Categorical: Contingency tables

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1 Goals

1.1 Goals

1.1.1 Goals of this section

- Contingency tables
  - a.k.a., crosstabs, frequency tables
  - 2-way (2 variables) and 3-way (3 variables)
  - Residuals
- Chi-square tests of independence
  - Fisher's Exact Test
  - Conditional, marginal, Simpson's paradox

1.1.2 Goals of this lecture

- Contingency tables
- Study design
- Measures of relationship
- Chi-square tests

2 Contingency tables

2.1 Layout and notation

2.1.1 Contingency table

- Shows the relationship between two (or more) variables
  - Frequency of responses in each category
- Do you believe in an afterlife?

	Yes	No
Female	509	116
Male	398	104

2.1.2 Notation for frequencies

	$J = 1$	$J = 2$	
$I = 1$	n_{11}	n_{12}	
$I = 2$	n_{21}	n_{22}	

2.1.3 Notation for frequencies

	$J = 1$	$J = 2$	
$I = 1$	n_{11}	n_{12}	n_{1+}
$I = 2$	n_{21}	n_{22}	n_{2+}
	n_{+1}	n_{+2}	n

- n_{11}, n_{12}, n_{21}, n_{22} are **joint** frequencies
- n_{1+}, n_{2+}, n_{+1}, n_{+2} are **marginal** frequencies

2.1.4 Notation for probabilities

	$J = 1$	$J = 2$	
$I = 1$	p_{11}	p_{12}	
$I = 2$	p_{21}	p_{22}	

2.1.5 Notation for probabilities

	$J = 1$	$J = 2$	
$I = 1$	p_{11}	p_{12}	p_{1+}
$I = 2$	p_{21}	p_{22}	p_{2+}
	p_{+1}	p_{+2}	1

- p_{11}, p_{12}, p_{21}, p_{22} are **joint** probabilities $= \frac{n_{ij}}{n}$
- p_{1+}, p_{2+}, p_{+1}, p_{+2} are **marginal** probabilities $= \frac{n_{i+}}{n}$ or $\frac{n_{+j}}{n}$
- Note: p used for **sample** values, π for corresponding **population** values

2.1.6 Marginal probability

- **Marginal probability**: Probability of X or Y, collapsing over the other
  - What is the distribution of X, ignoring Y?
  - What is the distribution of Y, ignoring X?

2.1.7 Marginal probability: Do you believe in an afterlife?

- Start with frequencies

	$Y = 1$: Yes	$Y = 2$: No	
$X = 1$:Female	$n_{11} = 509$	$n_{12} = 116$	$n_{1+} = 625$
$X = 2$: Male	$n_{21} = 398$	$n_{22} = 104$	$n_{2+} = 502$
	$n_{+1} = 907$	$n_{+2} = 220$	$n = 1127$

2.1.8 Marginal probability: Do you believe in an afterlife?

- Divide each value by n: The total sample size

	$Y = 1$: Yes	$Y = 2$: No	
$X = 1$:Female	$p_{11} = 0.452$	$p_{12} = 0.103$	$p_{1+} = 0.555$
$X = 2$: Male	$p_{21} = 0.353$	$p_{22} = 0.092$	$p_{2+} = 0.445$
	$p_{+1} = 0.805$	$p_{+2} = 0.195$	$p = 1$

- *Joint* probabilities sum to 1: $0.452 + 0.353 + 0.103 + 0.092 = 1$
- *Marginal* probabilities for *rows* sum to 1: $0.555 + 0.445 = 1$
- *Marginal* probabilities for *columns* sum to 1: $0.805 + 0.195 = 1$

2.1.9 Conditional probability

- Typically, X is an explanatory variable (predictor)
  - Y is an outcome variable
- **Conditional probability**: Probability of Y **at a given value of** X
  - When $X = 1$, what is the distribution of Y?
  - When $X = 2$, what is the distribution of Y?

