MATLAB Problem 3.1 - Beamforming Application

(a)
An optimal beamformer with unit response along $\theta_0 = 30°$ (DOI, direction of interest) is designed. The simulations are carried out for 50 different baseband signals, varying the baseband frequency f from 50 to 4950 Hz in 100 Hz steps. For the case that the direction of arrival (DOA) equals the DOI, we find the desired unit response of the beamformer. The more the DOA differs from the DOI, the more the incoming waves are suppressed.

Plotting the arriving signals at the 4 antennas for a DOA = 30° we observe the expected propagation delay.

(b)
In task (b) we simulate the scenario of two waves arriving (simultaneously) from different directions. The first is a cosine modulated carrier wave with DOA = DOI = 30°. The second is a sine modulated carrier wave with DOA = 45°. The cosine baseband frequency is denoted as $f_1 = 2 \cdot f_2$, where f_2 is the baseband frequency of the sine modulator. As expected the sine modulated wave arriving from 45 ° is suppressed. The following plot shows that the beamformer output and its envelope clearly correlate more with the cosine baseband signal (DOA=DOI).

(c)
Until now, the noise at the four antennas was assumed to be spatially uncorrelated. Now, we assume the noise signals across the antennas to be spatially correlated with an exponential autocorrelation function.

$$r_{vv}[k] = 0.85^{|k|} \tag{1}$$

The noise vector is obtained by applying a Cholesky-factorization to the autocorrelation matrix $\boldsymbol{R}_{vv} = \boldsymbol{P}\boldsymbol{P}^H$. We calculate the noise vector as $\boldsymbol{v} = \boldsymbol{P}\boldsymbol{e}$, where $\boldsymbol{e}$ is a vector that fulfills the whiteness property.
Repeating tasks a) and b) we find improvement in performance, but face computational problems, as the matrix R_{yy} gets close to being singular (non-invertible). If the noise across the antennas is correlated (in an extreme case, same noise across antennas), the difference in antenna signals is more due to the deterministic propagation delay which allows the beamformer to create "sharper" beams.

(d)

In task d) we generate polar plots of the beamformer output power. Therefore, the simulation varies the DOA from 1° to 180° in 1° steps. First, we carried out the simulations for a beamformer setup with 4 antennas. Increasing the number of antennas to 25 shows increased directivity/performance.

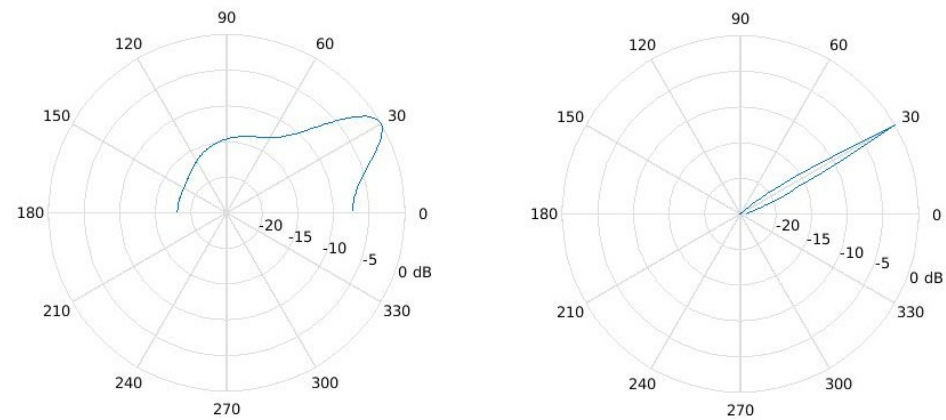

Figure 1: Output power in dB normalized with respect to the DOI = 30°, 4 antennas (left) vs. 25 antennas (right)

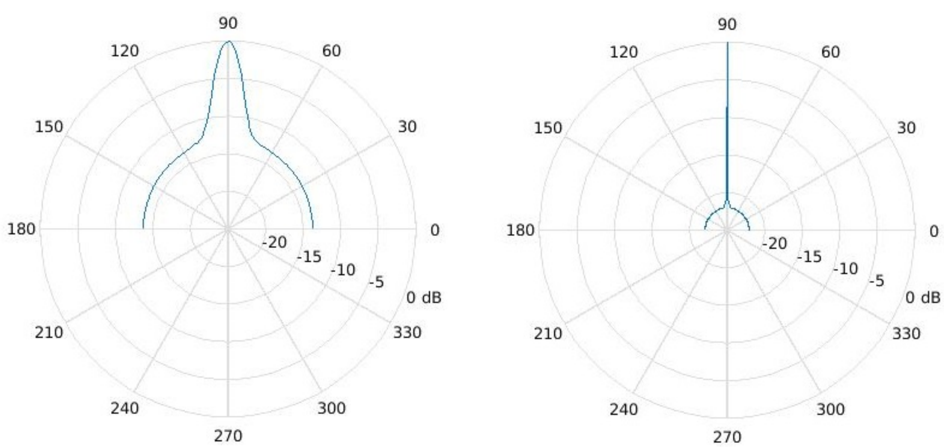

Figure 2: Output power in dB normalized with respect to the DOI = 90°, 4 antennas (left) vs. 25 antennas (right)