

Distributed, Real-Time Bayesian Learning in Online Services

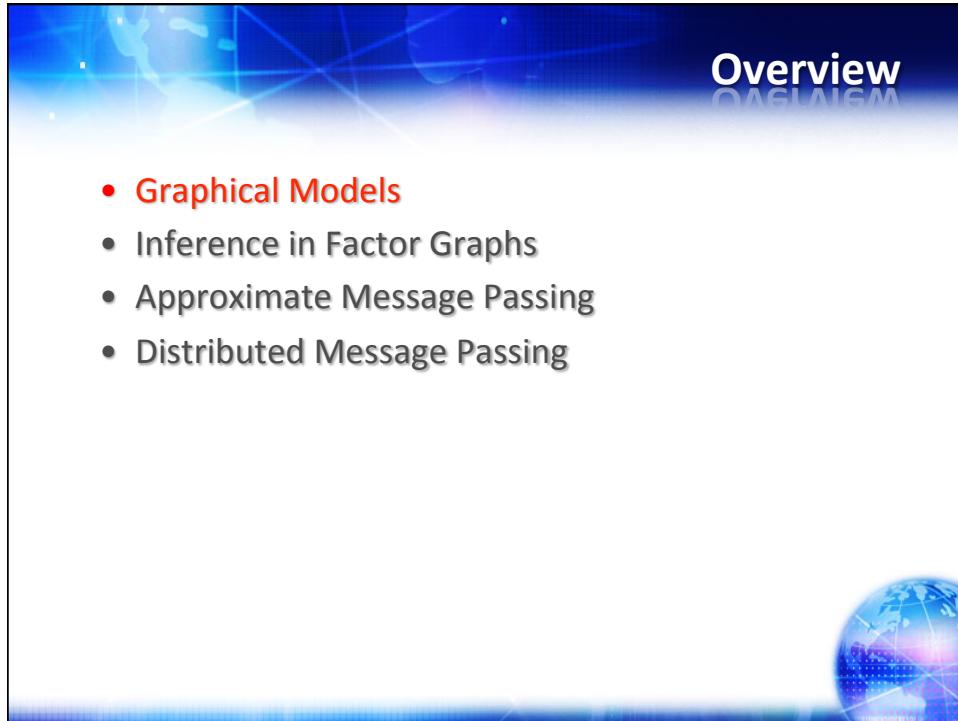
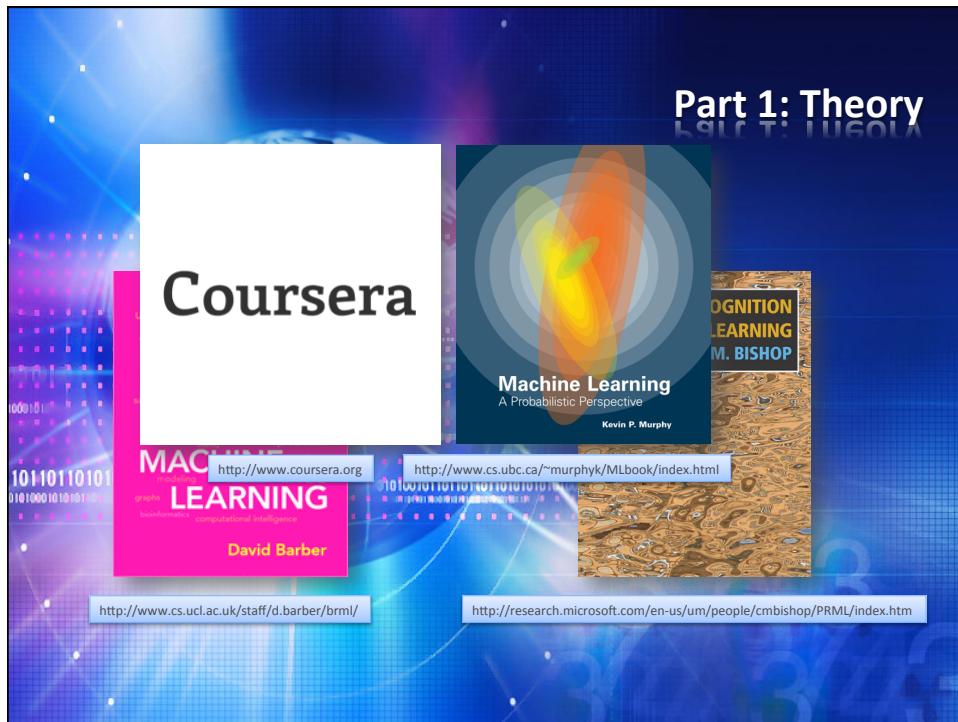
Ralf Herbrich
Amazon



Overview

- Part 1: Theory
 - Graphical Models
 - Inference in Factor Graphs
 - Approximate Message Passing
 - Distributed Message Passing
- Part 2: Applications
 - TrueSkill: Gamer Rating and Matchmaking
 - TrueSkill Through Time: History of Chess
 - Click-Through Rate Prediction in Online Advertising
 - Matchbox: Recommendation Systems





Probabilities and Beliefs

- **Design:** System must assign degree of plausibility $P(A)$ to each logical statement A.
- **Axiom:**

P must be a probability measure!

3. $P(A|C') > P(A|C)$ and $P(B|AC') = P(B|AC)$ then
 $P(AB|C') \geq P(AB|C)$



Infer-Predict-Decide Cycle

Decision Making:

$\text{Loss}(\text{Action}, \text{Data}) + P(\text{Data}) \rightarrow \text{Action}$

- Business-loss not learning-loss!
- Often involves optimization!

Inference:

$P(\text{Parameters}) + \text{Data} \rightarrow P(\text{Parameters} | \text{Data})$

- Requires a (structural) model $P(\text{Data} | \text{Parameters})$
- Allows to incorporate prior information $P(\text{Parameters} | \text{Data})$

Prediction:

$P(\text{Parameters}) + \text{Data} \rightarrow P(\text{Data})$

- Requires integration/summation of parameter uncertainty
- Does not change state!



Graphical Models

- **Definition:** Graphical representation of joint probability distribution
 - Nodes:  = Variables
 - Edges: Relationship between variables
- **Variables:**
 - Observed Variables: Data
 - Unobserved Variables: ‘Causes’ + Temporary/Latent
- **Key Questions:**
 - (Conditional) Dependency: $p(a, b|c) \stackrel{?}{=} p(a|c) \cdot p(b|c)$
 - Inference/Marginalisation: $p(a, b) = \sum_c p(a, b, c)$



Directed Models: Bayesian Networks

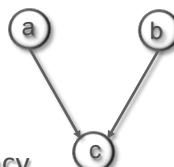
- **Definition:** Graphical representation of joint probability distribution (Pearl, 1988)
 - Nodes:  = Variables
 - Directed Edges: Conditional probability distribution

- **Semantic:**

$$p(\mathbf{x}) = \prod_i p(x_i | \mathbf{x}_{\text{parents}(i)})$$

- Ancestral relationship of dependency

$$p(a, b, c) = p(a) \cdot p(b) \cdot p(c|a, b)$$



Undirected Models: Markov Networks

- **Definition:** Graphical representation of joint probability distribution (Pearl, 1988)

– Nodes: = Variables

– Edges: Dependency between variables

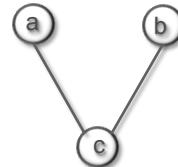
- **Semantic:**

$$p(\mathbf{x}) = \frac{1}{Z} \cdot \prod_c \phi(x_c) \quad \phi \geq 0$$

– Local potentials over cliques

$$p(a, b, c) = \frac{1}{Z} \cdot \phi_{ac}(a, c) \cdot \phi_{bc}(b, c)$$

$$Z = \sum_a \sum_b \sum_c \phi_{ac}(a, c) \cdot \phi_{bc}(b, c)$$



Factor Graphs

- **Definition:** Graphical representation of product structure of a function (Wiberg, 1996)

– Nodes: = Factors = Variables

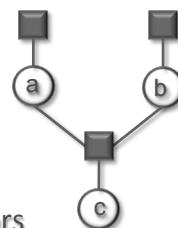
– Edges: Dependencies of factors on variables.

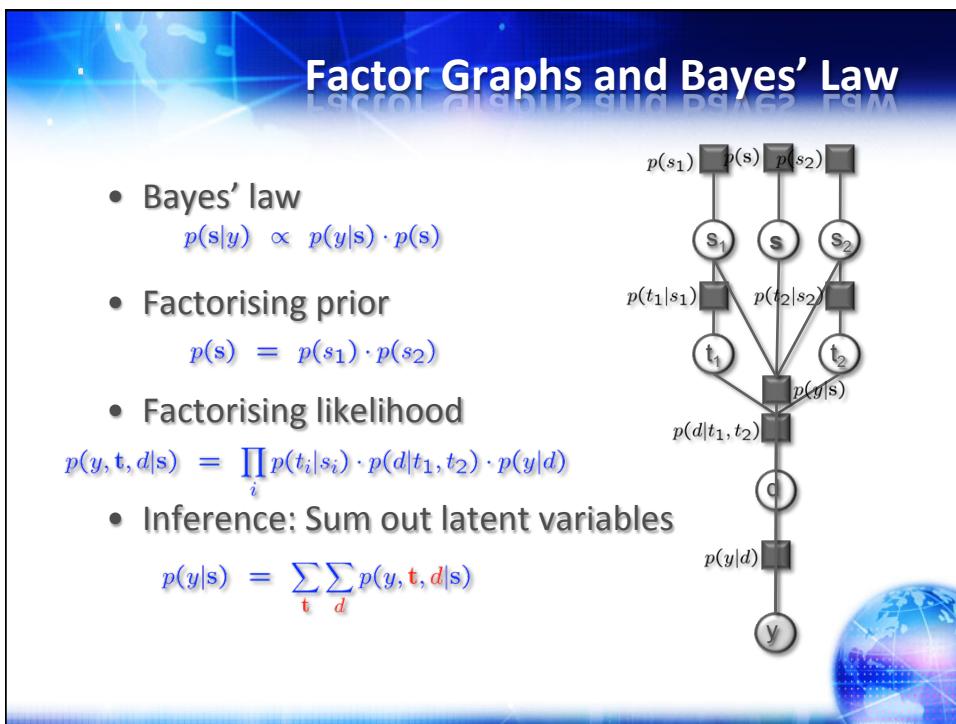
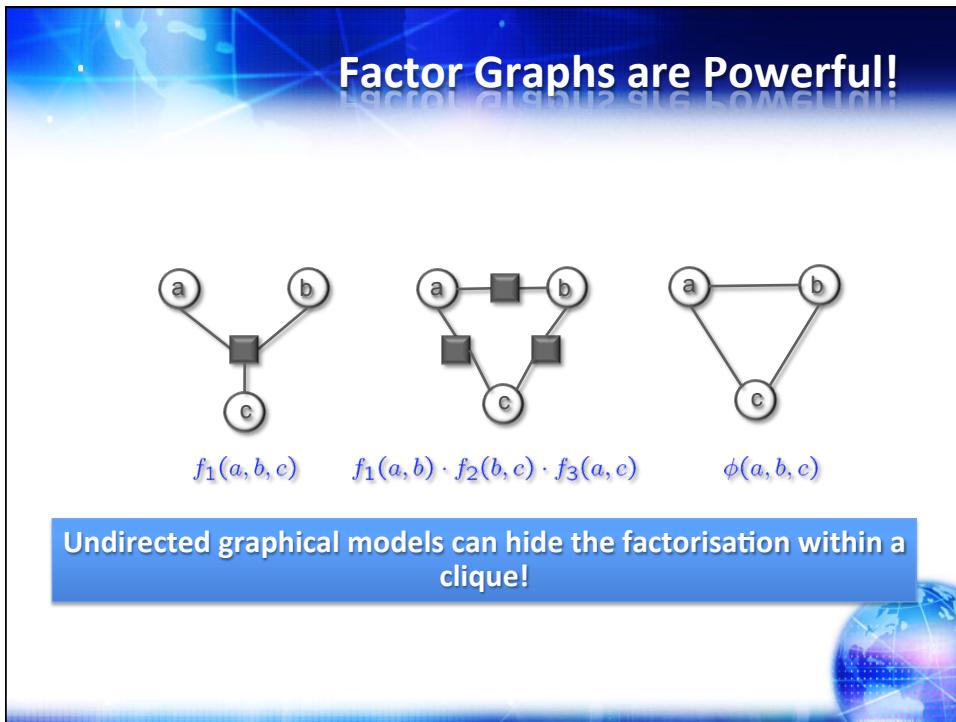
- **Semantic:**

$$p(\mathbf{x}) = \prod_f f(\mathbf{x}_{V(f)})$$

– Local variable dependency of factors

$$p(a, b, c) = f_1(a) \cdot f_2(b) \cdot f_3(a, b, c)$$





Summary

	Dependency	Efficient Inference	Usage
Bayesian Networks	Yes	Somewhat	Ancestral Generative Process
Markov Networks	Yes	No	Local Couplings and Potentials
Factor Graphs	No	Yes	Efficient, distributed inference

Overview

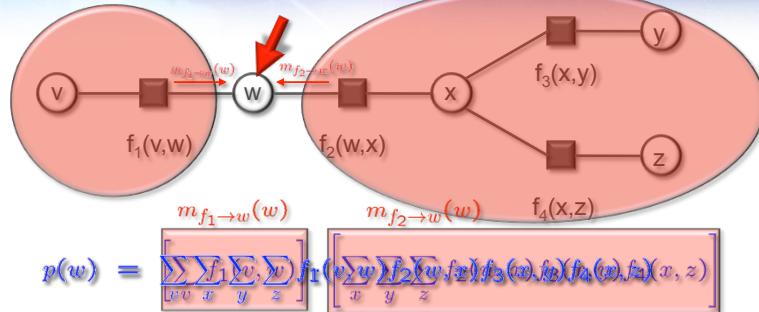
- Graphical Models
- Inference in Factor Graphs
- Approximate Message Passing
- Distributed Message Passing

Factor Graphs and Factor Trees

- **Factor Graphs:** Arbitrary functions
 - Bayesian Networks
 - Markov Networks
- **Factor Trees:** Functions where the variable indices never decrease from left to right
- **Factor Graph → Factor Tree:**
 1. Pick an arbitrary node
 2. Build the spanning tree



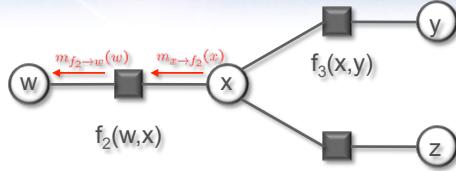
Factor Trees: Separation



Observation: Sum of products becomes product of sums of all messages from neighbouring factors to variable!



