

Asset Purchases, Limited Asset Markets Participation and Inequality

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Abstract

This paper examines the impact of quantitative easing (QE) on aggregate demand and inequality in a restricted financial participation economy. It shows that when wages are sticky and asset market participation is high, QE stimulates aggregate demand and reduces income and consumption inequality. Conversely, if wages are flexible and asset market participation is low, QE can reduce aggregate demand and raise inequality. To study these phenomena, I build and calibrate a New-Keynesian dynamic, general equilibrium model with sticky wages for the Euro Area (EA) that incorporates limited assets market participation, financial frictions and allows central bank purchases from banks and households. Bond purchases increase aggregate demand and benefit financially restricted households more due to the dominance of QE's indirect effects, reducing income and consumption inequality. The stimulating effects are conditional on the level of wage stickiness and thus the cyclicity of profits. When wages are flexible and thus profits countercyclical, low financial participation levels invert QE's positive effects. Using an external instrument SVAR, I find that QE was stimulative and profits in the EA move pro-cyclically supporting the sticky wage specification of the model. This result combined with the high level of asset markets participation in the EA make the QE a stimulating and redistributive tool for the region.

JEL classification: E12; E44; E52; E58; E61

Keywords: Quantitative Easing; Inequality; Financial Participation; DSGE; SVAR; External Instrument

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1. Introduction

Asset purchase programmes following the Great Recession aimed to hold down long-term interest rates and stimulate aggregate demand. Although empirical literature has shown that the goal of the programmes has been achieved (see [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) among others), a question lately posed by policy makers ([Yellen, 2016](#); [Bernanke, 2015](#); [Draghi, 2016](#)) and gained the media attention is whether and in what extent asset purchase programmes have contributed to the increase of inequality.¹ In this paper I answer two questions: firstly whether QE increases aggregate demand and secondly how it affects inequality.

I show that a QE policy does not only reduce the structural wedges in the Euro Area (EA) economy but also provides redistribution. I develop a DSGE model with banks, financially constrained and unconstrained households and a central bank that can purchase assets from banks and households which is calibrated to the Euro Area. QE induces more lending through the exchange of banks' government bonds with reserves and thus stimulates the economy. Turning to the inequality impact of the QE, I show that the indirect (i.e general equilibrium) effects outweigh the direct effects leading to a reduction in income and consumption inequality. The economic intuition of the QE impact on inequality is as follows. Consider an increase in the bond holdings of the central bank in an economy with two types of consumers, asset holders (optimizers) and hand to mouth consumers. The outcome of this operation will have direct and indirect effects. The direct effects, namely the reduction of the interest rates and the asset price increases will harm and benefit the bond holders respectively. On the other hand, the indirect or general equilibrium effects such as the employment level and real wage increases will benefit hand to mouth consumers.

The second result of the paper is on the adverse effects of QE in an economy under a high share of financially constrained agents. I show that the sign of the QE's impact depends on the asset markets participation level and the cyclicalities of profits, a by-product of the wage stickiness in the model. QE's impact can be negative for a low asset markets participation level and flexible wages. This extends the result of [Bilbiie \(2008\)](#) for the conventional monetary policy. Differently from Bilbiie's work, instead of a nominal rate reduction, the central bank buys a fraction bank's bond holdings and exchanges them with reserves under a constant nominal rate which leads to a similar increase in the marginal costs under flexible wages and to a drop in profits, as in the class of NK models after a monetary easing (see [Christiano, Eichenbaum, and Evans \(1997\)](#)). This, under a low asset markets participation rate leads to an economic downturn. In an economy with sticky wages and high market participation as I show in an empirical exercise that the EA is, marginal costs do not increase as much and profits are procyclical leading to stimulating impact of the QE. I lastly demonstrate that the negative impact of the QE in a flexible wage economy can be alleviated by fiscal redistribution. In detail, a fiscal

¹Does Quantitative Easing Mainly Help the Rich? (CNBC), Debate rages on quantitative easing's effect on inequality (Financial Times), Quantitative easing helped vulnerable more than rich, says ECB (Financial Times).

rule that redistributes share of the profits as transfers to hand to mouth consumers can be welfare improving. The higher the redistribution, the less likely for QE to be contractionary for a low level of financial participation. This highlights the importance of monetary and fiscal policy coordination.

