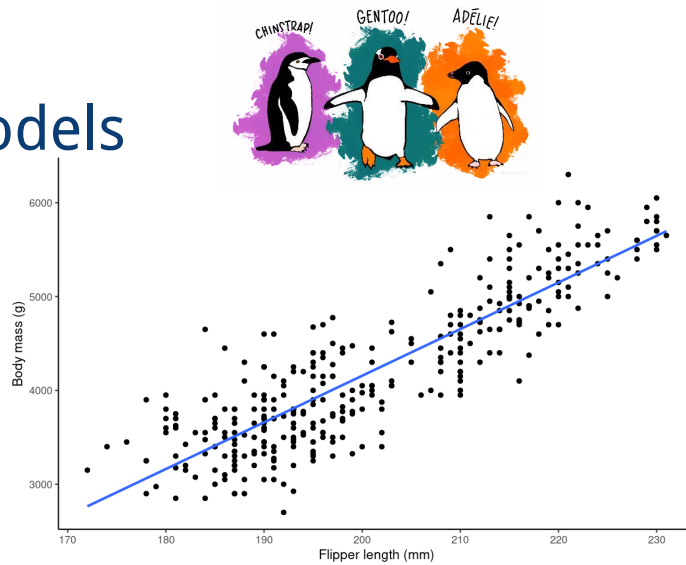


# Linear Models

Regressions,  
ANOVAs, and  
Model assumptions

Artwork by [@allison\\_horst](#)



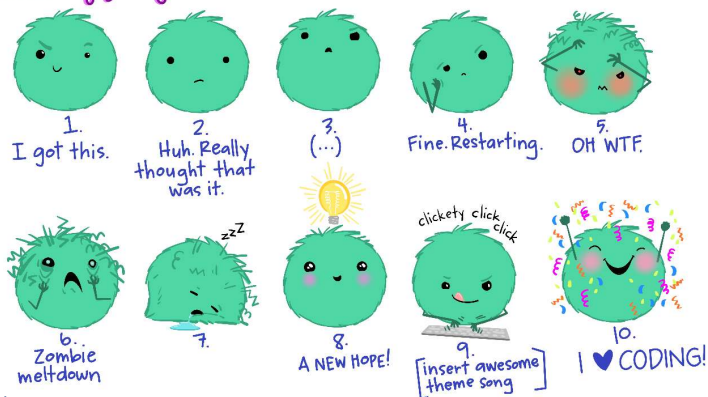
## Getting started (again)

Open RStudio  
Open your NRI project  
Open a **new** script for today:  
File > New File > R Script

Make sure to load packages at the top:  
`library(tidyverse)`  
`library(palmerpenguins)`

## How Are we Doing?

*debugging*



Artwork by [@allison\\_horst](#)

[@allison\\_horst](#)

3 / 58

# Linear Models

## Linear Models

### Running models in R

```
lm(y ~ x1 + x2, data = data)
```

- **y** is the **response** variable (**dependent**)
- **x** are the **explanatory** variables (**independent, predictor**)

Here we're assuming a **continuous y**

5 / 58

## Linear Models

### Running models in R

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- **x** are the **explanatory** variables (**independent, predictor**)

Here we're assuming a **continuous y**

### Different types of models

- If we only have one **x** which is continuous, this is a **simple linear regression**
- If both **x** are continuous, this is a **multiple linear regression**
- If both **x** are categorical, this is an **ANOVA**
- If **x1** is continuous and **x2** is categorical, this is an **ANCOVA**

5 / 58

# Linear Models

## Running models in R

```
lm(y ~ x1 + x2, data = data)
```

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- **x** are the **explanatory** variables (**independent**, **predictor**)

Here we're assuming a **continuous y**

## Different types of models

- If we only have one **x** which is continuous, this is a **simple linear regression**
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- If both **x** are categorical, this is an **ANOVA**
- If **x1** is continuous and **x2** is categorical, this is an **ANCOVA**

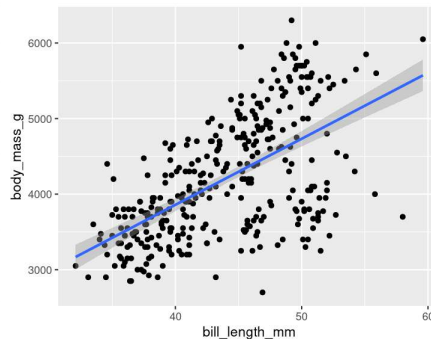
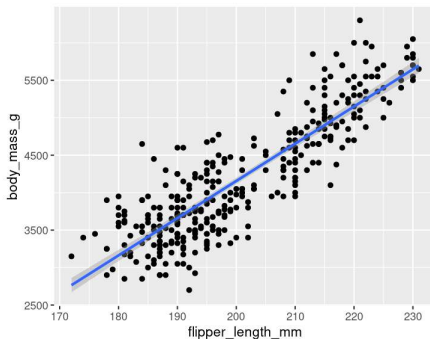
R will figure it out for you

5 / 58

# Regressions

## Real example

- Is penguin body mass a function of skeletal size?
- Can it be predicted by flipper length and bill length?



6 / 58

# Regressions

## Real example

```
lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
##  
## Call:  
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,  
##     data = penguins)  
##  
## Coefficients:  
##      (Intercept)  flipper_length_mm  bill_length_mm  
##      -5736.897         48.145         6.047
```

As we have **two continuous** predictors, this is technically a *multiple* regression

7 / 58

# Regressions

## Real example

```
lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
##  
## Call:  
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,  
##     data = penguins)  
##  
## Coefficients:  
##      (Intercept)  flipper_length_mm  bill_length_mm  
##      -5736.897         48.145         6.047
```

Hmm, not a lot of detail...  
Only **Intercept** and Slopes  
(**flipper\_length\_mm** and **bill\_length\_mm**)

As we have **two** predictors, this is technically a *multiple* regression

8 / 58

# Regressions

## Assign model to **m** (or any other name you want to give it)

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

**m** is a model object

```
class(m)
```

```
## [1] "lm"
```

This contains all the information about the model

9 / 58

# Regressions

Use **summary()** to show summary table:

```
summary(m)  
##  
## Call:  
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,  
##     data = penguins)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -1090.5  -285.7   -32.1    244.2   1287.5   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept)  -5736.897    307.959  -18.629  <2e-16 ***  
## flipper_length_mm  48.145      2.011   23.939  <2e-16 ***  
## bill_length_mm    6.047       5.180    1.168    0.244   
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 394.1 on 339 degrees of freedom  
## (2 observations deleted due to missingness)  
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585   
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

10 / 58

# Regressions

Use `summary()` to show summary table:

summary(m)

