Offering Strategy of a Price-Maker Energy Storage System in Day-Ahead and Balancing Markets

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This document serves as an electronic companion to the paper "Offering Strategy of a Price-Maker Energy Storage System in Day-Ahead and Balancing Markets" presented at IEEE PES PowerTech 2017. It provides supplementary material relative to the mathematical formulation of the bilevel optimization problem, the corresponding Mathematical Program with Equilibrium Constraints (MPEC) and its reformulation as Mixed-Integer Linear Programming Problem (MILP).

Section 1 provides the bilevel optimization problem formulation of an energy storage system (ESS) operator which participates strategically in both day-ahead and balancing markets. This problem is recast as a single-level MPEC in Section 2. In addition, the linearization of the objective function is shown in Section 3 and the linearization of the complementarity constraints is presented in Section 4. Finally, Section 5 provides the equivalent MILP.

Nomenclature

Sets and Indices

$i \in I$	Index of dispatchable units from 1 to I.
$t \in T$	Index of time periods from 1 to T.
$\omega \in \Omega$	Index of wind power scenarios from 1 to Ω.
Ξ	Decision set of market clearing problems.

Parameters

C_i	Cost of power production by unit i [\$/MWh].
$C_i^{R^+}$	Cost of upward reserve capacity by unit i [\$/MWh].
$C_i^{R^-}$	Cost of downward reserve capacity by unit i [\$/MWh].
C_i^+	Cost of upward regulation by unit i [\$/MWh].
C_i^-	Cost of downward regulation by unit i [\$/MWh].
RR_t^+	System wide upward reserve capacity requirements at time t [MW].
RR_t^-	System wide downward reserve capacity requirements at time t [MW].
$R_i^{+,\max}$	Maximum upward reserve capacity by unit i [MW].
$R_i^{-,\max}$	Maximum downward reserve capacity by unit i [MW].
W_t^{E}	Expected wind power production at time t [MW].
$W_{\omega,t}$	Wind power production in scenario ω at time t [MW].
π_ω	Probability of occurrence of scenario ω.
C^{shed}	Cost of lost load [\$/MWh].
L_t	Aggregated electrical energy demand at time t [MW].
$P^{\downarrow,\max}$	Maximum discharging power of ESS [MW].
$P^{\uparrow,\max}$	Maximum charging power of ESS [MW].
E^{ini}	Initial state of charge of ESS [MWh].
$E^{\max}$	Maximum state of charge of ESS [MWh].
η^{sd}	Self-discharging efficiency of ESS.
$\eta^{\uparrow}$	Charging efficiency of ESS.
$\eta^{\downarrow}$	Discharging efficiency of ESS.
M	Large enough constant for Fortuny-Amat linearisation.

Primal Variables

$p_{i,t}$	Power production by unit i at time t [MW].
$R_{i,t}^+$	Upward reserve capacity by unit i at time t [MW].
$R_{i,t}^-$	Downward reserve capacity by unit i at time t [MW].
$r_{i,\omega,t}^+$	Upward regulation by unit i in scenario ω at time t [MW].

$r^{-}_{i,\omega,t}$	Downward regulation by unit i in scenario ω at time t [MW].
w_t	Day-ahead schedule of wind power at time t [MW].
$w^{\text{spill}}_{\omega,t}$	Wind power spillage at time t [MW].
$l^{\text{shed}}_{\omega,t}$	Load shedding in scenario ω at time t [MW].
e_t	State of charge of ESS at time t [MWh].
$p^{\uparrow}_t$	ESS power charging at time t [MW].
$p^{\downarrow}_t$	ESS power discharging at time t [MW].
$\overline{p}^{\uparrow}_t$	Strategic charging offer by ESS at time t [MW].
$\overline{p}^{\downarrow}_t$	Strategic discharging offer by ESS at time t [MW].
$\alpha^{\uparrow}_t$	Strategic offering price of charging by ESS at time t [\$/MWh].
$\alpha^{\downarrow}_t$	Strategic offering price of discharging by ESS at time t [\$/MWh].
Dual Variables	
λ^{DA}_t	Day-ahead market-clearing prices in scenario ω at time t [\$/MWh].
$\lambda^{\text{BA}}_{\omega,t}$	Balancing market-clearing prices in scenario ω at time t [\$/MWh].
μ	Dual variable of balancing market clearing problem constraints.
ν	Dual variable of day-ahead market clearing problem constraints.
Binary Variables	
$x^{\uparrow}_t$	Charging of ESS at time t.
$x^{\downarrow}_t$	Discharging of ESS at time t.
u	Binary variable for the Fortuny-Amat Linearisation.
Symbols	
DA	Day-ahead market.
BA	Balancing market.
RC	Reserve capacity market.
UL	Upper-level problem.
LL	Lower-level problem.
P	Primal variables.
D	Dual variables.

1 Bilevel Optimization Problem for S-DABA offering strategy

This section presents the mathematical formulation of a bilevel optimization problem of an ESS operator which participates strategically as price-maker in the day-ahead and balancing markets. The upper-level problem (UL) corresponds to the expected profit maximization, i.e., negative profit minimization, of the ESS operator. The lower level (LL,DA) reproduces the day-ahead market-clearing that aims at minimizing day-ahead operational system cost. Similarly, the lower level problem simulates the balancing market-clearing that aims at minimizing the balancing operational system cost. This bilevel problem writes as:

$$\underset{\Theta}{\text{Min.}} \sum_{t=1}^{N_T} \Big[\lambda^{\text{DA}}_t(p^{\uparrow,\text{DA}}_t - p^{\downarrow,\text{DA}}_t) + \sum_{\omega=1}^{N_\Omega} \pi_\omega \Big(\lambda^{\text{BA}}_{\omega,t}(p^{\uparrow,\text{BA}}_{\omega,t} - p^{\uparrow,\text{DA}}_t) - \lambda^{\text{BA}}_{\omega,t}(p^{\downarrow,\text{BA}}_{\omega,t} - p^{\downarrow,\text{DA}}_{s,t})\Big)\Big] \tag{1a}$$

subject to

$$e^{\text{DA}}_t = \eta^{\text{sd}} E^{\text{ini}} + \eta^{\uparrow} p^{\uparrow,\text{DA}}_t - \frac{1}{\eta^{\downarrow}} p^{\downarrow,\text{DA}}_t, t = 1 \tag{1b}$$

$$e^{\text{DA}}_t = \eta^{\text{sd}} e^{\text{DA}}_{(t-1)} + \eta^{\uparrow} p^{\uparrow,\text{DA}}_t - \frac{1}{\eta^{\downarrow}} p^{\downarrow,\text{DA}}_t, \forall t > 1 \tag{1c}$$

$$e^{\text{DA}}_t = E^{\text{ini}}, t = T \tag{1d}$$

$$0 \leq e^{\text{DA}}_t \leq E^{\max}, \forall t \tag{1e}$$

$$0 \leq \overline{p}^{\uparrow,\text{DA}}_t \leq (1 - x^{\uparrow,\text{DA}}_t) P^{\uparrow,\max}, \forall t \tag{1f}$$

$$0 \leq \overline{p}^{\downarrow,\text{DA}}_t \leq (1 - x^{\downarrow,\text{DA}}_t) P^{\downarrow,\max}, \forall t \tag{1g}$$

