Intro

Couplage (ex par le bois) :
-Quenching -> suppression des oscillations
-(Mutual) Synchro (ajustement de rythmes par faible intéraction -> frequency entrainment/locking) si 2 systèmes oscillent de manière :

- autonome (self-sustained, src d'En interne transformée en mvt qui compense dissipation, pas de dépendance au temps explicite, sinon résonance par ex chpM)
- indépendante (oscilllations si seul, pas proie-prédateur))
- couplage faible(diff à mesurer)
- frequency detuning faible (f1-f2)
- ≠ synchronous motion / synchronisation (weak coupling, sinon pas décomposable, déf weak ? doit pas changer qualitativement le comportement)
- synchr est un processus dynamique, pas un état

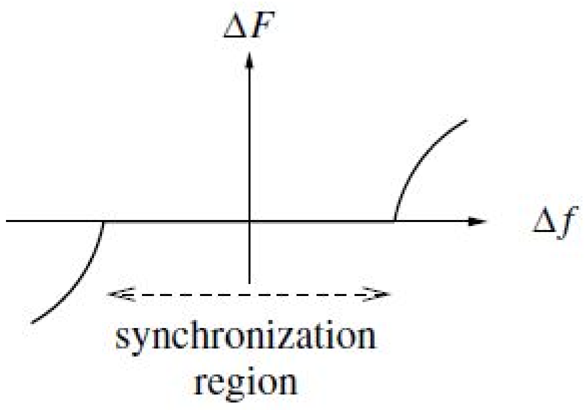


Figure 1.9. Frequency vs. detuning plot for a certain fixed strength of interaction. The difference of frequencies ΔF of two coupled oscillators is plotted vs. the detuning (frequency mismatch) Δf of uncoupled systems. For a certain range of detuning the frequencies of coupled oscillators are identical ($\Delta F = 0$), indicating synchronization.

-Synchro 2 pendules/ forme des oscillations dépend pas CI (rythme interne, stable aux perturbations, retourne origine)
-> in-phase synchro (phase-shift=0) ou anti-phase -> 2 sont possibles selon couplage (mini additionnal shift) -> onset des phases=phase locking

Types synchro

-Par force externe
- -> synchro ac lumière etc mais phase shift ≠ selon personnes, chgt horaire=jet lag

-Ensemble d'oscillateurs -> global coupling
-Chaotic oscillators -> mean frequency=Nt/t, pour t large intervalle
- -> coincidence of mean f but not equal, A remains chaotic/uncorrelated, f adjusted
- -> peut parler de phase shift = phase synchro of chaotic systems
- -> complete synchro/ influence sur A si couplage très fort

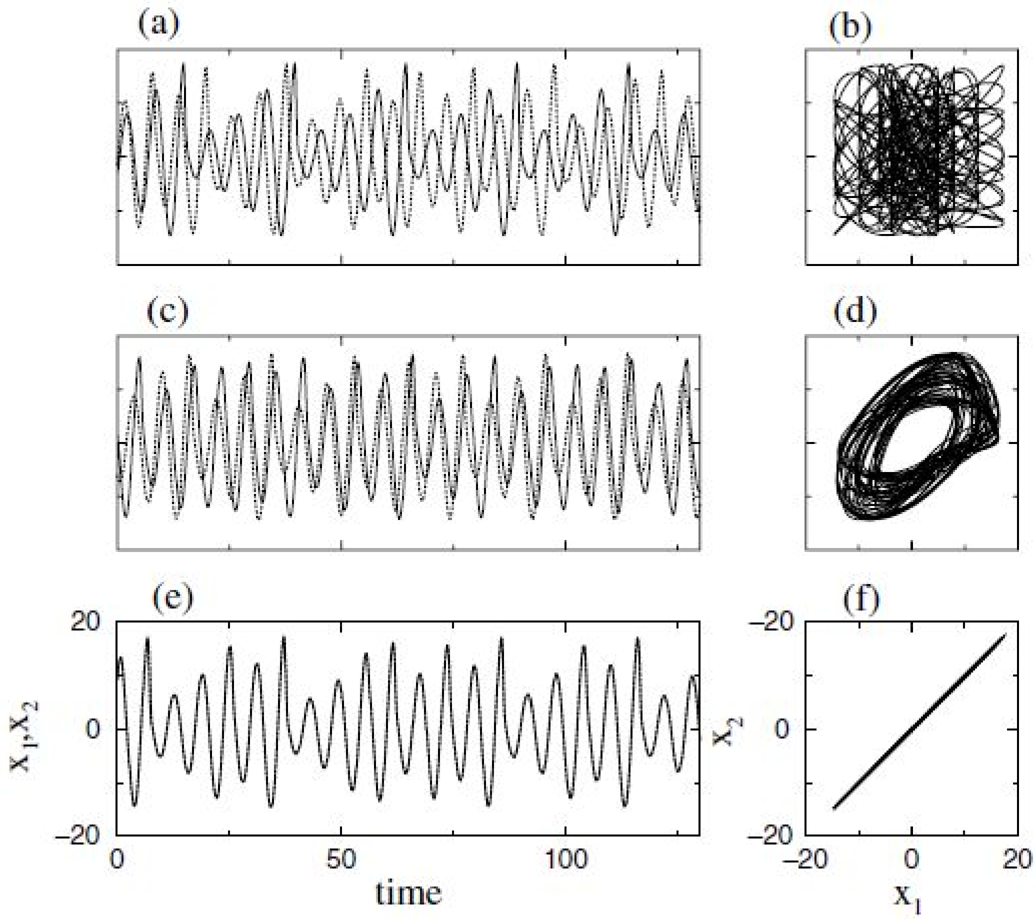


Figure 1.16. Two chaotic signals x_1, x_2 originating from nonidentical uncoupled systems are shown in (a). Within the time interval shown they have 21 and 22 maxima, respectively, hence the mean frequencies are different. Introduction of coupling between the oscillators adjusts the frequencies, although the amplitudes remain different (c). The plot of x_1 vs. x_2 (d) now shows some circular structure that is typical in the case of two signals with equal frequencies and a constant phase shift (compare with (b), where no structure is seen). Strong coupling makes the two signals nearly identical ((e) and (f)).

- Integrate-and-fire oscillators : alternance silence/activité

expe : peut pas changer force du couplage -> détecte interactions par free oscillations

<u>Déf</u>
synchro : ajustement de rythmes par intéraction (pas seulement complète coincidence)
locking=entrainment -> phase locking ≠ = des phases : f1,f2 sont n:m locked si |nf1-mf2|<K
syst chaotiques : ≠ nivx de synchro :

-phase synchro/phase locking : relation entre les phases s’instaure/ A chaotique

-complete synchro : devient =

oscillator : self-sustained system = self-oscillatory = self-excited
limit cycle : attractor of the selfS oscillator, not of the forced system
dynamical systems : deterministic -> connait t = connait futur-> idéal : pd pas cpte B
connaître état du système : x + y (ex dérivée) -> coordinates in phase space

<u>Chapitre 2</u>

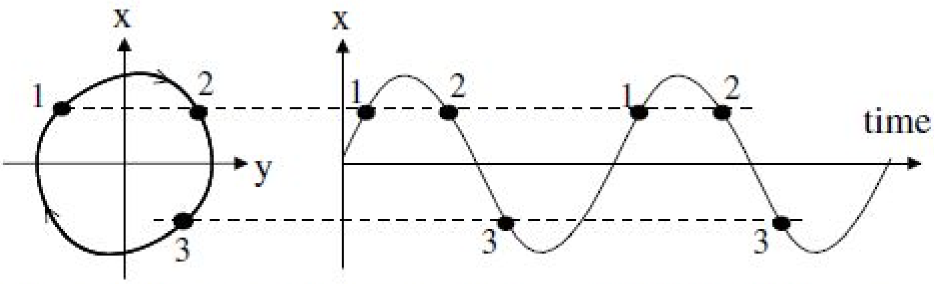


Figure 2.1. Periodic oscillation is represented by a closed curve in the phase space of the system: equivalent states $x(t)$ and $x(t+T)$ (denoted by a number on the time plot) correspond to the same point on that curve (denoted by the same number). On the contrary, the states with the same $x(t)$ are different if we take the second coordinate.

phase portrait -> phase points

Limit circle for quasi-linear oscillator = cercle : x(t)=Asin(w0t+Phi0)

Phase Phi(t)=w0t+Phi0 -> même état quand x=y mod 2Pi

A dépend pas Ci, phase si, peut choisir Phi0=0

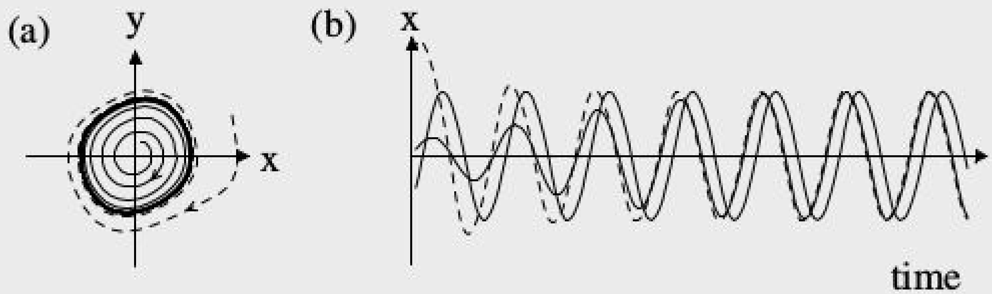


Figure 2.2. (a) The closed curve (bold curve) in the phase plane attracts all the trajectories from its neighborhood, and is therefore called the **limit cycle**. The same trajectories are shown in (b) as a time plot.

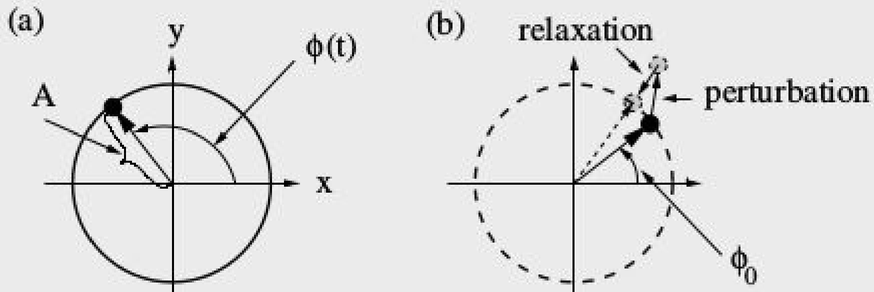


Figure 2.3. (a) A stationary self-sustained oscillation is described by the rotation of the phase point along the limit cycle; its polar coordinates correspond to the phase $\phi(t)$ and amplitude A of the oscillation. (b) In the rotating coordinate system the stationary oscillations correspond to the resting point (shown by a filled circle). If this point is kicked off the limit cycle, the perturbation of the amplitude decays, while the perturbation of the phase remains; the perturbed state and the state after the perturbation decays are shown as small dashed circles.

Amplitude is stable, phase is free/facilement ajustable (ni stable, ni instable) -> après perturbation

Φ(t)=Φ0+2π(t-t0)/T → uniforme, même si le mvt l'est pas A=déviation de la trajectoire par rapport au cycle

direction radiale : cvg, dir tan : ni cvg ni dvg → Lyapunov exponents (cvg=expo -,dvg = expo +,|expo|=cvg rate)

→ phase of oscillator corresponds to 0 of Lyapunov exponent

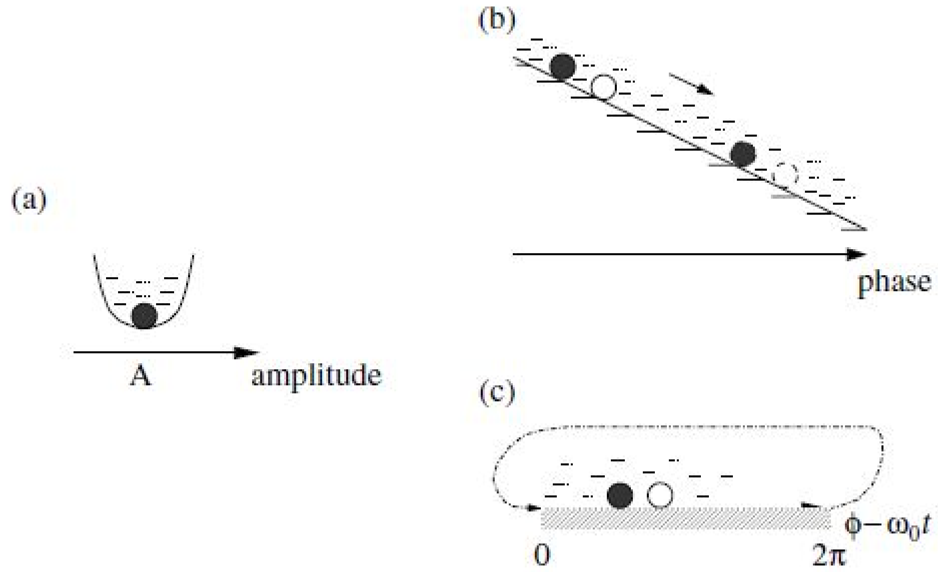


Figure 2.4. Stability of a point on the limit cycle illustrated by the dynamics of a light particle in a viscous fluid. (a) Transversely to the cycle, the point is in a state of stable equilibrium: perturbation of the amplitude rapidly decays to the stable value A. (b) Perturbation along the cycle, i.e., that of the phase, neither grows nor decays. Steady growth of the phase can be represented by a light particle sliding in a viscous fluid along an inclined plane. Two such particles (corresponding to the unperturbed and perturbed phases, filled and open circles) slide with a constant lag. (c) In a rotating (with velocity ω_0) reference frame the phase is constant (see also the discussion in Section 3.1). This corresponds to a particle on a horizontal plane in the state of neutral (indifferent) equilibrium. The dashed line reminds us that the phase is a 2π-periodic variable.

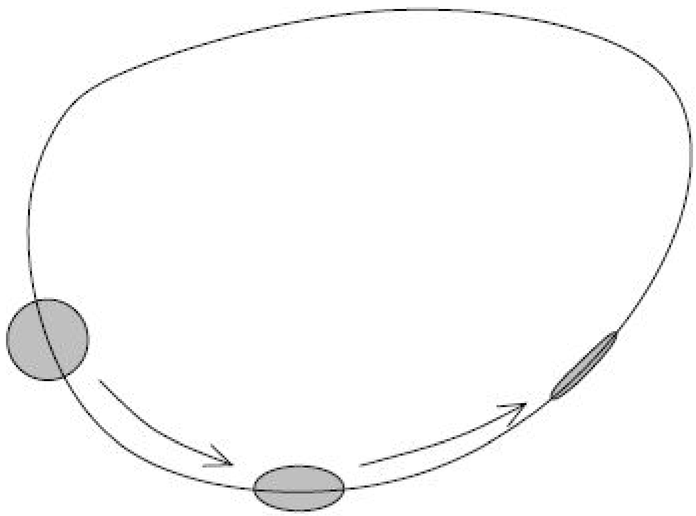


Figure 2.6. Convergence and divergence of trajectories is characterized by Lyapunov exponents. Suppose we consider a cloud of initial conditions around some point on the limit cycle (shaded circle). This phase volume decreases during the evolution, resulting in an elliptical form. This corresponds to a negative Lyapunov exponent in the direction transversal to the cycle, and to a zero Lyapunov exponent in the tangential direction.

systèmes conservatifs sensibles aux CI /perturbation reste → stabilité des SSOscillatorsI
nonlinéarité → mvt périodique que pour certaines A (sinon aA ok)
linéaire = conservatif (augmente/diminue infini mais pas de limit cycle)
Phase of a forced system is not free! → peut pas se synchroniser (balançoire)
Relaxation oscillators → Intervalles of slow/fast motion (integrate and fire, ex accumulation eau)
coeur self-sustained car intéraction avec respi etc mais pluss perturbation que cause rythme cardiaque
→ pacemaker = synchro de plusieurs parties

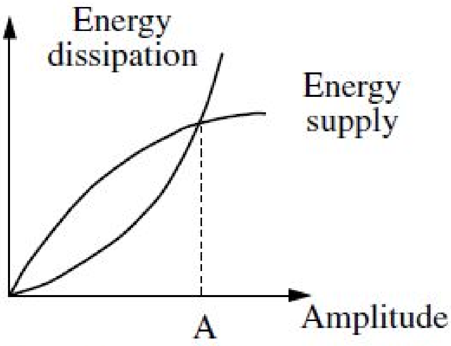


Figure 2.8. Interplay of the dissipation and supply of energy from a source determines the amplitude A of the steady oscillations. In the example shown here this amplitude is stable: if it is increased due to some perturbation, the dissipation starts to prevail over the energy supply, and this leads to a decrease of the amplitude. Similarly, the occasional decrease of amplitude results in an excess of supply over the loss, and therefore the initial amplitude A is re-established.

Chapitre 3

effet de la force sur le phase point dépend de Φ-Φe

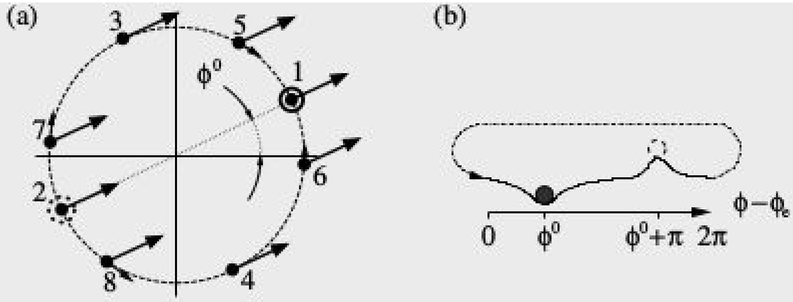


Figure 3.3. (a) A weak external force cannot influence the amplitude of the limit cycle but can shift the phase ϕ of the oscillator. The effect of the forcing depends on the phase difference $\phi - \phi_e$: at points 3 and 4 this effect is maximal, whereas at points 1 and 2 the force acts in the radial direction only, and cannot therefore shift the phase point along the cycle. Note that in the vicinity of point 2 the force propels the point away from point 2. On the contrary, in the vicinity of point 1 the phase point is pushed towards this equilibrium position. Hence, the external force creates a stable (open circle at point 1) and unstable (dotted bold circle at point 2) equilibrium positions on the cycle. These equilibrium positions are also shown in (b) (cf. Fig. 2.4c).

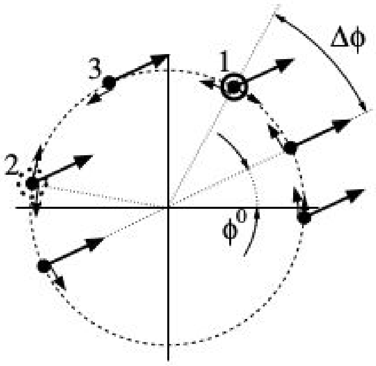


Figure 3.4. Small detuning. The rotation of the phase point is affected by the external force that accelerates or decelerates the rotation depending on the phase difference $\phi - \phi_e$. The rotation is pictured by arrows inside the cycle; here it is counter-clockwise, corresponding to $\omega_0 > \omega$. At point 1 the rotation is compensated by the force. This point then becomes the stable equilibrium position. Unstable equilibrium is then at point 2 (cf. Fig. 3.3a). The phase point is stopped by the force and a stable phase shift $\Delta\phi$ is maintained. Possible values of $\Delta\phi$ lie in the interval $-\pi/2 < \Delta\phi < \pi/2$.

