Assignment 1

1. Find the solution for the following difference equations:

restart :

$$sol\_a := unapply\left(rsolve\left(\left\{x(n+1) = \left(\frac{n+1}{n+2}\right)^2 x(n) + \frac{1}{n+2}, x(0) = 1\right\}, x(n)\right), n\right)$$

$$sol\_a := n \mapsto \frac{n+2}{2\cdot(n+1)} \tag{1}$$

$$sol\_b := unapply\left(rsolve\left(\{x(n+3) = 4\cdot x(n+2) - x(n+1) - 6\cdot x(n) + 60\cdot 4^n, x(0) = 2, x(1) = 12, x(2) = 12\}, x(n)\right), n\right)$$

$$sol\_b := n \mapsto -4\cdot(-1)^n - 16\cdot 3^n + 16\cdot 2^n + 6\cdot 4^n \tag{2}$$

$$eq\_c := x(n+1) = \frac{2\cdot x(n)}{1+4\cdot x(n)}$$

$$eq\_c := x(n+1) = \frac{2\,x(n)}{1+4\,x(n)} \tag{3}$$

$$eq\_c\_y := \frac{1}{subs\left(x(n) = \frac{1}{y(n)}, x(n+1) = \frac{1}{y(n+1)}, eq\_c\right)}$$

$$eq\_c\_y := y(n+1) = \frac{y(n)\left(1+\frac{4}{y(n)}\right)}{2} \tag{4}$$

$$sol\_c := unapply\left(\frac{1}{rsolve\left(\left\{eq\_c\_y, \mathrm{y}(0) = \frac{1}{1}\right\}, y(n)\right)}, n\right)$$

$$sol\_c := n \mapsto \frac{1}{-3\cdot\left(\frac{1}{2}\right)^n + 4} \tag{5}$$

2. Let us consider the difference equation

$$x(n+1) = \frac{x(n)^2+7}{x(n)} \tag{6}$$

(a) Find the equilibrium points and study their stability.

restart :

$$f := x \rightarrow \frac{x^2+7}{2\,x}$$

$$f := x \mapsto \frac{x^2+7}{2\cdot x} \tag{7}$$

$$solve(x = f(x), x)$$

$$\sqrt{7}, -\sqrt{7} \tag{8}$$

$$evalf(\mathrm{D}(f)(\mathrm{sqrt}(7))), evalf(\mathrm{D}(f)(-\mathrm{sqrt}(7)))$$