$$x^{\uparrow,\text{DA}}_t + x^{\downarrow,\text{DA}}_t = 1, \forall t \tag{1h}$$

$$0 \leq \alpha^{\uparrow,\text{DA}}_t, \alpha^{\downarrow,\text{DA}}_t, \forall t \tag{1i}$$

$$x^{\uparrow,\text{DA}}_t, x^{\downarrow,\text{DA}}_t \in \{0,1\}, \forall t \tag{1j}$$

$$e^{\mathrm{BA}}_{\omega,t} = \eta^{\mathrm{sd}} E^{\mathrm{ini}} + \eta^{\uparrow} p^{\uparrow,\mathrm{BA}}_{\omega,t} - \frac{1}{\eta^{\downarrow}} p^{\downarrow,\mathrm{BA}}_{\omega,t}, \forall\omega, t = 1 \tag{1k}$$

$$e^{\mathrm{BA}}_{\omega,t} = \eta^{\mathrm{sd}} e^{\mathrm{BA}}_{\omega,(t-1)} + \eta^{\uparrow} p^{\uparrow,\mathrm{BA}}_{\omega,t} - \frac{1}{\eta^{\downarrow}} p^{\downarrow,\mathrm{BA}}_{\omega,t}, \forall\omega, \forall t > 1 \tag{1l}$$

$$e^{\mathrm{BA}}_{\omega,t} = E^{\mathrm{ini}}, \forall\omega, t = T \tag{1m}$$

$$0 \le e^{\mathrm{BA}}_{\omega,t} \le E^{\max}, \forall\omega, \forall t \tag{1n}$$

$$0 \le \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t} \le (1 - x^{\uparrow,\mathrm{BA}}_{\omega,t}) P^{\uparrow,\max}, \forall\omega, \forall t \tag{1o}$$

$$0 \le \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t} \le (1 - x^{\downarrow,\mathrm{BA}}_{\omega,t}) P^{\downarrow,\max}, \forall\omega, \forall t \tag{1p}$$

$$x^{\uparrow,\mathrm{BA}}_{\omega,t} + x^{\downarrow,\mathrm{BA}}_{\omega,t} = 1, \forall\omega, \forall t \tag{1q}$$

$$0 \le \alpha^{\uparrow,\mathrm{BA}}_{\omega,t}, \alpha^{\downarrow,\mathrm{BA}}_{\omega,t}, \forall\omega, \forall t \tag{1r}$$

$$x^{\uparrow,\mathrm{BA}}_{\omega,t}, x^{\downarrow,\mathrm{BA}}_{\omega,t} \in \{0, 1\}, \forall\omega, \forall t \tag{1s}$$

$$\tag{1t}$$

where $p^{\uparrow,\mathrm{DA}}_t, p^{\downarrow,\mathrm{DA}}_t, p^{\uparrow,\mathrm{BA}}_{\omega,t}, p^{\downarrow,\mathrm{BA}}_{\omega,t} \lambda^{\mathrm{DA}}_t, \lambda^{\mathrm{BA}}_{\omega,t} \in \arg\{$

$$\begin{aligned} \underset{\Xi^{\mathrm{LL,BA,P}}_{\omega,t} \cup \Xi^{\mathrm{LL,BA,D}}_{\omega,t}}{\mathrm{Min.}} \quad & \sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} (C^{+}_i r^{+}_{i,\omega,t} - C^{-}_i r^{-}_{i,\omega,t}) + C^{\mathrm{shed}} l^{\mathrm{shed}}_{\omega,t} \\ & - \alpha^{\uparrow,\mathrm{BA}}_{\omega,t} (p^{\uparrow,\mathrm{BA}}_{\omega,t} - p^{\uparrow,\mathrm{DA}}_t) + \alpha^{\downarrow,\mathrm{BA}}_{\omega,t} (p^{\downarrow,\mathrm{BA}}_{\omega,t} - p^{\downarrow,\mathrm{DA}}_t) \Big] \end{aligned} \tag{2a}$$

subject to

$$\sum_{i=1}^{N_I} (r^{+}_{i,\omega,t} - r^{-}_{i,\omega,t}) + l^{\mathrm{shed}}_{\omega,t} + (W_{\omega,t} - W^{\mathrm{E}}_t - w^{\mathrm{spill}}_{\omega,t}) - (p^{\uparrow,\mathrm{BA}}_{\omega,t} - p^{\uparrow,\mathrm{DA}}_t) + (p^{\downarrow,\mathrm{BA}}_{\omega,t} - p^{\downarrow,\mathrm{DA}}_t) = 0 \quad : \lambda^{\mathrm{BA}}_{\omega,t}, \tag{2b}$$

$$0 \le r^{+}_{i,\omega,t} \le R^{+}_{i,t}, \quad : \underline{\mu}^{+}_{i,\omega,t}, \overline{\mu}^{+}_{i,\omega,t}, \forall i \tag{2c}$$

$$0 \le r^{-}_{i,\omega,t} \le R^{-}_{i,t}, \quad : \underline{\mu}^{-}_{i,\omega,t}, \overline{\mu}^{-}_{i,\omega,t}, \forall i \tag{2d}$$

$$0 \le w^{\mathrm{spill}}_{\omega,t} \le W_{\omega,t}, \quad : \underline{\mu}^{\mathrm{spill}}_{\omega,t}, \overline{\mu}^{\mathrm{spill}}_{\omega,t} \tag{2e}$$

$$0 \le p^{\uparrow,\mathrm{BA}}_{\omega,t} \le \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t}, \quad : \underline{\mu}^{\uparrow,\mathrm{BA}}_{\omega,t}, \overline{\mu}^{\uparrow,\mathrm{BA}}_{\omega,t} \tag{2f}$$

$$0 \le p^{\downarrow,\mathrm{BA}}_{\omega,t} \le \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t}, \quad : \underline{\mu}^{\downarrow,\mathrm{BA}}_{\omega,t}, \overline{\mu}^{\downarrow,\mathrm{BA}}_{\omega,t} \tag{2g}$$

$$0 \le l^{\mathrm{shed}}_{\omega,t} \le L_t, \quad : \underline{\mu}^{\mathrm{shed}}_{\omega,t}, \overline{\mu}^{\mathrm{shed}}_{\omega,t} \tag{2h}$$

where $p^{\uparrow,\mathrm{DA}}_t, p^{\downarrow,\mathrm{DA}}_t \in \arg\{$

$$\underset{\Xi^{\mathrm{LL,DA,P}}_t \cup \Xi^{\mathrm{LL,DA,D}}_t}{\mathrm{Min.}} \quad \sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} C_i p_{i,t} - \alpha^{\uparrow,\mathrm{DA}}_t p^{\uparrow,\mathrm{DA}}_t + \alpha^{\downarrow,\mathrm{DA}}_t p^{\downarrow,\mathrm{DA}}_t \Big] \tag{3a}$$

subject to

$$\sum_{i=1}^{N_I} p_{i,t} + W^{\mathrm{E}}_t + (-p^{\uparrow,\mathrm{DA}}_t + p^{\downarrow,\mathrm{DA}}_t) - L_t = 0 \qquad : \lambda^{\mathrm{DA}}_t, \tag{3b}$$

$$R^{-}_{i,t} \le p_{i,t} \le P^{\max}_i - R^{+}_{i,t}, \quad : \underline{\nu}^{\mathrm{p}}_{i,t}, \overline{\nu}^{\mathrm{p}}_{i,t}, \forall i \tag{3c}$$