Thirdly, to accompany the theoretical results on the QE impact for the Euro Area and also to investigate the behaviour of profits, I use an external instrument SVAR with high frequency identification. Using an identified QE shock for the Euro Area I show that QE was stimulating. Also the VAR shows that profits move pro-cyclically in the Euro Area giving motivation for the sticky wages specification of the model.

This study introduces limited asset markets participation (LAMP), agency problems associated with financial intermediation and a QE framework in an otherwise standard business cycles model with sticky prices and wages. By combining Galí, López-Salido, and Vallés (2007) (GLS hereafter), Bilbiie (2008) and Gertler and Karadi (2013) a setting is developed where central bank purchases of government bonds or private assets and the exchange of those with reserves, create direct and indirect effects on the real economy affecting differently those with and without access to financial markets. I evaluate inequality between the two groups in terms of consumption and income inequality following Krueger and Perri (2006).

Financial frictions play a prominent role in the analysis. There is a moral hazard problem between the banks and their depositors due to the bankers ability to divert assets back to their household members. This implies an incentive constraint for the banks. Central bank bond purchases relax the bank's constraint and stimulate the demand for loans. QE in the model works as a credit stimulating mechanism to the real economy. Furthermore, the existence of the banker's incentive constraint together with households' transaction costs eliminate the perfect substitutability of assets and break the neutrality result of open market operations first shown by Wallace (1981) and more recently by Curdia and Woodford (2010).²

To evaluate the behaviour of profits after a QE shock in the Euro Area I employ an external instrument SVAR approach. This is based on the work of Mertens and Ravn (2013) and the high frequency identification approach of Gertler and Karadi (2015). To identify QE policy surprises I make use of the Euro Area Monetary Policy Event Study Database by Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019), I develop and use the QE factor as external instrument. Results show that profits move pro-cyclically supporting the specification of sticky wages in the model.

Financial Inclusion in the Euro Area. I use household-level data, the Eurosystem Household Finance and Consumption Survey (HFCS), and document the fraction of the Euro Area households that are hand to mouth consumers. I restrict attention to the first wave of the HFCS data conducted mainly in 2009 and 2010. This is done in order to eliminate as possible the effects of the 2008 financial crisis. Also the survey was performed well before the start of ECB's QE in March 2015. The data has been collected

²Wallace (1981) was the first to show that open market operations are not effective under the assumption of the same return between money and assets purchased and a fixed fiscal policy stance. The result remains the same in future studies built on more relaxed assumptions. See also Sargent and Smith (1987); Chamley and Polemarchakis (1984); Eggertsson and Woodford (2003); Curdia and Woodford (2010).

from 15 Euro Area member states for a sample of more than 62,000 households.

Figure 1 reports the distribution of financial and real asset holdings of the Euro Area residents.³ As the Figure shows, 20-30% of the Euro Area households hold a total value of financial assets that is close to zero (green bar). In comparison all percentiles of Euro Area households holding real assets hold substantial values of real assets (yellow bar).

