Heuristics for Advertising Revenue Optimization in Online Social Networks

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Agenda

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4. Overview of heuristics
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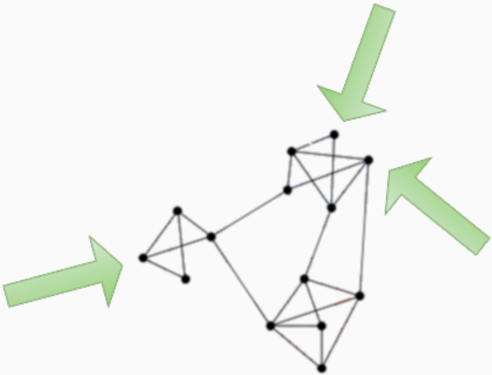
6. Heuristic for Impression Vector Allocation
7. Performance of Impression Vector Allocation Heuristic
6. Future work and Conclusion

Context

Advertisers are turning more to the Internet to disseminate advertisements.

OSNs are one of the most popular uses of the Internet.



Hosein and Lawrence [1]

Assume that knowing that your friend clicked raises. your probability of clicking, and that knowing that your friend didn't click lowers your probability of clicking.

$$p \leftarrow min\ \{1,\ max\ \{0,\ p_{init} + \alpha\frac{f}{n} - \beta\frac{g}{n}\}\}$$

Assumed that friendships are reciprocal.

Assumed a fixed number of stages, k.

Assume a fixed number of impressions, M

$x_k[i] = 1$ if i was previously given an impression else it is 0

$c_k[i] = 1$ if i clicked a given past impression else it is 0

$u_k[i] = 1$ if i is given an impression in this stage else it is 0

$$p_k[i] = \begin{cases} \text{prob}(c_{k-1}[i] = 1 | u_k[i] = 1, \vec{x}_k, \vec{c}_k) & \text{if } x_k[i] = 0 \\ 0 & \text{otherwise} \end{cases}$$

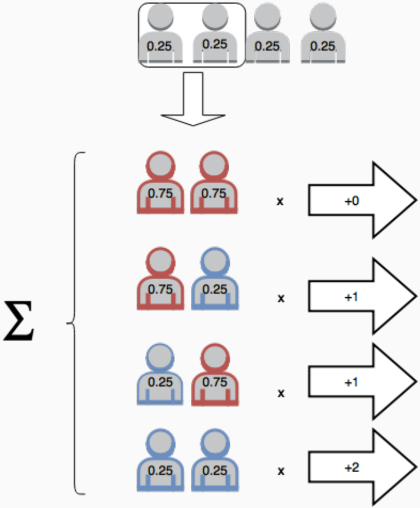
$$J_k^* = \max_{\vec{u}\in\{0,1\}^N} \sum_{\vec{v}\in\mathcal{V}|\vec{u}} Pr(\vec{v}) J_{k-1}^*(\vec{x}^k + \vec{u}, \vec{c}_k + \vec{v}, \vec{p}_{k-1}) \tag{1}$$

$$\text{subject to:} \quad \sum_{i=1}^{N} u[i] = m_k \quad \text{and} \quad \vec{u} + \vec{x}_k \leq 1.$$

In the final stage we have:

$$J_0^* = |\vec{c}_0| + \max_{\vec{u}\in\{0,1\}^N} \sum_{i=1}^{N} p_0[i]u[i] \tag{2}$$

$$\text{subject to:} \quad \sum_{i=1}^{N} u[i] = m_0 \quad \text{and} \quad \vec{u} + \vec{x}_0 \leq 1.$$


0.25
0.25
0.25
0.25
Σ
0.75
0.75
x
+0
0.75
0.25
x
+1
0.25
0.75
x
+1
0.25
0.25
x
+2

Hosein and Lawrence [1]

$$\binom{n}{m_k} 2^{m_k}$$

This grows large very quickly!