- Conditional probability is the *joint value* divided by the *marginal value* **for that value of X**
  - It is **conditional** on that value of X

2.1.10 Conditional probability: Do you believe in an afterlife?

	$Y = 1$: Yes	$Y = 2$: No	
$X = 1$:Female	$n_{11} = 509$	$n_{12} = 116$	$n_{1+} = 625$
$X = 2$: Male	$n_{21} = 398$	$n_{22} = 104$	$n_{2+} = 502$
	$n_{+1} = 907$	$n_{+2} = 220$	$n = 1127$

2.1.11 Conditional probability: Do you believe in an afterlife?

	$Y = 1$: Yes	$Y = 2$: No	
$X = 1$:Female	$n_{11} = 509$	$n_{12} = 116$	$n_{1+} = 625$
$X = 2$: Male	$n_{21} = 398$	$n_{22} = 104$	$n_{2+} = 502$
	$n_{+1} = 907$	$n_{+2} = 220$	$n = 1127$

- When $X = 1$ (female):
  - $P(Yes) = \frac{509}{625} = 0.8144$
  - $P(No) = \frac{116}{625} = 0.1856$
- When $X = 2$ (male):
  - $P(Yes) = \frac{398}{502} = 0.7928$
  - $P(No) = \frac{104}{502} = 0.2072$

2.2 Sensitivity and specificity

2.2.1 True state vs test result

	Positive	Negative	
Diseased	1	0	1
Not diseased	12	87	99
	13	87	100

2.2.2 Sensitivity and specificity

- Sensitivity
  - Probability of **positive** for individuals who actually are **positive**
  - "True positive"
  - Statistical power
- Specificity
  - Probability of **negative** for individuals who actually are **negative**
  - "True negative"
  - 1 - type I error rate

2.2.3 Sensitivity and specificity

	Positive	Negative	
Diseased	1	0	1
Not diseased	12	87	99
	13	87	100

- Sensitivity $= P(positive\ test|diseased) = \frac{n_{11}}{n_{1+}} = \frac{1}{1} = 1.0$
- Specificity $= P(negative\ test|not\ diseased) = \frac{n_{22}}{n_{2+}} = \frac{87}{99} = 0.88$

2.2.4 Sensitivity and specificity

- Ideally, both sensitivity and specificity are *high*
  - They are probabilities, so near 1
- Several things are related
  - Sensitivity and specificity
  - Joint probabilities / frequencies
  - Marginal distribution of the X variable (**base rate**)

2.2.5 Base rate (with some rounding)

- Sensitivity = 0.86
- Specificity = 0.88
- **Base rate =** 0.01

	Positive	Negative	
Diseased	1	0	1
Not diseased	12	87	99
	13	87	100

- Sensitivity = 0.86
- Specificity = 0.88
- **Base rate** = 0.30

	Positive	Negative	
Diseased	26	4	30
Not diseased	8	62	70
	32	66	100

3 Study design

3.1 Fixed and random

3.1.1 Designing a study

- When designing a study
  - What do we *constrain* vs allow to *vary*?
  - What comes *first*?
- Design leads to analysis
  - Two groups, each with 3 time points
    - Repeated measures ANOVA or mixed model
  - Observe two continuous variables
    - Correlation or linear regression

3.1.2 Fixed vs random

- The **marginal** frequencies of a contingency table can be **fixed** or **random**
- **Fixed**: Chosen by the researcher
  - e.g., Collect data on 50 men and 50 women

- **Random**: Vary depending on the sample
  - e.g., Collect data on gender in the sample
- Note 1: “Fixed” and “random” are kind of (but not exactly) like “manipulated” and “measured”
  - More complicated: “fixed” isn't always X – sometimes it's Y
- Note 2: This is just *one of many definitions* of “fixed vs random”

3.1.3 Why do we care if they're fixed or random?

- Probability = random divided by *fixed*
  - **If there are no fixed marginals, we can't calculate a probability**
- Ratio = random divided by random
  - We can always calculate ratios (e.g., odds ratios...)
- Basically, what is “fixed” is what you can “condition on”