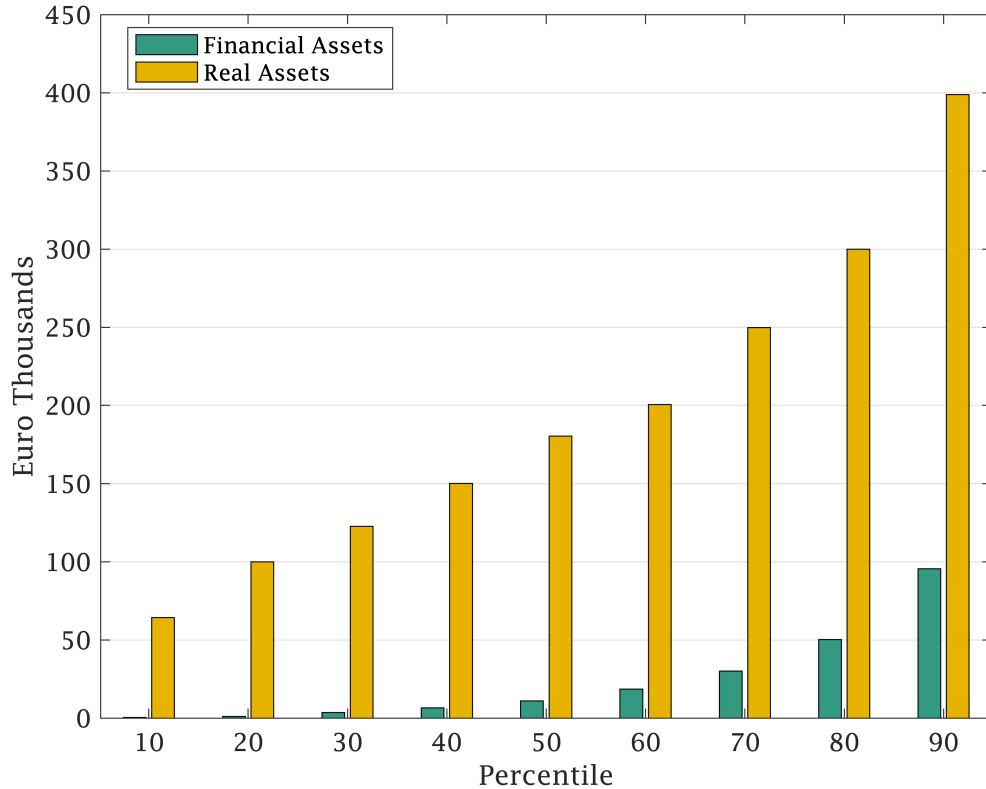


Fig. 1. Total Financial and Real Assets among EA Households

Heterogeneity in asset holdings is also present between the Euro Area member countries. Using the same dataset, I show the volume of financial assets held by households for each country of the Euro Area in Figure 2. Countries of the core of the EA such as Belgium, Luxembourg and Netherlands report financial assets holdings that are way above those in countries in the periphery such as Greece, Italy and Spain. I will show later that the same monetary policy can lead to different impact effects conditionally on a country's level of financial participation.

³Financial assets include deposits (sight and saving accounts), mutual funds, bonds, shares, money owed to the households, value of voluntary pension plans and whole life insurance policies of household members and other financial assets item - which includes private non-self-employment businesses, assets in managed accounts and other types of financial assets. Real assets include the value of household's main residence.

