KD Ex 1 – Solutions Neacșu-Miclea Liviu-Ștefan ICA 246/2

G1

a)

1. {Bean}' = {needs water to live, lives on land, needs chlorophyll to produce food, two seed leaves}
2. {lives on land}' = {Frog, Reed, Bean, Maize}
3. {two seed leaves}' = {Bean}
   {two seed leaved}'' = {Bean}' = {needs water to live, lives on land, needs chlorophyll to produce food, two seed leaves}
4. {Frog}' = {needs water to live, lives in water, lives on land, can move around, has limbs, suckles its offspring}
   {Maize}'={needs water to live, lives on land, needs chlorophyll to produce food, one seed leaf} => {Frog, Maize}' = {Frog}' ∩ {Maize}' = {needs water to live, lives on land}
5. {needs chlorophyll to produce food}’ = {Spike-Weed, Reed, Bean, Maize}
   {can move around}’ = {Leech, Bream, Frog}
   => {needs chlorophyll to produce food, can move around }’
   = {needs chlorophyll to produce food}’ ∩ {can move around}’ = ∅
6. {lives in water, lives on land}’ = {lives in water}’ ∩{lives on land}’={Frog, Reed}
7. Same as 5?

b)

$K_{new} = (G_{new}, M_{new}, I_{new})$, where:

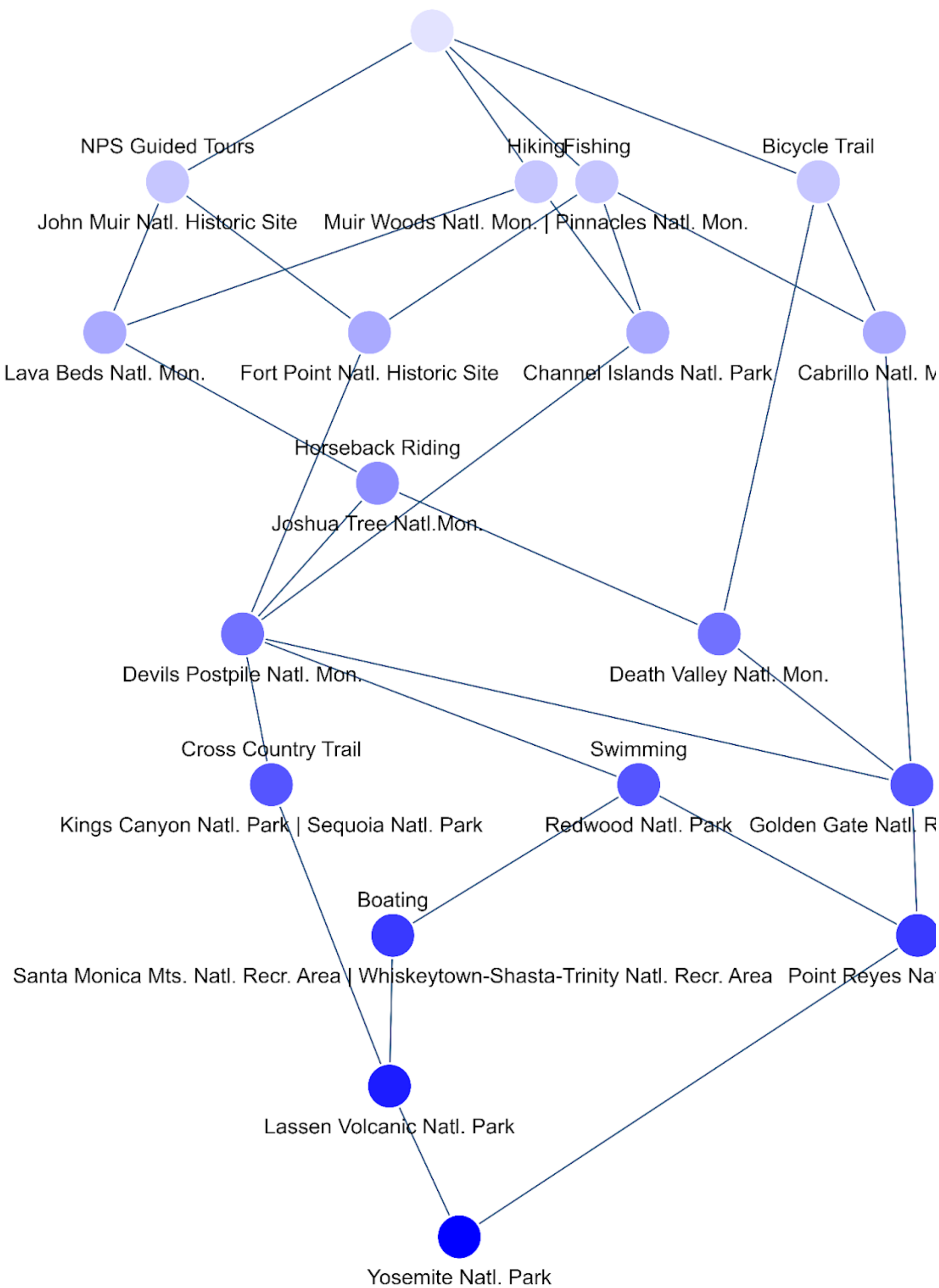
$G_{new} = G \cup \{Dolphin\}$

$M_{new} = M \cup \{is\ mammal\}$

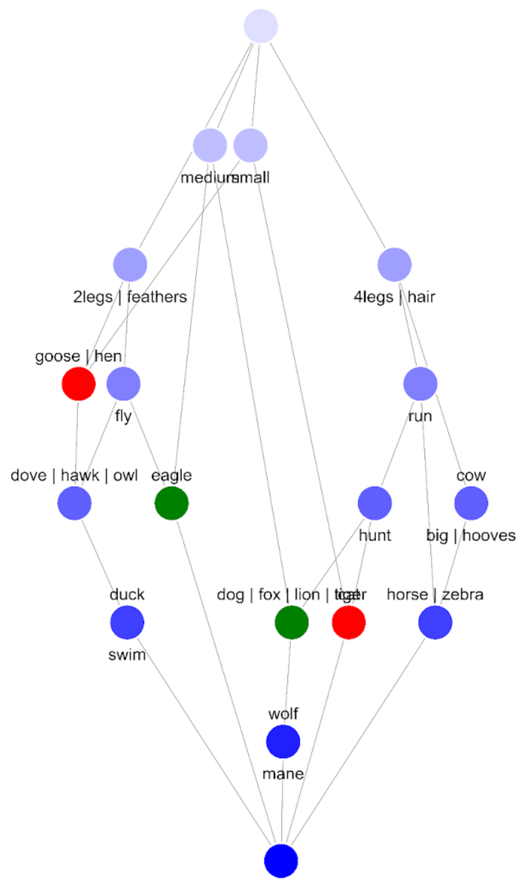
$I_{new} = I \cup \{(Dolphin, needs\ water\ to\ live), (Dolphin, lives\ in\ water),$

$(Dolphin, can\ move\ around), (Dolphin, is\ mammal)\}$.