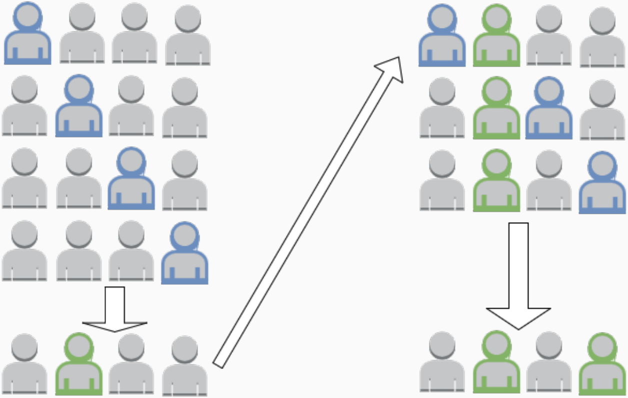
Hosein and Lawrence's Heuristic

1. Consider each user and allocate an impression to them, considering the expected number of clicks generated in that stage.

2. Find the best such user, termed u_1.

3. Find users $u_2, u_3, \ldots u_{m_k}$ in the same manner

4. Repeat for remaining stages.



$$\sum_{j=1}^{m_k} (n - j + 1)\, 2^{m_k} = 2^{m_k+1}\,(2 + n + m_k) - 2n - 4$$

Maximum Influence Heuristic

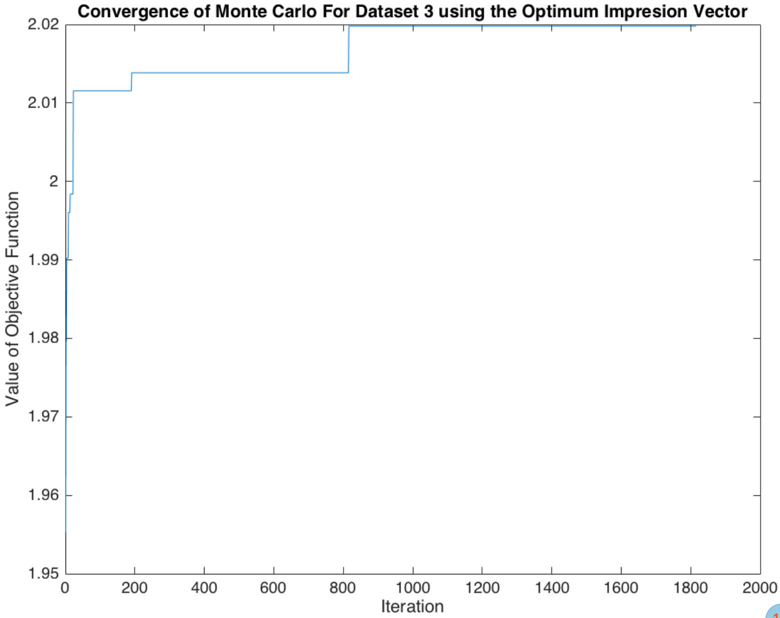
At stage k multiply probability of clicking and degree, selecting the best m_k users. Expected value requires 2^{m_k} sub-problems per stage.

$$2^{m_k} \text{subproblems per stage}$$

LSMC

Using Monte Carlo Simulation as our objective function, J, we approximated the expected number of clicks generated by a particular allocation of impressions to users. We randomly generated a seed allocation, S, and drew a random adjacent point in the search space, S' by swapping allocations. If $J(S') > J(S)$, we retained S', and continued until convergence.

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


Convergence of Monte Carlo For Dataset 3 using the Optimum Impresion Vector
Value of Objective Function
Iteration
2.02
2.01
2
1.99
1.98
1.97
1.96
1.95
0
200
400
600
800
1000
1200
1400
1600
1800
2000

Evaluation of Allocation Heuristics

Table: Dataset Parameters

Data	Users	Impressions	Stages	Avg No Friends	p_{K-1}
1	6	5	2	2.7	0.25
2	7	5	3	2.0	0.25
3	15	7	3	4.4	0.25
4	50	10	3	3.5	0.25
5	100	20	3	5.5	0.25
6	1000	20	2	100	0.20

