The Art of Electronics, 3rd Edition - Solutions

This is an ongoing project with solutions to problems in "The Art of Electronics, 3rd edition" by Paul Horowitz and Winfield Hill. There is no guarantee for the correctness of the solutions and any suggestions for improvement are more that welcome.

Contents

1 Foundations

Exercise 1.1

(a) $R = 5\text{k} + 10\text{k} = \boxed{\mathbf{15}\text{k}\boldsymbol{\Omega}}$

(b) $R = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{5\text{k} \cdot 10\text{k}}{5\text{k} + 10\text{k}} = \boxed{\mathbf{3.33}\text{k}\boldsymbol{\Omega}}$

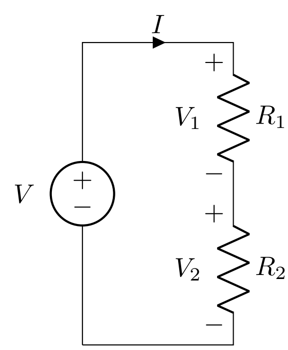
Exercise 1.2

$$P = IV = \left(\frac{V}{R}\right) V = \frac{(12\text{V})^2}{1\Omega} = \boxed{\mathbf{144}\text{W}}$$

Exercise 1.3

Consider a simple series resistor circuit.

Figure 1: A basic series circuit.



By KVL and Ohm's law

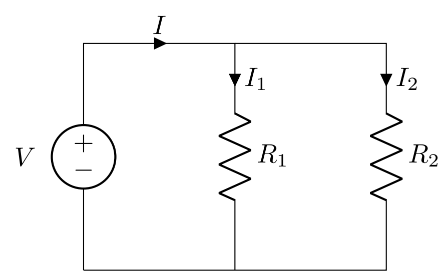
$$V = V_1 + V_2 = R_1 \cdot I + R_2 \cdot I = (R_1 + R_2) \cdot I = R \cdot I$$

where

$$\boxed{\mathbf{R = R_1 + R_2}}$$

is the resistance of R_1 and R_2 in series. Now, consider a simple parallel resistor circuit.

Figure 2: A basic parallel circuit.



By KCL and Ohm's law

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \cdot V$$

solving for V as a function of I we get

$$V = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot I = \frac{R_1 R_2}{R_1 + R_2} \cdot I = R \cdot I$$

where

$$\boxed{\mathbf{R} = \frac{\mathbf{1}}{\frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}}} = \frac{\mathbf{R_1 R_2}}{\mathbf{R_1 + R_2}}}$$

is the resistance of R_1 and R_2 in parallel.

Exercise 1.4

We known that the resistance R_{12}[1] of two resistors R_1 and R_2 in parallel is given by

$$R_{12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Now, the resistance R_{123} of three resistors R_1, R_2 and R_3 in parallel is equal to the resistance of two resistors R_{12} (the resistance between R_1 and R_2 in parallel) and R_3 in parallel, then

$$R_{123} = \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_3}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

We will prove by induction that the resistance $R_{1\cdots n}$ of n resistances $R_1, R_2, \ldots, R_n$ in parallel is given by

$$R_{1\cdots n} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

First, it's trivial to show that with $n = 1$ the equality holds. Now, we will assume that the equality is satisfied for $n = k$, that is

$$R_{1\cdots k} = \frac{1}{\sum_{i=1}^{k} \frac{1}{R_i}}$$

Then, we must show that equality holds for $n = k + 1$. Thus, the resistance $R_{1\cdots(k+1)}$ of $(k + 1)$ resistances $R_1, R_2, \ldots, R_{k+1}$ in parallel is equal to the resistance of two resistors $R_{1\cdots k}$ and R_{k+1} in parallel, then

$$R_{1\cdots(k+1)} = \frac{1}{\frac{1}{R_{1\cdots k}} + \frac{1}{R_{k+1}}} = \frac{1}{\sum_{i=1}^{k} \frac{1}{R_i} + \frac{1}{R_{k+1}}} = \frac{1}{\sum_{i=1}^{k+1} \frac{1}{R_i}}$$

where we have proved that equality holds for $n = k + 1$. Finally, the resistance of n resistors in parallel is given by

$$\boxed{\mathbf{R_{1\cdots n}} = \frac{\mathbf{1}}{\sum_{\mathbf{i=1}}^{\mathbf{n}} \frac{\mathbf{1}}{\mathbf{R_i}}} = \frac{\mathbf{1}}{\frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}} + \ldots + \frac{\mathbf{1}}{\mathbf{R_n}}}}$$

[1] Here we have only assigned a name to the resistance in parallel between R_1 and R_2.

