Unsupervised Learning of Disentangled and Interpretable Representations from Sequential Data

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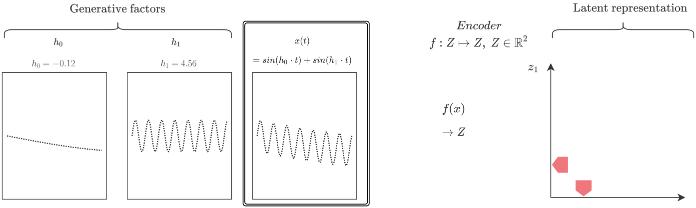
Overview

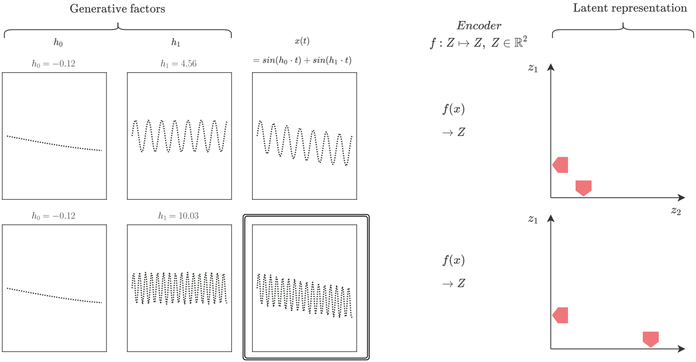
- Introduction
- What are Disentangled Representations? (intuition)
- Why Disentangled Representations
- Formal Description of Disentangled Representations
- Disentanglement in the Context of the Paper
- The Factorized Hierarchical VAE Model
- Results

- Propose Sequential Factorized Hierarchical VAE (**FHVAE**)
- Focus on speech data
  - Sequence level (Speaker, ...) attributes
  - Segment level (content, noise, ...) attributes
- Reflect different temporal scales in latent space

- Encode distinct generating factors in separate subsets of latent space dimensions



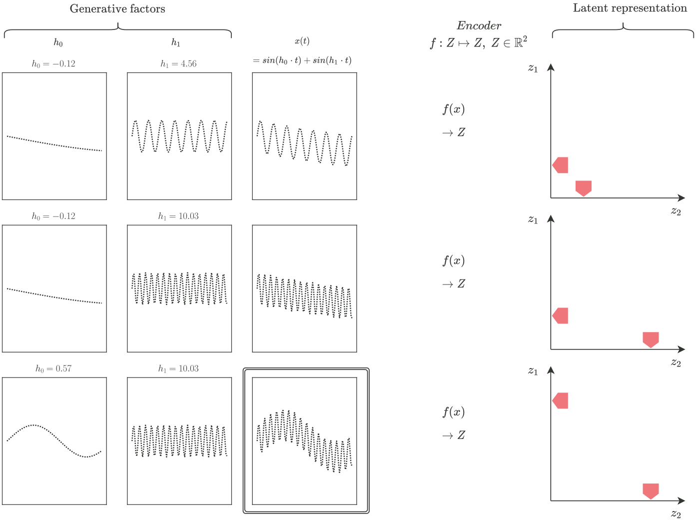
- Encode distinct generating factors in separate subsets of latent space dimensions



What is disentanglement?

Intuition

- Encode distinct generating factors in separate subsets of latent space dimensions



- Explainability/Interpretability
- Fairness
- Scientific modeling
- Speaker verification
- Denoising

Disentangled Representations Formally

A field-trip to group theory: important concepts

- Group
  - Operation and non-empty set $G = (\circ, G)$
  - Set closed under operation, identity element, inverse elements, associativity
- Symmetry group
  - Set of transformations that leave object X invariant
  - Operation is composition of transformations
- Group action
  - Results of symmetry transformations on object X
  - I.e. set of changed order
- Direct product
  - $G = G_1 \times ... \times G_n$

Disentangled Representations Formally

A field-trip to group theory: What is disentanglement in terms of group theory?

- Disentangled group actions
  - Result of subset of symmetries G_i that only change subset X_i of object, but leave other $X_{j \neq i}$ invariant
- $\rightarrow$ If we observe disentangled group actions in the world, we want to model those
- We can assume G can be decomposed into direct product of symmetry subgroups G_i

Disentangled Representations Formally

A field-trip to group theory: What is disentanglement in terms of group theory?

- We want to find symmetry preserving mapping $f : X \mapsto Z$

$$
\begin{array}{ccc}
X & \xrightarrow{G} & X \\
f \downarrow & & f \downarrow \\
Z & \xrightarrow{G} & Z
\end{array}
$$

- $\rightarrow$ Equivariant map $g \cdot f(x) = f(g \cdot x)$
- Result is disentangled representation
  - Decomposition $Z = Z_1 \times ... \times Z_n$
  - Z_i only affected by symmetry G_i on X
  - Z_i invariant to all $G_{j \neq i}$

Back to the paper

Did they achieve disentanglement?