Table: Performance and Runtime Comparisons

Data	Method	Optimal $\vec{m}$	Value	Time (ms)
1	Optimal	[2, 3]	1.40	64
	Hosein and Lawrence	[2, 3]	1.40	32
	Maximum Influence	[3, 2]	1.38	8
2	Optimal	[2, 2, 1]	1.56	56841
	Hosein and Lawrence	[1, 2, 2]	1.54	4678
	Maximum Influence	[1, 2, 2]	1.52	48
3	Optimal	[2, 2, 3]	2.03	170967777
	Hosein and Lawrence	[2, 2, 3]	2.03	211516
	Maximum Influence	[2, 2, 3]	1.99	378
4	LSMC	[2, 2, 6]	3.22	7229250
	Maximum Influence	[2, 1, 7]	3.09	10444
5	LSMC	[2, 2, 16]	6.54	23426641
	Maximum Influence	[2, 3, 15]	6.39	40097214
6	LSMC	[10, 10]	4.32	302874000
	Maximum Influence	[8, 12]	4.04	49838538

Heuristic For Impression Vector

Since m_k impressions are allocated in the 1^{st} stage, the expected number of users to click is $p_K m_K$.
$p_K m_K d$ users will be effected

$$p_K m_K d = m_{K-1}$$

Heuristic For Impression Vector

$$m_2 = \frac{m_1}{p_2 d} \quad \text{and} \quad m_1 = \frac{m_0}{p_1(d-1)} \tag{3}$$

Since the total number of impressions is M we have

$$M = m_0 + m_1 + m_2 = m_0 \left[1 + \frac{1}{p_1(d-1)} + \frac{1}{p_2 p_1 d(d-1)}\right]$$

Therefore

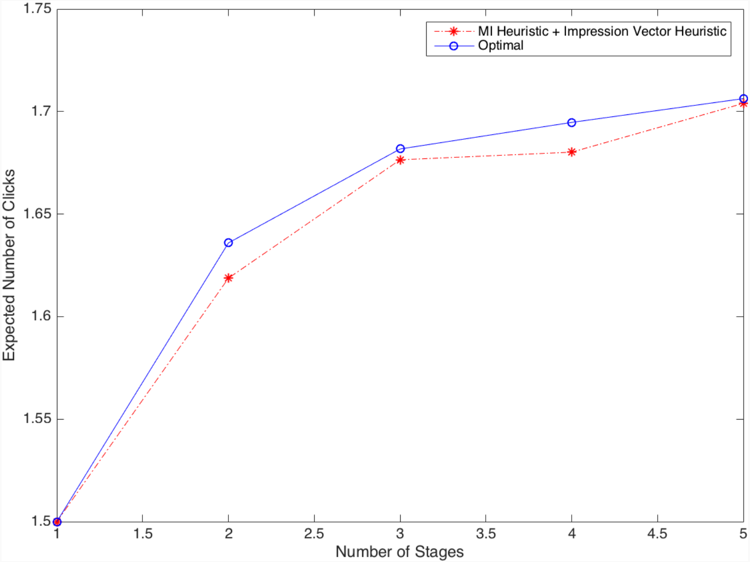
$$m_0 = \frac{M}{1 + \frac{1}{p_1(d-1)} + \frac{1}{p_2 p_1 d(d-1)}} \quad \text{where} \quad p_1 = p_2 + \frac{\alpha}{d} \tag{4}$$

Performance of Heuristic

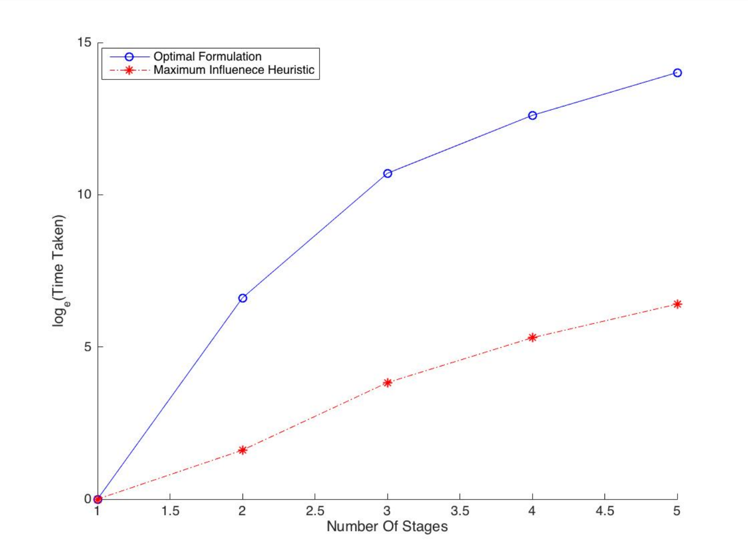
Table: Performance of Impression Vector Heuristic

