

Exercise 18.12

$$X = U \Delta V^T$$

$$\downarrow R V^T$$

$$(18.16) \quad \hat{\beta}_0, \hat{\beta} = \arg \min_{\beta_0, \beta} \sum_{i=1}^n L(y_i, \beta_0 + \underline{x_i^T} \beta) + \lambda \beta^T \beta$$

$$(18.17) \quad \hat{\theta}_0, \hat{\theta} = \arg \min_{\theta_0, \theta} \sum_{i=1}^n L(y_i, \theta_0 + \underline{r_i^T} \theta) + \lambda \theta^T \theta$$

$$\underline{X_B} = \underline{R_B} V^T$$

$$\underline{x_i^T} \leftrightarrow \underline{r_i^T}$$

$$X \begin{pmatrix} \equiv \\ \equiv \\ \equiv \end{pmatrix} R \begin{pmatrix} \equiv \\ \equiv \\ \equiv \end{pmatrix}$$

Exercise 18.16

$$\alpha \leftrightarrow \frac{\alpha}{M}$$

M number of tests

$$a) P(A) \leq \alpha$$

$$P(A) = P\left(\bigcup_{j=1}^M A_j\right)$$



$$S_1 \geq S_2 \geq \dots \geq S_M$$

$$= \sum_{j=1}^M P(A_j) - \sum_{j < j'} P(A_j \cap A_{j'}) + \dots + (-1)^{n+1} \sum_{j_1, \dots, j_n} P(A_{j_1} \cap \dots \cap A_{j_n})$$

$$\leq \sum_{j=1}^M P(A_j) = M \frac{\alpha}{M} = \alpha$$

$$P(A) \leq \alpha$$

$$b) P(A) = 1 - P(A^c) = 1 - \prod_{j=1}^M P(A_j^c) = 1 - \left(1 - \frac{\alpha}{M}\right)^M$$

$$\text{Binomial approximation: } (1+x)^r \approx 1+rx$$

$$x = -\frac{\alpha}{M}, \quad r = M \Rightarrow 1 - \left(1 - \frac{\alpha}{M}\right)^M \approx 1 - \left(1 + M\left(-\frac{\alpha}{M}\right)\right) = 1 - (1 - \alpha) = \alpha$$