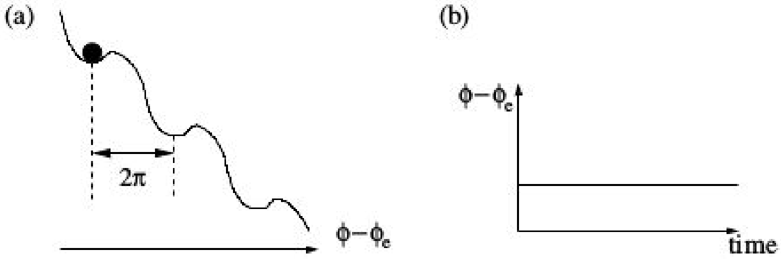


Figure 3.5. Small detuning. The external force creates minima in the tilted potential (a) and the particle rests in one of them. This corresponds to a stable constant phase difference, $\phi - \phi_e = \phi^0 + \Delta\phi$ (b).

Point 1 : synchronisation, fréquences deviennent égales, phase shift (non nul, majoré) stable
VS pas synchro : phase ≠ dvg → synchro = phase locking
si detuning trop important (vitesse rot), force peut plus compenser la rotation
si detuning=0 : $\Delta\Phi=\Phi_0=0$ (change rien)

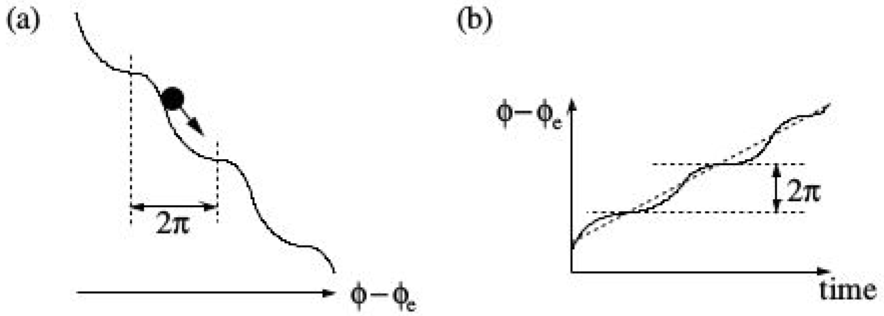


Figure 3.6. Large detuning. The force cannot entrain the particle: there are no local minima in the tilted potential and the particle slides down (a). Under-threshold force modulates the velocity of the particle, so that the phase difference grows nonuniformly (b). The average growth rate, shown by the dashed line, determines the *beat frequency*. From (a) one can see that the period of rotation (shown by two dashed lines) coincides with the period of modulation.

beat frequency Ω_b /ω_0 oscillateur/naturel, ω force →ici, oscillateur oscille à $\Omega_b+\omega<\omega_0$
force rend rotation non uniforme/fait décelérer car s'arrête presque aux mini de pot→ $\Omega_b<\omega_0-\omega$ ($\omega_0>\omega$)
régime quasi-périodique : caractérisé par Ω_b et $\omega+\Omega_b$
frequency locking → range de detuning pour avoir synchro
large detuning → besoin grande force pour synchro

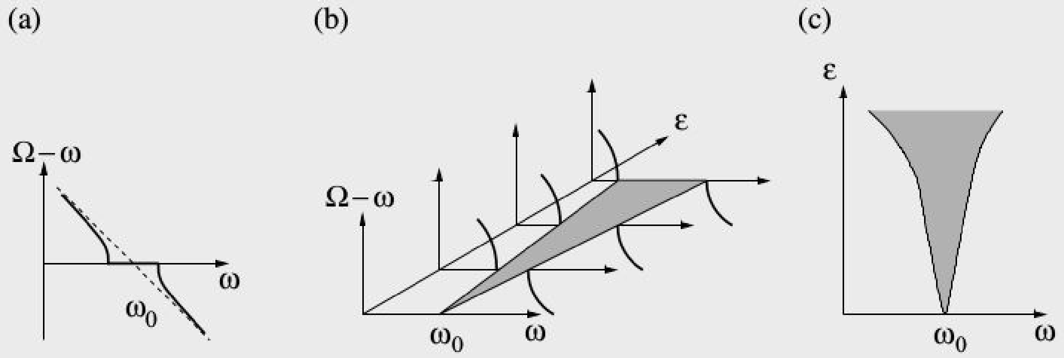


Figure 3.7. (a) Difference of the frequencies of the driven oscillator Ω and the external force ω as a function of ω for a fixed value of the forcing amplitude ε. In the vicinity of the frequency of the autonomous oscillator ω_0, $\Omega - \omega$ is exactly zero; this is denoted **frequency locking**. With the breakdown of synchronization the frequency of the driven oscillator remains different from ω_0 (the dashed line shows $\omega_0 - \omega$ vs. ω): the force is too weak to entrain the oscillator, but it "pulls" the frequency of the system towards its own frequency. (b) The family of $\Omega - \omega$ vs. ω plots for different values of the driving amplitude ε determines the domain where the frequency of the driven oscillator Ω is equal to that of the drive ω. This domain, delineated by gray in (c), is known as the **synchronization region** or **Arnold tongue**.

ici Ω = oscillateur et ω = force
touche axe → besoin force infinitésimale

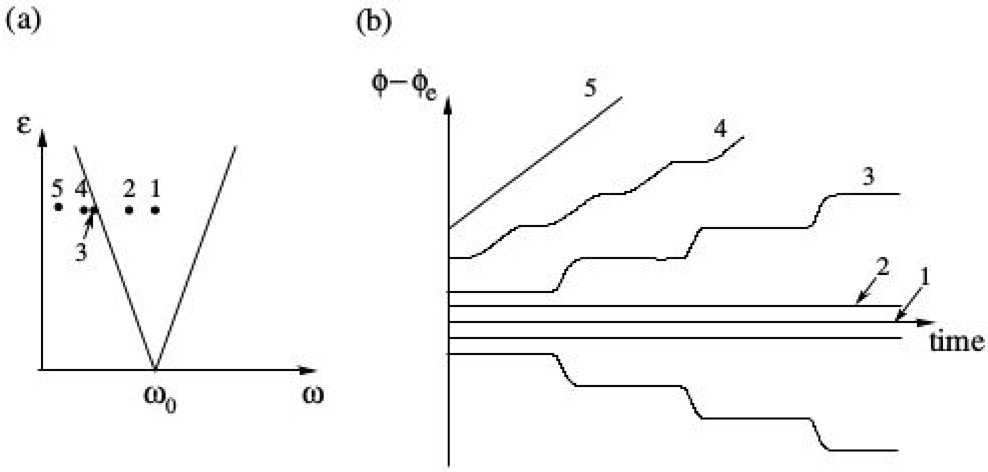


Figure 3.8. Dynamics of the phase at the synchronization transition. The phase difference is shown in (b) for different values of the frequency of the driving force; these values are indicated in (a) by points 1–5, within and outside the synchronization region. In the synchronous state (points 1 and 2) the phase difference is constant (lines 1 and 2 in (b)); it is zero in the very center of the tongue and nonzero otherwise. Just outside the tongue the dynamics of the phase are intermittent: the phase difference appears as a sequence of rapid jumps (slips) intermingled with epochs of almost synchronous behavior (point and curve 3). As one moves away from the border of the tongue the dynamics of the phase tend towards uniform growth (points and curves 4 and 5). The transition at the right border of the tongue occurs in a similar way, only the phase difference now decreases.

phase slip puis très lent ↲ potentiel quasi horizontal → alternance quasi-synchro/slip

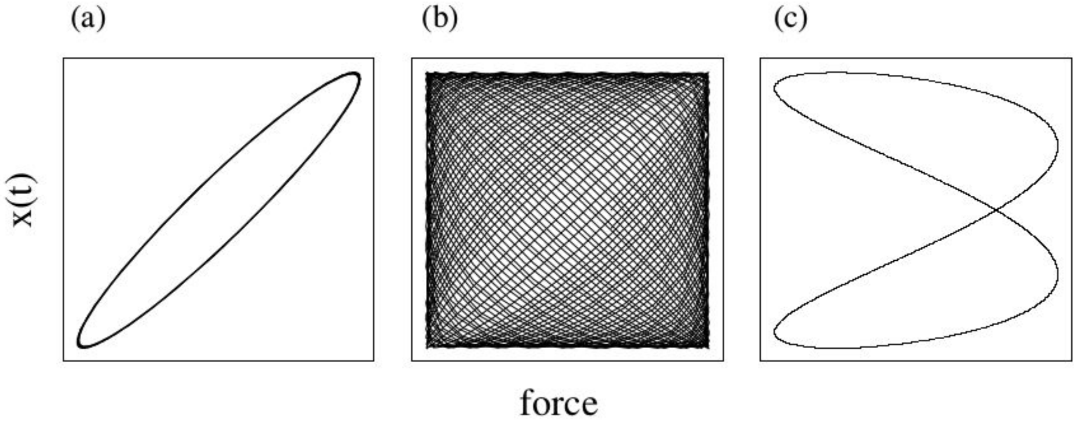


Figure 3.9. Lissajous figures indicate synchronization. An observable $x(t)$ of the forced oscillator is plotted vs. the force. (a) Synchronous state. The periods of oscillations along the force and x axes are identical, therefore the plot is a closed curve. (b) Quasiperiodic state. The point never returns to the same coordinates and the plot fills the region. (c) The 8-shaped Lissajous figure corresponds to the case when the force performs exactly two oscillations within one cycle of the oscillator. This is an example of high-order (1 : 2) synchronization discussed in Section 3.2. (Note that the scales along the axes are chosen differently in order to obtain clearer presentation; in fact, the amplitude of the force is much smaller than the amplitude of the signal (weak forcing).)

Lissajous figures aussi pour évaluer phase shift : Φ(t)=Φe(t) → ligne, cercle = ±π/2

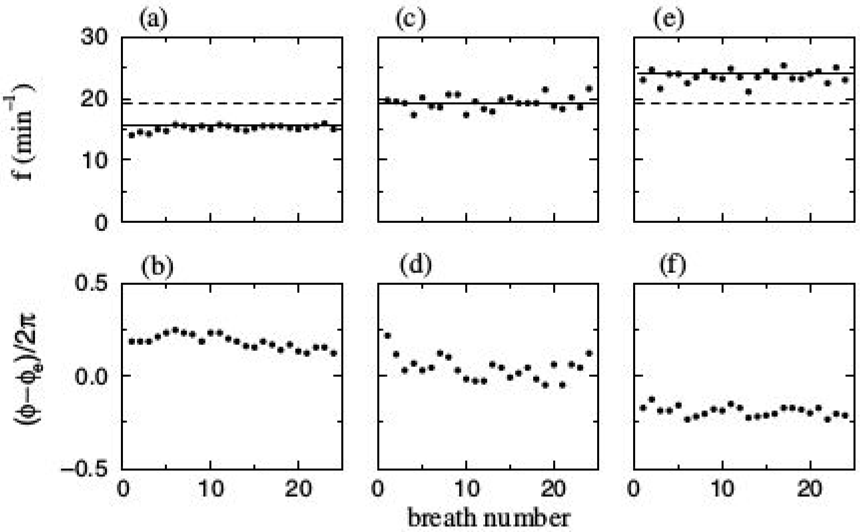
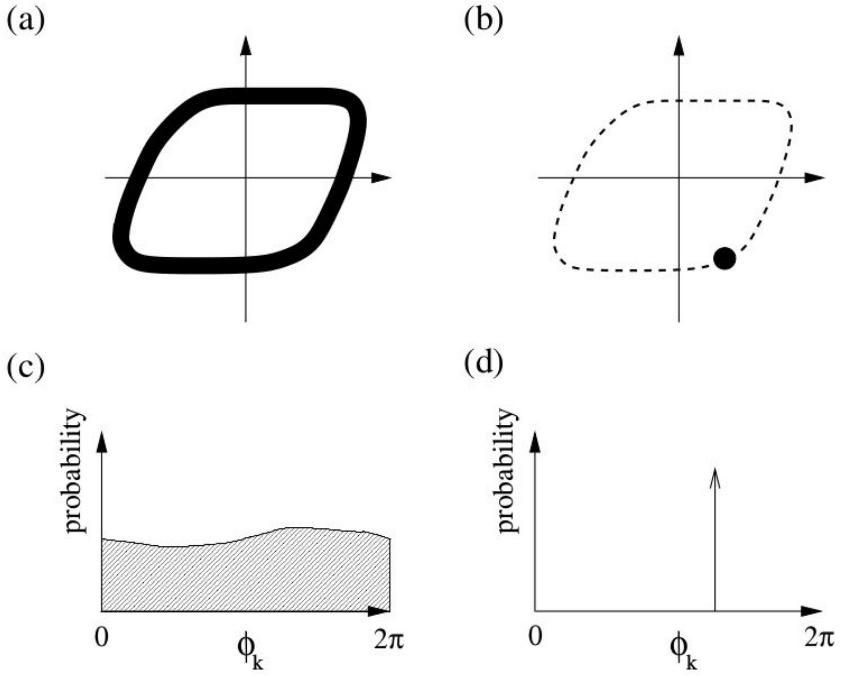


Figure 3.11. Frequency of respiration during mechanical ventilation (a, c, and e) and the difference between the phase of the ventilator and that of spontaneous breathing (b, d and f) computed for 24 consecutive breaths. The plots correspond to three locked states where the frequency of the external force (solid line) is smaller than (a, b), equal to (c, d) or larger than (e, f) the "off pump" frequency (dashed line). The frequency of respiration is entrained and fluctuates around the value of the external frequency. The average phase shift is respectively positive (b), close to zero (d) and negative (f). From [Graves *et al.* 1986].

figure pour petit detuning : noise ⇨ x oscillateur périodique parfait ⇨ f/Φ≠ fluctuent + respi ≠ sinus
signe du shift dépend du signe du detuning : f < ⇨ Φ ⊕
limit cycle pou nonlinear → pas tjs un cercle
stroboscopic : observations à kT → phase de la force externe fixée → point qd synchro
peut étudier systèmes chaotiques → met sur un cercle même si A ≠, on s'en fout, intérêt pour Φ



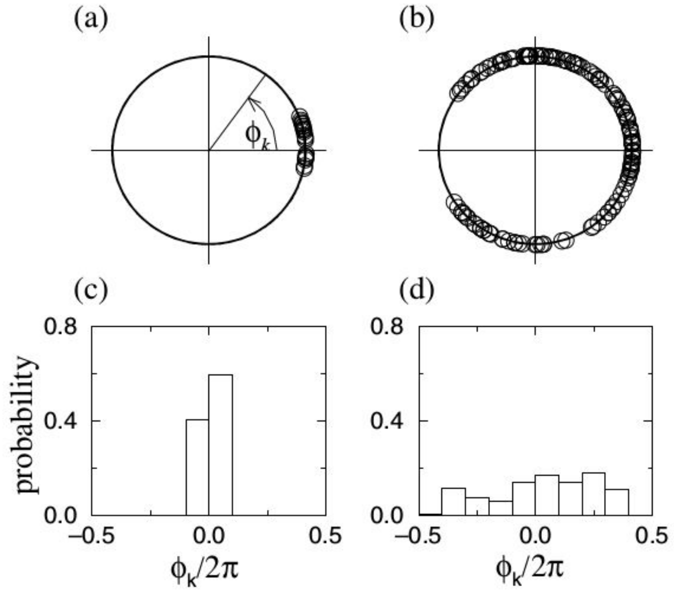


Figure 3.15. Stroboscopic observation of the phase of the flashes of a periodically stimulated firefly. The distribution of phases for the time interval $0\ \text{s} < t < 22\ \text{s}$ is sharp (a and c), whereas the distribution for $22\ \text{s} < t < 130\ \text{s}$ is broad (b and d), reflecting loss of synchronization at $t \approx 22$ s (cf. Fig. 3.13). Plotted using data from [Ermentrout and Rinzel 1984].

peut avoir un pulse de force périodique ≠ force en continu → ex periodic stimulation pacemaker
pas de forcing : ω0>ω → avance v (phase decrepancy)
1 pulse : Φ → Φ+Δ ⇨ peut compenser v (si bonne A, bon w), puis // autonome, puis compense ..
forcing : A> ⇨ déphasage >, v>⇨F nécessaire >⇨ΔΦ augmente
si Φ0 force arbitraire, + long pour ramener de 3 à 1 (3.17)

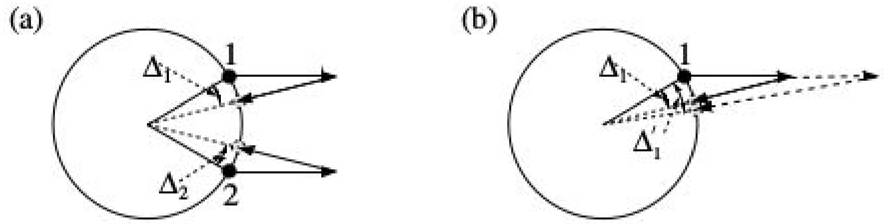


Figure 3.16. Resetting of the oscillator phase by a single pulse, $\phi \to \phi + \Delta$. (a) The effect of the pulse depends on the oscillation phase at the instant of stimulation. If the pulse is applied to the point at position 1 then the phase is delayed ($\Delta_1 < 0$); perturbation applied to the point 2 advances the phase ($\Delta_2 > 0$). (b) Effect of the strength of the pulse: the pulse of a larger amplitude (the vector shown by a dashed line) enlarges the resetting ($|\Delta_1'| > |\Delta_1|$).

