OPTIMAL PI-CONTROL OF A PELTIER OVEN
FYS4420
EXPERIMENTAL TECHNIQUES IN CONDENSED MATTER PHYSICS

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Abstract. The aim of this work is to perform PI-control of a Peltier oven connected to a box filled with water and polyester. I shall study the temperature-response over time thanks to a thermistor connected with the box through a copper foil. I will first make some measurements and study the response under step and pulse-excitation. From this data, I will then compute some parameters needed to perform PI-control. Finally, I shall run PI-control in different configurations, e.g. using a step reference. Finally, I will show both an asymptotic deviation and how to make the system unstable. The results will show how this kind of servo-control acts on the system and what the responses are.

Contents

1. Introduction

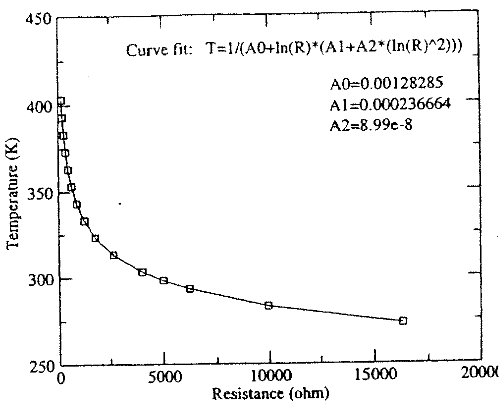
To begin with, some measurements are needed in order to proceed with the PI-control. Therefore, room temperature in detected and step- and pulse-excitation performed. These enable us to compute all the parameters related to the PI-control. These parameters are either computed directly from a theoretical relation or through graphic methods.

Secondly, PI-control of the system is carried out in different configurations. After running with one reference temperature, a step-reference is added. Moreover, the sampling rate is modified and an asymptotic behaviour observed. Finally, the system is made unstable by increasing the corrective feedback.

The purpose is to study the system response and compare it to the theoretical previsions. After a section concerning the theoretical background of the experiment and the methods used to perform it, I present the results and the discuss them.

2. Theory and Methods

In order to perform the measurement, the mixture container is put on the Peltier element and on its top a copper layer is connected to a temperature sensor, say a thermistor type ACC-003 (ELFA). An excitation current passes through the Peltier oven, causing an increase or decrease in temperature. An external voltage source keeps $V_{CC} \simeq 5\ V$ and provides the excitation current. The source has a large resistor in series, $R_{source} = 1.5984\ M\Omega$. To register a temperature value, the voltage across the thermistor, V_t, is measured (in a 4-wire configuration to avoid voltage drop) thanks to a 16 bit DAQ card (NI6036E). The thermistor characteristic is shown in fig. 1.



Temperature (°C)	Temperature (K)	Resistance (Ω)
0	273.15	16325
10	283.15	9950
20	293.15	6245
25	298.15	5000
30	303.15	4028.5
40	313.15	2663.3
50	323.15	1801.5
60	333.15	1244
70	343.15	876
80	353.15	627.5
90	363.15	457.7
100	373.15	339.2
110	383.15	255.2
120	393.15	194.7
130	403.15	150.5

Figure 1. **Detected temperature with respect to the thermistor resistance**; thermistor type ACC-003 (ELFA).

The box temperature can therefore be obtained as[1]:

$$T = \frac{I}{A_0 + \ln\left(A_1 + A_2\left(\ln R_t\right)^2\right)} \tag{1}$$

where the current I and the thermistor resistance R_t are:

$$I = \frac{V_{CC} - V_t}{R_{source}}, \qquad R_t = \frac{V_t}{I} \tag{2}$$

All measurements are performed thanks to a computer, connected to the instruments, where the codes are run and the data saved. All instruments are checked to be connected and working properly before the measurements begin.

In order to detect the room temperature, the pre-written Matlab code `temp-check` is run with excitation set to zero. To perform step-excitation, the code is run with a fixed excitation level. The awaited response is computed thanks to linear response theory:

$$r(t) = \Delta E H_0 \left(1 - \frac{T_1}{T_1 - T_2} e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} e^{-\frac{t}{T_2}}\right) \tag{3}$$

As for the pulse-excitation, the algorithm is modified to excite for a given time and then change the excitation to zero. The response is expected to be:

$$r(t) = \frac{\Delta E H_0}{T_1 - T_2}\left[T_1\left(e^{\frac{\epsilon}{T_1}} - 1\right)e^{-\frac{t}{T_1}} - T_2\left(e^{\frac{\epsilon}{T_2}} - 1\right)e^{-\frac{t}{T_2}}\right] \tag{4}$$

All the responses are saved, including the figures and all the output data.