Messages: From Factors To Variables

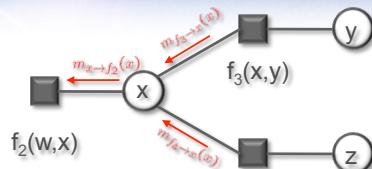


$$m_{f_2 \rightarrow w}(w) = \sum_x \sum_y \sum_z f_2 \left(w \sum_y \sum_z f_3(x, y) f_4(x, z) \right)$$

Observation: Factors only need to sum out all their local variables!



Messages: From Variables To Factors



$$m_{x \rightarrow f_2}(x) = \left[\sum_y \sum_z f_3(y) \right] y \cdot \left[\sum_z f_4(z) \right]$$

Observation: Variables pass on the product of all incoming messages!



The Sum-Product Algorithm

- Three update equations (Aji & McEliece, 1997)

$$\begin{aligned} p(t) &= \prod_{f \in F_t} m_{f \rightarrow t}(t) \\ m_{f \rightarrow t_1}(t_1) &= \sum_{t_2} \sum_{t_3} \cdots \sum_{t_n} f(t_1, t_2, t_3, \dots) \prod_{i>1} m_{t_i \rightarrow f}(t_i) \\ m_{t \rightarrow f}(t) &= \prod_{f_j \in F_t \setminus \{f\}} m_{f_j \rightarrow t}(t) \end{aligned}$$

- Update equations can be directly derived from the distributive law.
- Calculate all marginals at the same time!
- Only need to pass messages twice along each edge!



Practical Considerations I

- **Log-Transform:** $\lambda_{f \rightarrow t}(t) := \log [m_{f \rightarrow t}(t)]$

$$\begin{aligned} \log [p(t)] &= \sum_{f \in F_t} \lambda_{f \rightarrow t}(t) \\ \lambda_{f \rightarrow t_1}(t_1) &= \sum_{t_2} \sum_{t_3} \cdots \sum_{t_n} f(t_1, t_2, t_3, \dots) \exp \left[\sum_{i>1} \lambda_{t_i \rightarrow f}(t_i) \right] \\ \lambda_{t \rightarrow f}(t) &= \sum_{f_j \in F_t \setminus \{f\}} \lambda_{f_j \rightarrow t}(t) \end{aligned}$$

- **Exponential Family Messages:**

$$m(t) \propto \exp (\psi(t) \cdot \theta)$$

- Message updates are just additions of the parameters θ !



Exponential Families

- (Univariate) Gaussian: $\theta := \left(\frac{\mu}{\sigma^2}, \frac{1}{\sigma^2} \right)$
- Bernoulli: $\theta := \log \left(\frac{p}{1-p} \right)$
- Binomial: $\theta := \log \left(\frac{p}{1-p} \right)$
- Beta: $\theta := (\alpha, \beta)$
- Gamma: $\theta := \left(\alpha, \frac{1}{\beta} \right)$



Practical Considerations II

- Redundant computations:

$$\begin{aligned} p(t) &= \prod_{f \in F_t} m_{f \rightarrow t}(t) \\ m_{t \rightarrow f}(t) &= \prod_{f_j \in F_t \setminus \{f\}} m_{f_j \rightarrow t}(t) \end{aligned} \quad \Rightarrow \quad p(t) = m_{t \rightarrow f}(t) \cdot m_{f \rightarrow t}(t)$$

- Caching: Only store $p(t)$ and $m_{f \rightarrow t}(t)$, then

$$m_{t \rightarrow f}(t) = \frac{p(t)}{m_{f \rightarrow t}(t)}$$



Overview

- Graphical Models
- Inference in Factor Graphs
- **Approximate Message Passing**
- Distributed Message Passing

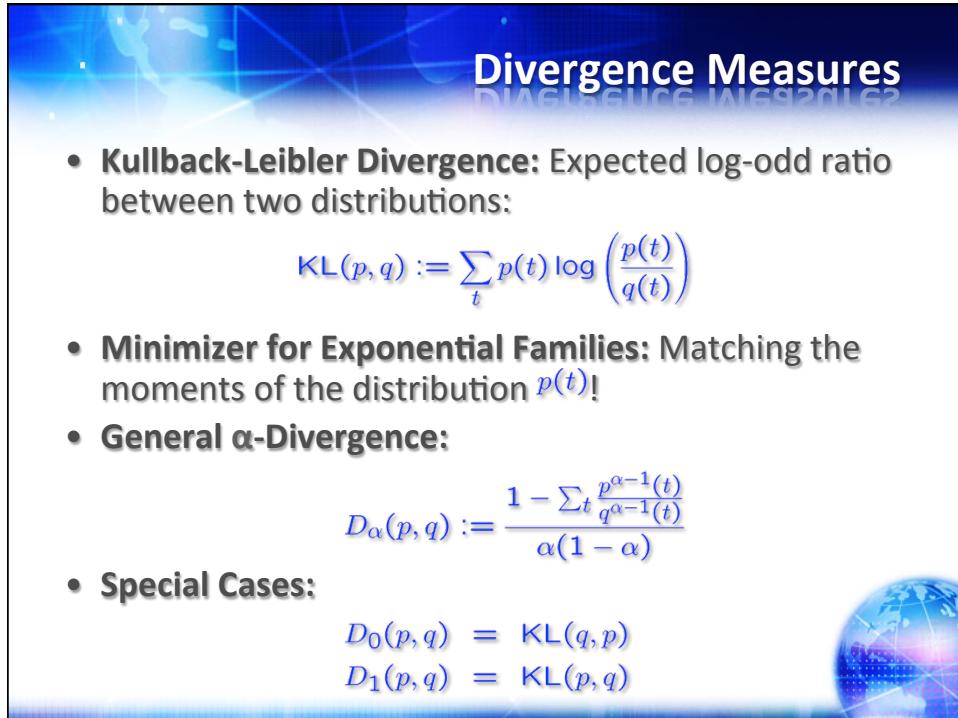
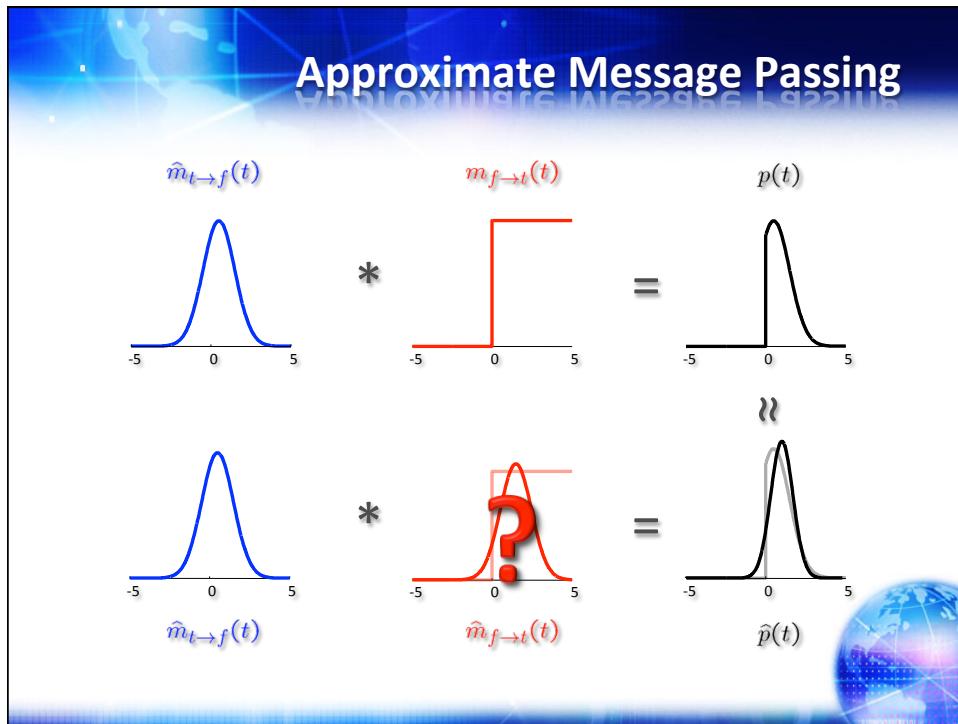


Approximate Message Passing

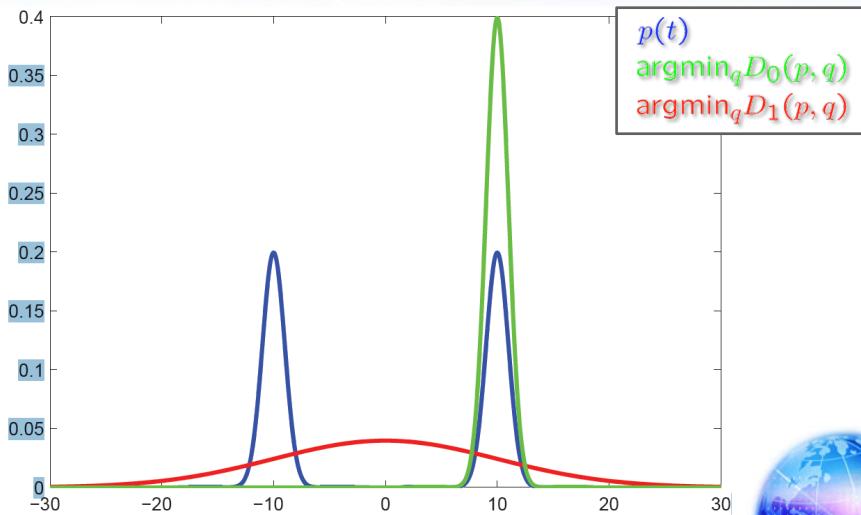
- **Problem:** The exact messages from factors to variables may not be closed under products.
- **Solution:** Approximate *each* marginal as well as possible in using a divergence measure on beliefs.
- **General Idea:** Leave-one out approximation

$$\begin{aligned}\hat{p}(t) &= \operatorname{argmin}_{\hat{p}}, D[m_{f \rightarrow t} \cdot \hat{m}_{t \rightarrow f}, \hat{p}] \\ \hat{m}_{f \rightarrow t}(t) &= \frac{\hat{p}(t)}{\hat{m}_{t \rightarrow f}(t)}\end{aligned}$$





α -Divergence in Pictures



Overview

- Graphical Models
- Inference in Factor Graphs
- Approximate Message Passing
- **Distributed Message Passing**

Large-Data Challenge

- **Large Data (e.g. Facebook user actions)**
 - 500m daily users
 - 3 bln daily likes & comments
- **Two types of variables**
 - Observed → Data Factors
 - Latent → Model parameters
- **Discriminative Models**
 - Given the model parameters, data variables are independent

$$p(\theta | \mathbf{X}, \mathbf{Y}) \propto \prod_i p(y_i | \theta, \mathbf{x}_i) \cdot \prod_j p(\theta_j)$$