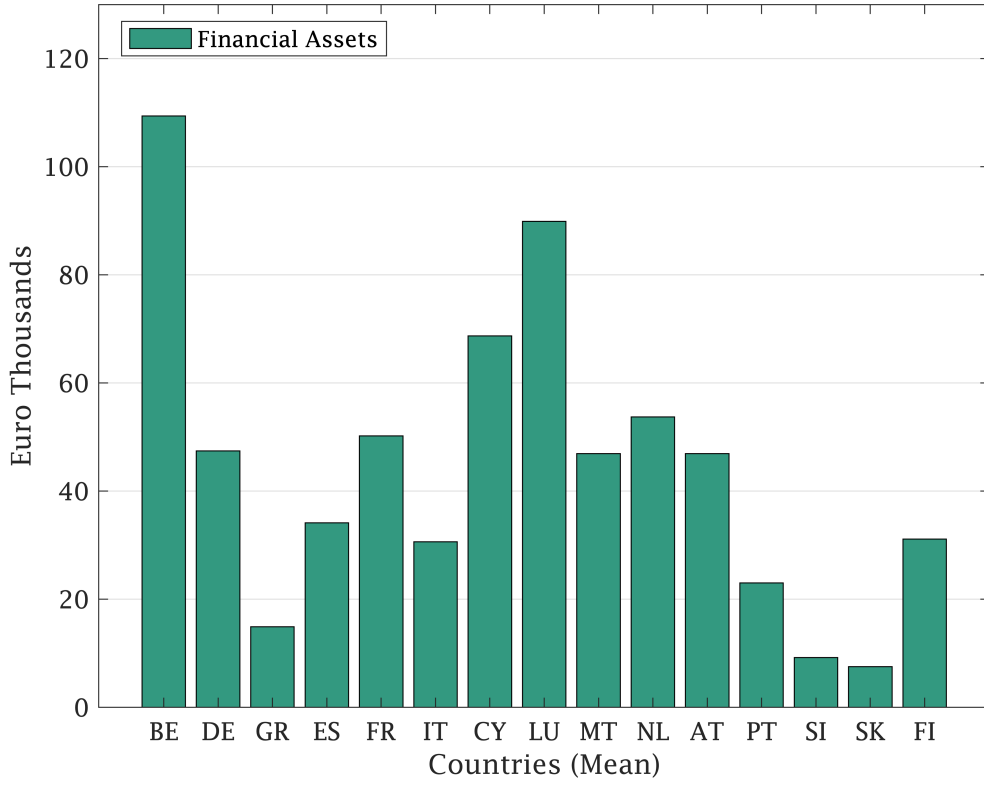


Fig. 2. Total Financial Assets among EA Households by Country

Related Literature. This study relates to several strands of the macro-finance literature. Firstly, it builds upon similar works using the Two Agent Neo-Keynesian (TANK) framework. [Galí et al. \(2007\)](#) firstly introduced a TANK framework to study the effects of government spending on consumption. [Bilbiie \(2008\)](#) using a simple NK model with two agents shows that the change in the level of asset market participation can have different impact on the monetary policy transmission mechanism: an expansionary monetary policy shock can have contractionary effects when asset markets participation is low. The present study brings this result to a DSGE model with financial frictions and QE as the monetary easing tool. [Colciago \(2011\)](#) proves that this result no longer holds for conventional monetary policy when wages are sticky while [Broer, Harbo Hansen, Krusell, and Öberg \(2020\)](#) identify as well the importance of wage rigidities on the cyclicity of profits in a two agent model and their distributional consequences. My result complements [Broer et al. \(2020\)](#) for a more realistic model with unconventional monetary policy for the Euro Area.⁴

[Kaplan, Moll, and Violante \(2018\)](#), [McKay and Reis \(2016\)](#) and [Ravn and Sterk \(2016\)](#) develop an Aiyagari-type heterogeneous agent framework with New Keynesian nom-

⁴Studies using a TANK framework also include [Monacelli \(2009\)](#) and [Bilbiie, Monacelli, and Perotti \(2013\)](#), [Galí, López-Salido, and Vallés \(2004\)](#).

inal rigidities (HANK) making the characterization and study of the full income and wealth distribution feasible. As shown by [Debortoli and Galí \(2017\)](#), a two agents framework is able to identify differences in average consumption *between* the constrained and unconstrained agents but lacks on characterising consumption heterogeneity *within* the subset of unconstrained households. Since the main focus on this paper is on the interactions between the two types of agents, it suffices to use a less rich setting of heterogeneity.

