum_{k\ odd} \phi^k l_k^i) = \delta_i (E_i - O_i). \end{aligned} \tag{A.30}$$

where l_k^i is the number of loops from (to) firm i that involve chains of length k in the credit network.

A similar derivation can be done for $G_i'U$:

$$\begin{aligned} G_i'U &= D_i'\phi AM[X\beta + \Delta + \Gamma_b + \Gamma_{-b}] + (M+I)\epsilon \\ &= \gamma_b (\sum_{k\ even} \phi^k l_k^b - \sum_{k\ odd} \phi^k l_k^b) = \gamma_b (E_b - O_b), \end{aligned} \tag{A.31}$$

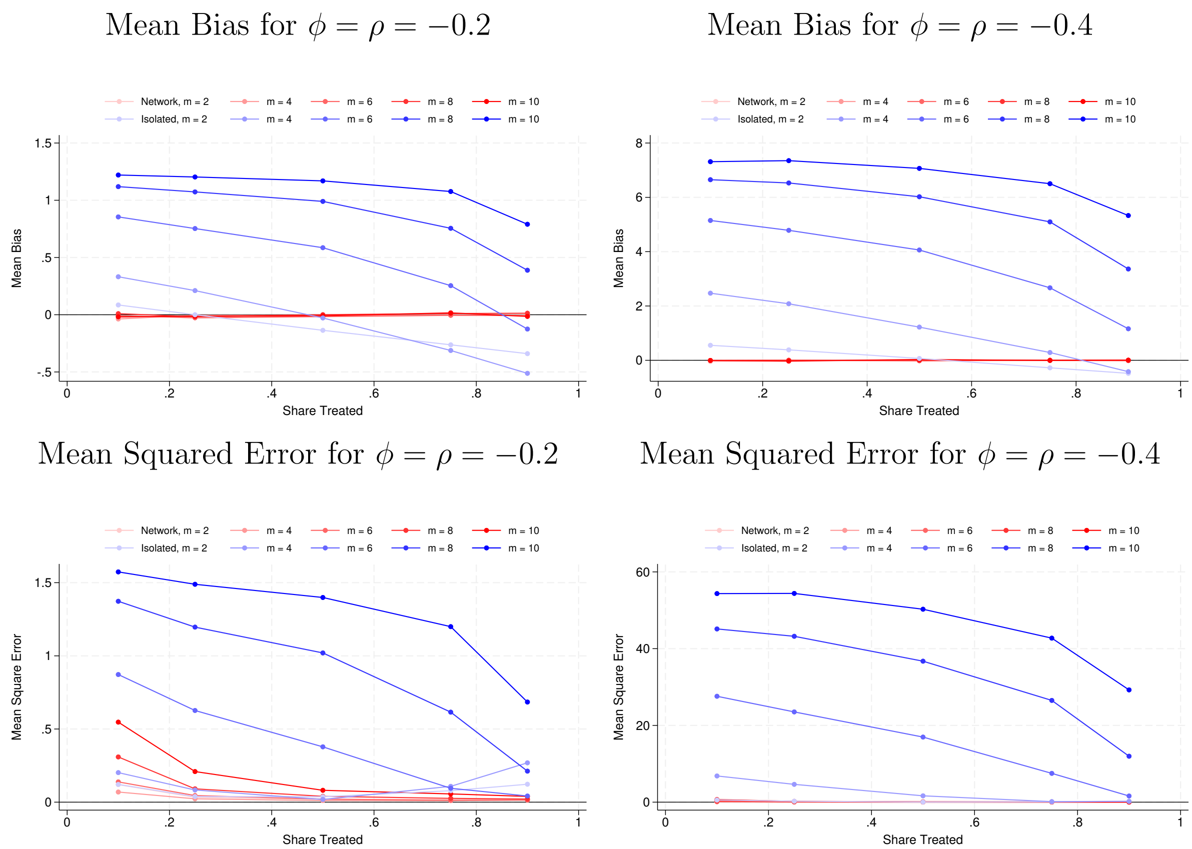
[55] Such a result would not hold if the order of the $OPIV$ used to approximate the TIV is higher, because $X'A^kX$ could contain powers of the same x_i when $k \geq 2$. Using a first order approximation helps to avoid this.

[56] Simulation in Section 4 confirms the result, while we show that our method can easily accommodate an instrumental variable strategy in Section A.4.

where l_k^b is the number of loops from (to) bank b that involve chains of length k in the credit network. Given that both O_b and E_b are positive, the sign of the bias depends on the credit network topology. In indirect networks, we may expect $E_b > O_b$, as loops generate paths of even length going back and forth at each step.

A.2 Additional Monte Carlo Experiments

Figure A.1: Treatment Effect Estimates with Fixed Effects - ICM and CNM Performance



Notes: Mean bias and mean squared error for treatment effect (β). We compute the mean bias across the 500 simulated samples, where the data generating process includes a set of fixed effects. For each sample, the number of nodes N in the network is 200, 100 firms and 100 banks. In black, we plot the zero line. In different shades of blue, we plot how the mean bias (upper two figures) and the mean square error (lower two figures) of the ICM's estimate change. In red, we do the same for the CNM's estimates. Darker shades signal denser networks (m=2,4,6,8,10). In left panels $\phi = -0.2$, in right panels $\phi = -0.4$, for both panels $\theta = 0.1$. We display the plots' underlying data in Appendix Table A.2.