The parameters needed to perform PI-control can be computed using the previous measurements results. The value of H_0 is given by:

$$H_0 = \frac{T_{saturation} - T_{room}}{\Delta E} \tag{5}$$

where $T_{saturation}$ is the saturation temperature from the step-excitation, T_{room} is the room temperature and ΔE is the excitation amplitude.

T_1 and T_2 can be estimated with a graphical method. First, we shall plot the difference between the temperature detected during the pulse-excitation and the room temperature (the response) against time. Then, we localise the peak and infer its coordinates, say $\left(t_p^{max}, r_p^{max}\right)$. Last, we obtain T_1 and T_2 from the two intersections of the response function $r_p(T_{12})$:

$$r_p(T_{12}) = \Delta E H_0 e^{-\frac{t_p}{T_{12}}}\left(e^{\frac{\epsilon}{T_{12}}} - 1\right) \tag{6}$$

and the straight line $r(t) = r_p^{max}$. The abscissas of the intersection points are the time constants T_1 and T_2. The values are inferred from the graph.

The constants K_I and K_P in eq. (8) are obtained from P and I as follow:

$$P = \frac{1}{3}\frac{\left(T_1 + T_2\right)^2}{T_1 T_2} - 1, \qquad I = \frac{1}{27}\frac{\left(T_1 + T_2\right)^3}{\left(T_1 T_2\right)^2}, \qquad K_P = \frac{P}{H_0}, \qquad K_I = \frac{I}{H_0} \tag{7}$$

[1] A_0, A_1 and A_2 are curve fitting constants. The values are specified in fig. 1.

The PI-controller measurements are performed thanks to the `pid-control` algorithm. In system control, when the reference $r_{ref}(t)$ varies over time, we talk about servo-control. The aim is the response to follow a given path, a reference. In this case, the feedback is determined by the difference between the reference response $r_{ref}(t) = T_{ref}(t) - T_{room}$ and the actual response $r(t)$ over time. This is defined as the error: $\mathrm{err}(t) = r_{ref}(t) - r(t)$. The feedback is given as:

$$e(t) = K_P \,\mathrm{err}(t) + K_I \int_0^t \mathrm{err}(\tau) d\tau \tag{8}$$

In the previous function, the two contributions are weighted by the two constants K_I and K_P. The first addend is proportional to the error at time t, while the other term takes into account the error at previous times. We refer to this kind of feedback as PI-control. In our specific case r_{ref} (so T_{ref}) is held constant, so we call it a regulator.

Additionally, in order to obtain an asymptotic static deviation between T_{ref} and T, we set $K_I = 0$. Then, as time goes to infinity, the following relation holds:

$$\lim_{t\to\infty} r(t) = \Delta r \left(\frac{K_P}{H_0^{-1} + K_P} \right) \equiv r(\infty) = r_{ref} \tag{9}$$

Thus, an expression for the theoretical ratio of the average saturation temperature (here T_{mean}) and the reference is:

$$\frac{r_{ref}}{\Delta r} = \frac{T_{ref} - T_{room}}{T_{mean} - T_{room}} = \frac{K_P}{H_0^{-1} + K_P} \tag{10}$$

Finally, thanks to feedback strategy, the response is modified and the system made unstable. It is know that the second term in eq. (8) is necessary for the response to reach the reference temperature and stabilise. Therefore, K_P is given a high value in eq. (8), so that the integral term contribution becomes negligible with respect to the first. Then, the system is expected to become unstable.

3. Results

3.1. **Room temperature.** So as to perform the room temperature measurement for approximately 20 min, the code was set to run 2850 samples (each measurement taking roughly 0.75 s). The result is shown in fig. 2. The computed average temperature is[2]:

$$T_{room} = 296.09 \pm 0.05\ K \tag{11}$$

[2]The uncertainty on the value is given by the standard deviation, as in all the following data.

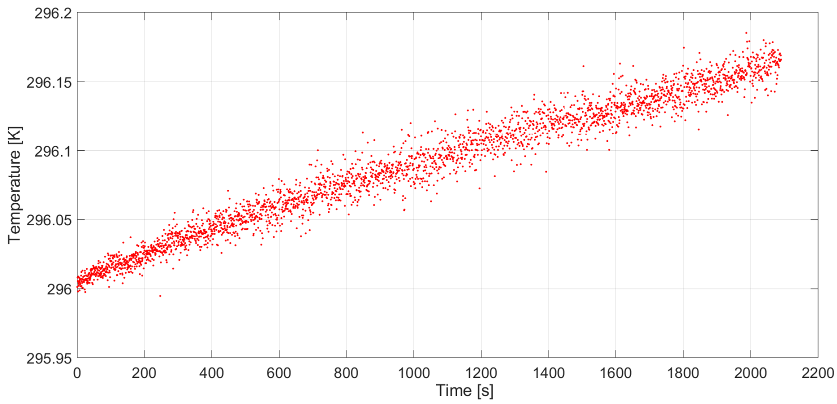


FIGURE 2. **Room temperature measurement.** The plot shows the trend of temperature over time. Each dot represents a single detection.

