# Unsupervised Learning of Disentangled and Interpretable Representations from Sequential Data

Wei-Ning Hsu, Yu Zhang, and James Glass
Talk by Stefan Wezel

Seminar ML4S

January 11, 2021

# Overview

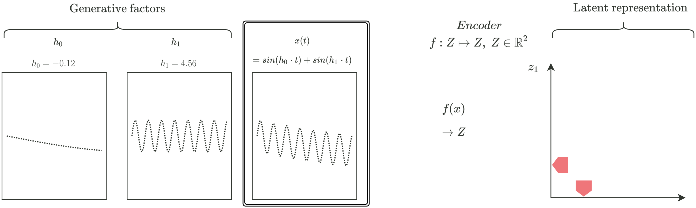
- Introduction
- What are Disentangled Representations? (intuition)
- Why Disentangled Representations
- Formal Description of Disentangled Representations
- Disentanglement in the Context of the Paper
- The Factorized Hierarchical VAE Model
- Results

- Propose Sequential Factorized Hierarchical VAE (**FHVAE**)
- Focus on speech data
  - Sequence level (Speaker, ...) attributes
  - Segment level (content, noise, ...) attributes
- Reflect different temporal scales in latent space

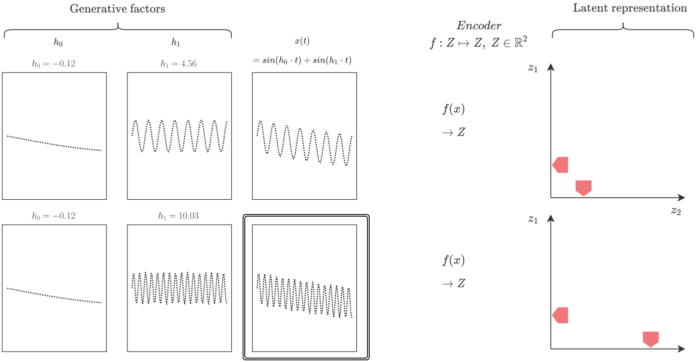
# What is disentanglement?

## Intuition

- Encode distinct generating factors in separate subsets of latent space dimensions



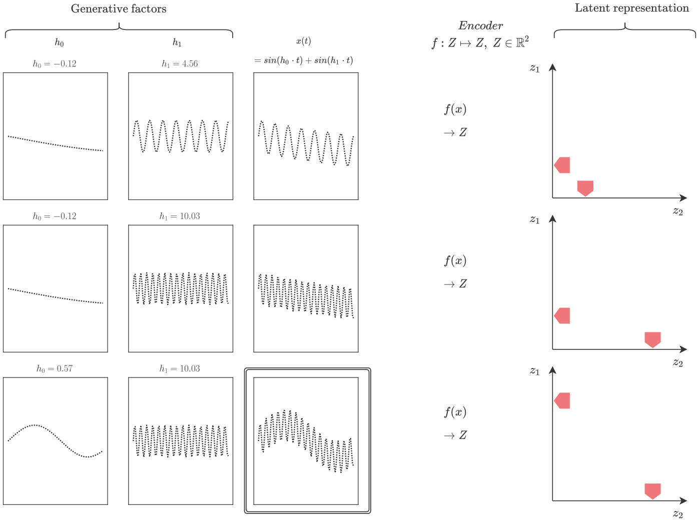
- Encode distinct generating factors in separate subsets of latent space dimensions



# What is disentanglement?

## Intuition

- Encode distinct generating factors in separate subsets of latent space dimensions



# Why learn disentangled representations?

## Motivation

- Explainability/Interpretability
- Fairness
- Scientific modeling
- Speaker verification
- Denoising

# Disentangled Representations Formally

## A field-trip to group theory: important concepts

- Group
  - Operation and non-empty set $G = (\circ, G)$
  - set closed under operation, identity element, inverses elements, associativity
- Symmetry group
  - Set of transformations that leave object $X$ invariant
  - Operation is composition of transformations
- Group action
  - Results of symmetry transformations on object $X$
  - i.e. set of changed order (permuation)
- Direct product
  - $G = G_1 \times ... \times G_n$

# Disentangled Representations Formally

A field-trip to group theory: What is disentanglement in terms of group theory?

