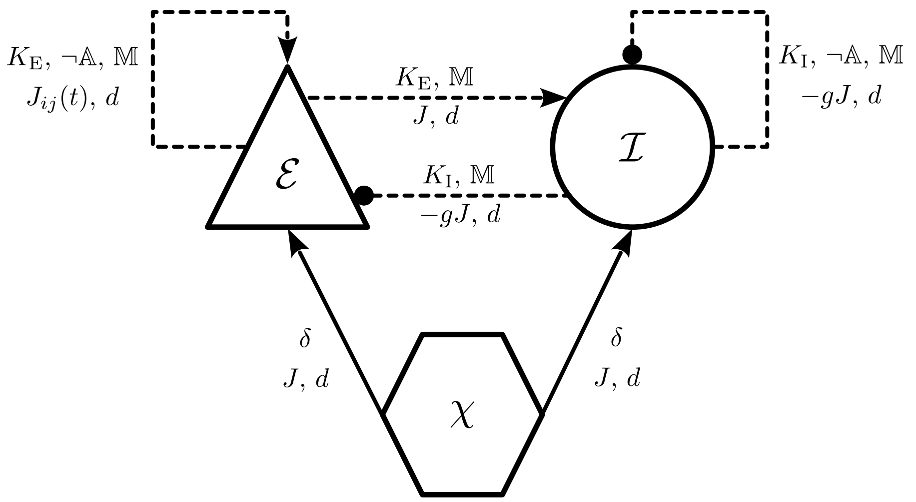
Model description:
TwoPopulationNetworkPlastic

1 Model description

Summary	
Populations	excitatory population $\mathcal{E}$, inhibitory population $\mathcal{I}$, external Poissonian spike sources $\mathcal{X}$
Connectivity	sparse random connectivity respecting Dale's principle
Neurons	leaky integrate-and-fire (LIF)
Synapses	linear input integration with alpha-shaped postsynaptic currents (PSCs), spike-timing dependent plasticity (STDP) for connections between excitatory neurons
Input	stationary, uncorrelated Poissonian spike trains



(see legend)

Populations		
Name	**Elements**	**Size**
$\mathcal{E}$	LIF neurons	$N_{\mathsf{E}} = \beta N$
$\mathcal{I}$	LIF neurons	$N_{\mathsf{I}} = N - N_{\mathsf{E}}$
$\mathcal{X}$	realizations of a Poisson point process	N

Table 1: Description of the network model (continued on next page).

Connectivity		
Source	**Target**	**Pattern**
$\mathcal{E}$	$\mathcal{E}$	• random, independent; homogeneous in-degree $K_{\mathsf{E},i} = K_{\mathsf{E}}$ ($\forall i \in \mathcal{E}$) • plastic synaptic weights $J_{ij}(t)$ ($\forall i \in \mathcal{E}, j \in \mathcal{E}$) • homogeneous spike-transmission delays $d_{ij} = d$ ($\forall i \in \mathcal{E}, j \in \mathcal{E}$)
$\mathcal{E}$	$\mathcal{I}$	• random, independent; homogeneous in-degree $K_{\mathsf{E},i} = K_{\mathsf{E}}$ ($\forall i \in \mathcal{I}$) • fixed synaptic weights $J_{ij} \in \{0, J\}$ ($\forall i \in \mathcal{I}, j \in \mathcal{E}$) • homogeneous spike-transmission delays $d_{ij} = d$ ($\forall i \in \mathcal{I}, j \in \mathcal{E}$)
$\mathcal{I}$	$\mathcal{E} \cup \mathcal{I}$	• random, independent; homogeneous in-degree $K_{\mathsf{I},i} = K_{\mathsf{I}}$ ($\forall i \in \mathcal{E} \cup \mathcal{I}$) • fixed synaptic weights $J_{ij} \in \{-gJ, 0\}$ ($\forall i \in \mathcal{E} \cup \mathcal{I}, j \in \mathcal{I}$) • homogeneous spike-transmission delays $d_{ij} = d$ ($\forall i \in \mathcal{E} \cup \mathcal{I}, j \in \mathcal{I}$)
$\mathcal{X}$	$\mathcal{E} \cup \mathcal{I}$	• one-to-one • fixed synaptic weights $J_{ij} = J$ ($\forall i \in \mathcal{E} \cup \mathcal{I}, j \in \mathcal{X}$) • homogeneous spike-transmission delays $d_{ij} = d$ ($\forall i \in \mathcal{E} \cup \mathcal{I}, j \in \mathcal{X}$)
no self-connections ("autapses"), multiple connections ("multapses") are permitted		
all unmentioned connections $\mathcal{E} \cup \mathcal{I} \rightarrow \mathcal{X}$, $\mathcal{X} \rightarrow \mathcal{X}$ are absent		