Dataset	Optimal Vector (value)	Heuristic Vector (value)
1	[2, 3] (1.40)	[2, 3] (1.40)
2	[2, 2, 1] (1.56)	[3, 1, 1] (1.51)
3	[2, 2, 3] (2.03)	[2, 2, 3] (2.03)
4	[2, 2, 6] (3.22)	[3, 3, 4] (3.14)
5	[2, 2, 16] (6.54)	[4, 6, 10] (6.41)
6	[10, 10] (4.32)	[1, 19] (4.22)

Performance

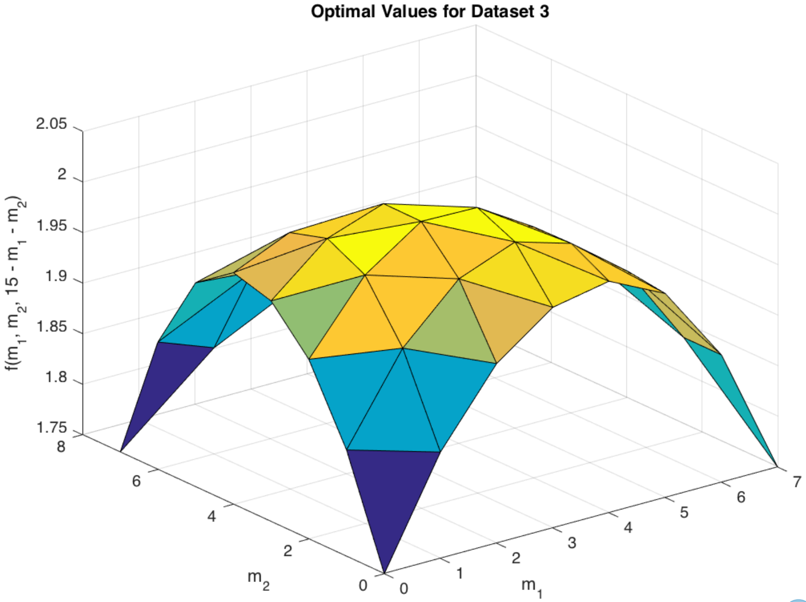


Performance



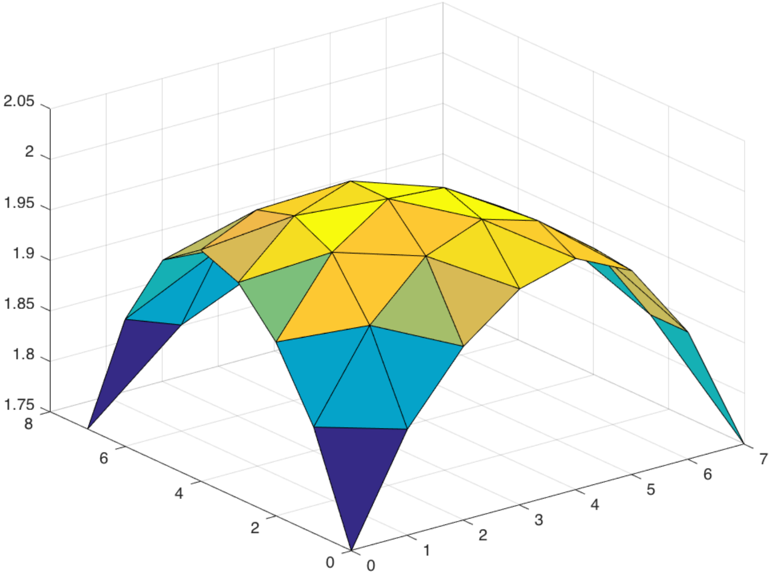
Conclusion

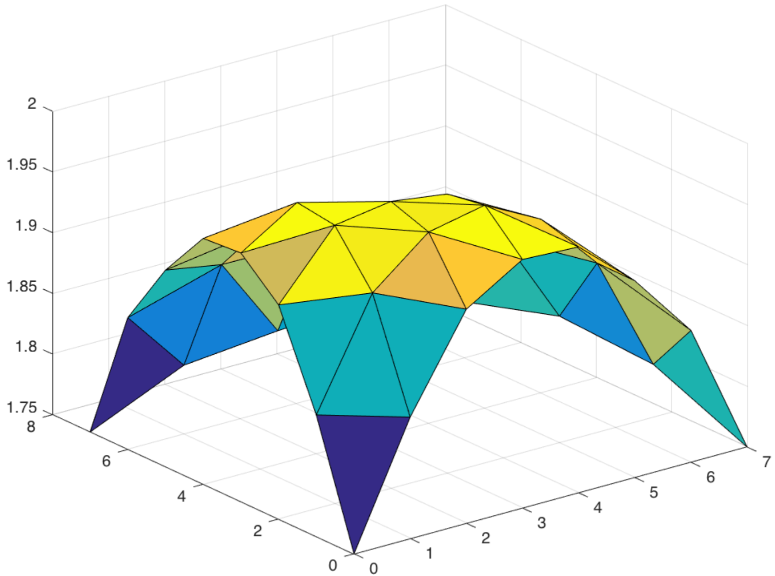
We presented two heuristics that can approximate solutions to the problem posed in [1] in less time by minimizing the number of possibilities checked.

