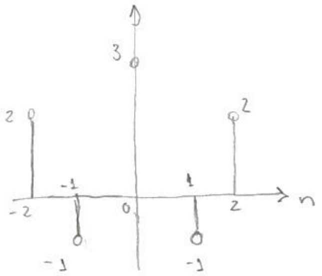
2) $x(n) = 2\,\delta(n+2) - \delta(n+1) + 3\,\delta(n) - \delta(n-1) + 2\,\delta(n-2)$



a) $X(e^{j\omega})\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}\Big|_{\omega=0} = x(n+2)\,\cancelto{1}{e^{-j0(2)}} + x(n+1)\,\cancelto{1}{e^{-j0(-1)}} +$

$x(n)\,\cancelto{1}{e^{-j0.0}} + x(n-1)\,\cancelto{1}{e^{-j0.1}} + x(n-2)\,\cancelto{1}{e^{-j0.2}} = 2 + (-1) + 3 + (-1) +$

$X(e^{j\omega})\Big|_{\omega=0} = 5$

b) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$

Se : $x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$

então : $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 2\pi \sum_{n=-\infty}^{\infty} x(n) = 2\pi\big(2+(-1)+3+(-1)+2\big) = 10\pi$

c) $X(e^{j\omega})\Big|_{\omega=\pi} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\pi n} = \sum_{-\infty}^{\infty} (-1)^n\, x(n) =$

$= (-1)^{-2} \cdot 2 + (-1)^{-1} \cdot (-1) + (-1)^0 \cdot 3 + (-1)^1 \cdot (-1) + (-1)^2 \cdot 2 = 9$

d) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x(n)|^2 = 2\pi(4+1+9+1+4) = 38\pi$

3) a) $x_1(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & n = 0, 2, 4, \dots \\ 0, & \text{caso contrário} \end{cases}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \frac{1}{1 - \frac{1}{4} e^{-j\omega 2}}$$

b) $x_2(n) = \alpha^n \operatorname{sen}(n\omega_0) u(n)$

$$\operatorname{sen}(n\omega_0) = \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}$$

$$x_2(n) = \frac{1}{2j}\left[\alpha^n e^{j\omega_0 n} - \alpha^n e^{-j\omega_0 n}\right] u(n)$$

Usando:

$$a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{TFTD} a_1 X_1(e^{j\omega}) + a_2 X_2(e^{j\omega})$$

$$x(n) e^{j\omega_0 n} \xleftrightarrow{TFTD} X(e^{j(\omega-\omega_0)})$$

$$X_2(e^{j\omega}) = \frac{1}{2j}\left[\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}} - \frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}\right]$$

$$X_2(e^{j\omega}) = \frac{1}{2j}\left[\frac{1+\alpha e^{-j(\omega-\omega_0)} - 1 + \alpha e^{-j(\omega+\omega_0)}}{1-\alpha^{-j(\omega-\omega_0)} - \alpha^{-j(\omega+\omega_0)} + \alpha^2 e^{-j2\omega}}\right]$$

$$X_2(e^{j\omega}) = \frac{\alpha \operatorname{sen}(\omega_0) e^{-j\omega}}{1 - 2\alpha\cos(\omega_0) e^{-j\omega} + \alpha^2 e^{-j2\omega}}$$

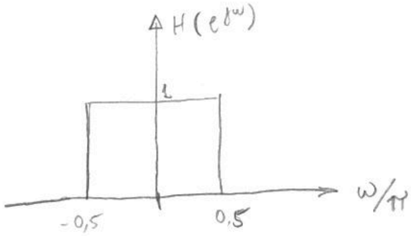
4) a) $x(n) = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} 1\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1 . e^{j\omega n}\, d\omega =$

