Università della Svizzera italiana

Institute of Computational Science ICS

Numerical Computing **2020**

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Due date: Wednesday, 14 October 2020, 11:55 PM

Solution for Project 2

Submission instructions
(Please, notice that following instructions are mandatory: submissions that don't comply with, won't be considered)

- Assignments must be submitted to Moodle (i.e. in electronic format).
- Provide both executable package (single .class or .jar file) and sources (.java files). If you are using non-sdk libraries, please add them in the file. Sources must be organized in packages called:
ch.usi.inf.ncc12.assignment<assignmentNumber>.exercise<exerciseNumber>.<name>.<surname>
and the jar file must be called:
assignment<AssignmentNumber>.<Name>.<Surname>.jar
Projects exported directly from Eclipse would be much appreciated (Please, be sure that you are including also the sources in the jar file).
- The produced files (one pdf and one jar file) must be collected into a single archive file (.zip) named:
assignment<AssignmentNumber>.<Name>.<Surname>.zip

The purpose of this assignment[1] is to learn the importance of sparse linear algebra algorithms to solve fundamental questions in social network analyses. We will use the coauthor graph from the Householder Meeting and the social network of friendships from Zachary's karate club [1]. These two graphs are one of the first examples where matrix methods were used in computational social network analyses.

[1]This document is originally based on a blog from Cleve Moler, who wrote a fantastic blog post about the Lake Arrowhead graph, and John Gilbert, who initially created the coauthor graph from the 1993 Householder Meeting. You can find more information at `http://blogs.mathworks.com/cleve/2013/06/10/lake-arrowhead-coauthor-graph/`. Most of this assignment is derived from this archived work.

1. The Reverse Cuthill McKee Ordering [10 points]

We can see that the original matrix is symmetric and from the structure, we know this matrix will produce a disastrous fill-in. The Reverse Cuthill McKee algorithm is an effective way to reduce bandwidth and reduces the expected fill-in. In the RCM matrix we can see this effect and we'll have much less expensive computations.