Table A.1: Simulation study - ICM treatment effect bias under different share of treated, spillovers and density

		m=2		m=4		m=6		m=8		m=10	
$\phi = \rho$	% of treated										
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	0.059	0.124	0.198	0.110	0.299	0.143	0.390	0.199	0.456	0.251
	0.25	0.007	0.036	0.037	0.026	0.080	0.029	0.117	0.031	0.155	0.039
-0.2	0.50	-0.142	0.035	-0.238	0.067	-0.295	0.095	-0.314	0.106	-0.328	0.114
	0.75	-0.266	0.079	-0.516	0.272	-0.660	0.441	-0.749	0.565	-0.813	0.664
	0.90	-0.339	0.123	-0.685	0.473	-0.879	0.774	-1.006	1.014	-1.095	1.201
	0.10	0.253	0.213	0.660	0.574	0.958	1.016	1.206	1.546	1.352	1.909
	0.25	0.105	0.063	0.397	0.208	0.588	0.388	0.766	0.629	0.901	0.848
-0.3	0.50	-0.093	0.031	-0.067	0.026	-0.013	0.020	0.064	0.022	0.113	0.029
	0.75	-0.313	0.110	-0.517	0.277	-0.601	0.369	-0.642	0.421	-0.661	0.444
	0.90	-0.436	0.197	-0.791	0.630	-0.961	0.929	-1.056	1.118	-1.133	1.288
	0.10	0.622	0.653	2.029	4.629	3.048	9.756	3.766	14.635	4.245	18.382
	0.25	0.410	0.280	1.512	2.479	2.334	5.623	2.890	8.530	3.295	11.025
-0.4	0.50	0.052	0.043	0.638	0.491	1.117	1.337	1.457	2.199	1.723	3.042
	0.75	-0.273	0.095	-0.201	0.073	-0.081	0.042	0.032	0.032	0.099	0.040
	0.90	-0.478	0.238	-0.728	0.541	-0.807	0.665	-0.840	0.718	-0.867	0.765

Notes. The mean bias and the MSE are copmuted across the 500 simulated samples. The number of nodes N in the network is 200, 100 firms and 100 banks, in each sample. The column spillovers intensity reports the value of $\phi = \rho$ in the simulations.

Table A.2: Simulation study - ICM treatment effect bias with FEs under different share of treated, spillovers and density

$\phi = \rho$	% of treated	m=2		m=4		m=6		m=8		m=10	
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	0.085	0.121	0.332	0.202	0.855	0.872	1.119	1.373	1.221	1.574
	0.25	0.000	0.038	0.211	0.083	0.753	0.627	1.073	1.196	1.204	1.489
-0.2	0.50	-0.136	0.037	-0.028	0.019	0.586	0.378	0.990	1.020	1.169	1.399
	0.75	-0.263	0.078	-0.312	0.108	0.254	0.095	0.755	0.615	1.077	1.200
	0.90	-0.340	0.123	-0.512	0.269	-0.125	0.042	0.389	0.212	0.790	0.684
	0.10	0.284	0.217	0.896	0.994	1.936	4.023	2.529	6.666	2.785	7.962
	0.25	0.116	0.064	0.663	0.522	1.776	3.282	2.433	6.035	2.773	7.776
-0.3	0.50	-0.112	0.035	0.271	0.114	1.443	2.182	2.255	5.170	2.663	7.165
	0.75	-0.306	0.104	-0.235	0.075	0.816	0.748	1.833	3.480	2.430	6.000
	0.90	-0.439	0.201	-0.564	0.328	0.142	0.084	1.099	1.347	1.933	3.900
	0.10	0.552	0.564	2.471	6.830	5.149	27.597	6.650	45.138	7.313	54.346
	0.25	0.385	0.245	2.082	4.646	4.786	23.521	6.530	43.221	7.351	54.391
-0.4	0.50	0.069	0.043	1.222	1.657	4.062	16.962	6.022	36.735	7.067	50.276
	0.75	-0.277	0.094	0.287	0.146	2.669	7.507	5.097	26.517	6.503	42.740
	0.90	-0.480	0.240	-0.418	0.204	1.162	1.610	3.363	11.971	5.331	29.257

Notes. The mean bias and the MSE are computed across the 500 simulated samples. The number of nodes N in the network is 200, 100 firms and 100 banks, in each sample. The column spillovers intensity reports the value of $\phi = \rho$ in the simulations.

Table A.3: Simulation study - CNM estimator performance under different share of treated, spillovers and density

		m=2		m=4		m=6		m=8		m=10	
$\phi = \rho$	% of treated										
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	-0.058	0.118	-0.022	0.059	-0.013	0.039	0.000	0.030	0.006	0.024
	0.25	-0.006	0.034	-0.002	0.017	-0.005	0.013	-0.009	0.009	-0.006	0.008
-0.2	0.50	-0.020	0.017	-0.002	0.009	-0.005	0.007	-0.003	0.005	-0.002	0.004
	0.75	-0.010	0.015	-0.004	0.009	-0.013	0.007	-0.006	0.005	-0.005	0.004
	0.90	-0.007	0.021	-0.018	0.015	-0.013	0.011	-0.007	0.008	-0.007	0.007
	0.10	-0.009	0.098	-0.016	0.058	0.000	0.045	0.000	0.032	-0.011	0.033
	0.25	-0.012	0.035	-0.005	0.020	-0.009	0.013	-0.007	0.011	-0.004	0.009
-0.3	0.50	-0.010	0.016	0.004	0.008	0.001	0.006	0.002	0.005	-0.002	0.004
	0.75	-0.009	0.015	-0.003	0.007	-0.004	0.005	-0.008	0.003	0.002	0.003
	0.90	-0.012	0.014	-0.011	0.010	-0.012	0.007	0.000	0.005	-0.008	0.005
	0.10	-0.019	0.096	-0.014	0.070	0.001	0.058	-0.019	0.045	-0.007	0.040
	0.25	0.009	0.034	-0.001	0.022	0.006	0.017	0.001	0.011	-0.009	0.012
-0.4	0.50	-0.013	0.017	-0.004	0.009	-0.004	0.006	0.001	0.005	0.000	0.004
	0.75	-0.008	0.011	-0.004	0.005	-0.001	0.004	0.001	0.000	0.000	0.002
	0.90	-0.003	0.011	-0.005	0.006	-0.004	0.004	-0.003	0.003	-0.003	0.002

Notes. The mean bias and the MSE are computed across the 500 simulated samples. The number of nodes n in the network is 200, 100 firms and 100 banks, in each sample. The first column reports the spillovers intensity, i.e. the value of $\phi = \rho$ in the simulations.

