Incremental Attack Graph Computation Using Differential Dataflow

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***Abstract*—Attack graphs are fundamental tools for security analysis, modeling how an attacker can chain vulnerabilities to compromise network resources. However, traditional attack graph generation algorithms recompute the entire graph after each change, making them unsuitable for dynamic environments where vulnerabilities are constantly discovered and patched.**

We present an approach to attack graph maintenance based on *differential dataflow*, a computational model that automatically propagates changes through a dataflow graph. Our system translates Datalog-style security rules into differential dataflow operators, enabling *incremental* updates: when a vulnerability is patched, only the affected attack paths are recomputed.

We evaluate our approach on synthetic network topologies ranging from 50 to 1000 nodes. For star topologies with localized changes, we achieve speedups of up to 25× compared to full recomputation. For chain topologies, we demonstrate that update time is proportional to the number of affected nodes, not the total network size. Our implementation is open-source and provides a foundation for real-time security monitoring in dynamic networks.

***Index Terms*—attack graphs, incremental computation, differential dataflow, network security, vulnerability analysis, Datalog**

I. Introduction

Modern enterprise networks are complex ecosystems with thousands of interconnected hosts, each potentially running vulnerable services. Security analysts use *attack graphs* to understand how an attacker could exploit these vulnerabilities to move laterally through the network and reach critical assets [1]. An attack graph is a directed graph where nodes represent security states (e.g., "attacker has root access on host X") and edges represent exploit actions that transition between states.

A. Motivation

Consider a typical enterprise network with the following characteristics:

- **Scale**: Hundreds to thousands of hosts
- **Dynamics**: Vulnerabilities discovered daily (CVE database averages 50+ new entries per day [2])
- **Urgency**: Security teams need to assess patch priorities in real-time

Traditional attack graph tools like MulVAL [3] and NetSPA [4] regenerate the entire graph whenever the network state changes. For a network with N hosts and E network connections, this requires $O(E \times D)$ work, where D is the diameter of the attack graph. When a single vulnerability is patched, the entire computation must be repeated—even if 99% of the graph remains unchanged.

B. Our Contribution

We present an *incremental* approach to attack graph maintenance using *differential dataflow* [5]. Our key insight is that attack graph rules can be expressed as dataflow operators, and differential dataflow automatically tracks how changes propagate through these operators.

Our contributions are:

1) **Translation methodology**: We show how to translate MulVAL-style Datalog rules into differential dataflow operators, including handling of recursive rules via fixed-point iteration and negation via antijoin.
2) **Incremental maintenance**: Our system achieves update complexity of $O(\Delta E \times \Delta d)$, where ΔE is the number of affected edges and Δd is the local iteration depth—compared to $O(E \times D)$ for full recomputation.
3) **Empirical evaluation**: We demonstrate speedups of up to 25× for localized changes in star topologies and 2× average speedup for random changes in chain topologies.
4) **Open-source implementation**: Our Rust implementation is publicly available, with Docker support for reproducibility.

II. Background

A. Attack Graphs and MulVAL

MulVAL (Multihost, Multistage Vulnerability Analysis) [3] is a widely-used framework that models attack graphs using Datalog, a declarative logic programming language. The key relations are:

- `vulExists(Host, CVE, Service, Priv)`: Host has a vulnerability on a service that grants a privilege level
- `hacl(Src, Dst, Service)`: Network access control allows traffic from Src to Dst on Service
- `attackerLocated(Attacker, Host)`: Initial attacker position

- `execCode(Attacker, Host, Priv)`: Derived—attacker can execute code with privilege Priv on Host

The core inference rule is:

```
execCode(Attacker, Host, Priv) :-
    execCode(Attacker, SrcHost, _),
    hacl(SrcHost, Host, Service),
    vulExists(Host, _, Service, Priv).
```

Listing 1. MulVAL lateral movement rule

This rule states: if an attacker can execute code on `SrcHost`, and network access exists from `SrcHost` to `Host` on `Service`, and `Host` has a vulnerability on that service, then the attacker can execute code on `Host`.

