**FACULITY OF ENGINEERING**

**A PURKINJE CELL MODEL THAT SIMULATES COMPLEX SPIKES**

**BY**

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Declaration and statements of student in respect of their submitted work

I declare that the work in this dissertation was carried out in accordance with the requirements of the University’s Regulations and Code of Practice for Taught Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, this work is my own work. Work done in collaboration with, or with the assistance of others, is indicated as such. I have identified all material in this dissertation which is not my own work through appropriate referencing and acknowledgement. Where I have quoted or otherwise incorporated material, which is the work of others, I have included the source in the references. Any views expressed in the dissertation, other than referenced material, are those of the author.

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**Abstract**

Purkinje cells are the principal neurons of the cerebellar cortex. One of their distinguishing features is that they fire two distinct types of action potential, called simple and complex spikes, which interact with one another. Simple spikes are stereotypical action potentials that are elicited at high, but variable, rates (0 − 100 Hz) and have a consistent waveform. Complex spikes are composed of an initial action potential followed by a burst of lower amplitude spikelets. Complex spikes occur at comparatively low rates (∼ 1 Hz) and have a variable waveform. Although, they are critical to all motor coordination in the cerebellar cortex, a simple model that describes the complex spike waveform is lacking. ++

**Keywords: Purkinje cells, neurons, mathematical model, complex spikes, simulation**

Word count: N/A

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1. ***Introduction***

Located in the cerebellum and being amongst the largest neurons in the central nervous system [1], Purkinje cells are extraordinary and instantly distinguishable from other brain neurons for their complex flat dendritic trees and for their ability to fire to distinct types of action potential, namely simplex and complex spikes. Purkinje cells have a crucial role in the motor coordination and other areas of function such as cognition and emotion. The purpose of this paper is to describe a simple mathematical model that can simulate these complex spikes. ++

1. ***Methods***

In this paper a model of the Purkinje cell somatic voltage dynamics is presented with a three-current model, which has a leak channel and two voltage-gated channels:

( 1 )

where Cm = 1 μF / cm2 is the membrane capacitance, is a leak current,

( 2 )

with = 0.3 mS / cm2 and = 10.613 mV. INa and IK stand for the sodium current and the potassium current. The most commonly used sign convention in neural modeling is that ionic current flowing out of the cell is positive and ionic current flowing into the cell is negative [2].

* 1. **Ion channels**

The current through the sodium channel was modelled using a Markovian scheme developed by Raman and Bean [3] , as seen in Table 1. This scheme models the dynamics of both the transient and resurgent gate. The sodium current is:

( 3 )

Where *o* is the fraction of gates in the open state O, = 120.0 mScm−2 and VNa = 115 mV

Diagram, schematic

Description automatically generated with medium confidence

Table 1: Current through the resurgent sodium channel is described using a Markovian Scheme.

Where

( 4 )

( 5 )

( 6 )

and γ = 150 ms−1 , δ = 40 ms−1 , ε = 1.75 ms−1 , D = 0.005 ms−1 , U = 0.5 ms−1 , N = 0.75 ms−1 , F = 0.005 ms−1 with

( 7 )

The potassium current is:

( 8 )

Where , = 36.0 mScm−2 and VK = -12 mV. *n* is described with the usual Hodgkin-Huxley channel equations(Eq 17, Eq 18) described in 2.2.

* 1. **Hodgkin-Huxley model**

The **Hodgkin–Huxley model**, or **conductance-based model**, is a mathematical model that describes how action potentials in neurons are initiated and propagated. It is a set of nonlinear differential equations that approximates the electrical characteristics of excitable cells such as neurons. It is a continuous-time dynamical system.

The typical Hodgkin–Huxley model treats each component of an excitable cell in this case a Purkinje cell as an electrical element. The lipid bilayer is represented as a capacitance (Cm). Voltage-gated ion channels are represented by electrical conductances (, for the potassium channel and for the sodium channel) that depend explicitly on voltage. Leak channels are represented by linear conductances (). Finally, the membrane potential is denoted by *Vm,* and it is time dependent.

Thus, for a cell with sodium and potassium channels, the total current through the membrane is given by:

( 9 )

The Hodgkin and Huxley developed a model in which the properties of an excitable cell are described by a set of four ordinary differential equations. Together with the equation for the total current mentioned above, these are:

( 10 )

( 11 )

( 12 )

( 13 )

where *I* is the current per unit area, and α{\displaystyle \alpha \_{i}} and {\displaystyle \beta \_{i}}β are rate constants for the each ion channel, which depend on voltage but not time. There are three gating variables: n for the Potassium channel activation, m for the Sodium channel activation (opening) and h for the Sodium channel inactivation (closing). The value of a gating variables is dimensionless and will vary between 0 and 1; 0 indicates that the channel is closed, whereas 1 indicates that the channel is open. The gating variable fraction is an indication of the conductance of a certain ion at a given time and membrane voltage

( 14 )

( 15 )

( 16 )

(n,m,h)∞ are the steady state values for activation and are usually represented as a function of Vm.