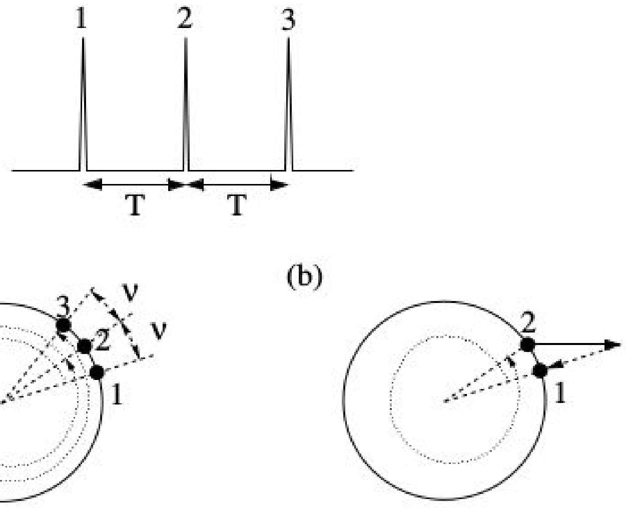


Figure 3.17. (a) Stroboscopic observation of an *autonomous oscillator* at the instants of pulse occurrence; when pulse 1 comes, the point is found in position 1, and so on. The frequency of the oscillator is chosen to be larger than the frequency of pulses, $\omega_0 > \omega = 2\pi/T$; therefore, the stroboscopically observed point moves along the cycle. (b) Synchronization by resetting. Periodic pulses applied to the point in position 2 reset it to position 1, thus compensating the phase discrepancy due to detuning. Between the pulses the phase evolves as in the autonomous system.

oscillateur perturbé périodiquement : $\Phi_{n+1}=\Phi_n+\nu+\Delta(\Phi_n+\nu)$ → circle map = (Φ_n→ Φ_{n+1}), synchro si id

si perturbation 1*/2 : pulse 2 * plus fort suffit pour compenser, dépd pas forme du forcing (pulse,C0)

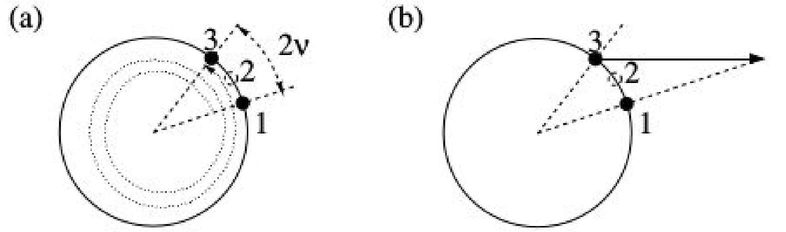


Figure 3.18. Synchronization of the oscillator by a $2T$-periodic pulse train.
(a) Between two pulses the point evolves from position 1 to position 3.
(b) Synchronization can be achieved even if every second pulse is skipped, but the amplitude of pulses must be larger (see Fig. 3.17b).

force $\omega_0/2$ → synchro order 2:1 → diff ↲ besoin A >

pulse ⇨ allongement (raccourcissement) du cycle T0

$n\omega=m\Omega$: Ω=oscillateur, ω=force

phase locking : $|n\Phi_e-m\Phi|<K$ (pas =0)

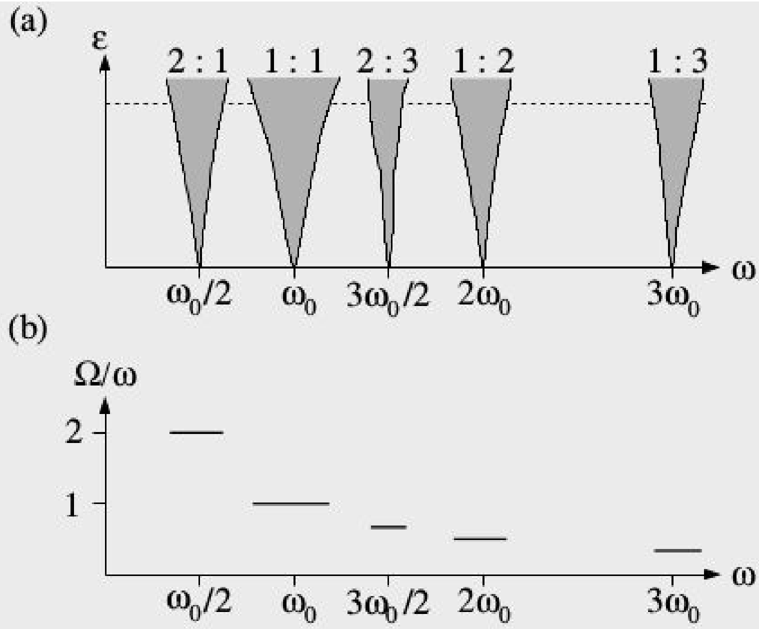


Figure 3.19. (a) Schematic representation of Arnold tongues, or regions of $n : m$ synchronization. The numbers on top of each tongue indicate the order of locking; e.g., 2 : 3 means that the relation $2\omega = 3\Omega$ is fulfilled. (b) The Ω/ω vs. ω plot for a fixed amplitude of the force (shown by the dashed line in (a)) has a characteristic shape, known as the *devil's staircase*, see Fig. 7.16 for full image (here the variation of the frequency ratio between the steps is not shown).

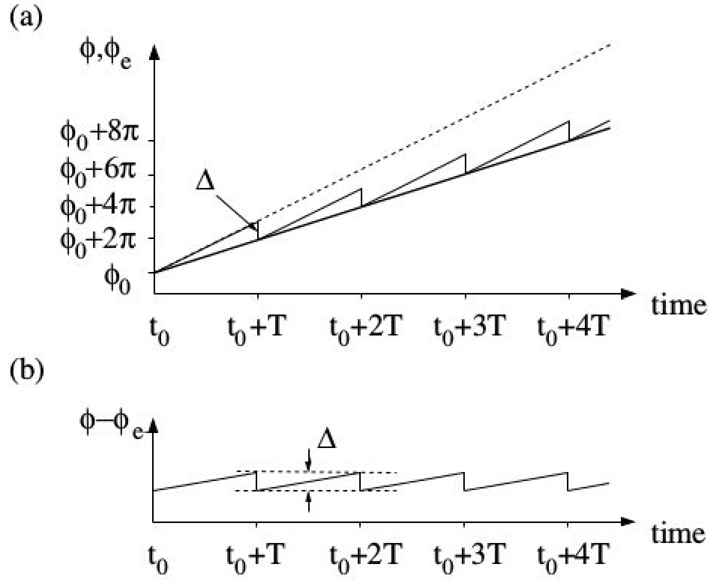


Figure 3.22. The phase difference in the synchronous state is not necessarily constant. (a) Phase of the external force ϕ_e increases linearly (bold line), whereas the phase ϕ (solid line) of the periodically kicked oscillator increases linearly between kicks, and is instantly reset, $\phi \to \phi - \Delta$, by each pulse. The phase of the autonomous oscillator would grow as shown by the dashed line. (b) The phase difference $\phi - \phi_e$ is bounded but oscillates.

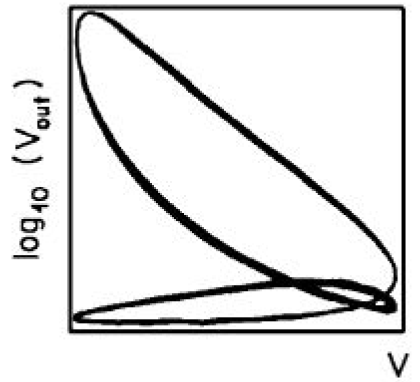


Figure 3.23. Intensity of the laser output log V_{out} plotted vs. the forcing voltage V. The closed curve, known as a Lissajous figure, indicates that one period of the output oscillations exactly corresponds to two periods of the modulating signal (external force), i.e., 1 : 2 synchronization takes place (cf. Fig. 3.9). From Simonet *et al.*, *Physical Review* E, Vol. 50, 1994, pp. 3383–3391. Copyright 1994 by the American Physical Society.

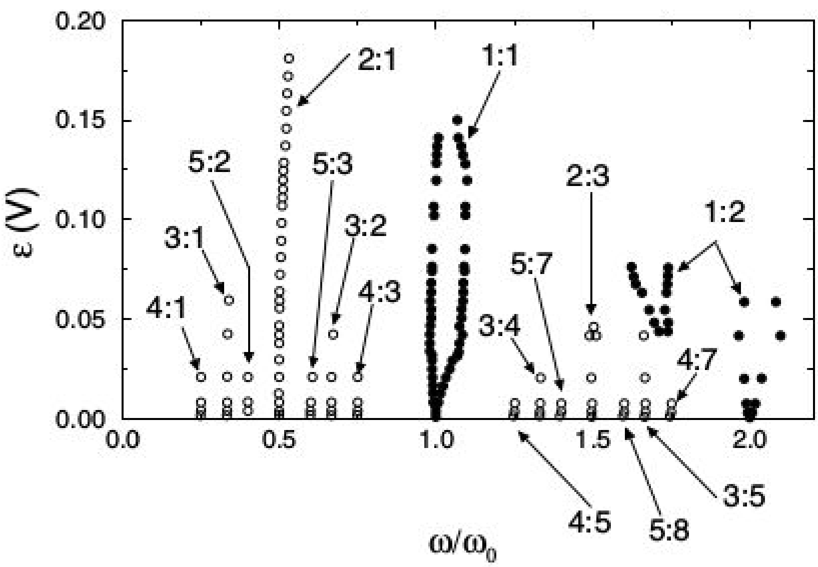


Figure 3.24. Synchronization tongues for the laser subject to external action. ε and ω are the amplitude and the frequency of the force, respectively. The borders of 1 : 1 and 1 : 2 tongues are shown by filled circles. The high-order tongues are very narrow; in the precision of the experiment they appear as lines that are shown by open circles. From Simonet *et al.*, *Physical Review* E, Vol. 50, 1994, pp. 3383–3391.

figures de Lissajous ⇨ synchronisation

synchronisations d’ordre > ⇨ arnold tongue plus étroite

Integrate and fire : atteindre le seuil, firing instantané EX ajoute eau/pacemaker
pulse ⇨ □ To (jms □ To) , influence Φ(oscillator), synchro rapide, si A>threshold, synchro en 1*

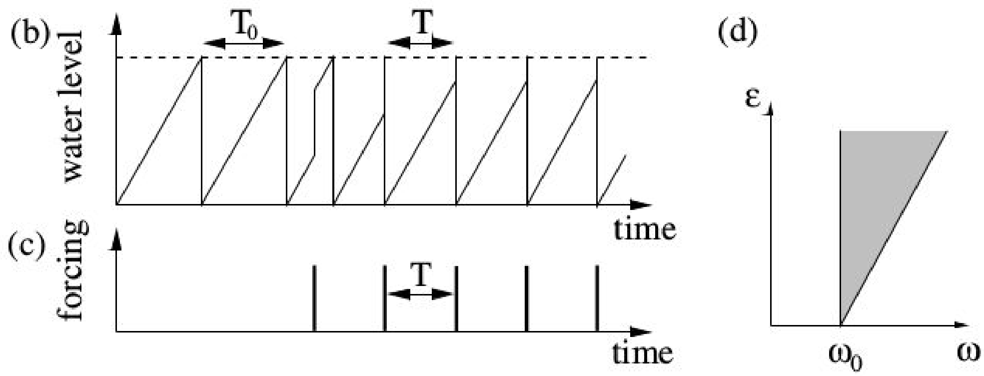


Figure 3.25. (a) Pulse forcing of the integrate-and-fire oscillator. An additional amount of water is periodically brought by a conveyor belt and instantly poured into the vessel. (b) The autonomous oscillator with the period T_0 is synchronized by the T-periodic sequence of pulses shown in (c). Each pulse results in an instant increase of the water level in the main vessel; hence the threshold value is reached earlier than in the absence of force. Note that synchronization sets in very quickly, within two cycles. (d) As the pulse force can only increase the frequency of the integrate-and-fire oscillator, the 1 : 1 synchronization region has a specific asymmetric shape.

pathologie du coeur : m contractions atria, puis n du ventricule (m>n) → n:m synchro
threshold périodique : U=Uthresh- ε sin(ωt) ⇨ synchro of the relaxation oscillator ↲ atteint + tôt
peut passer de synchro 1:n à 1:n+1 → oscillation passe de ω à ω/2 (ω=ω0, puis ωo=ω/2 ac cptance □)
↲ overlap régions de synchro

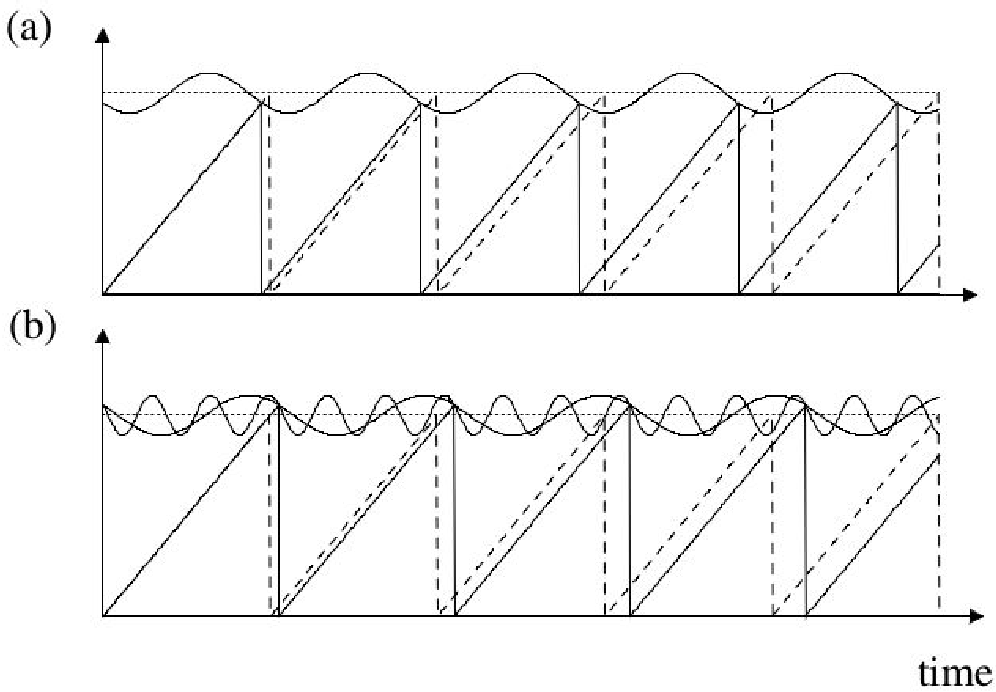


Figure 3.28. Synchronization of a relaxation oscillator via variation of the threshold. The threshold of the unforced system is shown by the dotted line; the time course of autonomous oscillation is shown by the dashed line. Periodic variation of the threshold can force the system to oscillate with the frequency of variation; this action can both decrease (a) or increase (b) the frequency of oscillation. As is clearly seen in (b), synchronization of higher order (here it is 1 : 3) can also be easily achieved.

entre 2 fires, x=xthresh * ω0(t-ti)/2π → pente α ω0
force change fréquence ⇨ entraîne oscillateur → x=xthresh . (ω0+εsin(ωti+Φe))*(t-ti)/2π REVOIR

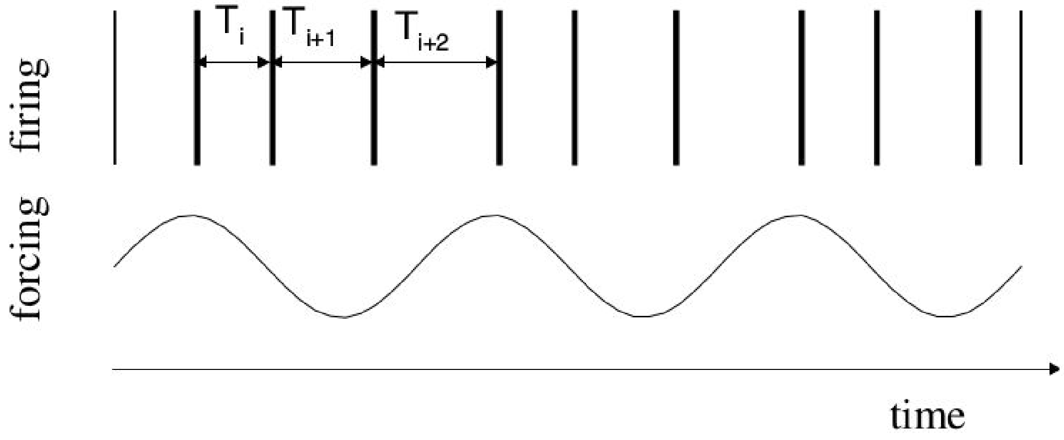


Figure 3.30. Synchronization of the integrate-and-fire oscillator via variation of its natural frequency. A regime of 3 : 1 locking is shown here: three spikes occur within one period of forcing. Note that the period of the entrained oscillator is varied (modulated) by the force, $T_i < T_{i+1} < T_{i+2}$, and the spikes are not evenly spaced in time.

modulation != synchronisation : peut moduler de façon non synchrone (mTi=T) si ajuste pas fréquences

NOISE : phase diffusion/ random walk, donc seulement une fréquence moy <T0>=T0 → accum des perturb
sans bruit : Φ=ω0t+Φ0

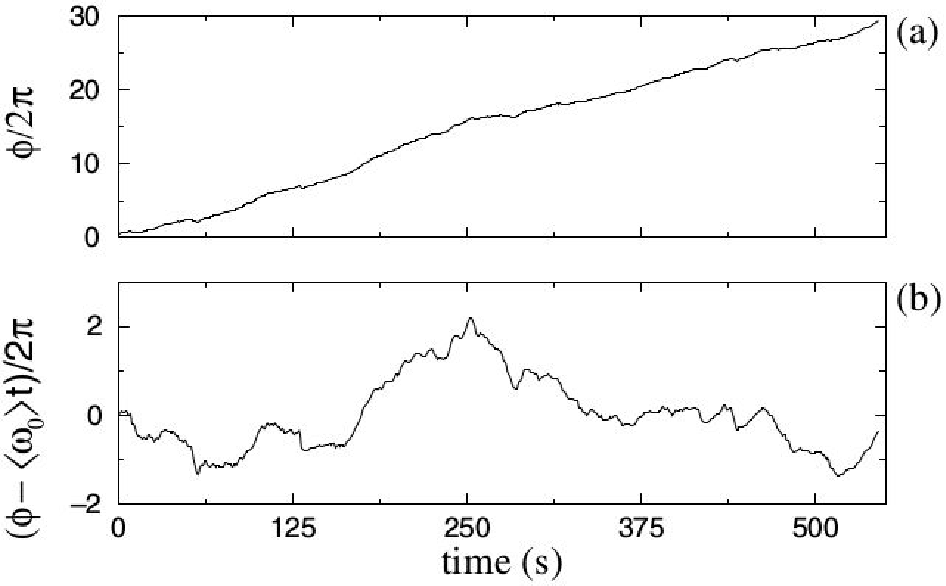


Figure 3.33. Random walk of the phase of respiration. Human respiration is governed by a rhythm generator situated in the brain stem. This system definitely cannot be regarded as an ideal, noise-free oscillator. The phase of respiration is computed from a record of the air flow measured by a thermistor at the nose. Due to noise, the increase of the phase is not linear (a) but can be regarded as a random walk. It is more illustrative to plot the deviation of the phase from linear growth $\langle\omega_0\rangle t$ (b). Note that this deviation is not small if compared with 2π.

