Ex. 18.16 *Bonferroni method for multiple comparisons.* Suppose we are in a multiple-testing scenario with null hypotheses H_{0j}, $j = 1, 2, \ldots, M$, and corresponding p-values p_j, $i = 1, 2, \ldots, M$. Let A be the event that at least one null hypothesis is falsely rejected, and let A_j be the event that the jth null hypothesis is falsely rejected. Suppose that we use the Bonferroni method, rejecting the jth null hypothesis if $p_j < \alpha/M$.

(a) Show that $\Pr(A) \leq \alpha$. [*Hint*: $\Pr(A_j \cup A_{j'}) = \Pr(A_j) + \Pr(A_{j'}) - \Pr(A_j \cap A_{j'})$]

(b) If the hypotheses $H_{0j}, j = 1, 2, \ldots, M$, are independent, then $\Pr(A) = 1 - \Pr(A^C) = 1 - \prod_{j=1}^{M} \Pr(A_j^C) = 1 - (1 - \alpha/M)^M$. Use this to show that $\Pr(A) \approx \alpha$ in this case.