3.2. **Step-excitation.** The code was then modified in order to perform step-excitation. The excitation value was set to $\Delta E = 3\ V$ and the samples number to 15000, so as to be sure the response curve reached saturation. The code was then stopped after approximately 3 hrs, having performed $\simeq 11400$ measurements. The resulting curve is shown in fig. 3.

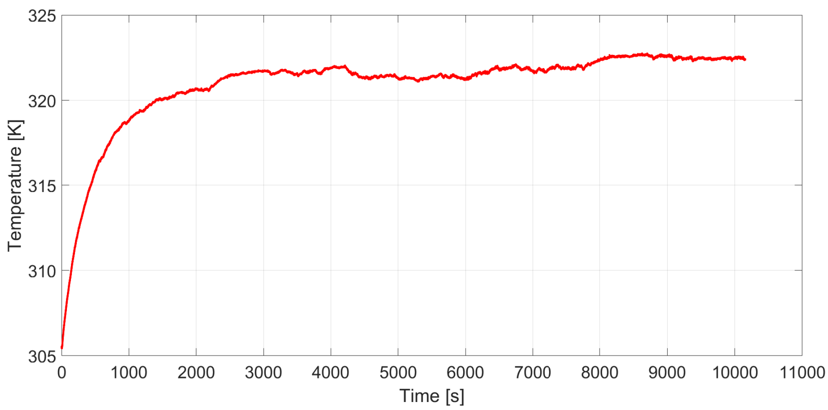


FIGURE 3. **Saturation curve for step-excitation.** The plot shows the trend of temperature against time.

A quick look after completing the measurement revealed that there were some issues within the saturation region. Essentially, the curve does not stabilise around a given temperature, but continues oscillating. In order to perform the remaining measurements, data were extracted from fig. 3. The likely cause of this issue was later discovered: a ventilation fan right above the instruments. Consequently, at the end of all the required tasks, the code was run again (12000 samples). The results of this second detection are displayed in fig. 4.

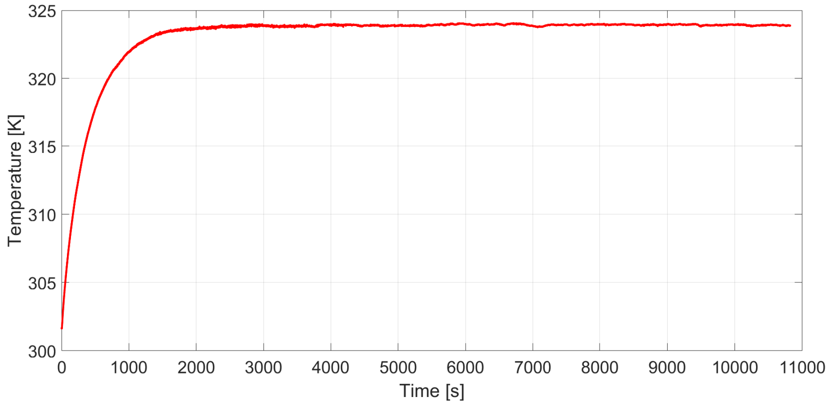


FIGURE 4. **Saturation curve for step-excitation: second measurement.** The plot shows the trend of temperature against time.

The saturation temperature was evaluated as the average over the last 2900 samples (first measurement):

$$T_{saturation_1} = 322.3 \pm 0.3\ K \tag{12}$$

As far as the second measurement is concerned, the average saturation temperature was computed over the last 6000 samples. The estimate is:

$$T_{saturation_2} = 323.93 \pm 0.05\ K \tag{13}$$

3.3. **Pulse-excitation.** Then, the code was changed with the aim of carrying out pulse-excitation. Excitation was set to $\Delta E = 3.5\ V$ for the first 30 samples (approx. 18 s) and then stopped. The algorithm was run for 300 samples. The result is presented in fig. 5.