3.2 Types of study design

3.2.1 Study designs

- Three study designs with different *fixed* and *random* marginals
  - Multinomial (or cross-sectional)
  - Retrospective
  - Prospective (or product binomial)
- Design of the contingency table determines
  - What is **conditioned on**
  - What are **probabilities**
  - How you can talk about the **relationship** in the table

3.2.2 Overall study design

	Heart attack	No heart attack
Placebo		
Aspirin		

- Relationship between aspirin use (vs placebo) and heart attack
  - X: Aspirin vs placebo
  - Y: Heart attack vs no heart attack

3.2.3 Multinomial

	Heart attack	No heart attack	
Placebo			Random
Aspirin			Random
	Random	Random	**Fixed**

- Collect data from n people
  - Measure aspirin vs placebo, heart attack vs not

3.2.4 Retrospective

	Heart attack	No heart attack	
Placebo			Random
Aspirin			Random
	Fixed	**Fixed**	

- Collect data from specific numbers of heart attack and non patients
  - Measure whether they took aspirin

3.2.5 Prospective

	Heart attack	No heart attack	
Placebo			**Fixed**
Aspirin			**Fixed**
	Random	Random	

- Collect data from specific number of aspirin and placebo people
  - Measure whether they have a heart attack

3.2.6 Other considerations

- Cross-sectional vs longitudinal
  - How long does it take to work?
  - Reverse causality
- Retrospective most useful for rare or unpredictable outcomes
  - Cancer, heart attack, extreme events
- Measures of association depend on the design
  - Some are only possible with *prospective* design

4 Measures of relationship

4.1 Design to relationship

4.1.1 Why does design matter?

- Probability vs ratio
  - What is fixed and what is random?
- Design of *study* and *contingency table* determine
  - What is **conditioned on**
  - What are **probabilities**
  - How you can talk about the **relationship** in the table

4.1.2 Why does design matter?

- Three ways to talk about relationships in contingency table
  1. Difference in proportion
     - Prospective only
  2. Relative risk
     - Prospective only
  3. Odds or odds ratio
     - Any design

4.1.3 Aspirin treatment for heart disease

	Heart attack	No heart attack	
Placebo	189	10845	**11034**
Aspirin	104	10933	**11037**
	293	21778	22071

- Relationship between aspirin use (vs placebo) and heart attack
  - X: Aspirin vs placebo
  - Y: Heart attack vs no heart attack
- **Prospective design**: The X marginals are fixed

4.2 Difference in proportion

4.2.1 Difference in proportion: General

- Null hypothesis
  - H_0: $\pi_1 = \pi_2$ or $\pi_1 - \pi_2 = 0$
- Observed difference
  - $p_1 - p_2 = \frac{n_{11}}{n_{1+}} - \frac{n_{21}}{n_{2+}}$
- Note: Only for prospective designs (X marginals fixed)

4.2.2 Difference in proportion: Aspirin example

- Null hypothesis
  - H_0: $\pi_{yes|placebo} = \pi_{yes|aspirin}$ or $\pi_{yes|placebo} - \pi_{yes|aspirin} = 0$
- Observed difference
  - $p_1 = p_{yes|placebo} = \frac{189}{11034} = 0.017$
  - $p_2 = p_{yes|aspirin} = \frac{104}{11037} = 0.009$
  - $p_1 - p_2 = 0.017 - 0.009 = 0.008$

4.2.3 Difference in proportion inference: General

- Standard error for the difference
  - $SE = \sqrt{\frac{p_1(1-p_1)}{n_{1+}} + \frac{p_2(1-p_2)}{n_{2+}}}$
- Large sample confidence interval on the difference
  - $(p_1 - p_2) \pm z_{\alpha/2}(SE)$

4.2.4 Difference in proportion inference: Aspirin example

- Standard error for the difference
  - $SE = \sqrt{\frac{p_1(1-p_1)}{n_{1+}} + \frac{p_2(1-p_2)}{n_{2+}}} = \sqrt{\frac{0.017(0.983)}{11034} + \frac{0.009(0.991)}{11037}} = 0.0015$
- Large sample confidence interval on the difference
  - $(p_1 - p_2) \pm z_{\alpha/2}(SE)$
  - $(0.017 - 0.009) \pm 1.96(0.0015)$
  - $[0.0047, 0.0107]$

