Exercise 7.4

$$Err_{in} = \frac{1}{N}\sum_{i=1}^{N} E_{Y_0}\left[(Y_{0i} - \hat{f}(x_i))^2\right]$$

$$\overline{err} = \frac{1}{N}\sum_{i=1}^{N} (y_i - \hat{f}(x_i))^2$$

Thesis: $E_Y[op] = \frac{2}{N} Cov(\hat{y}_i, y_i)$

$$E_Y[op] = E_Y\left[\frac{1}{N}\sum_{i=1}^{N} E_{Y_0}\left[(Y_{0i} - \hat{f}(x_i))^2\right] - \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{f}(x_i))^2\right]$$

$$= E_Y\left[\frac{1}{N}\sum_{i=1}^{N}\left(E_{Y_0}[Y_{0i}^2] - 2E_{Y_0}[Y_{0i}]\hat{f}(x_i) + \hat{f}(x_i)^2 - y_i^2 + 2y_i\hat{f}(x_i) - \hat{f}(x_i)^2\right)\right]$$

$E_Y[y] = \rightarrow$, $E_{Y_0}[y_{0i}]$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(E_Y[y_i^2] - 2E_Y[y]E_Y[\hat{f}(x_i)] - E_Y[y_i^2] + 2E_Y[y_i\hat{f}(x_i)]\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(-2E_Y[y]E_Y[\hat{f}(x_i)] + 2E_Y[y_i\hat{f}(x_i)]\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N} 2\,Cov(y_i, \hat{f}(x_i)) = \frac{2}{N}\sum_{i=1}^{N} Cov(y_i, \hat{y}_i)$$