- Disentangled with respect to what decomposition?
- Assume decomposition $G = G_{sequence} \times G_{segment}$
- Reflect decomposition in $Z = (z_1, z_2)$
- z_1: segment, z_2: sequence
- Find equivariant map $f : W \mapsto Z$, so that z_2 is only affected by symmetries $G_{sequence}$ and vice versa

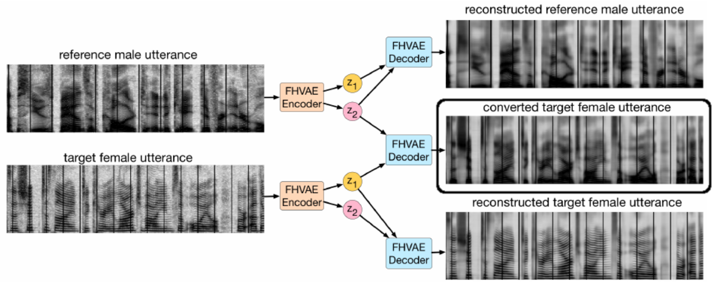
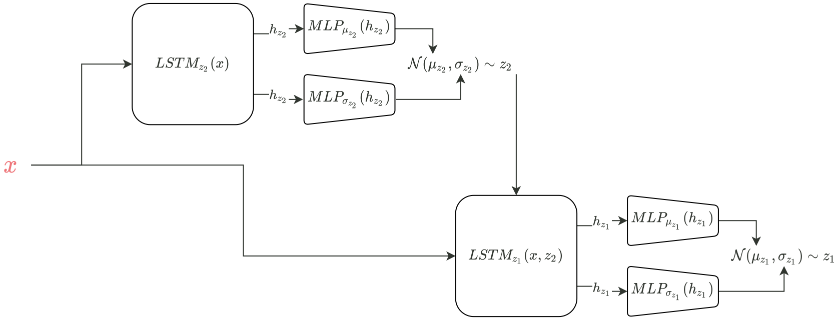


Image source: Hsu et al., 2017

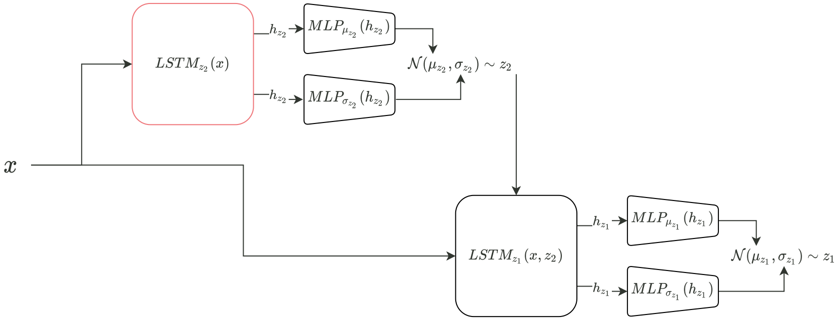
FHVAE

Encoder$_\theta$



FHVAE

Encoder$_\theta$



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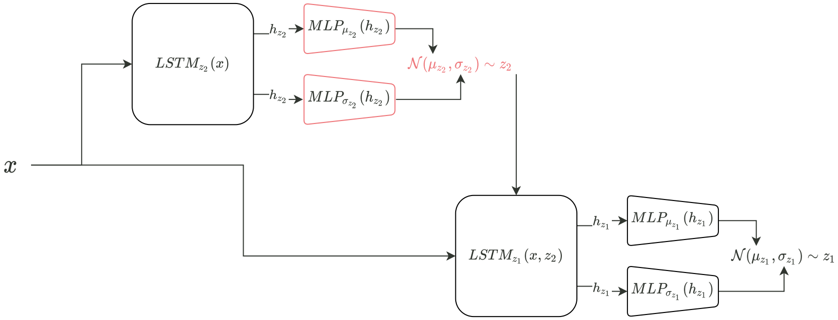
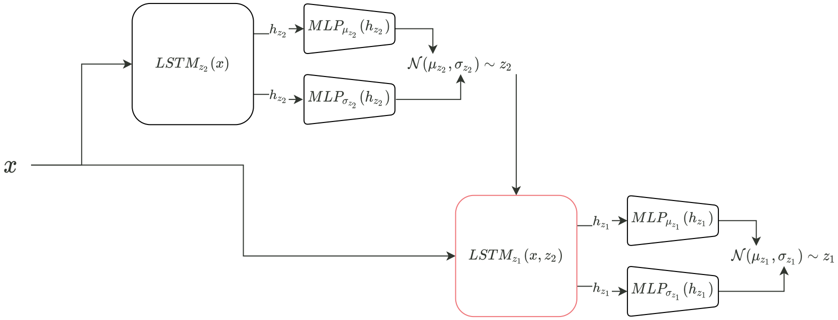
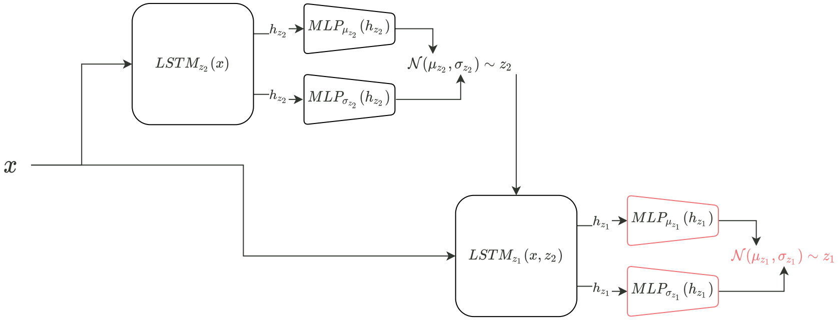


