1. Fie $f : [-2\pi, 2\pi] \to \mathbb{R}, f(x) = sin(x^2)$. Fie setul de noduri $\{x_k, k = \overline{0,9}\}$ Cebisev de peta I. Fie $t = \frac{\pi}{5}$. Se cer:

a. Reprezentati grafic functia f impreuna cu polinoamele de interpolare Lagrange si Hermite pentru acestea, pe acelasi grafic.

b. Aproximati $f(t)$ cu ajutorul polinoamelor de la subpunctul a.

c. Pentru fiecare interpolant, evaluati teoretic (cu ajutorul restului) si practic (comparand valorile functiei returnate de MATLAB cu valorile polinoamelor de interpolare).

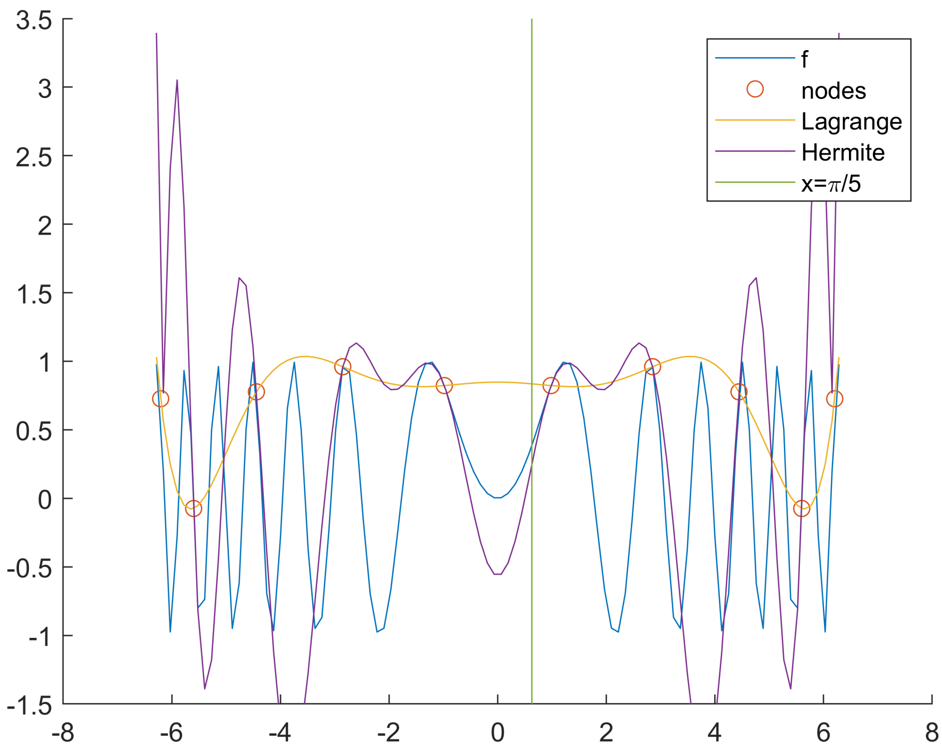
Solutie:

(a)

```
f = @(x) sin(x.*x);
df = @(x) cos(x.*x)*2.*x;
c = chebNodes(10, -2*pi, 2*pi);
x = c;
y = f(x);
dy = df(x);

graph_x = linspace(-2*pi,2*pi);
lagrange_y = LagrangeClassic(x, y, graph_x);
hermite_y = interpolareHermiteWithDerivative(x,y,dy, graph_x);


clf; hold on;
plot(graph_x, f(graph_x))
scatter(x, y)
plot(graph_x, lagrange_y)
plot(graph_x, hermite_y)
plot([pi/5, pi/5],[-1.5,3.5])
ylim([-1.5,3.5])
legend("f", "nodes", "Lagrange", "Hermite", "x=\pi/5")
hold off;
```



(b)

```
t = pi/5;
f_t = f(t)
```

```
f_t = 0.3846
```

```
lagrange_t = LagrangeClassic(x,y, t)
```

```
lagrange_t = 0.8353
```

```
hermite_t = interpolareHermiteWithDerivative(x,y,dy, t)
```

```
hermite_t = 0.2454
```

(c)

Masurarea teoretica erorii (Lagrange):

(Teorema) Daca $x_0, \ldots, x_m$ sunt $m+1$ noduri distincte in internalul $[a,b]$ si $f \in C^{n+1}[a,b]$, atunci pentru fiecare $x \in [a,b]$exista un numar $\xi(x) \in [a,b]$ astfel incat

$$f(x) = L_m f(x) + \frac{f^{(m+1)}(\xi(x))}{(m+1)!}(x-x_0)\ldots(x-x_m)$$

Dorim o margine superioara pentru eroarea

$$err(x) = \left|\frac{f^{(m+1)}(\xi(x))}{(m+1)!}(x-x_0)\dots(x-x_m)\right| \le \left|\frac{f^{(m+1)}(\xi(x))}{(m+1)!}\right| |(x-x_0)|\dots|(x-x_m)|$$

$$\le \left|\frac{f^{(m+1)}(\xi(x))}{(m+1)!}\right| |((x-x_1)(x_m-x))\cdot((x-x_2)(x_{m-1}-x))\cdot\dots$$

$$\le \left|\frac{f^{(m+1)}(\xi(x))}{(m+1)!}\right| \left(\frac{x_m-x_1}{2}\right)^2\left(\frac{x_2-x_{m-1}}{2}\right)^2\dots\left(\frac{x_{[m/2]}-x_{[m/2]+1}}{2}\right)^2$$