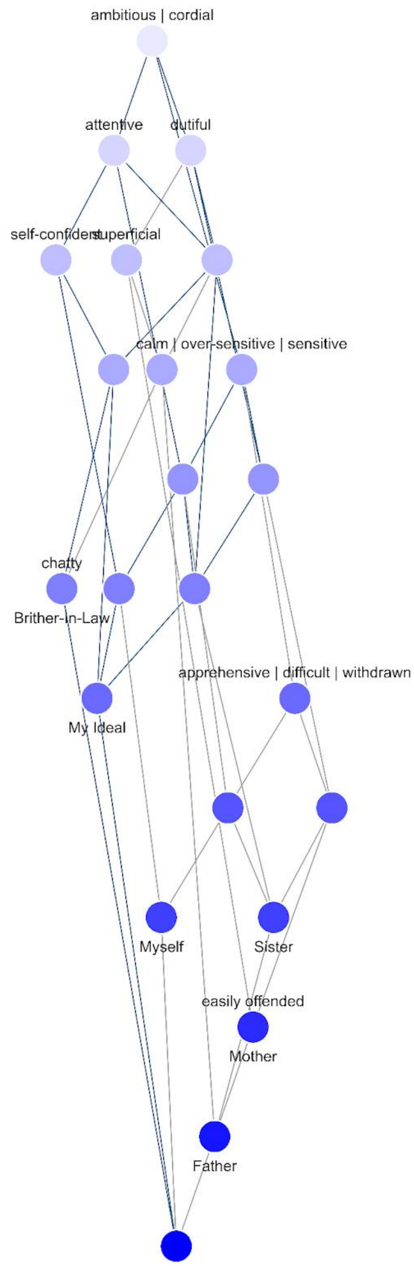
G2



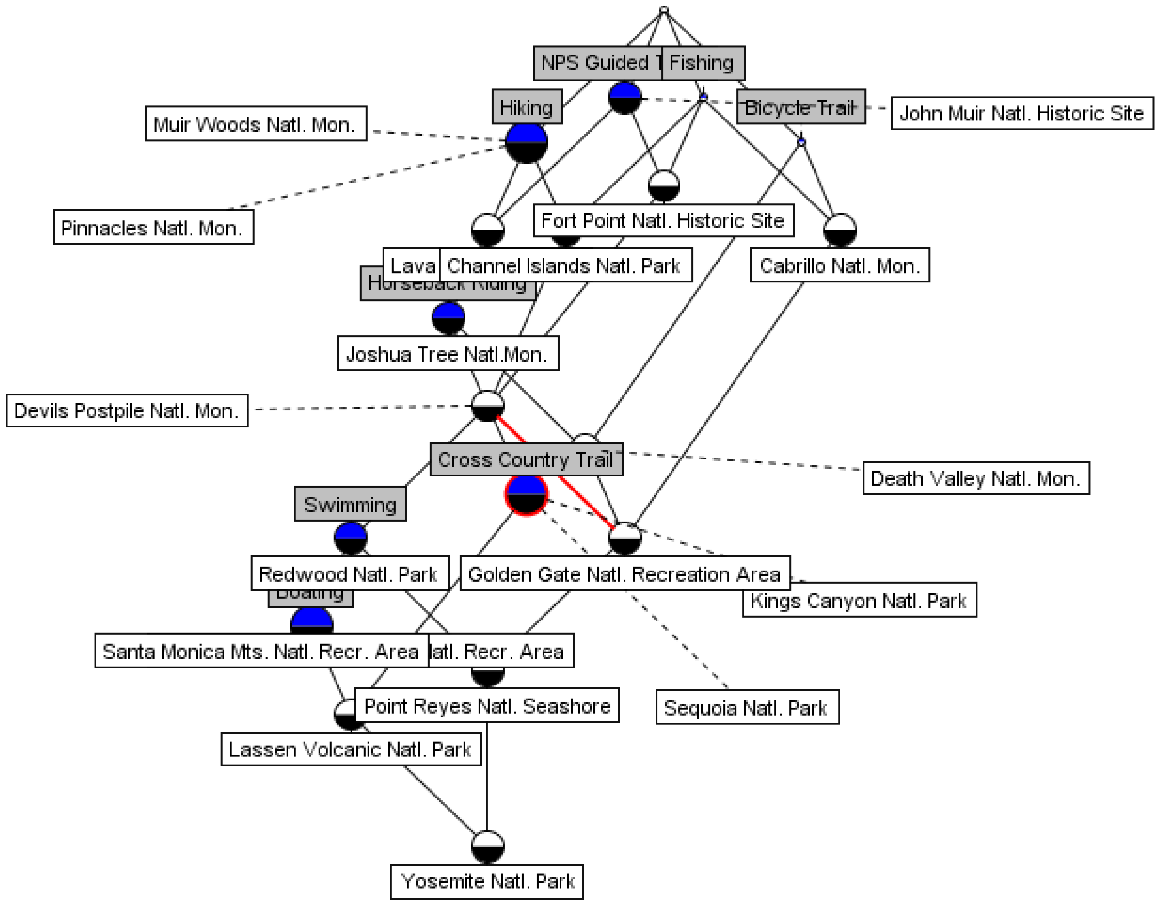
California National Parks – FCA tools bundle