- Disentangled group actions
  - Result of subset of symmetries $G_i$ that only change subset $X_i$ of object, but leave other $X_{j \neq i}$ invariant
- $\rightarrow$ If we observe disentangled group actions in the world, we want to model those
- We can assume $G$ can be decomposed into direct product of symmetry subgroups $g_i$

# Disentangled Representations Formally

## A field-trip to group theory: What is disentanglement in terms of group theory?

- We want to find symmetry preserving mapping $f : X \mapsto Z$

$$
\begin{array}{ccc}
X & \xrightarrow{G} & X \\
f \downarrow & & f \downarrow \\
Z & \xrightarrow{G} & Z
\end{array}
$$

- $\rightarrow$ Equivariant map $g \cdot f(x) = f(g \cdot x)$
- Result is disentangled representation
  - Decomposition $Z = Z_1 \times ... \times Z_n$
  - where $Z_i$ is only affected by transformations $G_i$ on $X$
  - and $Z_i$ invariant to all $G_{j \neq i}$ on $X$
  - $\rightarrow$ each subspace $Z_i$ can be transformed ONLY by the corresponding symmetry $G_i$ on $W$ (or on $Z$)

# Back to the paper

## Did they achieve disentanglement?

- Disentangled with respect to what decomposition?
- Assume decomposition $G = G_{sequence} \times G_{segment}$
- Reflect decomposition in $Z = (z_1, z_2)$
- $z_1$: segment, $z_2$: sequence
- Find equivariant map $f : W \mapsto Z$, so that $z_2$ is only affected by actions of $G_{sequence}$ and vice versa

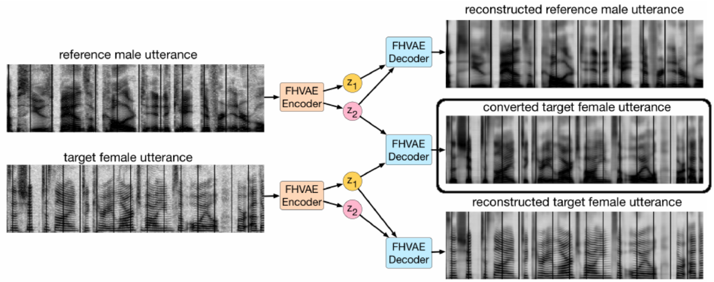
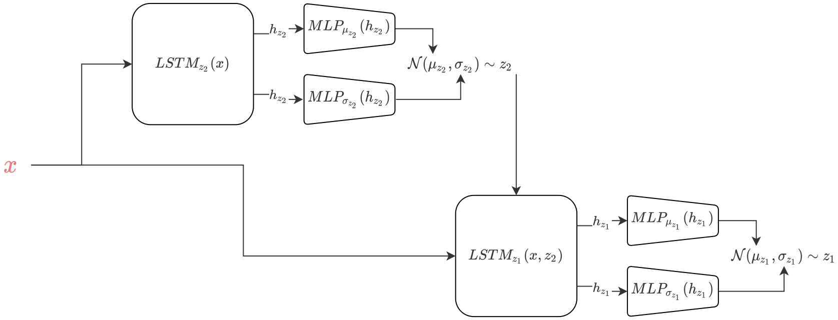