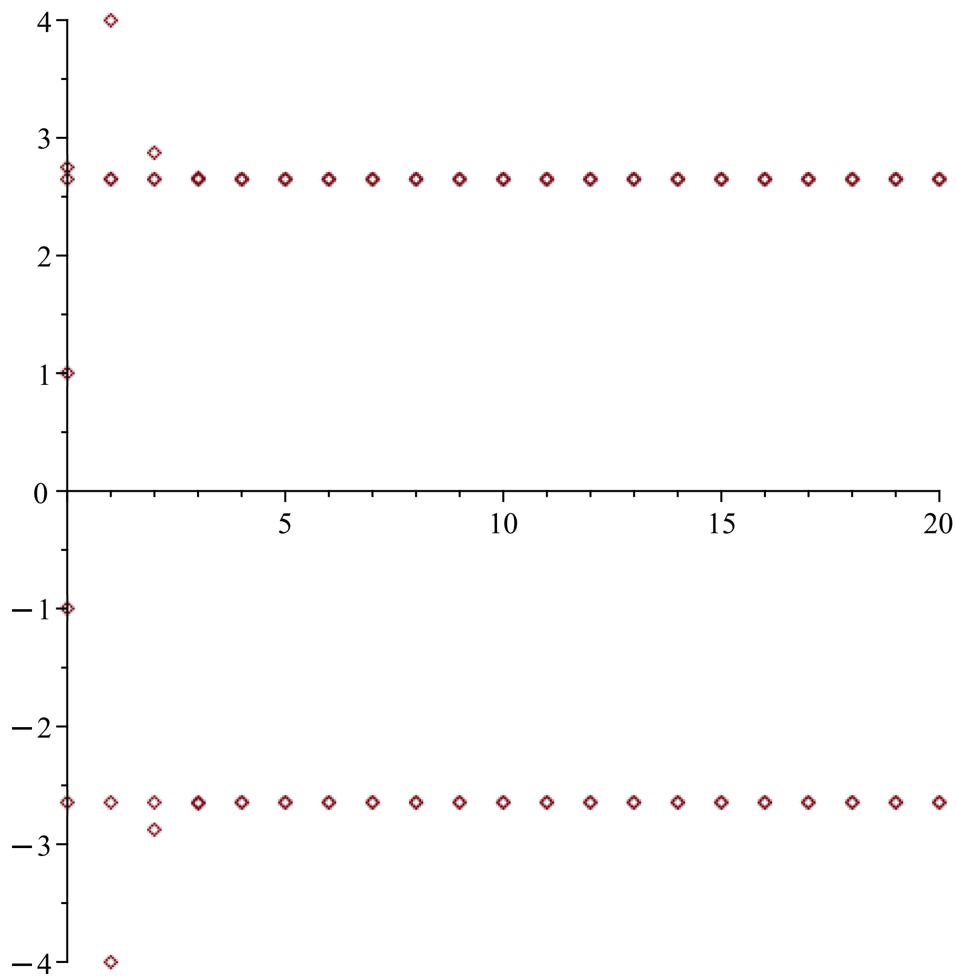
$$0., 0. \tag{9}$$

$\sqrt{7}, -\sqrt{7}$ locally asimptotically stable

(b) Make some numerical simulations.

```
gen_seq := proc(x0, N)
  local x, i;
  x[0] := x0;
  for i from 1 to N do x[i] := evalf(f(x[i - 1])); end:
  return x
end:
N := 20 :
a := gen_seq(1, N) :
b := gen_seq(sqrt(7) + 0.1, N) :
c := gen_seq(-1, N) : d := gen_seq(sqrt(7), N) : e := gen_seq(-sqrt(7), N) :
```

```
plot([[n, a[n]]$n = 0..N, [n, b[n]]$n = 0..N, [n, c[n]]$n = 0..N, [n, d[n]]$n = 0..N, [n, e[n]]$n = 0..N], style
    = point)
```



3. Let us consider the difference equation

$x(n+1) = x(n)^2 - 3$

(a) Find the 2-periodic cycle and study its stability

$f := x \rightarrow x^2 - 3$

$$f := x \mapsto x^2 - 3 \tag{10}$$

$\mathrm{f2} := \mathrm{unapply}(f(f(x)), x)$

$$f2 := x \mapsto (x^2 - 3)^2 - 3 \tag{11}$$

$b1, b2, b3, b4 := solve(f2(b) = b, b)$

$$b1, b2, b3, b4 := -2, 1, \frac{1}{2} - \frac{\sqrt{13}}{2}, \frac{1}{2} + \frac{\sqrt{13}}{2} \tag{12}$$

$b3$

$$\frac{1}{2} - \frac{\sqrt{13}}{2} \tag{13}$$

$evalf(\,\mathrm{abs}(\mathrm{D}(f)(b1) \cdot \mathrm{D}(f)(b2)) < 1)$

$$8. < 1. \tag{14}$$

$\mathrm{D}(f)(b1)\cdot\mathrm{D}(f)(b2)$

$$-8 \tag{15}$$

$evalf(\,\mathrm{abs}(\mathrm{D}(f2)(b3))\,<\,1)$

$$6.788897448 < 1. \tag{16}$$

$evalf(\,\mathrm{abs}(\mathrm{D}(f2)(b4))\,<\,1)$

$$21.21110256 < 1. \tag{17}$$

$solve(f(x) = x, x)$

$$\frac{1}{2} + \frac{\sqrt{13}}{2}, \frac{1}{2} - \frac{\sqrt{13}}{2} \tag{18}$$

All 4 equilibrium points are locally unstable.