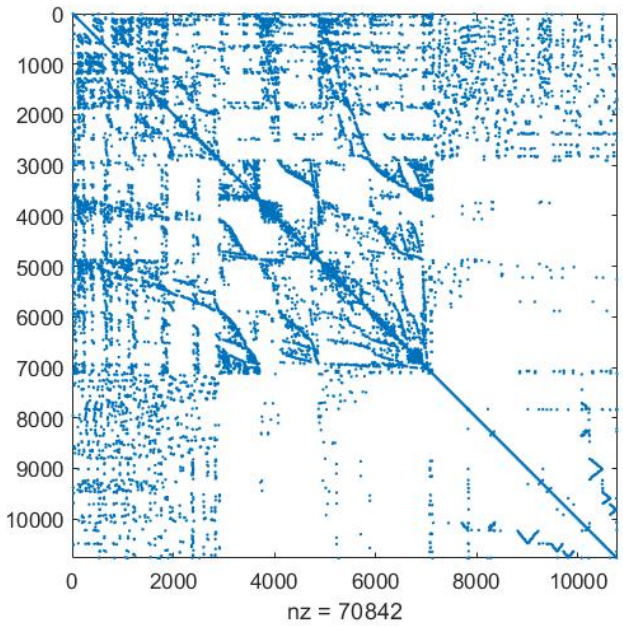

Figure 1. Original matrix A_SymPosDef

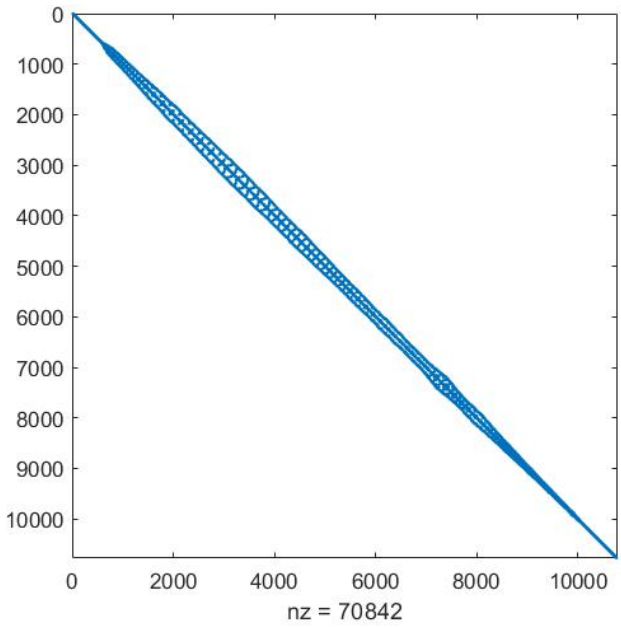

Figure 2. Reverse Cuthill McKee permuted matrix

With the Cholesky factor we have two upper triangular matrices, but since the RCM matrix has a lower bandwidth the overall number of nonzeros is smaller, improving the computations. Thus a good permutation will help performing the Cholesky factorization, which will result in space and time saving.

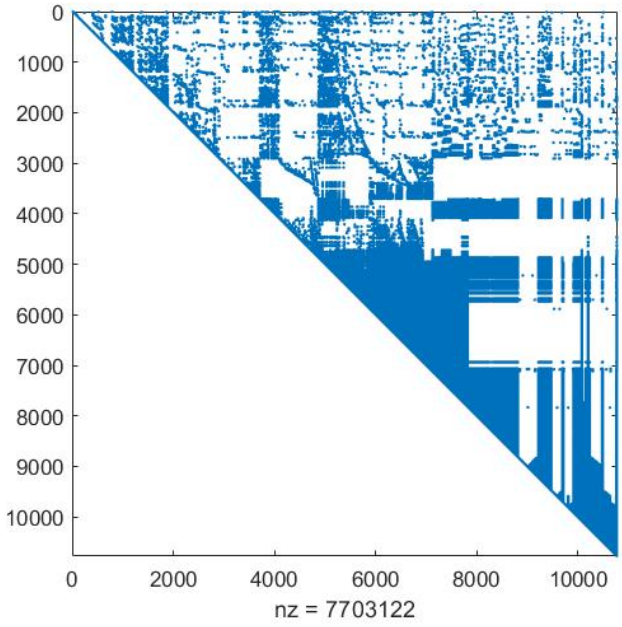

Figure 3. Cholesky factorization of A_SymPosDef

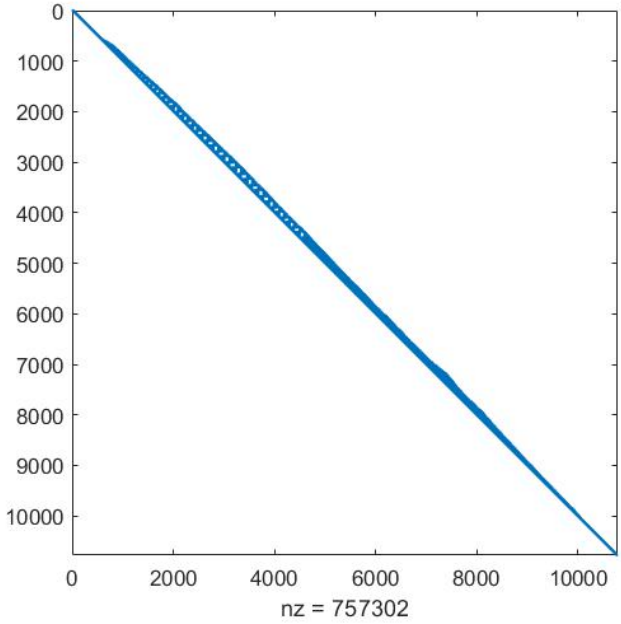

Figure 4. Cholesky factorization of the Reverse Cuthill McKee permuted matrix

2. Sparse Matrix Factorization [10 points]

- There are n^2 elements in matrix A. With $n < 4$ this holds because all elements are nonzero. For $4 \leq n$ we have non zero elements in the first row and column and the last row and column. We also have the diagonal with all nonzeros. We can see that the first row includes one element from the first column, one from the last column and one from the diagonal. If now we count the elements, we have n nonzero elements in the first row, n in the last row, $n-2$ in the first column and $n-2$ in the last column. We also add $n-2$ elements form diagonal, and summing everything we have:

$$n + n + (n-2) + (n-2) + (n-2) = n + n + 3(n-2) = n + n + 3n - 6 = 5n - 6$$

- $$A = \begin{bmatrix} 1 & \cdots & \cdots & \cdots & \cdots & 1 \\ \vdots & \ddots & 0 & 0 & 0 & \vdots \\ \vdots & 0 & 1 & 0 & 0 & \vdots \\ \vdots & 0 & 0 & 1 & 0 & \vdots \\ \vdots & 0 & 0 & 0 & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & \cdots & 1 \end{bmatrix}$$

- A matrix A with $n = 100000$ would be problematic to use chol(A) since it will produce a disastrous fill-in. Normally, Cholesky it's an optimal technique, but in this case the nonzero elements will blowup greatly increasing space complexity.