4.3 Relative risk

4.3.1 Relative risk: General

- Null hypothesis
  - H_0: $\frac{\pi_1}{\pi_2} = 1$
- Observed relative risk
  - $\frac{p_1}{p_2} = \frac{\frac{n_{11}}{n_{1+}}}{\frac{n_{21}}{n_{2+}}}$
- Note: Only for prospective designs (X marginals fixed)

4.3.2 Relative risk: Aspirin example

- Null hypothesis
  - H_0: $\frac{\pi_{yes|placebo}}{\pi_{yes|aspirin}} = 1$
- Observed relative risk
  - $p_1 = p_{yes|placebo} = \frac{189}{11034} = 0.017$
  - $p_2 = p_{yes|aspirin} = \frac{104}{11037} = 0.009$
  - $\frac{p_1}{p_2} = \frac{0.017}{0.009} = 1.818$

4.3.3 Relative risk inference: General

- Standard error for the **natural log** of the relative risk
  - $SE = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{21}} + \frac{1}{n_{1+}} + \frac{1}{n_{2+}}}$
- Large sample confidence interval on the relative risk
  - Calculate in terms of ln(relative risk)
    * $ln\left(\frac{p_1}{p_2}\right) \pm z_{\alpha/2}(SE)$
  - Then **exponentiate** to convert back to relative risk metric

4.3.4 Relative risk inference: Aspirin example

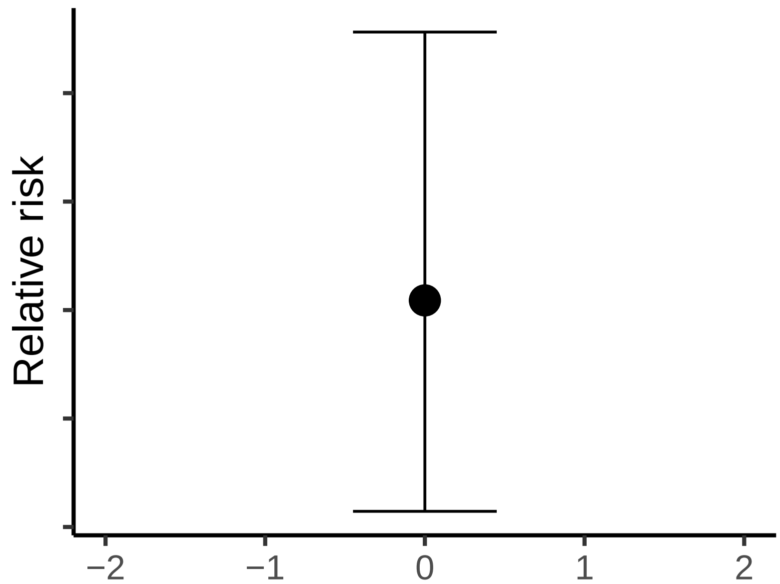
- Standard error for the **natural log** of the relative risk
  - $SE = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{21}} + \frac{1}{n_{1+}} + \frac{1}{n_{2+}}} =$
  - $\sqrt{\frac{1}{189} + \frac{1}{104} + \frac{1}{11034} + \frac{1}{11037}} =$
  - 0.1228

