COMPARTMENTAL MODELS USED IN EPIDEMIOLOGY

Presentation by

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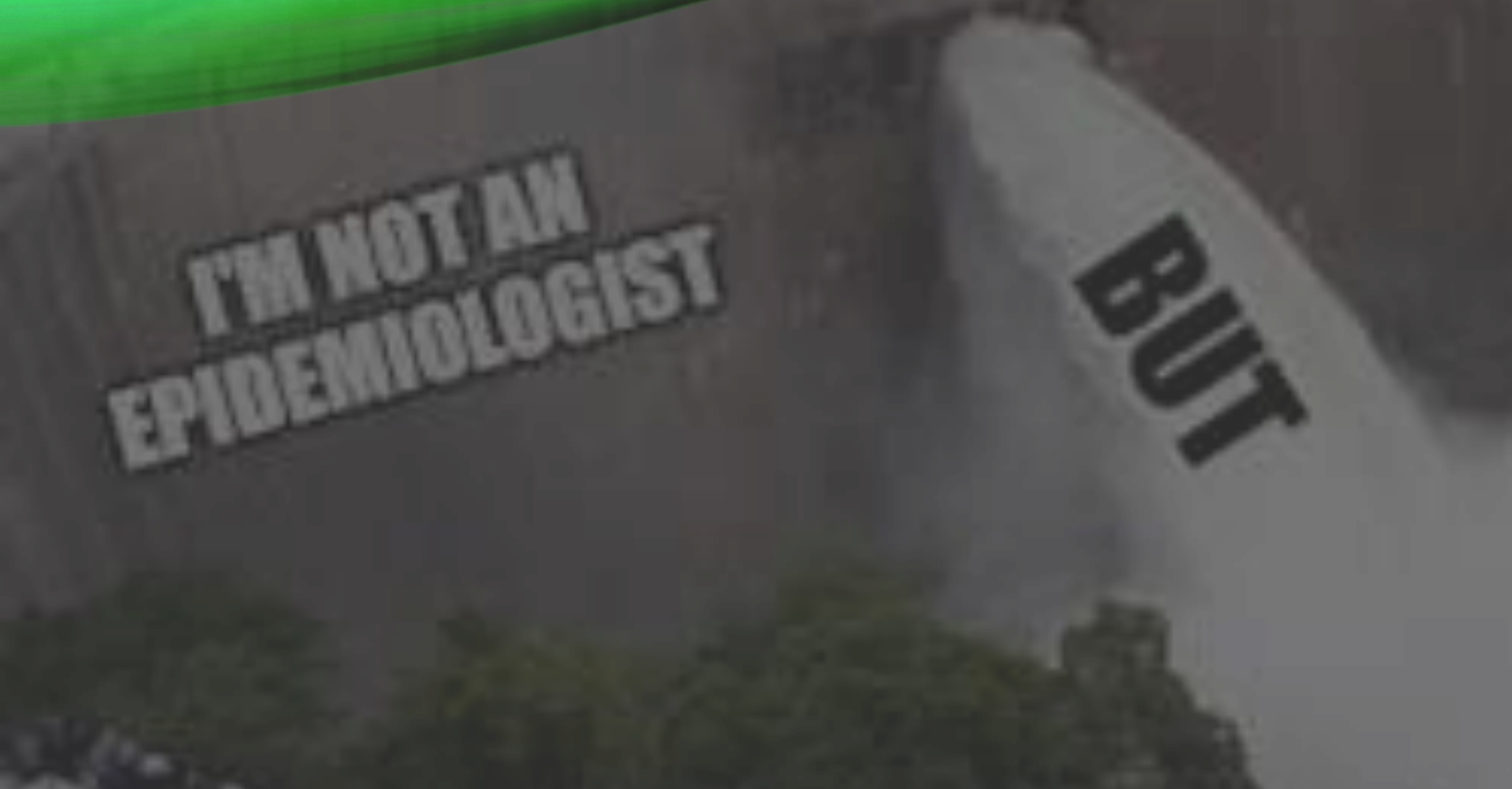
BACKGROUND



- Ongoing SARS – CoV – 2 outbreak
- Global impact on an unparalleled scale
- Missing information in terms of healthcare, political and societal implications
- Fake news, political benefit-forging

COMPARTMENTAL MODELS

= simplified technique to mathematically model infectious disease dynamics

Finds parameters for infectious diseases based on assumptions

Two main types: deterministic and stochastic

Deterministic: same parameters always return same results

Stochastic: basic equations as probability distributions

Benefits for laypeople:

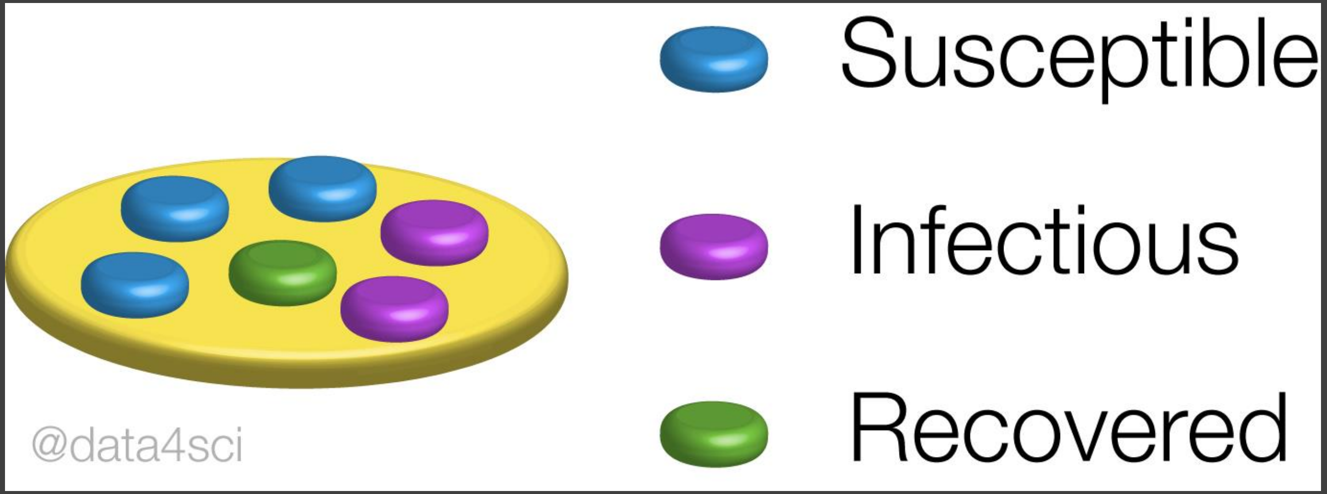
- Increased understanding of policies
- Correct interpretation of news

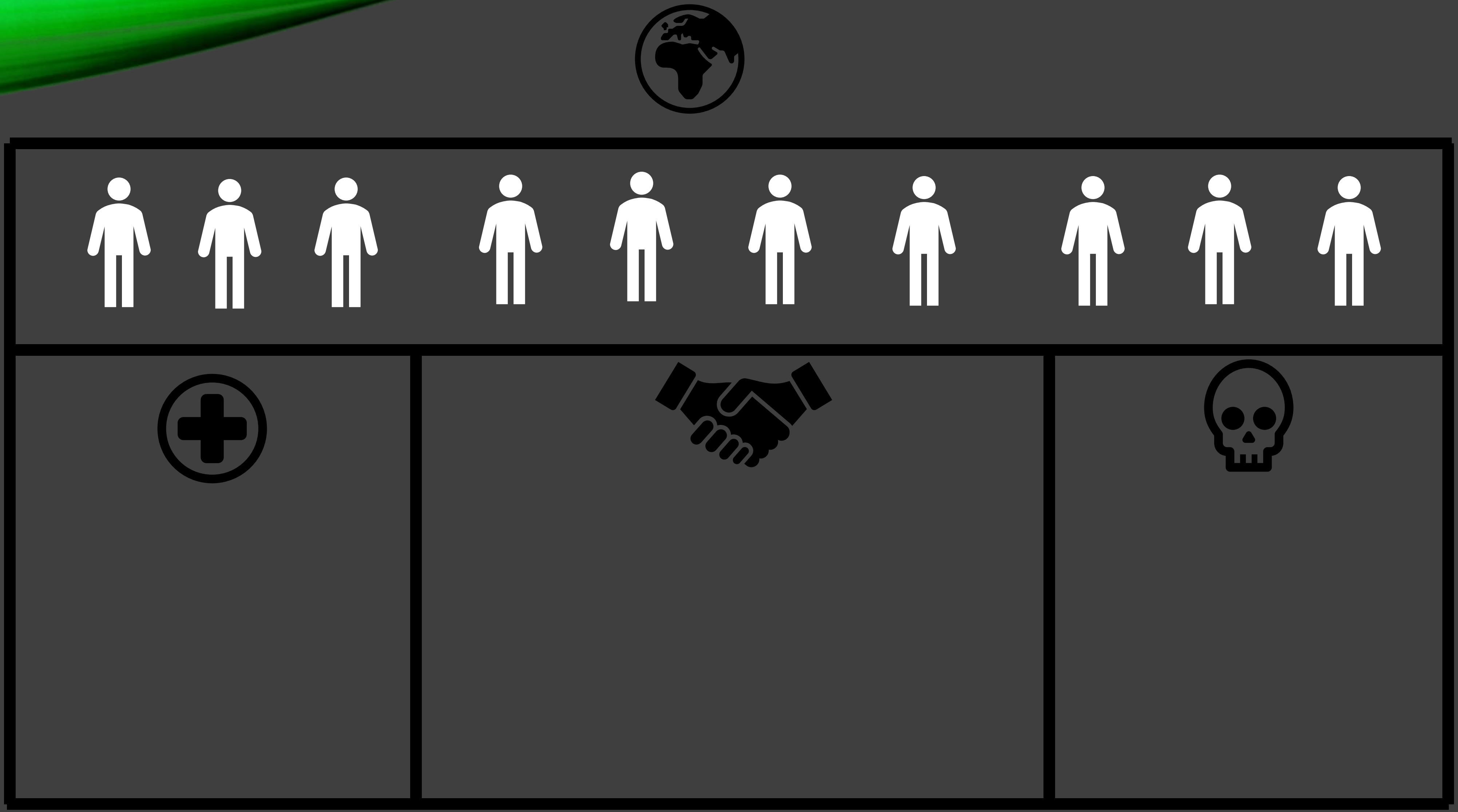
Benefits for specialists:

- Measure magnitude of epidemic
- Can plan restrictions and relaxations

WHAT DO WE MEAN BY ‘COMPARTMENTS’?