```
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1099.5   -385.7    -32.1     344.3    1287.5
##
## Wait!
## Shouldn't interpret until we know the
## model is solid
##
## (Standardized residuals. Not all are displayed because there are
## more than 20.)
##      Min       1Q   Median       3Q      Max
## -2e-16    2e-16    2e-16    2e-16    2e-16
##
## bill_length_mm      6.047      5.180      1.168      0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 394.1 on 339 degrees of freedom
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## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

10 / 58

# Model Diagnostics

## Model Assumptions

- Normality (of residuals)
- Constant Variance (no heteroscedasticity)

## Other cautions

- Influential observations (Cook's D)
- Multiple collinearity (with more than one **x** or explanatory variables)

There are other assumptions (independence, etc.) but they reflect experimental design, not patterns in the data

11 / 58

# Model Diagnostics

First let's get our relevant variables into a diagnostic data frame:

- **residuals** (regular and standardized)
- **fitted values**
- **cooks distance**
- **obs number**

```
d <- data.frame(residuals = residuals(m),
  std_residuals = rstudent(m),
  fitted = fitted(m),
  cooks = cooks.distance(m))

d <- mutate(d, obs = 1:n())
```

Remember! `n()` only works *inside* `summarize()/mutate()`

12 / 58

# Model Diagnostics

First let's get our relevant variables into a diagnostic data frame:

- **residuals** (regular and standardized)
- **fitted values**
- **cooks distance**
- **obs number**

```
d <- data.frame(residuals = residuals(m),
               std_residuals = rstudent(m),
               fitted = fitted(m),
               cooks = cooks.distance(m))
d <- mutate(d, obs = 1:n())
```

```
head(d)
```

##	residuals	std_residuals	fitted	cooks	obs
## 1	536.220898	1.368529806	3213.779	5.539153e-03	1
## 2	343.077607	0.873050231	3456.922	1.609402e-03	2
## 3	-645.064115	-1.644516798	3895.064	3.797384e-03	3
## 4	-327.003441	-0.833002736	3777.003	1.992863e-03	4
## 5	1.707668	0.004338503	3648.292	3.338060e-08	5
## 6	412.430396	1.051400111	3212.570	3.272886e-03	6

Remember! `n()` only works *inside* `summarize()`/`mutate()`

12 / 58

## Side Note: tidyverse functions

- From **dplyr** package (part of **tidyverse**)

```
d <- mutate(d, obs = 1:n())
```

### `mutate()`

- **tidyverse** functions always start with the **data**, followed by **other arguments**
- `mutate()` adds **new columns** to your data
- Also note: `1:5` is the same as `c(1,2,3,4,5)`

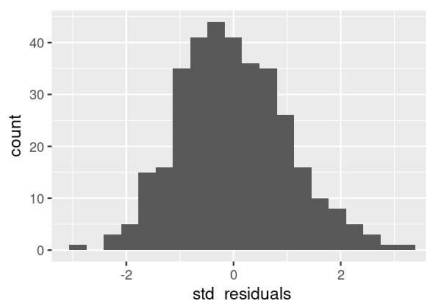
Remember! `n()` only works *inside* `summarize()`/`mutate()`

13 / 58

## Normality

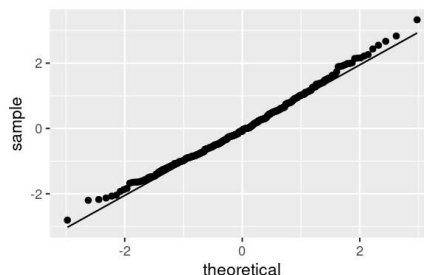
### Histogram of residuals

```
ggplot(data = d, aes(x = std_residuals)) +
  geom_histogram(bins = 20)
```



### QQ Normality plot of residuals

```
ggplot(data = d, aes(sample = std_residuals)) +
  stat_qq() +
  stat_qq_line()
```

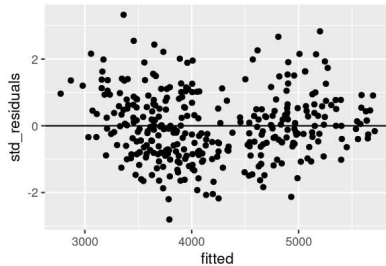


14 / 58

## Variance and Influence

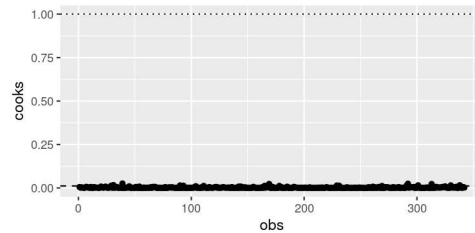
### Check heteroscedasticity

```
ggplot(d, aes(x = fitted, y = std_residuals)) +  
  geom_point() +  
  geom_hline(yintercept = 0)
```



### Cook's D

```
ggplot(d, aes(x = obs, y = cooks)) +  
  geom_point() +  
  geom_hline(yintercept = 1, linetype = "dotted") +  
  geom_hline(yintercept = 4/nrow(penguins),  
             linetype = "dashed")
```

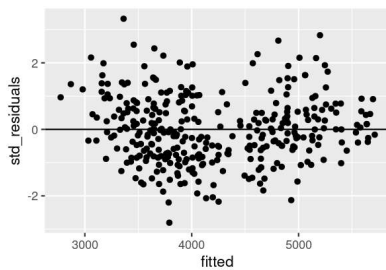


15 / 58

## Variance and Influence

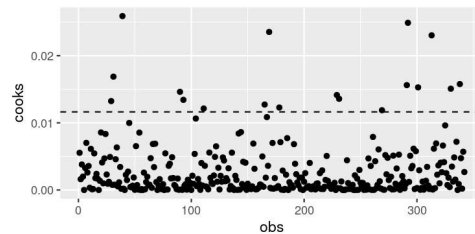
### Check heteroscedasticity

```
ggplot(d, aes(x = fitted, y = std_residuals)) +  
  geom_point() +  
  geom_hline(yintercept = 0)
```



### Cook's D

```
ggplot(d, aes(x = obs, y = cooks)) +  
  geom_point() +  
  geom_hline(yintercept = 4/nrow(penguins),  
             linetype = "dashed")
```