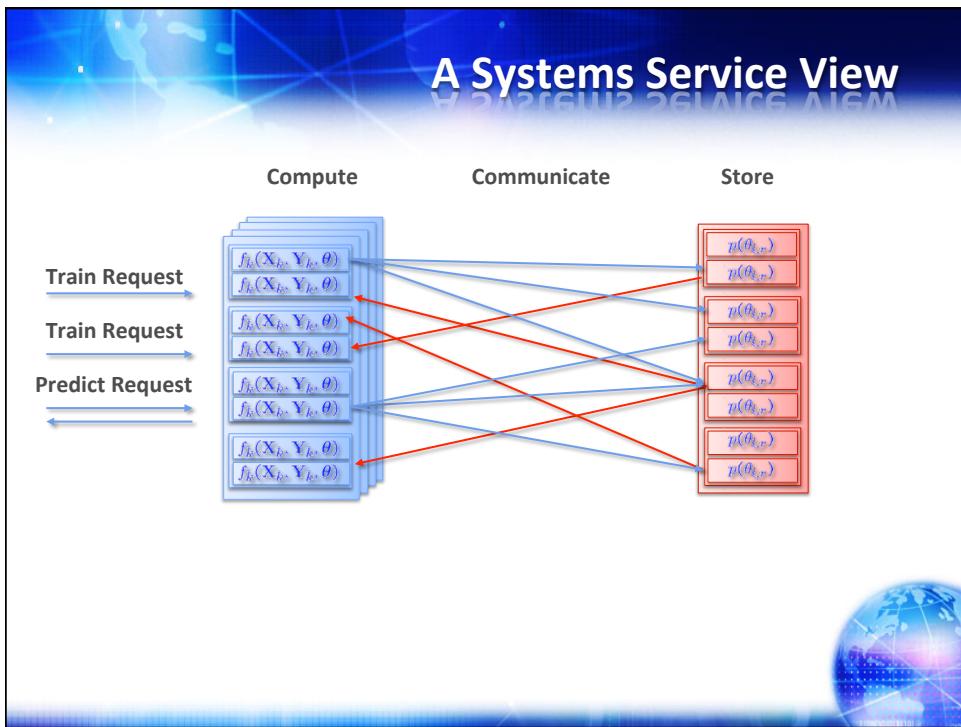
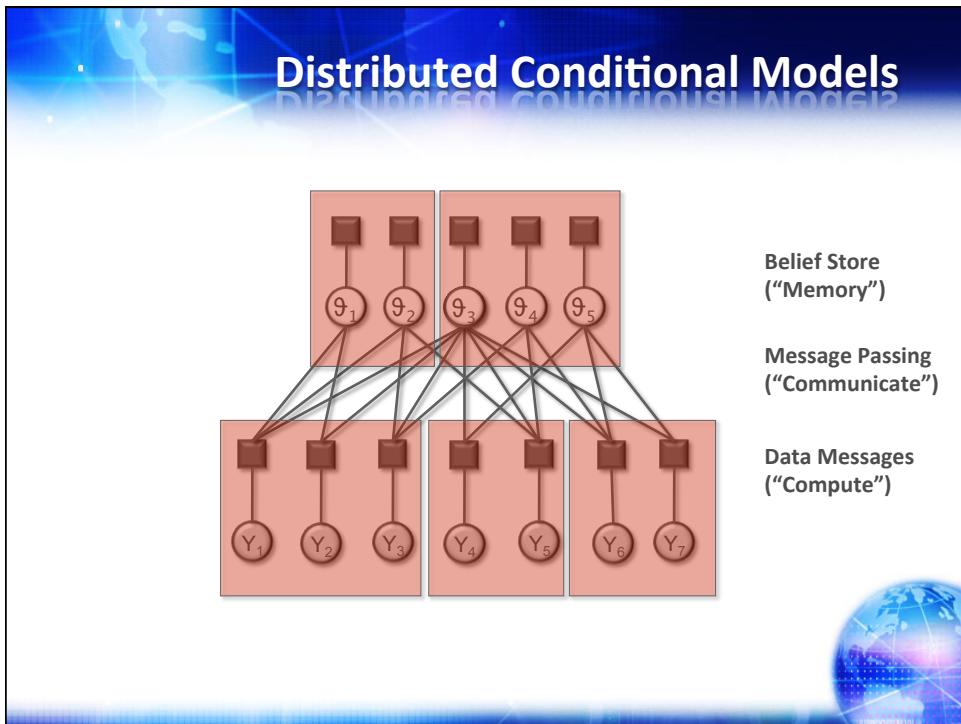
Distributed Message Passing

- **Idea:** Group variables and send messages across system boundaries

$$\prod_i p(y_i | \theta, \mathbf{x}_i) \cdot p(\theta) = \prod_k \left[\prod_{j=1}^{n_k} p(y_{k,j} | \theta, \mathbf{x}_{k,j}) \right] \cdot \prod_l \left[\prod_{r=1}^{m_l} p(\theta_{l,r}) \right]$$
$$f_k(\mathbf{X}_k, \mathbf{Y}_k, \theta) \qquad g_l(\theta_l)$$

- **Data factors:** $f_k(\mathbf{X}_k, \mathbf{Y}_k, \theta)$
 - Know exactly which model parameter messages get updated
- **Parameter factors:** $g_l(\theta_l)$
 - Need to keep track of which data factors need message update





Relation to Map-Reduce

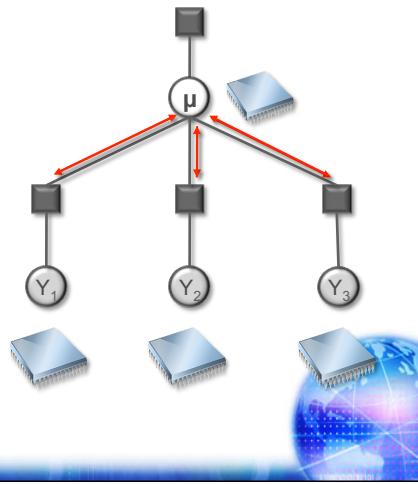
- **Map-Reduce**

- **Map:** Data nodes compute messages $m_{F_k \rightarrow \mu}$ from data y_i and $m_{\mu \rightarrow F_k}$
- **Reduce:** Combine messages $m_{F_k \rightarrow \mu}$ into p_μ by multiplication
- Vanilla MR is a single pass only!

- **Caveats:**

- Approximate data factors need all incoming message $m_{F_k \rightarrow \mu}$!
- Each machine needs to be able to store the belief over μ

$$p(\theta | \mathbf{x}, \mathbf{y}) \propto \prod_k f_k(Y_k | \theta, X_k) \cdot p(\theta)$$

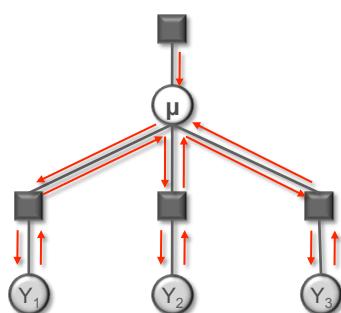


Approximation Quality

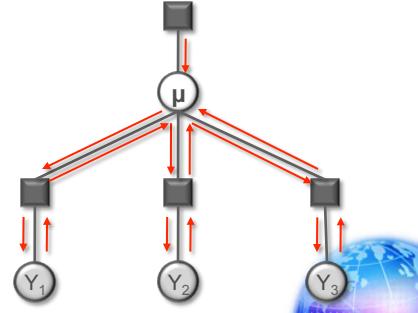
$$p(y_i | \theta, \mathbf{x}_i) = \Phi(y_i \theta^T \mathbf{x}_i)$$

$$p(\theta) = \prod_j \mathcal{N}(\theta_j; \mu_j, \sigma_j^2)$$

Sequential



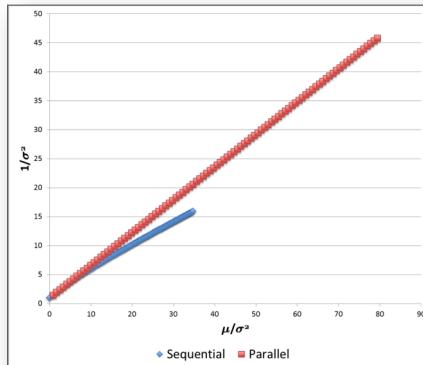
Parallel



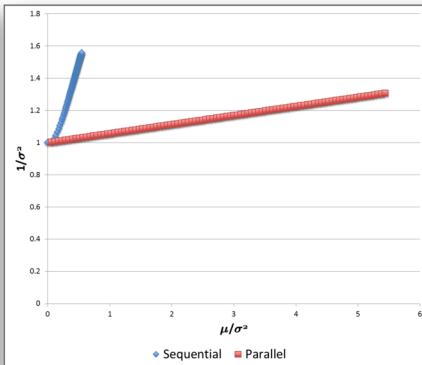
Approximation Quality

$$\mathbf{x} = [1; 1; \dots; 1]^T$$

Single Bias Feature



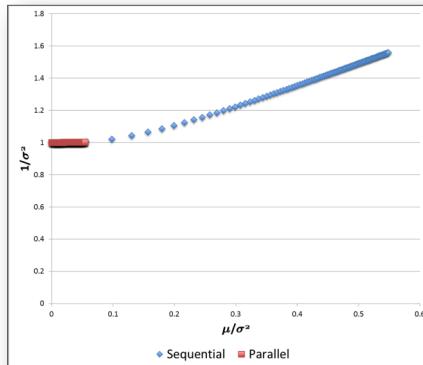
100 Bias Features



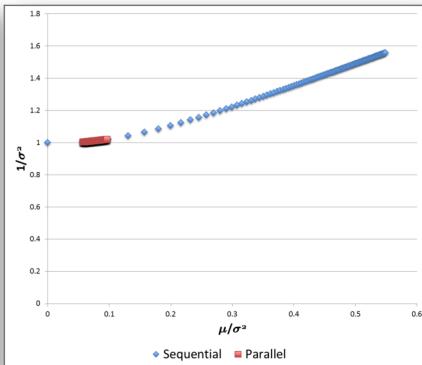
Solution : Dampening!

$$\lambda_{f \rightarrow \theta} \Rightarrow \alpha \cdot \lambda_{f \rightarrow \theta}$$

First Step



Second Step





Break!



Part 2: Applications

Overview

- TrueSkill: Gamer Rating and Matchmaking
- Click-Through Rate Prediction in Online Advertising
- Matchbox: Recommendation Systems



TrueSkill™

Joint work with Thore Graepel, Tom Minka & Phillip Treford



Motivation

- Competition is central to our lives
 - Innate biological trait
 - Driving principle of many sports
- Chess Rating for fair competition
 - ELO: Developed in 1960 by Árpád Imre Élő
 - Matchmaking system for tournaments
- Challenges of online gaming
 - Learn from few match outcomes efficiently
 - Support multiple teams and multiple players per team



The Skill Rating Problem

- Given:
 - Match outcomes: Orderings among k teams

• Given:

– Match outcomes: Orderings among k teams

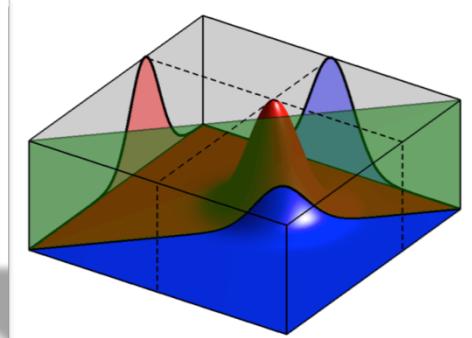
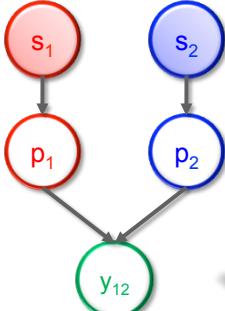
Team	Score
1st Red Team	50

Level	Gamertag	Avg. Life	Best Spree	Score
1st	N/A	N/A	N/A	50
2nd	SniperEye	N/A	N/A	25
3rd	N/A	N/A	N/A	24
4th	N/A	N/A	N/A	24
5th	N/A	N/A	N/A	15
6th	N/A	N/A	N/A	15
7th	N/A	N/A	N/A	11
8th	N/A	N/A	N/A	5

Rank	Player	Rating
1	SEWICSYDE OWNS	27
2	FATAL REVENGE	26
3	Paranoia 1	25
4	Pauk	25
5	IxX OMG Xxi	25
6	BittyTom	25
7	brian 2007	24
8	SEXY MOZES	24
9	droplets	24
10	jaCKdaSaMuRai	24
11	Il Me II	24
12	iamNightMare	24
13	a retarded007	24
14	Perfected Brit	24
15	THE MUFFIN MANx	24
16	TheVunit	23
17	Mr Sushi07	23