3. Degree Centrality [10 points]

output

```
Author: Golub                Degree: 31

Wilkinson
TChan
Varah
Overton
Ernst
VanLoan
Saunders
Bojanczyk
Dubrulle
George
Nachtigal
Kahan
Varga
Kagstrom
Widlund
OLeary
Bjorck
Eisenstat
Zha
VanDooren
Tang
Reichel
Luk
Fischer
Gutknecht
Heath
Plemmons
Berry
Sameh
Meyer
Gill
-------------------------------------------------
Author: Demmel               Degree: 15

Edelman
VanLoan
Bai
Schreiber
Kahan
Kagstrom
Barlow
NHigham
Arioli
```

```
Duff
Hammarling
Bunch
Heath
Greenbaum
Gragg
-------------------------------------------------
Author: Plemmons             Degree: 13

Golub
Nagy
Harrod
Pan
Funderlic
Bojanczyk
George
Barlow
Heath
Berry
Sameh
Meyer
Nichols
-------------------------------------------------
Author: Schreiber            Degree: 12

TChan
VanLoan
Moler
Gilbert
Pothen
NTrefethen
Bjorstad
NHigham
Eisenstat
Tang
Elden
Demmel
-------------------------------------------------
Author: Heath                Degree: 12

Golub
TChan
Funderlic
George
Gilbert
Eisenstat
Ng
Liu
Laub
```

```
Plemmons
Paige
Demmel
--------------------------------------------------
Author: TChan                Degree: 10

Golub
Saied
Ong
Kuo
Tong
Schreiber
Arioli
Duff
Heath
Hansen
--------------------------------------------------
Author: VanDooren            Degree: 10

Golub
Boley
Bojanczyk
Kagstrom
VanHuffel
Luk
Hammarling
Laub
Nichols
Paige
--------------------------------------------------
Author: Hammarling           Degree: 10

Wilkinson
Kaufman
Bai
Bjorck
VanHuffel
VanDooren
Duff
Greenbaum
Gill
Demmel
--------------------------------------------------
Author: Gragg                Degree: 9

Borges
Kaufman
Harrod
Reichel
```

```
Stewart
BunseGerstner
Ammar
Warner
Demmel
-------------------------------------------------
Author: VanLoan              Degree: 8

Golub
Moler
Schreiber
Kagstrom
Luk
Bunch
Paige
Demmel
-------------------------------------------------
Author: Moler                Degree: 8

Wilkinson
VanLoan
Gilbert
Schreiber
Henrici
Stewart
Bunch
Laub
-------------------------------------------------
Author: George               Degree: 7

Golub
Eisenstat
Ng
Liu
Tang
Heath
Plemmons
-------------------------------------------------
Author: Eisenstat            Degree: 7

Golub
Gu
George
Schreiber
Liu
Heath
Ipsen
-------------------------------------------------
Author: Luk                  Degree: 7
```

```
Golub
Overton
Boley
VanLoan
Bojanczyk
Park
VanDooren
-------------------------------------------------
Author: Gutknecht            Degree: 7

Golub
Ashby
Boley
NTrefethen
Nachtigal
Varga
Hochbruck
-------------------------------------------------
Author: Paige                Degree: 7

Anjos
VanLoan
Saunders
Bjorck
VanDooren
Laub
Heath
-------------------------------------------------
Author: NTrefethen           Degree: 6

Schreiber
Nachtigal
Reichel
Gutknecht
Greenbaum
ATrefethen
-------------------------------------------------
Author: Bjorck               Degree: 6

Golub
Park
Duff
Hammarling
Elden
Paige
-------------------------------------------------
Author: Reichel              Degree: 6
```

```
Golub
NTrefethen
Nachtigal
Fischer
Gragg
Ammar
-------------------------------------------------
Author: Stewart              Degree: 6

Moler
Bunch
Gragg
Meyer
Gill
Mathias
-------------------------------------------------
Author: Bunch                Degree: 6

LeBorne
Fierro
VanLoan
Moler
Stewart
Demmel
-------------------------------------------------
Author: Meyer                Degree: 6

Golub
Benzi
Funderlic
Stewart
Ipsen
Plemmons
-------------------------------------------------
Author: Gilbert              Degree: 5

Moler
Schreiber
Ng
Liu
Heath
-------------------------------------------------
Author: Varga                Degree: 5

Golub
Marek
Young
Gutknecht
Starke
```

```
------------------------------------------------
Author: Kagstrom             Degree: 5

Golub
VanLoan
VanDooren
Ruhe
Demmel
------------------------------------------------
Author: Barlow               Degree: 5

Zha
Ipsen
Plemmons
Nichols
Demmel
------------------------------------------------
Author: Widlund              Degree: 5

Golub
Bjorstad
OLeary
Smith
Szyld
------------------------------------------------
