Part (b): Here $\mathcal{X}$ is fixed, and $\mathcal{Y}$ varies. Also x_0 and $f(x_0)$ are fixed. So

$$\begin{aligned}\mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\left(f(x_0)-\hat{f}(x_0)\right)^2\right) &= f(x_0)^2 - 2.f(x_0).\mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\hat{f}(x_0)\right) + \mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\left(\hat{f}(x_0)\right)^2\right) \\ &= \left(f(x_0) - \mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\hat{f}(x_0)\right)\right)^2 + \mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\left(\hat{f}(x_0)\right)^2\right) - \left(\mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\hat{f}(x_0)\right)\right)^2 \\ &= (\text{bias})^2 + \mathrm{Var}(\hat{f}(x_0))\end{aligned}$$

Part (c): The calculation goes the same way as in (b), except that both $\mathcal{X}$ and $\mathcal{Y}$ vary. Once again x_0 and $f(x_0)$ are constant.

$$\begin{aligned}\mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\left(f(x_0)-\hat{f}(x_0)\right)^2\right) &= f(x_0)^2 - 2.f(x_0).\mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\hat{f}(x_0)\right) + \mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\left(\hat{f}(x_0)\right)^2\right) \\ &= \left(f(x_0) - \mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\hat{f}(x_0)\right)\right)^2 + \mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\left(\hat{f}(x_0)\right)^2\right) - \left(\mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\hat{f}(x_0)\right)\right)^2 \\ &= (\text{bias})^2 + \mathrm{Var}(\hat{f}(x_0))\end{aligned}$$

The terms in (b) can be evaluated in terms of the $\ell_i(x_0;\mathcal{X})$ and the distribution of ε_i. We need only evaluate $\mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\hat{f}(x_0)\right) = \sum \ell_i(x_0;\mathcal{X})f(x_i)$ and

$$\begin{aligned}\mathrm{E}_{\mathcal{Y}|\mathcal{X}}\left(\left(\hat{f}(x_0)\right)^2\right) &= \sum_{i,j} \ell_i(x_0;\mathcal{X})\ell_j(x_0;\mathcal{X})\mathrm{E}\left((f(x_i)+\varepsilon_i)(f(x_j)+\varepsilon_j)\right) \\ &= \sum_{i,j} \ell_i(x_0;\mathcal{X})\ell_j(x_0;\mathcal{X})f(x_i)f(x_j) + \sum_i \sigma^2 \ell_i(x_0;\mathcal{X})^2.\end{aligned}$$

The terms in (c) can be evaluated in terms of $\mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\hat{f}(x_0)\right)$ and $\mathrm{E}_{\mathcal{X},\mathcal{Y}}\left(\left(\hat{f}(x_0)\right)^2\right)$. This means multiplying the expressions just obtained by

$$h(x_1)\ldots h(x_n)dx_1\ldots dx_n$$