Solution 1: For this exercise we will derive the distribution function (CDF) for the Euclidean distance (denoted by y) from the origin to the *closest* of n points x_i where each point x_i is drawn uniformly from a p-dimensional unit ball centered at the origin.

For any given vector x_i (uniform in the unit ball) the distribution function of $y = ||x_i||$ is the ratio of the volume of a ball of radius y and the volume of a ball of radius one. This ratio is y^p and so $F(y) = y^p$. The distribution function for y is then $f(y) = py^{p-1}$.

Given N such vectors $\{x_i\}_{i=1}^N$ the distribution function for the smallest radius Y_1 (from all of them) is given by

$$F_{Y_1}(y) = 1 - (1 - F(y))^N = 1 - (1 - y^p)^N,$$

see [9] where the order statistics are discussed. The *median* distance for Y_1 is found by solving for y in

$$\frac{1}{2} = F_{Y_1}(y).$$

This gives

$$y = \left(1 - \left(\frac{1}{2}\right)^{1/N}\right)^{1/p} \equiv d_{\text{median}}(p, N),$$

which is the desired expression.