Author: Zha                  Degree: 5

Golub
Bai
Barlow
VanHuffel
Hansen
------------------------------------------------
Author: Park                 Degree: 5

Boley
Bjorck
VanHuffel
Luk
Elden
------------------------------------------------
Author: Liu                  Degree: 5

George
Gilbert
Eisenstat
Ng
Heath
------------------------------------------------
```

```
Author: Duff                Degree: 5

TChan
Bjorck
Arioli
Hammarling
Demmel
-------------------------------------------------
Author: Laub                Degree: 5

Kenney
Moler
VanDooren
Heath
Paige
-------------------------------------------------
Author: Ipsen               Degree: 5

Chandrasekaran
Barlow
Eisenstat
Meyer
Jessup
-------------------------------------------------
Author: Greenbaum           Degree: 5

Cullum
Strakos
NTrefethen
Hammarling
Demmel
-------------------------------------------------
Author: Nichols             Degree: 5

Byers
Barlow
VanDooren
Plemmons
BunseGerstner
-------------------------------------------------
Author: Wilkinson           Degree: 4

Golub
Dubrulle
Moler
Hammarling
-------------------------------------------------
Author: Boley               Degree: 4
```

```
Park
VanDooren
Luk
Gutknecht
--------------------------------------------------
Author: Harrod               Degree: 4

Plemmons
Gragg
Berry
Sameh
--------------------------------------------------
Author: Bojanczyk            Degree: 4

Golub
VanDooren
Luk
Plemmons
--------------------------------------------------
Author: Nachtigal            Degree: 4

Golub
NTrefethen
Reichel
Gutknecht
--------------------------------------------------
Author: VanHuffel            Degree: 4

Zha
Park
VanDooren
Hammarling
--------------------------------------------------
Author: Arioli               Degree: 4

TChan
MuntheKaas
Duff
Demmel
--------------------------------------------------
Author: Ng                   Degree: 4

George
Gilbert
Liu
Heath
--------------------------------------------------
Author: Hansen               Degree: 4
```

```
TChan
Fierro
OLeary
Zha
--------------------------------------------------
Author: BunseGerstner        Degree: 4

He
Byers
Gragg
Nichols
--------------------------------------------------
Author: Berry                Degree: 4

Golub
Harrod
Plemmons
Sameh
--------------------------------------------------
Author: Sameh                Degree: 4

Golub
Harrod
Plemmons
Berry
--------------------------------------------------
Author: Gill                 Degree: 4

Golub
Saunders
Hammarling
Stewart
--------------------------------------------------
Author: Kaufman              Degree: 3

Hammarling
Gragg
Warner
--------------------------------------------------
Author: Funderlic            Degree: 3

Heath
Plemmons
Meyer
--------------------------------------------------
Author: Saunders             Degree: 3

Golub
Paige
```

```
Gill
-------------------------------------------------
Author: Bai                  Degree: 3

Zha
Hammarling
Demmel
-------------------------------------------------
Author: Kahan                Degree: 3

Golub
Davis
Demmel
-------------------------------------------------
Author: Bjorstad             Degree: 3

Schreiber
Widlund
Boman
-------------------------------------------------
Author: OLeary               Degree: 3

Golub
Widlund
Hansen
-------------------------------------------------
Author: NHigham              Degree: 3

Schreiber
Pothen
Demmel
-------------------------------------------------
Author: Tang                 Degree: 3

Golub
George
Schreiber
-------------------------------------------------
Author: Fischer              Degree: 3

Golub
Modersitzki
Reichel
-------------------------------------------------
Author: Elden                Degree: 3

Schreiber
Bjorck
Park
```

```
------------------------------------------------
Author: Ammar                Degree: 3

He
Reichel
Gragg
------------------------------------------------
Author: He                   Degree: 2

BunseGerstner
Ammar
------------------------------------------------
Author: Overton              Degree: 2

Golub
Luk
------------------------------------------------
Author: Byers                Degree: 2

