Exercise 10.2

$$f^*(x) = \underset{f(x)}{\operatorname{argmin}}\ E_{Y|x}\left[e^{-f(x)Y}\right]$$

$$\frac{\partial E_{Y|x}\left[e^{-f(x)Y}\right]}{\partial f(x)} = E_{Y|x}\left[-Y e^{-f(x)Y}\right]$$

$$E_{Y|x}\left[-Y e^{-f(x)Y}\right] = 0$$

$$Y = \begin{cases} -1 & \text{with } \Pr[Y=-1 \mid X=x] \\ 1 & \text{with } \Pr[Y=1 \mid X=x] \end{cases}$$

$$-(-1)e^{-(-1)f(x)}\Pr[Y=-1 \mid X=x] - (1)e^{-(1)f(x)}\Pr[Y=1 \mid X=x] = 0$$

$$e^{f(x)}e^{f(x)}\Pr[Y=-1 \mid X=x] = e^{f(x)}e^{-f(x)}\Pr[Y=1 \mid X=x]$$

$$e^{2f(x)} = \frac{\Pr[Y=1 \mid X=x]}{\Pr[Y=-1 \mid X=x]}$$

odds

$$f(x) = \frac{1}{2}\log\frac{\Pr[Y=1 \mid X=x]}{\Pr[Y=-1 \mid X=x]}$$