Neuron	
Type	leaky integrate-and-fire (LIF) dynamics
Description	dynamics of membrane potential $V_i(t)$ and spiking activity $s_i(t)$ of neuron $i \in \{1, \ldots, N\}$: • emission of kth ($k = 1, 2, \ldots$) spike of neuron i at time t_i^k if $$V_i\left(t_i^k\right) \geq \theta$$ with spike threshold θ • reset and refractoriness: $$\forall k,\ \forall t \in \left(t_k^i,\ t_k^i + \tau_{\mathsf{ref}}\right] : \quad V_i(t) = V_{\mathsf{reset}}$$ with refractory period τ_{ref} and reset potential V_{reset} • spike train $s_i(t) = \sum_k \delta(t - t_i^k)$ • subthreshold dynamics of membrane potential $V_i(t)$: $$\forall k,\ \forall t \notin \left[t_i^k,\ t_i^k + \tau_{\mathsf{ref}}\right) : \quad \tau_{\mathsf{m}} \frac{\mathrm{d}V_i(t)}{\mathrm{d}t} = \left[E_{\mathsf{L}} - V_i(t)\right] + R_{\mathsf{m}} I_i(t)$$ with membrane time constant τ_{m}, membrane resistance R_{m}, resting potential E_{L}, and total synaptic input current $I_i(t)$

Table 1: Description of the network model (continued).

Synapse: transmission	
Type	current-based synapses with alpha-function shaped postsynaptic currents (PSCs)
Description	• total synaptic input current of neuron i $$I_i(t) = I_{\mathsf{E},i}(t) + I_{\mathsf{I},i}(t) + I_{\mathsf{X},i}(t)$$ • excitatory, inhibitory and external synaptic input currents $$I_{\mathsf{E},i}(t) = \sum_{j \in \mathcal{E}} \big(\mathsf{PSC}_{ij} * s_j\big)(t - d_{ij})$$ $$I_{\mathsf{I},i}(t) = \sum_{j \in \mathcal{I}} \big(\mathsf{PSC}_{ij} * s_j\big)(t - d_{ij})$$ $$I_{\mathsf{X},i}(t) = \sum_{j \in \mathcal{X}} \big(\mathsf{PSC}_{ij} * s_j\big)(t - d_{ij})$$ with spike trains $s_j(t)$ of local ($j \in \mathcal{E} \cup \mathcal{I}$) and external sources ($j \in \mathcal{X}$), spike transmission delays d_{ij}, and convolution operator "$*$": $\big(f * g\big)(t) = \int_{-\infty}^{\infty} \mathrm{d}s\, f(s) g(t-s)\big)$ • alpha-function shaped postsynaptic currents $$\mathsf{PSC}_{ij}(t) = \hat{I}_{ij} e \tau_{\mathsf{s}}^{-1} t e^{-t/\tau_{\mathsf{s}}} \Theta(t)$$ with synaptic time constant τ_{s} and Heaviside function $\Theta(\cdot)$ $\curvearrowright$ postsynaptic potential triggered by a single presynaptic spike $$\mathsf{PSP}_{ij}(t) = \hat{I}_{ij} \frac{e}{\tau_{\mathsf{s}} C_{\mathsf{m}}} \left(\frac{1}{\tau_{\mathsf{m}}} - \frac{1}{\tau_{\mathsf{s}}}\right)^{-2} \left(\left(\frac{1}{\tau_{\mathsf{m}}} - \frac{1}{\tau_{\mathsf{s}}}\right) t e^{-t/\tau_{\mathsf{s}}} - e^{-t/\tau_{\mathsf{s}}} + e^{-t/\tau_{\mathsf{m}}}\right) \Theta(t)$$ • PSC amplitude (synaptic weight) $$\hat{I}_{ij} = \mathsf{max}_t\big(\mathsf{PSC}_{ij}(t)\big) = \frac{J_{ij}}{J_{\mathsf{unit}}(\tau_{\mathsf{m}}, \tau_{\mathsf{s}}, C_{\mathsf{m}})}$$ parameterized by PSP amplitude $J_{ij} = \mathsf{max}_t\big(\mathsf{PSP}_{ij}(t)\big)$ with unit PSP amplitude (PSP amplitude for $\hat{I}_{ij} = 1$): $$J_{\mathsf{unit}}(\tau_{\mathsf{m}}, \tau_{\mathsf{s}}, C_{\mathsf{m}}) = \frac{e}{C_{\mathsf{m}}\left(1 - \frac{\tau_{\mathsf{s}}}{\tau_{\mathsf{m}}}\right)} \left(\frac{e^{-t_{\mathsf{max}}/\tau_{\mathsf{m}}} - e^{-t_{\mathsf{max}}/\tau_{\mathsf{s}}}}{\frac{1}{\tau_{\mathsf{s}}} - \frac{1}{\tau_{\mathsf{m}}}} - t_{\mathsf{max}} e^{-t_{\mathsf{max}}/\tau_{\mathsf{s}}}\right),$$ time to PSP maximum $$t_{\mathsf{max}} = \frac{1}{\frac{1}{\tau_{\mathsf{s}}} - \frac{1}{\tau_{\mathsf{m}}}} \left(-W_{-1}\left(\frac{-\tau_{\mathsf{s}} e^{-\frac{\tau_{\mathsf{s}}}{\tau_{\mathsf{m}}}}}{\tau_{\mathsf{m}}}\right) - \frac{\tau_{\mathsf{s}}}{\tau_{\mathsf{m}}}\right),$$ and Lambert-W function $W_{-1}(x)$ for $x \geq -1/e$