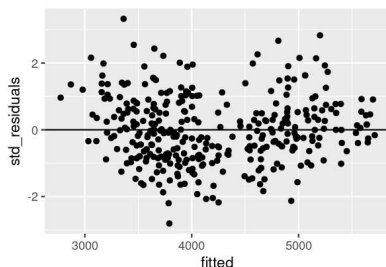


16 / 58

## Variance and Influence

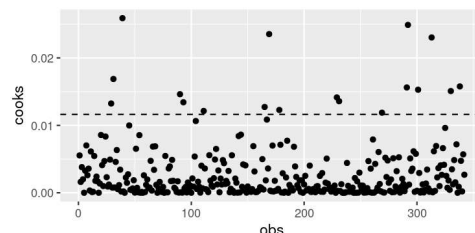
### Check heteroscedasticity

```
ggplot(d, aes(x = fitted, y = std_residuals)) +  
  geom_point() +  
  geom_hline(yintercept = 0)
```



### Cook's D

```
ggplot(d, aes(x = obs, y = cooks)) +  
  geom_point() +  
  geom_hline(yintercept = 4/nrow(penguins),  
             linetype = "dashed")
```

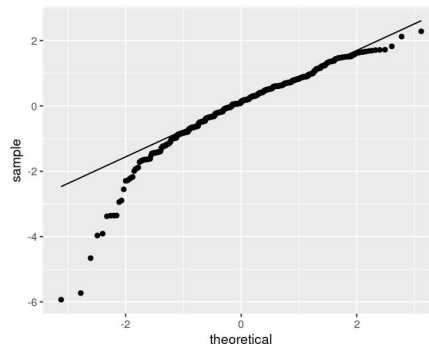
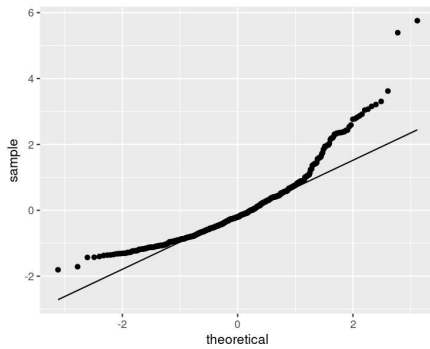


Pretty good!

16 / 58

## What is a 'Good' Normality Plot?

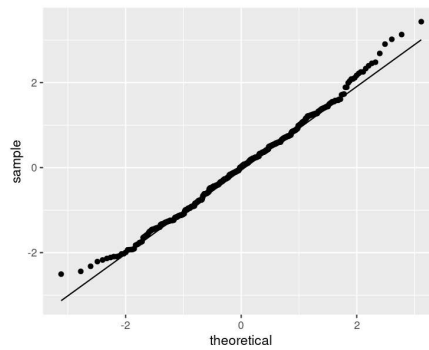
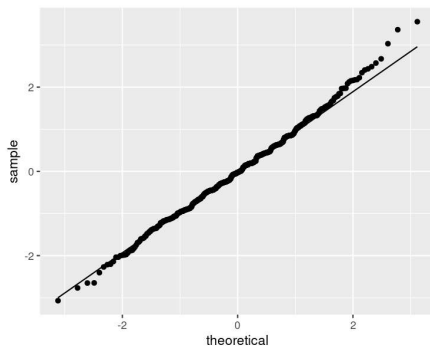
### Problematic



17 / 58

## What is a 'Good' Normality Plot?

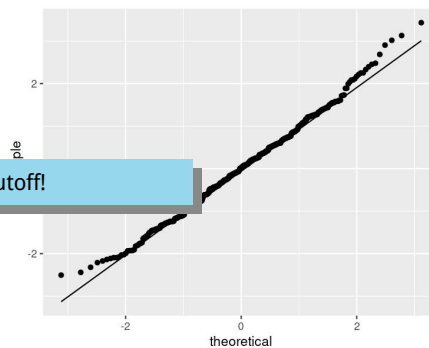
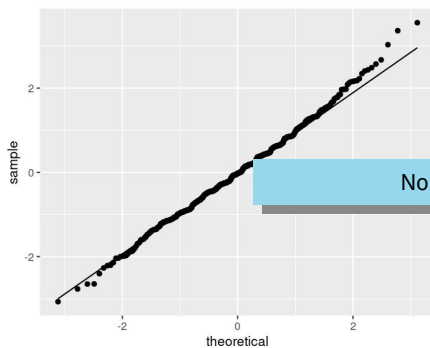
### Good



18 / 58

## What is a 'Good' Normality Plot?

### Good



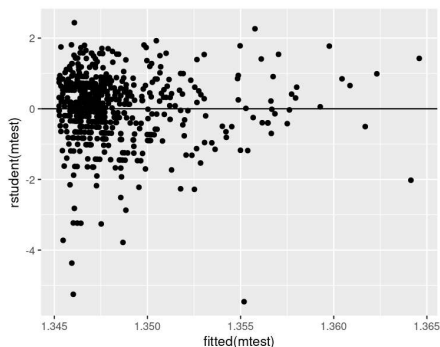
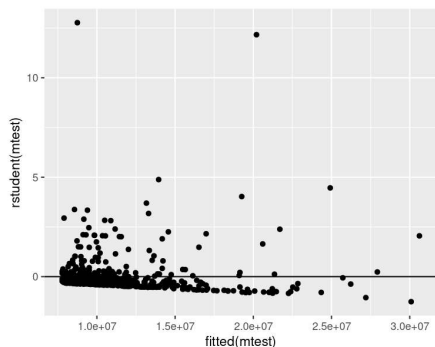
No clear cutoff!

18 / 58



## What is a 'Good' Heteroscedasticity Plot?

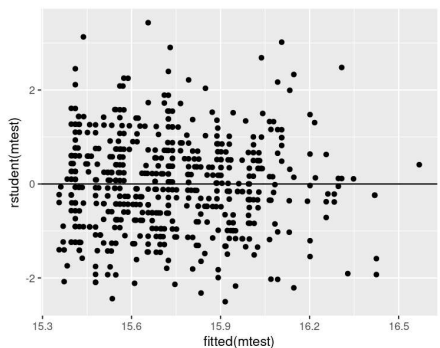
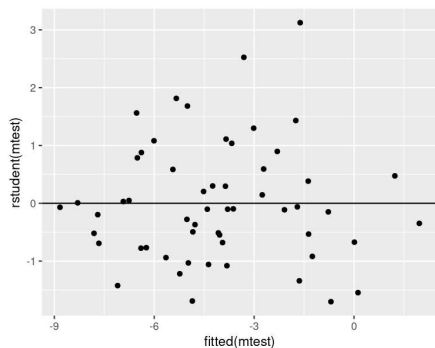
### Problematic