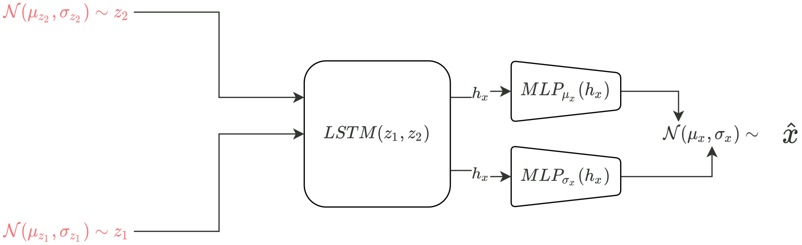


$LSTM_{z_2}(x)$
h_{z_2}
h_{z_2}
$MLP_{\mu_{z_2}}(h_{z_2})$
$MLP_{\sigma_{z_2}}(h_{z_2})$
$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
x
$LSTM_{z_1}(x, z_2)$
h_{z_1}
h_{z_1}
$MLP_{\mu_{z_1}}(h_{z_1})$
$MLP_{\sigma_{z_1}}(h_{z_1})$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$

FHVAE

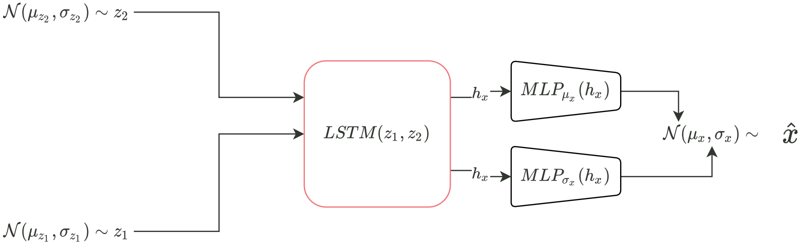
Encoder$_\theta$





$\mathcal{N}(\mu_{z_2}, \sigma_{z_2}) \sim z_2$
$\mathcal{N}(\mu_{z_1}, \sigma_{z_1}) \sim z_1$
$LSTM(z_1, z_2)$
h_x
h_x
$MLP_{\mu_x}(h_x)$
$MLP_{\sigma_x}(h_x)$
$\mathcal{N}(\mu_x, \sigma_x) \sim \hat{x}$

Decoder$_\phi$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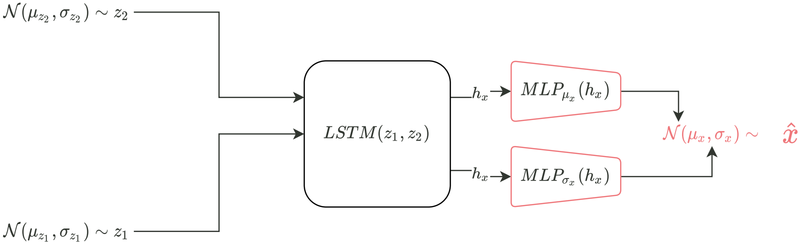
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$\mathcal{N}(\mu_x, \sigma_x) \sim \hat{x}$

$$\mathcal{L}(\theta,\phi,X) = \sum_{n=1}^{N} \underbrace{\mathcal{L}(\theta,\phi;x^{(n)}|\mu_2) + log\ p_\theta(\mu_2) + const.}_{\text{var. lower bound}} + \underbrace{\alpha \cdot log\ (i|z_2^{(i,n)})}_{\text{discrim.obj.}}$$

with $\mathcal{L}(\theta,\phi;x^{(n)}|\mu_2) = \mathbb{E}_{q_\phi(z_1^{(n)},z_2^{(n)}|x^{(n)})}\left[log\ p_\theta(x^{(n)}|z_1^{(n)},z_2^{(n)})\right]$

