

Exercise 7.5  $\hat{y} = Sy$ , show that  $\sum_{i=1}^n \text{Cov}(y_i, \hat{y}_i) = \text{trace}(S) \sigma_e^2$

$$\sum_{i=1}^n \text{Cov}(y_i, \hat{y}_i) = \text{trace}(\text{Cov}(y, \hat{y}))$$

$$\text{Cov}(y, \hat{y}) = \begin{pmatrix} \text{Cov}(y_1, \hat{y}_1) & \text{Cov}(y_1, \hat{y}_2) & \dots & \text{Cov}(y_1, \hat{y}_n) \\ \text{Cov}(y_2, \hat{y}_1) & \text{Cov}(y_2, \hat{y}_2) & \dots & \text{Cov}(y_2, \hat{y}_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(y_n, \hat{y}_1) & \text{Cov}(y_n, \hat{y}_2) & \dots & \text{Cov}(y_n, \hat{y}_n) \end{pmatrix}$$

$$= \text{trace}(\text{Cov}(y, Sy))$$

$$= \text{trace}(S \cdot \text{Cov}(y, y))$$

$$= \text{trace}(S \cdot \text{Var}(y))$$

$$= \text{trace}(S) \sigma_e^2$$