19 / 58

## What is a 'Good' Heteroscedasticity Plot?

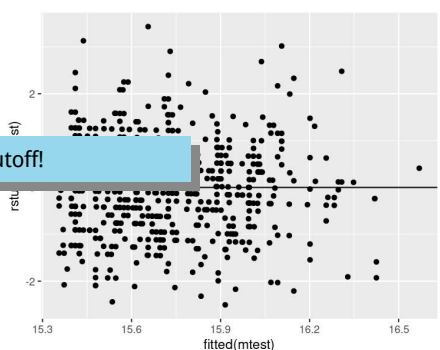
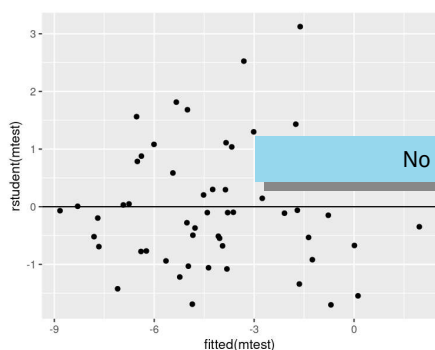
### Good



20 / 58

## What is a 'Good' Heteroscedasticity Plot?

### Good



No clear cutoff!

20 / 58

## Multicollinearity (collinearity)

- Only relevant with **more than one explanatory variable**
- Sometimes explanatory variables are so correlated they interfere with the model

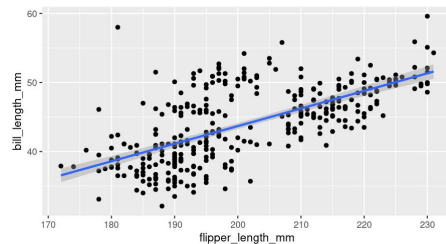
21 / 58

## Multicollinearity (collinearity)

- Only relevant with **more than one explanatory variable**
- Sometimes explanatory variables are so correlated they interfere with the model

### Look at our two explanatory variables

```
ggplot(data = penguins, aes(x = flipper_length_mm, y = bill_length_mm)) +  
  geom_point() +  
  stat_smooth(method = "lm")
```



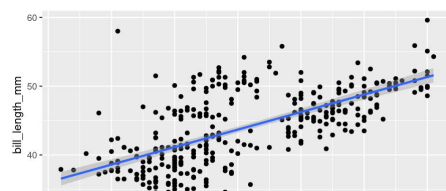
21 / 58

## Multicollinearity (collinearity)

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```
ggplot(data = penguins, aes(x = flipper_length_mm, y = bill_length_mm)) +  
  geom_point() +  
  stat_smooth(method = "lm")
```



Correlated, but not necessarily a problem

21 / 58

## Multicollinearity (collinearity)

- Only relevant with **more than one explanatory variable**
- Sometimes explanatory variables are so correlated they interfere with the model
- Correlations between variables *might* be problematic (but not necessarily)

Use **vif()** function from **car** package (vif = variance inflation factor\*)

```
library(car)
vif(m)
```

```
## flipper_length_mm  bill_length_mm
##           1.756154           1.756154
```

Hmm, that's pretty good (looking for < 10)

\* Can be interpreted as how much influence the variable has on the model

22 / 58

## Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5  -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897     307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145       2.011   23.939  <2e-16 ***
## bill_length_mm     6.047       5.180    1.168    0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 394.1 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

23 / 58

## Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

### Model

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
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##
## Residuals:
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## -1090.5  -285.7   -32.1    244.2   1287.5
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```

24 / 58

## Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

### Effects

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5   -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145      2.011   23.939  <2e-16 ***
## bill_length_mm      6.047      5.180    1.168    0.244
## ---
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```

25 / 58

## Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

### Missing observations

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5   -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145      2.011   23.939  <2e-16 ***
## bill_length_mm      6.047      5.180    1.168    0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

26 / 58

## Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

### $R^2$ and adjusted $R^2$

- Adjusted for the number of parameters

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5   -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145      2.011   23.939  <2e-16 ***
## bill_length_mm      6.047      5.180    1.168    0.244
## ---
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##
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## (2 observations deleted due to missingness)
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

27 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

- **Estimate**
  - Slope of the effect
- **Std. Error**
  - Variability in the estimates
- **t value**
  - Test statistic
  - Think of it as a holistic combination of estimate and variability
- **Pr(>|t|)**
  - **P-value**, significance of the results
  - Probability of getting **t-value** by chance

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5   -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145       2.011   23.939  <2e-16 ***
## bill_length_mm     6.047       5.180    1.168    0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 394.1 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

28 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

### Intercept

- Significant ( $P < 2e^{-16}$ )
- Penguins with a flipper length of 0 mm are predicted to have a body mass of -5736.9g
  - Not useful!

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5   -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145       2.011   23.939  <2e-16 ***
## bill_length_mm     6.047       5.180    1.168    0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## Residual standard error: 394.1 on 339 degrees of freedom
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## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

29 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

### Effect of Flipper Length

- Significant ( $P < 2e^{-16}$ )
- For each 1 mm increase in flipper length, body mass is predicted to increase by 48.14g

```
##
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
##     data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5   -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm    48.145       2.011   23.939  <2e-16 ***
## bill_length_mm     6.047       5.180    1.168    0.244
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## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## Residual standard error: 394.1 on 339 degrees of freedom
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## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

30 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

### Effect of Flipper Length

- Significant ( $P < 2e^{-16}$ )
- For each 1 mm increase in flipper length, body mass is predicted to increase by 48.14g

### Effect of Bill Length

- Non-significant ( $P = 0.244$ , i.e.  $P < 0.05$ )
- Therefore no effect (in this model) (and no interpretation of estimate)

\* $2e^{-16} = 0.0000000000000002$ , R uses this as the smallest number

```
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
## data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5  -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm  48.145       2.011   23.939  <2e-16 ***
## bill_length_mm    6.047       5.180    1.168    0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 394.1 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

31 / 58

# Interpreting Regressions

```
m <- lm(body
```

```
summary(m)
```

## Specific Details

### Effect of Flipper Length

- Significant ( $P < 2e^{-16}$ )
- For each 1 mm increase in flipper length, body mass is predicted to increase by 48.14g

### Effect of Bill Length

- Non-significant ( $P = 0.244$ , i.e.  $P < 0.05$ )
- Therefore no effect (in this model) (and no interpretation of estimate)

\* $2e^{-16} = 0.0000000000000002$ , R uses this as the smallest number

**Therefore**  
There is a significant relationship between flipper length and body mass  
But not between bill length and body mass (when including flipper length)

```
## data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5  -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm  48.145       2.011   23.939  <2e-16 ***
## bill_length_mm    6.047       5.180    1.168    0.244
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 394.1 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

