For $j=2$

$$b_2(x_0) = \sum_{i=1}^{N} \ell_i(x_0)\,(x_i - x_0)^{j=2} = \sum_{i=1}^{N} \ell_i(x_0)\,(x_i - x_0)^2$$

$$= \underbrace{\sum_{i=1}^{N} \ell_i(x_0)\,x_i^2}_{x_0^2} - 2x_0 \underbrace{\sum_{i=1}^{N} \ell_i(x_0)\,x_i}_{x_0} + x_0^2 \underbrace{\sum_{i=1}^{N} \ell_i(x_0)}_{1} \qquad \text{from Eq (1)}$$

$$= x_0^2 - 2x_0^2 + x_0 = 0$$

In general,

$$b_j(x_0) = \sum_{i=1}^{N} \ell_i(x_0)\,(x_i - x_0)^j$$

$$= \sum_{i=1}^{N} \ell_i(x_0) \sum_{m=0}^{j} \binom{j}{m} x_i^{j-m} (-x_0)^m$$

$$= \sum_{m=0}^{j} \binom{j}{m} (-x_0)^m \underbrace{\sum_{i=1}^{N} \ell_i(x_0)\,x_i^{j-m}}_{x_0^{j-m}}$$

$$= \sum_{m=0}^{j} \binom{j}{m} (-x_0)^m\, x_0^{j-m}$$

$$= \sum_{m=0}^{j} \binom{j}{m} x_0^j\, (-1)^m$$

$$= x_0^j \underbrace{\sum_{m=0}^{j} \binom{j}{m} (-1)^m (1)^{j-m}}_{(1-1)^j = 0} = 0$$