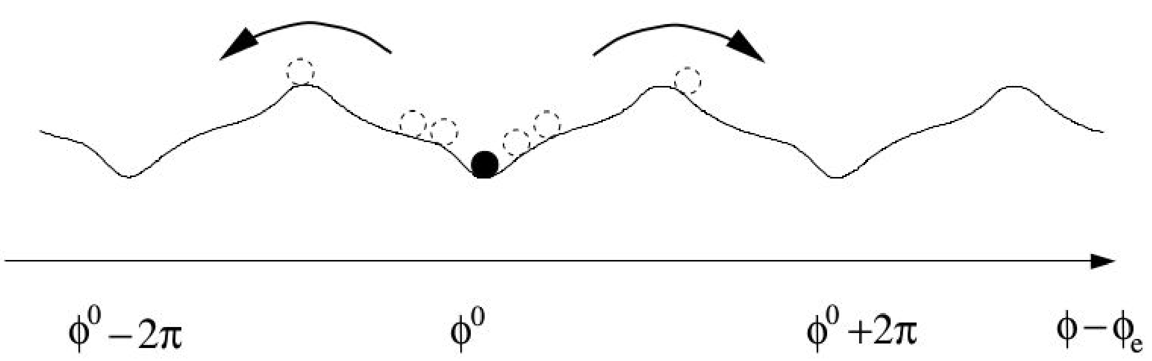


Figure 3.34. Phase diffusion in the oscillator that is forced without detuning: the phase point oscillates around the stable equilibrium point $\phi - \phi_e = \phi^0$ and sometimes jumps to physically equivalent states $\phi^0 \pm 2\pi k$.

si bruit pas assez élevé, reste autour de l'équilibre, sinon (non borné ou trop élevé, ex gaussien), peut sauter au puits de potentiel suivant → phase slip
Avec detuning : pas symétrique, tendance à tomber, synchro diff

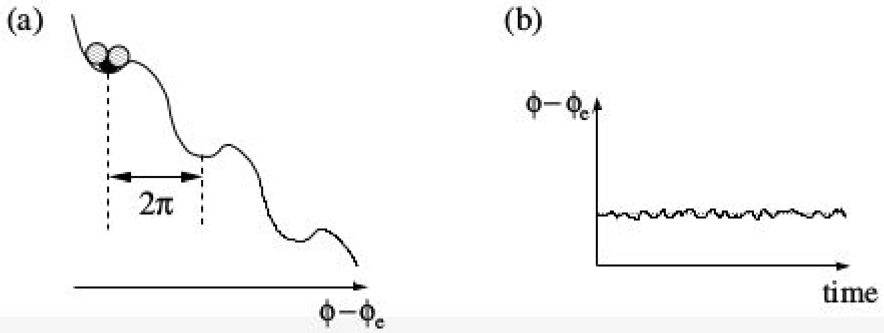


Figure 3.35. Phase dynamics of a forced oscillator perturbed by weak bounded noise. (a) The particle oscillates around a stable equilibrium point, but cannot escape from the minimum of the potential. Correspondingly, the phase difference fluctuates around a constant value (dotted line) that it would have in the absence of noise (b).

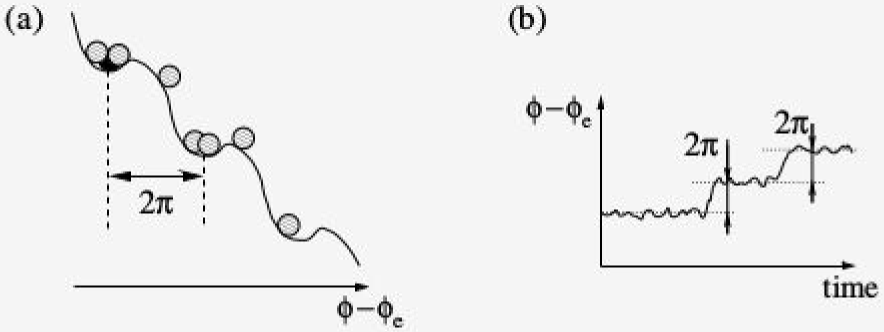


Figure 3.36. Phase dynamics of a forced oscillator subject to unbounded noise. (a) The particle oscillates around the stable equilibrium point ϕ^0 and sometimes jumps to physically equivalent states $\phi^0 \pm 2\pi k$. Although jumps in both directions are possible, the particle more frequently jumps over the small barrier, i.e., downwards. These jumps (phase slips) are clearly seen in (b). The time course of the phase difference resembles the phase dynamics of a noise-free oscillator at the desynchronization transition (see curve 3 in Fig. 3.8b), but in the noisy case the slips appear irregularly.

(a) : Ω = f(Force). pour gd detuning (+ petit que sans B), I(B) svt suffisant pour sauter barrière
(b) unbounded noise destroys synchro → ni phase locking, ni frequency locking, ok si assez faible, sinon attraction de Ω par ω

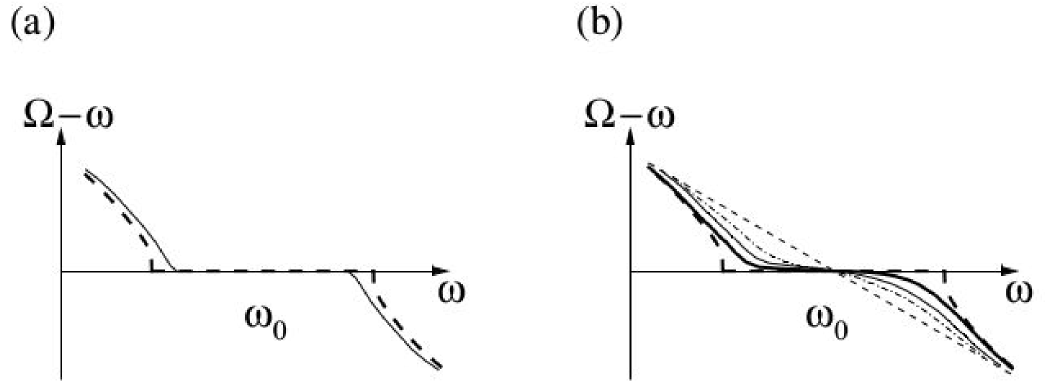


Figure 3.37. Frequency–detuning curves for noisy oscillators. (a) For weak bounded noise there exists a region of detuning where the frequencies of the oscillator Ω and the force ω are exactly equal. This region (solid line) is smaller than in the noise-free case (bold dashed line). (b) For unbounded noise the frequencies Ω and ω are equal only at one point, not in an interval. If the noise is very weak (bold line) then we can speak of approximate equality of frequencies in some range of detuning $\omega - \omega_0$. If the noise is stronger (solid and dashed-dotted lines) then this range shrinks to a point. Strictly speaking, synchronization appears only as a tendency: the external force "pulls" the frequency of the oscillator towards its own, but the noise prevents the entrainment. Very strong noise completely destroys synchronization (dashed line). The bold dashed line shows again the synchronization region for a noise-free oscillator.

Pas phase locking mais reste dans mini local de potentiel créé par la force : pic =ΔΦ sans bruit
distribution similaire quand modulation sans synchro ou désynchronisation (frontière detuning)

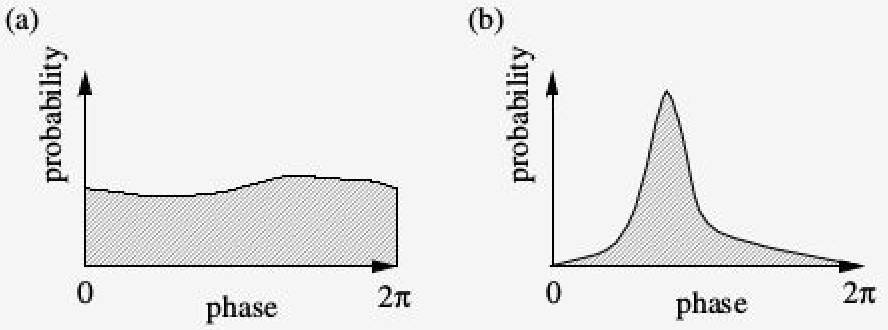


Figure 3.38. Broad (a) and unimodal (b) distributions of the phase difference in a noisy oscillator, without (a) and with (b) external forcing. If the forcing is absent, then the phase difference attains any value with almost the same probability, due to phase diffusion (a). The external force, creating minima in the potential, makes a certain value more probable (b). Similar distributions are obtained if we observe the phase of the oscillator stroboscopically, with the period of the external force. By such an observation, the phase of an entrained noise-free oscillator is always found in the same position (see. Fig. 3.13); the noise broadens the distribution but the maximum in it is preserved, indicating synchronization. (See also the experimental example shown in Fig. 3.15.) A distribution similar to (b) is also observed in the absence of synchronization, when the phase of a process is modulated.

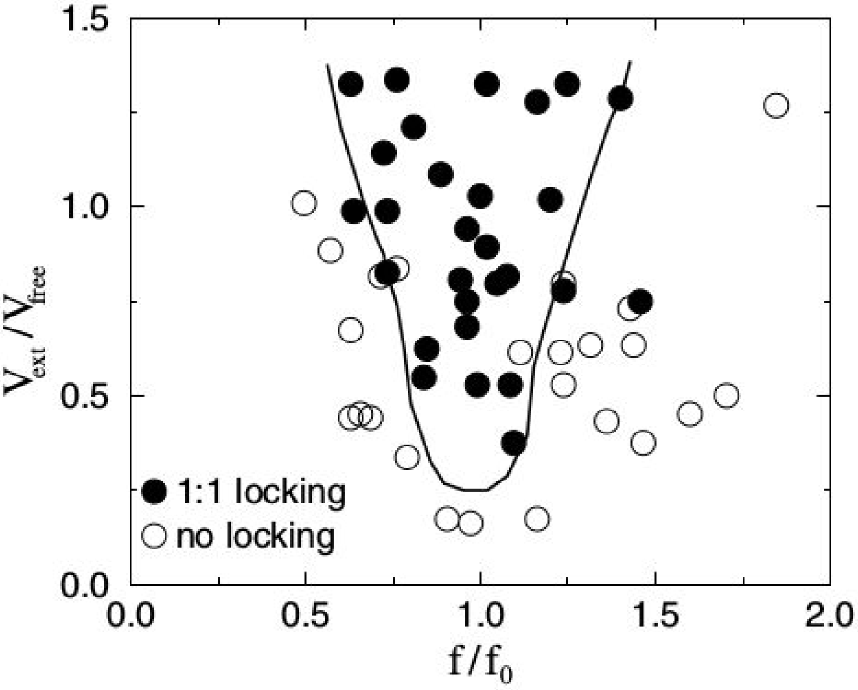


Figure 3.39. A region of 1 : 1 phase locking of spontaneous breathing by a mechanical ventilator obtained for seven subjects. The strength of the forcing is characterized by the volume V_{ex} of the inflation. Frequency and volume axes have been normalized with respect to the mean "off pump" frequency and tidal volume of separate experiments, respectively. The boundary of the synchronous regime has been drawn arbitrary. From Graves *et al.* [1986].

pas de frontière définie entre synchro/ x synchro
Expe : f0 sans stimulation externe

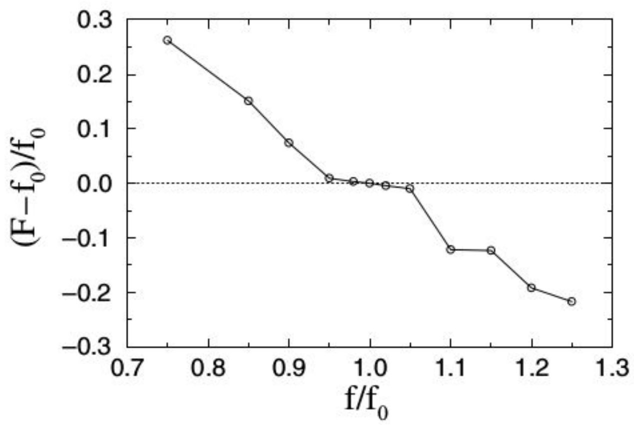


Figure 3.40. The observed mean frequency F of the heart rhythm of a human as a function of the frequency f of the weak external stimulation. This experimental curve is consistent with the corresponding frequency vs. detuning curves for a periodically driven noisy oscillator (cf. Fig. 3.37). From [Anishchenko *et al.* 2000].

rythmes circadiens : isolés à l'intérieur → période change : <24h = avance de Φ par rapport à la force ext, >24h = retard → si avance, de phase, c'est qu'on a naturellement une fréq + élevée mais qu'on est forcé?
ΔΦ dépend du detuning initial + so oscillateur est quasi-linéaire ou relaxation

force trop forte extérieure peut au contraire supprimer les oscillations/ Arnold Tongue forme ≠
→ idée pour supprimer les rythmes Tremor de Parkinson ?
driven system peut pas être synchronisé car phase relié à la phase de la force (pas de shift)
entre les 2 : noise induced oscillators → stochastic resonance

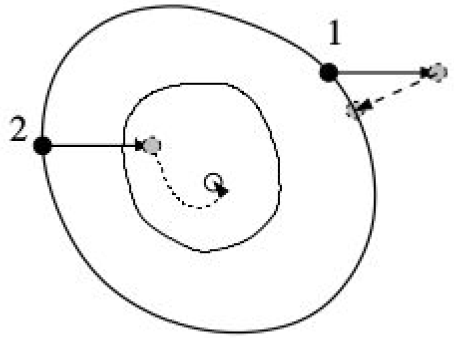


Figure 3.46. The effect of a single pulse on an oscillator with a coexisting limit cycle and stable equilibrium point depends on both amplitude and phase of stimulation. A pulse applied when the point is in position 1 just resets the phase; the pulse of the same amplitude applied at position 2 puts the phase point inside the basin of attraction of the stable equilibrium point (unfilled circle) thus quenching the oscillation; the basin boundary is shown by the solid curve.

threshold variable, quand seuil plus bas, proba de spike plus élevée -> phase locking

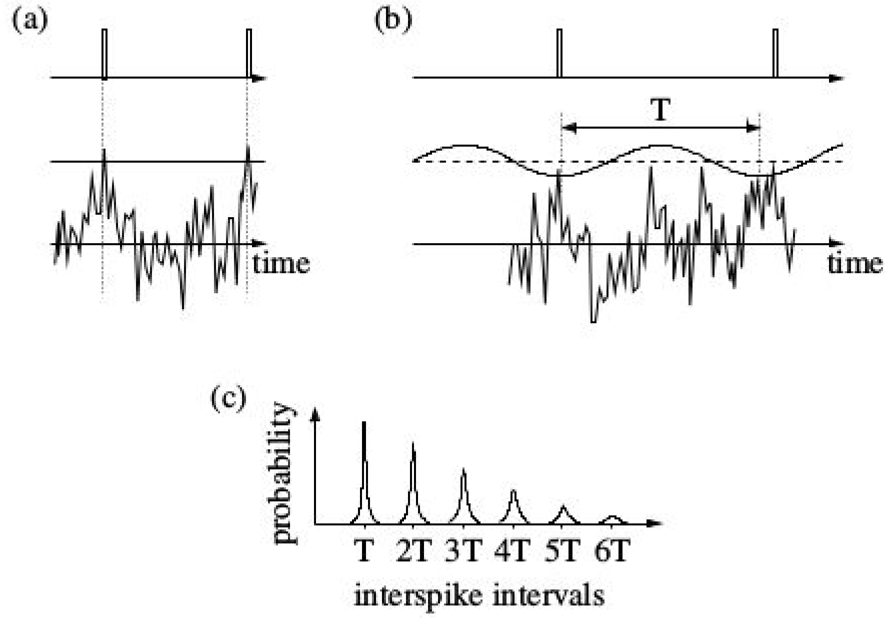


Figure 3.48. (a) Schematic representation of a noisy threshold system. This system produces a spike each time the noise crosses the threshold. (b) If the threshold is varied by a weak external signal, the firing predominantly occurs at a certain phase of the force, when the threshold is low. This results in the characteristic structure of the distribution of interspike intervals shown in (c). Such distributions have been observed in experiments with periodic stimulation of the primary auditory nerve of a squirrel monkey [Rose *et al.* 1967] and a cat [Longtin *et al.* 1994], as well as in periodically stimulated crayfish hair cells [Petracchi *et al.* 1995].

Plusieurs oscillateurs, une force → laser haute intensité par somme de lasers avec fréq=, Φ proches
Injection locking : un laser synchronise les autres

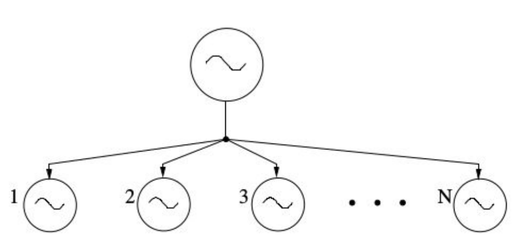


Figure 3.51. Ensemble of similar oscillators influenced by a common external force.

Chapitre 4 Ajustement mutuel des fréquences

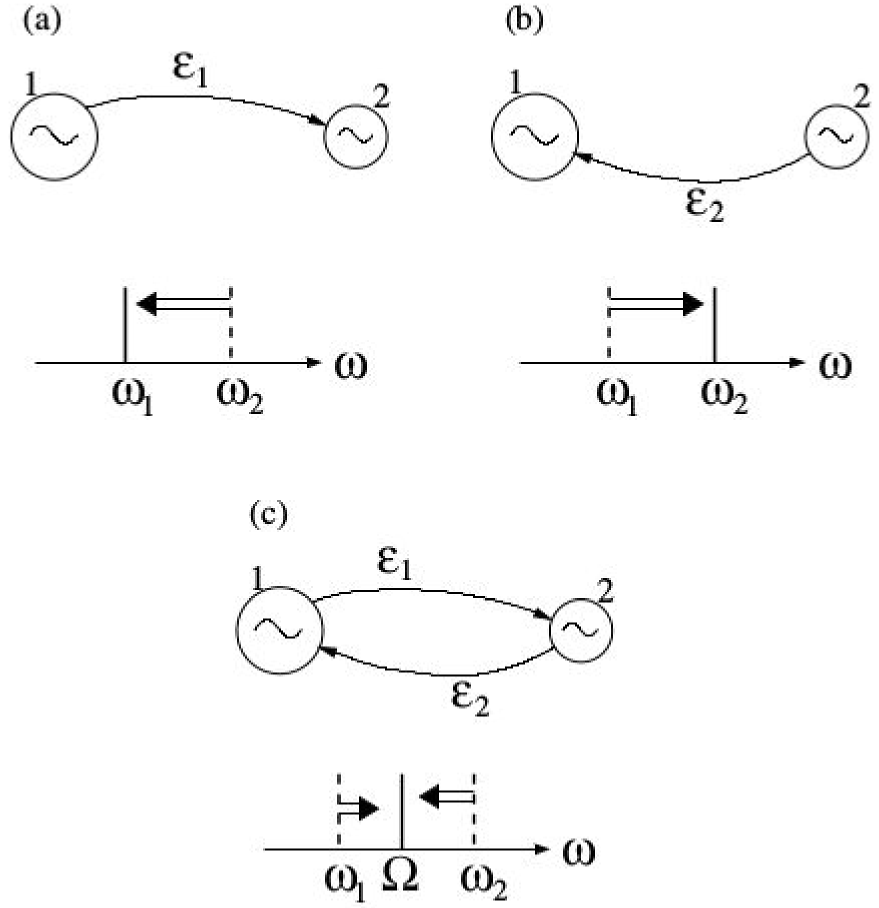


Figure 4.1. Adjustment of frequencies of two interacting oscillators (ω_1 and ω_2 are their natural frequencies). If the coupling goes in one direction (a and b), the frequency of the driven system (dashed vertical bar) is pulled towards the frequency of the drive. This is equivalent to the case of external forcing. If the interaction is bidirectional ($\varepsilon_{1,2} \neq 0$) then the frequencies of both systems change (c); the common frequency Ω of synchronized oscillators is typically in between ω_1 and ω_2. The frequency diagrams can be regarded as schematic drawings of the *power spectra* of oscillations.

synchronisation de 2 oscillateurs : si l'un est plus fort, seule une fréquence change (force extérieure), sinon les 2 se retrouvent au milieu -> frequency locking
faible intéraction → influence que la phase
synchro dépend des phases de 2 oscillateurs → in-phase (phase-attractive interaction) ou anti-phase synchro
peut avoir n:m synchro