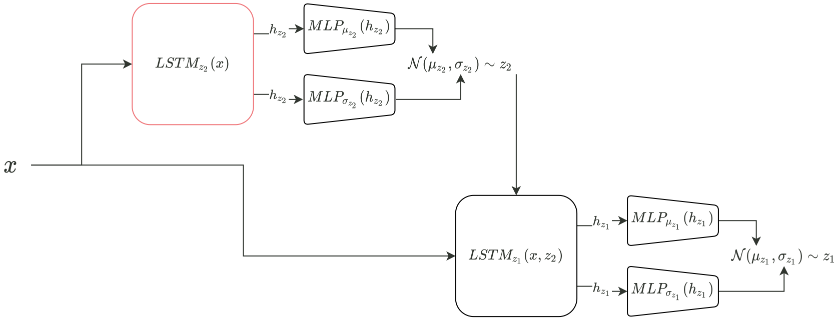

Image source: Hsu et al., 2017

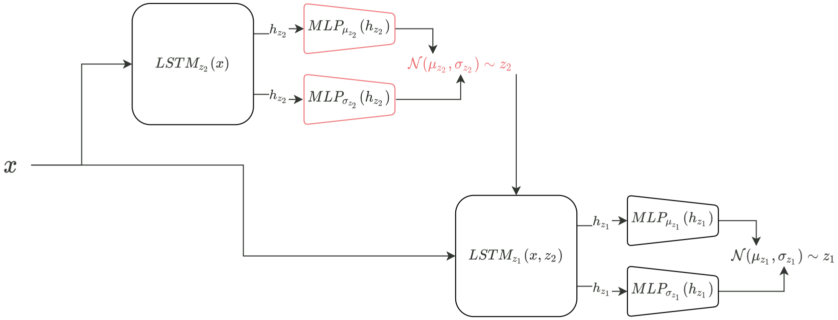
## Encoder$_\theta$

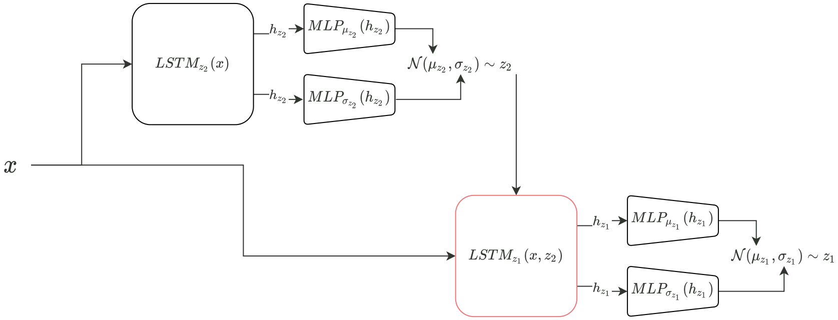


# FHVAE

## Encoder$_\theta$

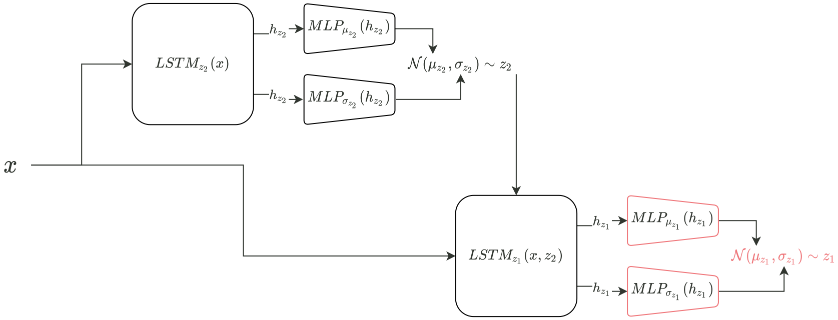


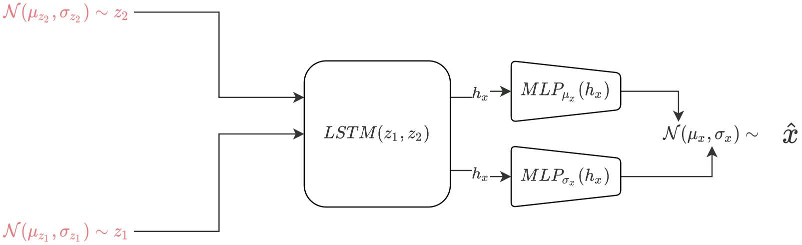

$x$
$LSTM_{z_2}(x)$
$h_{z_2}$
$h_{z_2}$
$MLP_{\mu_{z_2}}(h_{z_2})$
$MLP_{\sigma_{z_2}}(h_{z_2})$
$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
$LSTM_{z_1}(x, z_2)$
$h_{z_1}$
$h_{z_1}$
$MLP_{\mu_{z_1}}(h_{z_1})$
$MLP_{\sigma_{z_1}}(h_{z_1})$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$



