Lab 3 - Pb. 3

```
> f := (x1, x2, x3)->alpha*x1+beta*x3*exp(-c1*x3-c2*x1)
```

$$f := (x1, x2, x3) \mapsto \alpha \cdot x1 + \beta \cdot x3 \cdot e^{-c1 \cdot x3 - c2 \cdot x1} \tag{1}$$

a) Equilibrium points

```
> x01,x02 := solve(f(x,x,x)=x,x)
```

$$x01, x02 := 0, -\frac{\ln\left(-\frac{\alpha - 1}{\beta}\right)}{c1 + c2} \tag{2}$$

b) Stability

```
> study_stability :=proc(x0)
    local p1,p2,p3,lineq,chareq,cond1,cond2;
   p1 := D[1](f)(x0,x0,x0);
   p2 := D[2](f)(x0,x0,x0);
   p3 := D[3](f)(x0,x0,x0);
   lineq := y(n+1)=p1·y(n)+p2·y(n-1)+p3·y(n-2);
  chareq := r^3=p1·r^2+p2·r+p3;
  print("Char eq.:");
  print(chareq);
  # Schur-Cohn algorithm (degree 3)
  cond1 := simplify(abs(p3+p1)<1-p2);
  cond2 := simplify(abs(p1·p3+p2)<1-p3^2);
  print("Point ",x0," is stable if:");
 print(cond1);
  print(cond2);
  return cond1,cond2 :
  end;
```

```
study_stability := proc(x0)
    local p1, p2, p3, lineq, chareq, cond1, cond2;
    p1 := D[1](f)(x0,x0,x0);
    p2 := D[2](f)(x0,x0,x0);
    p3 := D[3](f)(x0,x0,x0);
    lineq := y(n+1)=p1*y(n)+p2*y(n-1)+p3*y(n-2);
    chareq := r^3=p1*r^2+p2*r+p3;
    print("Char eq.:");
    print(chareq);
    cond1 := simplify(abs(p1+p3)<1-p2);
    cond2 := simplify(abs(p3*p1+p2)<1-p3^2);
    print("Point ",x0," is stable if:");
    print(cond1);
    print(cond2);
```

(3)

```
    return cond1, cond2
end proc
```

```
> cond01, cond02 := study_stability(x01):
```

"Char eq.:"

$$r^3 = \alpha r^2 + \beta$$

"Point ", 0, " is stable if:"

$$|\alpha + \beta| < 1$$

$$|\beta\,\alpha| < -\beta^2 + 1 \tag{4}$$

```
> cond11, cond12 := study_stability(x02):
```

"Char eq.:"

$$r^3 = \left(\alpha + \frac{\beta \ln\left(-\frac{\alpha-1}{\beta}\right) c2\, e^{\frac{c1 \ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2} + \frac{c2 \ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2}}}{c1+c2}\right) r^2 + \beta\, e^{\frac{c1 \ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2} + \frac{c2 \ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2}} + \frac{\beta \ln\left(-\frac{\alpha-1}{\beta}\right) c1\, e^{\frac{c1 \ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2} + \frac{c2 \ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2}}}{c1+c2}$$

"Point ", $-\frac{\ln\left(-\frac{\alpha-1}{\beta}\right)}{c1+c2}$, " is stable if:"

$$\left|-1 + (\alpha-1) \ln\left(\frac{-\alpha+1}{\beta}\right)\right| < 1$$

$$\frac{\left|(\alpha-1)\left(\alpha(c1+c2) - \ln\left(\frac{-\alpha+1}{\beta}\right) c2\,(\alpha-1)\right)\left(\ln\left(\frac{-\alpha+1}{\beta}\right) c1 + c1 + c2\right)\right|}{|c1+c2|^2} < 1 - \frac{(\alpha-1)^2 \left(\ln\left(\frac{-\alpha+1}{\beta}\right) c1 + c1 + c2\right)^2}{(c1+c2)^2} \tag{5}$$