Table A.4: Simulation study - CNM estimator performance with FEs under different share of treated, spillovers and density

		m=2		m=4		m=6		m=8		m=10	
$\phi = \rho$	% of treated										
		mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse	mean bias	mse
	0.10	-0.025	0.120	-0.037	0.069	-0.008	0.139	0.010	0.309	-0.018	0.547
	0.25	-0.014	0.034	-0.005	0.023	-0.027	0.045	-0.018	0.091	-0.012	0.209
-0.2	0.50	-0.015	0.020	-0.005	0.011	-0.016	0.021	0.001	0.040	-0.009	0.081
	0.75	-0.005	0.017	-0.003	0.010	-0.005	0.013	0.010	0.026	0.016	0.056
	0.90	-0.003	0.022	-0.007	0.016	-0.007	0.014	0.013	0.021	-0.015	0.039
	0.10	-0.002	0.098	-0.011	0.072	-0.015	0.146	-0.035	0.309	-0.033	0.835
	0.25	-0.011	0.031	-0.016	0.025	-0.007	0.057	0.003	0.116	0.031	0.234
-0.3	0.50	-0.016	0.018	0.002	0.011	-0.004	0.024	0.009	0.052	0.012	0.107
	0.75	-0.001	0.012	-0.014	0.008	-0.006	0.015	-0.002	0.032	0.019	0.058
	0.90	-0.016	0.018	-0.005	0.009	-0.003	0.013	0.008	0.026	0.018	0.049
	0.10	-0.048	0.130	-0.007	0.081	-0.008	0.164	-0.004	0.333	-0.007	0.675
	0.25	-0.017	0.034	0.006	0.029	-0.006	0.053	-0.001	0.126	-0.030	0.220
-0.4	0.50	-0.004	0.016	-0.011	0.012	0.006	0.029	-0.016	0.049	0.029	0.102
	0.75	-0.008	0.011	-0.004	0.008	0.001	0.018	0.004	0.035	-0.006	0.087
	0.90	-0.015	0.014	-0.003	0.003	-0.012	0.015	0.005	0.034	0.005	0.065

Notes. The mean bias and the MSE are computed across the 500 simulated samples. The number of nodes n in the network is 200, 100 firms and 100 banks, in each sample. The first column reports the spillovers intensity, i.e. the value of $\phi = \rho$ in the simulations.

Table A.5: Simulation study - estimator performance under different spillovers, size and density

n	m		true			true			true			true			true		
			ϕ	ρ	β	ϕ	ρ	β	ϕ	ρ	β	ϕ	ρ	β	ϕ	ρ	β
200			-0.1	-0.1	2	-0.1	-0.2	2	-0.1	-0.3	2	-0.1	-0.4	2	-0.4	-0.4	2
	2	mean	-0.097	-0.100	1.977	-0.100	-0.209	1.981	-0.101	-0.306	1.979	-0.093	-0.414	1.979	-0.406	-0.406	1.987
		std	0.084	0.087	0.131	0.090	0.089	0.138	0.082	0.081	0.128	0.082	0.076	0.123	0.066	0.067	0.129
	4	mean	-0.099	-0.096	1.983	-0.095	-0.194	1.987	-0.094	-0.299	1.985	-0.096	-0.395	1.985	-0.402	-0.395	1.996
		std	0.040	0.041	0.107	0.040	0.040	0.095	0.040	0.044	0.101	0.040	0.042	0.096	0.039	0.040	0.092
	6	mean	-0.098	-0.098	1.991	-0.097	-0.197	1.985	-0.097	-0.295	1.989	-0.096	-0.398	1.992	-0.402	-0.395	1.996
		std	0.029	0.030	0.093	0.029	0.030	0.088	0.029	0.033	0.077	0.028	0.032	0.079	0.039	0.040	0.092
	8	mean	-0.101	-0.097	1.992	-0.101	-0.197	1.992	-0.100	-0.296	1.991	-0.098	-0.398	1.995	-0.403	-0.396	1.998
		std	0.024	0.022	0.079	0.025	0.026	0.070	0.024	0.026	0.070	0.024	0.028	0.069	0.033	0.032	0.073
	10	mean	-0.102	-0.096	1.999	-0.099	-0.198	1.994	0.000	-0.297	1.992	-0.098	-0.398	1.995	-0.402	-0.397	2.003
		std	0.021	0.020	0.066	0.023	0.023	0.068	0.022	0.024	0.062	0.021	0.024	0.060	0.031	0.030	0.063
800																	
	2	mean	-0.102	-0.098	1.999	-0.100	-0.201	1.998	-0.097	-0.301	1.999	-0.097	-0.401	1.998	-0.398	-0.401	2.000
		std	0.041	0.043	0.064	0.044	0.042	0.067	0.042	0.042	0.063	0.040	0.037	0.067	0.034	0.033	0.060
	4	mean	-0.098	-0.098	1.994	-0.099	-0.198	1.994	-0.098	-0.300	1.998	-0.097	-0.400	1.995	-0.398	-0.401	2.001
		std	0.022	0.021	0.054	0.021	0.022	0.051	0.021	0.022	0.048	0.018	0.020	0.047	0.021	0.020	0.047
	6	mean	-0.099	-0.100	1.998	-0.099	-0.200	1.998	-0.097	-0.300	1.996	-0.099	-0.400	1.999	-0.398	-0.401	2.001
		std	0.015	0.014	0.048	0.015	0.016	0.042	0.014	0.016	0.040	0.014	0.016	0.041	0.021	0.020	0.047
	8	mean	-0.100	-0.099	1.996	-0.100	-0.200	1.995	-0.099	-0.300	1.998	-0.099	-0.399	1.997	-0.401	-0.399	2.002
		std	0.012	0.012	0.041	0.012	0.014	0.036	0.013	0.014	0.035	0.012	0.014	0.036	0.015	0.016	0.036
	10	mean	-0.100	-0.099	1.997	-0.100	-0.200	2.001	-0.099	-0.300	1.998	-0.099	-0.400	1.996	-0.399	-0.400	2.000
		std	0.010	0.011	0.036	0.010	0.011	0.032	0.011	0.012	0.031	0.010	0.012	0.031	0.015	0.015	0.033

Notes. The mean and the std are computed across the 500 simulated samples. n is the number of nodes in the network, m regulates the network density as described in Section 4.1.

