

Exercise 15.5

$$f(x) = \frac{\text{Var}_{\theta} [E_{\theta} [T(x; \theta(z))]]}{\text{Var}_{\theta} [E_{\theta} [T(x; \theta(z))]] + E_{\theta} [\text{Var}_{\theta} [T(x; \theta(z))]]}$$

$$f(x) = \frac{\text{Cov}_{\theta, \theta} [\overbrace{T_1}^{T_1}(x; \theta(z)) \overbrace{T_2}^{T_2}(x; \theta(z))]}{\text{Var}_{\theta} [T(x; \theta(z))]} \leftarrow \sqrt{\text{Var}_{\theta, \theta} [T_1(x; \theta(z))]} \sqrt{\text{Var}_{\theta, \theta} [T_2(x; \theta(z))]}$$

$$\begin{aligned} \text{Cov}_{\theta, \theta} [T_1, T_2] &= E_{\theta, \theta} [T_1 T_2] - E_{\theta} [T_1] E_{\theta} [T_2] \\ &= E_{\theta} [E_{\theta, \theta} [T_1 T_2]] - E_{\theta} [E_{\theta, \theta} [T_1]] E_{\theta} [E_{\theta, \theta} [T_2]] \end{aligned}$$

Given 2, T_1 and T_2 are independent w.r.t. θ

$$= E_{\theta} [E_{\theta, \theta} [T]^2] - (E_{\theta} [E_{\theta, \theta} [T]])^2$$

$$= \text{Var}_{\theta} [E_{\theta, \theta} [T(x; \theta(z))]]$$

$$\begin{aligned} \text{Var}_{\theta} [\bar{T}] &= E_{\theta} [E_{\theta, \theta} [T^2]] - E_{\theta} [E_{\theta, \theta} [T]]^2 \\ &= E_{\theta} [\text{Var}_{\theta, \theta} [T] + (E_{\theta, \theta} [T])^2] - E_{\theta} [E_{\theta, \theta} [T]]^2 \\ &= E_{\theta} [\text{Var}_{\theta, \theta} [T]] + E_{\theta} [E_{\theta, \theta} [T]^2] - E_{\theta} [E_{\theta, \theta} [T]]^2 \\ &= E_{\theta} [\text{Var}_{\theta, \theta} [T(x; \theta(z))]] + \text{Var}_{\theta} [E_{\theta, \theta} [T(x; \theta(z))]] \end{aligned}$$

$$f(x) = \frac{\text{Var}_{\theta} [E_{\theta, \theta} [T(x; \theta(z))]]}{\underbrace{\text{Var}_{\theta} [E_{\theta, \theta} [T(x; \theta(z))]]}_{\text{within-}\theta \text{ variance}} + \underbrace{E_{\theta} [\text{Var}_{\theta, \theta} [T(x; \theta(z))]]}_{\text{within-}\theta \text{ variance}}}$$

$\text{Var}_{\theta} [\hat{f}_{\theta}(x)]$

within- θ variance

\uparrow with $n \downarrow$

\uparrow with $n \downarrow$