31 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

### Effect of Flipper Length

- Significant ( $P < 2e^{-16}$ )
- For each 1 mm increase in flipper length, body mass is predicted to increase by 48.14g

### Effect of Bill Length

- Non-significant ( $P = 0.244$ , i.e.  $P < 0.05$ )
- Therefore no effect (and no interpretation of estimate)

\* $2e^{-16} = 0.0000000000000002$ , R uses this as the smallest number

```
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
## data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1090.5  -285.7   -32.1    244.2   1287.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5736.897    307.959  -18.629  <2e-16 ***
## flipper_length_mm  48.145       2.011   23.939  <2e-16 ***
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## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 394.1 on 339 degrees of freedom
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## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

32 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

### Effect of Flipper Length

- Significant ( $P < 2e^{-16}$ )
- For each 1 mm increase in flipper length, body mass is predicted to increase by 48.14g

### Effect of Bill Length

- Non-significant ( $P = 0.244$ , i.e.  $P < 0.05$ )
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```
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
## data = penguins)
## Residuals:
## Min      1Q  Med      3Q      Max
## -1090.5  -285.7   -32.1    244.2   1287.5
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## (2 observations deleted due to missingness)
## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

32 / 58

# Interpreting Regressions

```
m <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
```

```
summary(m)
```

## Specific Details

### Effect of Flipper Length

- Significant ( $P < 2e^{-16}$ )
- For each 1 mm increase in flipper length, body mass is predicted to increase by 48.14g

### Effect of Bill Length

- Non-significant ( $P = 0.244$ , i.e.  $P < 0.05$ )
- Therefore no effect (and no interpretation of estimate)

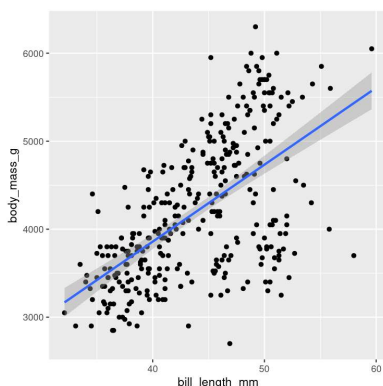
\* $2e^{-16} = 0.0000000000000002$ , R uses this as the smallest number

```
## Call:
## lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
## data = penguins)
## Residuals:
## Min      1Q  Med      3Q      Max
## -1090.5  -285.7   -32.1    244.2   1287.5
## Coefficients:
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## (Intercept)  -5736.897    307.959  -18.629  <2e-16 ***
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## Multiple R-squared:  0.76,    Adjusted R-squared:  0.7585
## F-statistic: 536.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

32 / 58

## Extra

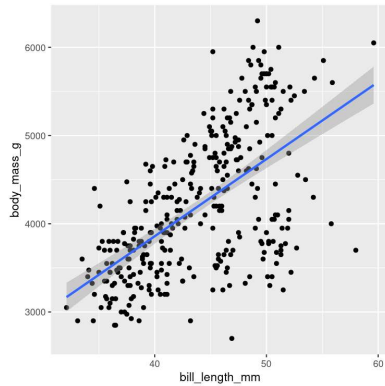
### Why no effect of Bill Length?



33 / 58

## Extra

### Why no effect of Bill Length?



```
m <- lm(body_mass_g ~ bill_length_mm, data = penguins)
summary(m)

##
## Call:
## lm(formula = body_mass_g ~ bill_length_mm, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1762.08  -446.98   32.59   462.31  1636.86
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   362.307    283.345   1.279   0.202
## bill_length_mm  87.415     6.402  13.654 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 645.4 on 340 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.3542,    Adjusted R-squared:  0.3523
## F-statistic: 186.4 on 1 and 340 DF,  p-value: < 2.2e-16
```

33 / 58

## Extra

### Why no effect of Bill Length?

- Hypothesis of *causation* but really just correlation
- Flipper length is the 'better' predictor of body mass
- When flipper length in the model, no extra variation explained by bill length
- When flipper length *not* in the model, some variation left to be explained

```
m <- lm(body_mass_g ~ bill_length_mm, data = penguins)
summary(m)

##
## Call:
## lm(formula = body_mass_g ~ bill_length_mm, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1762.08  -446.98   32.59   462.31  1636.86
##
## Coefficients:
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## (Intercept)   362.307    283.345   1.279   0.202
## bill_length_mm  87.415     6.402  13.654 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 645.4 on 340 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.3542,    Adjusted R-squared:  0.3523
## F-statistic: 186.4 on 1 and 340 DF,  p-value: < 2.2e-16
```

34 / 58

## Homework (Practice)\*

Consider **bill depth** your response and **bill length** your predictor

1. Plot the relationship
2. Create a linear regression model
3. Check your model diagnostics
  - Normality
  - Heteroscedasticity
  - Influential variables (i.e. Cook's distance)
4. Interpret the results of your model

\* Not to be handed in, answers posted in these slides next week



# ANOVAs

## Linear Models

### Running models in R

```
lm(y ~ x1 + x2, data = data)
```

- **y** is the **response** variable (**dependent**)
- **x** are the **explanatory** variables (**independent, predictor**)

Here we're assuming a **continuous y**

37 / 58

## Linear Models

### Running models in R

```
lm(y ~ x1 + x2, data = data)
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- **y** is the **response** variable (**dependent**)
- **x** are the **explanatory** variables (**independent, predictor**)

Here we're assuming a **continuous y**

### Different types of models

- If we only have one **x** which is continuous, this is a **simple linear regression**
- If both **x** are continuous, this is a **multiple linear regression**
- If both **x** are categorical, this is an **ANOVA**
- If **x1** is continuous and **x2** is categorical, this is an **ANCOVA**

37 / 58

# Linear Models

## Running models in R

