~~pentru~~ care are det $A=3$

A^{-1}

18.05.21

Ex. 7.

$A=\begin{pmatrix} 0 & m & 1 \\ m & -2 & 0 \\ 1 & -1 & m \end{pmatrix}$ $\det A = -m^3 - m + 2 \quad |\cdot(-1)$

$m^3 + m - 2 = 0 \Leftrightarrow m^3 + m - 1 - 1 = 0$

$(m^3 - 1) + (m - 1) = 0$

$(m-1)(m^2+m+1) + (m-1) = 0$

$(m-1)[m^2+m+2] = 0 \Leftrightarrow$

$m = 1$ $\quad \Delta < 0$

Să se calculeze det.

$d = \begin{vmatrix} 1 & 2 & 3 & 1 \\ 2 & 0 & 1 & -1 \\ 3 & 1 & 2 & 4 \\ 1 & 2 & 3 & -2 \end{vmatrix}$ col 2

Să se rezolve ecuația în C.

$\begin{vmatrix} 1 & 1 & 1 \\ -1 & x & x^2 \\ 1 & -1 & x \end{vmatrix} = 0$

Ex 8. p. 225

$d = \begin{vmatrix} 1 & 2 & 3 & 1 \\ 2 & 0 & 1 & -1 \\ 3 & 1 & 2 & 4 \\ 1 & 2 & 3 & -2 \end{vmatrix} = 2\cdot(-1)^3 \begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 4 \\ 1 & 3 & -2 \end{vmatrix} + \cancel{0\cdot(-1)^4} \ldots$

$+ 1\cdot(-1)^5 \begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} + 2\cdot(-1)^6 \begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 4 \end{vmatrix}$

$= -2(-8 - 8 + 4 + 2 - 24 + 6) - (-2 + 6 - 3 - 9 + 3 + 12) +$

$+ 2(4 + 4 - 9 - 3 + 2 - 24) = -2(29) - 15 + 2(-26) =$

$= -58 - 15 - 52 = -125$ R: $\det A = -125$

$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & -1 & x \end{vmatrix} = 0$ E. 8.

$A = \begin{pmatrix} 1 & a+1 & 0 \\ 0 & 1 & a+1 \\ a+1 & 0 & 1 \end{pmatrix} \in M_3(\mathbb{R})$

a) $\exists A^{-1}$ dacă $\det A \neq 0$

$\det A = \begin{vmatrix} 1 & a+1 & 0 \\ 0 & 1 & a+1 \\ a+1 & 0 & 1 \end{vmatrix} = 1 + (a+1)^3 + 0 + 0 =$

$= 1 + a^3 + 3a^2 + 3a + 1 = a^3 + 3a^2 + 3a + 2$

$a^3 + 3a^2 + 3a + 2 \neq 0$

$(a+1)^3 + 1 \neq 0$

$(a+1)^3 \neq -1$

$a + 1 \neq \sqrt[3]{-1}$

$a + 1 = -1$

$a = -2$

$\boxed{a \in \mathbb{R} \setminus \{-2\}}$

b) $a = 1$

$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ $\det A = 9$

$A_{11} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$ $A_{12} = \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = +4$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2$ $A_{21} = \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = -2$

$A_{22} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$ $A_{23} = \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = 4$

$A_{31} = \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = 4$ $A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = -2$

$A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$ $A^* = \begin{pmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{pmatrix}$

~~$A^{-1} = \frac{1}{9}\begin{pmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{pmatrix} = \ldots$~~

$A^{-1} = \frac{1}{9}\begin{pmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{pmatrix} =$

$= \begin{pmatrix} \frac{1}{9} & -\frac{2}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{1}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{4}{9} & \frac{1}{9} \end{pmatrix}$

c) $X \cdot A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 3 & 2 \end{pmatrix}$

$X = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & \frac{4}{9} \\ \frac{4}{9} & \frac{1}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{4}{9} & \frac{1}{9} \end{pmatrix} \Leftrightarrow \boxed{X = \begin{pmatrix} \frac{3}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{13}{3} & -\frac{7}{3} \end{pmatrix}}$

Să se rezolve în C

$\begin{vmatrix} 1 & 1 & 1 \\ -1 & x & x^2 \\ 1 & -1 & x \end{vmatrix} = 0$

$x^2 + 1 + x^2 - x + x^2 + x = 0$

$3x^2 + 1 = 0$

$3x^2 = -1$

$x^2 = -\frac{1}{3}$ $\quad x^2 = \frac{1}{3} i^2$ $\quad x = \pm i\sqrt{\frac{1}{3}}$

$a = -\frac{1}{3}$ $\quad b = 0$

$a < 0$

$S = \left\{ -i\sqrt{\frac{1}{3}};\ i\sqrt{\frac{1}{3}} \right\}$