Lab 3 - Pb4

$with(linalg):$

$f := (x, y) \rightarrow vector([b \cdot y, c \cdot x + s \cdot y - D \cdot y^2])$

$$f := (x, y) \mapsto vector([b \cdot y, c \cdot x + s \cdot y - D \cdot y^2]) \tag{1}$$

a) Equilibrium

$equil1, equil2 := solve([f(x, y)[1] = x, f(x, y)[2] = y], \{x, y\})$

$$equil1, equil2 := \{x = 0, y = 0\}, \left\{x = \frac{b\,(b\,c + s - 1)}{D}, y = \frac{b\,c + s - 1}{D}\right\} \tag{2}$$

b) Stability

$J := jacobian(f(x, y), [x, y]);$

$$J := \begin{bmatrix} 0 & b \\ c & -2\,D\,y + s \end{bmatrix} \tag{3}$$

$A1 := subs(equil1, jacobian(f(x, y), [x, y]))$

$$A1 := \begin{bmatrix} 0 & b \\ c & s \end{bmatrix} \tag{4}$$

$eigen1 := eigenvals(A1)$

$$eigen1 := \frac{s}{2} + \frac{\sqrt{4\,b\,c + s^2}}{2}, \frac{s}{2} - \frac{\sqrt{4\,b\,c + s^2}}{2} \tag{5}$$

$A2 := subs(equil2, jacobian(f(x, y), [x, y]))$

$$A2 := \begin{bmatrix} 0 & b \\ c & -2\,b\,c - s + 2 \end{bmatrix} \tag{6}$$

$eigen2 := eigenvals(A2)$

$$eigen2 := -b\,c - \frac{s}{2} + 1 + \frac{\sqrt{4\,b^2\,c^2 + 4\,b\,c\,s - 4\,b\,c + s^2 - 4\,s + 4}}{2}, -b\,c - \frac{s}{2} + 1 - \frac{\sqrt{4\,b^2\,c^2 + 4\,b\,c\,s - 4\,b\,c + s^2 - 4\,s + 4}}{2} \tag{7}$$

c) Simulation

$simulate := \mathbf{proc}(b0, c0, s0, D0, x0, y0, n)$

$\mathbf{local}\, x, y, f1, i, k, crt, e1, e2, params;$

$f1 := (x, y) \rightarrow vector([b0 \cdot y, c0 \cdot x + s0 \cdot y - D0 \cdot y^2]);$

$params := (b = b0, c = c0, s = s0, D = D0);$

$print(\text{"Eigenvalues for equil1"});$

$e1 := subs(params, eigen1[1]);$

```
e2 := subs(params, eigen1[2]);
print(evalf(e1), evalf(e2));
print("Eigenvalues for equil2");
e1 := subs(params, eigen2[1]);
e2 := subs(params, eigen2[2]);
print(evalf(e1), evalf(e2));

x[0] := x0; y[0] := y0;
for i from 1 to n do
  crt := f1(x[i − 1], y[i − 1]);
  x[i] := crt[1]; y[i] := crt[2];
end;
print('Final populations :')
print(x[n]);    print(y[n]);
plot([[x[k], y[k]]$k = 0 ..n])
end;
```

```
simulate := proc(b0, c0, s0, D0, x0, y0, n)                                (8)
    local x, y, f1, i, k, crt, e1, e2, params;
    f1 := (x, y)→linalg:-vector([b0*y, c0*x + s0*y − D0*y^2]);
    params := b = b0, c = c0, s = s0, D = D0;
    print("Eigenvalues for equil1");
    e1 := subs(params, eigen1[1]);
    e2 := subs(params, eigen1[2]);
    print(evalf(e1), evalf(e2));
    print("Eigenvalues for equil2");
    e1 := subs(params, eigen2[1]);
    e2 := subs(params, eigen2[2]);
    print(evalf(e1), evalf(e2));
    x[0] := x0;
    y[0] := y0;
    for i to n do crt := f1(x[i − 1], y[i − 1]); x[i] := crt[1]; y[i] := crt[2] end do;
    print('Final*populations; − 1')*print(x[n]);
    print(y[n]);
    plot([[x[k], y[k]]$(k = 0 ..n)])
end proc
```

```
simulate(1, 1, 1, 1, 2, 3, 5)
```

"Eigenvalues for equil1"