Exercise 1.5

Given that $P = \dfrac{V^2}{R}$, we know that the maximum voltage we can achieve is 15V and the smallest resistance we can have across the resistor in question is 1kΩ. Therefore, the maximum amount of power dissipated can be given by

$$P = \frac{V^2}{R} = \frac{(15\text{V})^2}{1\text{k}\Omega} = \boxed{\mathbf{0.225}\text{W}}$$

This is less than the 1/4W power rating.

Exercise 1.6

(a) The total current required by New York City that will flow through the cable is $I = \frac{P}{V} = \frac{10^{10}\ \text{W}}{115\ \text{V}} =$ 86.96 MA. Therefore, the total power lost per foot of cable can be calculated by:

$$P = I^2 R = (86.96 \times 10^6\ \text{A})^2 \times \left(5 \times 10^{-8}\ \frac{\Omega}{\text{ft}}\right) = \boxed{\mathbf{3.78 \times 10^8}\,\frac{\text{W}}{\text{ft}}}$$

(b) The length of cable over which all 10^{10} W will be lost is:

$$L = \frac{10^{10}\ \text{W}}{3.78 \times 10^8\ \frac{\text{W}}{\text{ft}}} = \boxed{\mathbf{26.45\ ft}}$$

(c) To calculate the heat dissipated by the cable, we can use the Stefan-Boltzmann equation $T = \sqrt[4]{\frac{P}{A\sigma}}$, with A corresponding to the cylindrical surface area of the 26.45 foot long section of 1 foot diameter cable. Note that σ is given in cm^2, so we will need to use consistent units.

$$A = \pi D L = \pi \times 30.48\ \text{cm}\ \times 806.196\ \text{cm} = 7.72 \times 10^4\ \text{cm}^2$$

Therefore,

$$T = \sqrt[4]{\frac{P}{A\sigma}} = \sqrt[4]{\frac{10^{10}\ \text{W}}{7.72 \times 10^4\ \text{cm}^2 \times 6 \times 10^{-12}\ \frac{\text{W}}{\text{K}^4\text{cm}^2}}} = \boxed{\mathbf{12,121\ K}}$$

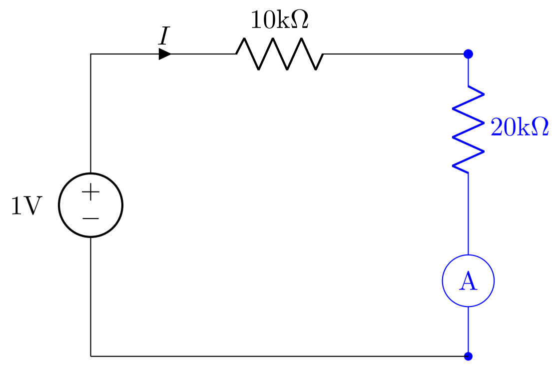
This is indeed a preposterous temperature, more than twice that at the surface of the Sun! The "solution" to this problem is to look at the melting point of copper, which is ∼1358 K at standard pressure. The copper cable will melt long before such a temperature is reached.

Exercise 1.7

A $20,000\Omega/\text{V}$ meter read, on its 1V scale, puts an $20,000\Omega/\text{V} \cdot 1\text{V} = 20,000\Omega = 20\text{k}\Omega$ resistor in series with an ideal ammeter (ampere meter). Also, a voltage source with an internal resistance is equivalent to an ideal voltage source with its internal resistance in series.

(a) In the first question, we have the following circuit:

Figure 3: A voltage source with internal resistance and a $20,000\Omega/\text{V}$ meter read in its 1V scale.

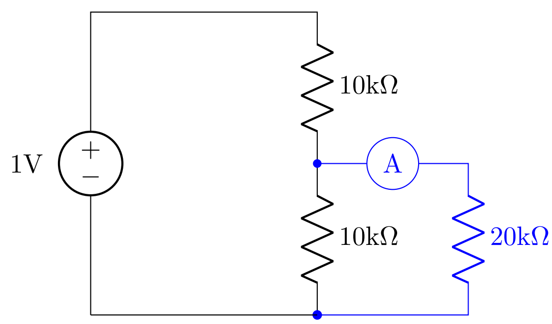


Then, we have that the current in the ideal ammeter and the voltage in the meter resistance are given by[2]

$$I = \frac{1\text{V}}{10\text{k}\Omega + 20\text{k}\Omega} = \boxed{\mathbf{0.0333}\text{mA}} \quad \text{and} \quad V = 0.0333\text{mA} \cdot 20\text{k}\Omega = \boxed{\mathbf{0.666}\text{V}}$$

(b) In the second question, we have the following circuit:

Figure 4: A $10\text{k}\Omega - 10\text{k}\Omega$ voltage divider and a $20,000\Omega/\text{V}$ meter read in its 1V scale.



