

- HONGWEI XI AND HANWEN WU, *Multirole Logic*.  
 Dept. of Comp. Sci., Boston Univ., 111 Cummington Mall, Boston MA 02215, USA.  
*E-mail:* [hwxi,hwwu@cs.bu.edu](mailto:hwxi,hwwu@cs.bu.edu).  
*URL Address:* <http://www.cs.bu.edu/~hwxi/>.

We formulate *multirole logic* [1] as a new form of logic and naturally generalize Gentzen's celebrated result of cut-elimination between two sequents into one between  $n$  sequents for any  $n \geq 1$ .

While the first and foremost inspiration for multirole logic came to us during a study on multiparty session types in distributed programming [2], it seems natural in retrospective to introduce multirole logic by exploring the well-known duality between conjunction and disjunction in classical logic. Let  $\overline{\mathcal{O}}$  be a (possibly infinite) underlying set of integers, where each integer is referred to as a role. In multirole logic, each formula  $A$  can be annotated with a set  $R$  of roles to form the  $i$ -formula  $[A]_R$ . For each *ultrafilter*  $\mathcal{U}$  on the power set of  $\overline{\mathcal{O}}$ , there is a (binary) logical connective  $\wedge_{\mathcal{U}}$  such that  $[A_1 \wedge_{\mathcal{U}} A_2]_R$  is interpreted as the conjunction (disjunction) of  $[A_1]_R$  and  $[A_2]_R$  if  $R \in \mathcal{U}$  ( $R \notin \mathcal{U}$ ) holds. Furthermore, the notion of negation is generalized to *endomorphisms* on  $\overline{\mathcal{O}}$ . We formulate both multirole logic (MRL) and linear multirole logic (LMRL) as natural generalizations of classical logic (CL) and classical linear logic (CLL), respectively. Among various meta-properties established for MRL and LMRL, we obtain one named *multiparty cut-elimination* stating that every cut involving one or more sequents can be eliminated. For instance, the cut-rule in CL is generalized to the following one:

$$\frac{\Gamma_1, [A]_{R_1} \quad \dots \quad \Gamma_n, [A]_{R_n}}{\Gamma_1, \dots, \Gamma_n}$$

where  $\overline{R_1} \uplus \dots \uplus \overline{R_n} = \overline{\mathcal{O}}$  is assumed. Note that Gentzen's cut-elimination is the special case where  $n = 2$ .

[1] HONGWEI XI AND HANWEN WU, *Multirole Logic (Extended Abstract)*, **arXiv**, arXiv:1703.06391 [math.LO], 2017.

[2] HONGWEI XI AND HANWEN WU, *Propositions in Linear Multirole Logic as Multiparty Session Types*, **arXiv**, arXiv:1611.08888 [cs.PL], 2016.