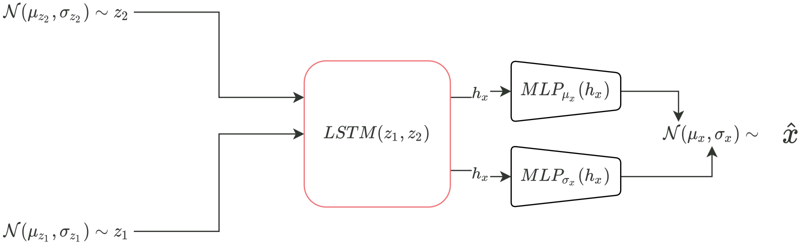
$LSTM_{z_2}(x)$
$h_{z_2}$
$h_{z_2}$
$MLP_{\mu_{z_2}}(h_{z_2})$
$MLP_{\sigma_{z_2}}(h_{z_2})$
$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
$x$
$LSTM_{z_1}(x, z_2)$
$h_{z_1}$
$h_{z_1}$
$MLP_{\mu_{z_1}}(h_{z_1})$
$MLP_{\sigma_{z_1}}(h_{z_1})$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$

Encoder$_\theta$

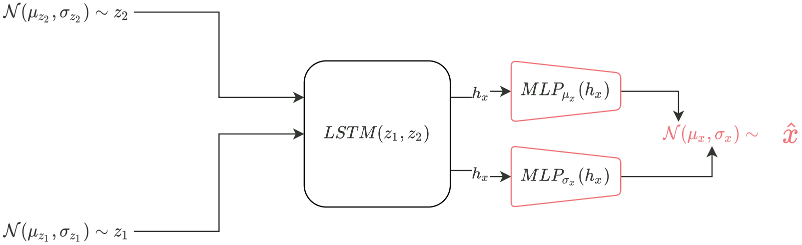




$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$
$LSTM(z_1, z_2)$
$h_x$
$h_x$
$MLP_{\mu_x}(h_x)$
$MLP_{\sigma_x}(h_x)$
$\mathcal{N}(\mu_x, \sigma_x) \sim \quad \hat{x}$



$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$
$LSTM(z_1, z_2)$
$h_x$
$h_x$
$MLP_{\mu_x}(h_x)$
$MLP_{\sigma_x}(h_x)$
$\mathcal{N}(\mu_x, \sigma_x) \sim \hat{x}$


$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$
$LSTM(z_1, z_2)$
$h_x$
$h_x$
$MLP_{\mu_x}(h_x)$
$MLP_{\sigma_x}(h_x)$
$\mathcal{N}(\mu_x, \sigma_x) \sim \hat{x}$

$$\mathcal{L}(\theta, \phi, X) = \sum_{n=1}^{N} \underbrace{\mathcal{L}(\theta, \phi; x^{(n)} | \tilde{\mu_2}) + log\ p_\theta(\tilde{\mu_2}) + const.}_{\text{var. lower bound}} + \underbrace{\alpha \cdot log\ (i | z_2^{(i,n)})}_{\text{discrim.obj.}}$$

with $\mathcal{L}(\theta, \phi; x^{(n)} | \tilde{\mu_2}) = \mathbb{E}_{q_\phi(z_1^{(n)}, z_2^{(n)} | x^{(n)})} \left[ log\ p_\theta(x^{(n)} | z_1^{(n)}, z_2^{(n)}) \right]$

$$-\mathbb{E}_{q_\phi(z_2^{(n)} | x^{(n)})} \left[ D_{KL}(q_\phi(z_1^{(n)} | x^{(n)}, z_2^{(n)}) || \underbrace{p_\theta(z_1^{(n)})}_{\text{sequ. ind.}}) \right]$$

$$-D_{KL}(q_\phi(z_2^{(n)} | x^{(n)}) || \underbrace{p_\theta(z_2^{(n)} | \tilde{\mu_2})}_{\text{seq. dep. prior}})$$

and $log\ (i | z_2^{(i,n)}) = log\ p(z_2^{(i,n)} | \tilde{\mu_2}^{(i)}) - log(\sum_{j=1}^{M} p(z_2^{(i,n)} | \mu_2^{(j)}))$