In particular, $m=9, x_i = 2\pi cos\left(\frac{(2i+1)\pi}{n}\right), i=\overline{0,9}$

```
syms x
f = f(x)
```

f = $\sin(x^2)$

```
d10f = diff(f, 10);
err1 = d10f/factorial(10)
```

err1 =

$$\frac{\cos(x^2)}{120} - \frac{x^4\cos(x^2)}{9} + \frac{2\,x^8\cos(x^2)}{315} - \frac{x^2\sin(x^2)}{12} + \frac{2\,x^6\sin(x^2)}{45} - \frac{4\,x^{10}\sin(x^2)}{14175}$$

$$\rightarrow \left|\frac{f^{(m+1)}(\xi(x))}{(m+1)!}\right| = |err1(\xi(x))| \le \frac{1}{120} + \frac{(2\pi)^4}{9} + \frac{2(2\pi)^8}{315} + \frac{(2\pi)^2}{12} + \frac{2(2\pi)^6}{45} + \frac{4(2\pi)^{10}}{14175}$$

```
M1 = 1/120+(2*pi)^4/9 + 2*(2*pi)^8/315 + (2*pi)^2/12 + 2*(2*pi)^6/45 + 4*(2*pi)^10/14175
```

M1 = 4.5394e+04

$\rightarrow |err1(\xi(x))| \le 5\cdot 10^4$

```
M2 = (((c(1)-c(10))/2)*((c(2)-c(9))/2)*((c(3)-c(8))/2)*((c(4)-c(7))/2)*((c(5)-c(6))/2))^2
```

M2 = 1.8730e+05

$\rightarrow$eroarea teoretica pentru aproximarea Lagrange este de ordinul 10^9 (?)

Eroarea practica Lagrange:

```
abs_err = max(double(subs(f, graph_x)) - lagrange_y)
```

abs_err = 0.9822

Masurarea teoretica erorii (Hermite):

Dorim o margine superioara pentru eroarea

$$err(x) = \left| \frac{f^{(m+1)}(\xi(x))}{(m+1)!} (x-x_0)^2 \ldots (x-x_m)^2 \right| \le \left| \frac{f^{(m+1)}(\xi(x))}{(m+1)!} \right| |(x-x_0)|^2 \ldots |(x-x_m)|^2$$

$$\le \left| \frac{f^{(m+1)}(\xi(x))}{(m+1)!} \right| ((x-x_0)\ldots(x-x_m))^2$$

In particular, $m = 9, x_i = 2\pi cos\left(\frac{(2i+1)\pi}{n}\right), i = \overline{0,9}$

```
M1 = 1/120+(2*pi)^4/9 + 2*(2*pi)^8/315 + (2*pi)^2/12 + 2*(2*pi)^6/45 + 4*(2*pi)^10/14175
```

```
M1 = 4.5394e+04
```

$\rightarrow |err1(\xi(x))| \le 5 \cdot 10^4$

```
M2 = M2^2 % ???
```

```
M2 = 3.5080e+10
```

Eroarea practica Hermite:

```
abs_err = max(double(subs(f, graph_x)) - hermite_y)
```

```
abs_err = 2.9346
```

```
function lk = L(nodes,k)
    % polinomul fundamental L_k pentru nodurile nodes_1,...,nodes_n
    % lk = prod_{j,j!=k}(x-xj)/prod_{j,j!=k}(xk-xj)
    m = size(nodes,2);
    roots = cat(2, nodes(1:k-1), nodes(k+1:m));
    p = poly(roots);
    lk = p ./ polyval(p,nodes(k));
end

function u = LagrangeClassic(x,y,t)
    m = size(x,2);
    n = size(t,2);
    pf = zeros('like', x);
    for k=1:m
        pf = pf + (L(x,k) .* y(k));
    end
    u = polyval(pf,t);
end

function c = chebNodes(n,a,b)
    c = (a+b)/2 + (b-a)/2*cos((2*[1:n]-1)*pi/(2*n));
```

```
end
```

```
function [ H, dH] = interpolareHermiteWithDerivative( x, f, fd, X )
    coefs=dif_div_duble(x,f,fd);

    coefs=coefs(1,:);

    z=repelem(x,2);
    H=X;
    dH=X;
    for k=1:length(X)
          P=1; DP=0;
      H(k)=coefs(1)*P; dH(k)=0;

      for i=2:length(coefs)
       DP=DP*(X(k)-z(i-1))+P;
       P=P*(X(k)-z(i-1));
       H(k)=H(k)+coefs(i)*P;
       dH(k)=dH(k)+coefs(i)*DP;
      end
    end
end

function T=dif_div_duble(x,f,df)
    T=NaN(2*length(x));
    z=repelem(x,2);
    T(:,1)=repelem(f,2);
    T(1:2:end-1,2)=df;
    T(2:2:end-2,2)=diff(f)./diff(x);
    for j=3:length(z)
         T(1:end-j+1,j)=diff(T(1:end-j+2,j-1))./(z(j:end)-z(1:end-j+1))';
    end
end
```