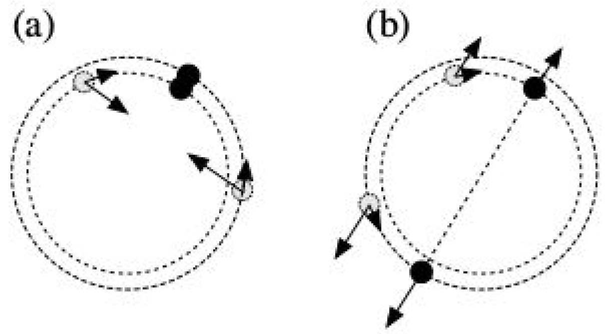


Figure 4.2. Two mutually coupled oscillators with phase-attractive (a) and phase-repulsive (b) interaction. The sketch shows the phases of two systems with equal natural frequencies; they are shown in a frame rotating with frequency $\omega = \omega_{1,2}$. Attraction (repulsion) of phases (shown by arrows) results in in-phase (anti-phase) synchronization. The components of the forces that push the phase points along the limit cycle are also shown by arrows. In the case of nonzero detuning (i.e., for nonidentical oscillators), the phase difference in the locked state is not exactly zero (or not exactly π); an additional phase shift arises, so that the interaction compensates the divergence of phases due to the difference in frequencies (cf. Fig. 3.4).

synchro en phase ou opposition de phase

synchronisation mutuelle pour integrate and fire : le plus rapide émet un PA, ensuite fait que les autres atteignent le seuil plus rapidement que sans couplage, donc le plus rapide impose le rythme

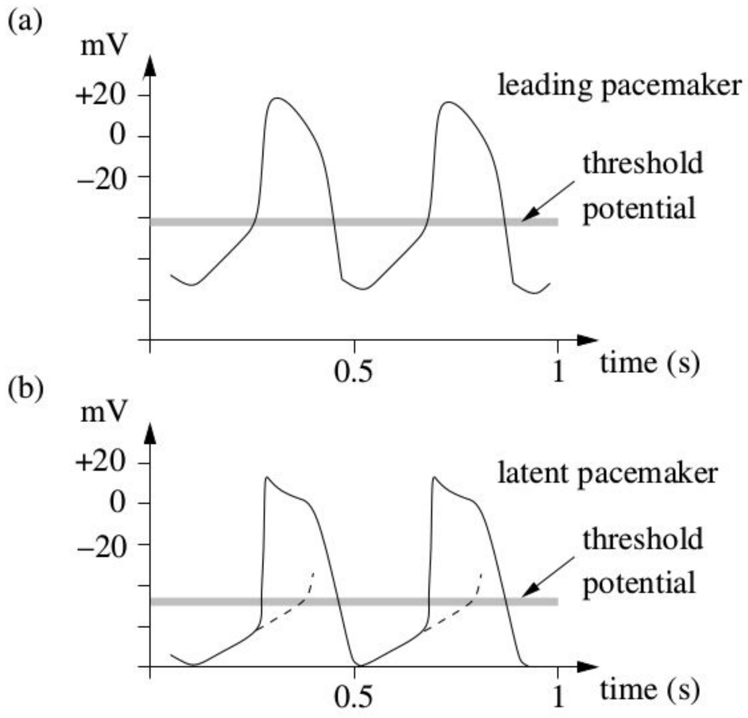


Figure 4.9. Synchronization of a true cell and a latent pacemaker cell of the sino-atrial node of the heart. The membrane potential of each cell slowly increases until it reaches the critical value; then, an action potential is generated, i.e., the cell fires. The threshold is first reached by the true pacemaker cell (a), and its discharge makes the latent pacemaker (b) generate the action potential before its threshold level is reached. Without interaction, the latent pacemaker would fire later (dashed curve). From [Dudel and Trautwein 1958], see also [Schmidt and Thews 1983].

Avec du bruit :

- There is no distinct border between synchronous and nonsynchronous states, the synchronization transition is smeared.

- In a synchronous state, epochs of almost constant but fluctuating phase difference are interrupted by phase slips when the phase difference comparatively rapidly increases or decreases by 2π.

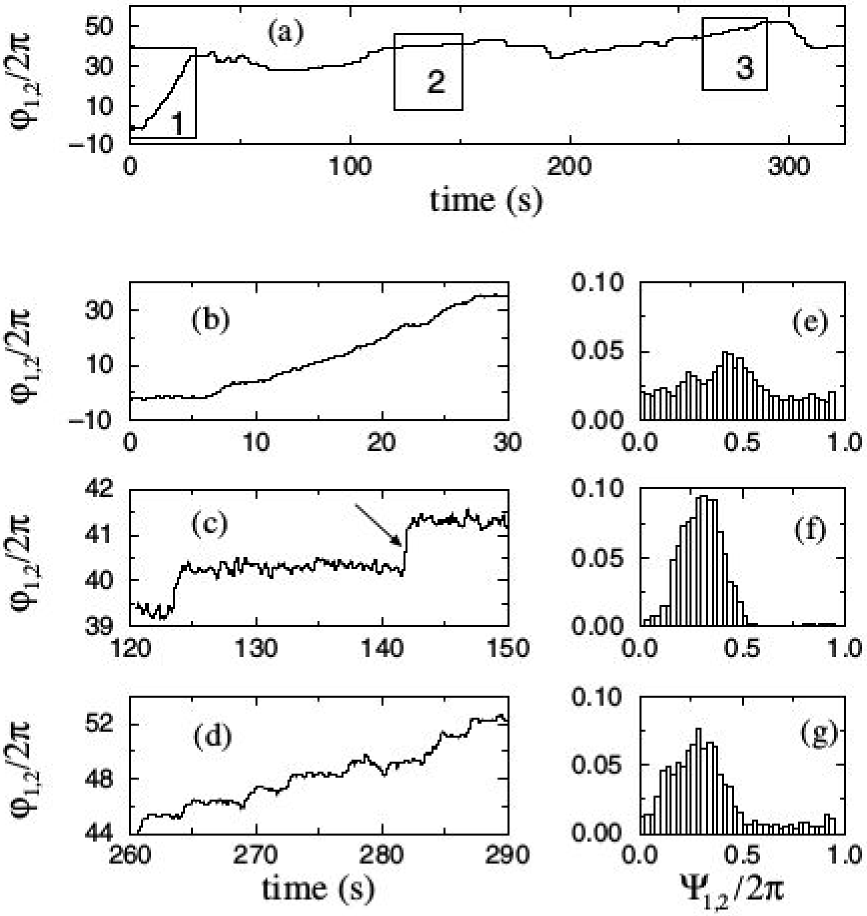


Figure 4.11. (a) The time course of the 1 : 2 phase difference $\varphi_{1,2} = \phi_{MEG} - 2\phi_{EMG}$ demonstrates the behavior that is typical of noisy interacting oscillators. Three 30 s intervals marked by boxes 1, 2 and 3 are enlarged in (b), (c) and (d), respectively. These intervals are essentially different due to the nonstationarity of the system (the tremor activity starts at ≈50 s). Indeed, within the first 30 s the oscillators can be considered as nonsynchronized (b): the phase difference increases and the distribution of the cyclic relative phase $\Psi_{1,2} = \varphi_{1,2} \mod 2\pi$ is almost uniform (e). The intervals shown in (c) and (d) and the corresponding unimodal distributions of Ψ in (f) and (g) reveal synchronization of interacting systems, although the strength of synchronization is different. The phase slip indicated by an arrow in (c) is enlarged in Fig. 4.12. From [Rosenblum *et al.* 2000], Fig. 2, Copyright Springer-Verlag.

Parkinson : un excès de synchronisation

For the particular data, the frequencies of the EMG and MEG signals are ≈6 Hz and ≈12 Hz, respectively,[5] therefore we expect to find synchronization of order 1 : 2.

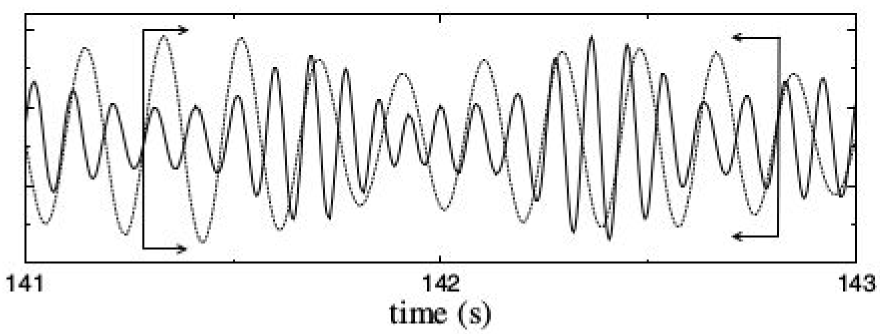


Figure 4.12. Phase slip takes several periods. The slow signal (dashed curve) is EMG, the fast signal (bold curve) is MEG (both are in arbitrary units). Within the marked region there are 8 and 17 cycles of EMG and MEG, respectively. On a larger time scale, the corresponding increase in the phase difference appears as a jump (cf. Fig. 4.11).

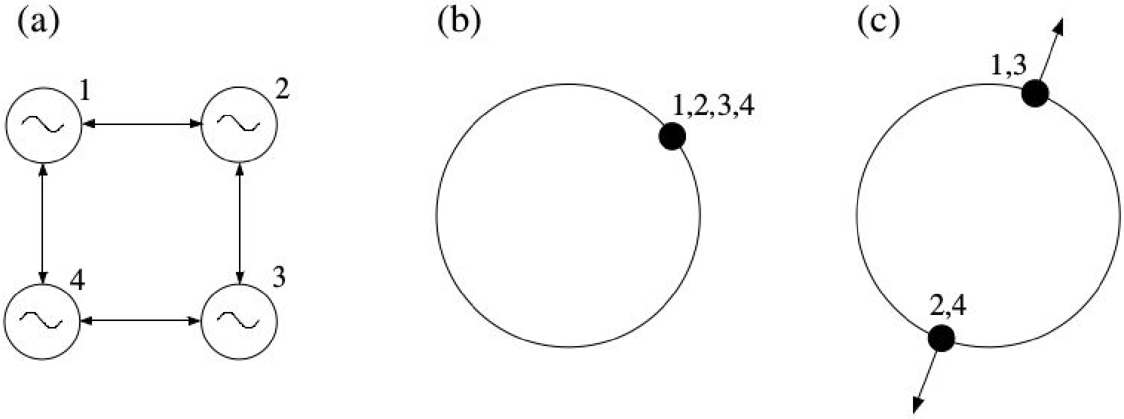


Figure 4.15. (a) Four identical oscillators arranged in a ring so that each system interacts with its two neighbors. (b) For a phase-attractive coupling, all oscillators are locked in-phase. (c) Phase-repulsive coupling results in the configuration when noninteracting elements (1 and 3, 2 and 4) have the same phases, and the interacting neighbors are in anti-phase.

anti-phase locking of these generators correspond to hopping and walking. Different gaits of quadrupeds (pace, trot, gallop, etc.) correspond to different synchronous states in a network of four systems.

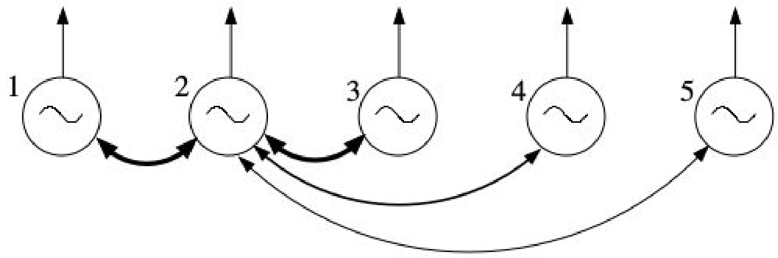


Figure 4.16. Schematic representation of a laser experiment by Glova *et al.* [1996]. The coupling strength in the array decreases with the distance between the lasers; this is reflected by the thickness of the arcs that denote coupling (shown only for laser 2).

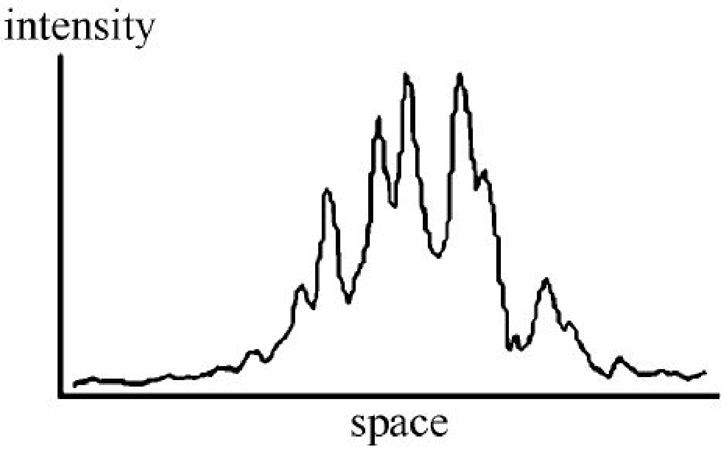


Figure 4.17. Radiation intensity in the far-field zone of a weakly coupled laser array. The interference occurs due to the phase locking of individual lasers. From [Glova *et al.* 1996].

si pas synchro, devrait être uniforme (interférences destructives)

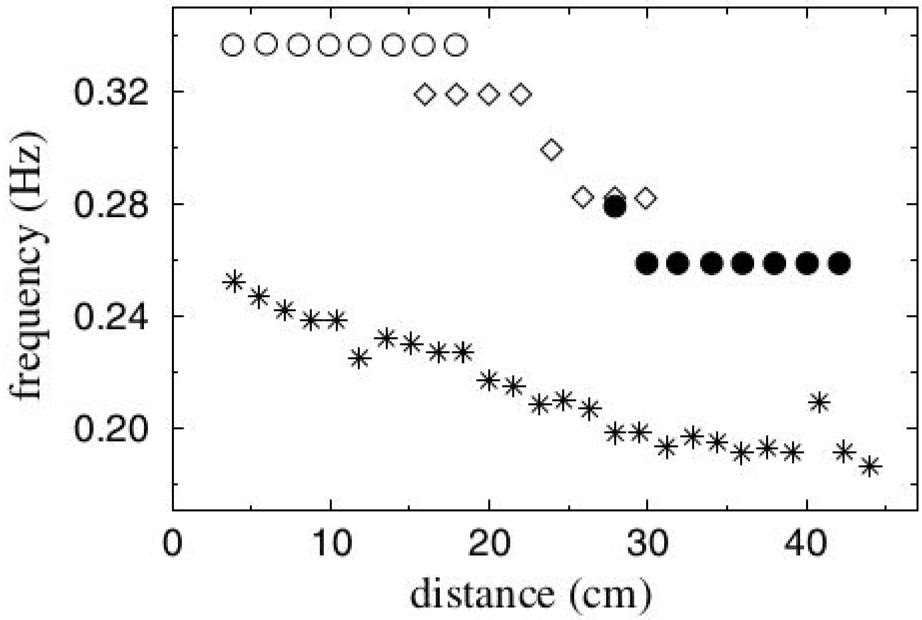


Figure 4.19. Synchronous clusters in a mammalian intestine. The frequency of slow electrical muscle activity plotted as a function of distance along the intestine typically shows a step-wise structure. (The distance is measured from the ligament of Treitz.) The ○, ◇, and • symbols represent the three consecutive (at 30 min intervals) measurements of frequency along the intestine *in situ*; for each measurement the electrodes were re-positioned. The stars show the frequency of the consecutive segments of the same intestine *in vitro*. From [Diamant and Bortoff 1969].

plusieurs clusters de synchronisation

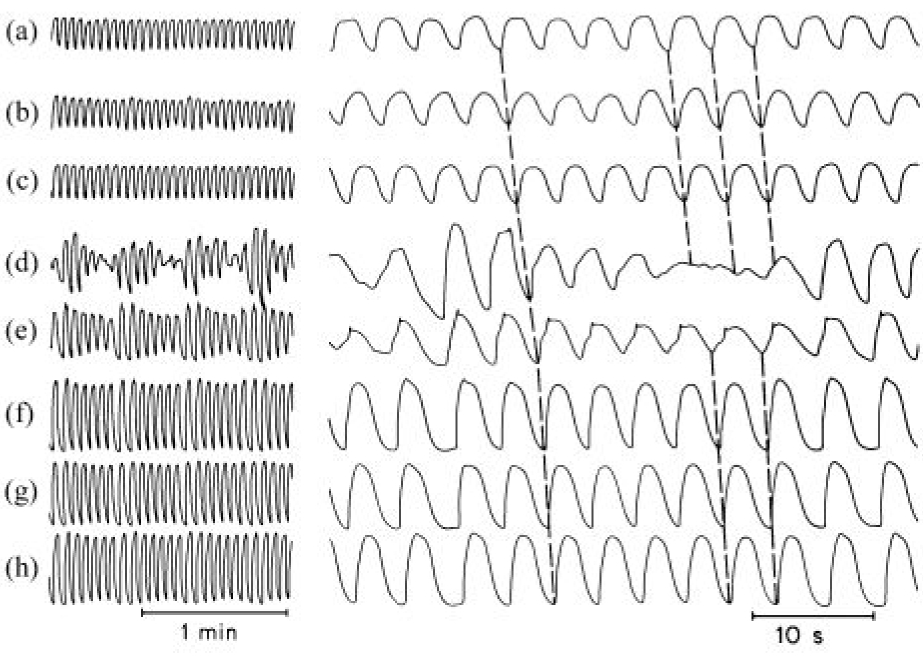


Figure 4.20. Electrical activity of the intestine of a cat, measured *in situ* by means of eight equidistantly spaced electrodes. The first three measurement points (a, b and c) belong to one cluster with the oscillation frequency 0.29 Hz. The three last electrodes (f, g and h) reflect the activity within another cluster, with a frequency of 0.26 Hz. In the zone between these two clusters an amplitude modulation of oscillation, or beats, is observed (d and e). Dashed lines show the lines of equal phase. From [Diamant and Bortoff 1969].