(b) Make numerical simulation

```
with(plots) :

cobweb := proc(f, xmin, xmax, a0, n)
local i, j, x, a, l, g1, g2;
a[0] := a0;
l[0] := [a[0], 0] :
 for i from 1 to n do
 a[i] := evalf(f(a[i−1])) :
 l[2*i−1] := [a[i−1], a[i]] :
 l[2*i] := [a[i], a[i]] :
 end do:
 g1 := plot([l[j]$j = 0 ..2*N], style = line, color = black) :

g2 := plot([f(x), x], x = xmin ..xmax, color = [red, blue], discont = true) :
 display(g1, g2);
end:

N := 10;
```

$$N := 10 \tag{19}$$

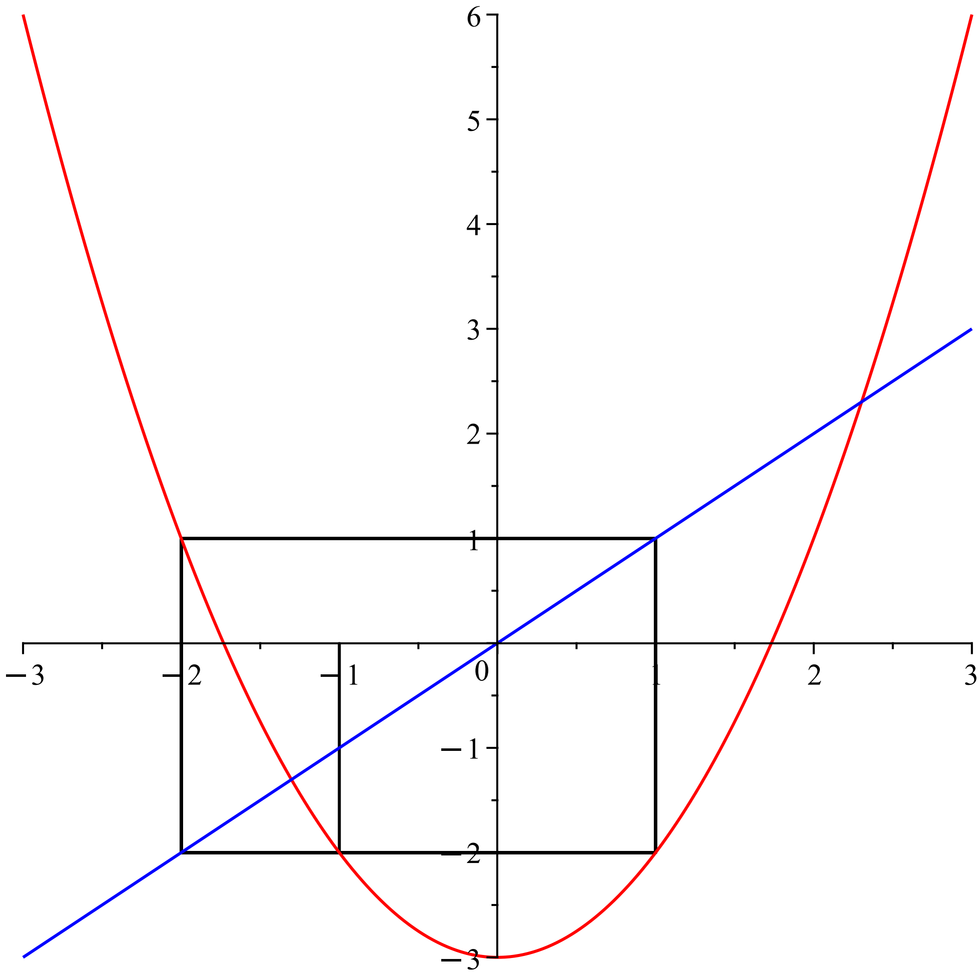
$a[0] := 2;$ **for** i **from** 1 **to** N **do** $a[i] := f(a[i-1]);$ **end**;

$$a_0 := 2$$
$$a_1 := 1$$
$$a_2 := -2$$
$$a_3 := 1$$
$$a_4 := -2$$
$$a_5 := 1$$
$$a_6 := -2$$
$$a_7 := 1$$
$$a_8 := -2$$