```
lm(y ~ x1 + x2, data = data)
```

- **y** is the **response** variable (**dependent**)
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## Different types of models

- If we only have one **x** which is continuous, this is a **simple linear regression**
- If both **x** are continuous, this is a **multiple linear regression**
- If both **x** are categorical, this is an **ANOVA**
- If **x1** is continuous and **x2** is categorical, this is an **ANCOVA**

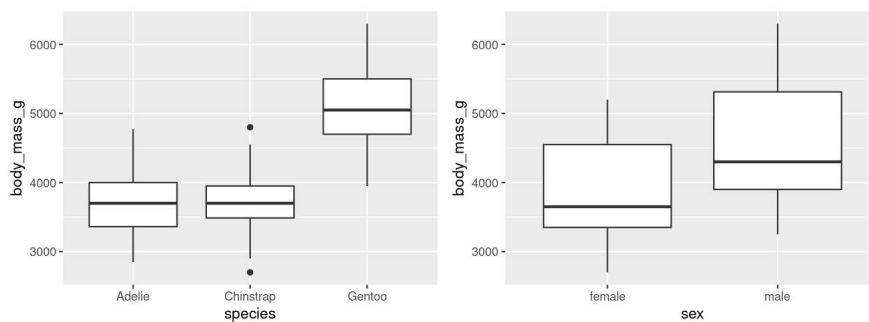
R will figure it out for you

37 / 58

# ANOVAs

## Real example

- Are male penguins larger than female penguins?
- Are different species different sizes?
- Can body mass be predicted by species and sex?



38 / 58

# ANOVAs

## Real example

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

As we have two **categorical** predictors, this is an ANOVA

But first diagnostics...

39 / 58

## Model Diagnostics

Same as before! Let's get our relevant variables into a diagnostic data frame:

- **residuals** (regular and standardized)
- **fitted values**
- **cooks distance**
- **obs number**

```
d <- data.frame(residuals = residuals(m),
               std_residuals = rstudent(m),
               fitted = fitted(m),
               cooks = cooks.distance(m))

d <- mutate(d, obs = 1:n())
```

Remember! `n()` only works *inside* `summarize()`/`mutate()`

40 / 58

## Model Diagnostics

Same as before! Let's get our relevant variables into a diagnostic data frame:

- **residuals** (regular and standardized)
- **fitted values**
- **cooks distance**
- **obs number**

```
d <- data.frame(residuals = residuals(m),
               std_residuals = rstudent(m),
               fitted = fitted(m),
               cooks = cooks.distance(m))

d <- mutate(d, obs = 1:n())
```

head(d)

##	residuals	std_residuals	fitted	cooks	obs
## 1	-289.94196	-0.9201062	4039.942	0.0021071103	1
## 2	427.61319	1.3590567	3372.387	0.0045831843	2
## 3	-122.38681	-0.3879729	3372.387	0.0003754346	3
## 4	77.61319	0.2460043	3372.387	0.0001509858	4
## 5	-389.94196	-1.2387413	4039.942	0.0038112292	5
## 6	252.61319	0.8013974	3372.387	0.0015994740	6

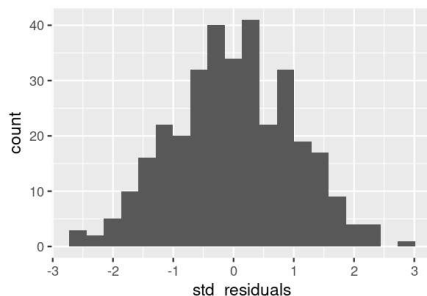
Remember! `n()` only works *inside* `summarize()`/`mutate()`

40 / 58

## Normality

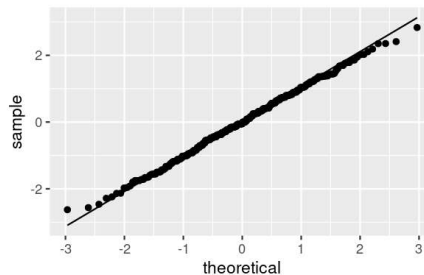
### Histogram of residuals

```
ggplot(data = d, aes(x = std_residuals)) +
  geom_histogram(bins = 20)
```



### QQ Normality plot of residuals

```
ggplot(data = d, aes(sample = std_residuals)) +
  stat_qq() +
  stat_qq_line()
```

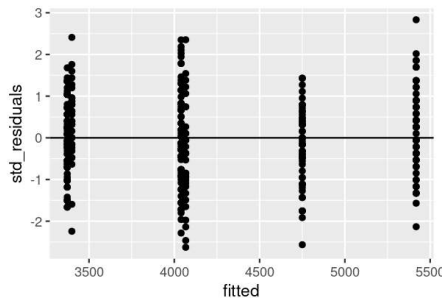


41 / 58

## Variance and Influence

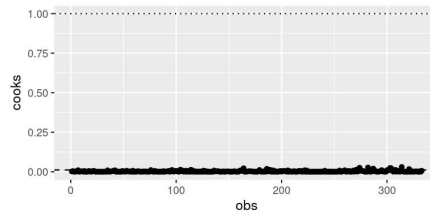
### Check heteroscedasticity

```
ggplot(d, aes(x = fitted, y = std_residuals)) +  
  geom_point() +  
  geom_hline(yintercept = 0)
```



### Cook's D

```
ggplot(d, aes(x = obs, y = cooks)) +  
  geom_point() +  
  geom_hline(yintercept = 1, linetype = "dotted") +  
  geom_hline(yintercept = 4/nrow(penguins),  
    linetype = "dashed")
```

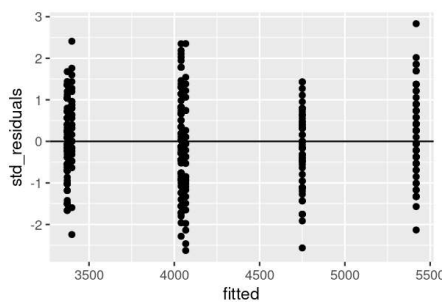


42 / 58

## Variance and Influence

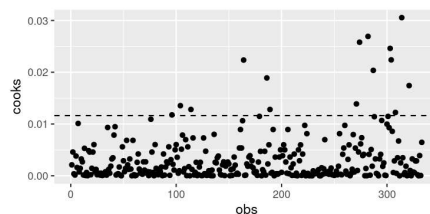
### Check heteroscedasticity

```
ggplot(d, aes(x = fitted, y = std_residuals)) +  
  geom_point() +  
  geom_hline(yintercept = 0)
```



### Cook's D

```
ggplot(d, aes(x = obs, y = cooks)) +  
  geom_point() +  
  geom_hline(yintercept = 4/nrow(penguins),  
    linetype = "dashed")
```

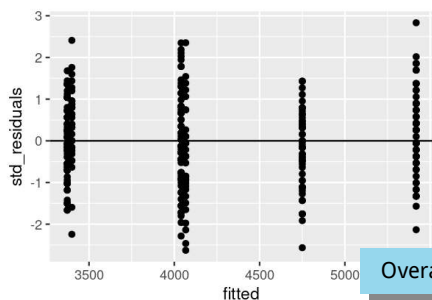


43 / 58

## Variance and Influence

### Check heteroscedasticity

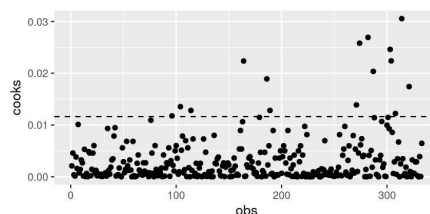
```
ggplot(d, aes(x = fitted, y = std_residuals)) +  
  geom_point() +  
  geom_hline(yintercept = 0)
```



Overall not too bad

### Cook's D