$$0 \le p^{\uparrow,\mathrm{DA}}_t \le \overline{p}^{\uparrow,\mathrm{DA}}_t, \quad : \underline{\nu}^{\uparrow,\mathrm{DA}}_t, \overline{\nu}^{\uparrow,\mathrm{DA}}_t \tag{3d}$$

$$0 \le p^{\downarrow,\mathrm{DA}}_t \le \overline{p}^{\downarrow,\mathrm{DA}}_t, \quad : \underline{\nu}^{\downarrow,\mathrm{DA}}_t, \overline{\nu}^{\downarrow,\mathrm{DA}}_t \tag{3e}$$

$$\}, \forall t\}, \forall\omega, \forall t$$

where

$$\Theta = \Xi^{\mathrm{UL}}_t \cup \Xi^{\mathrm{LL,DA,P}}_t \cup \Xi^{\mathrm{LL,DA,D}}_t \cup \Xi^{\mathrm{LL,BA,P}}_{\omega,t} \cup \Xi^{\mathrm{LL,BA,D}}_{\omega,t}$$

$$\Xi^{\mathrm{UL}}_{\omega,t} = \{\overline{p}^{\uparrow,\mathrm{DA}}_t, \overline{p}^{\downarrow,\mathrm{DA}}_t, x^{\uparrow,\mathrm{DA}}_t, x^{\downarrow,\mathrm{DA}}_t; \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t}, \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t}, x^{\uparrow,\mathrm{BA}}_{\omega,t}, x^{\downarrow,\mathrm{BA}}_{\omega,t}, \forall\omega\}, \forall t$$

$$\Xi^{\mathrm{LL,BA,P}}_{\omega,t} = \{r^+_{i,\omega,t}, r^-_{i,\omega,t}, \forall i; l^{\mathrm{shed}}_{\omega,t}, w^{\mathrm{spill}}_{\omega,t}, p^{\uparrow,\mathrm{BA}}_{\omega,t}, p^{\downarrow,\mathrm{BA}}_{\omega,t}\}, \forall \omega, \forall t$$

$$\Xi^{\mathrm{LL,BA,D}}_{\omega,t} = \{\underline{\mu}^+_{i,\omega,t}, \overline{\mu}^+_{i,\omega,t}, \underline{\mu}^-_{i,\omega,t}, \overline{\mu}^-_{i,\omega,t} \forall i; \lambda^{\mathrm{BA}}_{\omega,t}, \underline{\mu}^{\mathrm{spill}}_{\omega,t}, \overline{\mu}^{\mathrm{spill}}_{\omega,t}, \underline{\mu}^{\uparrow}_{\omega,t}, \overline{\mu}^{\uparrow}_{\omega,t}, \underline{\mu}^{\downarrow}_{\omega,t}, \overline{\mu}^{\downarrow}_{\omega,t}; \underline{\mu}^{\mathrm{shed}}_{\omega,t}, \overline{\mu}^{\mathrm{shed}}_{\omega,t}\}, \forall \omega, \forall t$$

$$\Xi^{\mathrm{LL,DA,P}}_{t} = \{p_{i,t}, \forall i; p^{\uparrow,\mathrm{DA}}_{t}, p^{\downarrow,\mathrm{DA}}_{t}\}, \forall t$$

$$\Xi^{\mathrm{LL,DA,D}}_{t} = \{\underline{\nu}^{\mathrm{p}}_{i,t}, \overline{\nu}^{\mathrm{p}}_{i,t}, \forall i; \lambda^{\mathrm{DA}}_{t}, \underline{\nu}^{\uparrow,\mathrm{DA}}_{t}, \overline{\nu}^{\uparrow,\mathrm{DA}}_{t}, \underline{\nu}^{\downarrow,\mathrm{DA}}_{t}, \overline{\nu}^{\downarrow,\mathrm{DA}}_{t}\}, \forall t.$$

The subscript P refers to the primal decision variables while D denotes to the dual decision variables of the respective problem.

2 Mathematical Program with Equilibrium Constraints

Considering that the hierarchical relationship between optimization problems (1) and (2)-(3) cannot be handled directly by available optimization solvers, the bilevel problem (1)-(3) is transformed into a single-level MPEC. Taking into account that the lower level problems (2)-(3) are linear, they can be replaced by their KKT conditions as follows:

$$\underset{\Theta}{\text{Min.}}\ (1a)$$

subject to

$$\text{Constraints (1b)} - \text{(1t)}$$

$$\Big\{\{\text{Constraint (2b)}$$

$$C^+_i - \lambda^{\mathrm{BA}}_{\omega,t} - \underline{\mu}^+_{i,\omega,t} + \overline{\mu}^+_{i,\omega,t} = 0, \forall i \tag{4a}$$

$$-C^-_i + \lambda^{\mathrm{BA}}_{\omega,t} - \underline{\mu}^-_{i,\omega,t} + \overline{\mu}^-_{i,\omega,t} = 0, \forall i \tag{4b}$$

$$\lambda^{\mathrm{BA}}_{\omega,t} - \underline{\mu}^{\mathrm{spill}}_{\omega,t} + \overline{\mu}^{\mathrm{spill}}_{\omega,t} = 0, \tag{4c}$$

$$-\alpha^{\uparrow,\mathrm{BA}}_{\omega,t} + \lambda^{\mathrm{BA}}_{\omega,t} - \underline{\mu}^{\uparrow}_{\omega,t} + \overline{\mu}^{\uparrow}_{\omega,t} = 0, \tag{4d}$$

$$\alpha^{\downarrow,\mathrm{BA}}_{\omega,t} - \lambda^{\mathrm{BA}}_{\omega,t} - \underline{\mu}^{\downarrow}_{\omega,t} + \overline{\mu}^{\downarrow}_{\omega,t} = 0, \tag{4e}$$

$$C^{\mathrm{shed}} - \lambda^{\mathrm{BA}}_{\omega,t} - \underline{\mu}^{\mathrm{shed}}_{\omega,t} + \overline{\mu}^{\mathrm{shed}}_{\omega,t} = 0, \tag{4f}$$

$$0 \le r^+_{i,\omega,t} \perp \underline{\mu}^+_{i,\omega,t} \ge 0, \forall i \tag{4g}$$

$$0 \le R^+_{i,t} - r^+_{i,\omega,t} \perp \overline{\mu}^+_{i,\omega,t} \ge 0, \forall i \tag{4h}$$

$$0 \le r^-_{i,\omega,t} \perp \underline{\mu}^-_{i,\omega,t} \ge 0, \forall i \tag{4i}$$

$$0 \le R^-_{i,t} - r^-_{i,\omega,t} \perp \overline{\mu}^-_{i,\omega,t} \ge 0, \forall i \tag{4j}$$

$$0 \le w^{\mathrm{spill}}_{\omega,t} \perp \underline{\mu}^{\mathrm{spill}}_{\omega,t} \ge 0, \forall q \tag{4k}$$

$$0 \le W_{\omega,t} - w^{\mathrm{spill}}_{\omega,t} \perp \overline{\mu}^{\mathrm{spill}}_{\omega,t} \ge 0, \tag{4l}$$