$$-\mathbb{E}_{q_\phi(z_2^{(n)}|x^{(n)})}\left[D_{KL}(q_\phi(z_1^{(n)}|x^{(n)},z_2^{(n)})||\underbrace{p_\theta(z_1^{(n)})}_{\text{sequ. ind.}})\right]$$

$$-D_{KL}(q_\phi(z_2^{(n)}|x^{(n)})||\underbrace{p_\theta(z_2^{(n)}|\mu_2)}_{\text{seq. dep. prior}})$$

and $log\ (i|z_2^{(i,n)}) = log\ p(z_2^{(i,n)}|\mu_2^{(i)}) - log(\sum_{j=1}^{M} p(z_2^{(i,n)}|\mu_2^{(j)}))$

- What is sequence dependent prior μ_2 (s-vector)?
  - Imagine a word vector
  - S-vector for every sequence
  - Similar sequence attributes $\rightarrow$ s-vectors close in euclidean space
  - $g(sequence\ id) = \mu_2$ as (differentiable) lookup table
    - $\rightarrow$ Embedding in pytorch, tensorflow

FHVAE

Objective - Sequence variational lower bound

$$\mathcal{L}(\theta, \phi, X) = \underbrace{log\ p(x|z_1, z_2)}_{\text{reconstruction}}$$

$$- \underbrace{D_{KL}(\mathcal{N}(\mu_{z_1}, \sigma_{z_1})||\mathcal{N}(0, 1))}_{\text{regularize } z_1 \text{ with global prior}}$$

$$- \underbrace{D_{KL}(\mathcal{N}(\mu_{z_2}, \sigma_{z_2})||\mathcal{N}(\mu_2, 0.5))}_{\text{regularize } z_2 \text{ with seq. dep. prior } \mu_2}$$

$$+ \underbrace{log\ p(\mu_2) \cdot \frac{1}{seq.\ length}}_{\text{prob. of } \mu_2 \text{ under standard Gaussian prior}}$$

$$log\ p(sequence\ id^{(i)} | z_2^{(i,n)}) = log\ \frac{p(\mu_{z_2}^{(i,n)} | \mu_2^{(i)})}{\sum_{j=1}^{num\ seqs} p(\mu_{z_2}^{(i,n)} | \mu_2^{(j)})}$$

- Encourage $z_2^{(i)}$ to be close to corresponding $\mu_2^{(i)}$
- and far from all other sequence's s-vectors $\mu_2^{(j \neq i)}$

FHVAE

Objective - Discriminative segment variational lower bound

$$\mathcal{L}^{dis}(\theta, \phi; x) = \mathcal{L}(\theta, \phi, X) + \alpha \cdot log\ p(sequence\ id|z_2)$$

- Joint objective to encourage factorization
- α hyperparameter to weigh discriminative objective

- Task: Speaker verification
  - Allows quantitative analysis of performance
  - Assess quality of disentanglement
  - Use s-vector μ_2 to predict speaker
- Compare i-vector baseline
  - i-vector used in SOTA speaker verification approaches
  - Low dimensional subspace of GMM universal background model
  - Contains speaker information (content-independent)

- Unsupervised speaker verification (Raw column)
- Metric: equal error rate (lower is better)
- μ_1 based on z_1 as sanity check

Features	Dimension	α	Raw	LDA (12 dim)	LDA (24 dim)
i-vector	48	-	10.12%	6.25%	5.95%
	100	-	9.52%	6.10%	5.50%
	200	-	9.82%	6.54%	6.10%
$\boldsymbol{\mu}_2$	16	0	5.06%	4.02%	-
	16	10^{-1}	4.91%	4.61%	-
	16	10^{0}	3.87%	3.86%	-
	16	10^{1}	**2.38%**	**2.08%**	-
	32	10^{1}	**2.38%**	**2.08%**	**1.34%**
$\boldsymbol{\mu}_1$	16	10^{0}	22.77%	15.62%	-
	16	10^{1}	27.68%	22.17%	-
	32	10^{1}	22.47%	16.82%	17.26%

- Evidence towards disentangling with respect to sequence-segment decomposition
  - Other decompositions may prove more challenging
- Good performance on speaker verification and denoising task
- I had trouble disentangling simple examples
- Questions for you:
  - Is learning disentangled representations worth the effort?
  - Have there been situations where you wished for an interpretable latent space?
  - Do you know any successful models where equivariant maps are used?

References

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- Scott, W.R., 2012. Group theory. Courier Corporation.
- Kingma, D.P. and Welling, M., 2013. Auto-encoding variational bayes. arXiv preprint arXiv:1312.6114.

Challenges

...

- If we really think about it, it is hard for us to define what a disentangled representation should actually be
- Precise biases of what the latent space should be decomposited into can be helpful as well as biases towards the ’form’ of these latent subspaces