Idiosyncratic Firm and Bank Shocks, Detailed Exposition. Here we deepen the properties firm and bank FEs' estimates, which account for idiosyncratic shocks, when using the isolated credit model (ICM) and the credit network model (CNM). To better assess the magnitude of the bias in finite samples, we let all FEs be positive by adding the minimum draw for each replication. To aggregate all the firms' and banks' specific parameters, we use the mean bias, the mean absolute bias and the root mean squared bias: $MB = 1/R\sum_{r=1}^{R}\{1/N[\sum_{j=1}^{F}(\hat{\delta}_j^r-\delta)+\sum_{k=1}^{B}(\hat{\gamma}_k^r-\gamma)]\}$, $MAB = 1/R\sum_{r=1}^{R}[1/N(\sum_{j=1}^{F}|\hat{\delta}_j^r-\delta|+\sum_{k=1}^{B}|\hat{\gamma}_k^r-\gamma|)]$, $RMSE = 1/R\sum_{r=1}^{R}[1/N\sqrt{\sum_{j=1}^{F}(\hat{\delta}_j^r-\delta)^2+\sum_{k=1}^{B}(\hat{\gamma}_k^r-\gamma)^2}]$.

Table A.6 reports these indicators for different network sizes (n = 200, 800 and 2000), network densities (m = 4,6,8 and 10) and magnitude of spillovers (ϕ and ρ). The bias for the ICM is increasing in both the density of the network and the magnitude of spillovers, while the bias of the CNM is always close to zero and converges to it as n tends to infinity.

Given that $\theta = 0.1$ in our pivotal setting and we constrained the FEs to be positive, the average FE is greater than 0.3 with a probability lower than 0.001. The bias of the ICM ranges from about 0.3 (when $m = 4$ and $\phi = \rho = -0.2$) and 3 (when $m = 10$ and

$\phi = \rho = -0.4$), which means that with low (high) density and small (large) spillovers the ICM estimate is on average about the double (ten times) the real idiosyncratic shock with very high probability. The intuition behind this result is that competitive interactions let positive idiosyncratic shocks diffuse through the credit network and amplify them, overestimating them, if such feedback loops are not accounted for.

Table A.6: Simulation study - ICM and CNM estimators performance under different spillovers, size and density

			$\phi = \rho = -0.2$			$\phi = \rho = -0.3$			$\phi = \rho = -0.4$		
n	m		MB	MAB	RMSE	MB	MAB	RMSE	MB	MAB	RMSE
200											
	4	ICM	0.303	0.394	61	0.412	0.460	73	0.628	0.648	98
		CNM	0.002	0.418	67	0.002	0.424	71	-0.005	0.420	69
	6	ICM	0.435	0.532	627	0.676	0.739	843	1.283	1.330	1,446
		CNM	-0.005	0.447	556	-0.009	0.448	556	-0.009	0.447	547
	8	ICM	0.650	0.732	1,472	1.066	1.125	2,163	2.230	2.281	4,316
		CNM	-0.013	0.472	1,025	0.001	0.466	1,002	-0.014	0.462	995
	10	ICM	0.820	0.891	2,234	1.414	1.462	3,599	3.074	3.138	7,628
		CNM	-0.027	0.498	1,376	-0.021	0.494	1,376	-0.018	0.474	1,312
800											
	4	ICM	0.276	0.377	484	0.376	0.429	527	0.594	0.620	729
		CNM	0.000	0.414	543	-0.002	0.415	535	0.001	0.417	539
	6	ICM	0.412	0.516	4,851	0.642	0.710	6,427	1.253	1.305	11,317
		CNM	-0.002	0.437	4,299	-0.002	0.436	4,284	-0.002	0.435	4,257
	8	ICM	0.615	0.701	11,235	1.039	1.100	16,944	2.195	2.256	34,081
		CNM	-0.004	0.448	7,707	0.000	0.448	7,664	-0.002	0.449	7,710
	10	ICM	0.789	0.862	17,467	1.374	1.430	28,187	3.015	3.081	59,562
		CNM	-0.007	0.458	10,089	-0.002	0.454	10,007	-0.009	0.457	10,041
2000											
	4	ICM	0.264	0.371	1,882	0.360	0.419	2,052	0.577	0.604	2,821
		CNM	0.000	0.415	2,144	-0.002	0.413	2,118	0.002	0.414	2,127
	6	ICM	0.399	0.507	18,845	0.632	0.703	25,099	1.238	1.295	44,536
		CNM	0.001	0.434	16,831	0.000	0.434	16,802	-0.001	0.435	16,858
	8	ICM	0.602	0.691	43,662	1.023	1.086	66,298	2.171	2.231	133,100
		CNM	-0.001	0.444	30,017	0.000	0.443	30,005	0.000	0.444	30,024
	10	ICM	0.775	0.851	68,219	1.354	1.412	109,951	3.001	3.069	234,902
		CNM	0.001	0.450	39,115	-0.005	0.450	39,012	-0.004	0.448	38,915

Notes. ICM and CNM stand respectively for isolated credit model and credit network model. n is the number of nodes in the network, m regulates the network density as described in Section 4.1. MB, MAB and RMSE stand respectively for mean bias, mean absolute bias and root mean square error. All are averaged across 500 replications and computed with the following formulas: $MB = 1/R\sum_{r=1}^{R}\{1/N[\sum_{j=1}^{F}(\hat{\delta}_j^r - \delta) + \sum_{k=1}^{B}(\hat{\gamma}_k^r - \gamma)]\}$, $MAB = 1/R\sum_{r=1}^{R}[1/N(\sum_{j=1}^{F}|\hat{\delta}_j^r - \delta| + \sum_{k=1}^{B}|\hat{\gamma}_k^r - \gamma|)]$, $RMSE = 1/R\sum_{r=1}^{R}[1/N\sqrt{\sum_{j=1}^{F}(\hat{\delta}_j^r - \delta)^2 + \sum_{k=1}^{B}(\hat{\gamma}_k^r - \gamma)^2}]$. $RMSE$ is reported without decimals.

A.2.1 Bias under Endogenous Treatment.

The analytical results in the supplementary online Appendix Section A.1.5 show that in the presence of endogenous treatments, both the CNM and ICM estimates of β can be biased. However, the difference between the two biases can inform about the presence of spillovers and the latter can be still estimated without distortion. In this section, we study the performance of both estimators when treatment endogeneity is introduced and sequentially increased in finite samples. To analyze this case, we start from the settings used above and generate $\epsilon = \iota X + V$, with V being a normal error term with mean equal to 0 and variance equal to σ. On the one hand, we increase sequentially the endogeneity of the treatment with $\iota = 0, 0.2, 0.5$. On the other hand, we increase the magnitude of spillovers $\phi = \rho = 0, -0.2, -0.4$, as in the previous exercises.

