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Øving 1

Diskret Matematikk

TMA4140

Oppgåver til seksjon 1.1

Oppgåve 12cf

La p, q og r vera proposisjonane

p : You have the flu.
q : You miss the final examination.
r : You pass the course.

Proposisjonar uttrykte som engelske setningar:

$q \to \neg r$: If you have the flu, you will not pass the course.
$(p \wedge q) \vee (\neg q \wedge r)$: You have the flu and you miss the final examination, or you do not miss the final examination and you pass the course.

Oppgåve 14

La p, q og r vera proposisjonane

p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

Proposisjonar uttrykte som logiske uttrykk:

a: You get an A in this class, but you do not do every exercise in this book.

$$r \wedge \neg q$$

e: Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$$(p \wedge q) \to r$$

Oppgåver til seksjon 1.3

Oppgåve 10

Skal vise at uttrykka er tautologiar.

a: $[\neg p \wedge (p \vee q)] \to q$

p	q	$\neg p$	$a : p \vee q$	$b : \neg p \wedge a$	$b \to q$
0	0	1	0	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	1	0	1	0	1

b: $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$a : p \to q$	$b : q \to r$	$c : a \wedge b$	$d : p \to r$	$c \to d$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

c: $[p \wedge (p \to q)] \to q$

p	q	$a : p \to q$	$b : p \wedge a$	$b \to q$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

d: $[(p \wedge q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$a : p \wedge q$	$b : p \to r$	$c : q \to r$	$d : a \wedge b \wedge c$	$d \to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	0	1	0	0	1
0	1	1	0	1	1	0	1
1	0	0	0	0	1	0	1
1	0	1	0	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Oppgåver til seksjon 1.4

Oppgåve 24

d: All students in your class can solve quadratic equations.

$P(x)$: student x kan løyse kvadratiske likningar
$Q(x)$: student x er i klassa

$x \in U = \{\text{alle elevar i klassa}\}$
$\forall x P(x)$
$x \in U = \{\text{alle mennesker}\}$
$\forall x(Q(x) \to P(x))$

e: Some students in your class does not want to be rich.

$P(x)$: student x har lyst å bli rik
$Q(x)$: student x er i klassa

$x \in U = \{\text{alle elevar i klassa}\}$
$\exists x P(x)$
$x \in U = \{\text{alle mennesker}\}$
$\exists x(Q(x) \to P(x))$

Oppgåver til seksjon 1.5

Oppgåve 12

$I(x)$: x has an Internet connection
$C(x, y)$: x and y have chatted over the Internet
$x \in U = \{\text{elevar i klassa}\}$

b: Rachel has not chatted over the Internet with Chelsea.

$$\exists x \exists y \neg C(x, y)$$

I klassa *eksisterer* ein person x, nemleg Rachel, og der *eksisterer* ein person y, Chelsea, slik at C *ikkje* held.

e: Sanjay has chatted with everyone except Joseph.

$$\exists x \exists y \neg C(x, y)$$

I klassa *eksisterer* ein person x, Sanjay, og der *eksisterer* ein person y, Joseph, slik at C *ikkje* held.

Oppgåve 30

c: $\neg\exists y(Q(y) \wedge \forall x \neg R(x,y))$

$$\begin{aligned}
&\neg\exists y(Q(y) \wedge \forall x \neg R(x,y)) \\
=\ &\forall y \neg(Q(y) \wedge \forall x \neg R(x,y)) \\
=\ &\forall y(\neg Q(y)) \vee (\neg\forall x \neg R(x,y)) \\
=\ &\forall y \neg Q(y) \vee \exists x R(x,y)
\end{aligned}$$

e: $\neg\exists y(\forall x \exists z T(x,y,z) \vee \exists x \forall z U(x,y,z))$

$$\begin{aligned}
&\neg\exists y(\forall x \exists z T(x,y,z) \vee \exists x \forall z U(x,y,z)) \\
=\ &\forall y \neg(\forall x \exists z T(x,y,z) \vee \exists x \forall z U(x,y,z)) \\
=\ &\forall y(\neg\forall x \exists z T(x,y,z)) \wedge (\neg\exists x \forall z U(z,y,z)) \\
=\ &\forall y(\exists x \forall z \neg T(x,y,z)) \wedge (\forall x \exists z \neg U(x,y,z))
\end{aligned}$$