Now, we can to obtain the Thévenin equivalent circuit of circuit in Figure 4 with

$$R_{\text{Th}} = \frac{10\text{k}\Omega \cdot 10\text{k}\Omega}{10\text{k}\Omega + 10\text{k}\Omega} = 5\text{k}\Omega$$

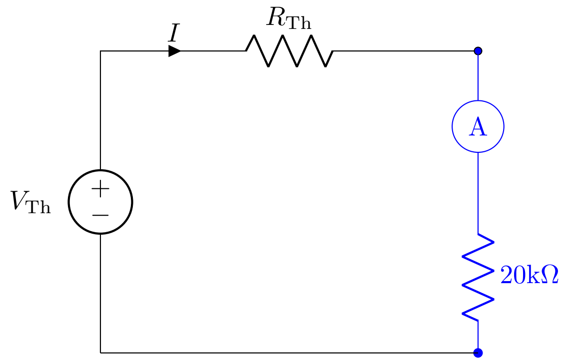
and

$$V_{\text{Th}} = 1\text{V} \cdot \frac{10\text{k}\Omega}{10\text{k}\Omega + 10\text{k}\Omega} = 0.5\text{V}$$

Then, we have the following equivalent circuit:

[2]When a meter only measures currents, it puts a resistance in series to measure the current through that resistance and internally converts that current into voltage to *measure voltages*.

Figure 5: Thévenin equivalent circuit of circuit in Figure 4.



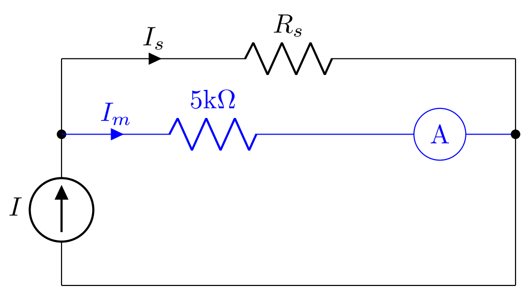
Finally, we have that the current in the ideal ammeter and the voltage in the meter resistance are given by

$$I = \frac{0.5\text{V}}{5\text{k}\Omega + 20\text{k}\Omega} = \boxed{\mathbf{0.02}\text{mA}} \quad \text{and} \quad V = 0.02\text{mA} \cdot 20\text{k}\Omega = \boxed{\mathbf{0.4}\text{V}}$$

Exercise 1.8

(a) In the first part, we have the following circuit:

Figure 6: 50μA ammeter with $5\text{k}\Omega$ internal resistance (shown in blue) in parallel with shunt resistor.



We want to measure I for 0-1 A, and the ideal ammeter measures up to 50 μA. To find what shunt resistance R_s allows us to do so, we set $I = 1\text{A}$ and $I_m = 50\mu\text{A}$. By KCL we know $I_s = 0.999950\text{A}$. To determine R_s, we still need to find the voltage across it. We can find this voltage by doing

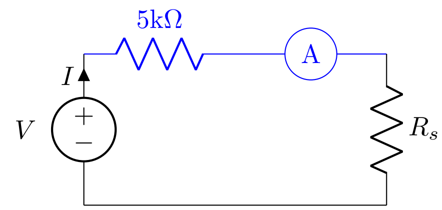
$$V = I_m R_m = 50\mu\text{A} \cdot 5\text{k}\Omega = 0.25\text{V}$$

Then we simply do

$$R_s = \frac{V}{I_s} = \frac{0.25\text{V}}{0.999950\text{A}} = \boxed{\mathbf{0.25\Omega}}$$

(b) In the second part, we have the following circuit:

Figure 7: 50μA ammeter with 5kΩ internal resistance (shown in blue) with a series resistor.