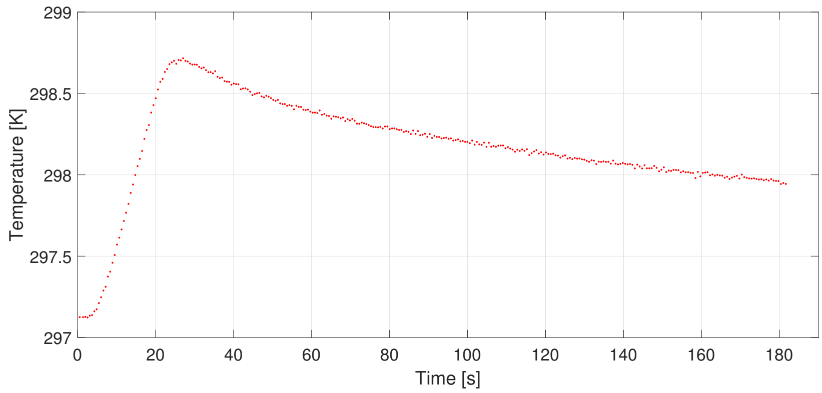


FIGURE 5. **Response to pulse-excitation.** The plot shows the trend of temperature over time.

3.4. **Computation of parameters.** In all the following measurements, the values I shall use are the ones obtained from the first step-excitation (fig. 3). The parameter H_0 computed from the second one is listed in section 3.4.1.

The parameter H_0 resulted to be (see eq. (5)):

$$H_0 = 8.7 \pm 0.1 \ \frac{K}{V} \tag{14}$$

To find T_1 and T_2, I first inferred the coordinates[3] of the maximum from fig. 6, say $\left(t_p^{max}, r_p^{max}\right) = (27.05 \pm 0.01 \ s, 2.628 \pm 0.001 \ K)$.

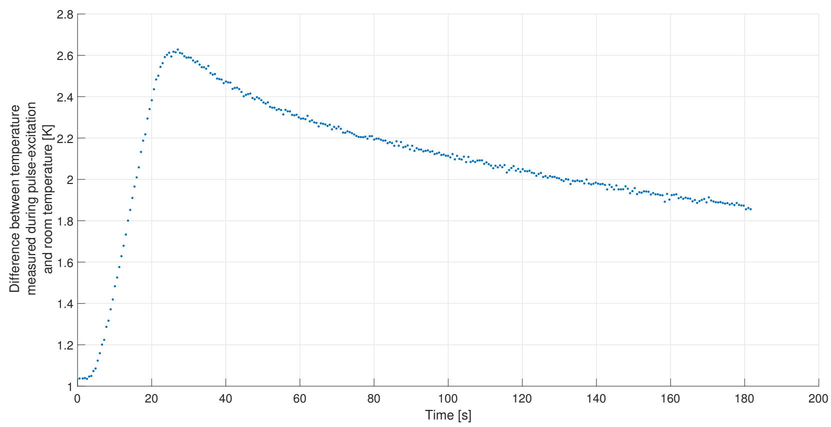


FIGURE 6. **Graphical method to find $\mathbf{T_1}$ and $\mathbf{T_2}$.** The plot shows the response $r(t) = (T_{pulse} - T_{room})$ against time.

[3]The uncertainty on all the parameters inferred from graphs is chosen to be the reading error on the last decimal digit considered.

Then, I plotted the response function $r_p = r_p(T_{12})$ (see eq. (6)) against time and superimposed it to the line $r(t) = r_p^{max}$, as shown in fig. 7. The two intersection points (the abscissas) in fig. 7 give the values of the time constants T_1 and T_2. The values have been inferred from the graph. I obtained:

$$T_1 = 190.81 \pm 0.01\ s, \qquad T_2 = 3.6977 \pm 0.0001\ s \tag{15}$$

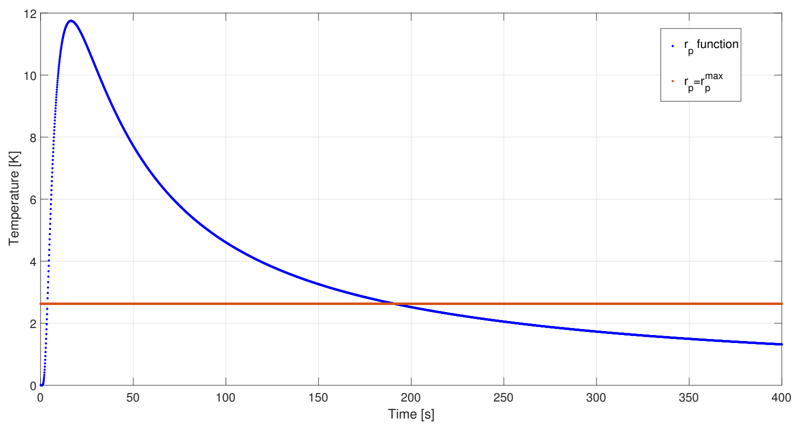


FIGURE 7. **Graphical method to find $\mathbf{T_1}$ and $\mathbf{T_2}$.** In blue is r_p as function of T_{12}, in red the straight line $r(t) = r_p^{max}$.

