

hw2 for stat341

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2E1. Q: Which of the expressions below correspond to the statement: the probability of rain on Monday? (1) $\Pr(\text{rain})$ (2) $\Pr(\text{rain}|\text{Monday})$ (3) $\Pr(\text{Monday}|\text{rain})$ (4) $\Pr(\text{rain}, \text{Monday})/\Pr(\text{Monday})$

Solve:

(2) correspond to the statement: the probability of rain on Monday.

2E2. Q: Which of the following statements corresponds to the expression: $\Pr(\text{Monday}|\text{rain})$?

(1) The probability of rain on Monday. (2) The probability of rain, given that it is Monday. (3) The probability that it is Monday, given that it is raining. (4) The probability that it is Monday and that it is raining

Solve: (3) corresponds to the expression: $\Pr(\text{Monday}|\text{rain})$

2E3. Q: Which of the expressions below correspond to the statement: the probability that it is Monday, given that it is raining? (1) $\Pr(\text{Monday}|\text{rain})$ (2) $\Pr(\text{rain}|\text{Monday})$ (3) $\Pr(\text{rain}|\text{Monday})\Pr(\text{Monday})$ (4) $\Pr(\text{rain}|\text{Monday})\Pr(\text{Monday})/\Pr(\text{rain})$ (5) $\Pr(\text{Monday}|\text{rain})\Pr(\text{rain})/\Pr(\text{Monday})$

Solve:

(1) and (4) correspond to the statement: the probability that it is Monday, given that it is raining

2H1. Q: Suppose there are two species of panda bear. Both are equally common in the wild and live in the same places. They look exactly alike and eat the same food, and there is yet no genetic assay capable of telling them apart. They differ however in their family sizes. Species A gives birth to twins 10% of the time, otherwise birthing a single infant. Species B births twins 20% of the time, otherwise birthing singleton infants. Assume these numbers are known with certainty, from many years of field research. Now suppose you are managing a captive panda breeding program. You have a new female panda of unknown species, and she has just given birth to twins. What is the probability that her next birth will also be twins?

Solve: Assume two species panda is equally likely. $P(\text{twins}) = 1 - P(\text{single}) = 1 - (0.9/2 + .8/2) = 0.15$ Based on the probability that, the probability for next birth of twins is 15%.

2H2. Q: Recall all the facts from the problem above. Now compute the probability that the panda we have is from species A, assuming we have observed only the first birth and that it was twins.

Solve:

Assume two species panda is equally likely. $P(A) = .5$ Probability is 10% for next is twins.

$$P(A|\text{twins}) = \frac{P(\text{twins}|A) \cdot P(A)}{P(\text{twins})} = \frac{.1 \cdot .5}{.15} = \frac{1}{3}$$

2H3. Q: Continuing on from the previous problem, suppose the same panda mother has a second birth and that it is not twins, but a singleton infant. Compute the posterior probability that this panda is species A.

Solve: Assume species are equally likely. For posterior probability, using

$$P(A|\text{secondnottwins}) = \frac{P(\text{secondnottwins}|A) \cdot P(A)}{P(\text{secondnottwins})} = \frac{.9 \cdot .5}{.9/2 + .8/2} = 52.9\%$$

2H4. Q: A common boast of Bayesian statisticians is that Bayesian inference makes it easy to use all of the data, even if the data are of different types. So suppose now that a veterinarian comes along who has a new genetic test that she claims can identify the species of our mother panda. But the test, like all tests, is imperfect. This is the information you have about the test: The probability it correctly identifies a species A panda is 0.8. The probability it correctly identifies a species B panda is 0.65. The vet administers the test to

your panda and tells you that the test is positive for species A. First ignore your previous information from the births and compute the posterior probability that your panda is species A. Then redo your calculation, now using the birth data as well.

Solve:

Based on the the posterior probability,

posterior can be identified as prior*likelihood

Based on the question there is $P(\text{test} + | A) = 0.8$ $P(\text{test} + | B) = 0.35$

Firstly, we did posterier for both species. Since species of A and B is equally likely.

Posterier for A: $\text{post}A = .8 \cdot .5 = 0.4$ Posterier for B: $\text{post}B = .35 \cdot .5 = 0.175$ Then normalize two probability, and it gives $\text{post}A = 69.57\%$ and $\text{post}B = 30.43\%$

Then use this posterior as the prior.

Posterier for A: $\text{post}A = 0.6957 \cdot P(\text{nexttwins} | A) \cdot P(\text{nextsingle} | A) = .6957 * 0.1 * .9 = 0.063$ Posterier for B: $\text{post}B = .3043 \cdot P(\text{nexttwins} | B) \cdot P(\text{nextsingle} | B) = .3043 * .2 * .8 = 0.049$ Therefore it is more likely to be species A.