- Splits whole population into subgroups (‘compartments’)
- Subgroups labelled according to basic property (e.g. S, I, R)
- *Every* inhabitant assigned into *one* of groups
- Progression between compartments ~ disease dynamics





POPULATION: 190

MODELLING OF EPIDEMIOLOGY:

- *Kermack-McKendrick Theory* (1927)
- Number & distribution of infections over time
- Only infection and removal events
- ✔ Simple epidemic (incl. threshold condition for an outbreak)
- ✖ Endemic / recurring disease

- β: transmission rate
- γ: removal rate

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

ASSUMPTIONS

00

No assumptions for: linearity, normality or homogeneity

01

Whole population belongs to a certain group

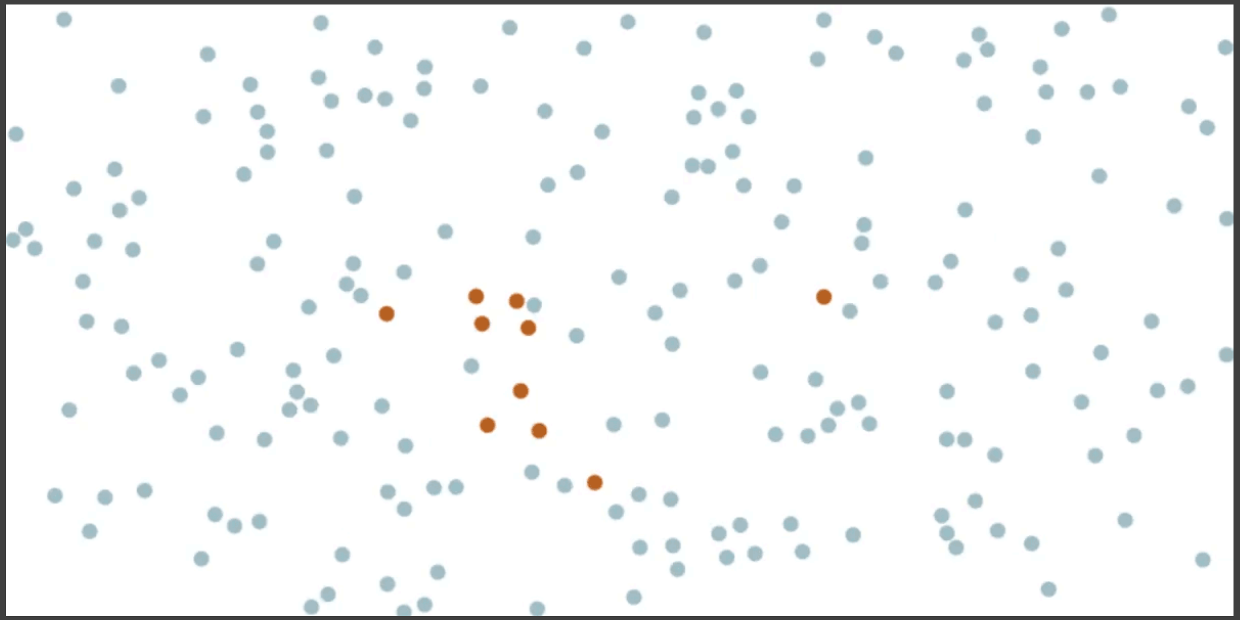
02

Rectangular and stationary age distribution

ASSUMPTIONS

03

Homogeneous, well mixed population where disease spreads evenly



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TYPES OF COMPARTMENTAL MODELS 1.

SIR (Susceptible – Infected - Recovered): Assumes lifelong immunity to the disease (most basic model), e.g. measles

SIS (Susceptible - Infected - Susceptible No immunity developed, e.g. common cold

SEIR (Susceptible – Exposed - Infected - Recovered) Non-infectious incubation period & lifelong immunity after recovery, e.g. chicken pox

TYPES OF COMPARTMENTAL MODELS 2.