We want to measure V for 0-10 V, and the ideal ammeter measures up to 50 μA. To find the series resistance R_s, we set $V = 10\text{V}$ and $I = 50\mu\text{A}$. Then we solve

$$\frac{V}{I} = 5\text{k}\Omega + R_s$$

$$R_s = \frac{10\text{V}}{50\mu\text{A}} - 5\text{k}\Omega = \boxed{\mathbf{195k\Omega}}$$

Exercise 1.10

(a) With two equal-value resistors, the output voltage is half the input voltage.

$$V_{out} = \frac{1}{2}V_{in} = \frac{30\text{V}}{2} = \boxed{\mathbf{15}\text{V}}$$

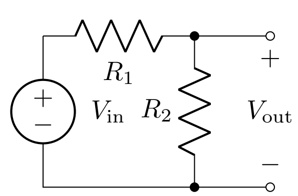
(b) To treat R_2 and R_{load} as a single resistor, combine the two resistors which are in parallel to find that the combined (equivalent) resistance is 5kΩ. Now, we have a simple voltage divider with a 10kΩ resistor in series with the 5kΩ equivalent resistor. The output voltage is across this equivalent resistance. The output voltage is given by

$$V_{out} = V_{in}\frac{5\text{k}\Omega}{10\text{k}\Omega + 5\text{k}\Omega} = \frac{30\text{V}}{3} = \boxed{\mathbf{10}\text{V}}$$

TODO: Add a diagram to make this clearer

(c) We can redraw the voltage divider circuit to make the "port" clearer.

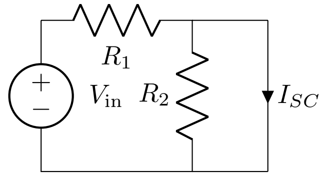
Figure 8: Voltage divider with port shown.



We can find V_{Th} by leaving the ports open (open circuit) and measuring V_{out}, the voltage across R_2. This comes out to be half the input voltage when $R_1 = R_2$, so $V_{\text{out}} = 15\text{V}$. Thus $V_{\text{Th}} = \boxed{\mathbf{15}\text{V}}$.

To find the Thévinen resistance, we need to find the short circuit current, I_{SC}. We short circuit the port and measure the current flowing through it.

Figure 9: Voltage divider with short circuit on the output.

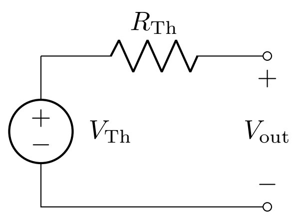


In this circuit, no current flows through R_2, flowing through the short instead. Thus we have $I_{SC} = \dfrac{V_{\text{in}}}{R_1}$. From this, we can find R_{Th} from $R_{\text{Th}} = \dfrac{V_{\text{Th}}}{I_{SC}}$. This gives us

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{SC}} = \frac{V_{\text{Th}}}{V_{\text{in}}/R_1} = \frac{15\text{V}}{30\text{V}/10\text{k}\Omega} = \boxed{\mathbf{5k\Omega}}$$

The Thévenin equivalent circuit takes the form shown below.

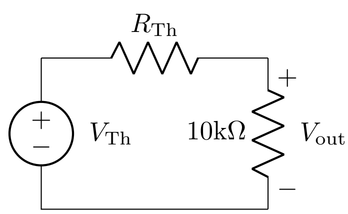
Figure 10: Thévenin equivalent circuit.



In terms of behavior at the ports, this circuit is equivalent to the circuit in Figure 8.

(d) We connect the 10kΩ load to the port of the Thévenin equivalent circuit in Figure 10 to get the following circuit.

Figure 11: Thévenin equivalent circuit with 10kΩ load.



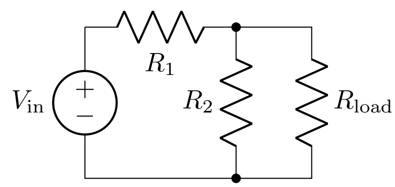
From here, we can find V_{out}, treating this circuit as a voltage divider.

$$V_{\text{out}} = \frac{10\text{k}\Omega}{R_{\text{Th}} + 10\text{k}\Omega} V_{\text{Th}} = \frac{10\text{k}\Omega}{5\text{k}\Omega + 10\text{k}\Omega} \cdot 15\text{V} = \boxed{\mathbf{10}\text{V}}$$

This is the same answer we got in part (b).

(e) To find the power dissipated in each resistor, we return to the original three-resistor circuit.

Figure 12: Original voltage divider with 10kΩ load attached.



