Simulation-based Inference

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Abstract

Simulators are used across different scientific domains where they serve as valuable tools to encode empirical knowledge about a system of interest. Inference in this setting often means finding parameters of the simulator for a given, real-world observation, which typically cannot be computed analytically.Papamakarios and Murray [2016] and Lueckmann et al. [2017] propose to learn a posterior over a simulators parameters by using neural density estimators. Here, we give an overview of their work and SBI in general.

Problem Setting and Related Work

Scientific fields, such as population genetics, particle physics, epidemiology, astrophysics among others make use of sophisticated simulators to model observed systems [Brehmer and Cranmer, 2020, de Witt et al., 2020, Delaunoy et al., 2020, Cranmer et al., 2020, Pritchard et al., 1999]. These simulators serve as forward models with strong inductive biases, encoding prior knowledge. Performing inference in this setting, however, would require a finding a posterior over parameters θ for a given observation x_0. This can be stated in closed form as

$$p(\theta \mid x = x_0) = \frac{p(x \mid \theta)\, p(\theta)}{p(x)} = \frac{\overbrace{\int p(x, z \mid \theta)\, \mathrm{d}z}^{\text{intractable}}\, p(\theta)}{\int p(x \mid \theta)\, p(\theta)\, \mathrm{d}\theta}, \tag{1}$$

where z is a nuisance parameter and x is observation data from experiments or simulations. If likelihood and evidence are intractable, which is typically the case, this cannot be computed analytically.
A collective term for methods that aim to solve this inverse problem is Approximate

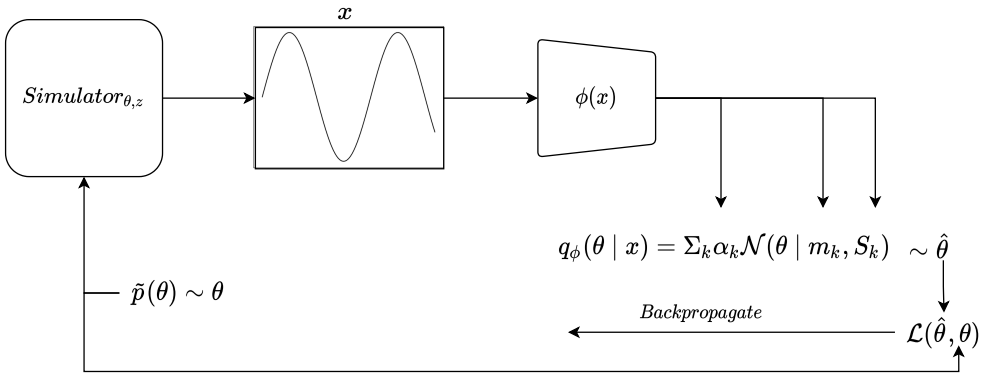

Figure 1: Given samples by the simulator, Multilayer perceptrons (MLP) are trained to parameterize a distribution over parameters generating the sample. A training signal is created by sampling from this distribution and comparing the sample to the actual parameters which are known.

Bayesian Computation (ABC). The most simple approach is rejection ABC [Pritchard et al., 1999] where samples produced by a simulator are discarded if not within an ϵ-ball around an observation of interest. For small values of ϵ this method may take unfeasibly many steps to even produce a single observation. For large ϵ it is not precise. Moreover, this method does not produce a full posterior over the parameter but merely yields point estimates that can be used to approximate a posterior. However, this approximated posterior $\hat{p}(\theta \mid\mid\mid x - x_0 \mid\mid < \epsilon)$ is not conditioned on an actual observation but rather on a value ϵ-close to it.
With Sampling ABC [Marjoram et al., 2003] and Sequential ABC [Beaumont et al., 2009, Bonassi et al., 2015] improvements over rejection ABC have been proposed. These methods produce samples more efficiently. While this can lead to faster convergence, they also do not produce a full posterior over parameters and thus suffer from the latter problem as well.

