Solutions for Chapter 2

Exercise 2.1

We assume a forward voltage for the LED of 1.5V. Then for I_{LED} we have

$$I_{LED} = \frac{V_R}{R} = \frac{3.3\text{V} - 1.5\text{V}}{330\Omega} \approx \boxed{\mathbf{5.5}\text{mA}}$$

To estimate the β_{min} we need the current entering the base

$$I_B = \frac{3.3\text{V} - 0.6\text{V}}{10\text{k}\Omega} = 0.27\text{mA}$$

Thus

$$\beta_{min} \geq \frac{I_{LED}}{I_B} = \boxed{\mathbf{20}}$$

Exercise 2.2

NOTE: According to the errata 0.63 **should be replaced by** 0.76 **and** $63\mu\text{sec}$ **by** $76\mu\text{sec}$**.**

Starting from the hint that the capacitor charges from -4.4V towards $+5$V, we would result to a total 9.4V for a full charge. However, the V_{BE} of Q_2 is clipping the charging process at only 5V of the total (from -4.4V to 0.6V). Thus, the capacitor will be 53% charged at the end.
Solving the voltage equation for a charging capacitor gives us

$$V_C(t) = V_f * (1 - e^{-\frac{t}{RC}})$$

set $V_C(t_1) = 0.53 * V_f$

$$0.53 = 1 - e^{-\frac{t_1}{R_3C_1}}$$

$$\Rightarrow$$

$$t_1 = -RC * ln(0.47) \approx \boxed{\mathbf{0.76 * R_3C_1}}$$

Exercise 2.3

The output voltage is reduced due to the $R_4 - R_5$ voltage divider

$$V_{\text{out}} = \frac{R_5}{R_4 + R_5} * (V_{CC} - 0.6\text{V}) \approx \boxed{\mathbf{4.18}\text{V}}$$

To estimate the minimum β_3, we need first to find the maximum (worst-case) collector current for which Q_3 should still be in saturation. For this we can assume a 0V drop across C and Q_3 while the current travels through the parallel connected resistors $R_2||R_3$.

$$I_{C_3,max} = \frac{V_{CC}}{R_2||R_3} = 5.5\text{mA}$$

$$\Rightarrow \beta_{3,min} = \frac{I_{C_3,max}}{I_{B_3}} = \frac{5.5\text{mA}}{\frac{(4.18\text{V}-0.6\text{V})}{20\text{k}\Omega}} \approx \boxed{\mathbf{31}}$$

Exercise 2.4

By using KCL and the fact that the transistor is in the active region we get

$$i_E = i_C + i_B = (\beta + 1) * i_B = (\beta + 1)\frac{v_B}{Z_{source}}$$

For small signals $Z_{\text{out}} = \frac{v_E}{i_E} = \frac{v_B}{i_E}$. Thus:

$$\boxed{\mathbf{Z}_{\text{out}} = \frac{\mathbf{v_B}}{(\boldsymbol{\beta} + \mathbf{1})\frac{\mathbf{v_B}}{\mathbf{Z_{source}}}} = \frac{\mathbf{Z_{source}}}{\boldsymbol{\beta} + \mathbf{1}}} \qquad q.e.d.$$

Note: In practice one will often see $Z_{\text{out}} \approx \frac{Z_{source}}{\beta}$. When $\beta \approx 100$ the "+1" part is often being ignored to simplify calculations.