From part (d), we know that the output voltage is 10V and that this is the voltage across the load resistor. Since $P = IV = \frac{V^2}{R}$, we find that the power through R_{load} is

$$P_{\text{load}} = \frac{V^2}{R_{\text{load}}} = \frac{(10\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{10}\text{mW}}$$

Similarly, we know that the power across R_2 is the same since the voltage across R_2 is the same as the voltage across R_{load}. Thus we have

$$P_2 = \boxed{\mathbf{10}\text{mW}}$$

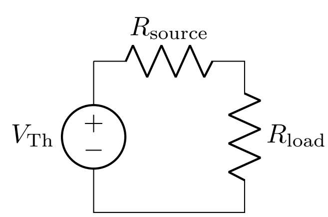
To find the power dissipated in R_1, we first have to find the voltage across it. From Kirchoff's loop rule, we know that the voltage around any closed loop in the circuit must be zero. We can choose the loop going through the voltage source, R_1, and R_2. The voltage supplied by the source is 30V. The voltage dropped across R_2 is 10V as discussed before. Thus the voltage dropped across R_1 must be $30\text{V} - 10\text{V} - 20\text{V}$. Now we know the voltage across and the resistance of R_1. We use the same formula as before to find the power dissipated.

$$P_1 = \frac{V^2}{R_1} = \frac{(20\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{40}\text{mW}}$$

Exercise 1.11

Consider the following Thévenin circuit where R_{source} is just another name for the Thévenin resistance, R_{Th}.

Figure 13: Standard Thévenin circuit with attached load.



We will first calculate the power dissipated in the load and then maximize it with calculus. We can find the power through a resistor using current and resistence since $P = IV = I(IR) = I^2R$. To find the total current flowing through the resistors, we find the equivalent resistance which is $R_{\text{source}} + R_{\text{load}}$. Thus the total current flowing is $I = \frac{V_{\text{Th}}}{R_{\text{source}}+R_{\text{load}}}$. The power dissipated in R_{load} is thus

$$P_{\text{load}} = I^2 R_{\text{load}} = \frac{V_{\text{Th}}^2 R_{\text{load}}}{(R_{\text{source}} + R_{\text{load}})^2}$$

To maximize this function, we take the derivative and set it equal to 0.

$$\begin{aligned}\frac{dP_{\text{load}}}{dR_{\text{load}}} &= V_{\text{Th}}^2 \frac{(R_{\text{source}} + R_{\text{load}})^2 - 2R_{\text{load}}(R_{\text{source}} + R_{\text{load}})}{(R_{\text{source}} + R_{\text{load}})^4} = 0 \\ &\implies R_{\text{source}} + R_{\text{load}} = 2R_{\text{load}} \\ &\implies R_{\text{source}} = R_{\text{load}}\end{aligned}$$

Exercise 1.12

(a) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{3/20} = \boxed{\mathbf{1.413}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{3/10} = \boxed{\mathbf{1.995}}$

(b) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{6/20} = \boxed{\mathbf{1.995}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{6/10} = \boxed{\mathbf{3.981}}$

(c) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{10/20} = \boxed{\mathbf{3.162}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{10/10} = \boxed{\mathbf{10}}$

(d) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{20/20} = \boxed{\mathbf{10}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{20/10} = \boxed{\mathbf{100}}$

Exercise 1.13

There are two important facts to notice from Exericse 1.12:

1. An increase of 3dB corresponds to doubling the power
2. An increase of 10dB corresponds to 10 times the power.

Using these two facts, we can fill in the table. Start from 10dB. Fill in 7dB, 4dB, and 1dB using fact 1. Then fill in 11dB using fact 2. Then fill in 8dB, 5dB, and 2dB using fact 1 and approximating 3.125 as π.

dB	ratio(P/P_0)
0	1
1	$\boxed{\mathbf{1.25}}$
2	$\boxed{\pi/\mathbf{2}}$
3	2
4	$\boxed{\mathbf{2.5}}$
5	$\boxed{\mathbf{3.125} \approx \pi}$
6	4
7	$\boxed{\mathbf{5}}$
8	$\boxed{\mathbf{6.25}}$
9	8
10	10
11	$\boxed{\mathbf{12.5}}$

Exercise 1.14

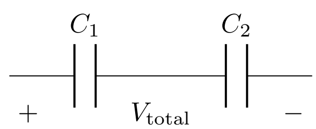
Recall the relationship between I, V, and C: $I = C\frac{dV}{dt}$. Now, we perform the integration:

$$\begin{aligned}
\int dU &= \int_{t_0}^{t_1} VI dt \\
U &= \int_{t_0}^{t_1} CV\frac{dV}{dt} dt \\
&= C\int_{0}^{V_f} V dV \\
U &= \frac{1}{2}CV_f^2
\end{aligned}$$

Exercise 1.15

Consider the following two capacitors in series.

Figure 14: Two capacitors in series.