$$0 \le p^{\uparrow,\mathrm{BA}}_{\omega,t} \perp \underline{\mu}^{\uparrow}_{\omega,t} \ge 0, \tag{4m}$$

$$0 \le \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t} - p^{\uparrow,\mathrm{BA}}_{\omega,t} \perp \overline{\mu}^{\uparrow}_{\omega,t} \ge 0, \tag{4n}$$

$$0 \le p^{\downarrow,\mathrm{BA}}_{\omega,t} \perp \underline{\mu}^{\downarrow}_{\omega,t} \ge 0, \tag{4o}$$

$$0 \le \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t} - p^{\downarrow,\mathrm{BA}}_{\omega,t} \perp \overline{\mu}^{\downarrow}_{\omega,t} \ge 0, \tag{4p}$$

$$0 \le l^{\mathrm{shed}}_{\omega,t} \perp \underline{\mu}^{\mathrm{shed}}_{\omega,t} \ge 0, \tag{4q}$$

$$0 \le L_t - l^{\mathrm{shed}}_{\omega,t} \perp \overline{\mu}^{\mathrm{shed}}_{\omega,t} \ge 0, \tag{4r}$$

$$\}, \forall \omega$$

$$\{\text{Constraint (3b)}$$

$$C_i - \lambda^{\mathrm{DA}}_{t} - \underline{\nu}^{\mathrm{p}}_{i,t} + \overline{\nu}^{\mathrm{p}}_{i,t} = 0, \forall i \tag{4s}$$

$$-\alpha^{\uparrow,\mathrm{DA}}_{t} + \lambda^{\mathrm{DA}}_{t} - \underline{\nu}^{\uparrow,\mathrm{DA}}_{t} + \overline{\nu}^{\uparrow,\mathrm{DA}}_{t} = 0, \tag{4t}$$

$$\alpha_t^{\downarrow,\mathrm{DA}} - \lambda_t^{\mathrm{DA}} - \underline{\nu}_t^{\downarrow,\mathrm{DA}} + \overline{\nu}_t^{\downarrow,\mathrm{DA}} = 0, \tag{4u}$$
$$0 \leq p_{i,t} - R_{i,t}^{-} \perp \underline{\nu}_{i,t}^{\mathrm{p}} \geq 0, \forall i \tag{4v}$$
$$0 \leq P_i^{\max} - R_{i,t}^{+} - p_{i,t} \perp \overline{\nu}_{i,t}^{\mathrm{p}} \geq 0, \forall i \tag{4w}$$
$$0 \leq p_t^{\uparrow,\mathrm{DA}} \perp \underline{\nu}_t^{\uparrow,\mathrm{DA}} \geq 0, \tag{4x}$$
$$0 \leq \overline{p}_t^{\uparrow,\mathrm{DA}} - p_t^{\uparrow,\mathrm{DA}} \perp \overline{\nu}_t^{\uparrow,\mathrm{DA}} \geq 0, \tag{4y}$$
$$0 \leq p_t^{\downarrow,\mathrm{DA}} \perp \underline{\nu}_t^{\downarrow,\mathrm{DA}} \geq 0, \tag{4z}$$
$$0 \leq \overline{p}_t^{\downarrow,\mathrm{DA}} - p_t^{\downarrow,\mathrm{DA}} \perp \overline{\nu}_t^{\downarrow,\mathrm{DA}} \geq 0, \tag{4aa}$$
$$\}\Big\}, \forall t.$$

3 Linearisation of the Objective Function

The objective function (5) of the bilevel problem (1)-(3) includes several non-linear terms which have to be substituted by equivalent linear formulations.

$$\sum_{t=1}^{N_T} \Big[\lambda_t^{\mathrm{DA}} (p_t^{\uparrow,\mathrm{DA}} - p_t^{\downarrow,\mathrm{DA}}) + \sum_{\omega=1}^{N_\Omega} \pi_\omega \Big(\lambda_{\omega,t}^{\mathrm{BA}} (p_{\omega,t}^{\uparrow,\mathrm{BA}} - p_t^{\uparrow,\mathrm{DA}}) - \lambda_{\omega,t}^{\mathrm{BA}} (p_{\omega,t}^{\downarrow,\mathrm{BA}} - p_t^{\downarrow,\mathrm{DA}}) \Big) \Big]. \tag{5}$$

Lower-Level BA

The objective function includes non-linear terms resulting from the lower-level BA since these are products of two decision variables, namely $\sum_{t=1}^{N_T} \Big[\lambda_{\omega,t}^{\mathrm{BA}} (p_{\omega,t}^{\uparrow,\mathrm{BA}} - p_{\omega,t}^{\downarrow,\mathrm{BA}}) \Big]$:

$$\sum_{t=1}^{N_T} \sum_{\omega=1}^{N_\Omega} \pi_\omega \Big[\lambda_{\omega,t}^{\mathrm{BA}} (-p_t^{\uparrow,\mathrm{DA}} + p_t^{\downarrow,\mathrm{DA}}) + \lambda_{\omega,t}^{\mathrm{BA}} (p_{\omega,t}^{\uparrow,\mathrm{BA}} - p_{\omega,t}^{\downarrow,\mathrm{BA}}) \Big], \forall \omega, \forall t. \tag{6}$$

Using the strong duality theorem, the objective function of lower-level BA problem (2a) can be reformulated as:

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} (C_i^{+} r_{i,\omega,t}^{+} - C_i^{-} r_{i,\omega,t}^{-}) + C^{\mathrm{shed}} l_{\omega,t}^{\mathrm{shed}} - \alpha_{\omega,t}^{\uparrow,\mathrm{BA}} (p_{\omega,t}^{\uparrow,\mathrm{BA}} - p_t^{\uparrow,\mathrm{DA}}) + \alpha_{\omega,t}^{\downarrow,\mathrm{BA}} (p_{\omega,t}^{\downarrow,\mathrm{BA}} - p_t^{\downarrow,\mathrm{DA}}) \Big] \\
&= \sum_{t=1}^{N_T} \Big[- \sum_{i=1}^{N_I} \overline{\mu}_{i,\omega,t}^{+} R_{i,t}^{+} - \sum_{i=1}^{N_I} \overline{\mu}_{i,\omega,t}^{-} R_{i,t}^{-} - \overline{\mu}_{\omega,t}^{\mathrm{spill}} W_{\omega,t} - \overline{\mu}_{\omega,t}^{\mathrm{shed}} L_t - \overline{\mu}_{\omega,t}^{\uparrow} \overline{p}_{\omega,t}^{\uparrow,\mathrm{BA}} - \overline{\mu}_{\omega,t}^{\downarrow} \overline{p}_{\omega,t}^{\downarrow,\mathrm{BA}} \\
&\qquad - \lambda_{\omega,t}^{\mathrm{BA}} (W_{\omega,t} - w_t + p_t^{\uparrow,\mathrm{DA}} - p_t^{\downarrow,\mathrm{DA}}) + (\alpha_{\omega,t}^{\uparrow,\mathrm{BA}} p_t^{\uparrow,\mathrm{DA}} - \alpha_{\omega,t}^{\downarrow,\mathrm{BA}} p_t^{\downarrow,\mathrm{DA}}) \Big], \forall i, \forall \omega, \forall t.
\end{aligned} \tag{7}$$

It should be noticed that the offering prices $\alpha_{\omega,t}^{\uparrow,\mathrm{BA}}$ and $\alpha_{\omega,t}^{\downarrow,\mathrm{BA}}$ for charging and discharging, respectively, are constants for the lower-level BA problem. Therefore, the products of $\alpha_{\omega,t}^{\uparrow,\mathrm{BA}} p_t^{\uparrow,\mathrm{DA}}$ and $\alpha_{\omega,t}^{\downarrow,\mathrm{BA}} p_t^{\downarrow,\mathrm{DA}}$ are also constants and thus can be removed from the objective function (2a).