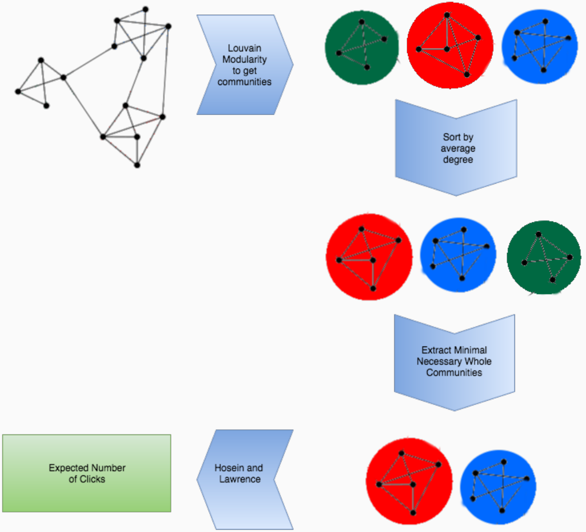


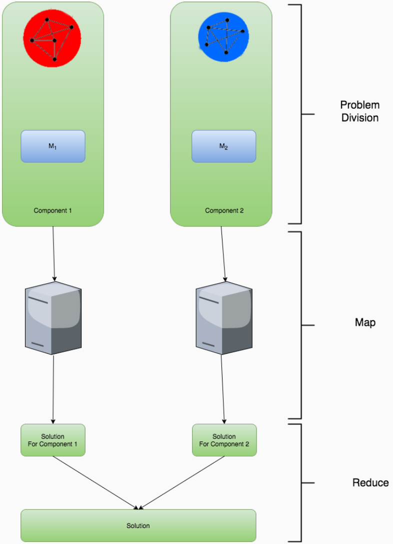
Optimal Values for Dataset 3
f(m1, m2, 15 - m1 - m2)
2.05
2
1.95
1.9
1.85
1.8
1.75
m2
8
6
4
2
0
m1
0
1
2
3
4
5
6
7

Expected Number of Clicks as Computed Using Heuristic 1 for Dataset 3




2
1.95
1.9
1.85
1.8
1.75
8
6
4
2
0
0
1
2
3
4
5
6
7


Louvain Modularity to get communities
Sort by average degree
Extract Minimal Necessary Whole Communities
Hosein and Lawrence
Expected Number of Clicks


Component 1
Component 2
M_1
M_2
Problem Division
Map
Solution For Component 1
Solution For Component 2
Reduce
Solution