Table A.7 reports our findings. The first interesting result is that spillovers are always correctly estimated by the CNM, even in the presence of high treatment endogeneity. This is because the spillovers among agents' outcomes can be still recovered even if the treatment is endogenous, as discussed in Section A.1.5 of the supplementary online Appendix. Moreover, we can see that when cross-elasticities are shut down there is no significant difference between the ICM and CNM, but, when they do, the estimator we propose performs strictly better than the ICM. The second result is that, while CNM's estimate bias increases steadily in ι, it is not sensitive to ϕ and ρ, at difference with the ICM estimates', where biases compound. Indeed, we can see that the bias of the ICM increases in both ι and $\phi = \rho$, almost doubling the magnitude of the estimated effect when spillovers and treatment endogeneity are high.

Table A.7: Performance of All Estimators When Treatment is Endogenous

			ι = 0			ι = -0.2			ι = -0.5		
			$\hat{\phi}$	$\hat{\rho}$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\rho}$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\rho}$	$\hat{\beta}$
$\phi = \rho = 0$	CNM	mean	0.001	-0.001	2.007	0.003	0.004	2.194	0.003	0.001	2.495
		std	0.033	0.035	0.076	0.031	0.032	0.075	0.027	0.027	0.077
	ICM	mean			2.008			2.193			2.494
		std			0.073			0.071			0.074
$\phi = \rho =$ -0.2	CNM	mean	-0.197	-0.197	2.002	-0.199	-0.197	2.203	-0.200	-0.199	2.500
		std	0.033	0.034	0.073	0.029	0.031	0.072	0.027	0.026	0.073
	ICM	mean			2.231			2.457			2.789
		std			0.085			0.084			0.087
$\phi = \rho =$ -0.4	CNM	mean	-0.398	-0.399	2.003	-0.400	-0.398	2.196	-0.399	-0.401	2.496
		std	0.030	0.030	0.077	0.027	0.028	0.083	0.023	0.023	0.076
	ICM	mean			2.915			3.197			3.649
		std			0.152			0.152			0.183

Notes: Over this Table, we compute the mean and the standard deviation across 500 simulated samples. The number of nodes is 800, $m = 2$ and $\beta = 2$.

A.3 Support Tables for the Empirical Application

Table A.8: Descriptive Statistics, Estimation Sample

	Mean	*S.D.*	*P.25*	*Median*	*P.75*	*N.*
			Credit Relationships Characteristics			
Δ ln(C)	0.032	0.416	-0.064	0.000	0.053	2,631,983
Revolving on Total	0.277	0.318	0.052	0.143	0.365	2,631,983
Granted on Total	0.293	0.207	0.129	0.240	0.417	2,631,983
Same Province	0.306	0.461	0.000	0.000	1.000	2,631,983
Rel. Age (1999)	3.859	2.578	2.000	3.000	6.000	2,631,983
			Firm Characteristics across Relationships			
N. Rel. Firm	4.162	2.304	2.000	4.000	5.000	2,631,983
I(Risk)	0.285	0.452	0.000	0.000	1.000	2,612,755
Risk Rating	5.274	1.794	4.000	5.000	7.000	2,612,755
			Bank Characteristics across Relationships			
N. Rel. Bank	19,305	1,7528	4,151	13,634	30,657	2,631,983
ln(100*Bank Eq./Asset)	1.915	0.284	1.720	1.844	2.082	2,228,595
I(Bank Id. Change)	0.024	0.154	0.000	0.000	0.000	2,631,983
			Aggregates			
Δ Overnight Rate (pp)	0.163	0.771	-0.510	0.014	1.130	2,631,983
			Count			
N. Firm	223,379					
N. Banks	696					
Years	2002-2008					

Notes: This Table presents descriptives for the estimation sample (2002-2008). The first panel, *Credit Relationships Characteristics*, describes the distribution of changes in granted credit, as well as that of the revolving intensity of credit relationships, the ratio between credit granted on the relationship and total credit granted to the firm, the dummy tracking whether firm and bank share the same headquarter province, and the age of the credit relationship since year 1999. The second panel, *Firm Characteristics across Relationships*, summarizes the distribution of number of credit relationships per firm, of the risk dummy (risk rating greater or equal to seven), and that of the underlying firm risk rating (there are 10 levels in increasing risk order) across credit relationships. The third panel, *Bank Characteristics across Relationships*, displays the distribution of the number of the credit relationships per bank, the log of the bank equity to assets ratio, and of the dummy tracking whether the id of the bank in the dataset changes (e.g., due to a merge) across credit relationships. Finally, we report descriptives for the Italian overnight rate in percentage points and the number of firms, banks and years covered in our sample. This is a support Table for Table 4.

Table A.9: First Stage for Table 4

	(1)	(2)	(3)	(4)
Dependent Variable:	$N_B\Delta\log(\text{granted}_{ibt})$		$N_F\Delta\log(\text{granted}_{ibt})$	
Instrumental Variables:	coeff.	std. err.	coeff.	std. err.
N_BTreat.$_{ibt-1}$	0.064	0.00005	-0.0014	0.0003
N_BRel. Age$_{ibt}$	0.0033	0.00002	-0.001	0.0001
N_BSame Province$_{ibt}$	-0.00038	0.0002	-0.0021	0.001
N_BRev. on Total$_{ibt-1}$	0.1782	0.0004	0.0178	0.0023
N_BGranted on Total$_{ibt-1}$	-0.1269	0.0005	-0.0002	0.0032
N_BI(Bank Id Change)$_{ibt}$	-0.0608	0.0002	-0.0033	0.0011
N_FTreat.$_{ibt-1}$	-0.0005	0.0001	0.0302	0.0009
N_FRel. Age$_{ibt-1}$	0.0006	0.00002	0.0039	0.0001
N_FSame Province$_{ibt-1}$	0.0021	0.0002	0.0025	0.001
N_FRev. on Total$_{ibt-1}$	-0.024	0.0003	0.0615	0.0017
N_FGranted on Total$_{ibt-1}$	-0.1119	0.0016	-0.2386	0.0096
N_FI(Bank Id Change)$_{ibt-1}$	0.0129	0.0007	0.0033	0.004
N	2,107,094		2,107,094	
F_{SW}	39,161		453	
Bank-FE	Yes		Yes	
Firm-FE	Yes		Yes	