Two Player Match Outcome Model

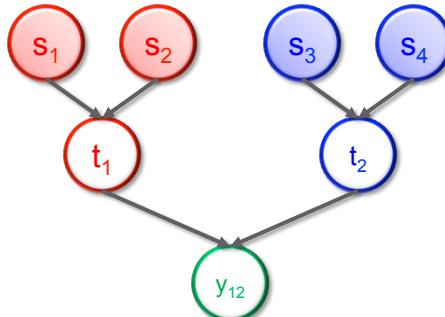
- Latent Gaussian performance model for fixed skills
- Possible outcomes: Player 1 wins over 2 (and vice versa)



$$\mathbf{P}(y_{12} = (1, 2) | p_1, p_2) = \mathbb{I}(p_1 > p_2)$$

Two Team Match Outcome Model

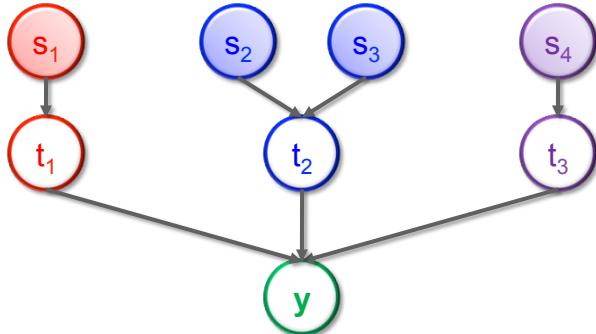
- Skill of a team is the sum of the skills of its members



$$\mathbf{P}(t_1 | s_1, s_2) = \mathcal{N}(t_1; s_1 + s_2, 2 \cdot \beta^2)$$

Multiple Team Match Outcome Model

- Possible outcomes: Permutations of the teams

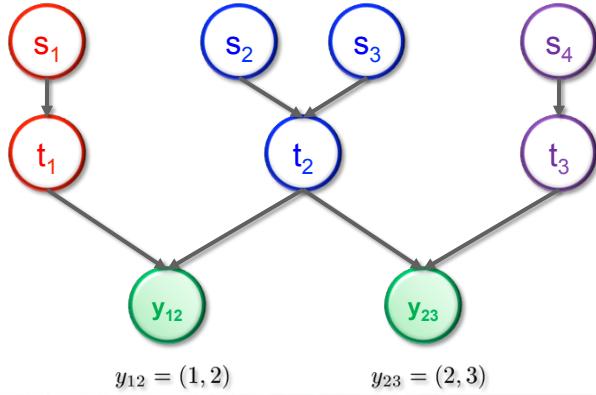


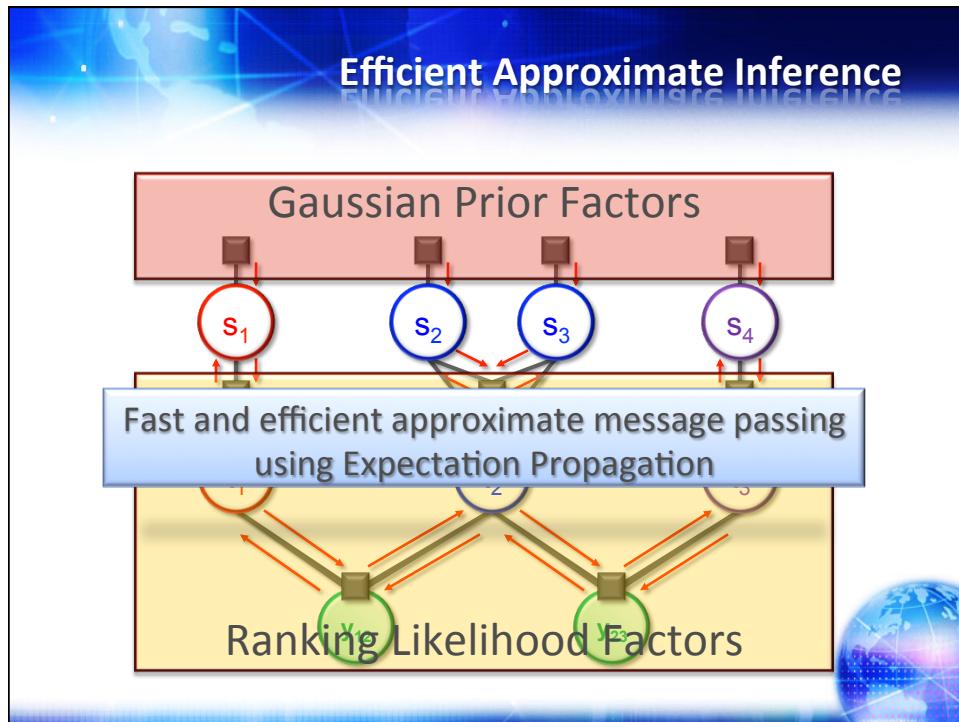
$$\mathbf{P}(y|t_1, t_2, t_3) = \mathbb{I}(y = (i, j, k)) \text{ where } t_i > t_j > t_k$$

Multiple Team Match Outcome Model

- But we are interested in the (Gaussian) posterior!

$$\mathbf{P}(s_i|y = (1, 2, 3)) = \mathcal{N}(s_i; \mu_i, \sigma_i^2)$$





Applications to Online Gaming

- **Leaderboard**
 - Global ranking of all players

$\mu_i - 3 \cdot \sigma_i$

- **Matchmaking**
 - For gamers: Most uncertain outcome

Level	Gamertag	Avg. Life	Best Spree	Score
1st	10 BlueBot	00:00:49	6	15
1st	7 SniperEye	00:00:41	4	14
1st	9 ProThepirate	00:01:07	3	13
1st	10 dazdemon	00:00:59	3	8
2nd	10 WastedHarry	00:00:41	4	17
2nd	3 Ascla	00:00:41	2	10
2nd	9 Amibot4Losing	00:00:41	2	9
2nd	12 BlackKings	00:00:48	3	0

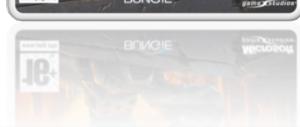
$P(p_i \approx p_j | \mu_i = \mu_j, \sigma_i^2 + \sigma_j^2)$

$P(p_i \approx p_j | \mu_i - \mu_j = 0, \sigma_i^2 + \sigma_j^2 = 0)$

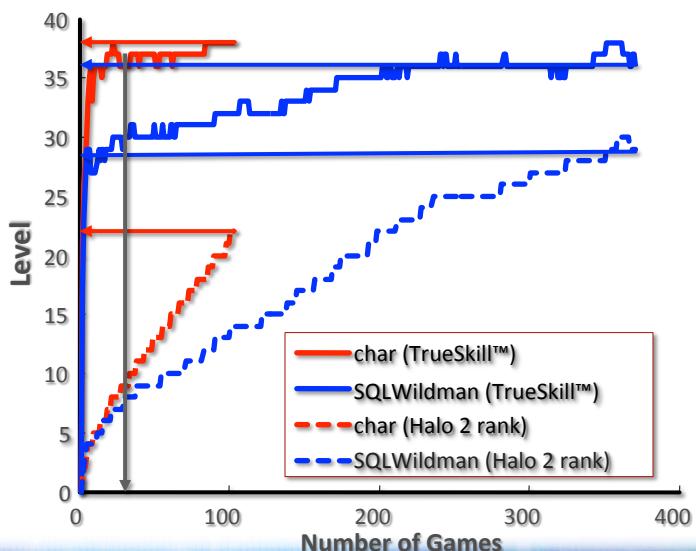
Experimental Setup

- Data Set: Halo 2 Beta

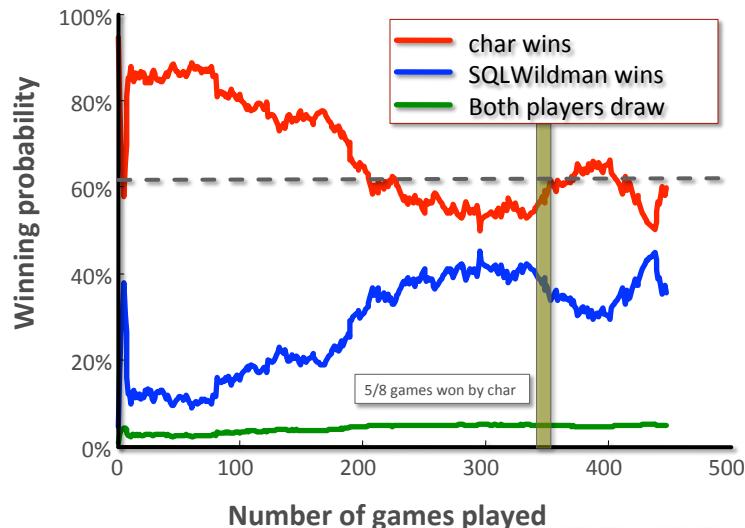
- 3 game modes
 - Free-for-All
 - Two Teams
 - 1 vs. 1
- > 60,000 match outcomes
- ≈ 6,000 players
- 6 weeks of game play
- Publically available



Convergence Speed



Convergence Speed (ctd.)



Xbox 360 & Halo 3

- **Xbox 360 Live**

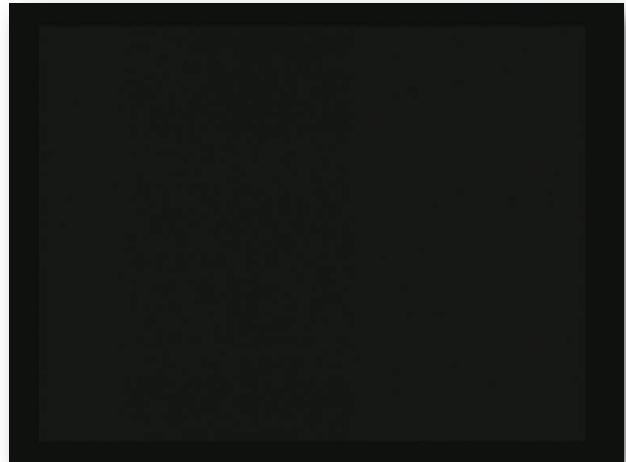
- Launched in September 2005
- Every game uses TrueSkill™ to match players
- > 10 million players
- > 2 million matches per day
- > 2 billion hours of gameplay



- **Halo 3**

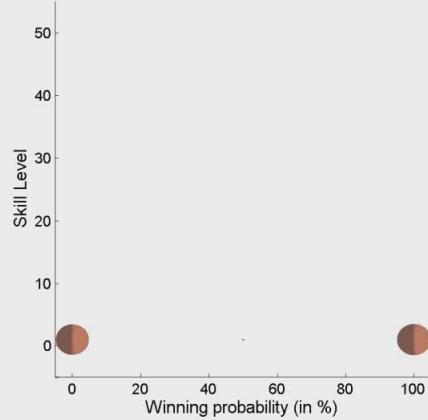
- Launched on 25th September 2007
- Largest entertainment launch in history
- > 200,000 player concurrently (peak: 1,000,000)

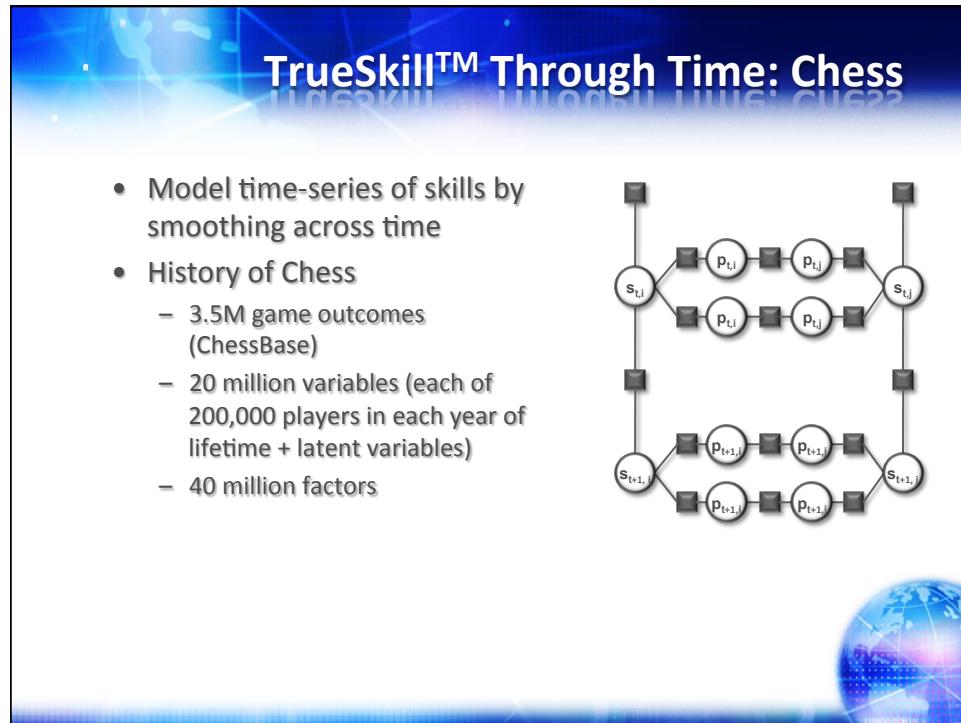
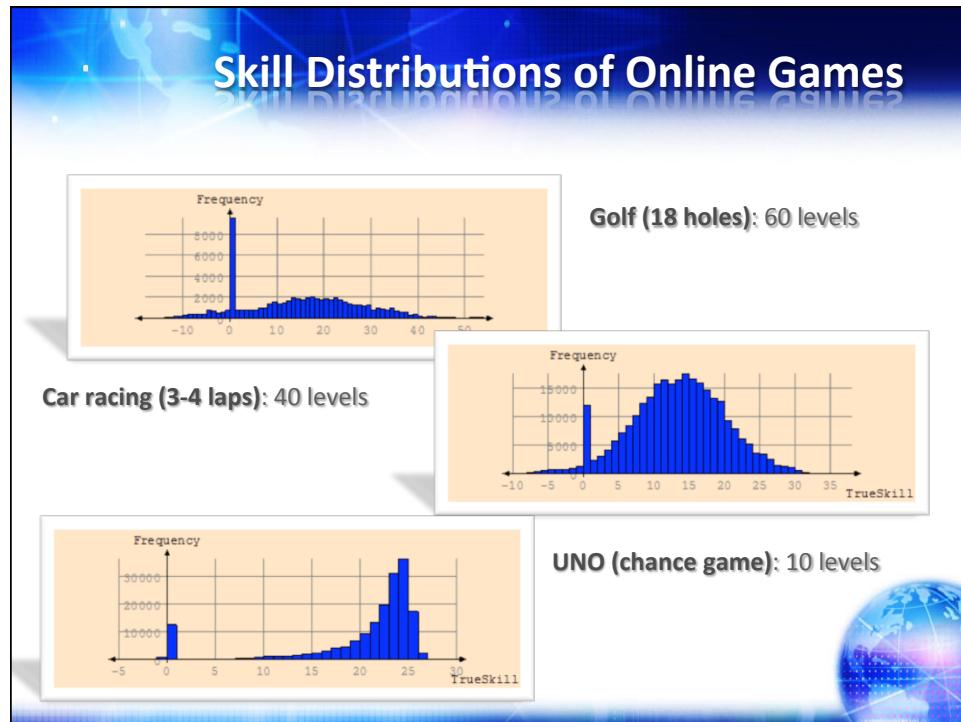
Halo 3 in Action