# FHVAE

## S-vector $\mu_2$

- What is sequence dependent prior $\mu_2$ (s-vector)?
  - imagine a word vector
  - s-vector for every sequence
  - similar sequence $\rightarrow$ in s-vectors close in euclidean space
  - $g(sequence\ id) = \mu_2$ as differentiable lookup table
    - $\rightarrow$ embedding in pytorch, tensorflow

Objective - Sequence variational lower bound

$$\mathcal{L}(\theta, \phi, X) = \underbrace{log\ p(x|z_1, z_2)}_{\text{reconstruction}}$$
$$- \underbrace{D_{KL}(\mathcal{N}(\mu_{z_1}, \sigma_{z_1})||\mathcal{N}(0,1))}_{\text{regularize } z_1 \text{ with global prior}}$$
$$- \underbrace{D_{KL}(\mathcal{N}(\mu_{z_2}, \sigma_{z_2})||\mathcal{N}(\tilde{\mu_2}, 0.5))}_{\text{regularize } z_2 \text{ with seq. dep. prior } \mu_2}$$
$$+ \underbrace{log\ p(\tilde{\mu_2}) \cdot \frac{1}{seq.\ length}}_{\text{prob. of } \tilde{\mu_2} \text{ under standard Gaussian prior}}$$

$$log\ p(sequence\ id|z_2) = CrossEntropy(\frac{-(\mu_{z_2} - \mu_2)^2}{\sigma_{z_2}^2},\ sequence\ id)$$

- $\rightarrow$ Try to predict sequence id with $z_2$

$$\mathcal{L}^{dis}(\theta, \phi; x) = \mathcal{L}(\theta, \phi, X) + \alpha \cdot log\ p(sequence\ id|z_2)$$

- Joint objective
  - encourage factorization
- discriminative objective can be adjusted through $\alpha$ hyperparameter
  - encourage $\mu_{z_2}$ to become more meaningful

- Task: Speaker verification
  - Allows quantitative analysis of performance
  - Assess quality of disentanglement
  - Use s-vector $\mu_2$ to predict speaker
- Compare i-vector baseline
  - i-vector used in SOTA speaker verification approaches
  - Low dimensional subspace of GMM universal background model
  - Contains speaker information (content-independent)

- Unsupervised speaker verification (Raw column)
- Metric: equal error rate (lower is better)
- $\mu_1$ based on $z_1$ as sanity check

| Features | Dimension | $\alpha$ | Raw | LDA (12 dim) | LDA (24 dim) |
|---|---|---|---|---|---|
| i-vector | 48 | - | 10.12% | 6.25% | 5.95% |
| | 100 | - | 9.52% | 6.10% | 5.50% |
| | 200 | - | 9.82% | 6.54% | 6.10% |
| $\boldsymbol{\mu}_2$ | 16 | 0 | 5.06% | 4.02% | - |
| | 16 | $10^{-1}$ | 4.91% | 4.61% | - |
| | 16 | $10^{0}$ | 3.87% | 3.86% | - |
| | 16 | $10^{1}$ | **2.38%** | **2.08%** | - |
| | 32 | $10^{1}$ | **2.38%** | **2.08%** | **1.34%** |
| $\boldsymbol{\mu}_1$ | 16 | $10^{0}$ | 22.77% | 15.62% | - |
| | 16 | $10^{1}$ | 27.68% | 22.17% | - |
| | 32 | $10^{1}$ | 22.47% | 16.82% | 17.26% |

- Evidence towards disentangling with respect to sequence-segment decomposition
  - other decompositions may prove more challenging
- Good performance on speaker verification and denoising task
- I had trouble disentangling simple examples
- Questions for you
  - Is learning disentangled representations worth the effort?
  - Have there been situations where you wished for an interpretable latent space?
  - Do you know any successful models where equivariant maps are used?