Chart, histogram

Description automatically generated

Figure 1: Potassium and Sodium channels as function of time{\displaystyle V\_{m}}

One representation of the Hodgkin–Huxley model can be thought of as a differential equation system with four state variables, {\displaystyle V\_{m}(t),n(t),m(t)}V(t), n(t), m(t) and h(t){\displaystyle h(t)}, that change with respect to time{\displaystyle t}. The system is difficult to study because it is a nonlinear system and cannot be solved analytically. However, certain properties and general behaviors, such as limit cycles(closed trajectories), can be proven to exist using numerical methods. A picture containing chart

Description automatically generated

Figure 2: Limit cycles of the gating variables as a function of Voltage

Channel kinetics are described with the original Hodgkin-Huxley channel equations and constants with, α, β being the forward and backwards rate in ms-1, respectively.

( 17 )

( 18 )

( 19 )

( 20 )

( 21 )

( 22 )

One other technical note is that certain function forms can become indeterminate at certain voltage values(Nelson05-ElectrophysModels). Given a specific voltage, an(Vm) and am (Vm) may evaluate to the indeterminate form 0/0. The solution to this problem is to apply L’Hospital’s rule [4], which states that if f(x) and g(x) approach zero as x approaches a, and f ‘(x)/ g’(x) approaches L as x approaches a, then the ratio f (x)/ g(x) approaches L as well. Using this rule, it can be shown that an(10) = 0.1 and am(25) = 0.430825375 . Also, the limits for every case are hand-tuned for simplicity and efficiency.

All the input parameters listed in Table 2, are defined in Python.

|  |  |  |
| --- | --- | --- |
| ***Parameters*** | ***Chosen Value*** | ***Reference*** |
| Cm (μF/cm2) | 1.0 | [5] |
| (mS/cm­­2) | 120.0 | [5] |
| (mS/cm­­2) | 36.0 | [5] |
| (mS/cm­­2) | 0.3 | [5] |
| Vm (mV) | 0.0 | [5] |
| VNa (mV) | 115.0 | \* [5] |
| VK (mV) | -12.0 | \* [5] |
| Vl (mV) | 10.613 | \* [5] |
| VThreshold (mV) | 55.0 | \* [5] |
| Iinj (μA/cm2) | 10.0 |  |
| Tmax (ms) | 35.0 |  |

Table 2: Experimental Constants for the Hodgkin and Huxley model. The parameters with a star (\*) have been hand-tuned to have their signs follow the sign convention see Section 3.1.

Chart, line chart

Description automatically generated

Figure 3: Neuron potential as a function of time

1. ***Implementation and fitting***

Simulations were run in Python 3.9.10, OS on a CPU. Models were simulated as ordinary differential equations and integration was performed explicitly using the scipy.integrate.odeint package suitable for stiff problems or non-stiff problems of first order ode-s. One second of Purkinje cell activity can be simulated in N/A minute N/A seconds.

* 1. **Sign Convention**

One issue while implementing the Hodgkin-Huxley model was the voltage sign convention. The modern convention is that depolarization makes the membrane potential Vm more positive [6]. However, Hodgkin and Huxley [5] used the opposite sign convention (depolarization negative) in their paper. Figures in this paper use the modern convention that depolarization is positive. The modern convention is chosen in order to display a more modern example of the Hodgkin-Huxley model. To achieve this, the rate constants and the voltage values have been adjusted, by multiplying with -1.

At a conceptual level, the choice of conventions for currents and voltages is inconsequential, however at the implementation level it matters a great deal, since inconsistencies will cause the model to behave incorrectly. The most important thing in choosing conventions is to ensure that the choices are internally consistent. One must pay careful attention Fig. 2 Simulated voltage-clamp data illustrating voltage-dependent properties of the K+ conductance in squid giant axon. The command voltage Vc(mV) is shown in the lower panel and the K+ current in the upper panel. Simulation parameters are from the Hodgkin and Huxley model (1952). Nelson, M.E. (2004) Electrophysiological Models In: Databasing the Brain: From Data to Knowledge. (S. Koslow and S. Subramaniam, eds.) Wiley, New York. to these issues when implementing a simulation using equations from a published model, since it may be necessary to convert the empirical results reported using one set of conventions into a form that is consistent with one’s own model conventions.--

* 1. **Error minimization**

Error function Markov and HH model

Optimization was performed using a mixture of error minimization and hand-tuning techniques.++

1. ***Results***
2. ***Discussion***

**Code Availability Statement:**

The Python source code used to create the figures and the modelling is available at github.com/PurkinjeCell.

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