c) Num sim

```
> simulate := proc(alpha0, beta0, c10, c20, x0, x1, x2, n)
    local fl, x, i, k, cond0, cond1;
    fl := subs(alpha = alpha0, beta = beta0, c1 = c10, c2 = c20, (x1, x2, x3)→alpha*x1 + beta*x3*exp(
      −c1*x3−c2*x1));
    x[0] := x0; x[1] := x1; x[2] := x2;
    cond0 := subs(alpha = alpha0, beta = beta0, c1 = c10, c2 = c20, cond01 and cond02);
    print("Cond x01 stable: ", (cond0)); print("Cond x01 stable: ", eval(cond0));
```

```
    cond1 := subs(alpha = alpha0, beta = beta0, c1 = c10, c2 = c20, cond11 and cond12);
    print("Cond x02 stable:", (cond1)); print("Cond x02 stable:", eval(cond1));
     for i from 3 to n - 1 do
        x[i] := f1(x[i - 3], x[i - 2], x[i - 1]);
     end;
     plot([[k, x[k]]$k = 0 ..n])
   end;
simulate := proc(α0, β0, c10, c20, x0, x1, x2, n)                                    (6)
    local f1, x, i, k, cond0, cond1;
    f1 := subs(α = α0, β = β0, c1 = c10, c2 = c20, (x1, x2, x3)→α*x1 + β*x3*exp( - c1*x3 - c2
    *x1));
    x[0] := x0;
    x[1] := x1;
    x[2] := x2;
    cond0 := subs(α = α0, β = β0, c1 = c10, c2 = c20, cond01 and cond02);
    print("Cond x01 stable: ", cond0);
    print("Cond x01 stable: ", eval(cond0));
    cond1 := subs(α = α0, β = β0, c1 = c10, c2 = c20, cond11 and cond12);
    print("Cond x02 stable:", cond1);
    print("Cond x02 stable:", eval(cond1));
    for i from 3 to n - 1 do x[i] := f1(x[i - 3], x[i - 2], x[i - 1]) end do;
    plot([[k, x[k]]$(k = 0 ..n)])
end proc
```

```
> simulate(0.4, 0.1, 0.1, 2, 0, 0.2, 0.4, 20)
```

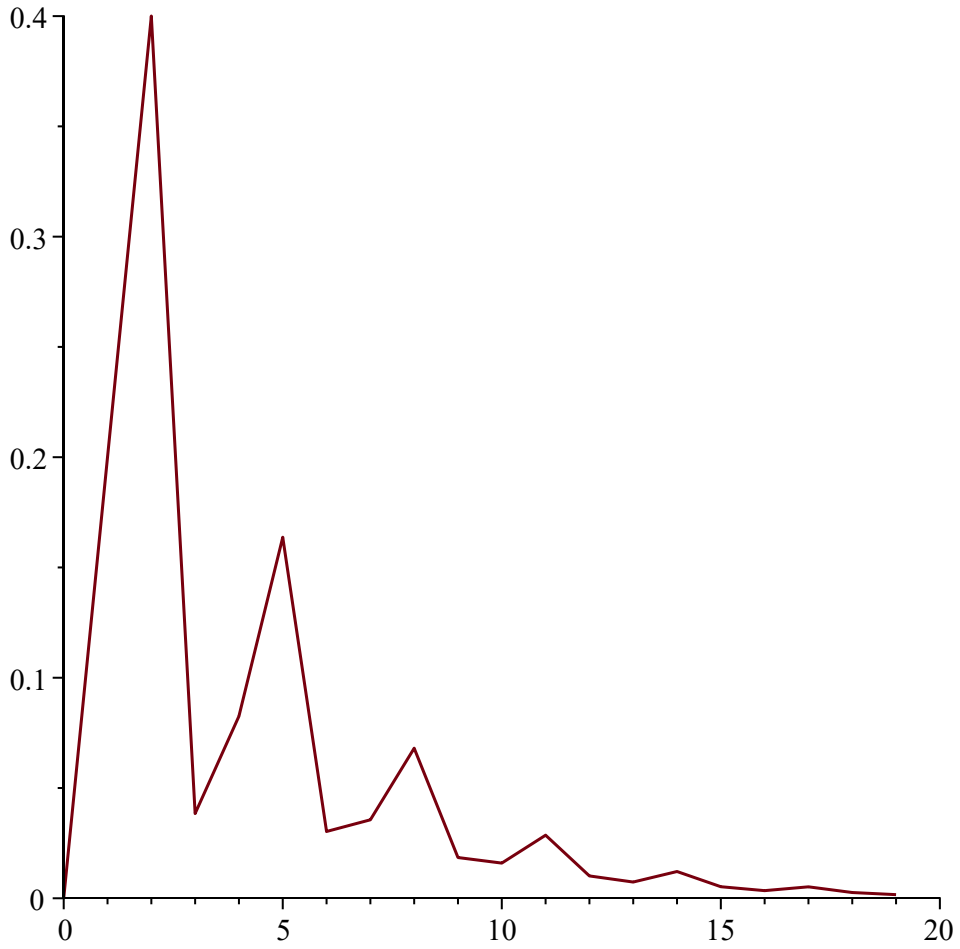
"Cond x01 stable: ", $|0.5| < 1$ **and** $|0.04| < 0.99$