SEIRS (Susceptible – Exposed - Infected – Recovered - Susceptible)
Only temporal immunity, disease becomes an *endemic*, e.g. malaria

MSIR (Maternally derived immunity – Susceptible – Infectious - Recovered)
Babies gain immunity thanks to the antibodies in the body of their mothers, e.g. measles

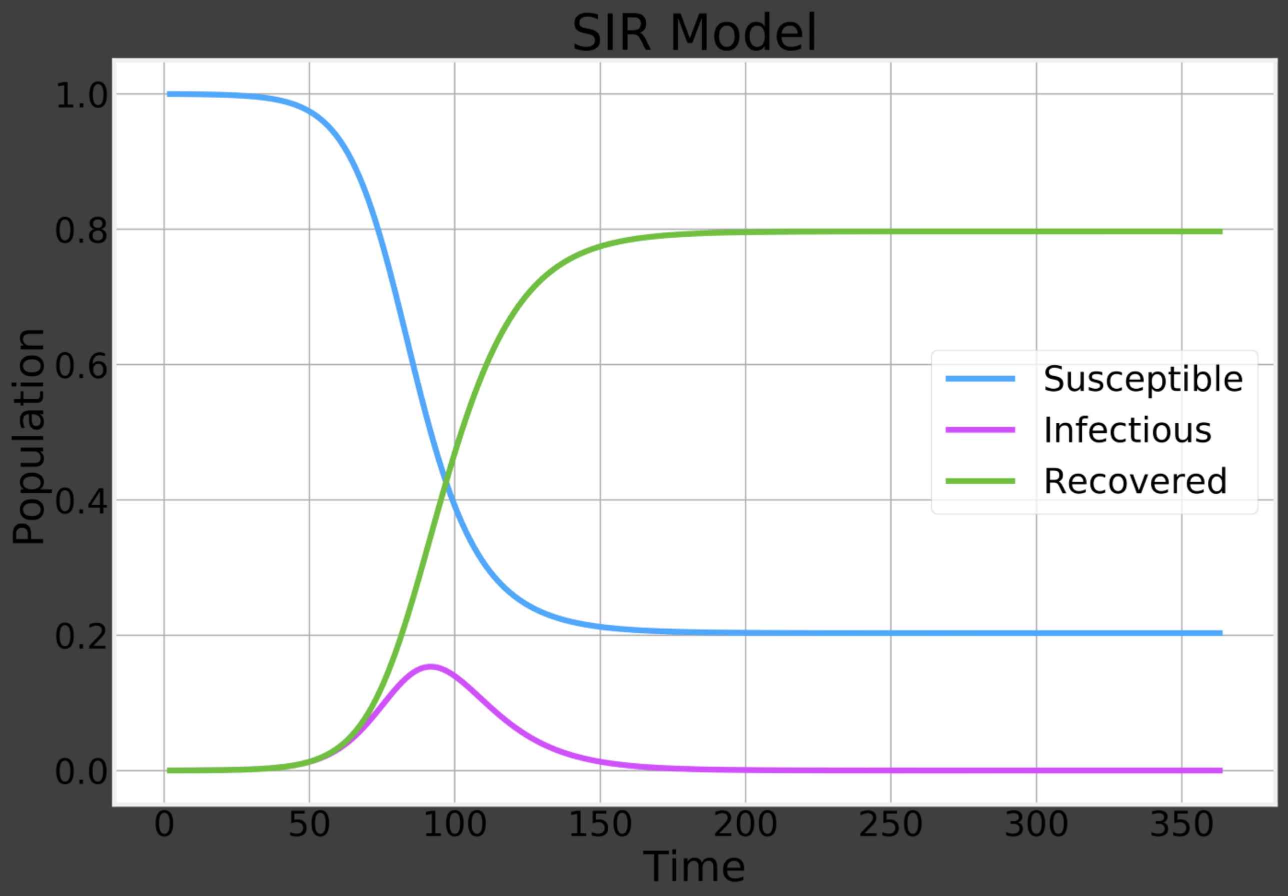
Etc.

THE **SIR** MODEL

- β = transmission rate (contact * possibility)
- γ = recovery rate ($\frac{1}{D}$: determined by avg. duration *D*)
- βSI = interaction (contact) between S and I
- γI = spontaneous (non-interacting) transition from *Infectious* to *Recovered* at a fixed rate γ

VISUALIZATION

- Susceptibles: only decrease
- Recovered: only increase
- Number of infectious individuals peak and decline
- Majority of the population becomes infected and then recovers