Neural Approach

Papamakarios and Murray [2016] build on recent advances in deep learning and propose to learn a posterior over parameters directly from samples produced by a simulator. They parameterize a Gaussian Mixture Model (GMM) with a Deep Neural Network (DNN) with parameters. They use the simulator to perform n simulations and store pairs of observations x_i and corresponding parameters θ_i, sampled from a proposal prior $\tilde{p}(\theta) = p(\theta)$, where $p(\theta)$ is a uniform or Gaussian distribution. This gives the set $\{(x_i, \theta_i)\}_{i \in n}$ which Pa-

pamakarios and Murray [2016] use as training data. A DNN with parameters ϕ is fitted to parameterize a GMM

$$q_\phi(\theta \mid x) = \sum_k \alpha_k \mathcal{N}(\theta \mid m_k, S_k) = \sum_k \phi_1(x)_k \mathcal{N}(\theta \mid \phi_2(x)_k, diag(\phi_3(x))_k) \tag{2}$$

, where k is the number of components, m the mean and S the covariance matrix of a Gaussian, and $\phi_{\{1,2,3\}}$ are distinct (but not necessarily mutually exclusive) subsets of a DNN.

To find a suiting function ϕ, $x_{0,\dots,n}$ are passed to it. Each respective output is used to parameterize $q_\phi(\theta \mid x)$. A sample $\hat{\theta}$ is then drawn from $\frac{p(\theta)}{\tilde{p}(\theta)} q(\theta \mid x)$. The difference between this sample $\hat{\theta}$ and the actual θ forms a loss which is backpropagated through ϕ. This completes an 'inner' training loop. Papamakarios and Murray [2016] further propose an 'outer' training loop, where once ϕ is fitted to n observations, $\frac{p(\theta)}{\tilde{p}(\theta)} q(\theta \mid x = x_0)$ replaces the proposal prior for a specific observation of interest. This procedure is summarized in Algorithm 1. This, however, requires $\tilde{p}(\theta)$ to be of a form so that the importance weights $\frac{p(\theta)}{\tilde{p}(\theta)}$ can be computed analytically.

$\tilde{p}(\theta) \leftarrow p(\theta)$
repeat
  for $n = 1..N$ **do**
    sample $\theta_n \sim \tilde{p}(\theta)$
    sample $x_n \sim p(x \mid \theta_n)$
  end
  train $q_\phi(\theta \mid x)$ on $\{x_n, \theta_n\}$
  $\tilde{p}(\theta) \leftarrow \frac{p(\theta)}{\tilde{p}(\theta)} q_\phi(\theta \mid x)$
until $\tilde{p}(\theta)$ *has converged*;

Algorithm 1: Training loop proposed by Papamakarios and Murray [2016].

Application in Neuroscience

Lueckmann et al. [2017] propose further enhancements to Papamakarios and Murray [2016]. They apply SBI to forward models widely used in neuro-science, such as the Hodgkin-Huxley model Hodgkin and Huxley [1952]. They state that simulators used in this field often produce nonsensical observations. To alleviate this, Lueckmann et al. [2017] additionally train a classifier that learns to detect parameter sets that will produce such 'bad' simulations, which are then discarded.

Further, they state that finding useful summary statistics of the often high-dimensional data is challenging. By using a recurrent neural network to extract features from observa-

tions, they find an informative, learned summary statistic.
Moreover, Lueckmann et al. [2017] formulate a loss such that it includes the importance weighting. This allows the use of more complex proposal priors, as they don't need to be computed analytically in their method.
Lueckmann et al. [2017] evaluate their method for different models and find that it reliably finds matching parameter settings for synthetic data. On in-vitro experiments they find parameter settings that match empirical data accurately.

Conclusion

Simulators are an important tool of reasarch scientiest across domains. SBI enables researches to leverage these inductive biases and perform inference with them.. Papamakarios and Murray [2016] build on recent advances in deep learning and propose a flexible and powerful SBI framework. Lueckmann et al. [2017] apply their method to a neuroscience setting and alleviate some encountered shortcomings by introducing further enhancements. An afterthought of our summary is that SBI methods can be applied in different settings where simulators are used but is not necessarily a fit-all solution. However, adaptions can be made accordingly, to suit a given task.

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