Using the stationarity conditions (4d) and (4e) we obtain:

$$\alpha_{\omega,t}^{\uparrow,\mathrm{BA}} = \lambda_{\omega,t}^{\mathrm{BA}} - \underline{\mu}_{\omega,t}^{\uparrow} + \overline{\mu}_{\omega,t}^{\uparrow}, \forall \omega, \forall t \tag{8a}$$
$$\alpha_{\omega,t}^{\downarrow,\mathrm{BA}} = \lambda_{\omega,t}^{\mathrm{BA}} + \underline{\mu}_{\omega,t}^{\downarrow} - \overline{\mu}_{\omega,t}^{\downarrow}, \forall \omega, \forall t, \tag{8b}$$

and multiplying by $p_{\omega,t}^{\uparrow,\mathrm{BA}}$ and $p_{\omega,t}^{\downarrow,\mathrm{BA}}$, respectively, we get:

$$\alpha_{\omega,t}^{\uparrow,\mathrm{BA}} p_{\omega,t}^{\uparrow,\mathrm{BA}} = \lambda_{\omega,t}^{\mathrm{BA}} p_{\omega,t}^{\uparrow,\mathrm{BA}} - \underline{\mu}_{\omega,t}^{\uparrow} p_{\omega,t}^{\uparrow,\mathrm{BA}} + \overline{\mu}_{\omega,t}^{\uparrow} p_{\omega,t}^{\uparrow,\mathrm{BA}}, \forall \omega, \forall t \tag{9a}$$
$$\alpha_{\omega,t}^{\downarrow,\mathrm{BA}} p_{\omega,t}^{\downarrow,\mathrm{BA}} = \lambda_{\omega,t}^{\mathrm{BA}} p_{\omega,t}^{\downarrow,\mathrm{BA}} + \underline{\mu}_{\omega,t}^{\downarrow} p_{\omega,t}^{\downarrow,\mathrm{BA}} - \overline{\mu}_{\omega,t}^{\downarrow} p_{\omega,t}^{\downarrow,\mathrm{BA}}, \forall \omega, \forall t. \tag{9b}$$

The complementarity constraints regarding charging of the ESS (4m) and (4n) give the following relations:

$$\underline{\mu}_{\omega,t}^{\uparrow} p_{\omega,t}^{\uparrow,\mathrm{BA}} = 0, \forall \omega, \forall t \tag{10a}$$
$$\overline{\mu}_{\omega,t}^{\uparrow} p_{\omega,t}^{\uparrow,\mathrm{BA}} = \overline{\mu}_{\omega,t}^{\uparrow} \overline{p}_{\omega,t}^{\uparrow,\mathrm{BA}}, \forall \omega, \forall t \tag{10b}$$

while the complementarity constraints regarding discharging of the ESS (4o), (4p) result in:

$$\underline{\mu}^{\downarrow}_{\omega,t} p^{\downarrow,\mathrm{BA}}_{t} = 0, \forall \omega, \forall t \tag{11a}$$

$$\overline{\mu}^{\downarrow}_{\omega,t} p^{\downarrow,\mathrm{BA}}_{\omega,t} = \overline{\mu}^{\downarrow}_{\omega,t} \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t}, \forall \omega, \forall t. \tag{11b}$$

Constraints (10a) and (10b) are substituted into (9a) and (11a), (11b) are substituted into (9b) which gives following equations:

$$\alpha^{\uparrow,\mathrm{BA}}_{\omega,t} p^{\uparrow,\mathrm{BA}}_{\omega,t} = \lambda^{\mathrm{BA}}_{\omega,t} p^{\uparrow,\mathrm{BA}}_{\omega,t} + \overline{\mu}^{\uparrow}_{\omega,t} \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t}, \forall \omega, \forall t \tag{12a}$$

$$\alpha^{\downarrow,\mathrm{BA}}_{\omega,t} p^{\downarrow,\mathrm{BA}}_{\omega,t} = \lambda^{\mathrm{BA}}_{\omega,t} p^{\downarrow,\mathrm{BA}}_{\omega,t} - \overline{\mu}^{\downarrow}_{\omega,t} \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t}, \forall \omega, \forall t. \tag{12b}$$

Equations (12a) and (12b) can now be replaced into equation (7) in order to obtain a formulation for $\sum_{t=1}^{N_T} \Big[\lambda^{\mathrm{BA}}_{\omega,t}(p^{\uparrow,\mathrm{BA}}_{\omega,t} - p^{\downarrow,\mathrm{BA}}_{\omega,t})\Big]$,

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} (C_i^+ r^+_{i,\omega,t} - C_i^- r^-_{i,\omega,t}) + C^{\mathrm{shed}} l^{\mathrm{shed}}_{\omega,t} \\
&\qquad - (-\alpha^{\uparrow,\mathrm{BA}}_{\omega,t} p^{\uparrow,\mathrm{DA}}_{t} + (\lambda^{\mathrm{BA}}_{\omega,t} p^{\uparrow,\mathrm{BA}}_{\omega,t} + \overline{\mu}^{\uparrow}_{\omega,t} \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t})) + (-\alpha^{\downarrow,\mathrm{BA}}_{\omega,t} p^{\downarrow,\mathrm{DA}}_{t} + (\lambda^{\mathrm{BA}}_{\omega,t} p^{\downarrow,\mathrm{BA}}_{\omega,t} - \overline{\mu}^{\downarrow}_{\omega,t} \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t}))\Big] \\
= &\sum_{t=1}^{N_T} \Big[- \sum_{i=1}^{N_I} \overline{\mu}^{+}_{i,\omega,t} R^+_{i,t} - \sum_{i=1}^{N_I} \overline{\mu}^{-}_{i,\omega,t} R^-_{i,t} - \overline{\mu}^{\mathrm{spill}}_{\omega,t} W_{\omega,t} - \overline{\mu}^{\mathrm{shed}}_{\omega,t} L_t - \overline{\mu}^{\uparrow}_{\omega,t} \overline{p}^{\uparrow,\mathrm{BA}}_{\omega,t} - \overline{\mu}^{\downarrow}_{\omega,t} \overline{p}^{\downarrow,\mathrm{BA}}_{\omega,t} \\
&\qquad - \lambda^{\mathrm{BA}}_{\omega,t}(W_{\omega,t} - w_t + p^{\uparrow,\mathrm{DA}}_{\omega,t} - p^{\downarrow,\mathrm{DA}}_{\omega,t}) + (\alpha^{\uparrow,\mathrm{BA}}_{\omega,t} p^{\uparrow,\mathrm{DA}}_{t} - \alpha^{\downarrow,\mathrm{BA}}_{\omega,t} p^{\downarrow,\mathrm{DA}}_{t})\Big], \forall i, \forall \omega, \forall t,
\end{aligned} \tag{13}$$