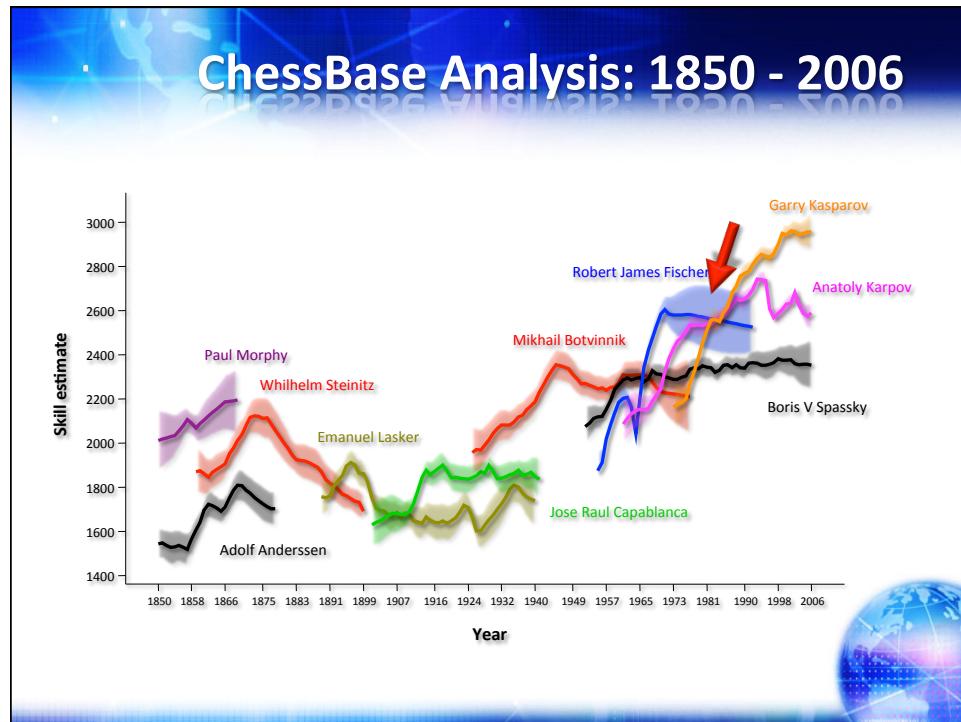


Halo 3 Public Beta Analysis

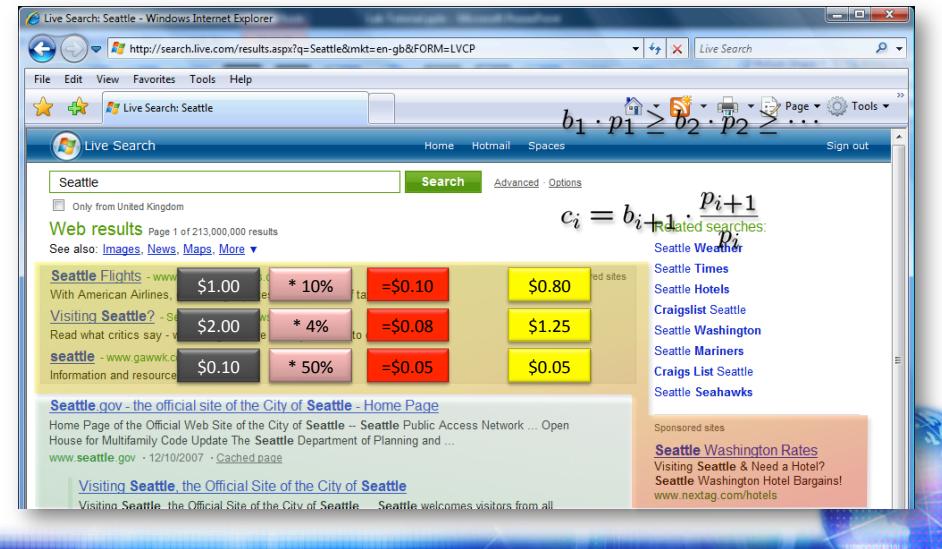
1 games played





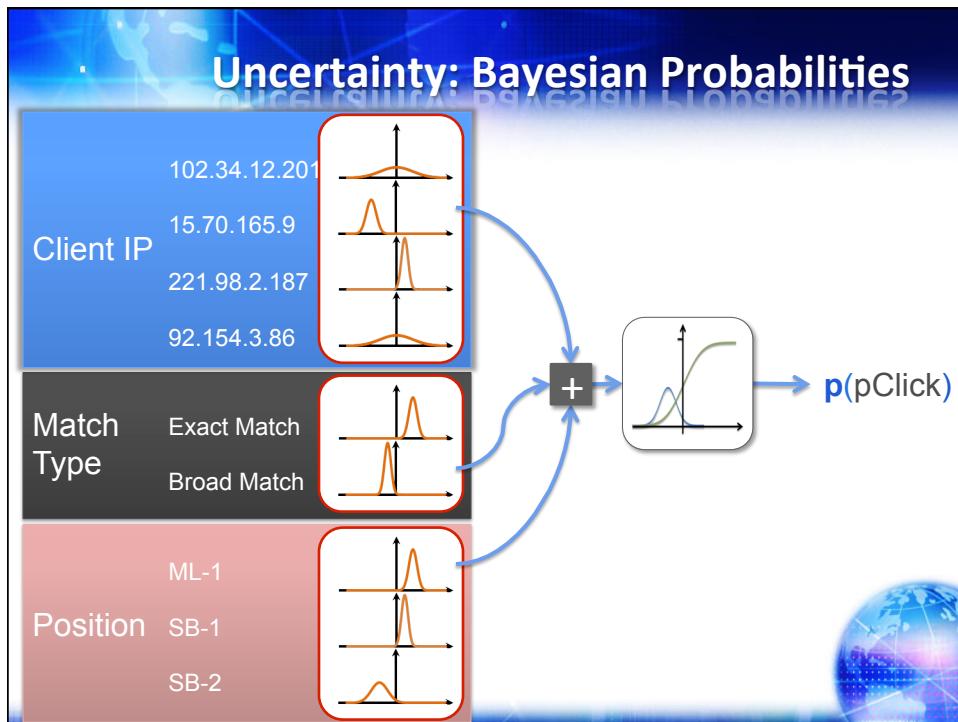
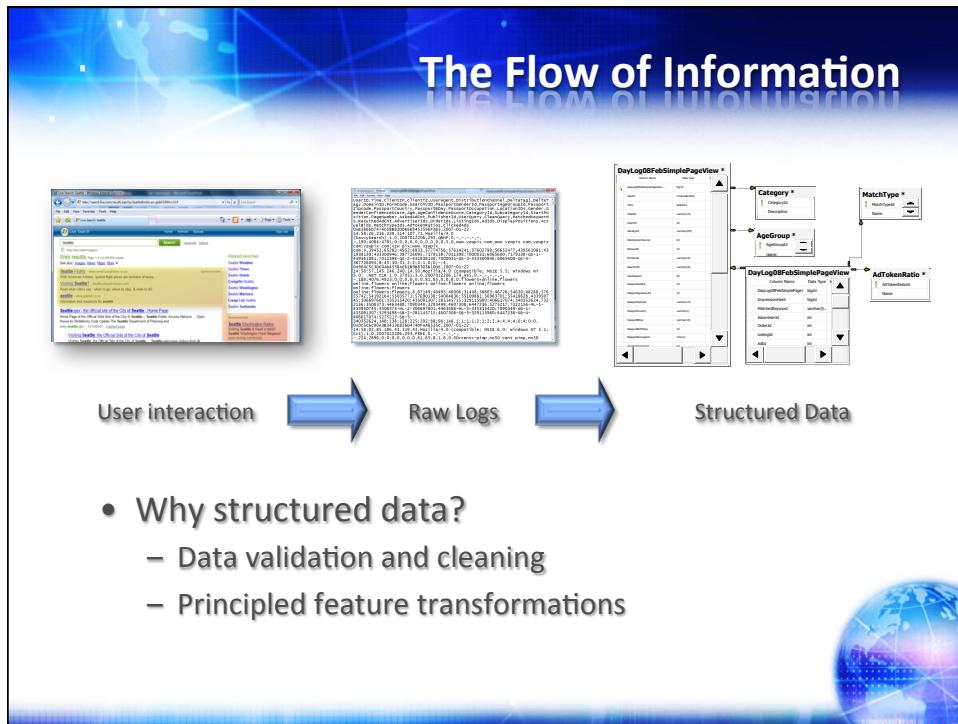


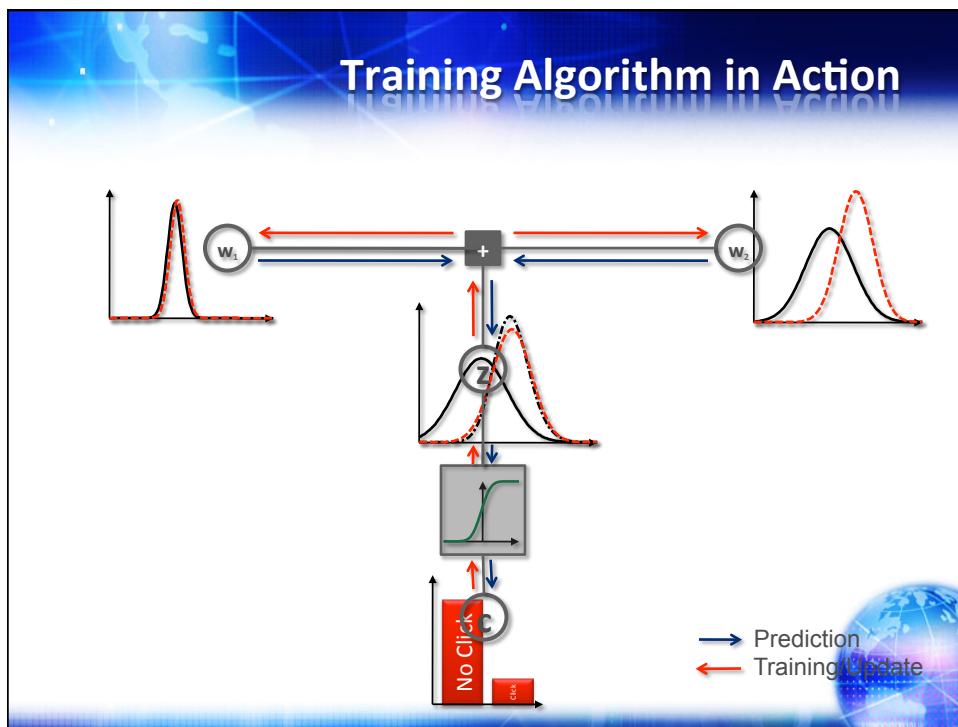
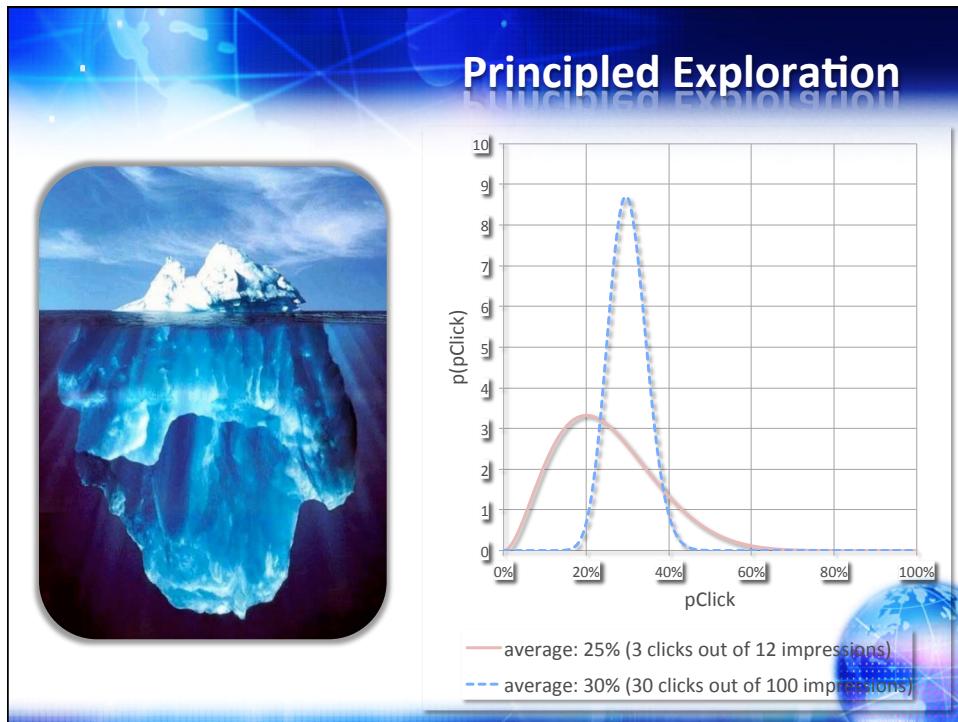
Why Predict Probability-of-Click?