"Cond x01 stable: ", *true*

"Cond x02 stable:", $|-1 - 0.6\ln(6.)| < 1$ **and** $\frac{|-0.6\,(0.84 + 1.2\ln(6.))\,(0.1\ln(6.) + 2.1)|}{|2.1|^2} < 1 - 0.08163265306\,(0.1\ln(6.) + 2.1)^2$

"Cond x02 stable:", *false*

Warning, data could not be converted to float Matrix




```
> simulate(0.4, 0.7, 0.1, 2, 0, 0.2, 0.4, 30)
```

"Cond x01 stable: ", $|1.1| < 1$ **and** $|0.28| < 0.51$

"Cond x01 stable: ", *false*

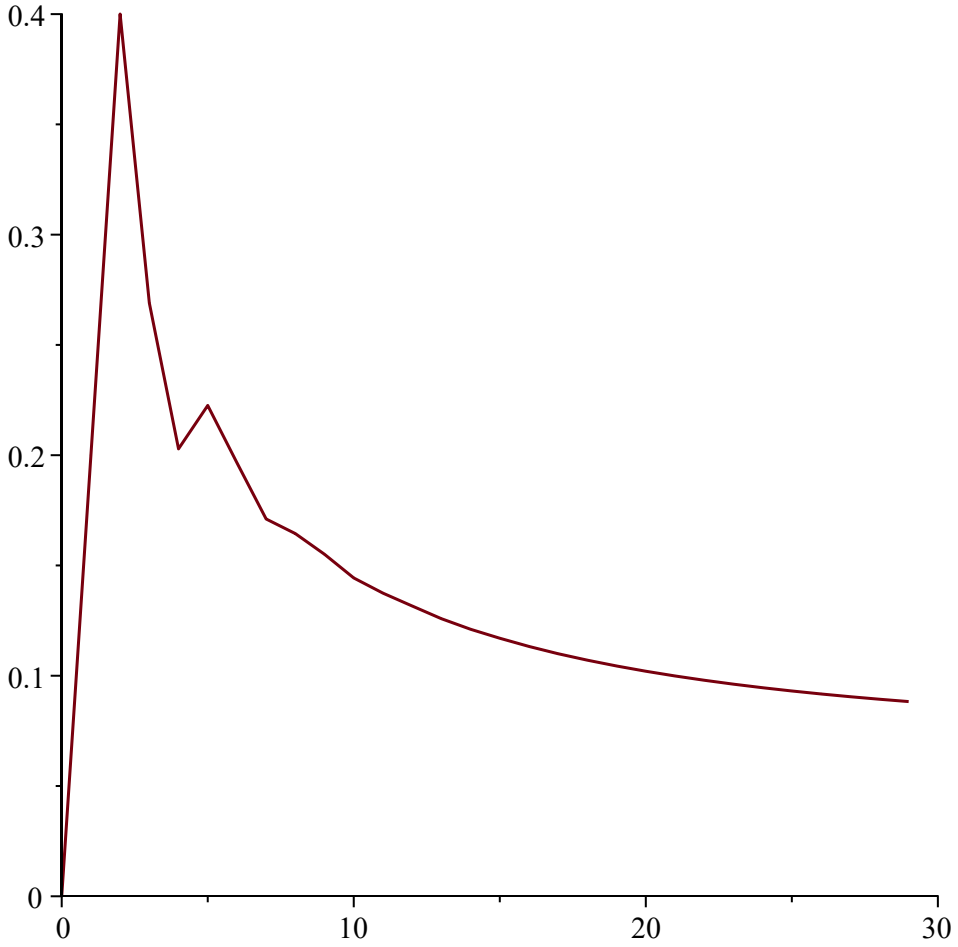
"Cond x02 stable:", $|-1 - 0.6\ln(0.8571428574)| < 1$ **and**