4.3.5 Relative risk inference: Aspirin example

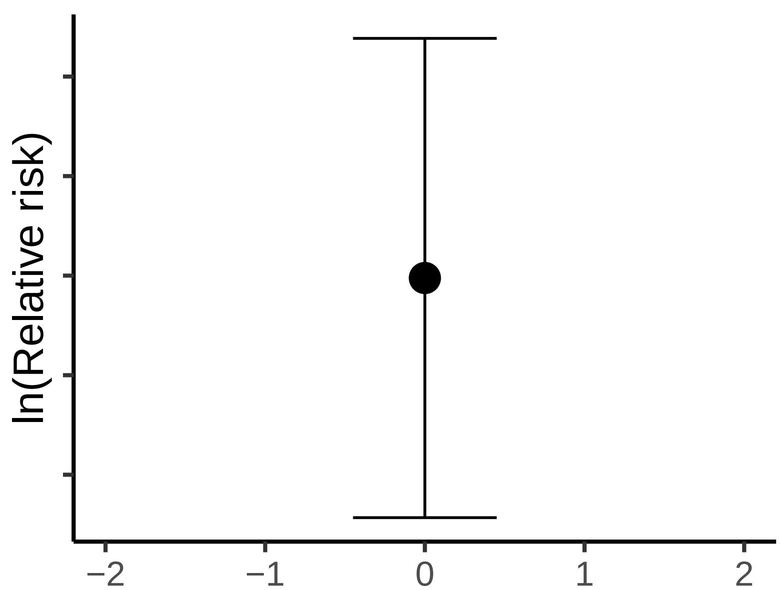
- Large sample confidence interval on the relative risk
  - Calculate in terms of ln(relative risk)
    * $ln\left(\frac{p_1}{p_2}\right) \pm z_{\alpha/2}(SE)$
    * $ln\left(\frac{0.017}{0.009}\right) \pm 1.96(0.1228)$
    * $[0.3569, 0.8384]$
  - Then **exponentiate** to convert back to relative risk metric
    * $[e^{0.3569}, e^{0.8384}]$
    * $[1.4289, 2.3126]$