To prove the capacitance formula, we need to express the total capacitance of both of these capacitors in terms of the individual capacitances. From the definition of capacitance, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}}$$

Notice that V_{total} is the sum of the voltages across C_1 and C_2. We can get each of these voltages using the definition of capacitance.

$$V_{\text{total}} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

The key observation now is that because the right plate of C_1 is connected to the left plate of C_2, the charge stored on both plates must be of equal magnitude.[3] Therefore, we have $Q_1 = Q_2$. Let us call this charge stored Q (i.e. $Q = Q_1 = Q_2$). Now, we know that the total charge stored is also Q.[4] Therefore, we know that $Q_{\text{total}} = Q$. Now, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}} = \frac{Q}{Q_1/C_1 + Q_2/C_2} = \frac{Q}{Q/C_1 + Q/C_2} = \frac{1}{1/C_1 + 1/C_2}$$

[4] If this were not true, then there would be a net charge on these two plates and the wire between them. Because we assume that the capacitors started out with no net charge and there is no way for charge to leave the middle wire or the two plates it connects, this is impossible.

[4] If you are having trouble seeing this, suppose we apply a positive voltage to the left plate of C_1 relative to the right plate of C_2. Suppose this causes the left plate of C_1 to charge to some charge q. We now must have a charge of $-q$ on the right plate of C_1 because q units of charge are now pushed onto the left plate of C_2. Now the left of C_2 has q units of charge which causes a corresponding $-q$ charge on the right side of C_2. Thus the overall total charge separated across these two capacitors is q.

Exercise 1.16

Equation 1.21 gives us the relationship between the time and the voltage (V_{out}) across the capacitor while charging. To find the rise time, subtract the time it takes to reach 10% of the final value from the time it takes to reach 90% of the final value.

$$\begin{aligned} V_{\text{out}} &= 0.1V_f = V_f(1 - e^{-t_1/RC}) \\ 0.1 &= 1 - e^{-t_1/RC} \\ t_1 &= -RC\ln(0.9) \end{aligned}$$

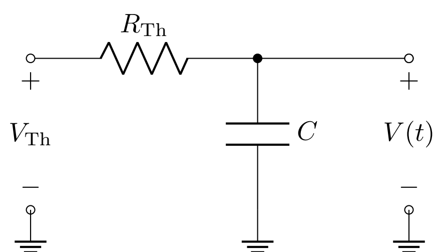
Similarly, we find that $t_2 = -RC\ln(0.1)$. Subtracting these two gives us

$$t_2 - t_1 = -RC(\ln(0.1) - \ln(0.9)) = 2.2RC$$

Exercise 1.17

The voltage divider on the left side of the circuit can be replaced with the Thévenin equivalent circuit found Exercise 1.10 (c). Recall that $V_{\text{Th}} = \frac{1}{2}V_{\text{in}}$ and $R_{\text{Th}} = 5\text{k}\Omega$. This gives us the following circuit.

Figure 15: Thévenin equivalent circuit to Figure 1.36 from the textbook.



Now we have a simple RC circuit which we can apply Equation 1.21 to. The voltage across the capacitor is given by

$$V(t) = V_{\text{final}}(1 - e^{-t/RC}) = V_{\text{Th}}(1 - e^{-t/R_{\text{Th}}C} = \boxed{\mathbf{\frac{1}{2}V_{\text{in}}(1 - e^{-t/5\times10^{-4}}})}$$

TODO: Add graph

Exercise 1.18

From the capacitor equation in the previous paragraph, we have

$$V(t) = (I/C)t = (1\text{mA}/1\mu\text{F})t = 10\text{V}$$

This gives us

$$\boxed{\mathbf{t = 0.01}\text{s}}$$

Exercise 1.19

The magnetic flux produced within the coil is proportional to the number of turns. Now, because the inductance of the inductor is proportional to the amount of magnetic flux that passes through all the coils, it is proportional to the product of the magnetic flux and the number of coils. Thus the inductance is propotional to the square of the number of turns. **TODO: Check/clarify this answer**

Exercise 1.20

We can use the formula for the full-wave rectifier ripple voltage to find the capacitance.

$$\frac{I_{\text{load}}}{2fC} = \Delta V \leq 0.1\text{V}_{\text{p-p}}$$

The maximum load current is 10mA and assuming a standard wall outlet frequency of 60 Hz, we have

$$C \geq \frac{10\text{mA}}{2 \times 60\text{Hz} \times 0.1\text{V}} = \boxed{\mathbf{833}\mu\text{F}}$$