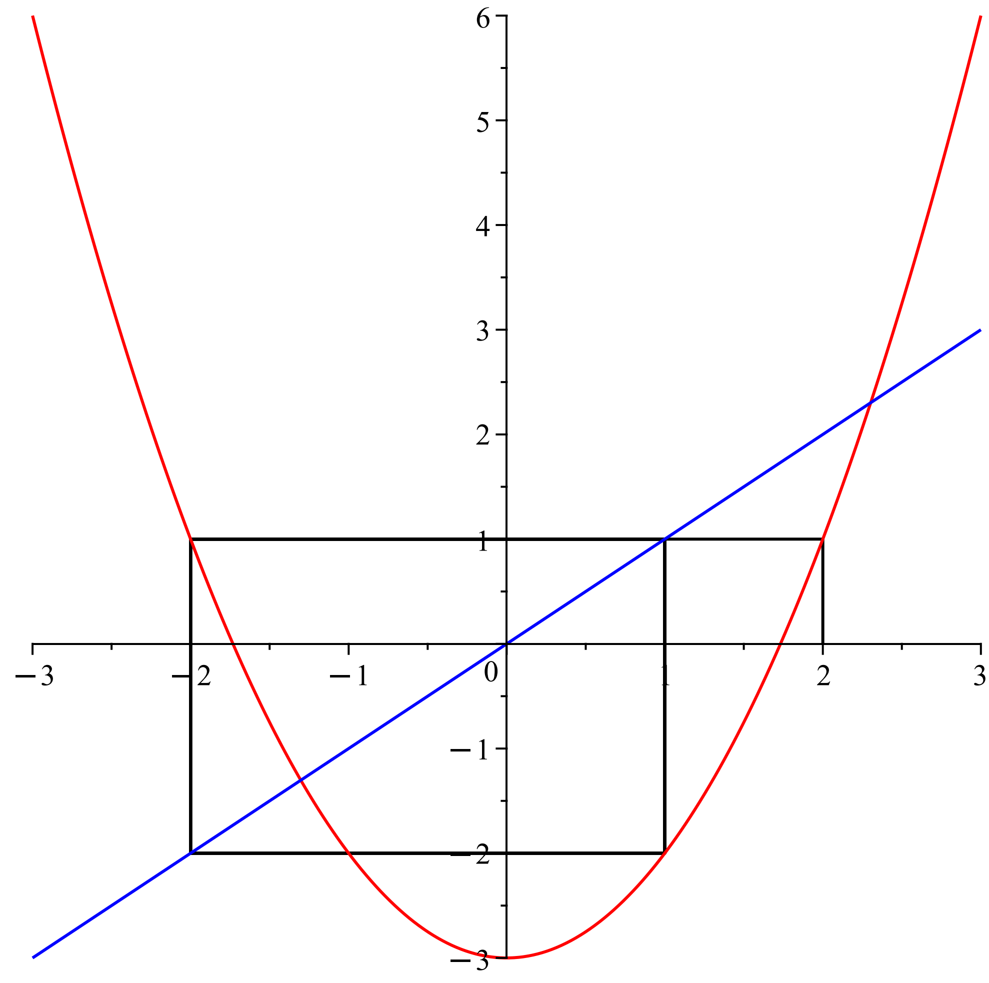
$$a_9 := 1$$

$$a_{10} := -2 \tag{20}$$

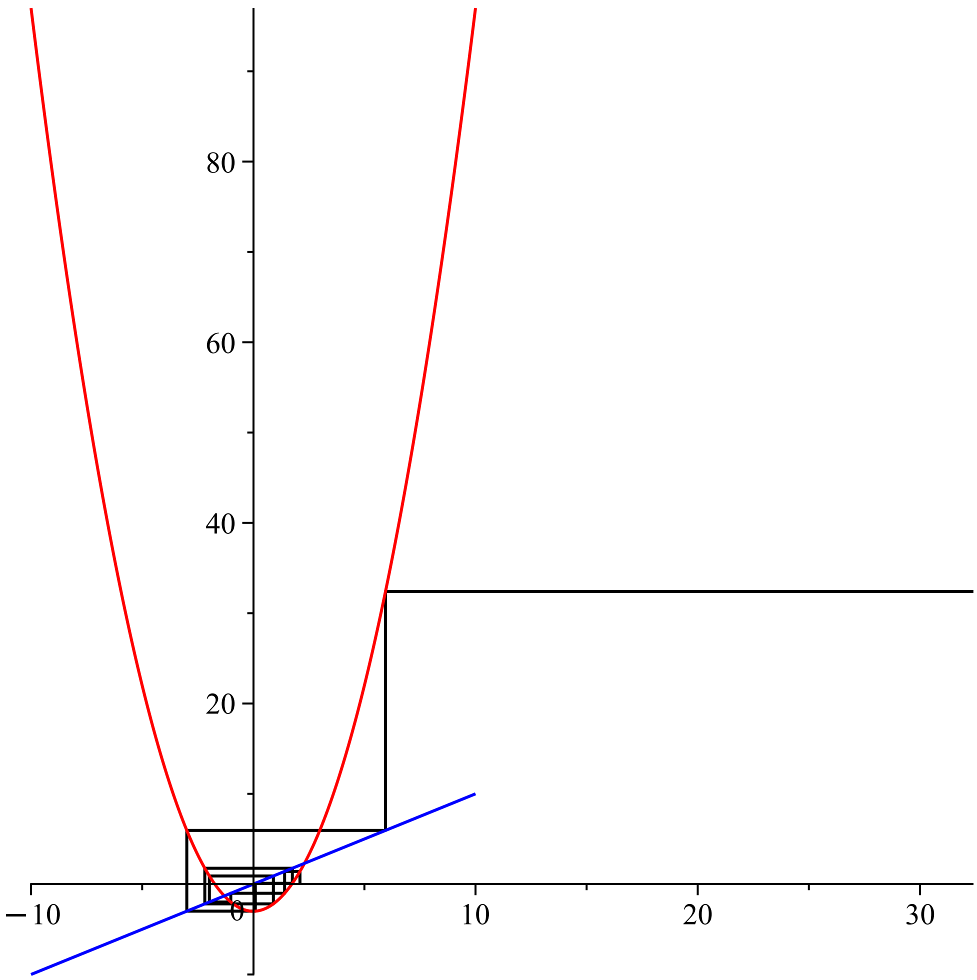