Notes: Estimated coefficients and standard errors for first stages of model (5) employing the 2SLS estimator in Equation (14), whose second stage is displayed in Table 4. The second stage dependent variable $\Delta\log(\text{granted}_{ibt})$ is the yearly log growth rate of the credit relationship. N_B bank-network lag operator, and it equals to $\sum_{j\in\mathbb{F}\setminus i} a_{ib,jb}x_{jbt}$ for every x covariate; N_F is the firm-network lag operator, and it equals to $\sum_{k\in\mathbb{B}\setminus b} a_{ib,fk}x_{ikt}$ for every x covariate. Treat.$_{ibt-1}$ is the the product between the change in the banks' overnight rate, the lag of the log-transformed equity over asset ratio in percentage points and the risky firm indicator variable. Bank-related coefficients are re-scaled to account for the disparity in the number of relationships between banks and firms, using the 25-th percentile of the distribution of banks' credit relationships as point of reference. In particular, coefficients and errors for covariates' bank-network lag in the first stage of $N_F\Delta\log(\text{granted}_{ibt})$ (Table's columns 3 and 4, first six lines) are multiplied by 4,402, while the coefficients and errors for the variables' firm-network lags are divided by 4,402 in the $N_B\Delta\log(\text{granted}_{ibt})$ first stage (Table's columns 1 and 2, last six lines). The F_{SW} is the F-test for weak instruments in linear IV models with multiple endogenous variables proposed by Sanderson and Windmeijer (2016). This is a support Table for Table 4.

A.4 Extensions and Discussions

In this section, we discuss issues related credit relationships that, while not materializing in practice, still played a role in equilibrium by their simple possibility, as unrealized outside options still affect the results on-path. In particular, we deepen our arguments regarding their lack of material relevance in the proposed empirical exercise in Section 5. Furthermore, we present an extension of the model to allow for heterogeneous ϕ and ρ. Finally, we provide and in-depth comparison of our model with other approaches used in the literature.

A.4.1 Heterogeneous FCC and BCC

The CNM can be augmented to have bank- and firm-type specific spillovers, for example if one is interested in studying the substitution of credit from a type of banks to another type of banks by firms, or the reallocation of credit from a certain type of firms to another one by banks, and eventually combinations of the two depending on the specific empirical questions.

A typical example for the FCC is the substitution of credit from low technology banks to high technology ones by firms demanding new and more modern financial services.[57] Another one, focusing this time on the BCC, is the reallocation of credit by banks from sectors hit by specific shocks to unaffected ones.[58] Let us focus on the first case and suppose that there are H (high tech) and L (low tech) banks.

If we have these two types, we will have four types of ρs. ρ^H captures the spillovers among relationships involving type H banks; ρ^L captures the spillovers among relationships involving type L banks; ρ^{LH} captures the spillovers from relationships with type H bank to those with type L; ρ^{HL} captures the spillovers from relationships with type L bank to those with type H. If, for example, one expects high substitution of credit from low tech banks to high tech banks, ρ^{HL} should be higher than the others.

Model (5) will then become:

$$sumgraC = (\rho^H A_F^H + \rho^L A_F^L + \rho^{HL} A_F^{HL} + \rho^{LH} A_F^{LH})C + \phi A_B C + Z\mu + \epsilon. \quad \text{(A.32)}$$

where the matrix A_F^H keeps track of connections among relationships involving the same firm and banks of type H; the matrix A_F^L keeps track of connections among relationships involving the same firm and banks of type L; A_F^{HL} and A_F^{LH} are symmetrically equal matrices that keep track of connections among relationships involving the same firm and banks of both types.

In such extension of the baseline model, the instrumental variables change accordingly to the different specification. First order approximations of the best IV for the five

[57] See Fuster et al. (2019), Fuster et al. (2022), Branzoli, Rainone, and Supino (2024), Core and De Marco (2021) and Kwan et al. (2021), among the others, for studies on the effects of bank technological adoption and credit.

[58] See Paravisini, Rappoport, and Schnabl (2023), Federico, Marinelli, and Palazzo (2023) and Federico, Hassan, and Rappoport (2023), among the others, for studies on lending behavior by banks more specialized or with loan portfolios concentrated in sectors more exposed to shocks.

endogenous variable are respectively:

$$\begin{aligned}
OPIV^1_{FH} &= A^H_F X, \\
OPIV^1_{FL} &= A^L_F X, \\
OPIV^1_{FLH} &= A^{LH}_F X, \\
OPIV^1_{FHL} &= A^{HL}_F X, \\
OPIV^1_{B} &= A_B X.
\end{aligned}$$

Second order approximations of the best IV for the five endogenous variable are respectively:

$$\begin{aligned}
OPIV^2_{FH} &= A^H_F [I, A_B] X, \\
OPIV^2_{FL} &= A^L_F [I, A_B] X, \\
OPIV^2_{FLH} &= A^{LH}_F [I, A_B] X, \\
OPIV^2_{FHL} &= A^{HL}_F [I, A_B] X, \\
OPIV^2_{B} &= A_B [I, A^H_F, A^L_F, A^{LH}_F, A^{HL}_F] X.
\end{aligned}$$

Similar derivations can be computed for higher order approximations.

If we are interested in heterogeneous reallocation policies by the banks, suppose within and between two sectors (say S and P), we will have again four types of ϕs. ϕ^S captures the spillovers among relationships involving firms in sector S; ϕ^T captures the spillovers among relationships involving firms in sector T; ϕ^{TS} captures the spillovers from relationships with firms in sector S to those in sector T; ϕ^{ST} captures the spillovers from relationships with firms in sector T to those in sector S. If, for example, one expects high substitution of credit from low tech banks to high tech banks, ρ^{LH} should be higher than the others.

Model (5) will then become:

$$C = (\phi^S A^S_B + \phi^T A^T_B + \phi^{ST} A^{ST}_B + \phi^{TS} A^{TS}_B) C + \rho A_F C + Z\mu + \epsilon. \qquad \text{(A.33)}$$

where the matrix A^S_B keeps track of connections among relationships involving the same bank and firms of sector S; the matrix A^T_B keeps track of connections among relationships involving the same bank and firms of sector T; A^{ST}_B and A^{TS}_B are symmetrically equal matrices that keep track of connections among relationships involving the same bank and firms of both sectors.