```
ggplot(d, aes(x = obs, y = cooks)) +  
  geom_point() +  
  geom_hline(yintercept = 4/nrow(penguins),  
    linetype = "dashed")
```



43 / 58

## Multicollinearity (collinearity)

### `vif()` function from `car` package

```
library(car)
vif(m)

##              GVIF Df GVIF^(1/(2*Df))
## species 1.000146  2      1.000036
## sex      1.000146  1      1.000073
```

Here we consider the `GVIF^(1/(2*Df))` value\*

Looks good!

\* See `?vif` and the reference therein: Fox, J. and Monette, G. (1992) Generalized collinearity diagnostics. JASA, 87, 178–183/ 58

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)

summary(m)

##
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80  -16.87   227.61   882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308  <2e-16 ***
## speciesChinstrap    26.92      46.48   0.579   0.563
## speciesGentoo    1377.86      39.10  35.236  <2e-16 ***
## sexmale           667.56      34.70  19.236  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)

summary(m)

Model

##
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80  -16.87   227.61   882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308  <2e-16 ***
## speciesChinstrap    26.92      46.48   0.579   0.563
## speciesGentoo    1377.86      39.10  35.236  <2e-16 ***
## sexmale           667.56      34.70  19.236  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

### Effects

```
##
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80  -16.87  227.61  882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308 <2e-16 ***
## speciesChinstrap    26.92      46.48   0.579   0.563
## speciesGentoo     1377.86      39.10  35.236 <2e-16 ***
## sexmale           667.56      34.70  19.236 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

47 / 58

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

### Missing observations

```
##
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80  -16.87  227.61  882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308 <2e-16 ***
## speciesChinstrap    26.92      46.48   0.579   0.563
## speciesGentoo     1377.86      39.10  35.236 <2e-16 ***
## sexmale           667.56      34.70  19.236 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

48 / 58

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

### $R^2$ and adjusted $R^2$

- Adjusted for the number of parameters

```
##
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80  -16.87  227.61  882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308 <2e-16 ***
## speciesChinstrap    26.92      46.48   0.579   0.563
## speciesGentoo     1377.86      39.10  35.236 <2e-16 ***
## sexmale           667.56      34.70  19.236 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

49 / 58

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

### Specific Details

- **Estimate**
  - Treatment contrasts
  - Average *differences* among categories compared to the base category
- **Std. Error**
  - Variability in the estimates
- **t value**
  - Test statistic
- **Pr(>|t|)**
  - **P-value**, significance of the *differences*
  - Probability of getting **t-value** by chance

```
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80 -16.87  227.61  882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308 <2e-16 ***
## speciesChinstrap    26.92       46.48    0.579  0.563
## speciesGentoo    1377.86      39.10   35.236 <2e-16 ***
## sexmale          667.56      34.70   19.236 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

50 / 58

## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

### Specific Details

- **Estimate**
  - Treatment contrasts
  - Average *differences* among categories compared to the base category
- **Std. Error**
  - Variability in the estimates
- **t value**
  - Test statistic
- **Pr(>|t|)**
  - **P-value**, significance of the *differences*
  - Probability of getting **t-value** by chance

Easier to interpret estimates if we consider a simpler model

```
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80 -16.87  227.61  882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308 <2e-16 ***
## speciesChinstrap    26.92       46.48    0.579  0.563
## speciesGentoo    1377.86      39.10   35.236 <2e-16 ***
## sexmale          667.56      34.70   19.236 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

50 / 58

## Interpreting ANOVA Summaries

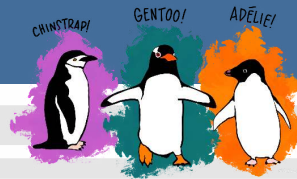
```
m <- lm(body_mass_g ~ species, data = penguins)
```

```
summary(m)
```

```
## Call:
## lm(formula = body_mass_g ~ species, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1126.02 -333.09 -33.09  316.91 1223.98
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3700.66      37.62   98.37 <2e-16 ***
## speciesChinstrap    32.43       67.51    0.48  0.631
## speciesGentoo    1375.35      56.15   24.50 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 462.3 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.6697,    Adjusted R-squared:  0.6677
## F-statistic: 343.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

51 / 58

# Interpreting ANOVA Summaries



```
m <- lm(body_mass_g ~ species, data = penguins)
```

```
summary(m)
```

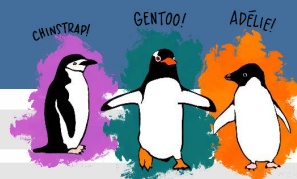
## Effect of Species

- **(Intercept)** represents base category (i.e. Adelie penguins)
- Adelie have mean body mass of 3700.66 g
- On average, Chinstrap penguins are 32.43 g heavier than Adelie penguins
- On average, Gentoo penguins are 1375.35 g heavier than Adelie penguins

```
##
## Call:
## lm(formula = body_mass_g ~ species, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1126.02  -333.09   -33.09   316.91  1223.98
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3700.66     37.62   98.37  <2e-16 ***
## speciesChinstrap    32.43     67.51    0.48   0.631
## speciesGentoo   1375.35     56.15   24.50  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 462.3 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.6697,    Adjusted R-squared:  0.6677
## F-statistic: 343.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

51 / 58

# Interpreting ANOVA Summaries



```
m <- lm(body_mass_g ~ species, data = penguins)
```

```
summary(m)
```

## Effect of Species

- **(Intercept)** represents base category (i.e. Adelie penguins)
- Adelie have mean body mass of 3700.66 g
- On average, Chinstrap penguins are 32.43 g heavier than Adelie penguins
- On average, Gentoo penguins are 1375.35 g heavier than Adelie penguins

Back to original model

```
##
## Call:
## lm(formula = body_mass_g ~ species, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1126.02  -333.09   -33.09   316.91  1223.98
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3700.66     37.62   98.37  <2e-16 ***
## speciesChinstrap    32.43     67.51    0.48   0.631
## speciesGentoo   1375.35     56.15   24.50  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 462.3 on 339 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.6697,    Adjusted R-squared:  0.6677
## F-statistic: 343.6 on 2 and 339 DF,  p-value: < 2.2e-16
```

51 / 58

# Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

## Effect of Species and Sex

- **(Intercept)** represents base category but is a combination of factors
- Much more complicated to interpret
- Comparisons are often not of interest anyway (unless you've set up contrasts, which are advanced stats but awesome!)

```
##
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
##  -816.87  -217.80  -16.87   227.61   882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39     31.43  107.308  <2e-16 ***
## speciesChinstrap    26.92     46.48    0.579   0.563
## speciesGentoo    1377.86     39.10   35.236  <2e-16 ***
## sexmale           667.56     34.70   19.236  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

52 / 58



## Interpreting ANOVA Summaries

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
summary(m)
```

### Effect of **Species** and **Sex**

- **(Intercept)** represents base category but is a combination of factors
- Much more complicated to interpret
- Comparisons are often not of interest anyway (unless you've set up contrasts, which are advanced stats but awesome!)