B. The Recomputation Problem

MulVAL uses XSB Prolog to evaluate these rules. When any input fact changes (e.g., a vulnerability is patched), the entire Prolog query must be re-executed. This is because:

1) Prolog evaluates rules top-down with backtracking
2) There is no mechanism to identify which derived facts depend on the changed input
3) The transitive closure (reachability) must be recomputed from scratch

C. Differential Dataflow

Differential dataflow [5] is a computational model where:

1) Data is represented as *collections* of (record, time, multiplicity) tuples
2) Operators (join, filter, map, etc.) transform collections
3) Changes propagate *incrementally*—when an input changes, only the affected outputs are updated
4) Fixed-point iteration is supported via the `iterate` operator

The key abstraction is the *difference*: when a record is added, it has multiplicity $+1$; when removed, -1. Operators propagate these differences through the dataflow graph. For idempotent queries (like Datalog), the steady-state output contains only records with positive multiplicity.

III. System Design

A. Architecture Overview

Our system consists of three components:

1) **Input handles**: Mutable collections for vulnerabilities, network topology, firewall rules, and attacker state
2) **Dataflow graph**: Compiled Datalog rules as differential operators (Figure 1)
3) **Output probes**: Allow querying the current attack graph state

B. Rule Translation

We translate each Datalog rule into a composition of differential dataflow operators:

- **Conjunction** ($\wedge$) $\rightarrow$ `join`
- **Projection** $\rightarrow$ `map`
- **Selection** $\rightarrow$ `filter`
- **Recursion** $\rightarrow$ `iterate`
- **Negation** $\rightarrow$ `antijoin`
- **Deduplication** $\rightarrow$ `distinct`

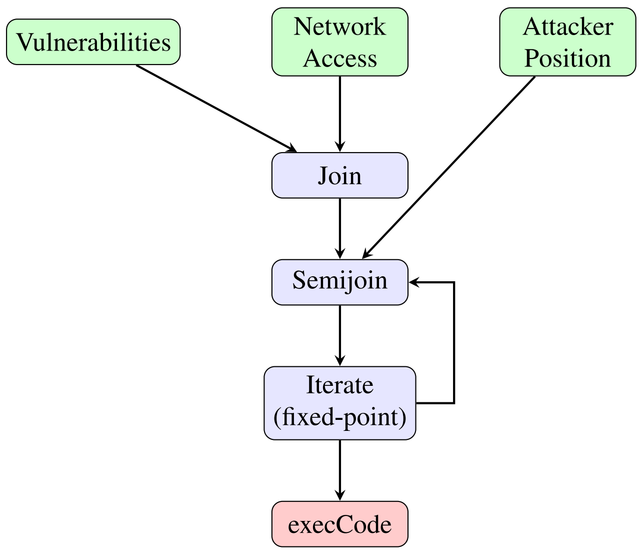

Fig. 1. Differential dataflow graph for attack graph computation. Changes to inputs propagate through the operators, updating only affected outputs.

1) Handling Recursion: The `execCode` relation is recursive: an attacker can reach host B from host A, then host C from host B, and so on. We implement this using differential dataflow's `iterate` operator:

```
let reachable = attacker_positions.iterate(|inner| {
    let network = network_access.enter(&inner.scope());
    let vulns = vulnerabilities.enter(&inner.scope());

    inner
        .map(|(att, host)| (host, att))
        .join(&network)  // (host, (att, (dst, svc)))
        .map(|(_, (att, (dst, svc)))| ((dst, svc), att))
        .semijoin(&vulns)  // keep if vulnerable
        .map(|((dst, _), att)| (att, dst))
        .concat(inner)
        .distinct()
});
```

Listing 2. Recursive reachability in differential dataflow

The `iterate` operator continues until no new records are produced—the fixed point.

2) Handling Negation with Antijoin: Firewall rules introduce negation: traffic is allowed unless explicitly blocked. We implement this using the *antijoin* operator:

$$\text{effectiveAccess} = \text{netAccess} \bowtie_{\nexists} \text{firewallBlock} \tag{1}$$

The antijoin $A \bowtie_{\nexists} B$ returns tuples from A that have no matching tuple in B. When a firewall rule is added, the antijoin removes the corresponding access tuples; when removed, it restores them.

