

$$l_k(r) = \frac{\partial \ell(r)}{\partial r_k} = \sum_{x_i \in R} y_{ik} - \sum_{x_i \in R} \frac{e^{p_k(x_i) + r_k}}{\sum_{n=1}^N e^{p_n(x_i) + r_n}}$$

$$l_{k,k}(r) = \frac{\partial^2 \ell(r)}{\partial r_k^2} = - \sum_{x_i \in R} \frac{e^{p_k(x_i) + r_k} \left( \sum_{n=1}^N e^{p_n(x_i) + r_n} \right) - (e^{p_k(x_i) + r_k})^2}{\left( \sum_{n=1}^N e^{p_n(x_i) + r_n} \right)^2}$$

$$b) \quad \gamma_k^* = \gamma_k^* - l_{k,k}(r_k^*)^{-1} l_k(r_k^*)$$

$$l_k(o) = \sum_{x_i \in R} y_{ik} - \sum_{x_i \in R} \frac{e^{p_k(x_i)}}{\sum_{n=1}^N e^{p_n(x_i)}} = \sum_{x_i \in R} (y_{ik} - p_k(x_i))$$

$$l_{k,k}(o) = - \sum_{x_i \in R} (p_k(x_i) - p_k^2(x_i))$$

$$\begin{aligned} \gamma_k^* &= 0 - \left( - \sum_{x_i \in R} (p_k(x_i) - p_k^2(x_i)) \right)^{-1} \sum_{x_i \in R} (y_{ik} - p_k(x_i)) \\ &= \frac{\sum_{x_i \in R} (y_{ik} - p_k(x_i))}{\sum_{x_i \in R} p_k(x_i) (1 - p_k(x_i))} \end{aligned}$$

c) subtract the mean