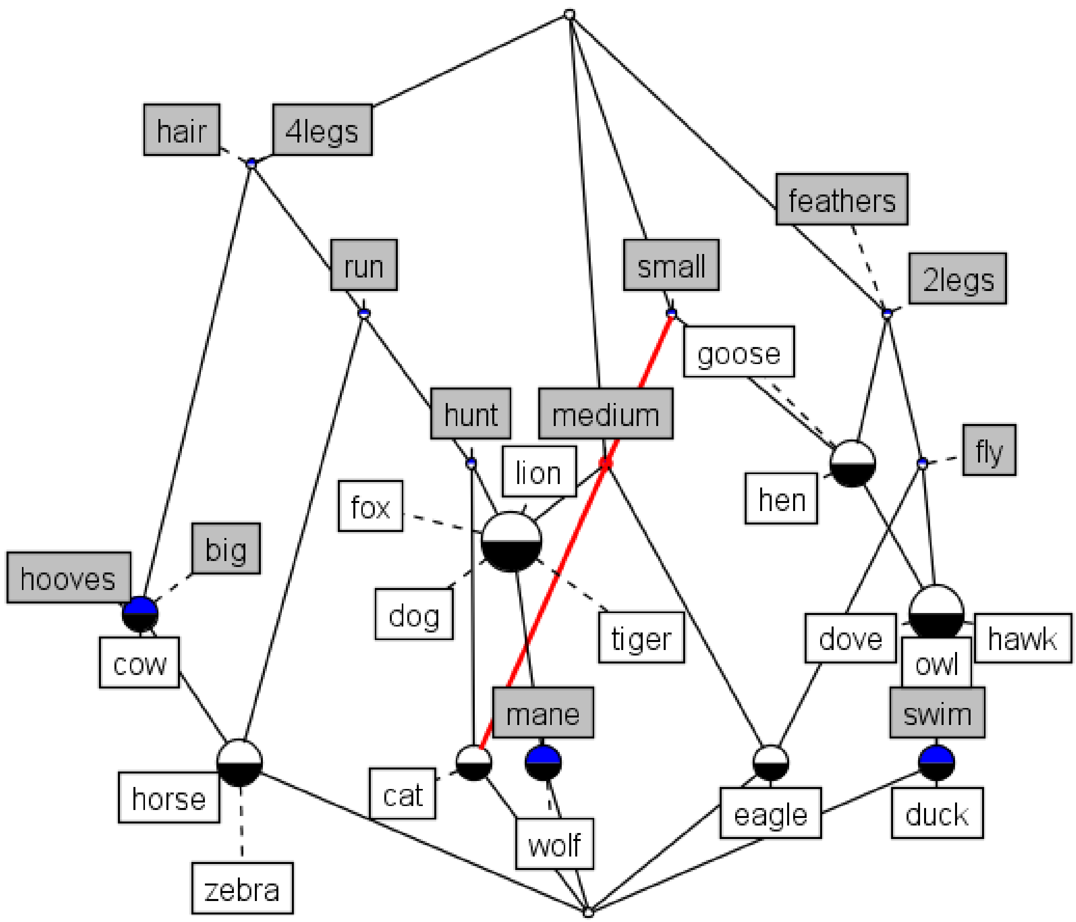
Birds & Animals – FCA tools bundle



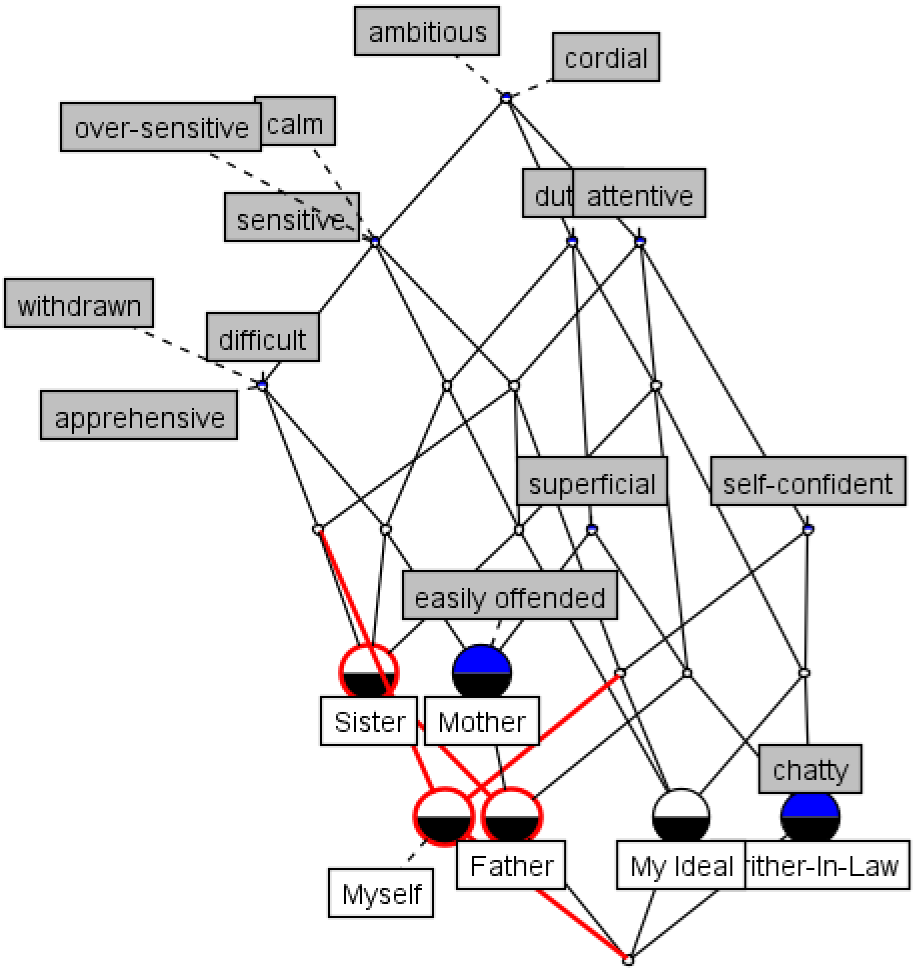
Psychological test – FCA tools bundle



National Parks California – conexp



Birds & Animals – conexp



Psychological test – conexp

G3

a)

Let $K = \{G, M, I\}$ be a format context, where G is the set of objects, M is the set of attributes,

$I \subseteq G \times M$ is the incidence relation.

For any subset $A \subseteq G$, the derivation operator is defined as:

$$A' = \{m \in M | \forall g \in A, (g, m) \in I\}.$$

For any subset $B \subseteq M$, the derivation operator is defined as:

$$B' = \{g \in G | \forall m \in B, (g, m) \in I\}.$$

b)

1. Suppose $A \subseteq B$. Then, there exists C such that $B = A \cup C$.

$$B' = \{m \in M | \forall g \in B, (g, m) \in I\}$$

$$A' = \{m \in M | \forall g \in A, (g, m) \in I\}$$

Let us arbitrarily choose an attribute $m \in B'$. Then, for each $g \in B$, we have $(g, m) \in I$. Since $A \subseteq B$, it follows that for each $g \in A$, $(g, m) \in I$ holds, thus $m \in A'$. Hence, $B' \subseteq A'$.

2.

$$A' = \{m \in M | \forall g \in A, (g, m) \in I\}$$

