KD Ex 2 – Neacșu-Miclea Liviu-Ștefan

G1.

a) A lattice is partially ordered set (poset) $(L, \leq)$ in which every pair of elements $a, b \in L$ has a greatest lower bound (GLB) $a \wedge b$ and a least upper bound (LUB) $a \vee b$.

b) The divisor lattice of 10 ordered by divisibility: $(L, \leq)$, where $L = \{1,2,5,10\}$ and $a \leq b$ if $a|b \ \ \forall a, b \in L$. This lattice has GLB=1 and LUB=10.

```
 10
 /\
2  5
 \/
 1
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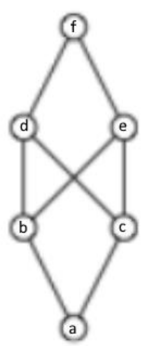
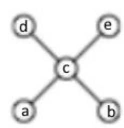
c) In the poset represented in diagram (i), f, d and e are upper bounds of the set {b,c}, but d and e are incomparable, so {b,c} has no upper bound, therefore (i) is not a lattice.

Diagram (v) does not represent a lattice, because incomparable elements a, b do not have lower bound, only the upper bound c. Therefore, (v) is not a lattice.





G2.

a) A lattice $(L, \leq)$ is complete if for every subset $S \subseteq L$:

- the least upper bound $\bigvee S$ exists
- the greatest lower bound $\bigwedge S$ exists.

Therefore, a complete lattice has a top element $\top = \bigvee L$ and a bottom element $\bot = \bigwedge L$. Unlike general lattices (which only require finite meets and joins), complete lattices guarantee meets and joins even for infinite subsets.

b) Every finite lattice is complete (since all subsets are finite). Therefore, all diagrams that represent lattices ((ii), (iii), (iv)) are complete.

c) The rational numbers in $[0,1]$ is a lattice, but it's not complete.
$(\mathbb{Q} \cap [0,1], \leq)$ with the usual order is a lattice: $\forall a, b \in L \; \exists \; a \wedge b = \min(a, b)\,, a \vee b = \max(a, b)$.
It is not a complete lattice, we can find a subset $\left\{\frac{[(\pi-3)\cdot 10^n]}{10^n} \; n \in \mathbb{N}^*\right\} \in \mathbb{Q} \cap [0,1]$ which is bounded by $\pi - 3 \notin \mathbb{Q} \cap [0,1]$, hence it has no upper bound in $\mathbb{Q}$.

G3

1	**Objects**	Tick Trick Track Donald Daisy Gustav Dagobert Annette Primus v. Quack
	Attributes	
2	**Objects**	Donald
	Attributes	middle male indebted
3	**Objects**	Dagobert
	Attributes	older male rich
4	**Objects**	Daisy Annette
	Attributes	female carefree
5	**Objects**	Tick Trick Track
	Attributes	younger male carefree
6	**Objects**	Donald Daisy Gustav
	Attributes	middle
7	**Objects**	Dagobert Annette Primus v. Quack
	Attributes	older
8	**Objects**	Tick Trick Track Daisy Gustav Annette Primus v. Quack
	Attributes	carefree
9	**Objects**	Tick Trick Track Donald Gustav Dagobert Primus v. Quack
	Attributes	male
10	**Objects**	Tick Trick Track Gustav Primus v. Quack
	Attributes	male carefree
11	**Objects**	Annette Primus v. Quack
	Attributes	older carefree
12	**Objects**	Dagobert Primus v. Quack
	Attributes	older male
13	**Objects**	Primus v. Quack
	Attributes	older male carefree
14	**Objects**	Daisy Gustav
	Attributes	middle carefree
15	**Objects**	Donald Gustav
	Attributes	middle male
16	**Objects**	Gustav
	Attributes	middle male carefree
17	**Objects**	Daisy
	Attributes	middle female carefree
18	**Objects**	Annette
	Attributes	older female carefree
19	**Objects**	
	Attributes	older middle younger male female rich carefree indebted

1. Write the attribute extents to a list

e1 = {older}’ = {Dagobert, Annette, Primus v. Quack}

e2 = {middle}’ = {Donald, Daisy, Gustav}

e3 = {younger}’ = {Tick, Trick, Track}

e4 = {male}’ = {Tick, Trick, Track, Donald, Gustav, Dagobert, Primus v. Quack}

e5 = {female}’ = {Daisy, Annette}

e6 = {rich}’ = {Dagobert}

e7 = {carefree}’ = {Tick, Trick, Track, Daisy, Gustav, Annette, Primus v. Quack}

e8 = {indebted}’ = {Donald}

2. Compute all pairwise intersections (if not already present), and 3. add G

e9 = e1 ∩ e4 = {Dagobert, Primus v. Quack}

e10 = e1 ∩ e6 = Ø

e11 = e1 ∩ e7 = {Primus v. Quack}

e12 = e2 ∩ e4 = {Donald, Gustav}

e13 = e2 ∩ e5 = {Daisy}

e14 = e2 ∩ e7 = {Daisy, Gustav}

e15 = e2 ∩ e8 = {Donald}

e16 = e3 ∩ e4 = {Tick, Trick, Track}

e17 = e3 ∩ e7 = {Tick, Trick, Track}

e18 = e4 ∩ e7 = {Tick, Trick, Track, Gustav, Primus v. Quack}

e19 = G = {Tick, Trick, Track, Donald, Daisy, Gustav, Dagobert, Annette, Primus v. Quack}

4. Compute the intents

(extent, intent = extend’)

1. ({Dagobert, Annette, Primus v. Quack}, {older})

2. ({Donald, Daisy, Gustav}, {middle})

3. ({Tick, Trick, Track}, {younger})

4. ({Tick, Trick, Track, Donald, Gustav, Dagobert, Primus v. Quack}, {male})

5. ({Daisy, Annette}, {female})

6. ({Dagobert}, {rich})

7. ({Tick, Trick, Track, Daisy, Gustav, Annette, Primus v. Quack}, {carefree})

8. ({Donald}, {indebted})

9. ({Dagobert, Primus v. Quack}, {older, male})

10. (∅, { older middle younger male female rich carefree indebted })

11. ({Primus v. Quack}, {older, male, carefree})

12. ({Donald, Gustav}, {middle, male})

13. ({Daisy}, {middle, female, carefree})

14. ({Daisy, Gustav}, {middle, carefree})

15. ({Donald}, {middle, male, indebted})