4.3.6 Assymetric confidence limits on relative risk

- Relative risk = 1.818

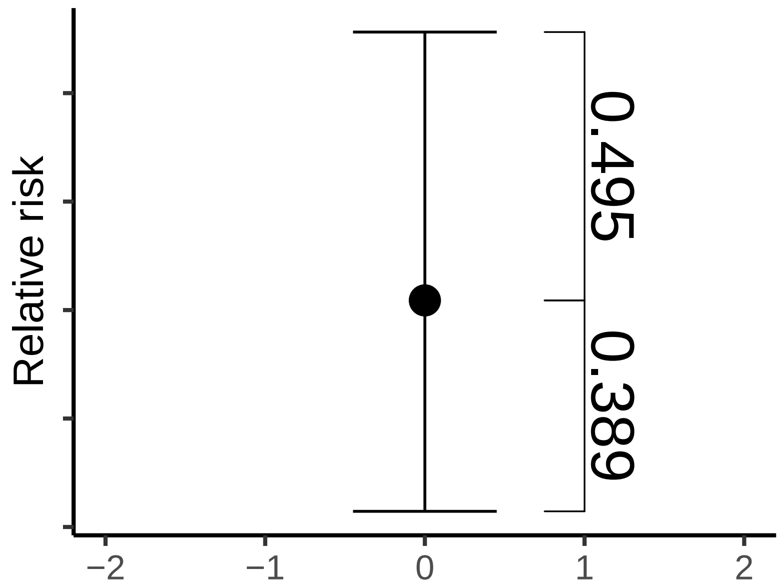


- ln(relative risk) = 0.598

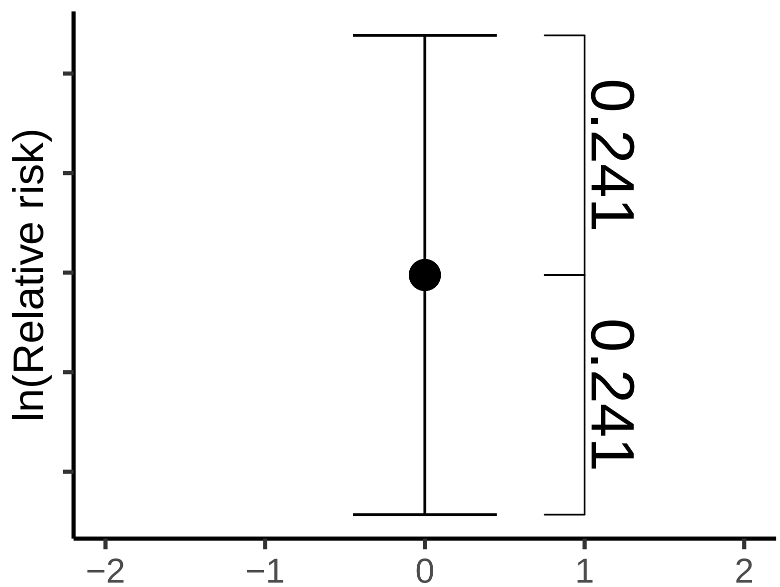


4.3.7 Assymetric confidence limits on relative risk

- Relative risk = 1.818



- ln(relative risk) = 0.598



4.4 Odds, odds ratio, logit

4.4.1 Odds

- Odds = probability of "success" divided by probability of "failure"

- Ranges from 0 to $+\infty$
    - Odds > 1: *success is more likely* than failure
    - Odds < 1: *failure is more likely* than success
    - Odds $= 1$: success and failure are *equally likely*

- **Odds** can be used with **any study design**

4.4.2 Odds

- Probability of a "success" divided by probability of "not a success"
  - $odds = \frac{p}{(1-p)}$

- $P(success) = 0.2$
- Odds of success =
  - $\frac{0.2}{(1-0.2)} =$
  - $\frac{0.2}{(0.8)} =$
  - 0.25 or $\frac{1}{4}$

- $P(success) = 0.5$
- Odds of success =
  - $\frac{0.5}{(1-0.5)} =$
  - $\frac{0.5}{(0.5)} =$
  - 1

- $P(success) = 0.8$
- Odds of success =
  - $\frac{0.8}{(1-0.8)} =$
  - $\frac{0.8}{(0.2)} =$
  - 4

4.4.3 Odds ratio

- Odds ratio is a **ratio of odds**: Ratio of ratios

$$\theta = \frac{odds_1}{odds_2} = \frac{p_1/(1-p_1)}{p_2/(1-p_2)} = \frac{n_{11}/n_{12}}{n_{21}/n_{22}} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

- Ranges from 0 to $+\infty$
  - Odds ratio > 1: *odds of success is more likely* in **group 1** than in **group 2**

- Odds ratio < 1: *odds of failure is more likely* in **group 1** than in **group 2**
- Odds ratio $= 1$: odds of success in **group 1** and **group 2** are equal

4.4.4 Odds ratio: General

- Null hypothesis
  - $H_0 : \frac{\pi_1/(1-\pi_1)}{\pi_2/(1-\pi_2)} = 1$
- Observed odds ratio
  - $\theta = \frac{p_1/(1-p_1)}{p_2/(1-p_2)} = \frac{n_{11}/n_{12}}{n_{21}/n_{22}} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$
- Note: **Odds** and **odds ratios** can be used with **any study design**

4.4.5 Odds ratio: Aspirin example

- Null hypothesis
  - H_0: $\frac{\pi_{yes|placebo}/\pi_{no|placebo}}{\pi_{yes|aspirin}/\pi_{no|aspirin}} = 1$
- Observed odds ratio
  - $\theta = \frac{n_{11}/n_{12}}{n_{21}/n_{22}} = \frac{189/10845}{104/10933} = \frac{0.0174}{0.00951} = 1.832$
  - Observed ln(odds ratio) $= ln(1.832) = 0.605$

4.4.6 Odds ratio inference: General

- Standard error for the **natural log** of the odds ratio
  - $SE = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$
- Large sample confidence interval on **natural log** of the odds ratio
  - Calculate in terms of ln(odds ratio)
    * $ln\left(\theta\right) \pm z_{\alpha/2}(SE)$
  - Then **exponentiate** to convert back to odds metric

4.4.7 Odds ratio inference: Aspirin example

- Standard error for the **natural log** of the odds ratio
  - $SE = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}} =$
  - $\sqrt{\frac{1}{189} + \frac{1}{10845} + \frac{1}{104} + \frac{1}{10933}} =$
  - 0.1228