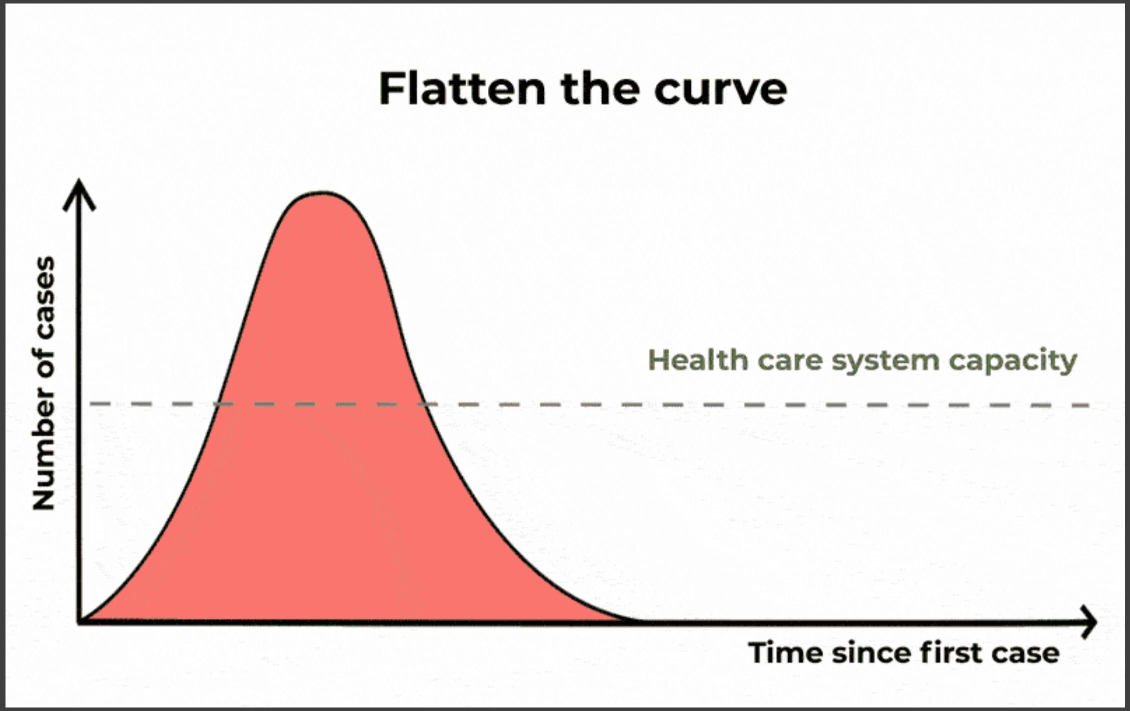


THE CURVE

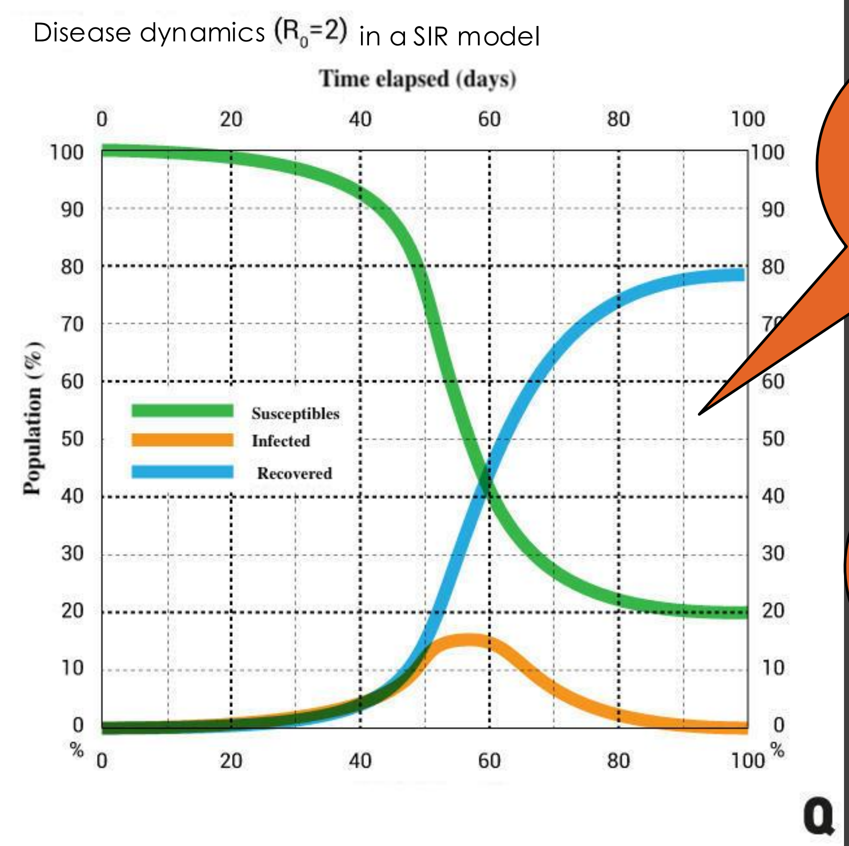
- $R_0 = \frac{\beta}{\gamma}$ = basic reproduction rate