BunseGerstner
Nichols
------------------------------------------------
Author: Fierro               Degree: 2

Bunch
Hansen
------------------------------------------------
Author: Dubrulle             Degree: 2

Golub
Wilkinson
------------------------------------------------
Author: Marek                Degree: 2

Varga
Szyld
------------------------------------------------
Author: Kuo                  Degree: 2

TChan
Tong
------------------------------------------------
Author: Tong                 Degree: 2

TChan
Kuo
------------------------------------------------
Author: Pothen               Degree: 2
```

```
Schreiber
NHigham
--------------------------------------------------
Author: Young                Degree: 2

Kincaid
Varga
--------------------------------------------------
Author: Szyld                Degree: 2

Marek
Widlund
--------------------------------------------------
Author: Ruhe                 Degree: 2

Wold
Kagstrom
--------------------------------------------------
Author: Warner               Degree: 2

Kaufman
Gragg
--------------------------------------------------
Author: Jessup               Degree: 2

Crevelli
Ipsen
--------------------------------------------------
Author: Hochbruck            Degree: 2

Gutknecht
Starke
--------------------------------------------------
Author: Starke               Degree: 2

Varga
Hochbruck
--------------------------------------------------
Author: Varah                Degree: 1

Golub
--------------------------------------------------
Author: Kenney               Degree: 1

Laub
--------------------------------------------------
Author: Ashby                Degree: 1

Gutknecht
```

```
------------------------------------------------
Author: LeBorne               Degree: 1

Bunch
------------------------------------------------
Author: Modersitzki           Degree: 1

Fischer
------------------------------------------------
Author: Ernst                 Degree: 1

Golub
------------------------------------------------
Author: Borges                Degree: 1

Gragg
------------------------------------------------
Author: Kincaid               Degree: 1

Young
------------------------------------------------
Author: Crevelli              Degree: 1

Jessup
------------------------------------------------
Author: Anjos                 Degree: 1

Paige
------------------------------------------------
Author: Benzi                 Degree: 1

Meyer
------------------------------------------------
Author: Gu                    Degree: 1

Eisenstat
------------------------------------------------
Author: Nagy                  Degree: 1

Plemmons
------------------------------------------------
Author: Pan                   Degree: 1

Plemmons
------------------------------------------------
Author: Edelman               Degree: 1

Demmel
------------------------------------------------
```

```
Author: Cullum               Degree: 1

Greenbaum
--------------------------------------------------
Author: Strakos              Degree: 1

Greenbaum
--------------------------------------------------
Author: Saied                Degree: 1

TChan
--------------------------------------------------
Author: Ong                  Degree: 1

TChan
--------------------------------------------------
Author: Wold                 Degree: 1

Ruhe
--------------------------------------------------
Author: Chandrasekaran       Degree: 1

Ipsen
--------------------------------------------------
Author: Boman                Degree: 1

Bjorstad
--------------------------------------------------
Author: MuntheKaas           Degree: 1

Arioli
--------------------------------------------------
Author: Smith                Degree: 1

Widlund
--------------------------------------------------
Author: Henrici              Degree: 1

Moler
--------------------------------------------------
Author: ATrefethen           Degree: 1

NTrefethen
--------------------------------------------------
Author: Davis                Degree: 1

Kahan
--------------------------------------------------
Author: Mathias              Degree: 1
```

```
Stewart
-------------------------------------------------
```

