UNIVERSITÀ DI PISA

Master Degree in Data Science and Business Informatics

Logistics project

Flight company problem

Francesca Scotti 657524 (f.scotti5@studenti.unipi.it)
Steffania Sierra 663718 (s.sierragalvis@studenti.unipi.it)

Academic year 2022-2023

Table of contents

1 Introduction 2

2 ILP Model 3

3 Optimal Solution 5

4 Modifications 6

5 Optimal solution with the modifications 6

1 Introduction

A flight company needs to decide where to open some new terminals and the number of flights to operate between the potentially opened terminals. There are 6 cities, namely Rome, Berlin, Paris, Vienna, Brussels, Amsterdam, where the company may open a terminal. The company may open at most 4 new terminals and the cost of opening depends on the location, as reported in the Table 1.

Terminal	Rome	Berlin	Paris	Vienna	Brussels	Amsterdam
Cost	4	3	9	7	6	8

Table 1: Opening cost of each terminal.

Depending on which terminals are opened, different flights may be operated by the company. Table 2 shows possible origin-destination pairs for the flights and, for each origin-destination pair, the profit that the company will have for each flight that will be operated from that origin to that destination.

A flight may be activated only if a terminal is opened both in the departure and in the arrival city. The total number of flights that can be operated by the company is 10. Additionally, some rules need to be considered:

1. at most 4 flights can be activated for each origin-destination pair, except for the pair Rome to Berlin, where the limit is set to 3 flights;
2. to Paris (it does not matter from where) at least 4 flights have to arrive.

The company wants to decide where to open the terminals, and how many flights to activate on each origin-destination pair, by respecting all the given rules and maximizing its revenue, given by the profit of operating flights minus the cost of opening terminals.

Origin Destination	Rome Berlin	Rome Paris	Rome Vienna	Berlin Rome	Berlin Amsterdam
Profit	7	5	4	7	6

Berlin Vienna	Paris Rome	Paris Vienna	Paris Brussels	Paris Amsterdam	Vienna Rome
4	5	2	4	1	6

Vienna Berlin	Vienna Paris	Brussels Paris	Brussels Amsterdam	Amsterdam Paris	Amsterdam Brussels
6	5	4	4	8	4

Table 2: Flights' profit

This report seeks to provide a solution to the airline logistics problem. The document is organized in several sections, each one described below.

1. In Section 2 is presented an ILP model for the stated problem.
2. In Section 3 is presented the solution of the model obtained after implement the model by means of the modelling language AMPL, and solve it using the optimization solver CPLEX.

3. Section 4 and 5 give solution to the airline logistic problem assuming that a tax equal to 1 must be payed by the company for each flight leaving from or arriving to Paris. Finally, after the presentation of the new solution, a brief discussion of how the optimal solution changes with respect to the one found in Section 3 is presented.

2 ILP Model

This problem is similar to a fixed charge location problem where the goal is to decide how many facilities to open, in our case terminals, having as input data an associated cost to the opening of each facility.

In order to formulate the problem, let us introduce the input data and the decision variables.

- Set of terminals in Table 1 enumerated from left to right, $N = \{1, 2, 3, 4, 5, 6\}$.
- Set A of possible origin - destination pairs of flights in Table 2.
- $G = (N, A)$: directed graph with nodes N and arcs A.
- c_i : cost of opening terminal i.
- $p_{i,j}$: profit of operating the flight with origin i and destination j.
- $y_i = \begin{cases} 1 & \text{if terminal } i \text{ is open} \\ 0 & \text{otherwise} \end{cases}, \forall i \in N.$
- x_{ij} : number of flights activated from i to j.

The graph at Figure 1 represents our location network.

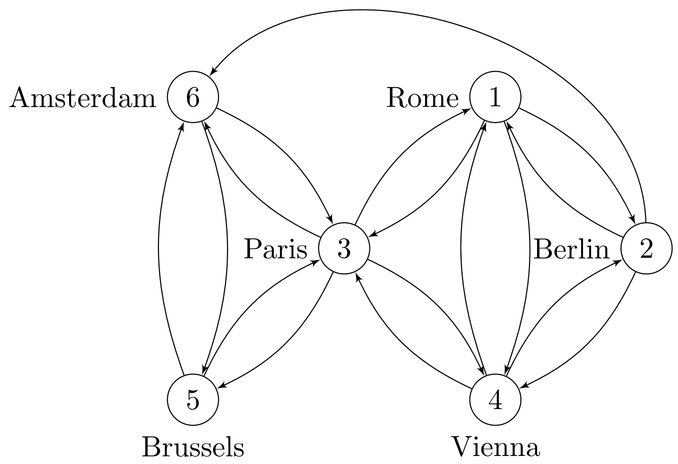

Figure 1: Graph G representing the possible flights to be activated between terminals.