Consequently, P, I, K_I and K_P were computed using eq. (7):

$$P = 16.88 \pm 0.01, \qquad I = 0.5475 \pm 0.0002\ \frac{1}{s} \tag{16}$$

$$K_P = 1.9 \pm 0.2\ \frac{V}{K}, \qquad K_I = 0.063 \pm 0.006\ \frac{V}{Ks} \tag{17}$$

3.4.1. *Second step-excitation.* The H_0 obtained from the second step-excitation is:

$$H_0 = 9.28 \pm 0.02\ \frac{K}{V} \tag{18}$$

This value is not compatible with the one computed above (see eq. (14)), but this can be attributed to the ventilation issue.

3.5. **PI-controller.** First, the values of the parameters obtained in section 3.4 were inserted into the specific algorithm to perform PI-control of the system. The reference temperature was fixed to $T_{ref_1} = 303.15\ K$, so as to keep it higher than room temperature. The samples number was set to 1000. The result is shown in fig. 8.

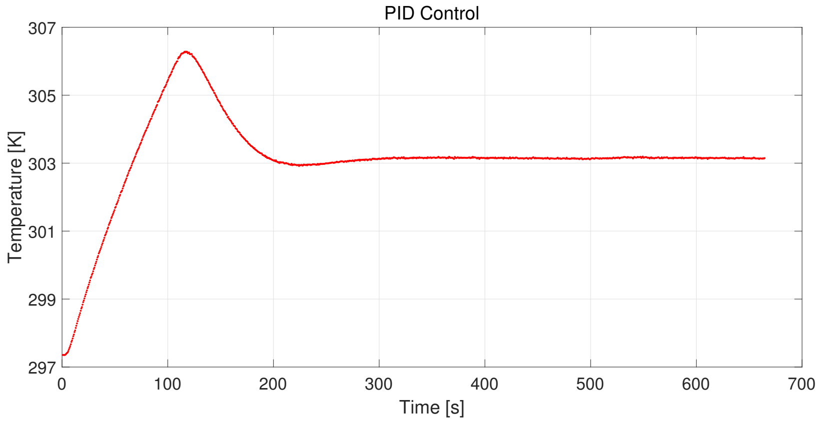


FIGURE 8. **PI-control.** The plot shows the trend of temperature over time.

3.6. **Step reference PI-controller.** With the aim of running a PI-controller with a step reference, a second reference temperature was set, $T_{ref_2} = 313.15\ K$. The code was then modified so that after 500 samples (approx. 300 s) the reference temperature would increase from T_{ref_1} to T_{ref_2}. The samples were fixed to 1500 (so as to reach saturation after the second peak). The outcome is presented in fig. 9.

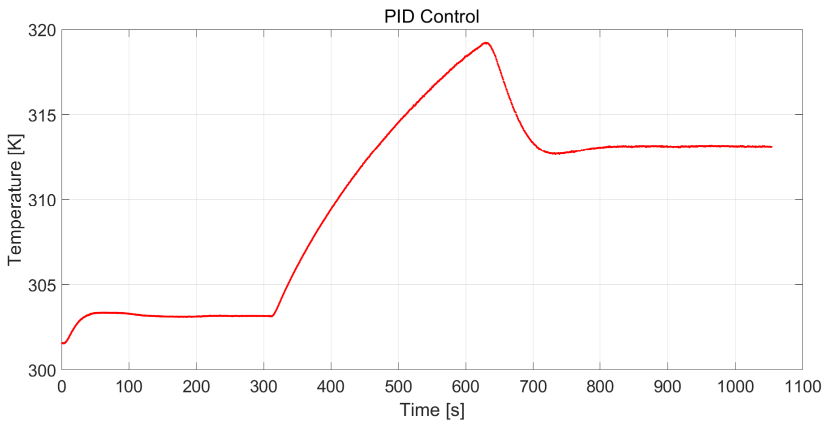


FIGURE 9. **Step reference PI-control.** The plot shows the trend of temperature over time. The first (lowest) reference temperature is set to $\simeq 303\ K$, the second (highest) to $\simeq 313\ K$.

3.6.1. *Longer sampling interval.* Then, the measurement was repeated in the same conditions except with ten times shorter sampling rate. The samples number was set to 180. For this task, the code was also changed so that the reference changed from T_{ref_1} to T_{ref_2} after 50 samples. The result is shown in fig. 10.