The Scale of Things

- Several weeks of data in training:
7,000,000,000 impressions
- 2 weeks of CPU time during training:
2 wks × 7 days × 86,400 sec/day =
1,209,600 seconds
- Learning algorithm speed requirement:
 - 5,787 impression updates / sec
 - 172.8 μ s per impression update

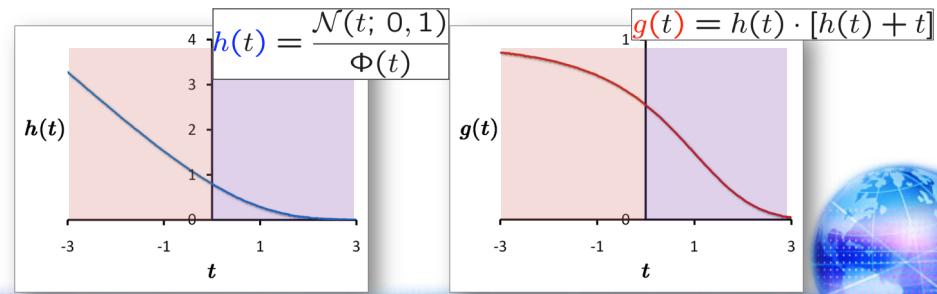




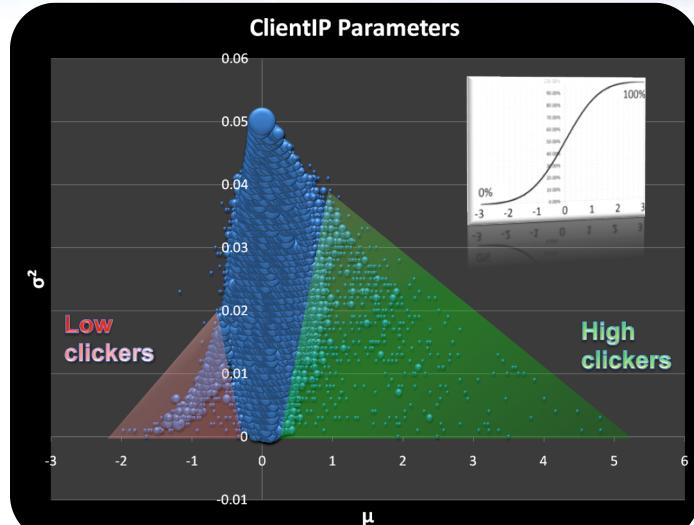
Inference: An Optimization View

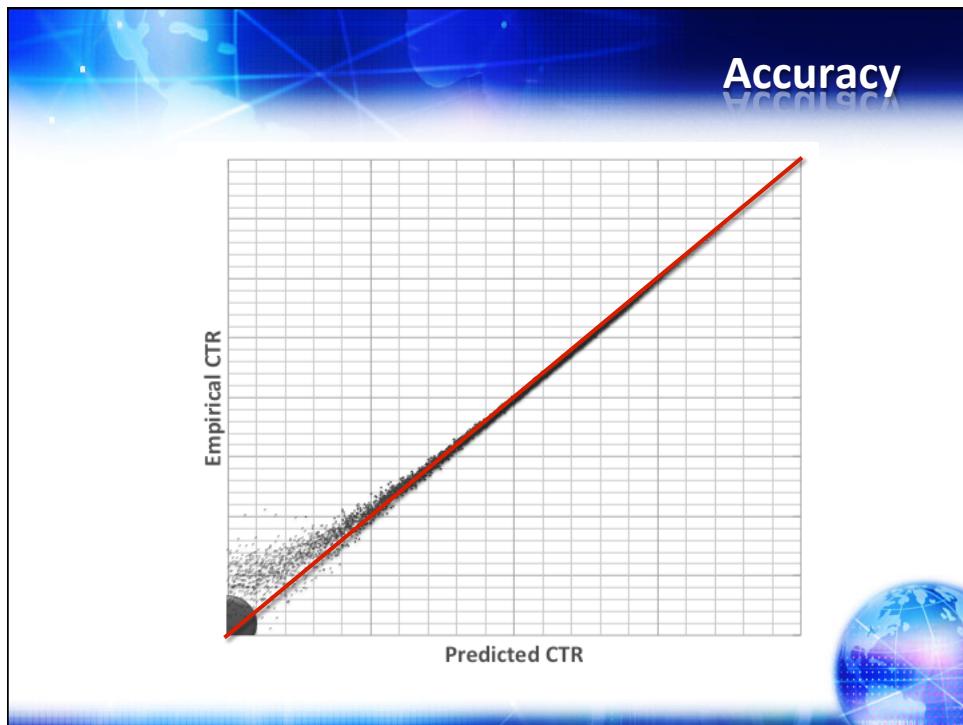
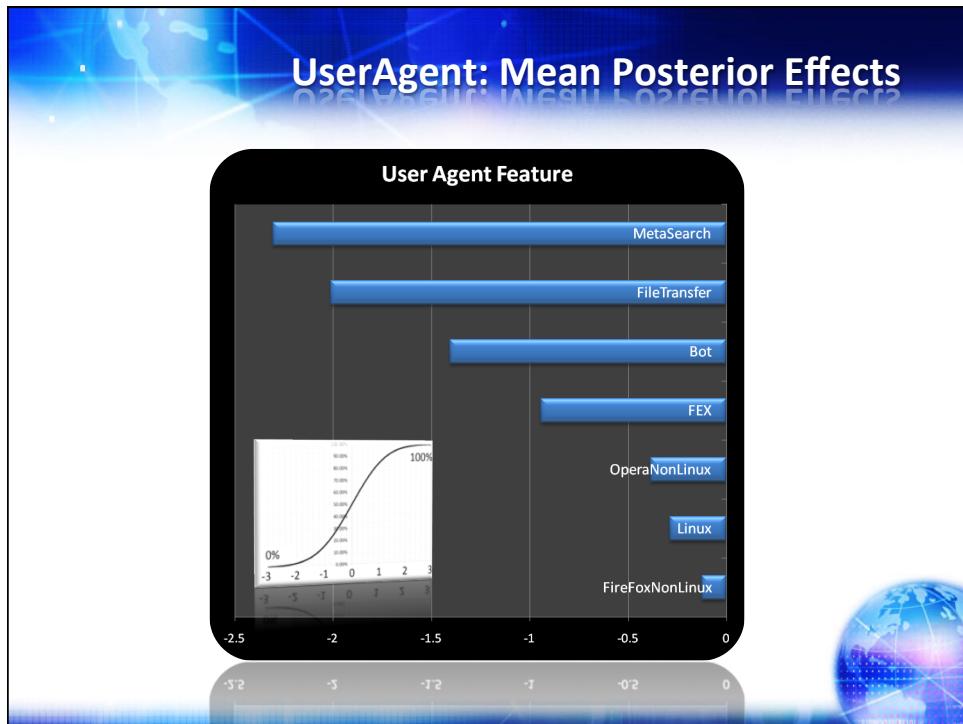
$$\mu_i \leftarrow \mu_i + \frac{\sigma_i^2}{s} \cdot h \left[\frac{\sum_{j=1}^d \mu_j}{s} \right] \quad \sigma_i^2 \leftarrow \sigma_i^2 \left(1 - \frac{\sigma_i^2}{s^2} \cdot g \left[\frac{\sum_{j=1}^d \mu_j}{s} \right] \right)$$

$$s^2 = \beta^2 + \sum_{j=1}^d \sigma_j^2$$



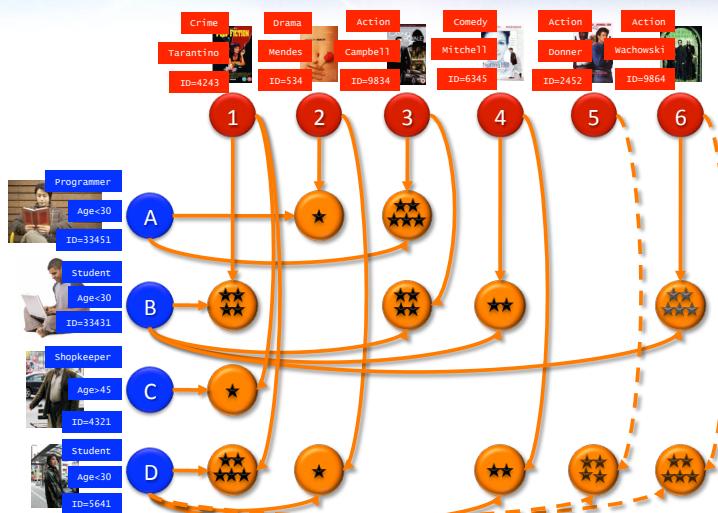
Client IP: Mean & Variance

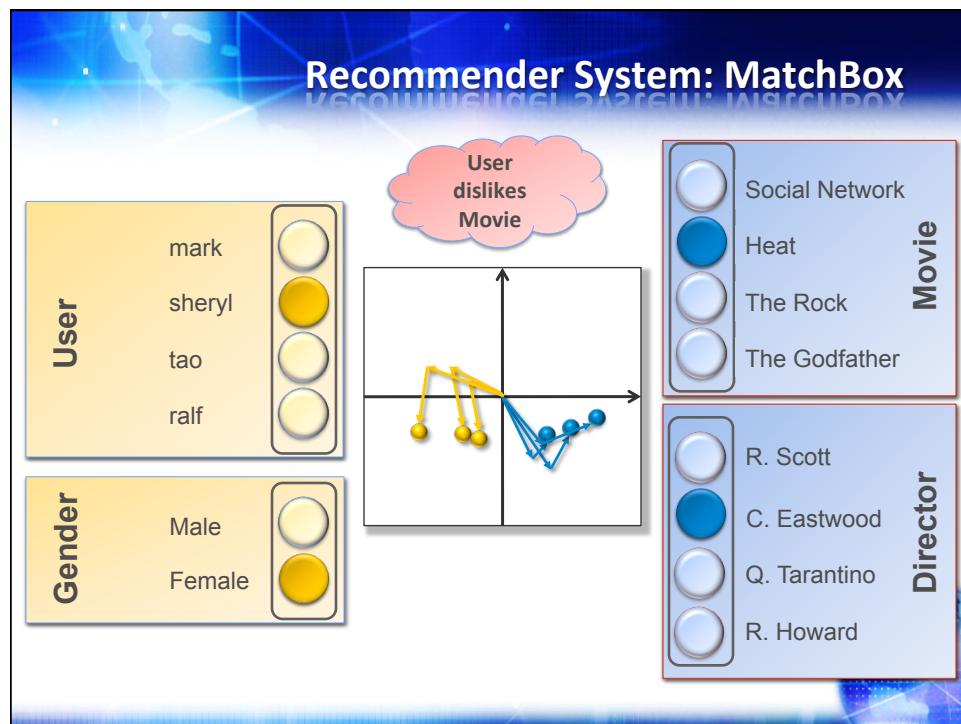
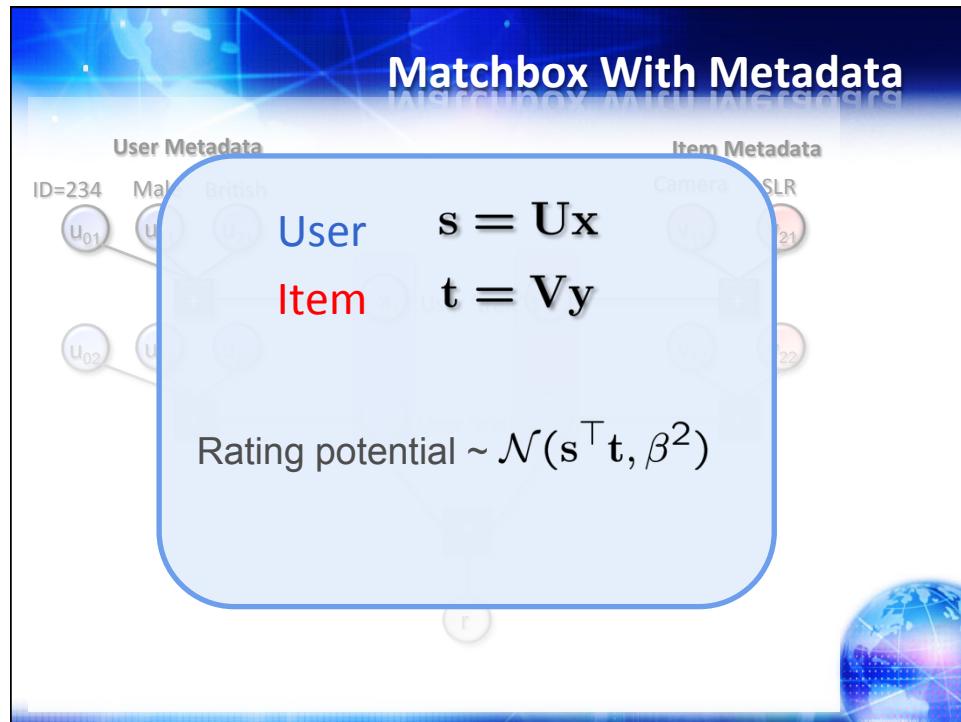