which simplifies in

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\lambda^{\mathrm{BA}}_{\omega,t}(p^{\uparrow,\mathrm{BA}}_{\omega,t} - p^{\downarrow,\mathrm{BA}}_{\omega,t})\Big] \\
= &\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} \overline{\mu}^{+}_{i,\omega,t} R^+_{i,t} + \sum_{i=1}^{N_I} \overline{\mu}^{-}_{i,\omega,t} R^-_{i,t} + \overline{\mu}^{\mathrm{spill}}_{\omega,t} W_{\omega,t} + \overline{\mu}^{\mathrm{shed}}_{\omega,t} L_t \\
&\qquad + \lambda^{\mathrm{BA}}_{\omega,t}(W_{\omega,t} - w_t + p^{\uparrow,\mathrm{DA}}_{\omega,t} - p^{\downarrow,\mathrm{DA}}_{\omega,t}) + \sum_{i=1}^{N_I} (C_i^+ r^+_{i,\omega,t} - C_i^- r^-_{i,\omega,t}) + C^{\mathrm{shed}} l^{\mathrm{shed}}_{\omega,t}\Big], \forall i, \forall \omega, \forall t.
\end{aligned} \tag{14}$$

Lower-Level DA

To derive an equivalent linear expression for the term $\sum_{t=1}^{N_T} \lambda^{\mathrm{DA}}_t (p^{\uparrow,\mathrm{DA}}_t - p^{\downarrow,\mathrm{DA}}_t)$ we apply the following steps.

The strong duality theorem is applied to reformulate the objective function (3a) of the lower-level DA problem. This relation is given by:

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} C_i p_{i,t} - \alpha^{\uparrow,\mathrm{DA}}_t p^{\uparrow,\mathrm{DA}}_t + \alpha^{\downarrow,\mathrm{DA}}_t p^{\downarrow,\mathrm{DA}}_t \Big] \\
= &\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} \underline{\nu}^{\mathrm{p}}_{i,t} R^-_{i,t} - \sum_{i=1}^{N_I} \overline{\nu}^{\mathrm{p}}_{i,t} (P^{\max}_i - R^+_{i,t}) - \overline{\nu}^{\uparrow,\mathrm{DA}}_t \overline{p}^{\uparrow,\mathrm{DA}}_t - \overline{\nu}^{\downarrow,\mathrm{DA}}_t \overline{p}^{\downarrow,\mathrm{DA}}_t - \lambda^{\mathrm{DA}}_t (W^{\mathrm{E}}_t - L_t), \forall i, \forall t.
\end{aligned} \tag{15}$$

The stationarity conditions (4t) and (4u) of the lower-level DA problem are reformulated as following:

$$\alpha^{\uparrow,\mathrm{DA}}_t = \lambda^{\mathrm{DA}}_t - \underline{\nu}^{\uparrow,\mathrm{DA}}_t + \overline{\nu}^{\uparrow,\mathrm{DA}}_t, \forall t \tag{16a}$$

$$\alpha^{\downarrow,\mathrm{DA}}_t = \lambda^{\mathrm{DA}}_t + \underline{\nu}^{\downarrow,\mathrm{DA}}_t - \overline{\nu}^{\downarrow,\mathrm{DA}}_t, \forall t. \tag{16b}$$

and multiplying them by $p^{\uparrow,\mathrm{DA}}_t$ and $p^{\downarrow,\mathrm{DA}}_t$, respectively, results in following equations:

$$\alpha^{\uparrow,\mathrm{DA}}_t p^{\uparrow,\mathrm{DA}}_t = \lambda^{\mathrm{DA}}_t p^{\uparrow,\mathrm{DA}}_t - \underline{\nu}^{\uparrow,\mathrm{DA}}_t p^{\uparrow,\mathrm{DA}}_t + \overline{\nu}^{\uparrow,\mathrm{DA}}_t p^{\uparrow,\mathrm{DA}}_t, \forall t \tag{17a}$$

$$\alpha^{\downarrow,\mathrm{DA}}_t p^{\downarrow,\mathrm{DA}}_t = \lambda^{\mathrm{DA}}_t p^{\downarrow,\mathrm{DA}}_t + \underline{\nu}^{\downarrow,\mathrm{DA}}_t p^{\downarrow,\mathrm{DA}}_t - \overline{\nu}^{\downarrow,\mathrm{DA}}_t p^{\downarrow,\mathrm{DA}}_t, \forall t \tag{17b}$$

The complementarity constraints regarding charging of the ESS (4x) and (4y) state

$$\underline{\nu}^{\uparrow}_t p^{\uparrow,\mathrm{DA}}_t = 0, \forall t \tag{18a}$$

$$\overline{\nu}_t^{\uparrow} p_t^{\uparrow,\mathrm{DA}} = \overline{\nu}_t^{\uparrow} \overline{p}_t^{\uparrow,\mathrm{DA}}, \forall t, \tag{18b}$$

while complementarity constraints regarding discharging of the ESS (4z), (4aa) give the relation regarding discharging analogously:

$$\underline{\nu}_t^{\downarrow} p_t^{\downarrow,\mathrm{DA}} = 0, \forall t \tag{19a}$$
$$\overline{\nu}_t^{\downarrow} p_t^{\downarrow,\mathrm{DA}} = \overline{\nu}_t^{\downarrow} \overline{p}_t^{\downarrow,\mathrm{DA}}, \forall t. \tag{19b}$$

Inserting the reformulated complementarity constraints (18a) and (18b) into (17a) and (19a), (19b) into (17b) results in

$$\alpha_t^{\uparrow,\mathrm{DA}} p_t^{\uparrow,\mathrm{DA}} = \lambda_t^{\mathrm{DA}} p_t^{\uparrow,\mathrm{DA}} + \overline{\nu}_t^{\uparrow,\mathrm{DA}} \overline{p}_t^{\uparrow,\mathrm{DA}}, \forall t \tag{20a}$$
$$\alpha_t^{\downarrow,\mathrm{DA}} p_t^{\downarrow,\mathrm{DA}} = \lambda_t^{\mathrm{DA}} p_t^{\downarrow,\mathrm{DA}} - \overline{\nu}_t^{\downarrow,\mathrm{DA}} \overline{p}_t^{\downarrow,\mathrm{DA}}, \forall t, \tag{20b}$$

which can now be substituted in the strong duality of the lower-level DA objective function (15):

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} C_i p_{i,t} - (\lambda_t^{\mathrm{DA}} p_t^{\uparrow,\mathrm{DA}} + \overline{\nu}_t^{\uparrow,\mathrm{DA}} \overline{p}_t^{\uparrow,\mathrm{DA}}) + (\lambda_t^{\mathrm{DA}} p_t^{\downarrow,\mathrm{DA}} - \overline{\nu}_t^{\downarrow,\mathrm{DA}} \overline{p}_t^{\downarrow,\mathrm{DA}}) \Big] \\
=&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} \underline{\nu}_{i,t}^{\mathrm{p}} R_{i,t}^{-} - \sum_{i=1}^{N_I} \overline{\nu}_{i,t}^{\mathrm{p}} (P_i^{\max} - R_{i,t}^{+}) - \overline{\nu}_t^{\uparrow,\mathrm{DA}} \overline{p}_t^{\uparrow,\mathrm{DA}} - \overline{\nu}_t^{\downarrow,\mathrm{DA}} \overline{p}_t^{\downarrow,\mathrm{DA}} - \lambda_t^{\mathrm{DA}} (W_t^{\mathrm{E}} - L_t), \forall i, \forall t.
\end{aligned} \tag{21}$$

