

Ex. 2.7 (forms for linear regression and k -nearest neighbor regression)

To simplify this problem let's begin in the case of simple linear regression where there is only one response y and one predictor x . Then the standard definitions of y and X state that

$$y^T = (y_1, \dots, y_n), \text{ and } X^T = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \end{bmatrix}.$$

Part (a): Let's first consider linear regression. We use (2.2) and (2.6), but we avoid just copying the formulas blindly. We have $\hat{\beta} = (X^T X)^{-1} X^T y$, and then set

$$\hat{f}(x_0) = [x_0 \ 1] \hat{\beta} = [x_0 \ 1] (X^T X)^{-1} X^T y.$$

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In terms of the notation of the question,

$$\ell_i(x_0; \mathcal{X}) = [x_0 \ 1] (X^T X)^{-1} \begin{bmatrix} 1 \\ x_i \end{bmatrix}$$

for each i with $1 \leq i \leq n$.

More explicitly, $X^T X = \begin{bmatrix} \sum_{i=1}^n 1 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{bmatrix}$ which has determinant $(n-1) \sum_{i=1}^n x_i^2 - 2n \sum_{i < j} x_i x_j$.

This allows us to calculate $(X^T X)^{-1}$ and $\ell_i(x_0; \mathcal{X})$ even more explicitly if we really want to. In the case of k -nearest neighbor regression $\ell_i(x_0; \mathcal{X})$ is equal to $1/k$ if x_i is one of the nearest k points and 0 otherwise.