The ILP model that describes the problem is the following:

$$\max \sum_{(i,j) \in A} p_{ij} x_{ij} - \sum_{i \in N} c_i y_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in A} y_i \leq 4 \tag{2}$$

$$\sum_{(i,j) \in A} x_{ij} = 10 \tag{3}$$

$$\sum_{(i,3) \in A} x_{i3} \geq 4 \tag{4}$$

$$x_{12} \leq 3y_1, \quad x_{12} \leq 3y_2 \tag{5}$$

$$x_{ij} \leq 4y_i, \quad x_{ij} \leq 4y_j, \quad \forall (i,j) \in A \setminus \{(1,2)\} \tag{6}$$

$$y_i \in \{0,1\}, \quad \forall i \in N \tag{7}$$

$$x_{ij} \geq 0, \quad \forall (i,j) \in A \tag{8}$$

The objective function (1) states that we want to maximize the revenue of the company, which is the profit $\sum_{(i,j)\in A} p_{ij}x_{ij}$ minus the fixed costs $\sum_{i\in N} c_i y_i$. The constraint (2) describes the limit of terminals to be opened, which is 4. Constraint (3) establishes the maximum of flights to be activated by the company, while constraint (4) describes rule 2 of the problem, which specifies that the terminal Paris has to receive minimum 4 flights. Constraint (7) sets the y_i to be binary for every $i \in N$ and constraint (8) sets the x_{ij} to be positive for every pair $(i,j) \in A$.

Finally, in order to understand why the constraints number (5) and (6) describe rule 1 of the problem, consider $(i,j) \in A$.

- If $y_i = y_j = 0$ then $x_{ij} = 0$. Equivalently, if terminals i and j are not open, then there are not flights between i and j.
- If $y_i = 0$ and $y_j = 1$ then $x_{ij} \leq 4$ ($x_{ij} \leq 3$ if $(i,j) = (1,2)$) and $x_{ij} = 0$. Therefore, x_{ij} has to be 0. Similarly, if $y_i = 1$ and $y_j = 0$. Equivalently, if either i or j is open and not the other one, then there are not flights between i and j.
- If $y_i = 1$ and $y_j = 1$ then $x_{ij} \leq 4$. This means that if the terminals i and j are open, then x_{ij} cannot be higher than 4 (or 3 if $(i,j) = (1,2)$).

Now, using the input data given in the presentation of the problem, the model that represents the problem is:

$$\begin{aligned} max\ & \overbrace{(7x_{12} + 5x_{13} + 4x_{14} + 7x_{21} + 4x_{24} + 6x_{26} + 5x_{31} + 2x_{34} + 4x_{35} + x_{36} +}^{\text{profit}} \\ & \qquad + 5x_{41} + 6x_{42} + 5x_{43} + 4x_{53} + 4x_{56} + 8x_{63} + 4x_{65}) \\ & \qquad - \underbrace{(4y_1 + 3y_2 + 9y_3 + 7y_4 + 6y_5 + 8y_6)}_{\text{fixed cost}} \end{aligned}$$

subject to

$$\left.\begin{aligned}
x_{12} \leq 3y_1, \quad x_{12} \leq 3y_2\\
x_{13} \leq 4y_1, \quad x_{13} \leq 4y_3\\
x_{14} \leq 4y_1, \quad x_{14} \leq 4y_4\\
x_{21} \leq 4y_2, \quad x_{21} \leq 4y_1\\
x_{24} \leq 4y_2, \quad x_{24} \leq 4y_4\\
x_{26} \leq 4y_2, \quad x_{26} \leq 4y_6\\
x_{31} \leq 4y_3, \quad x_{31} \leq 4y_1\\
x_{34} \leq 4y_3, \quad x_{34} \leq 4y_4\\
x_{35} \leq 4y_3, \quad x_{35} \leq 4y_5\\
x_{36} \leq 4y_3, \quad x_{36} \leq 4y_6\\
x_{41} \leq 4y_4, \quad x_{41} \leq 4y_1\\
x_{42} \leq 4y_4, \quad x_{42} \leq 4y_2\\
x_{43} \leq 4y_4, \quad x_{43} \leq 4y_3\\
x_{53} \leq 4y_5, \quad x_{53} \leq 4y_3\\
x_{56} \leq 4y_5, \quad x_{56} \leq 4y_6\\
x_{63} \leq 4y_6, \quad x_{63} \leq 4y_3\\
x_{65} \leq 4y_6, \quad x_{65} \leq 4y_5
\end{aligned}\right\}$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \leq 4$$

$$\begin{aligned}
x_{12} + x_{13} + x_{14} + x_{21} + x_{24} + x_{26} + x_{31} + x_{34} + x_{35} +\\
+ x_{36} + x_{41} + x_{42} + x_{43} + x_{53} + x_{56} + x_{63} + x_{65} = 10
\end{aligned}$$

$$x_{13} + x_{43} + x_{53} + x_{63} \geq 4$$

$$y_i \in \{0,1\} \quad \forall i \in N, \quad x_{ij} \geq 0, \quad \forall (i,j) \in A.$$