Table 1: Description of the network model (continued).

Synapse: plasticity	
Type	spike-timing dependent plasticity (STDP) with power-law weight dependence and all-to-all spike pairing scheme (Morrison et al., 2007) for connections between excitatory neurons
Description	dynamics of synaptic weights $J_{ij}(t)\ \forall i \in \mathcal{E}, j \in \mathcal{E}$: $\forall J_{ij} \geq 0:$ $$\frac{\mathrm{d}J_{ij}}{\mathrm{d}t} = \lambda^+ f^+(J_{ij}) \sum_k x_j^+(t)\delta\Big(t - [t_i^k + d_{ij}]\Big) + \lambda^- f^-(J_{ij}) \sum_l x_i^-(t)\delta\Big(t - [t_j^l - d_{ij}]\Big)$$ $\forall \{t \mid J_{ij}(t) < 0\}: \quad J_{ij}(t) = 0 \quad$ (clipping) with • pre- and postsynaptic spike times $\{t_j^l \mid l = 1, 2, \ldots\}$ and $\{t_i^k \mid k = 1, 2, \ldots\}$, • magnitude $\lambda^+ = \lambda$ of weight update for causal firing (postsynaptic spike following presynaptic spikes: $t_i^k > t_j^l$), • magnitude $\lambda^- = -\alpha\lambda$ of weight update for acausal firing (presynaptic spike following postsynaptic spikes: $t_i^k < t_j^l$), • power-law weight dependence $f^+(J_{ij}) = J_0 (J_{ij}/J_0)^{\mu^+}$ of weight update for causal firing with exponent μ^+ and reference weight J_0, • linear weight dependence $f^-(J_{ij}) = J_{ij}$ of weight update for acausal firing, • (dendritic) delay d_{ij}, • spike trace $x_j^+(t)$ of presynaptic neuron j, evolving according to $$\frac{\mathrm{d}x_j^+}{\mathrm{d}t} = -\frac{x_j^+(t)}{\tau^+} + \sum_l \delta(t - t_j^l)$$ with presynaptic spike times $\{t_j^l \mid l = 1, 2, \ldots\}$ and time constant τ^+, • spike trace $x_i^-(t)$ of postsynaptic neuron i, evolving according to $$\frac{\mathrm{d}x_i^-}{\mathrm{d}t} = -\frac{x_i^-(t)}{\tau^-} + \sum_k \delta(t - t_i^k)$$ with postsynaptic spike times $\{t_i^k \mid k = 1, 2, \ldots\}$ and time constant τ^- Note: The above weight update accounts for *all* pairs of pre- and postsynaptic spikes (all-to-all spike pairing scheme). The spike histories and the dependence of the weight update on the time lag of pre- and postsynaptic firing are fully captured by the spike traces $x_j^+(t)$ and $x_i^-(t)$.

Table 1: Description of the network model (continued).