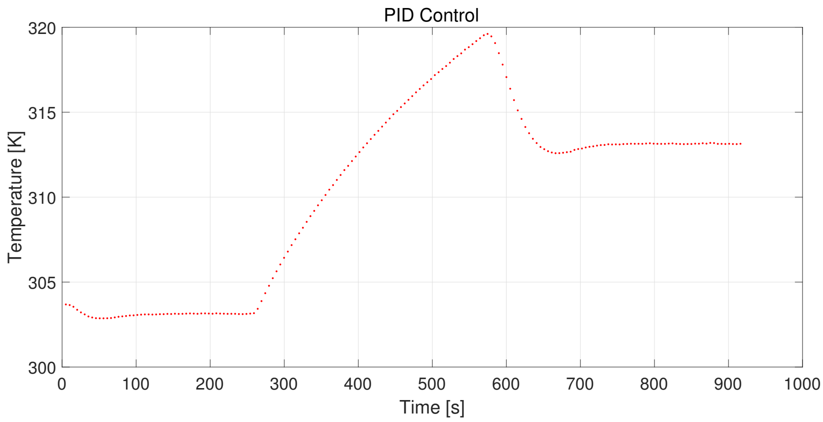


FIGURE 10. **Step reference PI-control; shorter sampling rate.** The plot shows the trend of temperature over time. The first (lowest) reference temperature is set to $\simeq 303\ K$, the second (highest) to $\simeq 313\ K$.

3.6.2. *Asymptotic deviation.* I wanted to verify the response presents an asymptotic deviation between T_{ref} and T when $K_I = 0$. The measurement was performed in the same conditions as section 3.6. The outcome is shown in fig. 11.

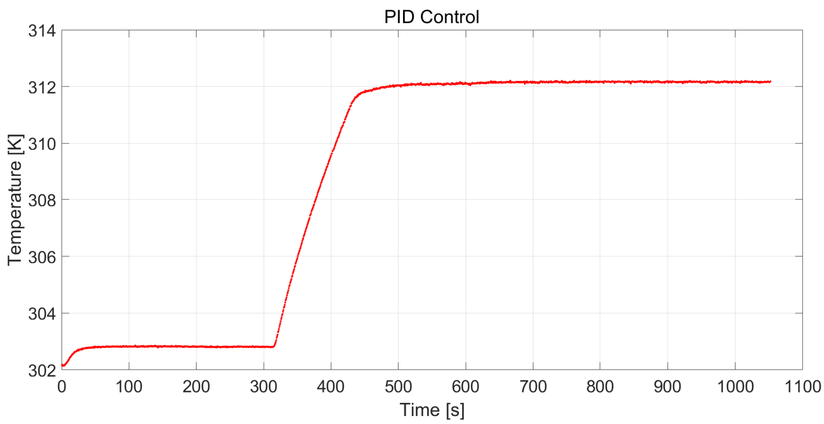


FIGURE 11. **Step reference PI-control; $\mathbf{K_I = 0}$.** The plot shows the trend of temperature over time. The first (lowest) reference temperature is set to $\simeq 303\ K$, the second (highest) to $\simeq 313\ K$.

3.7. **Feedback correction.** To make the system unstable, the corrective feedback was increased by setting $K_P = 100$. I carried out this measurement with one reference temperature $T_{ref_1} = 303.15\ K$ only. The system response is presented on fig. 12.

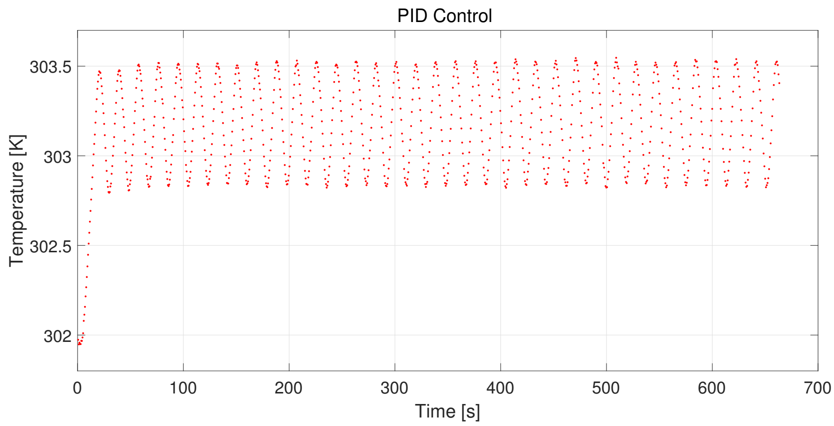


FIGURE 12. **PI-control; $\mathbf{K_P = 100}$.** The plot shows the trend of temperature over time. The reference temperature is set to $\simeq 303\ K$.