In such extension of the baseline model, the instrumental variables change accordingly to the different specification. First order approximations of the best IV for the five endogenous variable are respectively:

$$
\begin{aligned}
OPIV^1_{FH} &= A^S_B X, \\
OPIV^1_{FL} &= A^T_B X, \\
OPIV^1_{FLH} &= A^{TS}_B X, \\
OPIV^1_{FHL} &= A^{ST}_B X, \\
OPIV^1_{B} &= A_F X.
\end{aligned}
$$

Second order approximations of the best IV for the five endogenous variable are respectively:

$$
\begin{aligned}
OPIV^2_{FH} &= A^S_B [I, A_F] X, \\
OPIV^2_{FL} &= A^T_B [I, A_F] X, \\
OPIV^2_{FLH} &= A^{TS}_B [I, A_F] X, \\
OPIV^2_{FHL} &= A^{ST}_B [I, A_F] X, \\
OPIV^2_{B} &= A_F [I, A^S_B, A^T_B, A^{TS}_B, A^{ST}_B] X.
\end{aligned}
$$

Similar derivations can be computed for higher order approximations.

Overall, when we are interested in heterogeneous FCC and BCC, instrumental variables depend on the final specification and thus on the specific research question. We thus do not provide sufficient identification conditions as those in Proposition 4, for any possible configuration. Nevertheless, a more general condition for identification is that the matrix including the expected value of the endogenous variables and the other covariates in the model has full rank. In this example, we need the matrix

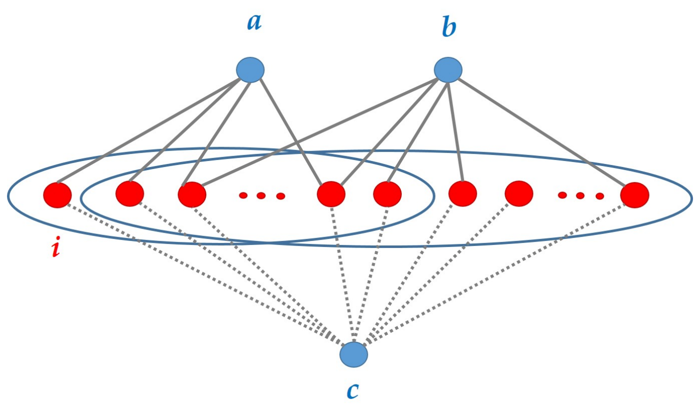
$$
[E(A^S_B C), E(A^T_B C), E(A^{ST}_B C), E(A^{TS}_B C), E(A_F C), Z]
$$

to have full rank. The intuition is that as long as there are intransitive quadriads for any combination of heterogeneous BCC and FCC resulting from the selected choices, this condition is always respected, the parameters are identified and the IVs can be constructed as linear combinations of the vectors appearing in the expected value of each endogenous terms.

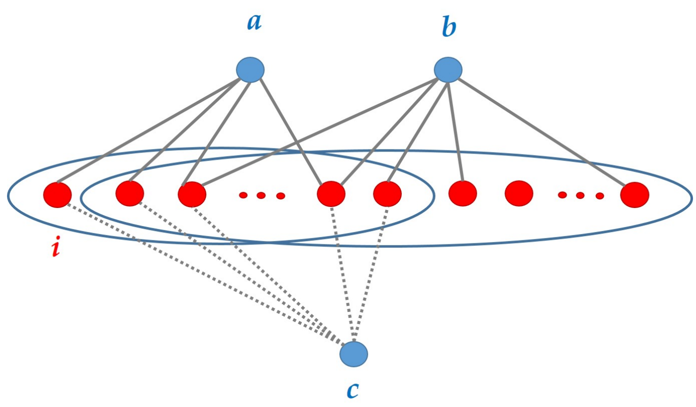
A.4.2 Unobserved credit relationships

The issue of unobservable links is pervasive in networks (Bramoullé, Djebbari, and Fortin, 2020b). Sampling and measurement error are the two main determinants. When just a sample of links is observed, for example when survey data is used to capture connections among agents omitted variable issues arise (Chandrasekhar and Lewis, 2011). As pointed out by Blume et al. (2015), incorrectly assuming an absence of connections invalidates the exclusion restriction for instrumental variables built from the network structure. From this standpoint, a reassuring feature of the credit register data we use is that it is not a sample. It covers the entire population of loans above a very low threshold: 30 thousand euros as total exposure in terms of granted credit. However, even though the issues of sampling and measurement error are no material for us, there could be market interactions, especially among banks, that may in principle be at work. Let us call them non-existing but potential credit relationships. Theoretically, a firm i served by bank a could potentially have gone to other banks in the market. Issues may arise if shocks to those banks influence the terms that bank a applies to firm i and the OPIV. If the firm borrows also from another bank, say b, these market interactions are visible and considered in our model. Indeed, it is what we focus on. If the firm does not borrow from another bank, say c, but its borrowing decisions with bank a are influenced by bank c's supply strategy, then the non-existing relationship between firm i and bank c may affect our estimates. The magnitude of this perturbation depends on the extent of the overlap between banks' portfolios, as in the OPIV, and the heterogeneity of their pricing strategies. We use this simple setting to provide examples in Figure A.2. Let us focus on a situation where we are interested in estimating the effect of a variation of the rest of bank a's relationships on relationship ia (ϕ), using our OPIV, i.e. variation in the relationships of bank b that have the same borrowers of those of bank a (say $-i_{ab}$). If the pricing strategy of bank c affects homogeneously firms that borrow from banks a and b (panel a)), the uniform (across relationships) reactions of both banks are absorbed by bank FEs and there are no issues. Some issues may arise if bank c targets exactly firm i and the whole set of borrowers that bank a and b share (panel b)). In this case, FEs will not capture the heterogeneous reaction within bank a's and b's pricing strategies and such non-existing, but potential, relationships would be omitted variables that are correlated with the instrumental variable. However, if bank c targets a fraction of borrowers in the market, say based on some characteristics, which include firm i and only a small fraction of bank b portfolio, variation in non-targeted bank b's loans can be still used as a valid instrument. If banks' pricing rules are known by the econometrician, she can easily assess which configuration is in place. If not, she has to assume that we are in the first or the last configuration. Fully homogeneous pricing rules are difficult to assume. However, as banks are usually risk adverse and diversify their portfolios, it is very unlikely that $-i_{ab}$ contains the very same type of firms of i and that bank c have a specific pricing rule only for them and i. In addition, in our model $-i_{ab}$ contains all the relationships of bank a but i, we do not focus on a subset of them, thus as long as the portfolio is diversified scenarios as depicted in (panel b)) are unlikely to materialize. In practice, it could happen only when the bank has only one type of borrower, which again is almost impossible to observe in the credit market. For this reason and given data unavailability of bank's pricing rules, we work under such assumption.