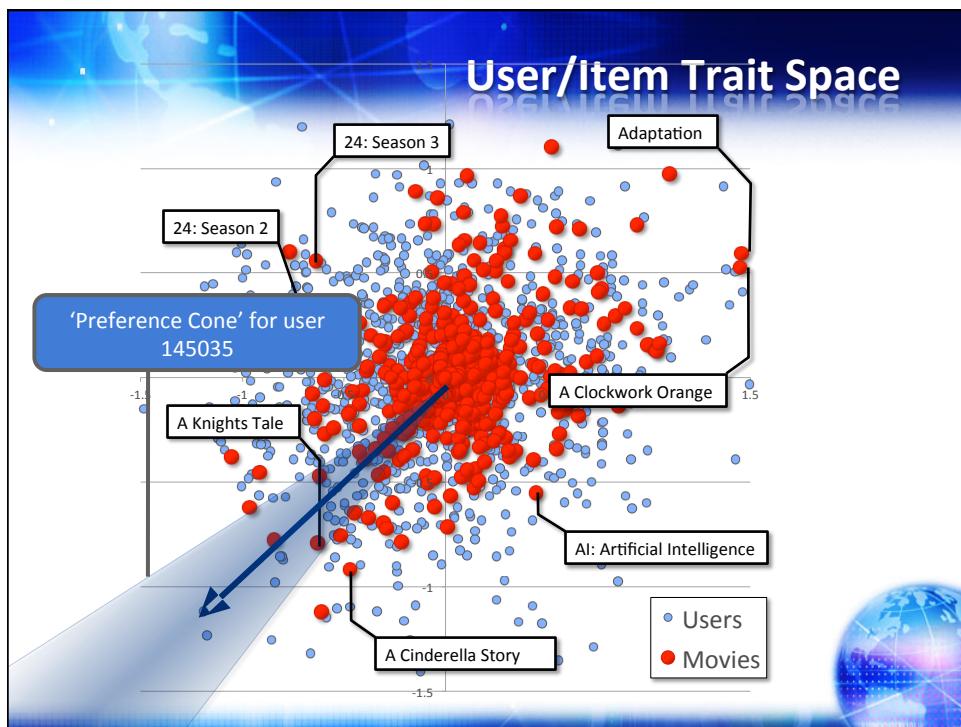
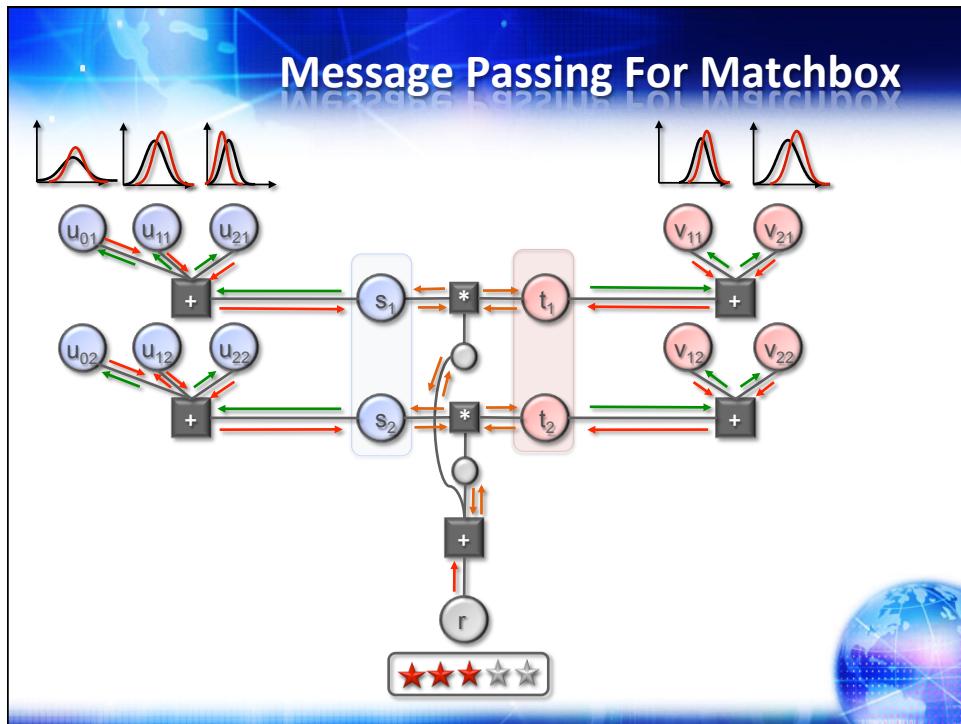


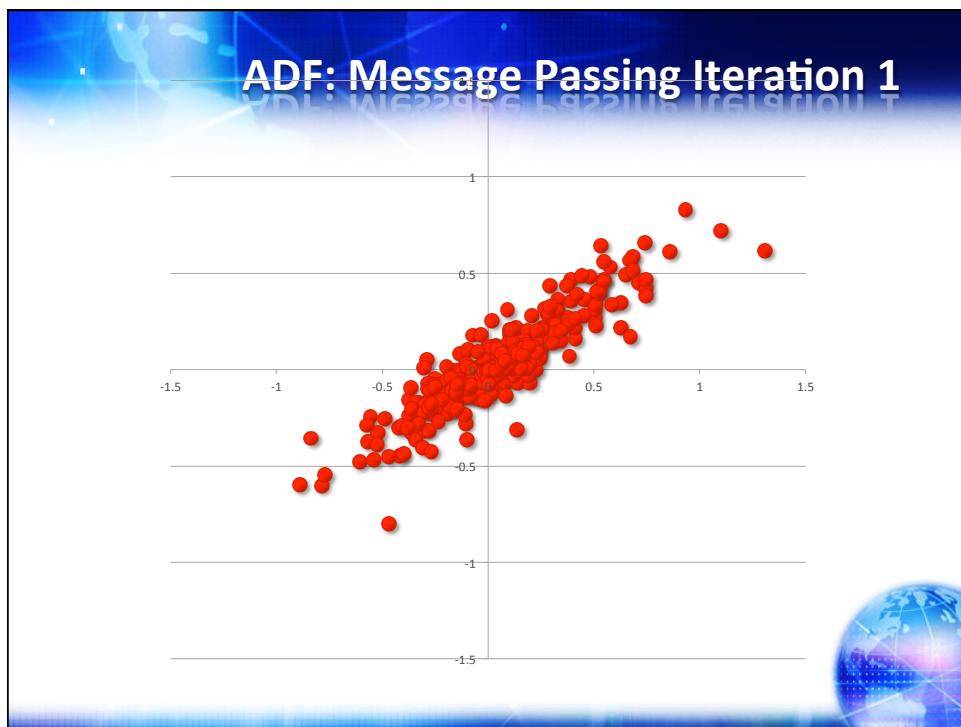
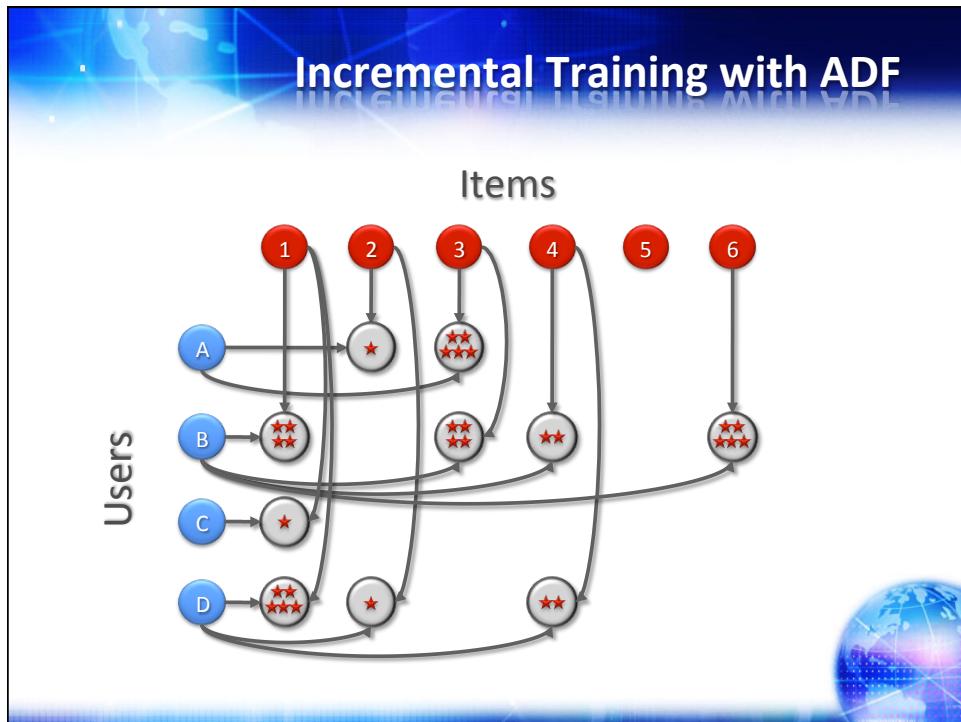
MatchBox