4. Discussion

4.1. **Room temperature.** A temperature increase was recorded, as it is evident from fig. 2. Temperature growth is approx. $0.15\ K$ per $2000\ s$, that means temperature rose about $0.045\ K$ every 10 min. Responsible for this trend can possibly be the ventilation still switched on and/or the difference in temperature between my body and the room.

4.2. **Step-excitation.** It is deductible from both fig. 3 and fig. 4 that the response to the step-excitation causes the temperature to reach and stabilise about a given value. This agrees with the behaviour predicted by eq. (3): as $t \to \infty$, $r(t) \to \Delta E H_0 = r(\infty)$ which is constant. Nevertheless, it is also evident temperature in fig. 3 fluctuates much more than in fig. 4.

In order to better understand the trends in fig. 3 and fig. 4, let me take into account fig. 13 and fig. 14. They are enlargements of the saturation region of the first and second step-excitation measurements, respectively. It is clear that the second measurement gave a better result than the first: temperature oscillates in a range of $\pm 0.7\ K$ in the first case and $\pm 0.15\ K$ in the second. Moreover, whereas in the second graph the value seem to establish around a given one (just keep oscillating about it) , in the first it is uneasy to infer an estimate of the saturation temperature.

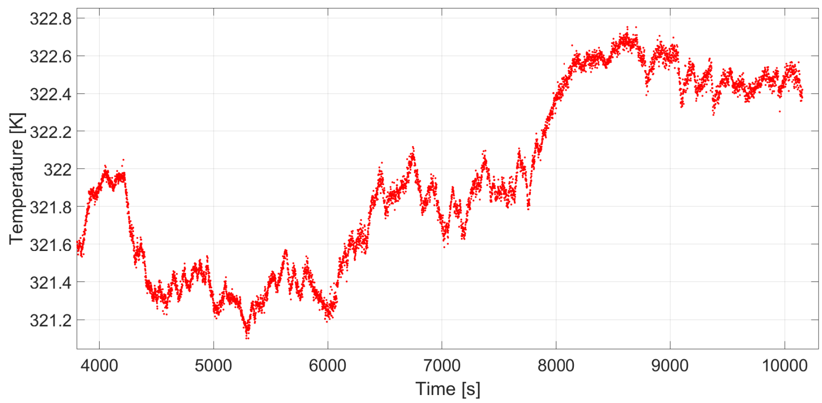


FIGURE 13. **Step-excitation; first measurement.** Detail from approx. $t = 4000\ s$ on.

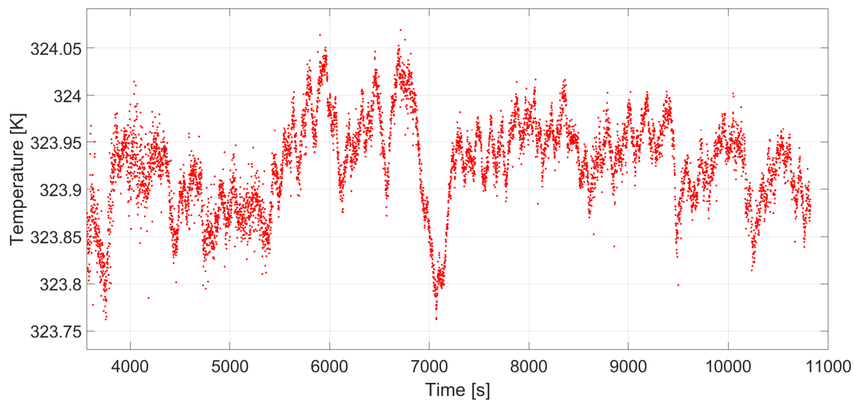


FIGURE 14. **Step-excitation; second measurement.** Detail from approx. $t = 3500\ s$ on.

4.3. **Pulse-excitation.** The temperature trend in fig. 5 respects the awaited response (see eq. (4)): as $t \to \infty$, $r(t) \to 0 \implies T \to T_{room}$. Moreover, when $t < \epsilon$ the trend is the same as in the step-excitation. Temperature starts growing as the measurement starts, reaching the peak value $T \simeq 298.72\ K$ at $t \simeq 27\ s$. This corresponds to the 46^{th} sample. So temperature continued to rise (about $9\ s$) after the excitation stopped. This is due to the fact that the Peltier oven and the thermistor are not in direct contact: indeed we seek to detect the container's temperature and not the Peltier element's one. Certainly, it takes heat some time to spread throughout the container and reach the thermistor after it is developed by the oven. The response after the maximum shows a decrease of T towards T_{room}, as expected.