$$\frac{|-0.6\,(0.84 + 1.2\ln(0.8571428574))\,(0.1\ln(0.8571428574) + 2.1)|}{|2.1|^2} < 1$$

$$-0.08163265306\,(0.1\ln(0.8571428574) + 2.1)^2$$

"Cond x02 stable:", *true*

Warning, data could not be converted to float Matrix




```
> simulate(0.4, 0.6, 2, 1, 0.1, 0.2, 0.4, 100)
```

"Cond x01 stable: ", $|1.0| < 1$ **and** $|0.24| < 0.64$

"Cond x01 stable: ", *false*

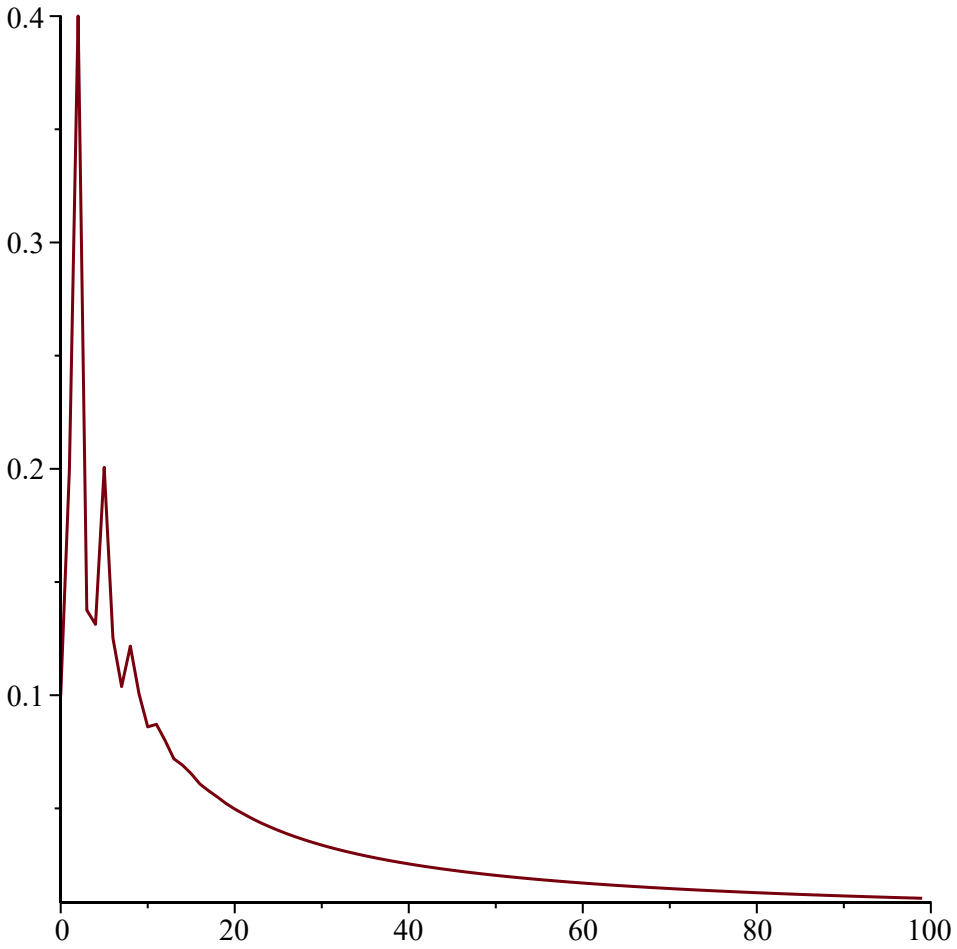
"Cond x02 stable:", $|-1 - 0.6\ln(1.000000000)| < 1$ **and**

$$\frac{|-0.6\,(1.2 + 0.6\ln(1.000000000))\,(2\ln(1.000000000) + 3)|}{|3|^2} < 1$$

$$-0.04000000000\,(2\ln(1.000000000) + 3)^2$$

"Cond x02 stable:", *false*

Warning, data could not be converted to float Matrix


0.4
0.3
0.2
0.1
0
20
40
60
80
100