Stimulus	
Type	stationary, uncorrelated Poisson spike trains
Description	$N = \|\mathcal{X}\|$ independent realizations $s_i(t)$ ($i \in \mathcal{X}$) of a Poisson point process with constant rate $\nu_{\mathsf{X}}(t) = \eta\nu_\theta$, where $$\nu_\theta = \frac{\theta - E_{\mathsf{L}}}{R_{\mathsf{m}}\hat{I}_X e\tau_{\mathsf{s}}}$$ denotes the rheobase rate, and η and $\hat{I}_X = J/J_{\mathsf{unit}}$ the relative rate and the synaptic weight (PSC amplitude) of external sources

Initial conditions	
Type	random initial membrane potentials, homogeneous initial synaptic weights and spike traces
Description	• membrane potentials: $V_i(t=0) \sim \mathcal{U}(V_{0,\mathsf{min}}, V_{0,\mathsf{max}})$ randomly and independently drawn from a uniform distribution between $V_{0,\mathsf{min}}$ and $V_{0,\mathsf{max}}$ ($\forall i$) • synaptic weights: $\hat{I}_{ij}(t=0) = J/J_{\mathsf{unit}}$ ($\forall i \in \mathcal{E}, j \in \mathcal{E}$) • spike traces: $x_{+,i}(t=0) = x_{-,i}(t=0) = 0$ ($\forall i \in \mathcal{E}$)

Table 1: Description of the network model (continued).

2 Model parameters

Network and connectivity		
Name	**Value**	**Description**
N	12500	total number of neurons in local network
β	0.8	relative number of excitatory neurons
N_{E}	$\beta N = 10000$	total number of excitatory neurons
N_{I}	$N - N_{\mathsf{E}} = 2500$	total number of inhibitory neurons
K	1250	total number of inputs per neuron (in-degree) from local network
K_{E}	$\beta K = 1000$	number of excitatory inputs per neuron (exc. in-degree) from local network
K_{I}	$K - K_{\mathsf{E}} = 250$	number of inhibitory inputs per neuron (inh. in-degree)
Neuron		
Name	**Value**	**Description**
θ	$20\,\mathrm{mV}$	spike threshold
E_{L}	$0\,\mathrm{mV}$	resting potential
τ_{m}	$20\,\mathrm{ms}$	membrane time constant
C_{m}	$250\,\mathrm{pF}$	membrane capacitance
R_{m}	$\tau_{\mathsf{m}}/C_{\mathsf{m}} = 80\,\mathrm{M\Omega}$	membrane resistance
V_{reset}	$0\,\mathrm{mV}$	reset potential
τ_{ref}	$2\,\mathrm{ms}$	absolute refractory period
Synapse		
Name	**Value**	**Description**
J	0.5 mV	(initial) weight (PSP amplitude) of excitatory synapses
g	10	relative strength of inhibitory synapses
J_{I}	$-gJ = -5$ mV	weight (PSP amplitude) of inhibitory synapses
J_{unit}	≈ 0.01567 mV/ pA	unit PSP amplitude
$\hat{I}_{\mathsf{E}}(0)$	$J/J_{\mathsf{unit}} \approx 31.9$ pA	(initial) weight (PSC amplitude) of excitatory synapses
$\hat{I}_{\mathsf{I}}$	$-gJ/J_{\mathsf{unit}} \approx -319$ pA	weight (PSC amplitude) of inhibitory synapses
$\hat{I}_{\mathsf{X}}$	$J/J_{\mathsf{unit}} \approx 31.9$ pA	weight (PSC amplitude) of external inputs
d	1.5 ms	spike transmission delay
τ_{s}	2 ms	synaptic time constant
$\lambda = \lambda^{+}$	20	magnitude of weight update for causal firing
μ^{+}	0.4	weight dependence exponent for causal firing
J_0	1 pA	reference weight
τ^{+}	15 ms	time constant of weight update for causal firing
α	0.1	relative magnitude of weight update for acausal firing
λ^{-}	$-\alpha\lambda = -2$	magnitude of weight update for acausal firing
τ^{-}	30 ms	time constant of weight update for acausal firing
Stimulus		
Name	**Value**	**Description**
η	1.2	relative rate of external Poissonian sources
ν_{θ}	$1442\,\mathrm{spikes/s}$	rheobase rate
ν_{X}	$\eta\nu_{\theta} \approx 1730\,\mathrm{spikes/s}$	rate of external Poissonian sources
Initial conditions		
Name	**Value**	**Description**
$V_{0,\mathsf{min}}$	$E_{\mathsf{L}} = 0$ mV	minimum initial membrane potential
$V_{0,\mathsf{max}}$	$\theta = 20$ mV	maximum initial membrane potential

Table 2: Model parameters. Parameters derived from other parameters are marked in gray.

References

Morrison, A., Aertsen, A., and Diesmann, M. (2007). Spike-timing-dependent plasticity in balanced random networks. *Neural Computation*, 19(6):1437–1467.