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Homework 5
1) Correct 2,3
2) Correct 1
3) Correct 3
4) Correct 2.
5) Correct 4.
6) Correct 4.
7) Correct 2,4
8) Correct 2
9) Correct 3
10) Correct 2,4.
11) Correct 4

12.

α) We have $p(x) = \vartheta^2 x e^{-\vartheta x} u(x)$

- Likelihood function is:
$$\mathcal{L}(\vartheta) = \prod_{i=1}^{N} p(x_i) = \prod_{i=1}^{N} \vartheta^2 x_i e^{-\vartheta x_i}$$

- Log-Likelihood function is:
$$\log(\mathcal{L}(\vartheta)) = \sum_{i=1}^{N} \log(\vartheta^2 x_i e^{-\vartheta x_i}) = \sum_{i=1}^{N} (2\log\vartheta + \log x_i - \vartheta x_i)$$

$$\Rightarrow \log \mathcal{L}(\vartheta) = 2N\log\vartheta + \sum_{i=1}^{N} \log x_i - \vartheta \sum_{i=1}^{N} x_i$$

for ML estimator
$$\frac{d}{d\vartheta} \log\mathcal{L}(\vartheta) = \frac{2N}{\vartheta} - \sum_{i=1}^{N} x_i = 0. \iff$$

$$\vartheta = \frac{2N}{\sum_{i=1}^{N} x_i} \Rightarrow \vartheta_{ML} = \frac{2N}{\sum_{i=1}^{N} x_i}$$

b) $\vartheta_{ML} = \sum_{i=1}^{5} x_i = 2 + 2.2 + 2.7 + 2.4 + 2.6 = 11.9.$

$\vartheta_{ML} = \frac{2 \cdot 5}{11.9} = \frac{10}{11.9} = 0.84$

$x = 2.3 \Rightarrow \hat{p}(2.3) = (0.84)^2 \cdot 2.3 e^{-0.84 \cdot 2.3} = 0.2351$

$x = 2.9 \Rightarrow \hat{p}(2.9) = (0.84)^2 \cdot 2.9 \cdot e^{-0.84 \cdot 2.9} = 0.1791.$