cobweb(f, −3, 3, −1, N) ;



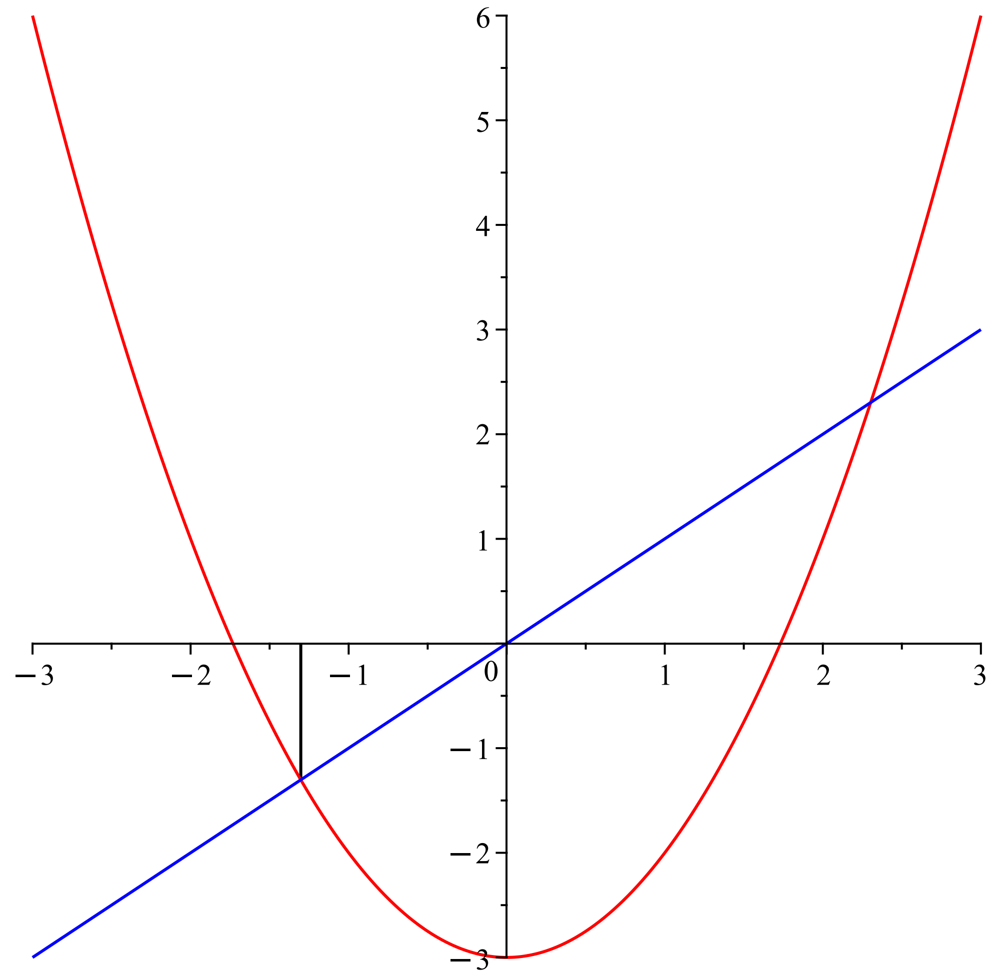
cobweb(f, −3, 3, 2, N) ;