So let's look at ANOVA tables instead

```
## Call:
## lm(formula = body_mass_g ~ species + sex, data = penguins)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -816.87 -217.80  -16.87  227.61  882.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3372.39      31.43  107.308  <2e-16 ***
## speciesChinstrap    26.92       46.48    0.579    0.563
## speciesGentoo     1377.86      39.10   35.236  <2e-16 ***
## sexmale           667.56      34.70   19.236  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 316.6 on 329 degrees of freedom
## (11 observations deleted due to missingness)
## Multiple R-squared:  0.8468,    Adjusted R-squared:  0.8454
## F-statistic: 606.1 on 3 and 329 DF,  p-value: < 2.2e-16
```

52 / 58

## Interpreting ANOVA Tables

### Type I

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
anova(m)
```

```
## Analysis of Variance Table
##
## Response: body_mass_g
##              Df Sum Sq Mean Sq F value    Pr(>F)
## species      2 145190219 72595110   724.21 < 2.2e-16
## ***
## sex          1  37090262 37090262   370.01 < 2.2e-16
## ***
## Residuals 329  32979185   100241
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

53 / 58

## Interpreting ANOVA Tables

### Type I

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
anova(m)
```

### Overall effects of **Species** and **Sex**

- Yes there are differences among **Species** ( $P < 2.2e^{-16}$ )
- Yes there are differences between **Sexes** ( $P < 2.2e^{-16}$ )

```
## Analysis of Variance Table
##
## Response: body_mass_g
##              Df Sum Sq Mean Sq F value    Pr(>F)
## species      2 145190219 72595110   724.21 < 2.2e-16
## ***
## sex          1  37090262 37090262   370.01 < 2.2e-16
## ***
## Residuals 329  32979185   100241
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

54 / 58

# Interpreting ANOVA Tables

## Type I

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
anova(m)
```

### Overall effects of **Species** and **Sex**

- Yes there are differences among **Species** ( $P < 2.2e-16$ )
- Yes there are differences between **Sexes** ( $P < 2.2e-16$ )

```
## Analysis of Variance Table
##
## Response: body_mass_g
##           Df Sum Sq Mean Sq F value    Pr(>F)
## species    2 145190219 72595110   724.21 < 2.2e-16 ***
## sex        1  37090262 37090262   370.01 < 2.2e-16 ***
## Residuals 329  32979185  100241
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

Not a whole lot of information...  
Stay tuned for **Post-Hoc** tests next week!

54 / 58

# Interpreting ANOVA Tables

## Type I

```
m1 <- lm(body_mass_g ~ species + sex, data =
penguins)
anova(m1)
```

```
## Analysis of Variance Table
##
## Response: body_mass_g
##           Df Sum Sq Mean Sq F value    Pr(>F)
## species    2 145190219 72595110   724.21 < 2.2e-16 ***
## sex        1  37090262 37090262   370.01 < 2.2e-16 ***
## Residuals 329  32979185  100241
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

```
m2 <- lm(body_mass_g ~ sex + species, data =
penguins)
anova(m2)
```

```
## Analysis of Variance Table
##
## Response: body_mass_g
##           Df Sum Sq Mean Sq F value    Pr(>F)
## sex        1  38878897 38878897   387.86 < 2.2e-16 ***
## species    2 143401584 71700792   715.29 < 2.2e-16 ***
## Residuals 329  32979185  100241
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

- For Type I ANOVAs, order matters with unbalanced samples
  - See that **Sum sq**, **Mean Sq** and **F value** all differ between the models
- Here, pretty minor, but important to remember with greater unbalances

55 / 58

# Interpreting ANOVA Tables

## Type III

```
m <- lm(body_mass_g ~ species + sex, data = penguins)
```

```
library(car)
Anova(m, type = "3")
```

```
## Anova Table (Type III tests)
##
## Response: body_mass_g
##           Sum Sq Df F value    Pr(>F)
## (Intercept) 1154266972 1 11514.96 < 2.2e-16 ***
## species    143401584 2   715.29 < 2.2e-16 ***
## sex        37090262 1   370.01 < 2.2e-16 ***
## Residuals  32979185 329
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

Type II ANOVAs do exist as well, but generally we use Type III in natural sciences

56 / 58

# Interpreting ANOVA Tables

## Type III

```
m1 <- lm(body_mass_g ~ species + sex, data =  
penguins)  
Anova(m1, type = "3")
```

```
## Anova Table (Type III tests)  
##  
## Response: body_mass_g  
##           Sum Sq Df F value    Pr(>F)  
## (Intercept) 1154266972  1 11514.96 < 2.2e-16 ***  
## species      143401584  2   715.29 < 2.2e-16 ***  
## sex          37090262  1   370.01 < 2.2e-16 ***  
## Residuals    32979185 329  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05  
'.' 0.1 ' ' 1
```

```
m2 <- lm(body_mass_g ~ sex + species, data =  
penguins)  
Anova(m2, type = "3")
```

```
## Anova Table (Type III tests)  
##  
## Response: body_mass_g  
##           Sum Sq Df F value    Pr(>F)  
## (Intercept) 1154266972  1 11514.96 < 2.2e-16 ***  
## sex          37090262  1   370.01 < 2.2e-16 ***  
## species      143401584  2   715.29 < 2.2e-16 ***  
## Residuals    32979185 329  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05  
'.' 0.1 ' ' 1
```

- Type III and unbalanced samples: Not dependent on variable order

57 / 58

## Homework (Practice)\*

Consider flipper length your response variable and species and sex your predictor variables

1. Plot the relationship between flipper length and species and between flipper length and sex
2. Create an ANOVA model of flipper length and species
3. Check diagnostics
4. Interpret the **summary table**
5. Interpret the **ANOVA Table**
6. Create an ANOVA model of flipper length and species and sex
7. Check diagnostics
8. Interpret the **ANOVA Table**

\* Not to be handed in, answers posted in these slides next week

58 / 58