4.4.8 Odds ratio inference: Aspirin example

- Large sample confidence interval on the log of the odds ratio
  - Calculate in terms of ln(odds ratio)
    - $ln\,(\theta) \pm z_{\alpha/2}(SE)$
    - $ln(1.832) \pm 1.96(0.1228)$
    - $[0.3647, 0.8462]$
  - Then **exponentiate** to convert back to odds ratio metric
    - $[e^{0.3647}, e^{0.8462}]$
    - $[1.44, 2.331]$

4.5 Comparisons

4.5.1 Compare: All measures

Measure	Study design	Calculation	Example value
Difference in proportion	Prospective	$p_1 - p_2$	0.008
Relative risk	Prospective	$\frac{p_1}{p_2}$	1.818
Odds ratio	Any design	$\frac{p_1/(1-p_1)}{p_2/(1-p_2)}$	1.832

- Note: $p_1 = n_{11}/n_{1+}$, $p_2 = n_{21}/n_{2+}$

4.5.2 Compare: Relative risk and odds ratio

- Relative risk = 1.818
- Odds ratio = 1.832

$$odds\ ratio = \frac{p_1/(1-p_1)}{p_2/(1-p_2)} = relative\ risk\frac{(1-p_1)}{(1-p_2)}$$

- When p_1 and p_2 are **both** close to 0 or **both** close to 1
  - Odds ratio and relative risk are very similar
  - In this example, $p_1 = 0.017$ and $p_2 = 0.009$

5 Chi-square tests

5.1 Independence

5.1.1 Independence

- Contingency table shows the **relationship** between **two variables**
- Example
  - X = Aspirin vs placebo
  - Y = Heart attack or not
- Question: Is there a *relationship* between X and Y?
  - Does knowing something about X tell you something about Y?
    - **Yes**: X and Y are related
    - **No**: X and Y are **independent**

5.1.2 Expected value under independence

- All statistical tests involve comparing a **test statistic** to an **expected value under the null hypothesis**
  - Null hypothesis here: Independence
- Independence is related to correlation, but stronger
  - Independence between variables → correlation = 0 → covariance = 0
- **Expected value under the null hypothesis**
  - Correlation and covariance between two variables are both 0

5.1.3 Expected value of joint frequencies

- $E(XY)$ is the **joint probability distribution** of X and Y
  - Cells in the table
- $E(X)$ and $E(Y)$ are their **marginal probability distributions**
  - Margins of the table

$$E(XY) = E(X)E(Y) - cov(XY)$$

- The joint frequencies depend on the **marginal frequencies** as well as the **covariance between the variables**

5.1.4 Expected value (under independence)

- If X and Y are independent, $cov(XY) = 0$ and

$$E(XY) = E(X)E(Y)$$

- If X and Y are independent, the **joint frequencies** are **completely** determined by the **marginal frequencies**
  - No covariance between X and Y

5.1.5 Expected value under independence

- Expected joint frequencies: $\mu_{ij} = \frac{n_{i+}n_{+j}}{n}$
- Expected joint frequencies

	Heart attack	No heart attack	
Placebo	$\mu_{11} = \frac{n_{1+}n_{+1}}{n}$	$\mu_{12} = \frac{n_{1+}n_{+2}}{n}$	n_{1+}
Aspirin	$\mu_{21} = \frac{n_{2+}n_{+1}}{n}$	$\mu_{22} = \frac{n_{2+}n_{+2}}{n}$	n_{2+}
	n_{+1}	n_{+2}	n

5.1.6 Observed values: Aspirin example

	Heart attack	No heart attack	
Placebo	189	10845	11034
Aspirin	104	10933	11037
	293	21778	22071

5.1.7 Expected values (independence): Aspirin example

	Heart attack	No heart attack	
Placebo	146.48	10887.52	11034
Aspirin	146.52	10890.48	11037
	293	21778	22071

- For example: $146.48 = \frac{11034*293}{22071}$

5.2 Chi-square test statistic

5.2.1 Chi-square test statistic

$$\chi^2 = \sum \left(\frac{(n_{ij} - \mu_{ij})^2}{\mu_{ij}} \right)$$

- Null hypothesis
  - Observed frequencies = expected frequencies
- If all observed = expected, then $\chi^2 = 0$
  - As variables become more related (not independent)
    * Differences between observed and expected increase
    * χ^2 gets larger