- $R_o<1$: disease stops
- $R_o>1$: *exponential* growth

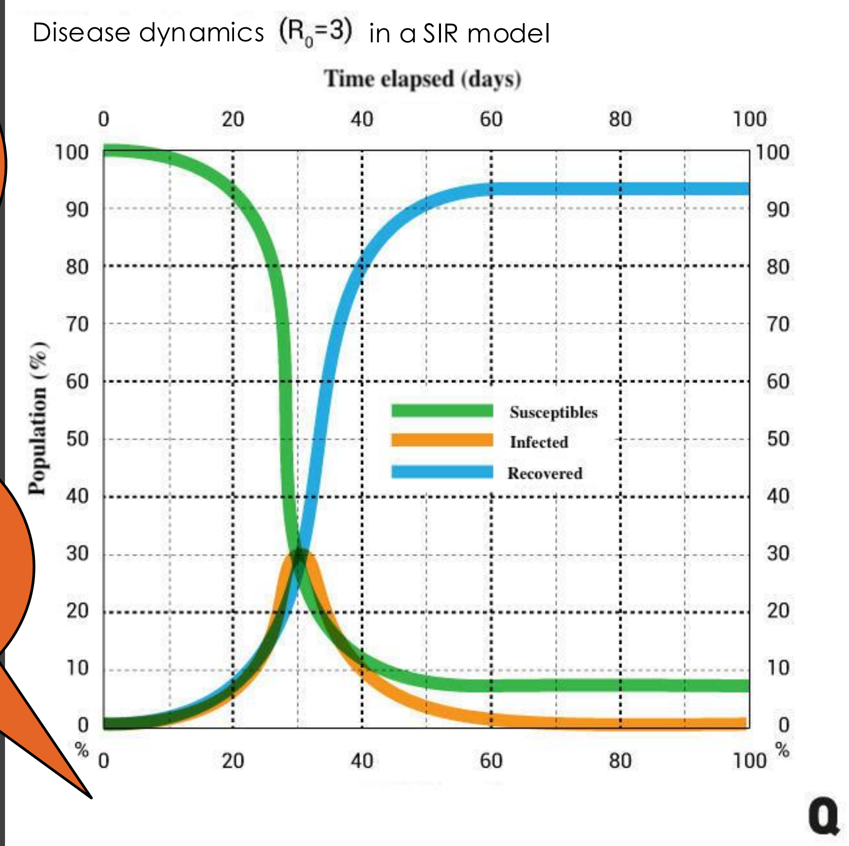
- $S_t < \frac{N}{R_0}$ = herd immunity treshold



EFFECT OF DIFFERENT R_0 VALUES



~80% infected



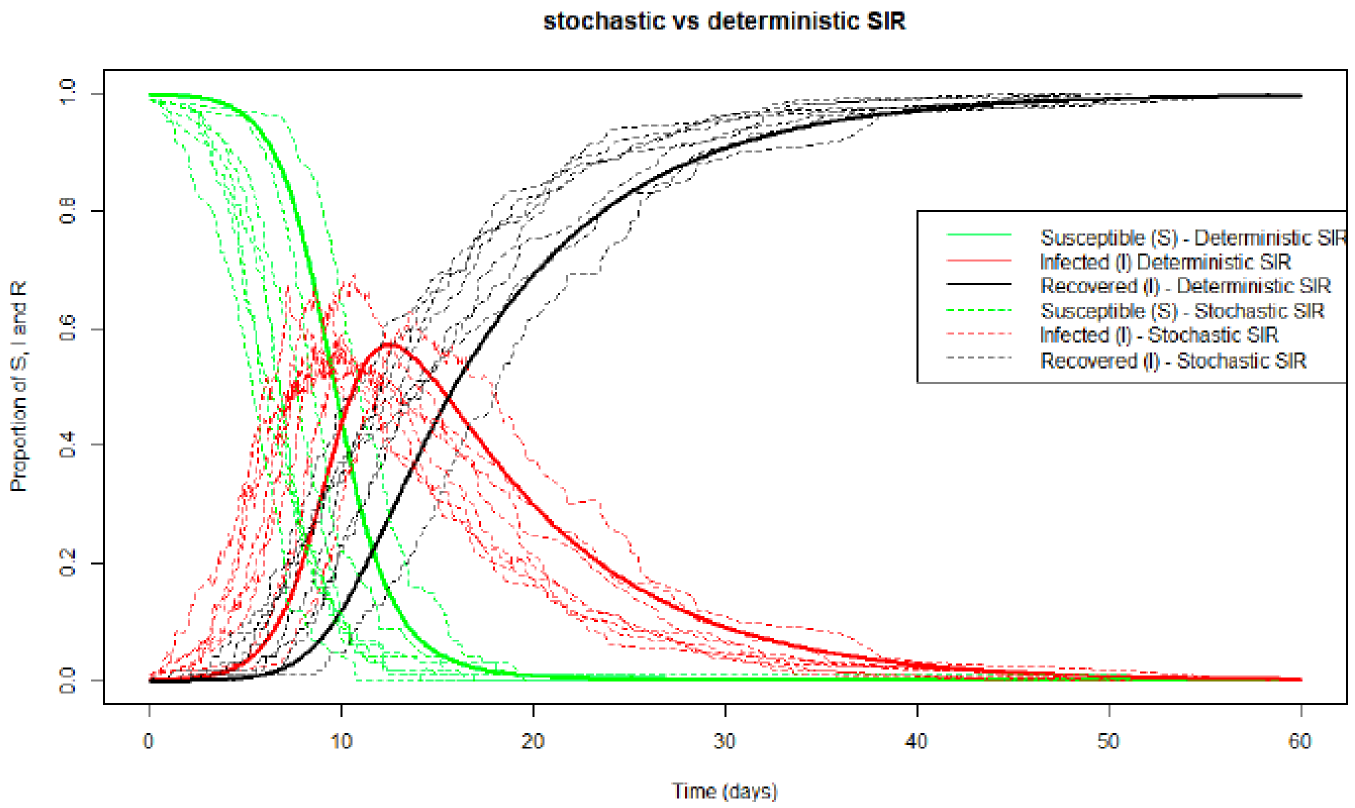
90%+ infected, faster peak

Source: Qubit. - qubit.hu

STOCHASTIC MODELS

- Differential equations → Probability Distributions
- **Pseudo-stochastic**: many independent deterministic models
- **Stochastic differential equations**: adds stochastic terms
- **Event driven approaches →**

