hw9 for stat341

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Q: 6E3,6E4 Additional Problems 1,2

Define a function working for the whole probblem set

```
H<-function(P){sum(-P*log(P))}# Function to compute entropy.
```

6E3. Suppose a four-sided die is loaded such that, when tossed onto a table, it shows "1" 20%, "2" 25%, "3" 25%, and "4" 30% of the time. What is the entropy of this die?

Solution:

Based on the equations ,the entropy of this die is 1.376

```
P<-c(.2,.25,.25,.3)
H(P)
```

```
## [1] 1.376227
```

6E4. Suppose another four-sided die is loaded such that it never shows "4". The other three sides show equally often. What is the entropy of this die?

Except(4),all the others inferentially equivalent ways to include the categorical variable in a regression.

Solution:

the entropy of this die is 1.098.

```
P2<-c(1/3,1/3,1/3)
H(P2)
```

```
## [1] 1.098612
```

Additional Problems 1 Making entropy larger. a) Which is larger: H(0.1, 0.3, 0.6) or H(0.2, 0.2,0.6)? b) Let p = and let q =

where p = (p1+p2)/2. Compute H(p) and H(q). Which is larger? c) Suppose a random process has n outcomes. Show that with one exception, there is always another random process that also has n outcomes, but has higher entropy? What is the one exception? (The exception is the random process with the maximal entropy among processes with n outcomes.)

Solution:

a)

```
I1<-c(.1,.3,.6) #H(0.1, 0.3, 0.6)
H(I1)
```

```
## [1] 0.8979457
```

```
I2<- c(.2,.2,.6) #H(0.2, 0.2,0.6)
H(I2)
```

```
## [1] 0.9502705
```

Based on the calculation, H(0.2, 0.2,0.6) has the larger value.

b) q gave a larger values. let's assume that $P_1 < P_2$ then there is $-log(P_1) - log(P_2) \geq -2 \cdot log(\frac{P_1+P_2}{2})$ Then there is $log(P_1) - log(\frac{P_1+P_2}{2}) \leq log(\frac{P_1+P_2}{2}) - log(P_2)$ $P_1 \cdot [log(P_1) - log(\frac{P_1+P_2}{2})] \leq P_2[log(\frac{P_1+P_2}{2}) - log(P_2)]$ $P_1 \cdot log(P_1) + P_2 \cdot log(P_2) \leq (P_1 + P_2) \cdot log(\frac{P_1+P_2}{2})$

THerefore, H(q)>H(p) for every $P_1 \neq P_2$ Notes: my poor Algerbra does not function well, suppose to give me H(q)>H(p)

c) For example,there is random set. There is always a larger entropy value if choose an average number of two values. ANd the maximum entropy of the system will be that every number is evenly-distributed. Therefore this one exception is choose to be everynumber is that Probability is 1/n and there are equally likely of the events.

```
k1<-c(.4,.2,.4)
H(k1)
```

```
## [1] 1.05492
```

```
k2<-c(.4,.3,.3)
H(k2)
```

```
## [1] 1.0889
```

2 Compute the entropy of tossing two coins two different ways: a) Consider the outcomes to be 0, 1 or 2 heads. b) Consider the outcomes to be HH, HT, TH, or TT. How do the results compare? Can you generalize?

Solution:

a)

```
d1<-c(1/4,1/2,1/4)
H(d1)
```

```
## [1] 1.039721
```

b)

```
d2<-c(1/4,1/4,1/4,1/4)
H(d2)
```

```
## [1] 1.386294
```

The second one gave a larger entropy values. Because second one considered how to get the one head specifically. And it provides more information.