Equation (21) can be simplified as:

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} C_i p_{i,t} - \lambda_t^{\mathrm{DA}} (p_t^{\uparrow,\mathrm{DA}} - p_t^{\downarrow,\mathrm{DA}}) \Big] \\
=&\sum_{t=1}^{N_T} \Big[\sum_{i=1}^{N_I} \underline{\nu}_{i,t}^{\mathrm{p}} R_{i,t}^{-} - \sum_{i=1}^{N_I} \overline{\nu}_{i,t}^{\mathrm{p}} (P_i^{\max} - R_{i,t}^{+}) - \lambda_t^{\mathrm{DA}} (W_t^{\mathrm{E}} - L_t), \forall i, \forall t,
\end{aligned} \tag{22}$$

which is equivalent to an expression for the non-linear term in the objective function of the single-level problem:

$$\begin{aligned}
&\sum_{t=1}^{N_T} \Big[\lambda_t^{\mathrm{DA}} (p_t^{\uparrow,\mathrm{DA}} - p_t^{\downarrow,\mathrm{DA}}) \Big] \\
=&\sum_{t=1}^{N_T} \Big[- \sum_{i=1}^{N_I} \underline{\nu}_{i,t}^{\mathrm{p}} R_{i,t}^{-} + \sum_{i=1}^{N_I} \overline{\nu}_{i,t}^{\mathrm{p}} (P_i^{\max} - R_{i,t}^{+}) + \lambda_t^{\mathrm{DA}} (W_t^{E} - L_t) + \sum_{i=1}^{N_I} C_i p_{i,t}, \forall i, \forall t.
\end{aligned} \tag{23}$$

Finally the Equation (14) and Equation (23) are substituted into the objective function (1a) and result in an overall linearised expression:

$$\begin{aligned}
&\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_T} \Big[- \sum_{i=1}^{N_I} \underline{\nu}_{i,t}^{\mathrm{p}} R_{i,t}^{-} + \sum_{i=1}^{N_I} \overline{\nu}_{i,t}^{\mathrm{p}} (P_i^{\max} - R_{i,t}^{+}) + \lambda_t^{\mathrm{DA}} (W_t^{\mathrm{E}} - L_t) + \sum_{i=1}^{N_I} C_i p_{i,t} \\
&\quad + \sum_{i=1}^{N_I} \overline{\mu}_{i,\omega,t}^{+} R_{i,t}^{+} + \sum_{i=1}^{N_I} \overline{\mu}_{i,\omega,t}^{-} R_{i,t}^{-} + \overline{\mu}_{\omega,t}^{\mathrm{spill}} W_{\omega,t} + \overline{\mu}_{\omega,t}^{\mathrm{shed}} L_t \\
&\quad + \lambda_{\omega,t}^{\mathrm{BA}} (W_{\omega,t} - W_t^{\mathrm{E}}) + \sum_{i=1}^{N_I} (C_i^{+} r_{i,\omega,t}^{+} - C_i^{-} r_{i,\omega,t}^{-}) + C^{\mathrm{shed}} l_{\omega,t}^{\mathrm{shed}} \Big].
\end{aligned} \tag{24}$$

4 Linearisation of Complementarity Constraints

The complementarity constraints of the MPEC (4g) to (4r) and (4v) to (4aa) are linearised by the Fortuny-Amat method [1]:

$$\{0 \leq \underline{\mu}_{i,\omega,t}^{+} \leq \underline{u}_{i,\omega,t}^{+} M, \forall i, \forall \omega \tag{25a}$$
$$0 \leq r_{i,\omega,t}^{+} \leq (1 - \underline{u}_{i,\omega,t}^{+}) M, \forall i, \forall \omega \tag{25b}$$
$$0 \leq \overline{\mu}_{i,\omega,t}^{+} \leq \overline{u}_{i,\omega,t}^{+} M, \forall i, \forall \omega \tag{25c}$$
$$0 \leq R_i^{+} - r_{i,\omega,t}^{+} \leq (1 - \overline{u}_{i,\omega,t}^{+}) M, \forall i, \forall \omega \tag{25d}$$

$$\underline{u}^{+}_{i,\omega,t}, \overline{u}^{+}_{i,\omega,t} = \{0,1\}, \forall i, \forall\omega \tag{25e}$$

$$0 \le \underline{\mu}^{-}_{i,\omega,t} \le \underline{u}^{-}_{i,\omega,t} M, \forall i, \forall\omega \tag{26a}$$
$$0 \le r^{-}_{i,\omega,t} \le (1-\underline{u}^{-}_{i,\omega,t})M, \forall i, \forall\omega \tag{26b}$$
$$0 \le \overline{\mu}^{-}_{i,\omega,t} \le \overline{u}^{-}_{i,\omega,t} M, \forall i, \forall\omega \tag{26c}$$
$$0 \le R^{-}_{i} - r^{-}_{i,\omega,t} \le (1-\overline{u}^{-}_{i,\omega,t})M, \forall i, \forall\omega \tag{26d}$$
$$\underline{u}^{-}_{i,\omega,t}, \overline{u}^{-}_{i,\omega,t} = \{0,1\}, \forall i, \forall\omega \tag{26e}$$

$$0 \le \underline{\mu}^{\text{spill}}_{\omega,t} \le \underline{u}^{\text{spill}}_{\omega,t} M, \forall\omega \tag{27a}$$
$$0 \le p^{\text{spill}}_{\omega,t} \le (1-\underline{u}^{spill}_{\omega,t})M, \forall\omega \tag{27b}$$
$$0 \le \overline{\mu}^{\text{spill}}_{\omega,t} \le \overline{u}^{\text{spill}}_{\omega,t} M, \forall\omega \tag{27c}$$
$$0 \le W_{\omega,t} - w^{\text{spill}}_{\omega,t} \le (1-\overline{u}^{\text{spill}}_{\omega,t})M, \forall\omega \tag{27d}$$
$$\underline{u}^{\text{spill}}_{\omega,t}, \overline{u}^{\text{spill}}_{\omega,t} = \{0,1\}, \forall\omega \tag{27e}$$

$$0 \le \underline{\mu}^{\uparrow,\text{BA}}_{\omega,t} \le \underline{u}^{\uparrow,\text{BA}}_{\omega,t} M, \forall\omega \tag{28a}$$
$$0 \le p^{\uparrow,\text{BA}}_{\omega,t} \le (1-\underline{u}^{\uparrow}_{\omega,t})M, \forall\omega \tag{28b}$$
$$0 \le \overline{\mu}^{\uparrow,\text{BA}}_{\omega,t} \le \overline{u}^{\uparrow,\text{BA}}_{\omega,t} M, \forall\omega \tag{28c}$$
$$0 \le \overline{p}^{\uparrow,\text{BA}}_{\omega,t} - p^{\uparrow,\text{BA}}_{\omega,t} \le (1-\overline{u}^{\uparrow,\text{BA}}_{\omega,t})M, \forall\omega \tag{28d}$$
$$\underline{u}^{\uparrow,\text{BA}}_{\omega,t}, \overline{u}^{\uparrow,\text{BA}}_{\omega,t} = \{0,1\}, \forall\omega \tag{28e}$$