4. The Connectivity of the Coauthors [10 points]

output

```
Golub and Moler common coauthors 2:

Wilkinson
VanLoan
-------------------------------------------------

Golub and Saunders common coauthors 1:

Gill
-------------------------------------------------

TChan and Demmel common coauthors 4:

Schreiber
Arioli
Duff
Heath
-------------------------------------------------
```

5. PageRank of the Coauthor Graph [10 points]

output

page-rank	in	out	author
0.0630	31	31	Golub
0.0312	15	15	Demmel
0.0269	13	13	Plemmons
0.0249	12	12	Schreiber
0.0236	10	10	TChan
0.0230	12	12	Heath
0.0207	9	9	Gragg
0.0203	10	10	Hammarling
0.0195	10	10	VanDooren
0.0171	8	8	Moler
0.0171	7	7	Gutknecht
0.0159	8	8	VanLoan
0.0150	7	7	Eisenstat

0.0147	7	7	Paige
0.0144	6	6	NTrefethen
0.0143	5	5	Varga
0.0141	6	6	Meyer
0.0140	6	6	Stewart
0.0139	7	7	Luk
0.0139	6	6	Bunch
0.0139	5	5	Widlund
0.0137	6	6	Reichel
0.0137	7	7	George
0.0137	5	5	Ipsen
0.0133	5	5	Greenbaum
0.0122	6	6	Bjorck
0.0113	5	5	Kagstrom
0.0112	5	5	Nichols
0.0112	5	5	Laub
0.0110	5	5	Barlow
0.0108	5	5	Zha
0.0108	5	5	Duff
0.0104	5	5	Park
0.0101	4	4	BunseGerstner
0.0101	4	4	Arioli
0.0101	5	5	Gilbert
0.0100	5	5	Liu
0.0098	4	4	Hansen
0.0092	4	4	Nachtigal
0.0090	3	3	Bjorstad
0.0089	4	4	Wilkinson
0.0088	4	4	Harrod
0.0088	4	4	Gill
0.0086	4	4	Berry
0.0086	4	4	Sameh
0.0086	4	4	Boley
0.0084	4	4	VanHuffel
0.0083	3	3	Fischer
0.0083	4	4	Bojanczyk
0.0081	4	4	Ng
0.0081	3	3	Kahan
0.0080	2	2	Young
0.0078	2	2	Jessup
0.0078	3	3	Ammar
0.0076	3	3	OLeary
0.0075	3	3	Kaufman
0.0072	3	3	NHigham
0.0072	2	2	Ruhe
0.0068	3	3	Saunders
0.0068	3	3	Funderlic
0.0068	3	3	Bai
0.0067	2	2	Marek

```
0.0067    3    3   Elden
0.0067    2    2   Szyld
0.0066    3    3   Tang
0.0065    2    2   Starke
0.0063    2    2   Hochbruck
0.0060    2    2   Kuo
0.0060    2    2   Tong
0.0058    2    2   He
0.0055    2    2   Warner
0.0055    2    2   Byers
0.0055    2    2   Fierro
0.0052    2    2   Pothen
0.0051    2    2   Dubrulle
```

---

6. Zachary's karate club: social network of friendships between 34 members [50 points]

output

- ```
  1: Node 34
     Degree: 17

  2: Node 1
     Degree: 16

  3: Node 33
     Degree: 12

  4: Node 3
     Degree: 10

  5: Node 2
     Degree: 9
  ```

---

output

- ```
       page-rank  in  out
  34     0.1009   17   17
   1     0.0970   16   16
  33     0.0717   12   12
   3     0.0571   10   10
   2     0.0529    9    9
  ```

- The rankings are identical because we are measuring the importance of the nodes using two different metrics. For Pagerank we have the nodes with highest inbound (and here every in edge is also an out edge) and the degree centrality measures the number of links incident upon a node, essentially the same.

output

- ```
  First Group:
    17   6   7   5  11  12   1  13  22  18   4   8   2  14  20   3
  Second Group:
     9  31  10  29  32  34  28  33  25  24  26  21  23  19  15  16  30  27
  ```

The spectral bisection divides the network in the same way as the real partition.

References

[1] The social network of a karate club at a US university, M. E. J. Newman and M. Girvan, Phys. Rev. E 69,026113 (2004) pp. 219-229.
  ```