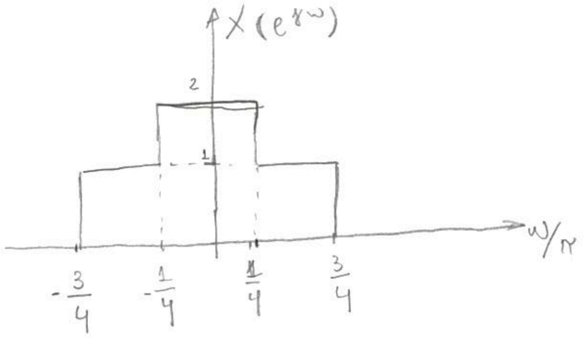
$$= \frac{1}{j2\pi n}\left(e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}\right) + \frac{1}{j2\pi n}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right) =$$

$$= \frac{\text{sen}\,(3/4\,\pi n)}{\pi n} + \frac{\text{sen}\,(1/4\,\pi n)}{\pi n}$$

$$= \frac{3}{4}\,\text{sinc}\left(\frac{3}{4}\pi n\right) + \frac{1}{4}\,\text{sinc}\left(\frac{1}{4}\pi n\right)$$

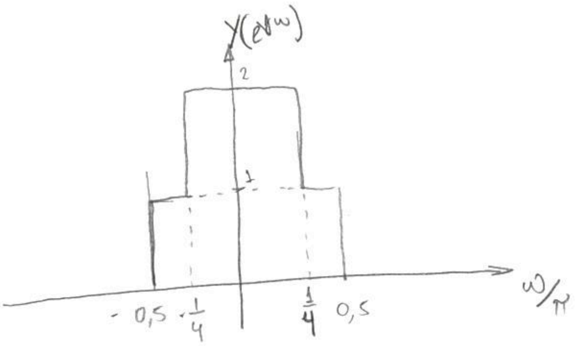
b)





Sendo:

$$Y(e^{j\omega}) = X(e^{j\omega}) . H(e^{j\omega})$$



c) $x_3(n) = x(n-K)$, sendo $x(n) = \mu(n) - \mu(n-K+1)$

Para $K > 1$

$$X(e^{j\omega}) = \sum_{n=0}^{K-2} e^{-j\omega n} = \frac{1 - e^{-j\omega(K-1)}}{1 - e^{-j\omega}} \cdot \frac{e^{j\omega\frac{(K-1)}{2}}}{e^{j\omega\frac{(K-1)}{2}}} \cdot \frac{e^{j\frac{\omega}{2}}}{e^{j\frac{\omega}{2}}}$$

$$= \frac{e^{j\omega\left(\frac{K-1}{2}\right)} - e^{-j\omega\left(\frac{K-1}{2}\right)}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} \cdot e^{-j\omega\left(\frac{K-1}{2}\right)} \cdot e^{j\frac{\omega}{2}} = \frac{e^{j\omega\left(\frac{K-1}{2}\right)} - e^{-j\omega\left(\frac{K-1}{2}\right)}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} \, e^{-j\omega\frac{K}{2}} \cdot e^{j\frac{\omega}{2}} \cdot e^{j\frac{\omega}{2}} =$$

$$= \frac{1}{2j} \cdot \frac{2j}{1} \cdot \frac{e^{j\omega\left(\frac{K-1}{2}\right)} - e^{-j\omega\left(\frac{K-1}{2}\right)}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} \cdot e^{-j\omega\frac{K}{2}} \cdot e^{j\omega} = \frac{\operatorname{sen}\left(\frac{\omega(K-1)}{2}\right)}{\operatorname{sen}(\omega/2)} e^{-j\frac{\omega K}{2}} \, e^{j\omega}$$

Usando a propriedade de deslocamento no tempo:

$$X_3(e^{j\omega}) = e^{-j\omega K} X(e^{j\omega}) = \frac{\operatorname{sen}\left(\frac{\omega(K-1)}{2}\right)}{\operatorname{sen}(\omega/2)} \cdot e^{-j\frac{3\omega K}{2}} \cdot e^{j\omega}$$

$$X_3(e^{j\omega}) = \frac{\operatorname{sen}\left(\frac{\omega(K-1)}{2}\right)}{\operatorname{sen}(\omega/2)} \cdot e^{-j\omega\left(\frac{3}{2}\omega - 1\right)}$$