1.618033988, −0.6180339880

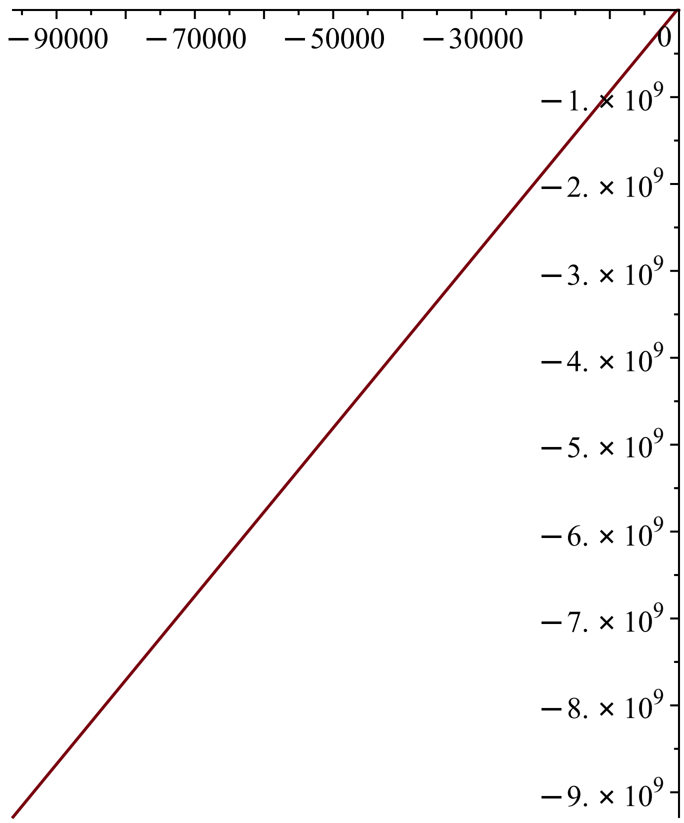
"Eigenvalues for equil2"

0.6180339880, -1.618033988

Final populations

-96427

-9298263066



simulate(0.1, 0.1, 2, 0.01, 100, 50, 200)

"Eigenvalues for equil1"

2.004987562, -0.004987562

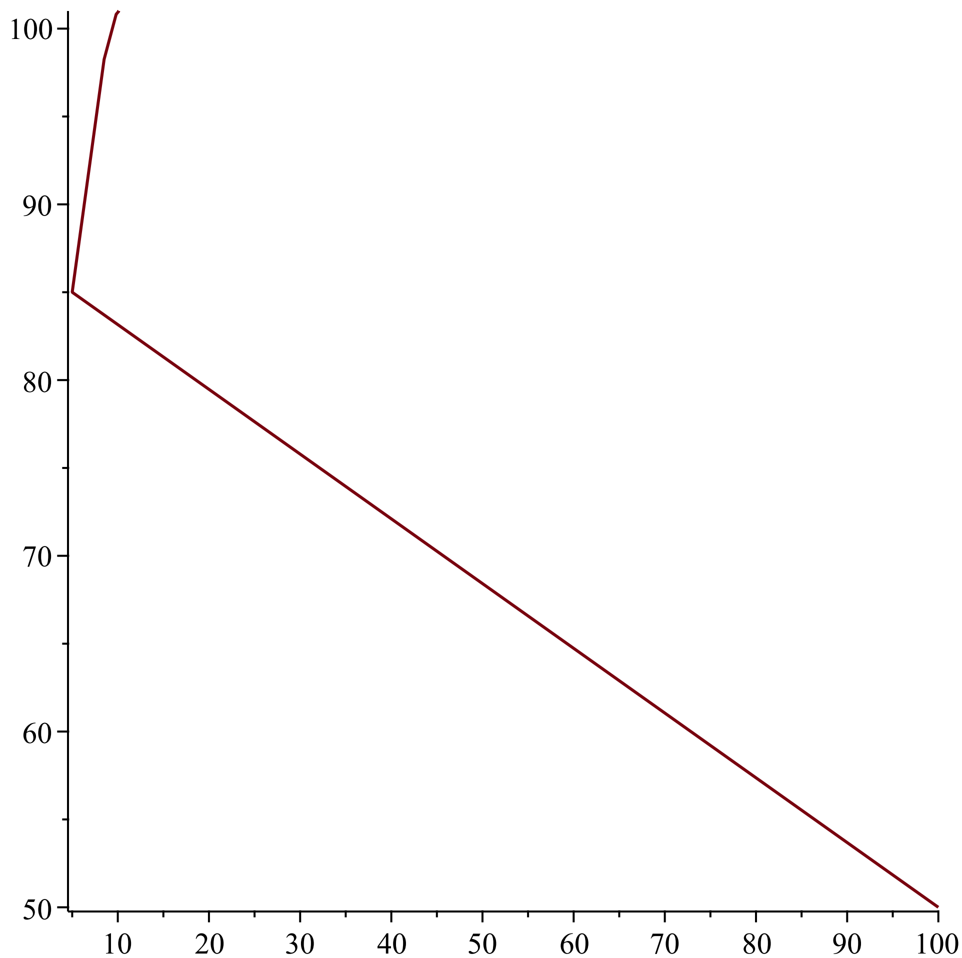
"Eigenvalues for equil2"

