Start with the second last term and "multiply by 1" and "add 0"

$$
\begin{aligned}
\theta_{K-1}(X-\xi_{K-1})_+^3 &= \frac{(X-\xi_K)_+^3}{(\xi_K-\xi_{K-1})}\left(\theta_{K-1}\xi_K - \theta_{K-1}\xi_{K-1}\right) \\
&= \frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})}\left(\theta_{K-1}\xi_K - \theta_{K-1}\xi_{K-1} + \underbrace{\theta_K\xi_K - \theta_K\xi_K}_{0}\right) \\
&= \frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})}\left(\xi_K(\theta_{K-1}+\theta_K) - \xi_{K-1}\theta_{K-1} - \xi_K\theta_K\right) \\
&= \frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})}\left(-\xi_K\sum_{k=1}^{K-2}\theta_k + \sum_{k=1}^{K-2}\theta_k\xi_k\right) \\
&= -\frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})}\sum_{k=1}^{K-2}\theta_k\left(\xi_K-\xi_k\right) \\
&= -\sum_{k=1}^{K-2}\theta_k\left(\xi_K-\xi_k\right)\frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})}
\end{aligned}
$$

Then, we combine the two expressions

$$
\begin{aligned}
\sum_{k=1}^{K}\theta_k(X-\xi_k)_+^3 &= \sum_{k=1}^{K-2}\theta_k(X-\xi_k)_+^3 + \theta_{K-1}(X-\xi_{K-1})_+^3 + \theta_K(X-\xi_K)_+^3, \\
&= \sum_{k=1}^{K-2}\theta_k(X-\xi_k)_+^3 - \sum_{k=1}^{K-2}\theta_k\left(\xi_K-\xi_k\right)\frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})} \\
&\quad + (X-\xi_K)_+^3\sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\left(\frac{1}{\xi_K-\xi_{K-1}} - \frac{1}{\xi_K-\xi_k}\right) \\
&= \sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\frac{(X-\xi_k)_+^3}{\xi_K-\xi_k} - \sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\frac{(X-\xi_{K-1})_+^3}{(\xi_K-\xi_{K-1})} \\
&\quad + \sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\left(\frac{(X-\xi_K)_+^3}{\xi_K-\xi_{K-1}} - \frac{(X-\xi_K)_+^3}{\xi_K-\xi_k}\right) \\
&\ \ \vdots
\end{aligned}
$$

Then, take the last term, and do the same

$$
\begin{aligned}
\theta_K(X-\xi_K)_+^3 &= \frac{(X-\xi_K)_+^3}{(\xi_K-\xi_{K-1})}\left(\theta_K\xi_K - \theta_K\xi_{K-1} + \underbrace{\theta_{K-1}\xi_{K-1} - \theta_{K-1}\xi_{K-1}}_{0}\right) \\
&= \frac{(X-\xi_K)_+^3}{(\xi_K-\xi_{K-1})}\left(-\xi_{K-1}(\theta_{K-1}+\theta_K) + \xi_{K-1}\theta_{K-1} + \xi_K\theta_K\right) \\
&= \frac{(X-\xi_K)_+^3}{(\xi_K-\xi_{K-1})}\left(\xi_{K-1}\sum_{k=1}^{K-2}\theta_k - \sum_{k=1}^{K-2}\theta_k\xi_k\right) \\
&= (X-\xi_K)_+^3\sum_{k=1}^{K-2}\theta_k\frac{\xi_K-\xi_k}{(\xi_K-\xi_{K-1})} \\
&= (X-\xi_K)_+^3\sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\frac{\xi_{K-1}-\xi_k+\xi_K-\xi_K}{(\xi_K-\xi_{K-1})(\xi_K-\xi_k)} \\
&= (X-\xi_K)_+^3\sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\left(\frac{1}{\xi_K-\xi_{K-1}} - \frac{1}{\xi_K-\xi_k}\right)
\end{aligned}
$$

$$
\vdots
$$

$$
\begin{aligned}
&= \sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\left(\frac{(X-\xi_k)_+^3}{\xi_K-\xi_k} - \frac{(X-\xi_{K-1})_+^3}{\xi_K-\xi_{K-1}} + \frac{(X-\xi_K)_+^3}{\xi_K-\xi_{K-1}} - \frac{(X-\xi_K)_+^3}{\xi_K-\xi_k}\right) \\
&= \sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\left(\frac{(X-\xi_k)_+^3-(X-\xi_K)_+^3}{\xi_K-\xi_k} - \left(\frac{(X-\xi_{K-1})_+^3-(X-\xi_K)_+^3}{\xi_K-\xi_{K-1}}\right)\right)
\end{aligned}
$$

Therefore,

$$
\sum_{k=1}^{K}\theta_k(X-\xi_k)_+^3 = \sum_{k=1}^{K-2}\theta_k(\xi_K-\xi_k)\left(d_k(X)-d_{K-1}(X)\right),
$$

where

$$
d_k(X) = \frac{(X-\xi_k)_+^3-(X-\xi_K)_+^3}{\xi_K-\xi_k}
$$