$$0 \le \underline{\mu}^{\downarrow,\text{BA}}_{\omega,t} \le \underline{u}^{\downarrow,\text{BA}}_{\omega,t} M, \forall\omega \tag{29a}$$
$$0 \le p^{\downarrow,\text{BA}}_{\omega,t} \le (1-\underline{u}^{\downarrow,\text{BA}}_{\omega,t})M, \forall\omega \tag{29b}$$
$$0 \le \overline{\mu}^{\downarrow,\text{BA}}_{\omega,t} \le \overline{u}^{\downarrow,\text{BA}}_{\omega,t} M, \forall\omega \tag{29c}$$
$$0 \le \overline{p}^{\downarrow,\text{BA}}_{\omega,t} - p^{\downarrow,\text{BA}}_{\omega,t} \le (1-\overline{u}^{\downarrow,\text{BA}}_{\omega,t})M, \forall\omega \tag{29d}$$
$$\underline{u}^{\downarrow,\text{BA}}_{\omega,t}, \overline{u}^{\downarrow,\text{BA}}_{\omega,t} = \{0,1\}, \forall\omega \tag{29e}$$

$$0 \le \underline{\mu}^{\text{shed}}_{\omega,t} \le \underline{u}^{\text{shed}}_{\omega,t} M, \forall\omega \tag{30a}$$
$$0 \le l^{\text{shed}}_{\omega,t} \le (1-\underline{u}^{\text{shed}}_{\omega,t})M, \forall\omega \tag{30b}$$
$$0 \le \overline{\mu}^{\text{shed}}_{\omega,t} \le \overline{u}^{\text{shed}}_{\omega,t} M, \forall\omega \tag{30c}$$
$$0 \le L_t - l^{\text{shed}}_{\omega,t} \le (1-\overline{u}^{\text{shed}}_{\omega,t})M, \forall\omega \tag{30d}$$
$$\underline{u}^{\text{shed}}_{\omega,t}, \overline{u}^{\text{shed}}_{\omega,t} = \{0,1\}, \forall\omega \tag{30e}$$

$$0 \le \underline{\nu}^{\text{p}}_{i,t} \le \underline{u}^{\text{p}}_{i,t} M, \forall i \tag{31a}$$
$$0 \le p_{i,t} - R^{-}_{i,t} \le (1-\underline{u}^{\text{p}}_{i,t})M, \forall i \tag{31b}$$
$$0 \le \overline{\nu}^{\text{p}}_{i,t} \le \overline{u}^{\text{p}}_{i,t} M, \forall i \tag{31c}$$
$$0 \le P^{max}_{t} - R^{+}_{i,t} - p_{i,t} \le (1-\overline{u}^{\text{p}}_{i,t})M, \forall i \tag{31d}$$
$$\underline{u}^{\text{p}}_{i,t}, \overline{u}^{\text{p}}_{i,t} = \{0,1\}, \forall i \tag{31e}$$

$$0 \le \underline{\nu}^{\uparrow,\text{DA}}_{t} \le \underline{u}^{\uparrow,\text{DA}}_{t} M, \tag{32a}$$
$$0 \le p^{\uparrow,\text{DA}}_{t} \le (1-\underline{u}^{\uparrow,\text{DA}}_{t})M, \tag{32b}$$
$$0 \le \overline{\nu}^{\uparrow,\text{DA}}_{t} \le \overline{u}^{\uparrow,\text{DA}}_{t} M, \tag{32c}$$
$$0 \le \overline{p}^{\uparrow,\text{DA}}_{t} - p^{\uparrow,\text{DA}}_{t} \le (1-\overline{u}^{\uparrow,\text{DA}}_{t})M, \tag{32d}$$
$$\underline{u}^{\uparrow,\text{DA}}_{t}, \overline{u}^{\uparrow,\text{DA}}_{t} = \{0,1\}, \tag{32e}$$

$$0 \leq \underline{\nu}_t^{\downarrow,\mathrm{DA}} \leq \underline{u}_t^{\downarrow,\mathrm{DA}} M, \tag{33a}$$
$$0 \leq p_t^{\downarrow,\mathrm{DA}} \leq (1 - \underline{u}_t^{\downarrow,\mathrm{DA}}) M, \tag{33b}$$
$$0 \leq \overline{\nu}_t^{\downarrow,\mathrm{DA}} \leq \overline{u}_t^{\downarrow,\mathrm{DA}} M, \tag{33c}$$
$$0 \leq \overline{p}_t^{\downarrow,\mathrm{DA}} - p_t^{\downarrow,\mathrm{DA}} \leq (1 - \overline{u}_t^{\downarrow,\mathrm{DA}}) M, \tag{33d}$$
$$\underline{u}_t^{\downarrow,\mathrm{DA}}, \overline{u}_t^{\downarrow,\mathrm{DA}} = \{0, 1\}, \tag{33e}$$
$$\}, \forall t,$$

where M is a large enough constant.

5 Mixed Integer Linear Programming Problem

Finally, a MILP is derived which is equivalent to the single-level problem:

$$\underset{\Theta \cup \Xi_{\omega,t}^{\mathrm{LL,BA,AUX}} \cup \Xi_t^{\mathrm{LL,DA,AUX}}}{\mathrm{Min.}} \quad (24)$$

subject to

$$\begin{aligned}
&\text{Constraints (1b)} - \text{(1t)} \\
&\Big\{\{\text{Constraint (2b)} \\
&\text{Constraints (4a)} - \text{(4f)} \\
&\text{Constraints (25a)} - \text{(30e)} \\
&\}, \forall \omega \\
&\{\text{Constraint (3b)} \\
&\text{Constraints (4s)} - \text{(4u)} \\
&\text{Constraints (31a)} - \text{(33e)} \\
&\}\Big\} \forall t,
\end{aligned}$$

where

$$\Xi_{\omega,t}^{\mathrm{LL,BA,AUX}} = \{\underline{u}_{i,\omega,t}^{+}, \overline{u}_{i,\omega,t}^{+}, \underline{u}_{i,\omega,t}^{-}, \overline{u}_{i,\omega,t}^{-} \forall i; \underline{u}_{\omega,t}^{\mathrm{spill}}, \overline{u}_{\omega,t}^{\mathrm{spill}}, \underline{u}_{\omega,t}^{\uparrow}, \overline{u}_{\omega,t}^{\uparrow}, \underline{u}_{\omega,t}^{\downarrow}, \overline{u}_{\omega,t}^{\downarrow}, \underline{u}_{\omega,t}^{\mathrm{shed}}, \overline{u}_{\omega,t}^{\mathrm{spill}}\}, \forall \omega, \forall t,$$

and

$$\Xi_{\omega,t}^{\mathrm{LL,DA,AUX}} = \{\underline{u}_{i,t}^{\mathrm{p}}, \overline{u}_{i,t}^{\mathrm{p}}, \forall i; \underline{u}_t^{\uparrow}, \overline{u}_t^{\uparrow}, \underline{u}_t^{\downarrow}, \overline{u}_t^{\downarrow}\}, \forall t.$$

References

[1] J. Fortuny-Amat and B. McCarl, "A representation and economic interpretation of a two-level programming problem," *J. Oper. Res. Soc.*, vol. 32, no. 9, pp. 783-792, 1981.