5.2.2 Chi-square test statistic

- $\chi^2 = \sum \left(\frac{(n_{ij} - \mu_{ij})^2}{\mu_{ij}} \right) =$
- $\frac{(189-146.48)^2}{146.48} + \frac{(10845-10887.52)^2}{10887.52} + \frac{(104-146.52)^2}{146.52} + \frac{(10933-10890.48)^2}{10890.48} =$
- $12.343 + 0.166 + 12.339 + 0.166 = 25.014$

5.2.3 Chi-square test inference

- Null hypothesis: **Independence**
  - Observed frequencies = expected frequencies
- Degrees of freedom = $(I - 1)(J - 1) = (2 - 1)(2 - 1) = 1$
  - $\chi^2_{critical}(1) = 3.86$
- $25.014 > 3.86$
  - Reject H_0 that the variables are **independent**
    * Placebo vs aspirin is **related to** heart attack status

5.3 Summary and Assumptions

5.3.1 Assumptions of chi-square test

- All **expected** joint frequencies are at least 5
  - Often incorrect stated as all **observed** joint frequencies
    - OK to have observed cells < 5
    - Just can't have **expected** values < 5
- Chi-square is a **large sample** test
  - χ^2 distribution is continuous
    - Sampling distribution of the test statistic doesn't start looking like χ^2 until the sample is quite large

5.3.2 Chi-square tests are everywhere

- Some are easy to see how they relate to the $(O - E)^2/E$ format here
  - SEM tests of model fit compare the **observed** covariance matrix to the **expected** covariance matrix
- Some are less obvious
  - Likelihood ratio test
  - χ^2 tests of regression coefficients

5.3.3 Chi-square test vs odds ratio

- χ^2 test and odds ratio
  - Two different ways of assessing the **relationship** between 2 variables
- χ^2 tells you about *statistical significance*
- Odds ratio tells you about *effect size*
- Use both to get a complete picture of the relationship

5.4 Fisher's Exact Test

5.4.1 Fisher's Exact Test

- "Lady tasting tea" problem from Fisher
    - Can the lady tell if tea or milk was put in first?
    - Very small expected values

	Guess tea	Guess milk	
Tea first	n_{11}	n_{12}	**4**
Milk first	n_{21}	n_{22}	**4**
	4	**4**	8

5.4.2 Fisher's Exact Test

- Exact test: Not an approximation via the χ^2 distribution
  - All the possible ways that observations can be distributed in cells
  - Is observed way **less likely** than we would expect if variables are independent?
- "Small sample test" for 2×2 contingency tables
  - Often used as alternative when cells < 5
  - But there are additional assumptions

5.4.3 Exact test

- Flip a coin 2 times
  - First flip: Head or tail
  - Second flip: Head or tail
  - Both flips: HH, HT, TH, TT
- Probability of HH is 0.25: 1 option out of 4 equally likely options

5.4.4 Fisher's Exact Test

- **All marginal frequencies are fixed**
  - Multinomial: Total fixed
  - Retrospective: Column (outcome) marginals fixed
  - Prospective: Row (predictor) marginals fixed
- When all marginal frequencies fixed
  - Once you know 1 joint frequency (one cell)
  - You know all the other joint frequencies
  - Test statistic = n_{11}

5.4.5 Fisher's Exact Test

	Guess tea	Guess milk	
Tea first	3	1	**4**
Milk first	1	3	**4**
	4	**4**	8

6 Summary

6.1 Summary

6.1.1 Summary of this week

- Contingency tables
  - What they are and how they work
- Study design
  - Relates to contingency table
  - Relates to measures of relationship
- Chi-square tests
  - Fisher's Exact Test

6.1.2 Next week

- More complicated contingency tables
  - 2×3 (and larger) tables
  - 3-way tables: $2 \times 2 \times 2$ tables
- Chi-square tests for these table
  - Probing the tables
  - Residuals