4.4. **PI-controller.** The PI-control of the system is clearly shown in fig. 8. First, temperature increases towards the reference value T_{ref_1}. It then continues to grow for another 3 K before reaching the peak value and decreases. This behaviour is again partly attributed to the fact that the Peltier oven and the thermistor are not in direct contact (see 4.3). Moreover, it shows how the regulator works: when the thermistor reads a temperature higher than the reference (which happens with a delay, as said above), it tells the oven to decrease the temperature. Consequently, the container temperature decreases towards T_{ref_1}, now performing a small inflection with downwards concavity. It then stabilise around the reference value. This is possible thanks to the integral term in eq. (8), which keeps track of the whole previous behaviour.

4.5. **Step reference PI-controller.** In the step-reference measurement (fig. 9), it is firstly evident that the starting temperature is not the room value. This happened because the code was run after the previous PI-control, leaving the Peltier element heated above room temperature. The trend respects the awaited behaviour, initially growing towards the first reference temperature, T_{ref_1}, and then saturating around it. In a more detailed way, it first reaches a peak and then makes a small downwards inflection: it replicates the trend in fig. 8. Since these are scarcely visible due to the picture scaling, I enlarged the region concerning the first reference in fig. 15.

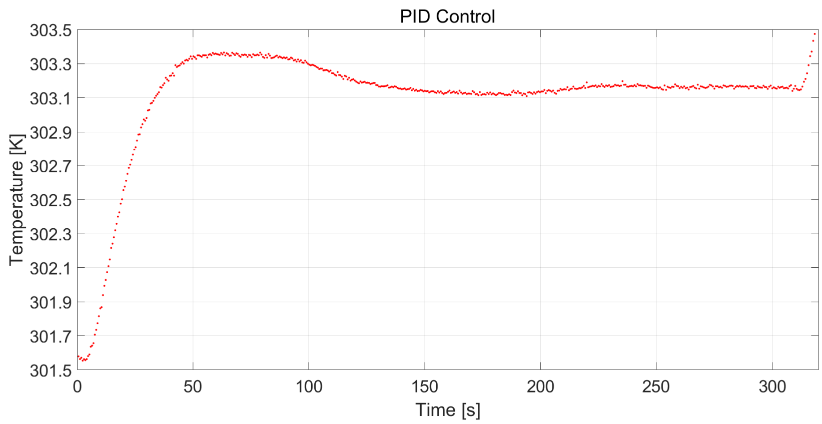


FIGURE 15. **Step-reference PI-control; detail.** The visible data are those around the first reference temperature $T_{ref_1} = 303.15\ K$.

Afterwards, when the reference is changed to T_{ref_2}, temperature starts increasing again. The trend then replicates again the one in fig. 8.

Applying longer sampling intervals (fig. 10) does not to affect the response. Temperature shows the same trend as in fig. 9. Simply, the samples are taken 10 times less frequently. Hence, I set the code to 180 samples instead of 1500 (previous task).

As for the asymptotic deviation shown in fig. 11, let me compute the ratio between $r_{ref} = T_{ref} - T_{room}$ and the average response $r_{mean} = T_{mean} - T_{room}$ detected after approaching saturation:

$$\left(\frac{r_{ref}}{r_{mean}}\right)_1 = 0.952 \pm 0.009, \qquad \left(\frac{r_{ref}}{r_{mean}}\right)_2 = 0.942 \pm 0.004 \tag{19}$$

and compare it to the theoretical value, given by eq. (10):

$$\left(\frac{r_{ref}}{r_{mean}}\right)_{theoretical} = 0.944 \pm 0.005 \tag{20}$$

Both the ratios obtained are compatible with the expected value, within uncertainty.

4.6. **Feedback correction.** The system is definitely unstable when $K_P = 100$. The response shown in fig. 12 oscillates about $\pm 0.35\ K$ around the reference temperature $T_{ref} = 303.15\ K$. The amplitude of the oscillation remains approximately constant throughout the measurement.

5. Conclusion

The experiment as a whole gave results in agreement with the expected behaviour. The room temperature measurement and the step and pulse-excitation allowed me to compute the parameters to perform the PI-control.

The system response under PI-control was consistent with the awaited trend. Also, I verified how the response varies when using a step-reference. Moreover, I displayed that a longer sampling frequency does not alter the result of the regulator. Finally, I was able to study how the feedback works by changing the values of the constants K_I and K_P in eq. (8), first showing the asymptotic behaviour of the response in absence of the integral term and then making the system unstable by letting the first term dominate.

The positive results of the PI-control can be considered a test of the goodness of the parameters obtained in the first part of the experiment.

References

[1] Course Material available on the Dropbox folder,
https://www.dropbox.com/sh/ao9oht81avxxaef/AABGRsi3zy2zq$_d LIJ4X5i9da?dl = 0$