0.0904987562, -0.1104987562

Final populations

10.10000000

101.0000000



simulate(0.1, 0.1, 0.5, 0.01, 100, 50, 200) # *stable, decay to 0,0*

"Eigenvalues for equil1"

0.5192582404, −0.0192582404

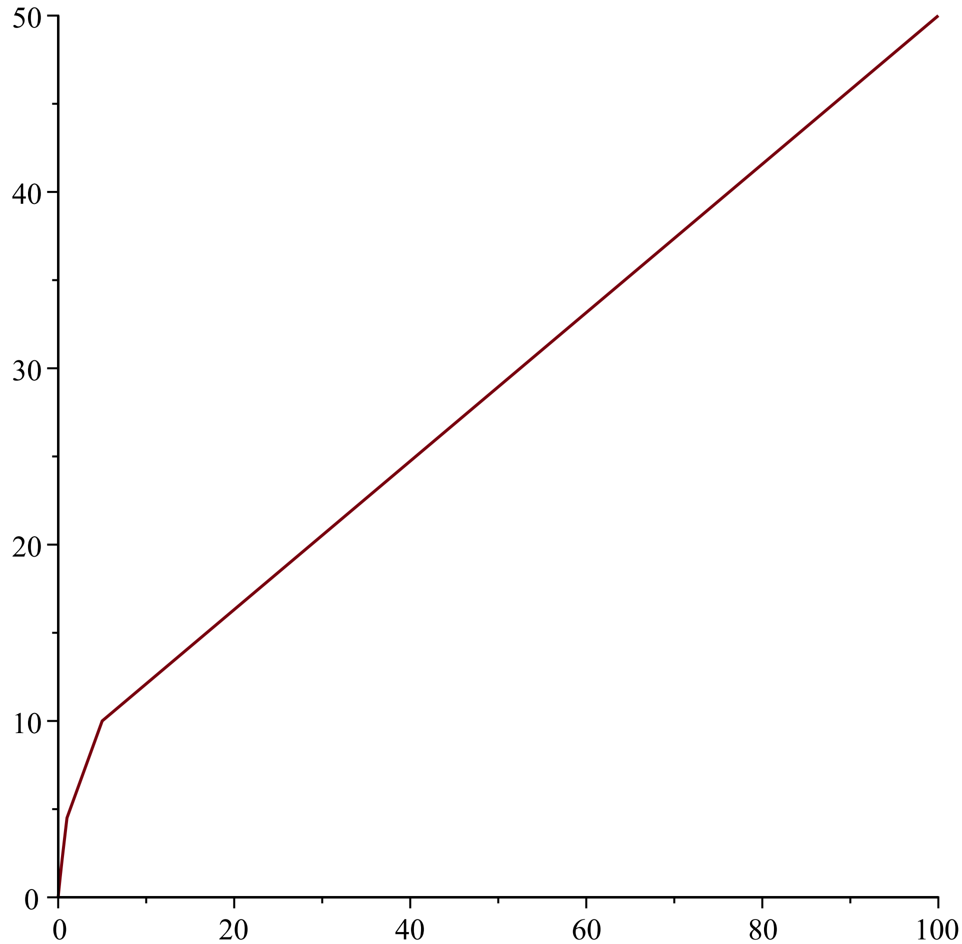
"Eigenvalues for equil2"

1.486726188, −0.0067261880

Final populations

$3.258184219 \times 10^{-57}$

$1.691839005 \times 10^{-56}$



50
40
30
20
10
0
0
20
40
60
80
100