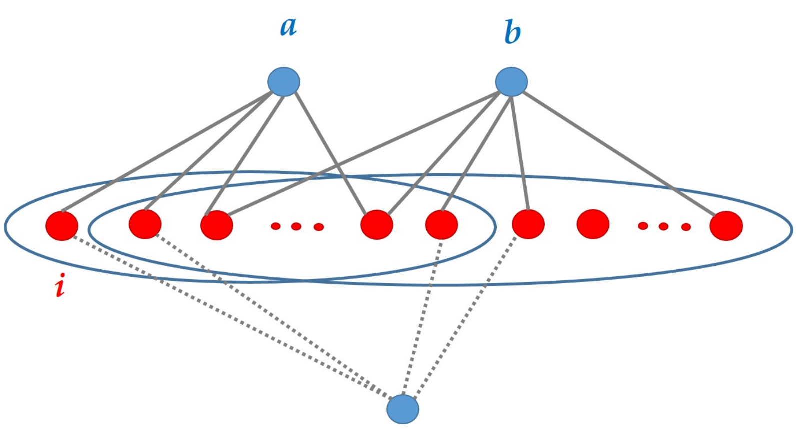
Figure A.2: Non-existing but potential credit relationships



(a) Full portfolio strategy



(b) Large overlapping partial portfolio strategy



(c) Small overlapping partial portfolio strategy

Notes: Blue nodes a, b and c are banks, red nodes are firms, solid edges are existent credit relationships, dashed ones are non-existing but potential relationships.

A.4.3 Comparison between OPIVs, SSIVs, and GIVs

In general terms, OPIVs are related to other approaches decomposing market's aggregate outcomes to derive instrumental variables, such as shift share instrumental variables (Bartik, 1991; Blanchard et al., 1992; Borusyak, Hull, and Jaravel, 2022; Goldsmith-Pinkham, Sorkin, and Swift, 2020, SSIVs, see), and granular instrumental variables (GIVs, Gabaix and Koijen, 2024). However, our approach differs from the GIVs and SSIVs approaches substantially. OPIVs and both these approaches are actually complements, because they can be used in different types of markets.

The GIVs and SSIVs are procedures designed to estimate price elasticities in centralized markets, where there is only one price, while we design OPIVs to estimate objects more similar to elasticities of substitution in decentralized markets, where the price varies at the pair level and the identity of counterparties matters. For example, GIVs exploit the fact that in centralized markets single agents demand (or supply) depends on the aggregate price, but the aggregate price does not depend on the demand of the single agent, rather the aggregate demand. The instrumental variable is obtained when few large actors account for a substantial fraction of aggregate demand (supply) and idiosyncratic shocks are volatile relative to the volatility of aggregate shocks. Although being deeply different from each other, both GIVs and SSIVs derive instruments by decomposing aggregate quantities (like the aggregate demand, for example) and using exogenous components, under different assumptions.

The OPIV does not decompose any aggregate quantity, it instead derives instrumental variables for endogenous disaggregated outcomes, exploiting intransitivity in decentralized markets. The OPIVs exploits the fact that in decentralized markets agents demand (or supply) in a single contract depends on what happens in other contracts involving the same parties. We obtain OPIVs under the condition that agents have not fully overlapping portfolios of counterparties. Whereas GIVs use players' size disparities to derive exclusion restrictions, OPIVs use intransitivity. GIVs need that idiosyncratic shocks to large players can be separated from systemic ones (granularity). What OPIVs need is that not all banks lend to all firms.

A.4.4 Comparison with Papers on Spillovers in Corporate Finance

In this section we discuss the main differences between our model and those of the family which Huber (2022), Berg, Reisinger, and Streitz (2021) belong to.

The first difference is about the main outcome variables and the source of spillovers considered. We study the formation of outcomes in the credit market, and specifically we focus on quantities of loans. They study the effects of shocks to banks to firms' outcomes, such as employment. They assume that spillovers come from firms operating in the same region or sector. Our spillovers come from relationships that share the very same counterparty. In other words, they provide tools to account for spillovers among firms when the effects of financial shocks on real outcomes are analyzed. We provide a tool to model spillovers among credit relationships and account for them when the effects of financial shocks on credit outcomes are analyzed.

Focusing on credit outcomes implies another difference: we study outcomes at the bilateral level, because we look at credit market outcomes that are bilateral by construction, the other papers look at individual outcomes. This feature allows us to exploit the micro structure of the market for identification and infer on counterparties specific behaviors. It comes with non trivial differences in the complexity of the model and in the interpretation of the results. Indeed, we can provide a structural model that not only controls for spillovers, but also allows to recover parameters that have a direct behavioral interpretation in terms of credit substitution and reallocation, as discussed above.

Whereas these works focus on the impact of peers' treatment status (being more or less hit by shocks, i.e., contextual effects) in reduced form, we mainly focus on identifying and controlling for the effects of peers' outcomes (endogenous effects). It follows that our approach recovers parameters that can be precisely mapped to the primitives of a structural model, thus having a direct behavioral interpretation. Measuring these primitives is especially interesting in the case of credit relationships. The bank-firm relationship is a key determinant of financial shocks' pass-through, but its working is still a black box. Our model allows us to unpack the black box and uncover the importance of bank supply and firm demand reallocation across different relationships to determine the effect of shocks in equilibrium.

In summary, we focus on endogenous effects and they focus on exogenous effects. The endogenous effects capture agents' choices/outcomes that depend on that of others, and can be interpreted through assumed response functions of agents, while exogenous (or contextual) effects control for others' exogenous characteristics or treatment status. From this perspective, the two approaches can be useful complements to study the effects of financial shocks.