$$A'' = \{g \in G | \forall m \in A', (g, m) \in I\}$$

Let $a \in A$ be an arbitrary object. According to the definition of the derivation operator, $(a, m) \in I$ for each $m \in A'$. That means that $a \in A''$. Therefore, $A \subseteq A''$.

3. From 2, $A \subseteq A''$. Applying 1, we find

$$A''' \subseteq A'.$$

Similarly to 2, we can prove that $S \subseteq S''$ for each $S \subseteq M$. Since $A' \subseteq M$, it follows that

$$A' \subseteq A'''.$$

Finally, by the double inclusion principle, we obtain $A' = A'''$.

4.

direct proof. (C, D) formal concept $=> \exists E \in G, C = E'', D = E'$

If (C, D) is a formal concept, then C consists of exactly those objects that share all attributes in D, and D consists of exactly those attributes that apply to all objects in C.

In other words, we have $C' = D$ and $D' = C$.

From $C' = D$ results $C'' = D' = C$.

Taking $E = C$, we get $E'' = C$ and $E' = D$.

inverse proof. From $C = E''$ derive once get $C' = E''' = E'$ using property 3.

But $D = E'$ too so $C' = D$.

From $D = E'$ derive once and obtain $D' = E'' = C$.

Thus we found $C' = D$ and $D' = C$, which means that (C, D) resembles a formal concept.\