# References


- Hsu, W.N., Zhang, Y. and Glass, J., 2017. Unsupervised learning of disentangled and interpretable representations from sequential data. In Advances in neural information processing systems (pp. 1878-1889).
- Higgins, I., Amos, D., Pfau, D., Racaniere, S., Matthey, L., Rezende, D. and Lerchner, A., 2018. Towards a definition of disentangled representations. arXiv preprint arXiv:1812.02230.
- Scott, W.R., 2012. Group theory. Courier Corporation.
- Kingma, D.P. and Welling, M., 2013. Auto-encoding variational bayes. arXiv preprint arXiv:1312.6114.

# Disentangled Representations Formally

## A field-trip to group theory: Disentangle our example formally

- Signal can get shifted or warped
- the set of these transformations make up a symmetry group
- This can be decomposed into shifts and warps/subsets of original set (all shifted $\times$ all warped)
- Either content is preserved, or speaker is preserved
- the resulting set of transformed signals are the actions of the symmetry group on the world state

# Disentangled Representations Formally

## A field-trip to group theory

- This symmetry group can be decomposed into symmetry subgroups
- One affects location
- the other affects frequence

- Group action $G \times X \mapsto X$
- Group decomposes into direct product $G = G_{shifts} \times G_{warps}$
- Is disentangled with respect to decomposition of $G$
  - if there is decomposition $X = X_{shifted} \times X_{warped}$
  - and actions $G_{shifts} \times X_{shifted} \mapsto X_{shifted}$
  - and actions $G_{warps} \times X_{warped} \mapsto X_{warped}$

# What are disentangled representations formally?



- Let $W$ be the set of world states (all shifts and warps of signal)
- Generative process $b : W \mapsto O$ (voice to audio processing unit)
- Inference process $h : O \mapsto Z$ (observation to latent space)
- $f : W \mapsto Z, f = h \circ b$
- Now, we know, there is a symmetry group acting on $W$ ($G \times W \mapsto W$)
- We want to find corresponding $G \times Z \mapsto Z$ to reflect symmetry structure of W in Z
- More formal: $g \cdot f(w) = f(g \cdot w)$
- This is whats called an equivariant map (famous example: convnet)

# What are disentangled representations formally?

## Disentangled Representation

- Assume symmetry transformations $G$ of $W$ decompose into direct product $G = G_1 \times \ldots \times G_n$
- Representation is disentangled if
  - equivariant map $f : W \mapsto Z, g \cdot f(w) = f(g \cdot w) \forall g \in G, w \in W$
  - such a map would split $Z$ into independent subspaces, thus satisfying:
    - Decomposition $Z = Z_{shifted} \times Z_{warped}$
    - where $Z_{shifted}$ is only affected by shifts in $W$ ($G_{shifts}$)
    - and $Z_{warped}$ is only affected by warps in $W$ ($G_{warps}$)
    - Thus each subspace can be transformed by the corresponding symmetry (like shift or warp independently)
- There may be more criteria (preserving group structure, isomorphisms, ...) but for the intuition this is sufficient

# Did they achieve disentanglement

...

- With respect to a decomposition into two
- Setting: 10 sentences, 630 speakers
- How can we formulate this in group theory terms?

- With respect to a decomposition into two
- regularize z2 by sequence dependant prior (lookup table of s-vectors)
- and z1 by sequence independant prior

# How did they do it?

## Methods

- Sample batch at segment level (instead of sequence level)
- Maximize segment variational lower bound
- (Force z2 to be close to mu2)
- approximation of mu2 is closed form equation (concave function, set derivative to 0)

# Challenges

...

- If we really think about it, it is hard for us to define what a disentangled representation should actually be
- Precise biases of what the latent space should be decomposited into can be helpful as well as biases towards the 'form' of these latent subspaces