Exercise 6.2

k is the degree of the polynom

Show that $\sum_{i=1}^{N}(x_i - x_0)\,l_i(x_0) = 0$ for local linear regression

Define $b_j(x_0) = \sum_{i=1}^{N}(x_i - x_0)^j\,l_i(x_0)$. Show that $b_0(x_0) = 1$ $\forall k$

$b_j(x_0) = 0$ $\forall j \leq k$

Define $L(x_0)$ the column vector of the $l_i(x_0)$ ← equivalent kernels

$$L(x_0)^T = [\,l_1(x_0) \;\dots\; l_N(x_0)\,]$$

From eq (6.8) and (6.9)

$$\sum_{i=1}^{N} l_i(x_0)\,\cancel{y_i} = \underbrace{L(x_0)^T\cancel{y}} = b(x_0)^T\left(B^T W(x_0) B\right)^{-1} B^T W(x_0)\,\cancel{y}$$

where $B = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^k \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_N & x_N^2 & \dots & x_N^k \end{bmatrix}$ and $b(x_0)^T = (1 \; x_0 \;\dots\; x_0^k)$

$$L(x_0)^T = b(x_0)^T\left(B^T W(x_0) B\right)^{-1} B^T W(x_0)$$

$$L(x_0)^T B = b(x_0)^T \cancel{\left(B^T W(x_0) B\right)^{-1} B^T W(x_0) B}$$

$$[\,l_1(x_0) \;\dots\; l_N(x_0)\,]\begin{bmatrix} 1 & x_1 & \dots & x_1^k \\ \vdots & \vdots & & \\ 1 & x_N & \dots & x_N^k \end{bmatrix} = [\,1 \; x_0 \;\dots\; x_0^k\,]$$

Eq (1)

$$\Rightarrow \sum_{i=1}^{N} l_i(x_0) = 1 \quad \text{and} \quad \underbrace{\sum_{i=1}^{N} l_i(x_0)\,x_i^j = x_0^j}_{\text{if } j=m} \quad \forall j = 1, \dots, k$$

In particular

$$\sum_{i=1}^{N} l_i(x_0)\,x_i^j = x_0^j = x_0^m\,x_0^{j-m} = \left(\sum_{i=1}^{N} l_i(x_0)\,x_i^m\right) x_0^{j-m} \qquad m \in [0, j]$$

For $j=1$

both derive from Eq (1)

$$b_1(x_0) = \sum_{i=1}^{N} l_i(x_0)\,(x_i - x_0)^{j=1} = \underbrace{\sum_{i=1}^{N} l_i(x_0)\,x_i}_{x_0} - x_0 \underbrace{\sum_{i=1}^{N} l_i(x_0)}_{1}$$

$$= x_0 - x_0 = 0$$