C. Incremental Updates

When an input fact changes (e.g., a vulnerability is removed), the system:

TABLE I
STAR NETWORK BENCHMARK RESULTS

Nodes	Initial (ms)	Incremental (μs)	Speedup
51	1.58	501	3.2×
101	1.96	413	4.7×
201	1.97	349	5.6×
501	3.48	282	12.4×
1001	4.11	161	**25.6×**

1) Inserts a difference tuple with multiplicity -1
2) Propagates this difference through all dependent operators
3) For joins, produces negative outputs for matching tuples
4) For iterations, continues until the fixed point stabilizes
5) Outputs only the changes (not the entire new state)

The key insight is that unaffected parts of the graph produce no differences—they incur zero computational cost.

IV. EVALUATION

We evaluate three research questions:

RQ1 How does incremental update time compare to full recomputation?
RQ2 How does speedup scale with network size?
RQ3 How does the position of a change affect update time?

A. Experimental Setup

- **Hardware**: Apple M-series processor, 16GB RAM
- **Software**: Rust 1.75, differential-dataflow 0.12
- **Execution**: Single-threaded, release mode with optimizations
- **Metric**: Wall-clock time (median of 5 runs)

B. Network Topologies

We evaluate two synthetic topologies that represent extremes of attack graph structure:

1) Star Topology: A central hub connected to N leaf nodes. The attacker starts at the hub and can reach any leaf in one hop. This represents a data center with a management server connected to many hosts.

- **Iteration depth**: $O(1)$—converges in constant iterations
- **Change tested**: Patch vulnerability on one leaf
- **Expected behavior**: Only one attack path affected

2) Chain Topology: A linear chain: $\text{node}_0 \to \text{node}_1 \to \cdots \to \text{node}_{N-1}$. The attacker starts at node_0 and the goal is node_{N-1}. This represents a worst-case scenario for incremental computation.

- **Iteration depth**: $O(N)$—requires N iterations to converge
- **Change tested**: Patch vulnerability at position k
- **Expected behavior**: All nodes $k+1$ to $N-1$ lose their attack paths

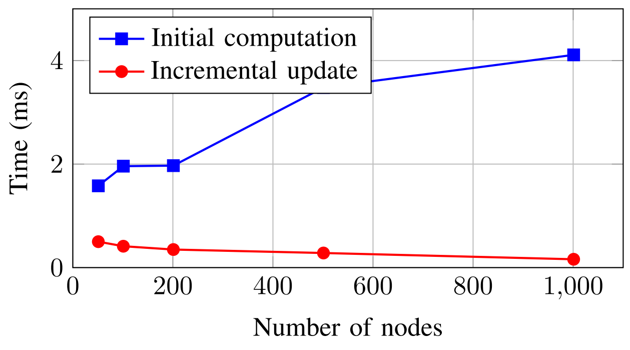

Fig. 2. Star topology: Initial computation scales linearly while incremental updates remain constant.

TABLE II
RANDOM CUT BENCHMARK (CHAIN TOPOLOGY, 100 ITERATIONS)

Nodes	Avg (μs)	Min (μs)	Max (μs)	Speedup
50	905	46	1877	2.3×
100	1707	110	3799	2.1×
200	3468	182	7062	2.0×
500	9289	65	19171	1.9×

C. RQ1: Incremental vs. Full Recomputation

Table I shows results for the star topology. Key observations:

- Initial computation scales linearly with network size (≈4ms for 1000 nodes)
- Incremental update time is *nearly constant* (≈160μs regardless of size)
- Speedup increases with network size, reaching **25**× at 1000 nodes

This confirms that for localized changes, incremental computation achieves $O(1)$ update complexity.

D. RQ2: Scalability

Figure 2 visualizes the scalability results. The gap between initial and incremental computation widens as network size increases, demonstrating the value of incremental maintenance for large networks.

