Problema 5. Deduceti seria Taylor pentru $\ln(1+x)$ si aproximati $\ln(2)$ folosind primii 8 termeni. Cati termeni sunt necesari pentru a obtine $\ln(2)$ cu 5 zecimale corecte? La fel pentru $\ln\left(\frac{1+x}{1-x}\right)$.

In [58]:
```
T8 = taylor(ln(1+x),x,0,8)
show(T8)
```

$$-\frac{1}{8}x^8 + \frac{1}{7}x^7 - \frac{1}{6}x^6 + \frac{1}{5}x^5 - \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{2}x^2 + x$$

In [26]:
```
ln2_approx = T8(1)
show("By Taylor: ln(2) ≈ ", ln2_approx, " = ", numerical_approx(ln2_appro
x))
show("By built-in log: ln(2) ≈ ", numerical_approx(log(2)))
```

$$\text{By Taylor: } \ln(2) \approx \frac{533}{840} = 0.634523809523809$$

$$\text{By built-in log: } \ln(2) \approx 0.693147180559945$$

In [56]:
```
true_5_decimals = floor(log(2)*(10**5))
true_5_decimals

ln2_approx_n = ln2_approx.n()
appx_5_decimals = 0

k=8
while appx_5_decimals != true_5_decimals:
    k+=1
    ln2_approx_n += ((-1)**(k+1)) / k
    appx_5_decimals = floor(ln2_approx_n * (10**5))
    if k%10000==0:
        print(f"T{k} => 0.{appx_5_decimals}")
print(f"T{k} => 0.{appx_5_decimals}")
```

```
T10000 => 0.69309
T20000 => 0.69312
T30000 => 0.69313
T40000 => 0.69313
T50000 => 0.69313
T60000 => 0.69313
T69632 => 0.69314
```

Testam cativa termeni pentru $\ln\left(\frac{1+x}{1-x}\right)$

In [67]:
```
T8 = taylor(ln((1+x)/(1-x)),x,0,8)
show(T8)
```

$$\frac{2}{7}x^7 + \frac{2}{5}x^5 + \frac{2}{3}x^3 + 2x$$

In [68]:
```
T10 = taylor(ln((1+x)/(1-x)),x,0,10)
show(T10)
```

$$\frac{2}{9}x^9 + \frac{2}{7}x^7 + \frac{2}{5}x^5 + \frac{2}{3}x^3 + 2x$$

In [69]:
```
show("T8(1/3) = ", numerical_approx(T8(1/3)))
show("T10(1/3) = ", numerical_approx(T10(1/3)))
```

T8(1/3) =0.693134757332288

T10(1/3) =0.693146047390827

Primii 10 termeni sunt suficienti pentru a aproxima $\ln(2) = \ln\left(\frac{1+1/3}{1-1/3}\right)$ (huge improvement from the previous attempt)