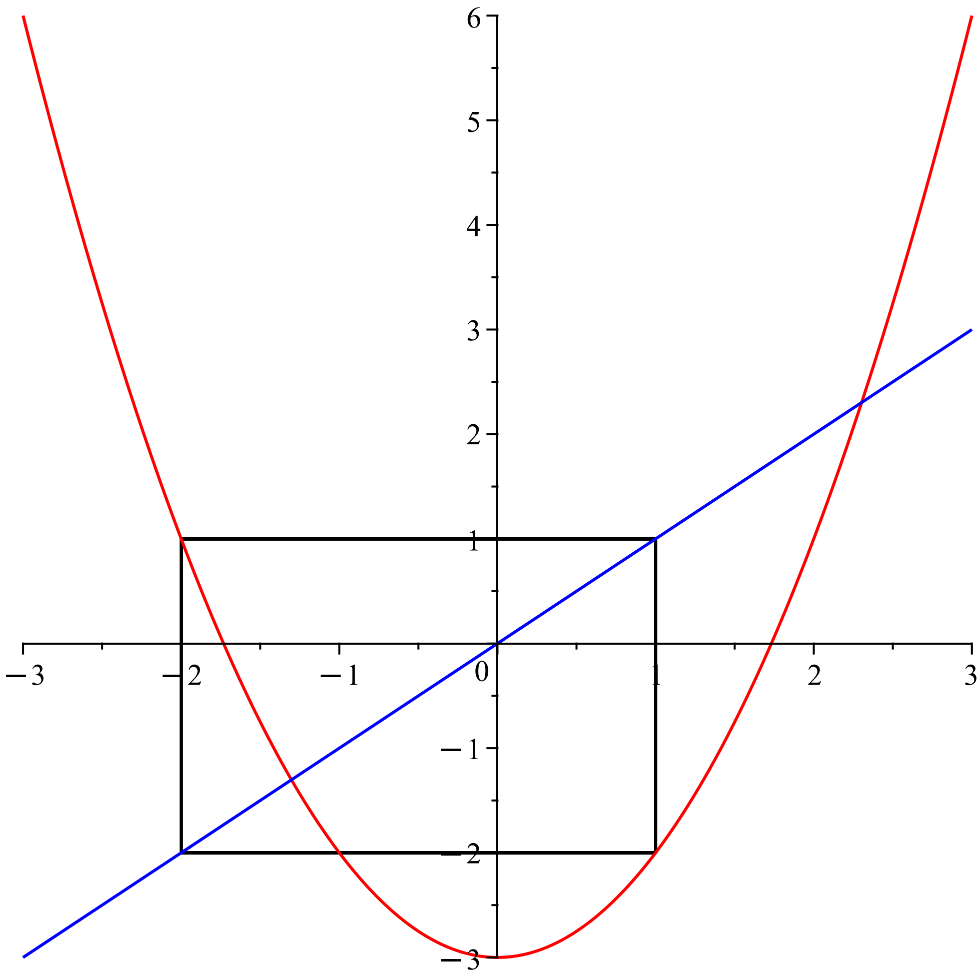
cobweb(f, −10, 10, 2.1, N) ;



$$cobweb\left(f, -3, 3, \frac{1}{2} - \frac{\sqrt{13}}{2}, N\right)$$



$cobweb(f, -3, 3, -2, N)$



4.