Joint work with Thore Graepel, Joaquin Quiñonero Candela, David Stern

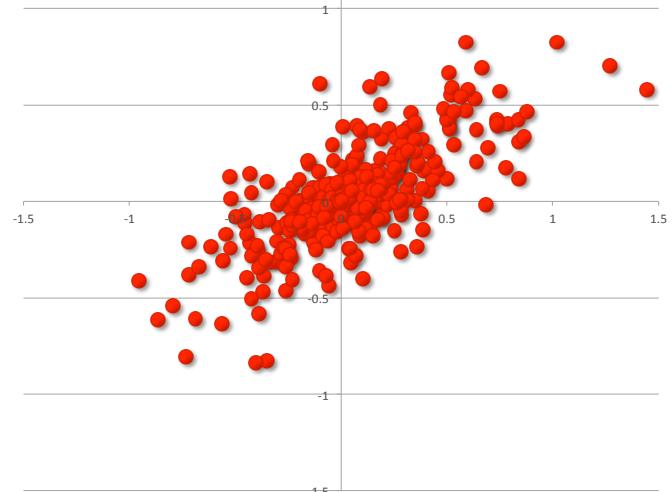




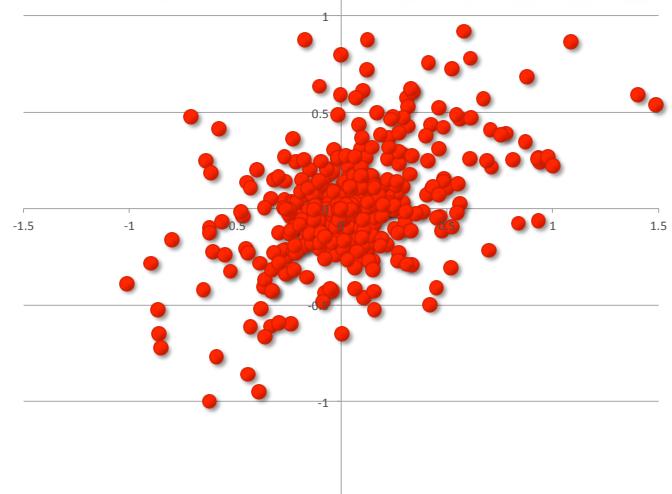


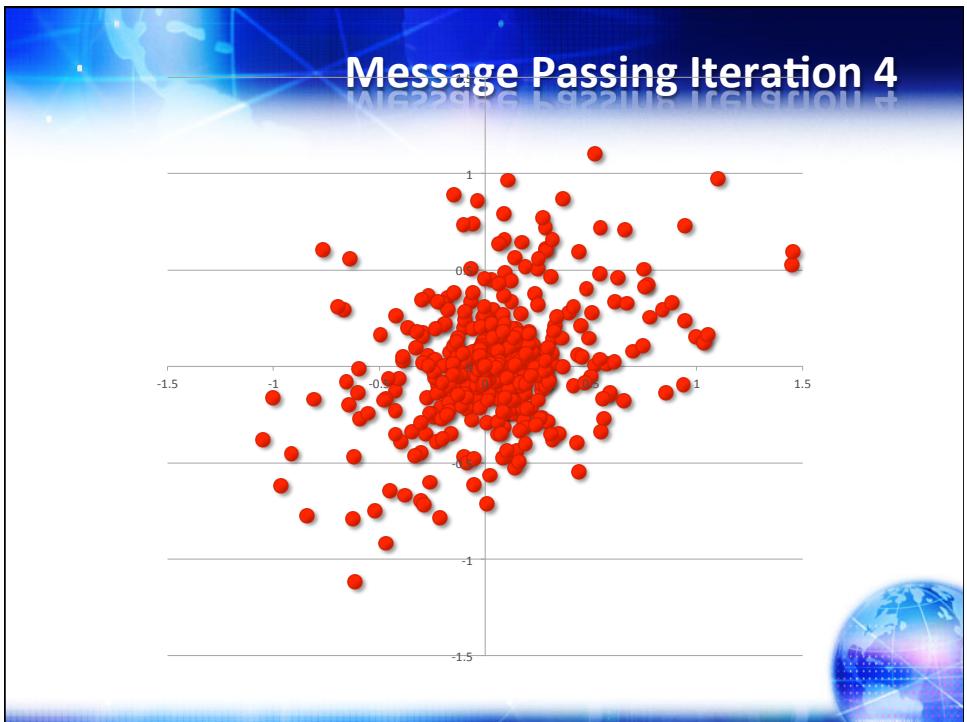


Message Passing Iteration 2

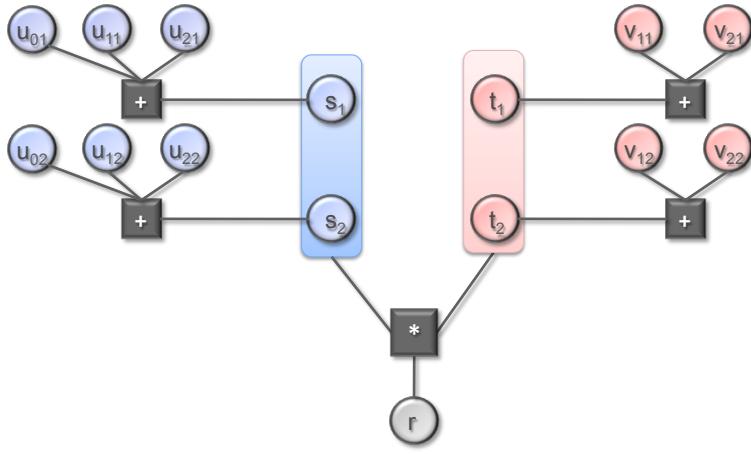


Message Passing Iteration 3

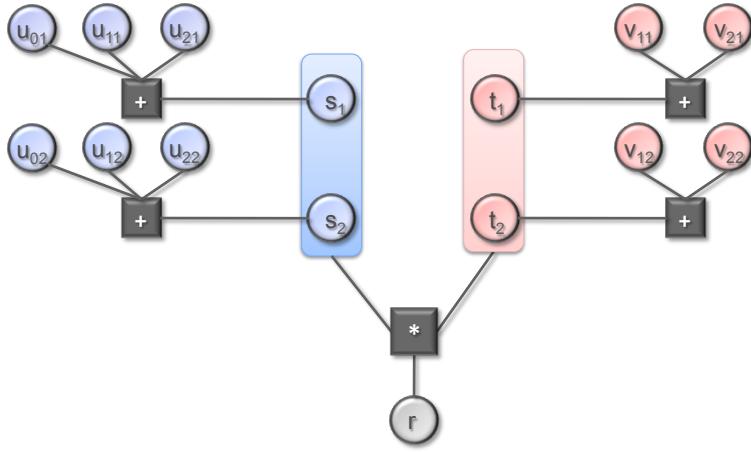




Feedback Models



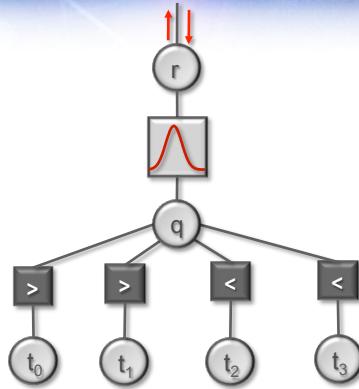
Feedback Models

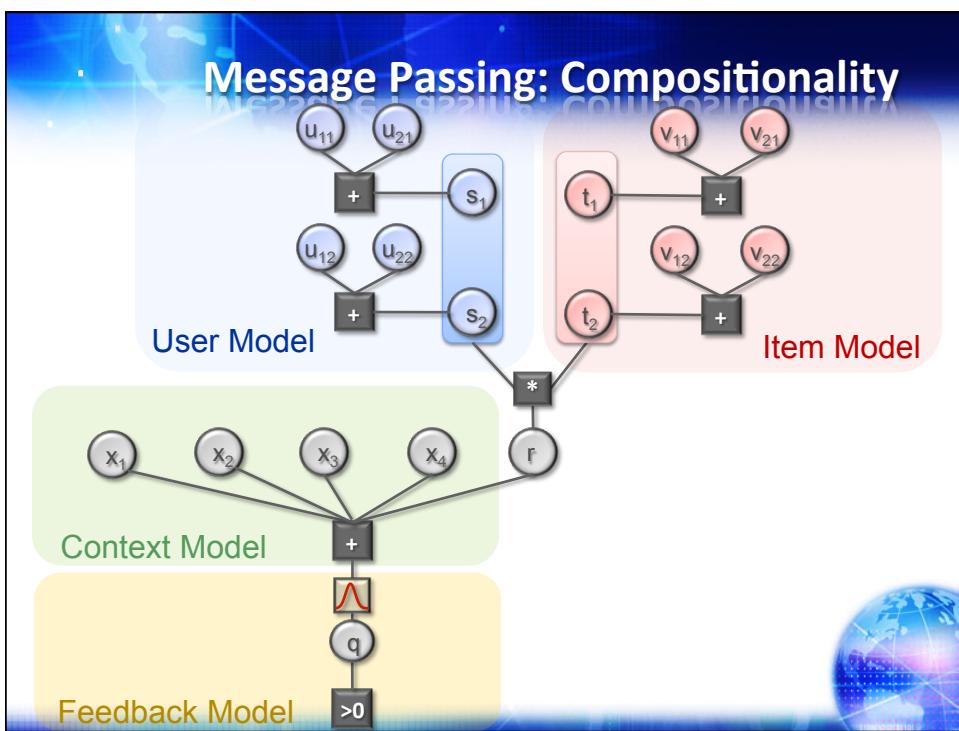
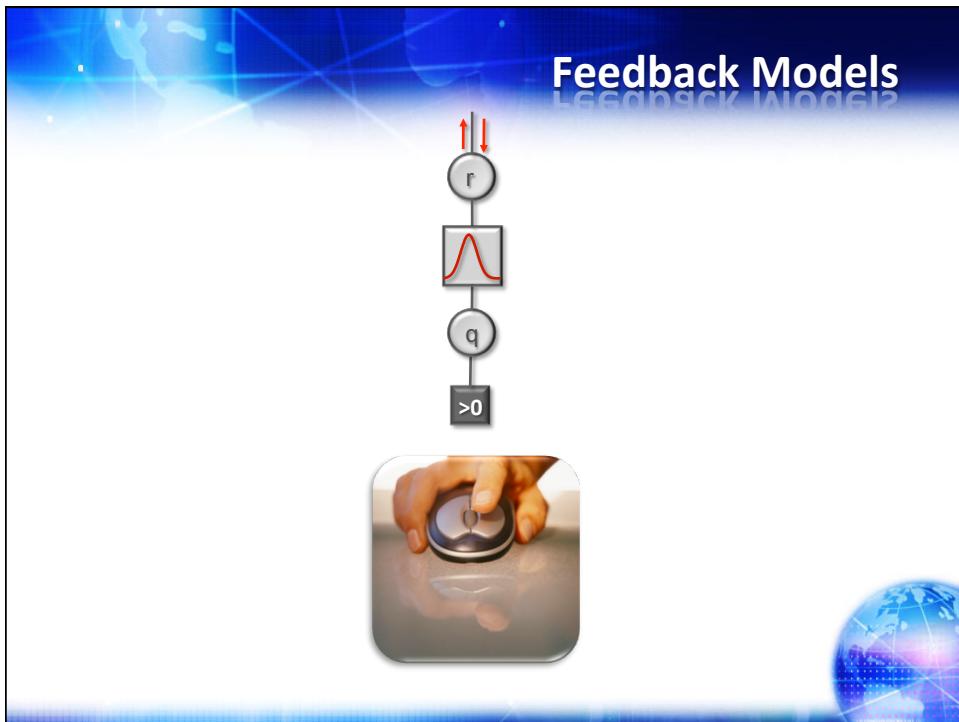


Feedback Models



Feedback Models







accuracy



Performance and Accuracy



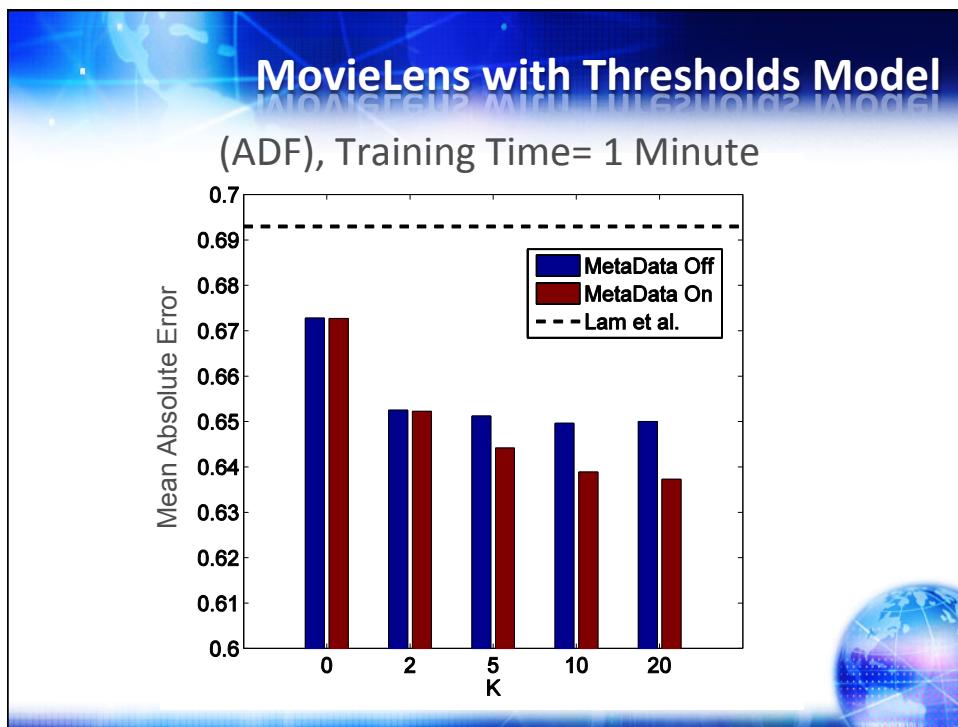
MovieLens Data

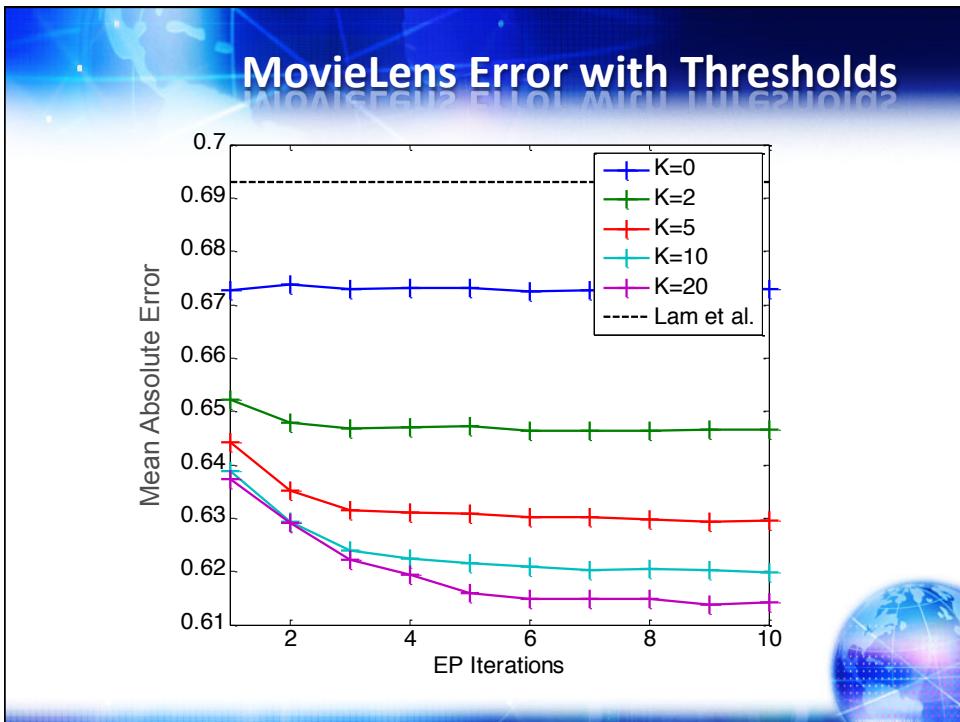
- 1 million ratings
- 3,900 movies / 6,040 users
- User / movie metadata

MovieLens – 1,000,000 ratings

6,040 users 3,900 movies

User ID				Movie ID	
User Job		User Age	Movie Genre		
Other	Lawyer	<18	Action	Horror	
Academic	Programmer	18-25	Adventure	Musical	
Artist	Retired	25-34	Animation	Mystery	
Admin	Sales	35-44	Children's	Romance	
Student	Scientist	45-49	Comedy	Thriller	
Customer Service	Self-Employed	50-55	Crime	Sci-Fi	
Health Care	Technician	>55	Documentary	War	
Managerial	Craftsman		Drama	Western	
Farmer	Unemployed		Fantasy	Film Noir	
Homemaker	Writer				
			User Gender		
			Male		
			Female		





Recommendation Speed

- **Goal:**
find N items with highest predicted rating.
- **Challenge:**
potentially have to consider all items.
- Two approaches to make this faster:
 - Locality Sensitive Hashing
 - KD Trees
- **Locality Sensitive Hash:**

$$P(h(x) = h(y)) = \text{sim}(x, y)$$



Random Projection Hashing

- Random Projections:
 - Generate random hyper planes
(m random vectors, a_i).
 - Gives m bit hash, $\{x_0, x_1, \dots, x_m\}$, by:
$$x_i = \mathbf{1}[a_i \cdot t > 0]$$
- $p(\text{all bits match}) \propto \text{cosine similarity}.$
- Store items in buckets indexed by keys.
- Given a user trait vector:
 1. Generate key, q.
 2. Search buckets by hamming distance from q until find N items.