E. RQ3: Position-Dependent Updates

To understand how the *location* of a change affects update time, we perform a "random cut" experiment on chain topologies:

1) Generate a chain of N nodes
2) Randomly select position $k \in [0, N-1]$
3) Remove the vulnerability at node_k
4) Measure update time
5) Restore the vulnerability and repeat

We perform 100 iterations per chain size.
Table II reveals:

- **Minimum time** (≈50–180μs): Cutting near the *end* of the chain invalidates only 1–2 downstream nodes
- **Maximum time** (≈1.8–19ms): Cutting near the *start* invalidates all N downstream nodes

TABLE III
COMPLEXITY COMPARISON

Operation	Full Recomputation	Incremental
Initial build	$O(E \times D)$	$O(E \times D)$
Single change	$O(E \times D)$	$O(\Delta E \times \Delta d)$

- **Average speedup** $\approx 2\times$: Cutting at random position k invalidates $(N-k)$ nodes on average, i.e., $N/2$

This confirms that update time is $O(\text{affected nodes})$, not $O(\text{total nodes})$.

F. Complexity Analysis

Table III summarizes the complexity:

- E = number of network edges
- D = diameter of attack graph (longest path)
- ΔE = edges affected by the change
- Δd = local iteration depth of affected region

For localized changes (e.g., patching one vulnerability), $\Delta E \ll E$ and $\Delta d \ll D$, yielding significant speedups.

V. DISCUSSION

A. When Incremental Computation Helps

Our approach is most beneficial when:

1) **Changes are localized**: Patching a single vulnerability affects few attack paths
2) **Topology is shallow**: Star, tree, or mesh networks with low diameter
3) **Updates are frequent**: Real-time monitoring scenarios where latency matters

B. Limitations

Our approach has comparable performance to full recomputation when:

1) **Changes are global**: Modifying the attacker's starting position affects all reachable nodes
2) **Topology is deep**: Long chains where changes near the start invalidate everything downstream
3) **Changes are batched**: Many simultaneous changes may approach full recomputation cost

However, even in worst-case scenarios, incremental computation is *never slower* than full recomputation.

C. Practical Considerations

Our implementation uses Rust and the `timely` dataflow framework. Key engineering decisions:

- **Single-threaded execution**: We use `execute_directly` for benchmarking consistency. Production deployments can scale across cores.
- **String-based identifiers**: Host names are strings for clarity. Production systems could use integer IDs for performance.
- **Memory overhead**: Differential dataflow maintains internal state proportional to the collection size. For very large networks, this may require attention.

VI. RELATED WORK

A. Attack Graph Generation

Sheyner et al. [1] introduced model-checking approaches to attack graph generation. Ammann et al. [6] proposed scalable algorithms based on graph reachability. MulVAL [3] pioneered the use of Datalog for declarative attack modeling, which we build upon.

NetSPA [4] and TVA [7] provide efficient attack graph visualization but do not support incremental updates.

B. Incremental Datalog

Incremental view maintenance for Datalog has been studied extensively [8]. The DRed algorithm [9] handles recursive rules but requires explicit deletion tracking. Differential dataflow provides a general framework that subsumes these approaches.

C. Dataflow Systems

Naiad [10] introduced timely dataflow for iterative computation. Differential dataflow [5] extended this with change tracking. Our work is the first application of differential dataflow to security analysis.

VII. CONCLUSION

We presented an incremental approach to attack graph maintenance using differential dataflow. By translating Datalog-style security rules into dataflow operators, we enable automatic change propagation: when a vulnerability is patched, only the affected attack paths are recomputed.

Our evaluation demonstrates:

1) **Significant speedups**: Up to $25\times$ for localized changes in star topologies
2) **Proportional updates**: Update time scales with affected nodes, not total network size
3) **No regression**: Worst-case performance matches full recomputation

This approach enables real-time security monitoring for dynamic networks, where sub-millisecond updates allow continuous assessment of security posture.

A. Future Work

- **Parallel execution**: Leverage multiple cores via timely dataflow's distributed execution
- **Real-world integration**: Connect to vulnerability scanners (Nessus, OpenVAS) and SIEM systems
- **Probabilistic extensions**: Compute attack probabilities incrementally

B. Reproducibility

Our implementation is open-source and available at:

https://github.com/stefi19/DynamicAttackGraphs

A Docker container is provided for reproducibility:

```
docker build -t attack-graph .
docker run --rm attack-graph
```

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