$S1(n, p, S0) := (1 + n \cdot p)S0$

$$S1 := (n, p, S0) \mapsto (1 + p \cdot n) \cdot S0 \tag{21}$$

$S2(n, p, r, S0) := \left(1 + \frac{p}{r}\right)^n S0$

$$S2 := (n, p, r, S0) \mapsto \left(1 + \frac{p}{r}\right)^n \cdot S0 \tag{22}$$

$SA := unapply(S1(n, 0.04, S0), n, S0)$

$$SA := (n, S0) \mapsto (1 + 0.04 \cdot n) \cdot S0 \tag{23}$$

$SB := unapply(S2(12 \cdot n, 0.03, 12, S0), n, S0)$

$$SB := (n, S0) \mapsto 1.002500000^{12 \cdot n} \cdot S0 \tag{24}$$

(a)

$SA(5, 1000), SA(10, 1000), SA(15, 1000), SA(20, 1000)$

$$1200.00, 1400.00, 1600.00, 1800.00 \tag{25}$$

$SB(5, 1000), SB(10, 1000), SB(15, 1000), SB(20, 1000)$

$$1161.616782, 1349.353547, 1567.431725, 1820.754995 \tag{26}$$

(b)

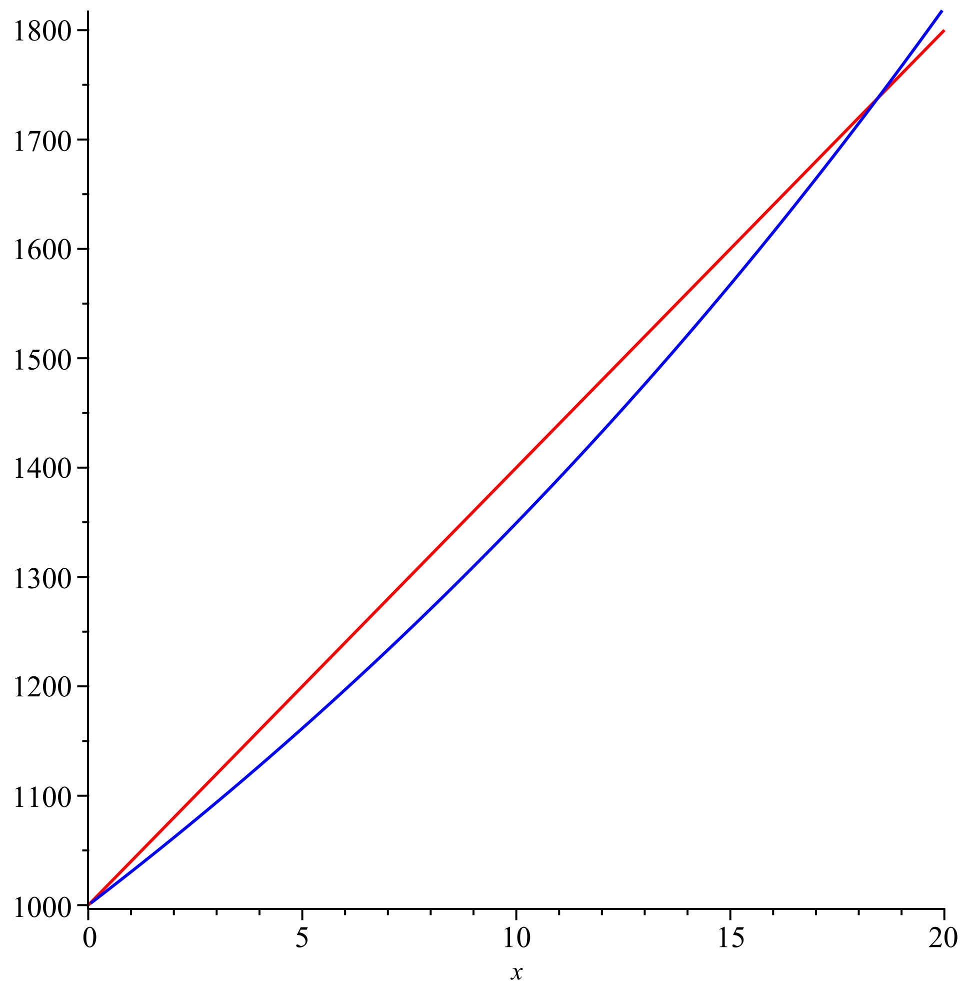
$solve(SA(n, S0) = SB(n, S0), n)$

$$2.669998613 \times 10^{-792}, 18.43930863 \quad \textbf{(27)}$$

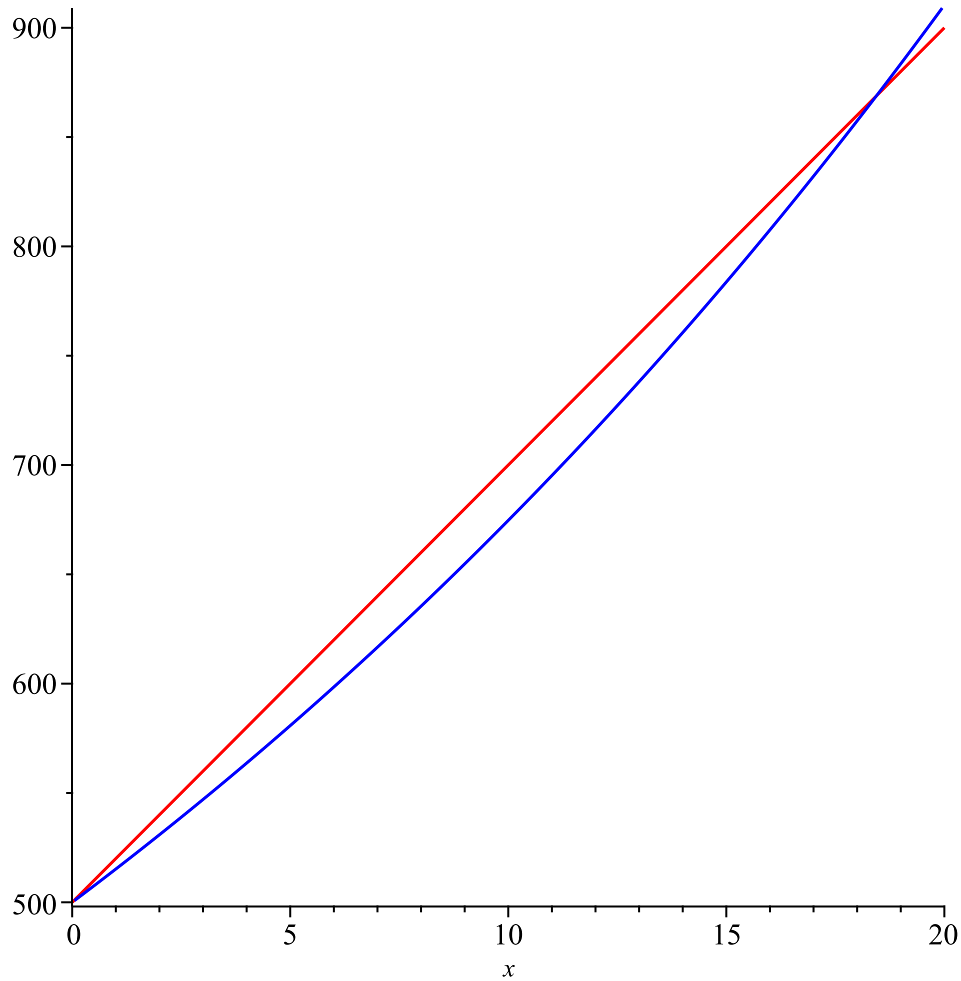
$n1 := 18.43930863$

$$n1 := 18.43930863 \quad \textbf{(28)}$$

$plot([SA(x, 1000), SB(x, 1000)], x = 0..20, color = [red, blue]);$



$plot([SA(x, 500), SB(x, 500)], x = 0..20, color = [red, blue]);$



900
800
700
600
500
0
5
10
15
20
x