Exercise 5.1

$$f(x) = \sum_{m=1}^{6} \beta_m h_m(x)$$

$$f(x) = \beta_1 + \beta_2 \underline{x} + \beta_3 \underline{x^2} + \beta_4 \underline{x^3} + \beta_5 \underline{(x-\xi_1)_+^3} + \beta_6 \underline{(x-\xi_2)_+^3}$$

1) piecewise cubic polynomial

$x < \xi_1 \Rightarrow f(x) = \beta_1 + \beta_2 x + \beta_3 x^2 + \beta_4 x^3$

$\xi_1 \le x < \xi_2 \rightarrow f(x) = \beta_1 + \beta_2 x + \beta_3 x^2 + \beta_4 x^3 + \beta_5 x^3 - 3\beta_5 \xi_1 x^2 + 3\beta_5 \xi_1^2 x - \beta_5 \xi_1^3$

$\overset{!}{=} (\beta_1 - \beta_5 \xi_1^3) + (\beta_2 + 3\beta_5 \xi_1^2) x + (\beta_3 - 3\beta_5 \xi_1) x^2 + (\beta_4 + \beta_5) x^3$

$x \ge \xi_2 \rightarrow$ same

2) continuity (ξ_1, same for ξ_2)

- $\lim_{x \to \xi_1^-} f(x) = \beta_1 + \beta_2 \xi_1 + \beta_3 \xi_1^2 + \beta_4 \xi_1^3$
  $\lim_{x \to \xi_1^+} f(x) = \beta_1 + \beta_2 \xi_1 + \beta_3 \xi_1^2 + \beta_4 \xi_1^3 + \overbrace{\beta_5 \lim_{x \to \xi_1^+} (x-\xi_1)_+^3}^{0}$ $\Rightarrow$ equal / $\rightarrow$ continuous
- $\lim_{x \to \xi_1^-} f'(x) = \beta_2 + 2\beta_3 \xi_1 + 3\beta_4 \xi_1^2$
  $\lim_{x \to \xi_1^+} f'(x) = \beta_2 + 2\beta_3 \xi_1 + 3\beta_4 \xi_1^2 + \overbrace{3 \lim_{x \to \xi_1^+} \beta_5 (x-\xi_1)_+^2}^{0}$ $\Rightarrow$ equal / continuous in $f'(x)$
- $\lim_{x \to \xi_1^-} f''(x) = 2\beta_3 + 6\beta_4 \xi_1$
  $\lim_{x \to \xi_1^+} f''(x) = 2\beta_3 + 6\beta_4 \xi_1 + \overbrace{6 \lim_{x \to \xi_1^+} \beta_5 (x-\xi_1)_+}^{0}$ $\Rightarrow$ continuous in $f''(x)$

Ex 5.9 $\quad S_\lambda = (I + \lambda K)^{-1}$ $\quad \hat{y} = S_\lambda y$

$S_\lambda = N(N^T N + \lambda \Omega_N)^{-1} N^T$ $\quad$ N invertible

$\overset{!}{=} \left((N^T)^{-1} (N^T N + \lambda \Omega_N) N^{-1}\right)^{-1}$

$= \Big(\underbrace{(N^T)^{-1} N^T}_{I} \underbrace{N N^{-1}}_{I} + \lambda \underbrace{(N^T)^{-1} \Omega_N N^{-1}}_{K}\Big)^{-1}$

$= (I + \lambda K)^{-1}$ $\quad$ ↳ K does not depend on λ