Ex 3.14 orthogonal case $x_j^T x_{j'} = 0 \quad \forall j \neq j'$

1) $x_j^T x_j = 1 \quad \forall j \quad , \quad \frac{1}{N}\sum_{i=1}^{N} x_{ij} = 0 \quad \forall j \quad , \quad y^{(0)} = \bar{y} \quad , \quad x_j^{(0)} = x_j \ \forall j$

2) $m = 1$

a) $z_1 = \sum_{j=1}^{p} \hat{\varphi}_{1j} x_j^{(0)}$, where $\hat{\varphi}_{1j} = \underbrace{\langle x_j^{(0)}, y \rangle}$

b) $\hat{\theta}_1 = \frac{\langle z_1, y \rangle}{\langle z_1, z_1 \rangle}$

$$\langle z_1, y \rangle = \langle \sum_{j=1}^{p} \hat{\varphi}_{1j} x_j^{(0)}, y \rangle = \sum_{j=1}^{p} \hat{\varphi}_{1j} \overbrace{\langle x_j^{(0)}, y \rangle} = \sum_{j=1}^{p} \hat{\varphi}_{1j}^2$$

$$\langle z_1, z_1 \rangle = \langle \sum_{j=1}^{p} \hat{\varphi}_{1j} x_j^{(0)}, \sum_{j'=1}^{p} \hat{\varphi}_{1j'} x_{j'}^{(0)} \rangle$$

$x_j^T x_{j'}$

$$= \sum_{j=1}^{p}\sum_{j'=1}^{p} \hat{\varphi}_{1j} \hat{\varphi}_{1j'} \langle x_j^{(0)}, x_{j'}^{(0)} \rangle$$

$$= \sum_{j=1}^{p} \hat{\varphi}_{1j}^2 \implies \hat{\theta}_1 = 1$$

c) $\hat{y}_{(1)} = \hat{y}_{(0)} + 1 z_1 = \hat{y}_{(0)} + \sum_{j=1}^{p} \hat{\varphi}_{1j} x_j^{(0)}$

d) $x_j^{(1)} = x_j^{(0)} - \frac{\langle z_1, x_j^{(0)} \rangle}{\langle z_1, z_1 \rangle} z_1$

$$\langle z_1, x_j^{(0)} \rangle = \langle \sum_{j'=1}^{p} \hat{\varphi}_{1j'} x_{j'}^{(0)}, x_j^{(0)} \rangle = \sum_{j'=1}^{p} \hat{\varphi}_{1j'} \langle x_{j'}^{(0)}, x_j^{(0)} \rangle = \hat{\varphi}_{1j}$$

$$x_j^{(1)} = x_j^{(0)} - \frac{\hat{\varphi}_{1j}}{\sum_{j=1}^{p} \hat{\varphi}_{1j}^2} z_1 = x_j^{(0)} - \frac{\hat{\varphi}_{1j}}{\sum_{j=1}^{p} \hat{\varphi}_{1j}^2} \sum_{j=1}^{p} \hat{\varphi}_{1j} x_j^{(0)}$$

$m = 2 \quad z_2 = \sum_{j=1}^{p} \hat{\varphi}_{2j} x_j^{(1)}$, where $\hat{\varphi}_{2j} = \langle x_j^{(1)}, y \rangle$

$$\hat{\varphi}_{2j} = \langle x_j^{(1)}, y \rangle$$

$$= \langle x_j^{(0)} - \frac{\hat{\varphi}_{1j}}{\sum_{j=1}^{p} \hat{\varphi}_{1j}^2} \sum_{j=1}^{p} \hat{\varphi}_{1j} x_j^{(0)}, y \rangle$$

$$= \underbrace{\langle x_j^{(0)}, y \rangle}_{\hat{\varphi}_{1j}} - \frac{\hat{\varphi}_{1j}}{\sum_{j=1}^{p} \hat{\varphi}_{1j}^2} \sum_{j=1}^{p} \hat{\varphi}_{1j} \overbrace{\langle x_j^{(0)}, y \rangle}^{\hat{\varphi}_{1j}}$$

$$= \hat{\varphi}_{1j} - \frac{\hat{\varphi}_{1j}}{\cancel{\sum_{j=1}^{p} \hat{\varphi}_{1j}^2}} \cancel{\sum_{j=1}^{p} \hat{\varphi}_{1j}^2} = 0$$