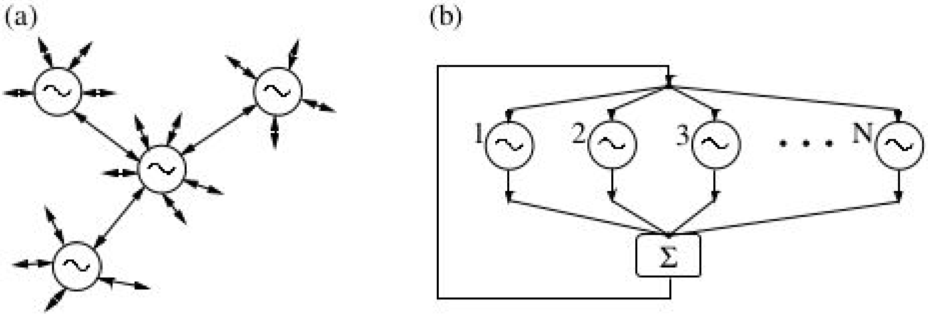


Figure 4.24. (a) Each oscillator in a large population interacts with all others. Such an interaction is denoted all-to-all, or global coupling. (b) An equivalent representation of globally coupled oscillators: each element of the ensemble is driven by the mean field that is formed by all elements.

synchro d'une large population → couplage global (common medium)
Etude hand clapping : onset synchro précédée de T*2
vitesse < : + stable (- noisy individuellement) et - de dispersion des fréquences → synchro transition
Etude running : vitesse élevée, quadrupèdes (1:1 respi/mvt au trop et galop) et humains (4:1,3:12:1,1:1.5,5:2,3:2)
mais 2:1 favorisé → synchro respi/mvt améliore perfo? ou juste csqce d'oscillateurs couplés

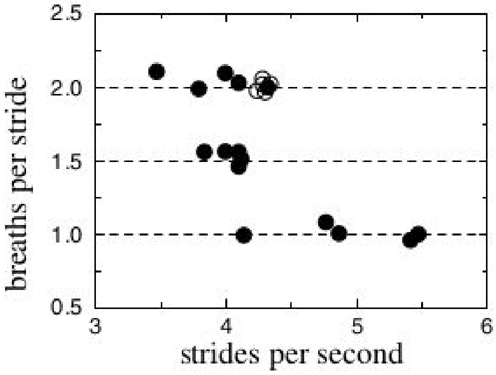


Figure 4.27. Relation between breathing and stride frequency in a young hare *Lepus californicus* running on a treadmill. Each data point is the mean of three consecutive strides. Filled circles are data obtained when the animal was immature; open circles were obtained when it was sub-adult. At lower speeds, there were two complete breathing cycles per locomotor cycle. At higher speeds, the hare abruptly switched to 1 : 1 synchronous regime, thereby halving its breathing frequency. When forced to run near the transition speed of approximately four strides per second, the animal repeatedly alternated two patterns, thus exhibiting 3 : 2 locking. Plotted using data from [Bramble and Carrier 1983].

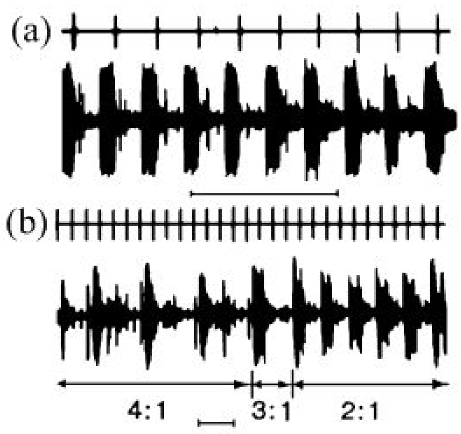
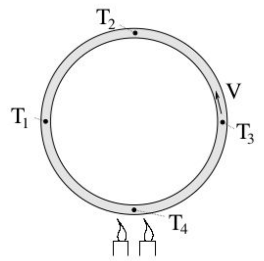


Figure 4.28. Oscilloscope record of gait and breathing in free running mammals. The upper trace is the footfall of the right forelimb (horse) or leg (human). The lower trace is breathing as recorded from a microphone in a face mask. (a) Horse at gallop. (b) Human during shift from 4 : 1 to 2 : 1 pattern. Horizontal bars denote the time scale (2 s). Reprinted with permission from Bramble and Carrier, *Science*, Vol. 219, 1983, pp. 251–256. Copyright 1983 American Association for the Advancement of Science.

Synchro maintient rythmes vitaux, comme respiration, mais responsable de maladies (épilepsie, Parkinson)
anesthésie découple les oscillateurs individuels → - synchro EEG quand anesth.

Chapitre 5 Synchro of chaotic systems

chaotique : peut pas prédire long terme, même sans variation des paramètres du système → irrégulier et imprédictible
résulte de la dynamique du système
dans phase space, strange attractors
ex convection Lorenz : si augm chauffage par en-dessous → rotation devient instable
besoin au moins 3 coordonnées
distinction self-sustained : equations autonomes/tous instants équivalents
forced → breaks the continuous time symetry → becomes discrete : equivalent si mod(Tforce)=0



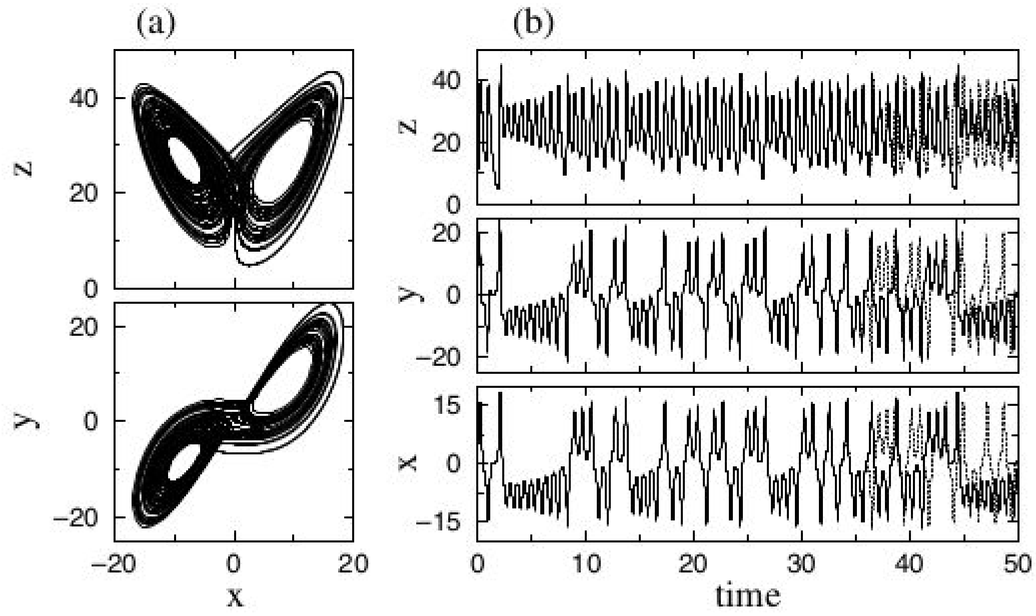


Figure 5.2. The dynamics of the Lorenz system (see Eqs. (10.4)). (a) Projections of the phase portrait on planes (x, y) and (x, z). Note the symmetry $x \to -x$, $y \to -y$. (b) The time series of the variables x, y, z (solid curves). In this figure we also demonstrate the sensitivity to small perturbations: at time $t = 25$ a perturbation 10^{-4} is added to the x variable. The perturbed evolution, shown with the dashed curve, very soon diverges from the original one and demonstrates a different pattern of oscillations.

irrégularité mais peut prédire petit pas de temps → régi par eq diff, mais très sensible aux petites perturbations CI → Simu compliquées : précision finie suffit jamais

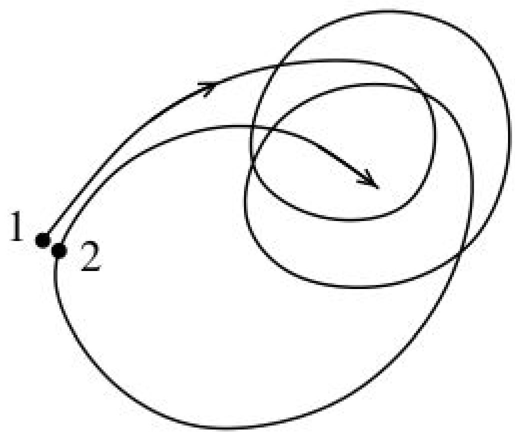


Figure 5.3. An illustration of why the instability of trajectories leads to irregularity: nearly repeated states 1 and 2 eventually diverge, making all repetitions temporary.

instabilité mesurée par l'exposant de lyapunov le plus grand., 1/exposant = tps caractéristique d'instabilité
CI proches, après tps prop 1/expo, points éparpillés
nb variables indép de la dynamiques → n exposants de lyap.
rappel : oscill périodiques on 1 expo nul, tous les autres négatifs (cvg vers le limit cycle/shift du point le long du cycle pour 0→ perturbations ni augm ni diminuent) → phase pour chaotic oscillators : coordonnées le long de la trajectoire

→ variable qui correspond à l’exposant 0 → possiblement phase synchro

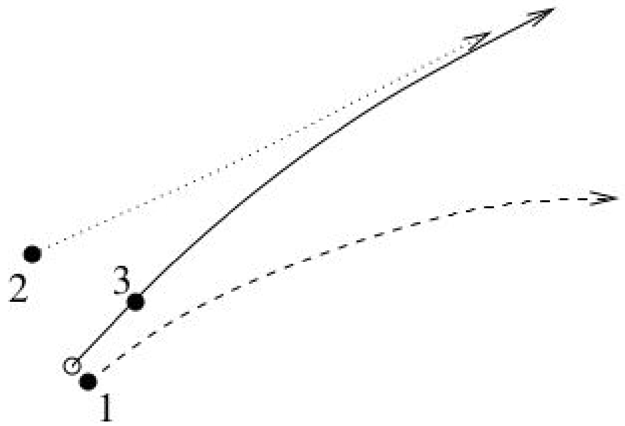


Figure 5.4. Diverging, converging and neutral perturbations of a chaotic system. The state of a three-dimensional chaotic system (open circle) is perturbed in one of three ways corresponding to three directions in the phase space (filled circles). The unperturbed trajectory is shown with the solid curve. The perturbation 1 grows: the corresponding trajectory (the dashed curve) moves away from the original one. The perturbation 2 decays: its trajectory (the dotted line) converges to the unperturbed one. The perturbation 3 lies on the same trajectory and neither grows nor decays.

certains oscillateurs chaotiques pê caractérisés par time-dependant phase et fréquence → synchro (FrL) possible
svt, chaotic oscillations ressemble à quasi-pér modulé de manière irrégulière → A/T varie

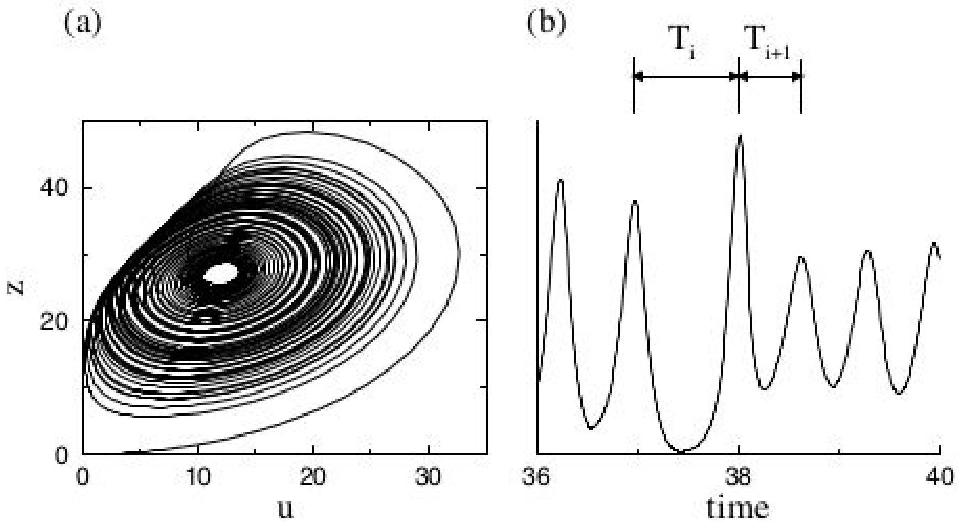
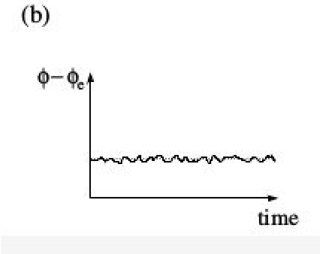


Figure 5.5. (a) In the variables z, u the dynamics of the Lorenz model look like rotations around a center (at $u \approx 12$, $z \approx 27$), with an irregular amplitude and an irregular return time. (b) The return times T_i are shown in the plot of z vs. time as the distance between the maxima; T_i can be considered as instantaneous periods of chaotic oscillation.

définition tps entre 2 évts similaires (max variable) → construction Poincaré map avec condition z max
return times = périodes instantanées <T>=n(tau)/tau, avec tau>>1, <w>=2*pi*N(tau)/tau
→ fréq pê entrainée par force exte/autre système chaotique couplé
fréquence irrégulière → pente phase aussi → phase diffuse (cf B)
phase dynamique = rotation ac <f> + diffusion (random walk) (rate of diffusion prop to variation of return time)



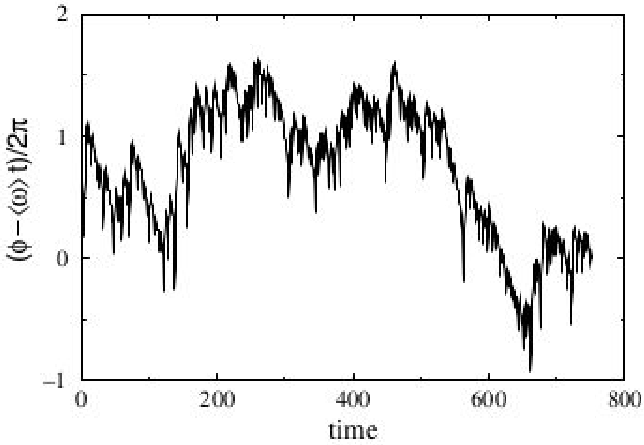


Figure 5.6. The deviation of the phase of the Lorenz system from uniform rotation demonstrates a typical pattern of a diffusion process (random walk).

// Fig 3.35

diffusion phase < dvg trajectoires voisines
oscillations cohérentes : #return times <<1 → projection dans phase plane = rotations assez régulières ac modulation chaotique de l'amplitude
calculer phase = non linéaire (néglige variations A)
external forcing : phased locked by external force + mean freq = freq F → même si pas tous les points entraînés
synchro pour A modérée de forcing (sinon suppression chaos), mais doit qd même supprimer phase diffusion

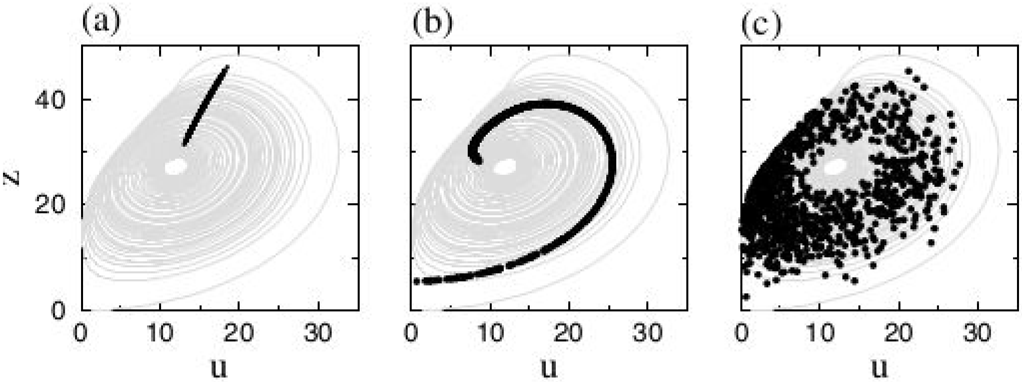


Figure 5.7. Phase diffusion in the Lorenz model. The attractor is shown in gray. (a) A set of initial conditions (black dots) is chosen on the Poincaré surface of section (variable z is maximal), all the phases are zero. The evolution of this set is followed in (b) and (c). (b) After time $t = 0.75$ (one mean return time) some points lag while others advance, the phases are distributed in a finite range less than 2π. (c) After a further time interval $t = 3.75$ the trajectories become distributed along the attractor, which means that the phases are nearly uniformly distributed from 0 to 2π.

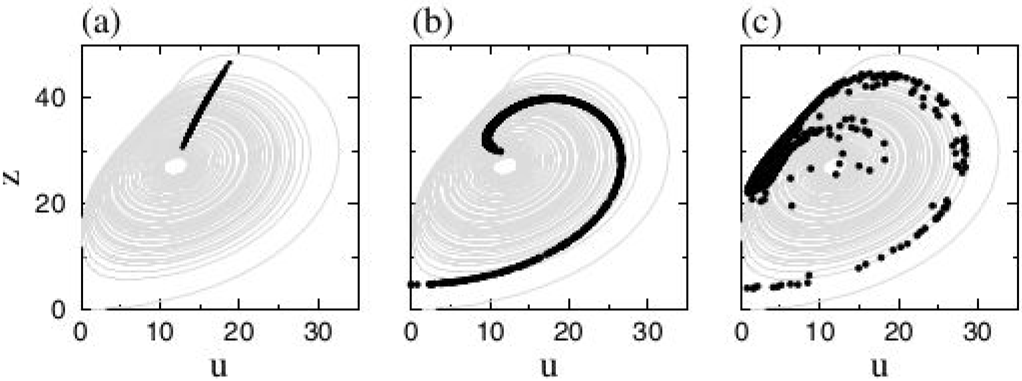


Figure 5.8. The same as Fig. 5.7, but in the periodically forced (with dimensionless frequency 8.3) Lorenz model. (a) Initial conditions are the same as in Fig. 5.7a. (b) After one period of forcing the phases are slightly more concentrated around the mean value; in fact, a difference to Fig. 5.7b can hardly be seen. Nevertheless, the effect accumulates and after a further 50 forcing periods this concentration is clearly seen in (c). A large number of points rotate synchronously with forcing, although some are not entrained.