Now we need to find the AC input voltage. The peak voltage after rectification must be 10V (per the requirements). Since each phase of the AC signal must pass through 2 diode drops, we have to add this to find out what our AC peak-to-peak voltage must be. Thus we have

$$V_{\text{in,p-p}} = 10\text{V} + 2(0.6\text{V}) = \boxed{\mathbf{11.2}\text{V}}$$

Exercise 1.30

V_{out} is the voltage at the output of the impedance voltage divider. We know that $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$. So we have

$$V_{\text{out}} = \frac{Z_C}{Z_R + Z_C} V_{\text{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{\text{in}} = \frac{1}{1 + j\omega RC} V_{\text{in}}$$

The magnitude of this expression follows:

$$|V_{\text{out}}| = \sqrt{V_{\text{out}} V_{\text{out}}^*} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} V_{\text{in}}$$

where V_{out}^* is the complex conjugate of V_{out}.

Exercise 1.37

The Norton equivalent circuit can be found by measuring I_{short} and V_{open}:

$$I_{\text{Norton}} = I_{\text{short}} = \frac{10\text{V}}{10\text{k}\Omega} = \boxed{\mathbf{1}\text{mA}}$$

and since $V_{\text{open}} = 5\text{V}$, we have

$$R_{\text{Norton}} = \frac{5\text{V}}{1\text{mA}} = \boxed{\mathbf{5}\text{k}\boldsymbol{\Omega}}$$

2 Bipolar Transistors

Exercise 2.1

We assume a forward voltage for the LED of 1.5V. Then for I_{LED} we have

$$I_{LED} = \frac{V_R}{R} = \frac{3.3\text{V} - 1.5\text{V}}{330\Omega} \approx \boxed{\mathbf{5.5}\text{mA}}$$

To estimate the β_{min} we need the current entering the base

$$I_B = \frac{3.3\text{V} - 0.6\text{V}}{10\text{k}\Omega} = 0.27\text{mA}$$

Thus

$$\beta_{min} \geq \frac{I_{LED}}{I_B} = \boxed{\mathbf{20}}$$

Exercise 2.2

NOTE: According to the errata 0.63 **should be replaced by** 0.76 **and** $63\mu\text{sec}$ **by** $76\mu\text{sec}$**.**

Starting from the hint that the capacitor charges from -4.4V towards $+5$V, we would result to a total 9.4V for a full charge. However, the V_{BE} of Q_2 is clipping the charging process at only 5V of the total (from -4.4V to 0.6V). Thus, the capacitor will be 53% charged at the end.
Solving the voltage equation for a charging capacitor gives us

$$V_C(t) = V_f * (1 - e^{-\frac{t}{RC}})$$

set $V_C(t_1) = 0.53 * V_f$

$$0.53 = 1 - e^{-\frac{t_1}{R_3 C_1}}$$

$$\Rightarrow$$

$$t_1 = -R_3 C_1 * ln(0.47) \approx \boxed{\mathbf{0.76 * R_3 C_1}}$$

Exercise 2.3

The output voltage is reduced due to the $R_4 - R_5$ voltage divider

$$V_{\text{out}} = \frac{R_5}{R_4 + R_5} * (V_{CC} - 0.6\text{V}) \approx \boxed{\mathbf{4.18}\text{V}}$$

To estimate the minimum β_3, we need first to find the maximum (worst-case) collector current for which Q_3 should still be in saturation. For this we can assume a 0V drop across C and Q_3 while the current travels through the parallel connected resistors $R_2||R_3$.

$$I_{C_3,max} = \frac{V_{CC}}{R_2||R_3} = 5.5\text{mA}$$

$$\Rightarrow \beta_{3,min} = \frac{I_{C_3,max}}{I_{B_3}} = \frac{5.5\text{mA}}{\frac{(4.18\text{V}-0.6\text{V})}{20\text{k}\Omega}} \approx \boxed{\mathbf{31}}$$

Exercise 2.4

By using KCL and the fact that the transistor is in the active region we get

$$i_E = i_C + i_B = (\beta + 1) * i_B = (\beta + 1)\frac{v_B}{Z_{source}}$$

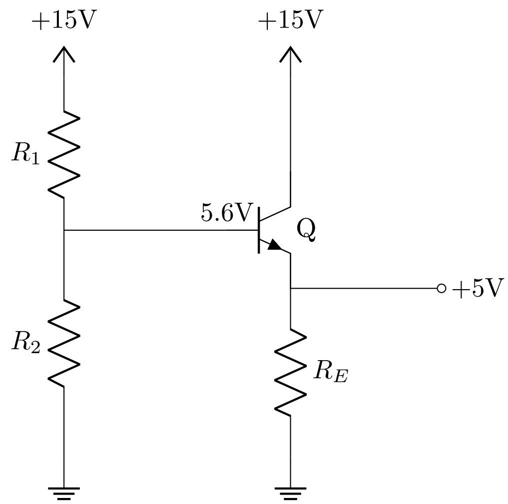
For small signals $Z_{\text{out}} = \frac{v_E}{i_E} = \frac{v_B}{i_E}$. Thus:

$$\boxed{\mathbf{Z}_{\text{out}} = \frac{\mathbf{v_B}}{(\beta + \mathbf{1})\frac{\mathbf{v_B}}{\mathbf{Z_{source}}}} = \frac{\mathbf{Z_{source}}}{\beta + \mathbf{1}}} \qquad q.e.d.$$

Note: In practice one will often see $Z_{\text{out}} \approx \frac{Z_{source}}{\beta}$. When $\beta \approx 100$ the "+1" part is often being ignored to simplify calculations.

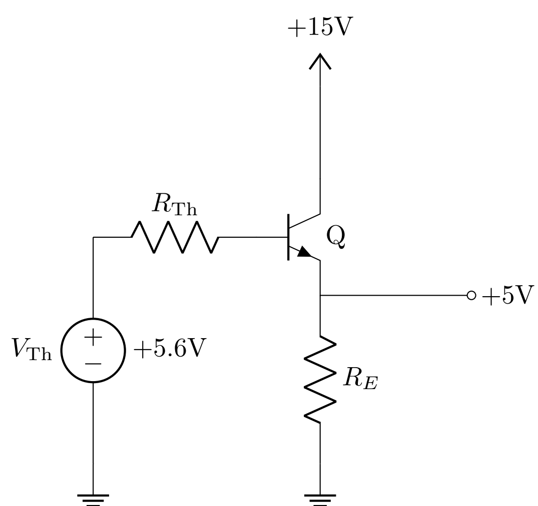
Exercise 2.5

Figure 16: Follower driven by voltage divider



We can simplify the voltage divider with it equivalent Thévenin voltage source depicted in Figure 17 below.

Figure 17: Follower driven by equivalent Thévenin source



With $R_{\text{Th}} = R_1||R_2$ which is also our R_{source}. The output impedance of our circuit then is

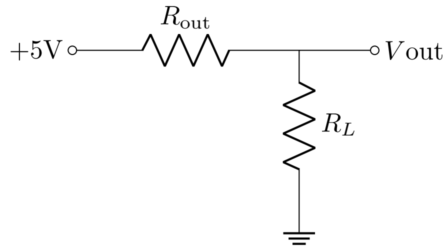
$$R_{\text{out}} = \frac{R_{\text{Th}}}{\beta + 1} \approx \frac{R_{\text{Th}}}{100} = \frac{R_1||R_2}{100} \qquad \text{(assuming } \beta \approx 100)$$

We also know that the following condition needs to be true in order to achieve the wished 5.6V at the base:

$$\frac{R_2}{R_1 + R_2} = \frac{5.6\text{V}}{15\text{V}} \Rightarrow R_2 \approx 0.6 * R_1$$

Now let's observe the equivalent circuit *with* load.

Figure 18: Equivalent voltage source of emitter follower with load



Our goal is to have a maximum voltage drop of 5% with maximum load:

$$V_{\text{out}} = \frac{R_L}{R_{\text{out}} + R_L} * 5\text{V} \geq 0.95 * 5\text{V}$$

$\Rightarrow$

$$R_{\text{out}} \leq \frac{R_L}{19} \leq \frac{\frac{4.75\text{V}}{25\text{mA}}}{19} = 10\Omega$$

$\Rightarrow$

$$\frac{R_{\text{Th}}}{100} \leq 10\Omega$$

$\Rightarrow$

$$\frac{R_1||R_2}{100} \leq 10\Omega$$

$\Rightarrow$

$$R_1 \leq 2.7\text{k}\Omega \text{ and } R_2 \leq 1.62\text{k}\Omega$$

We choose the following values for R_1 and R_2:

$$\boxed{\mathbf{R_1 = 2.7\text{k}\Omega}} \text{ and } \boxed{\mathbf{R_2 = 1.6\text{k}\Omega}}$$

NOTE 1: We could have picked more conservative (smaller) values for the resistors. However, this would increase the idle power consumption.

NOTE 2: We could also define a value for R_E i.e. to limit the quiescent current, but this is out of the scope of this exercise. Basically we see now R_E as our load or put differently, our total load is $R_E||Z_{whatever-the-user-wants}$.