Typical graph of a stochastic SIR model



THE BUILDING OF A COMPARTMENTALISED MODEL

- COVID-19 : novel pneumonia caused by a zoonotic virus
- Outbreak: Wuhan, Hubei province, China (Dec 2019)
- Origin #1: Natural selection in animal host before zoonotic transfer?
- Origin #2: Natural selection in humans following zoonotic transfer?

IS SIR ENOUGH?

MISSING PARAMETERS?

Asymptomatic or mildly symptomatic spreaders?

Window period before positive test?

Recovery confers immunity?

Chance of mortality?

THE **SEIR** MODEL

- ✔ latent period ✖ mortality
- For diseases with a significant incubation period: not yet infectious (compartment E)
- σ: rate of latent to infectious (avg. duration of incubation: $\frac{1}{D}$)

SEIR (Susceptible – Exposed - Infected - Recovered)
Non-infectious incubation period & lifelong immunity after recovery, e.g. chicken pox

$$\frac{dS(t)}{dt} = -\frac{\beta \cdot I(t) \cdot S(t)}{N}$$

$$\frac{dE(t)}{dt} = \frac{\beta \cdot I(t) \cdot S(t)}{N} - \sigma \cdot E(t)$$

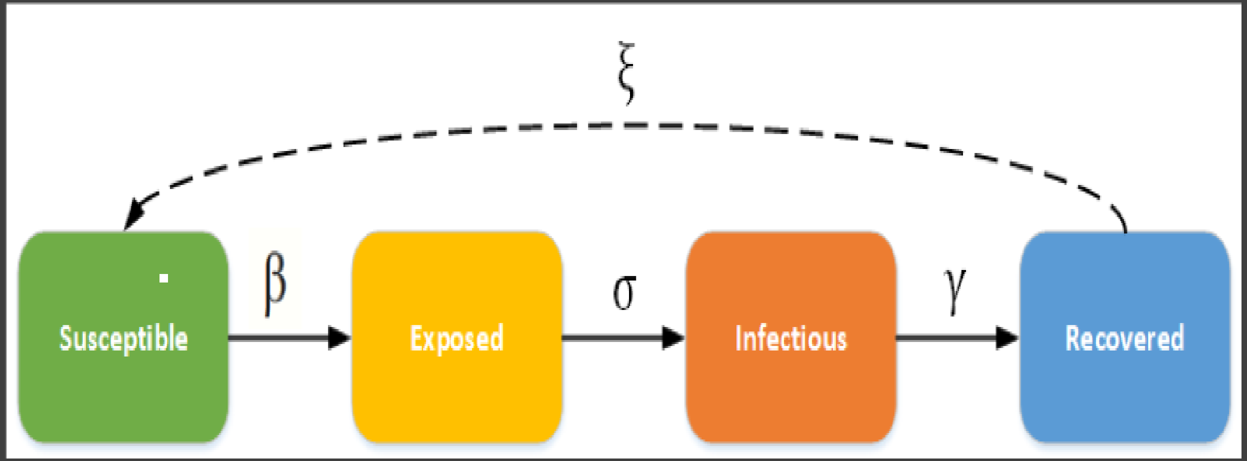
$$\frac{dI(t)}{dt} = \sigma \cdot E(t) - \gamma \cdot I(t)$$

$$\frac{dR(t)}{dt} = \gamma \cdot I(t)$$

THE **SEIRS** MODEL

- ξ: rate at which recovered lose immunity
- ✔ latent period ✖ immunity

SEIRS (Susceptible – Exposed - Infected – Recovered - Susceptible)
Only temporal immunity, disease becomes an *endemic*, e.g. malaria