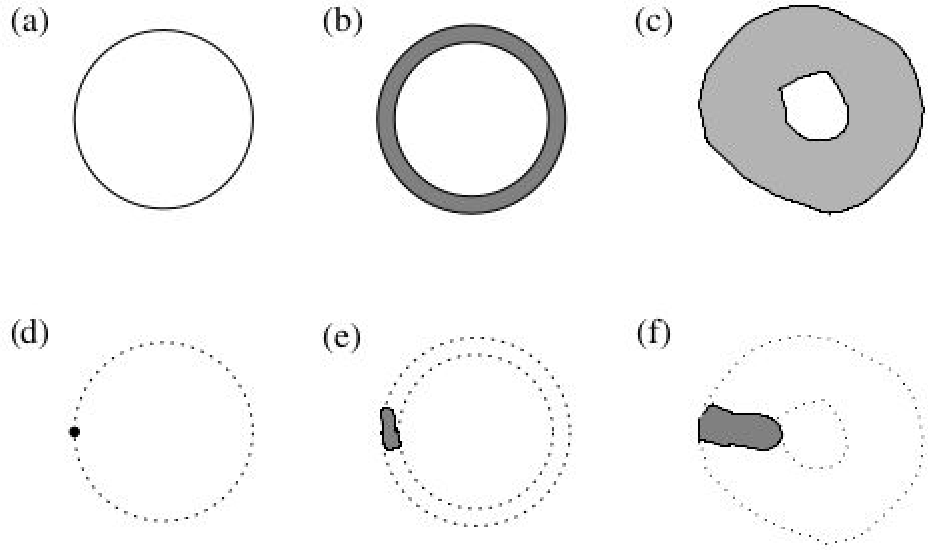


Figure 5.9. A sketch of common features of phase synchronization in periodic (a), (d), noisy (b), (e), and chaotic (c), (f) oscillators. When a periodic oscillator (a) is entrained by a periodic force, the stroboscopically (with the period of the force) observed phase has a definite value (d). This perfect picture is partially spoiled by a bounded noise, but nevertheless entrainment is possible: the fluctuations of the phase are bounded (e). A chaotic oscillator (c) resembles the noisy one (b); here the diffusion of the phase can be also suppressed, yielding a state with a chaotic amplitude but bounded phase (f).

periodic : phase locking pour F arbitrairement petite
noisy/chaotic : besoin suppression phase diffusion → synchro threshold (reste qd même noisy/chaotic)
noisy/chaotic très similaires pour phase synchro → pour détecter synchro, pas besoin décider 1/autre
Pikowsky 1985 : forcing → pic spectral + fin (largeur du pic caractérise phase diffusion, et F supprime diffusion)

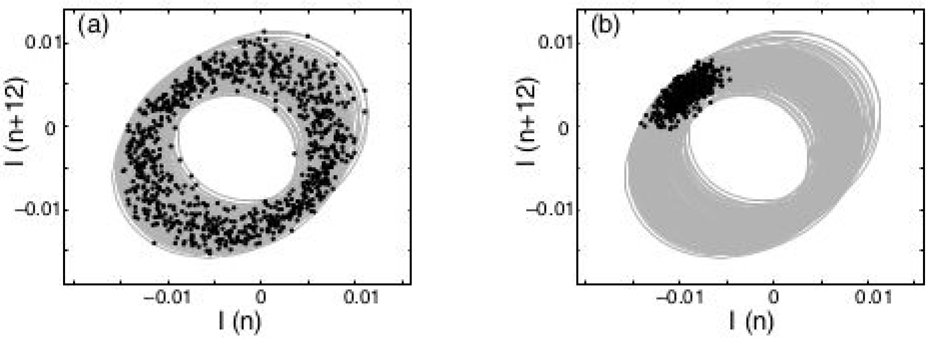


Figure 5.10. Stroboscopic observation of the unforced (a) and forced (b) chaotic gas discharge. The attractor is shown in gray, and the states of the system observed with the period of the force are shown by circles. Concentration of the points in (b) indicates phase synchronization (cf. Figs. 5.8 and 5.9). From [Rosa Jr. *et al.* 2000].

couplage mutuel fort peut mener à synchro complète (contrairement à phase synchro, peut être observé pour tout système chaotique,pas juste autonome)
Couplage de 2 systèmes identiques :
 # synchro classique car pas ajustement de rythmes mais suppression de rythmes d’oscillateurs couplés
→ entrainement/locking : si états égaux, le reste dans le temps → complete synchro même si chaotique
possible pour toute F de coupling, mais stable que si F assez fort (si petite perturbation, dvg ds tps) → thresh phenom
thresh prop to Lyap expo of individual system

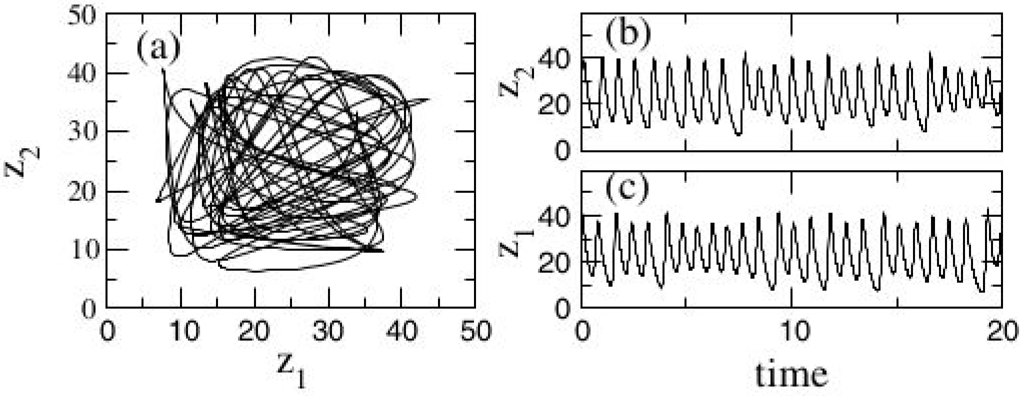


Figure 5.11. Weakly coupled Lorenz models (below the synchronization threshold). The simplest way to see the difference between the two systems is to plot the variables of one vs. the variables of the other one (a Lissajous-type plot), as in (a). The difference between two chaotic states can be also seen from the time series (b) and (c).

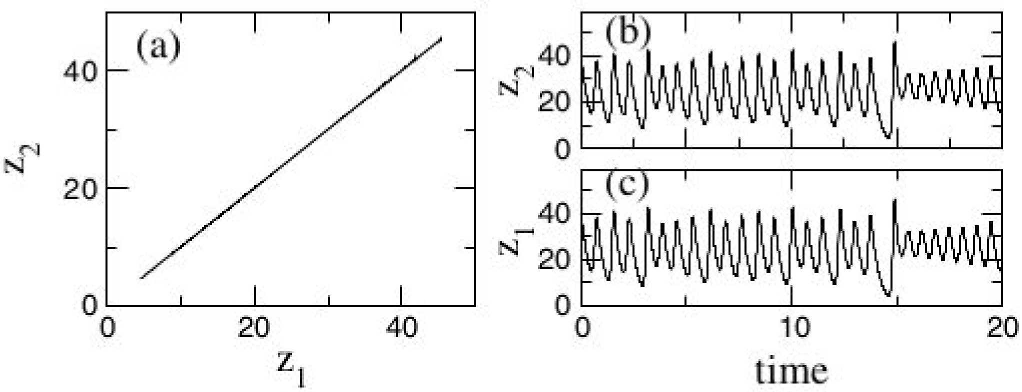


Figure 5.12. Complete synchronization in coupled Lorenz models. (a) The states of the systems are identical, as can be easily seen on the plane z_1 vs. z_2: the trajectory lies on the diagonal $z_1 = z_2$. The time series of two systems are chaotic in time, but completely coinciding ((b) and (c)).

Couplage de 2 systèmes non identiques : relation x2=F(x1) → generalized synchro, svt master/slave
symétrie entre variables (z→ v et inv change rien), z=v est une solution possiblement chaotique
chaos destroying synchro : peut faire osciller périodiquement le système

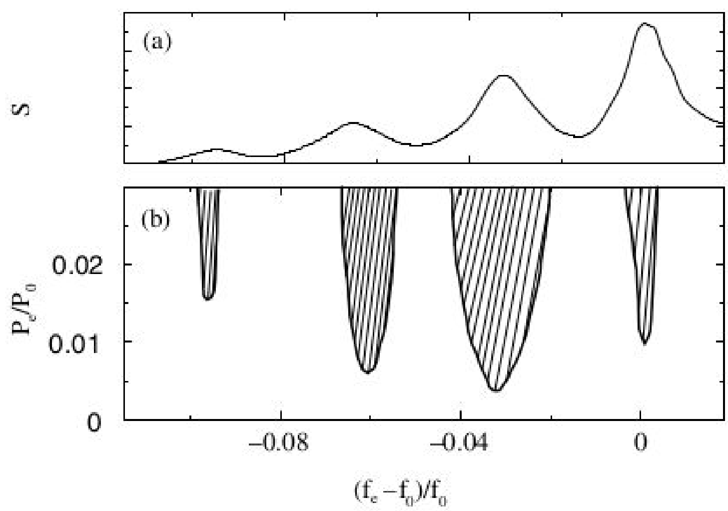


Figure 5.15. Chaos-destroying synchronization of an electron beam–backscattered electromagnetic wave system by a periodic force. (a) Power spectrum of the autonomous chaotic system shows several broad maxima. If a periodic force with a frequency close to that of one of the maxima acts on the system, it can suppress chaos. As a result, periodic oscillations with the frequency of the external force are observed within the shaded regions (b). Note that the force should be sufficiently strong. From [Bezruchko 1980; Bezruchko *et al.* 1981].

Chapitre 6 : Détection de synchronisation dans les expériences
→ déterminer synchro properties d'un oscillateur expérimentalement
→ utlisation idée synchro pour analyser interdépendance entre 2 signaux → Appendix A2

ECG : R-peak
phase peut être interpolation linéaire entre les valeurs : $\phi(t)=2\pi k+2\pi*(t-tk)/(tk+1-tk)$ → si marqueur clair, mais influence du bruit + 1 évt par période → mauvaises stats
ou avec transformée de Hilbert : $f(t)=s(t)+ish(t)=A\exp(i\phi(t))$ → trouve la phase, mais slmt si s à bande étroite
donne phase à tous les points → peut lisser influence bruit

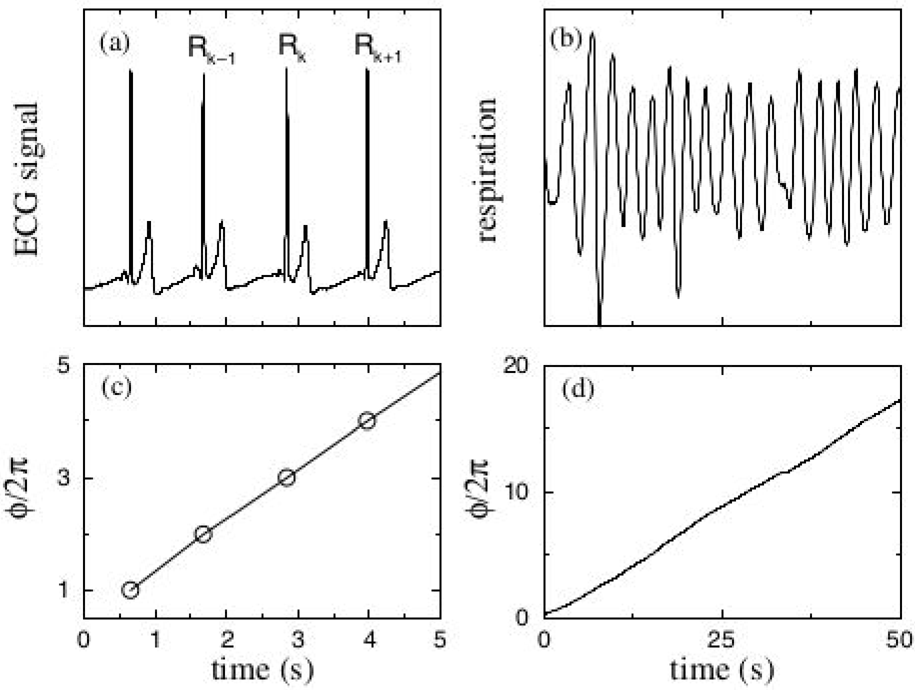


Figure 6.1. Short segments of (a) an electrocardiogram (ECG) with R-peaks marked and (b) of a respiratory signal; both signals are shown in arbitrary units. (c) Phase of the ECG computed according to Eq. (6.1) is a piece-wise linear function of time; the instants when the R-peaks occur are shown by circles. (d) Phase of respiration obtained via the Hilbert transform (Eq. (6.2)).

hilbert sensible à low-frequency trend
incertitude sur l'estimation de la phase dûe aux observations
fréquence : nbCycles/unit ou pente du phase growth
<u>Expériences actives</u> : peut être entraîné par une force ?
1/ modifier detuning/varier couplage → regarde phase/fréq → Arnold tongue
si varie un seul param et observe transition, sûr qu'on a synchro

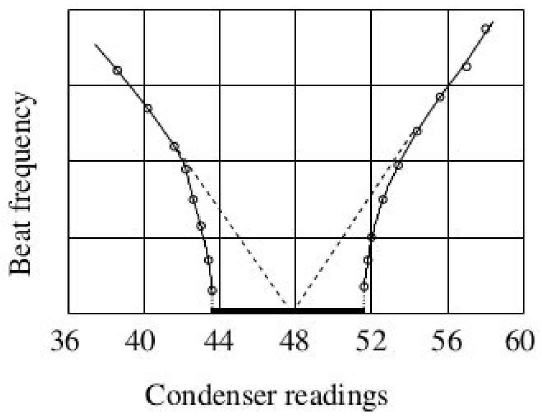


Figure 4.4. Results of the experiment with coupled triode generators. If no synchronization effects are taken into account, the theoretical change of the beat frequency with capacity is indicated by the dotted lines. The continuous lines are drawn through the experimental values. Synchronization region is shown by the horizontal bar. From [Appleton 1922].

synchro transition = Om1-Om2 VS paramètre varié
si synchro quasi-périodique → Lissajous figures ie x1 VS x2 : freq locked=closed curve

Expériences passives : 2 oscillateurs peuvent se synchroniser pour un certain type d'intéraction ? → couplées ou indép

ex ECG/respi : synchro is not a state, but a process of adjustements of freq/phases

+ noisy systems transition of synchro is smeared : pas de # distincte entre synchro/X
+ Mais peut révéler relations phases/fréquences dans les données
+ Synchro analysis of bivariate data donne infos sur interrelation entre les systèmes qui génèrent les signaux
+ linear cross-correlation/ nonlinear statistical measures comme info mutuelle ou maxi correlation → pb #
+ ici hyp : data générée par systèmes qui intéragissent
+ interdépendance de phase peut venir de stochastic résonance
+ détails 6.4

fréquences proches → quand coincidence/dû intéraction? pas de solution, slmt indicateur indirect

- freq ratio aide pas : peut avoir <Om1>/<Om2>=1.05 même si 1:1 locking, car phase slips chage Rés
- B → phase diffusion/random walk/$\varphi 1-\varphi 2$ uniforme → intéraction se manifeste par pic dans distrib → PL
    - analyse de phase # par technique stroboscopique

- Non-stationnarité (variation des param) complique analyse, mais donne aussi des preuves d'intéraction :
    - fréq instantanées changent mais relation cste → peu probable que ça arrive par pure chance → FL
    - variation des ratio de ex env 5/2 à env 3 : transition entre Arnold tongue adjacentes

→ peut révéler seulement de l'intéraction

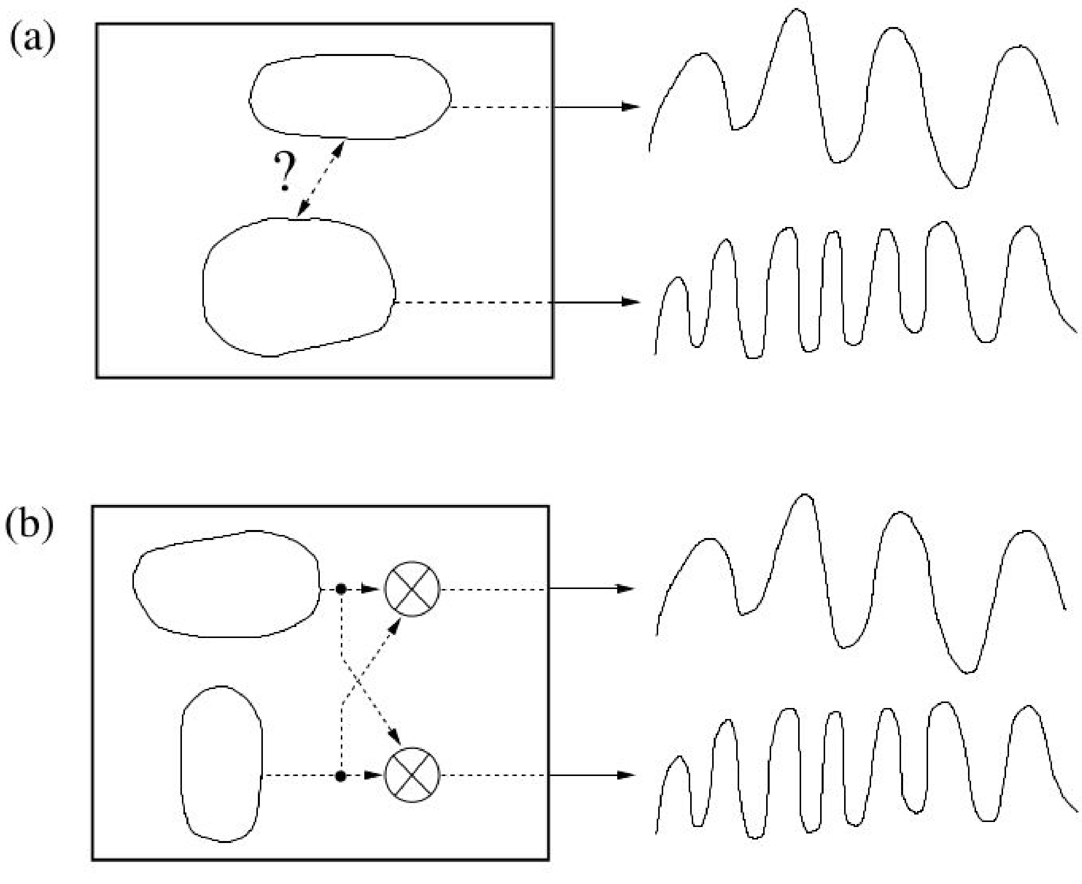


Figure 6.2. Illustration of the synchronization approach to the analysis of bivariate data. The goal of this analysis is to reveal the presence of a weak interaction between two subsystems from the signals at their outputs only. The assumption made is that the data are generated by two oscillators having their own rhythms (a). An alternative hypothesis is a mixture of signals generated by two uncoupled systems (b). These hypotheses cannot be distinguished by means of traditional techniques. In the former case, the interaction between the systems can be revealed by analysis of phases estimated from the data. Note that interdependence of phases can result not only from coupling of self-sustained oscillators, capable of synchronization, but also from modulation (cf. Fig. 3.31) or stochastic resonance (cf. Fig. 3.48).

<u>Analyser relations entre phases</u>
generalized phase difference : φnm=nφ1-mφ2 → straight forward method
ex posture control 2 groupe noisy/oscillatory patterns (+ fréq pathology) → suggère excitation d'un selfSO responsable controle posture
stabilogrammes clairement oscillatoires

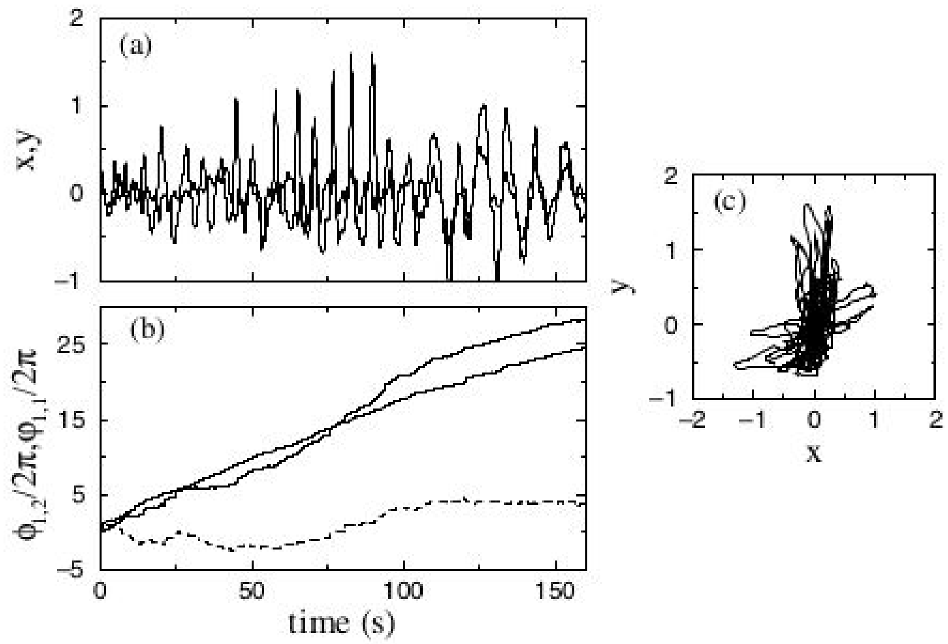


Figure 6.3. (a) Stabilogram of a neurological patient. x (bold curve) and y (solid curve) represent body sway while the subject was in a quiet stance with open eyes in anterior–posterior and lateral directions, respectively. The phases of these signals, and the phase difference are shown in (b) by bold, solid and dashed curves, respectively. The transition to a regime where the phase difference fluctuates around a constant is clearly seen at ≈110 s. A typical plot of y vs. x shows no structure that could indicate the interrelation between the signals (c). From [Pikovsky *et al.* 2000].