This paper is closely related to the literature on distributional effects of monetary policy. The seminal paper of this fast growing literature focuses on the impact of conventional monetary policy in the US is the work by [Coibion, Gorodnichenko, Kueng, and Silvia \(2017\)](#). For the Euro Area, [Lenza and Slacalek \(2018\)](#) employ a Bayesian VAR with sign restrictions to identify the effects of asset purchases show that QE reduces income and wealth inequality. [Ampudia, Georgarakos, Slacalek, Tristani, Vermeulen, and Violante \(2018\)](#) show that indirect effects of monetary policy outweigh the direct ones in their study for the Euro Area. [Slacalek, Tristani, and Violante \(2020\)](#) show that in the Euro Area consumption responses are different after a monetary policy change with the earnings heterogeneity channel being of importance. Most empirical studies agree that the QE effects benefit mostly the lower end of income distribution in line with this paper's results. Empirical studies by [Bunn, Pugh, Yeates et al. \(2018\)](#) using UK data and [Bivens \(2015\)](#) also concur on the relatively bigger effect to lower income households.⁵ Studies using structural models do mostly focus on conventional monetary policy. [Dolado, Motyovszki, and Pappa \(Forthcoming\)](#) focus on labour frictions and conclude that a monetary policy easing increases income inequality between skilled and unskilled workers. [Gornemann, Kuester, and Nakajima \(2016\)](#) use a heterogeneous agents framework, as developed by [Kaplan et al. \(2018\)](#), accompanied with matching frictions and propose an addition of unemployment stabilization to the dual mandate of the central banks to achieve higher welfare. Finally, [Cui and Sterk \(2019\)](#) build a heterogeneous agents model and show that QE is dominated by conventional policy in welfare terms. This paper contributes to this literature by showing the effects of a QE shock to aggregate variables and inequality in the Euro Area using new approaches in a both theoretical and empirical framework.

Related to this paper's empirical specification there is a vast literature on SVARs using a set of different identification methods. A very comprehensive summary is included in [Ramey \(2016\)](#). This paper makes use of the proxy - or external instruments - SVAR approach using the high frequency identified monetary policy surprises included in the Euro Area Monetary Policy Event Study (EA-MPD) database by [Altavilla et al. \(2019\)](#). The external SVAR method has been pioneered by [Stock and Watson \(2012\)](#) and [Mertens and Ravn \(2013\)](#). [Gertler and Karadi \(2015\)](#) developed this further by using a high frequency identification approach for monetary policy changes in the US. This paper follows their method using the EA-MPD database to identify the factors and then use them as external instruments for QE shocks. The present study is the first one to employ an external SVAR with the monetary policy surprises as in [Altavilla et al. \(2019\)](#)

⁵For a comprehensive literature review see [Colciago, Samarina, and de Haan \(2018\)](#).

to identify the QE's impact in the Euro Area.

[Hohberger, Priftis, and Vogel \(2019a\)](#) in parallel work conduct a similar study where they evaluate the effects of QE to consumption and income inequality in a standard NK setting with two agents. They show that consumption and income inequality fall after a QE policy, in line with this paper's results. In their analysis, the perfect substitutability of assets and the QE neutrality are eliminated through portfolio costs. In the present paper the Wallace neutrality is endogenously eliminated through the banks moral hazard problem. QE shifts the economy to its first best allocation. Furthermore, this study explores the effects of QE when asset market participation is low in two different labour market settings and provides evidence on the procyclicality of profits after a QE shock using a proxy SVAR approach. In my knowledge there is no other study employing a TANK model with financial frictions and an explicit framework for asset purchases by the central bank that measures changes in consumption and income inequality.

2. The DSGE Model

The economy is populated by two types of households: Rule of thumb and optimising households that differ in their ability to participate in the assets market. A continuum of firms and financial intermediaries owned by the optimizers, labour wide unions that set the wages, capital goods producers and retailers, a monetary authority and the treasury complete the model economy. There is a moral hazard problem between the savers and the banks. Banks can steal a fraction of their funds and return them to their families. This problem introduces an intensive constraint to the model to be followed by the banks. Finally, the central bank performs its conventional monetary policy under a Taylor rule, but can also engage in asset purchases and pay the investors back the same value in newly created reserves.