3 Optimal Solution

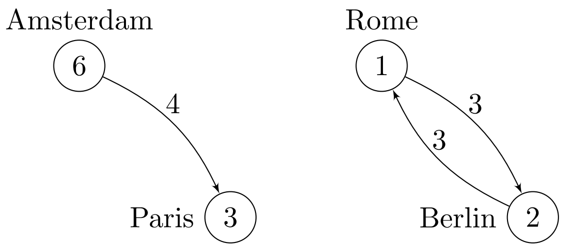

Figure 2: Graph representing the optimal solution.

The optimal solution was obtained implementing the model described in section 2 in AMPL and choosing as solver CPLEX. The terminals that should be open are in Rome, Berlin, Paris and Amsterdam, so

$$y_1 = 1, y_2 = 1, y_3 = 1, y_4 = 0, y_5 = 0, y_6 = 1.$$

The flights activated are Amsterdam-Paris with 4 flights, Berlin- Rome with 3 flights and lastly Rome-Berlin with 3 flights. So the results on the x_{ij} variable are:

$$x_{12} = 3, x_{21} = 3, x_{63} = 4$$

The total revenue is 50 and the results are represented in Figure 2.

4 Modifications

Now, if a tax fee of 1 is added to the flights arriving to or leaving from Paris, the objective function has to be modified. We subtract one unit of profit for each flights arriving or leaving Paris, which is node number 3. We can use the variable x_{ij} since it represents the number of fliths, since the cost of the tax is 1, it is not needed to multiply the variable for the tax cost. The resulting objective functions is stated as:

$$\max \sum_{(i,j)\in A} p_{ij}x_{ij} - \sum_{i\in N} c_i y_i - \sum_{(i,3)\in A} x_{i3} - \sum_{(3,i)\in A} x_{3i}$$

Using the input data given in the presentation of the problem, the objective functions is formulated as follows:

$$\begin{aligned} max\ &\overbrace{(7x_{12} + 5x_{13} + 4x_{14} + 7x_{21} + 4x_{24} + 6x_{26} + 5x_{31} + 2x_{34} + 4x_{35} + x_{36}+ \\ &+5x_{41} + 6x_{42} + 5x_{43} + 4x_{53} + 4x_{56} + 8x_{63} + 4x_{65})}^{\text{profit}} \\ &-(\underbrace{4y_1 + 3y_2 + 9y_3 + 7y_4 + 6y_5 + 8y_6}_{\text{fixed cost}}) \\ &-(\underbrace{x_{13} + x_{43} + x_{53} + x_{63} + x_{31} + x_{34} + x_{35} + x_{36}}_{\text{tax fee of Paris terminal}}) \end{aligned}$$

The constraints to which this optimization problem is subject to are exactly the same to the original constraints mentioned in Section 2.

5 Optimal solution with the modifications

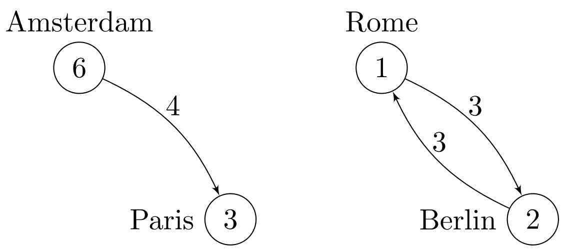

Figure 3: Graph representing the optimal solution with additional constraints.

The modified ILP model was implemented in AMPL and solved with CPLEX. It gives the following optimal solution:

The terminals that should be open do not change and they are in Rome, Berlin, Paris and Amsterdam, so the values of the decision variables y_i are still:

$$y_1 = 1, y_2 = 1, y_3 = 1, y_4 = 0, y_5 = 0, y_6 = 1$$

Also the flights activated do not change and are Amsterdam-Paris with 4 flights, Berlin-Rome with 3 flights and lastly Rome-Berlin with 3 flights. So the results on the x_{ij} variable are, still:

$$x_{12} = 3, x_{21} = 3, x_{63} = 4$$

The total revenue decreases and is now 46. The tax is paid four times since there are just 4 flights arriving to Paris and none starting from Paris. Since there is still the constraint that at least 4 flights have to arrive to Paris, 4 is the minimum amount of tax that the company has to pay. The graph of this optimal solution does not change and it is shown in Figure 3.