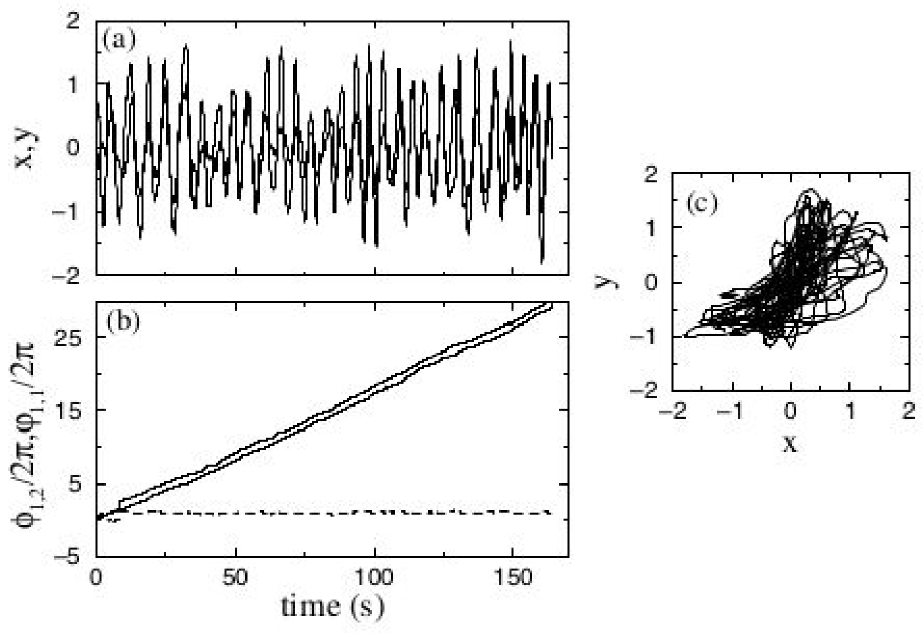
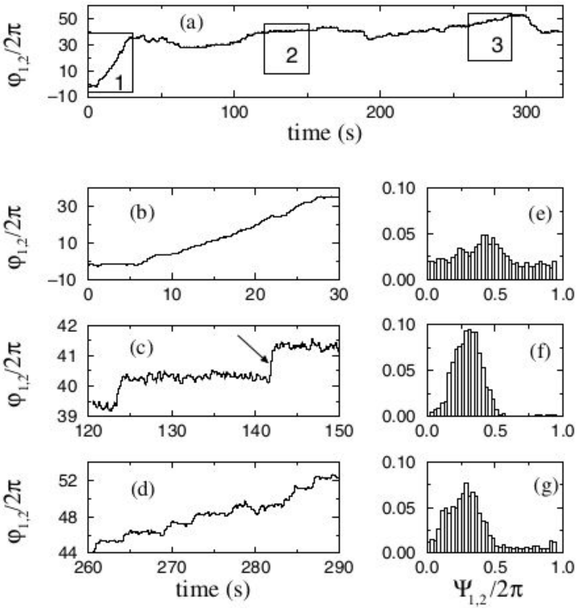


Figure 6.4. Stabilogram of the same patient as described in Fig. 6.3 obtained during the test with the eyes closed. All the notations are the same as in Fig. 6.3. From the phase difference one can see that the phases of the body sway in two directions are tightly interrelated within the whole test, although the amplitudes are irregular and essentially different. From [Pikovsky *et al.* 2000].

- \+ : peut tracer transitions entre # régimes (dû à la non-stationnarité des paramètres des 2 systèmes) en peu de périodes, normalement pas assez pour méthode conventionnelles de time series
- \- : n:m+1 apparaît non synchrone → doit trouver n et m par essai erreur
    - en pratique, peut estimer n et m à partir du power spectra des signaux

- Ac bcp de B, phase slips masque présence de plateaux(4.11), et synchro peut être révélée slmt par stat approche, ie distribution de la phase relative cyclique



→ si fait mod 2π, états équivalents avant/après phase slip : intéraction = max dans la distrib
→ données non stationnaires → running window analysis, distributions [t-tau/2,t+tau/2]
→ calcul de la déviation de la distrib d'une uniforme : [Tass et al. 1998;Rosenblum et al. 2001].
→ Rq : strong/unbounded noise : impossible de dire sans ambiguité si synchro
→ quantification of the strength of the interaction
→ synchro index as fct of time can reveal time course of interaction pour conclure à C responsable de tremor
→ synchro index pour multichannels data → d° relatif d'interrelation entre # signaux

technique stroboscopique : phase of driven oscillator observée ac période force : φk=φ0+kT
oscillateur entrainé = dirach (périodique) ou bande étroite (noisy/chaotic)
Ici, 1 des 2 = time marker → chaque fois que oscillateur prend 2π → phase stroboscope (eq strob si oscill périodique)
Phase stroboscope when nOm1=mOm2
synchrogramme : φ1(tk) mod 2π = f(t) (figure b)

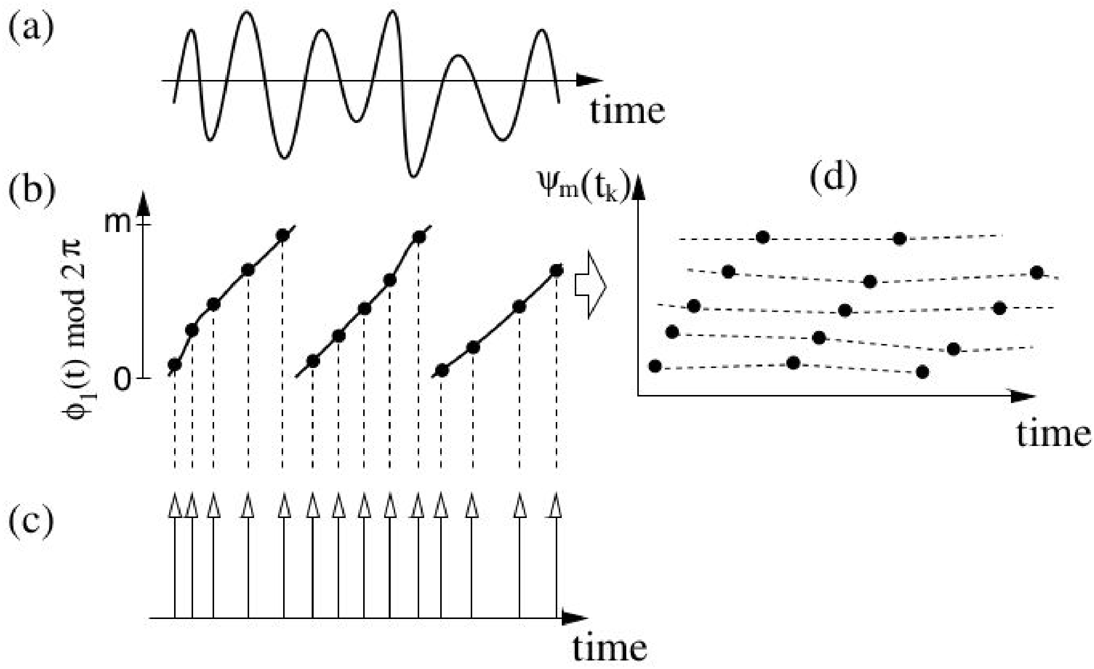


Figure 6.5. Principle of the phase stroboscope, or synchrogram. Here a slow signal (a) is observed in accordance with the phase of a fast signal (c). Measured at these instants, the phase ϕ_1 of the slow signal wrapped modulo $2\pi m$, (i.e., m adjacent cycles are taken as a one longer cycle) is plotted in (d); here $m = 2$. In this presentation $n : m$ phase synchronization shows up as n nearly horizontal lines in (d); a similar picture appears in the case of modulation.

pour n:1 locking (point à chaque spike) → s'attend à n # points avant d'atteindre les 2π en +
n:m car lorsque le 1er croît de 2πm, l'autre croit de 2πn, on s'attend à n points en m cycles
choisit slmt m par essai/erreur, on voit n sur le synchrogramme

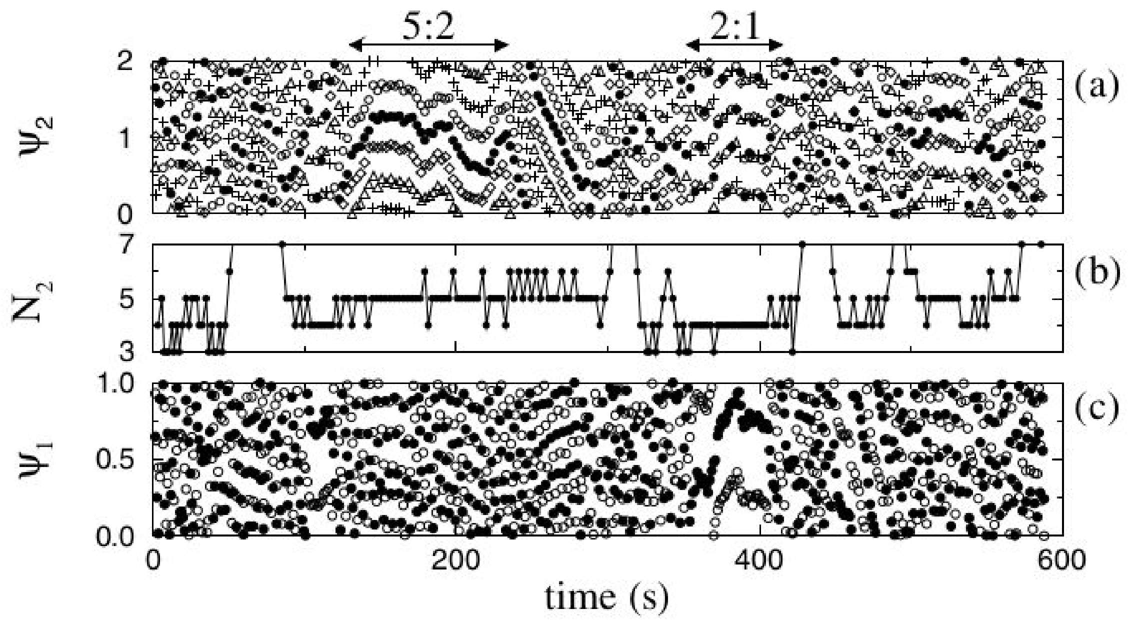


Figure 6.6. An example of alternation of interaction with approximate frequency relations 2 : 1 and 5 : 2 between heart rate and respiration of a healthy baby. (a) Two adjacent respiratory cycles are taken as one cycle. Therefore, epochs of 2 : 1 and 5 : 2 relation between phases appear as four- and five-line patterns. (b) Plot of the number of heartbeats within two respiratory cycles N_2 also indicates epochs of interdependence. (c) If the respiratory phase is wrapped modulo 2π then only the 2 : 1 relation is seen. The data points are shown by different symbols in cyclic order (five and two symbols in (a) and (c), respectively) for better visual impression. Plotted using data from [Mrowka *et al.* 2000].

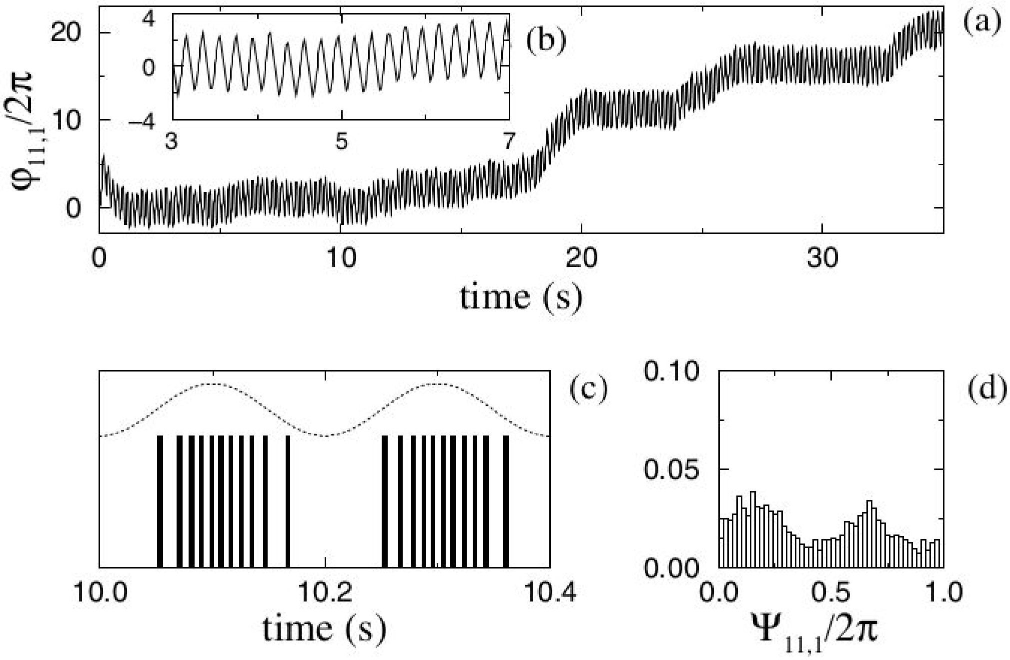


Figure 6.7. Phase relation between a neuron spiking and external electric field. (a, b) Plot of the phase difference supports the hypothesis of complex interrelation between the data. The raw signals are plotted in (c), two periods of the external force are shown; within this time interval there are two groups of 11 spikes. Note that due to strong modulation the fluctuation of the phase difference (a, b) is very strong, and the respective distribution (d) of the cyclic relative phase in not unimodal. From these plots it is not clear whether the signals are indeed interrelated; the synchrogram technique (Fig. 6.8) is much more effective here. Data courtesy of D. F. Russell, A. B. Neiman and F. Moss.

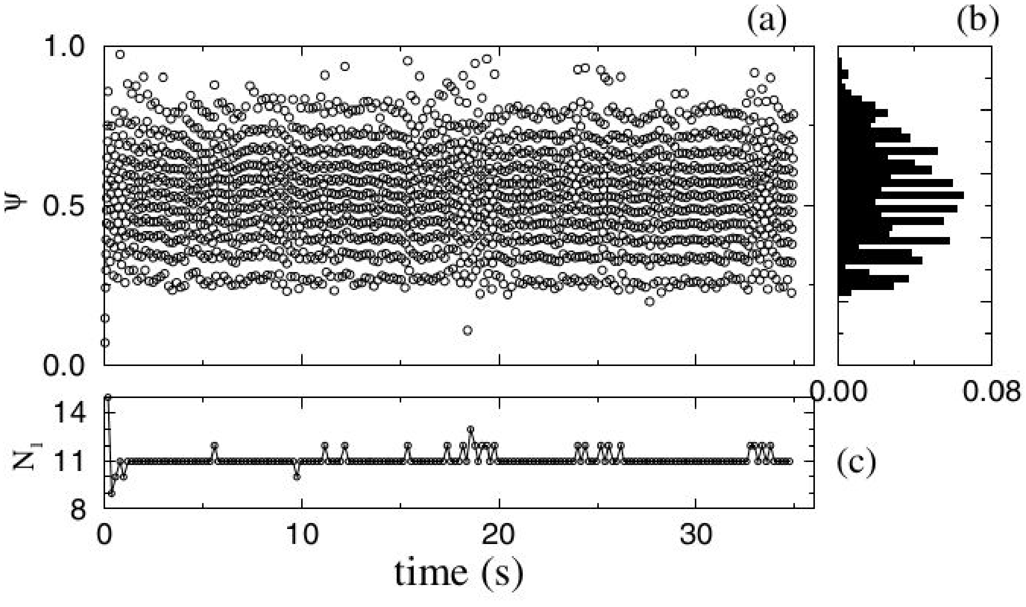


Figure 6.8. The stroboscopic technique (synchrogram) is very effective in an analysis of interrelations when the spiking frequency of the electrosensitive receptor of a paddlefish is much higher than that of an external electric field. (a) The 11-stripe structure of the synchrogram makes interrelation (with the exception of short epochs) obvious; this is confirmed by the corresponding distribution in (b). (c) Plot of the number of spikes per cycle of external force also confirms an approximate 11 : 1 relation between frequencies. Data courtesy of D. F. Russell, A. B. Neiman and F. Moss.

c/ 11 cycles de la force extérieure, 1 cycle de l’oscillateur → 11:1 synchro
d’abord $w1/w2 = 11$
$\varphi_{1,11}(t_k)=11 \cdot w_e \cdot t_k-2\pi k$, avec t_k le kème spike, $w_e=2\pi \cdot 5$ (f=5), $\Psi=\varphi_{1,11} \bmod 2\pi$

Rq :
ne pas oublier hyp : 2 selfSO qui intéragissent ou sont indépendants + détecte slmt intéraction
ex pas de sens si complètement indépendants :

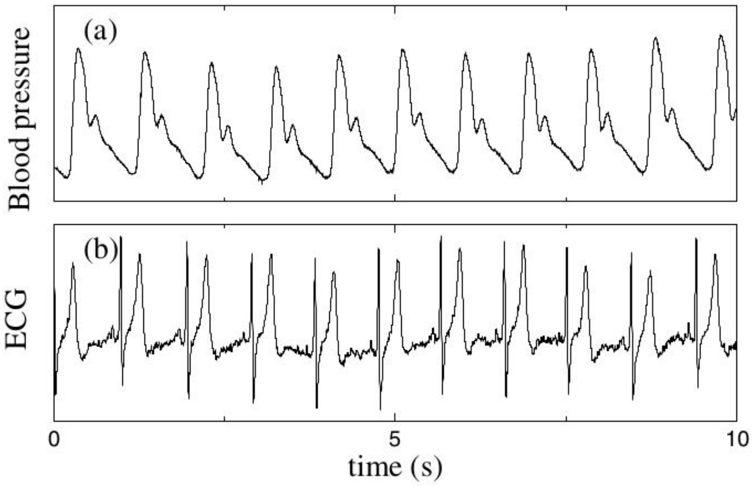


Figure 6.9. Records of blood pressure measured with the Finapress system at the index finger (a) and an electrocardiogram (b). One pulse in the blood pressure corresponds to one contraction of the heart, but it cannot be otherwise. The data represent two variables coming from one self-sustained system and one passive system. This is an example of the case when synchronization analysis is useless. Data courtesy of R. Mrowka.

Quantification of interrelation : (voir références)

- mutual info between 2 phases
- Entropy pour déviation de la distribution de la phase relative from uniform
- intensité du premier mode de Fourier de la distribution
- P(φ1=k|φ2 cst) → équivalent à la quantification de la distribution de la phase stroboscopique
- flatness of stripes (bandes) in the synnchrogramm
- coefficient of phase diffusion (besoin very long time series) → plutôt dans active expe