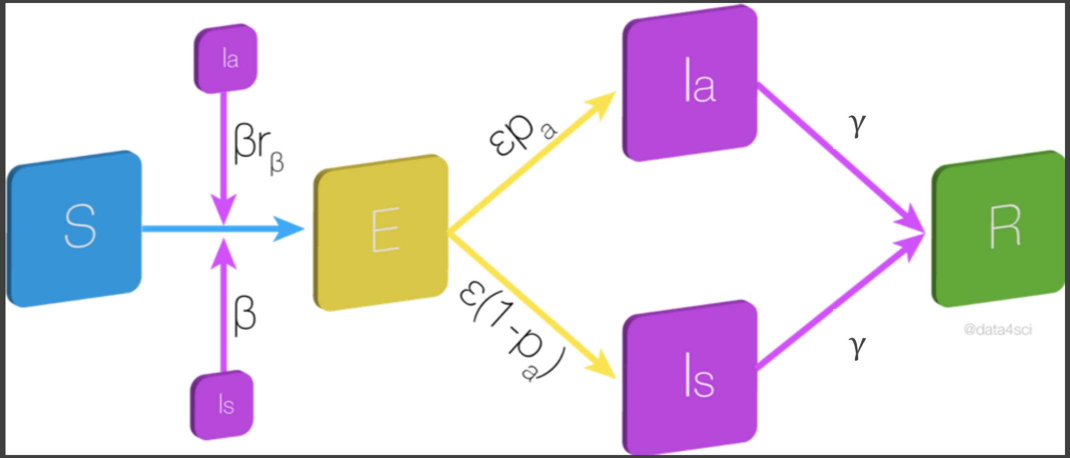


THE SEIIR MODEL

- Asymptomatic: less infectious by fraction r_β.
- p_a becomes asymptomatic
- $1-p_a$ develops symptoms

SEIIR (Susceptible – Exposed – Infected [asymptomatic] – Infected - Recovered)

Covid-19: asymptomatic carriers ~ 40%



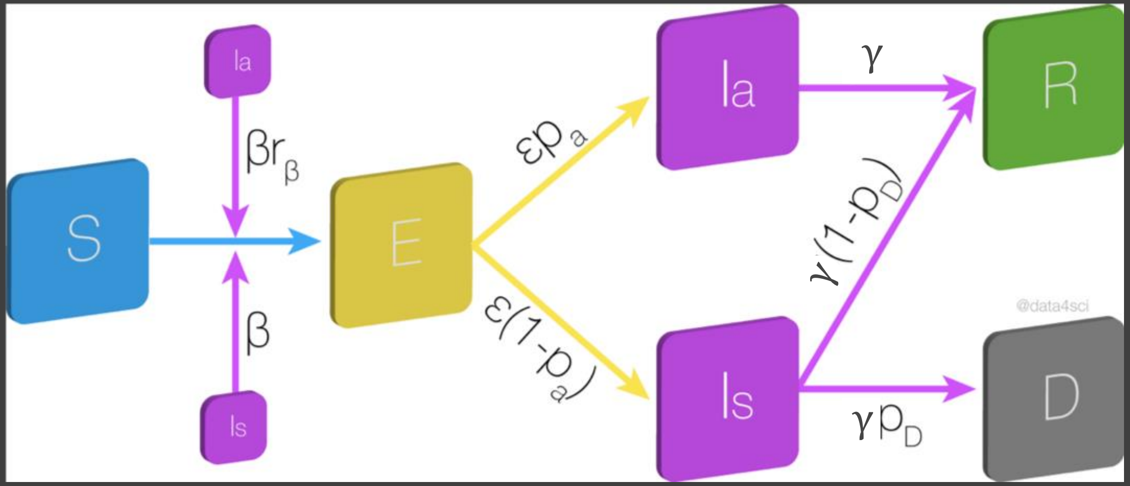
Source: data4sci *

THE **SEIIRD** MODEL

- Fraction of infected die by γp_D (the inverse of avg. time from onset to death)

- Remaining fraction recovers with rate $\gamma(1-p_D)$ (the inverse of avg. time to full recovery)

SEIIRD (Susceptible – Exposed – Infected [asymptomatic] – Infected – Recovered - Deceased)



*Source: data4sci **

STILL MORE POSSIBLE PARAMETERS

- Confirmed vs. Real number of cases
- Quarantine & social distancing
- Vaccination etc…

THANK YOU FOR YOUR ATTENTION!

RESOURCES

- Allthiswasfield, *Blogspot*

http://allthiswasfield.blogspot.com/2020/04/another-flatten-covid-19-curve.html

- Doeschl-Wilson, Andrea : Modelling Epidemics

https://jvanderw.une.edu.au/L7_ModellingEpidemics3.pdf

- Goncalves, Bruno. *Medium:*

https://medium.com/data-for-science/epidemic-modeling-101-or-why-your-covid19-exponential-fits-are-wrong-97aa50c55f8

https://medium.com/data-for-science/epidemic-modeling-102-all-covid-19-models-are-wrong-but-some-are-useful-c81202cc6ee9

- Idmod.org

https://idmod.org/docs/hiv/model-seir.html

- Miller, Joel C. A note on the derivation of epidemic final sizes, *Bull Math Biol*. 2012 September ; 74(9): 2125–2141. doi:10.1007/s11538-012-9749-6.
- Nesse, Hans - Global Health - SEIR Model

https://www.public.asu.edu/~hnesse/classes/seir.html?Beta=0.7878&Gamma=0.303&Sigma=0.196&Mu=0&Nu=0&initialS=9773000&initialE=4089&initialI=2000&initialR=0&iters=200

* image slightly modified by creator of slide