2.1. Households - The Two Agents Framework

All households are assumed to have identical preferences, given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i}^s) - \frac{\chi}{1+\epsilon} L_{t+i}^{1+\epsilon, s}], \quad (1)$$

C_{t+i}^s denotes the per capita consumption of the household members and L_{t+i}^s the supply of labour. The super-index $s \in [o, r]$ specifies the household type (o for "optimizers" or r for "rule of thumb"). $\beta \in [0, 1]$ is the discount factor. Due to the stochastic setting, households make expectations for the future based on what they know in time t and \mathbb{E}_t is the expectation operator at time t . Finally, ϵ is the inverse Frisch elasticity of labour supply and χ is the relative utility weight of labour.

Optimizers. Optimizers account to a measure of $(1-\lambda)$ of the economy's population. Their portfolio includes one period government bonds B_t^o , bank deposits D_t^o and firm shares S_t^o . They can freely adjust their deposit holdings. However, they are not experts

in trading bonds and shares. Transactions above or below a frictionless level \bar{S}_t^o and \bar{B}_t^o for shares and bonds respectively require broker expertise and this induces costs. Costs equal to $\frac{1}{2}\kappa(S_t^o - \bar{S}_t^o)^2$ for shares and $\frac{1}{2}\kappa(B_t^o - \bar{B}_t^o)^2$ for bonds deviating from their respective frictionless level.⁶

Optimizing households budget constraint then is

$$\begin{aligned} C_t^o + T_t^o + D_t^o + q_t[B_t^o + \frac{1}{2}\kappa(B_t^o - \bar{B}_t^o)^2] + Q_t[S_t^o + \frac{1}{2}\kappa(S_t^o - \bar{S}_t^o)^2] \\ = W_t L_t^o + \Pi_t + R_{d,t} D_{t-1}^o + R_{b,t} q_{t-1} B_{t-1}^o + R_{k,t} Q_{t-1} S_{t-1}^o, \end{aligned} \quad (2)$$

Total deposits D_t^o are the sum of households' private deposits and deposits created by the exchange of securities with reserves when the central bank purchases those during a QE. They are remunerated at the risk-free rate $R_{d,t}$. $R_{b,t}$ and $R_{k,t}$ are the gross returns for the bonds and shares respectively in period t . W_t is the real wage which both types of households take as given. T_t^o are taxes (or transfers if negative) that optimizing households pay every period. Finally, optimizers receive income Π_t from the ownership of both non-financial firms and financial intermediaries.

The problem of the optimizing household is to choose $C_t^o, L_t^o, D_t^o, B_t^o, S_t^o$ in order to maximize its expected utility (1) subject to the budget constraint (2) at every period. Let $u_{c,t}^o$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ denote the optimizing household's stochastic discount factor (the intertemporal marginal rate of substitution)

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{c^o,t+1}^o}{u_{c^o,t}^o}. \quad (3)$$

Maximizing optimizers' utility with respect to deposits yields their intertemporal optimality condition

$$\mathbb{E}_t \Lambda_{t,t+1} R_{d,t+1} = 1. \quad (4)$$

The choices for private securities and long-term government bonds are given by:

$$\begin{aligned} S_t^o &= \bar{S}_t^o + \frac{\mathbb{E}_t \Lambda_{t,t+1} (R_{k,t+1} - R_{t+1})}{\kappa} \\ B_t^o &= \bar{B}_t^o + \frac{\mathbb{E}_t \Lambda_{t,t+1} (R_{b,t+1} - R_{t+1})}{\kappa} \end{aligned} \quad (5)$$

It follows that households hold always the frictionless amount of each asset. Their demand for extra units is increasing in the excess returns relative to the respective curvature parameter that governs the marginal transaction cost κ . As marginal transaction costs go to zero, excess returns disappear: There is frictionless arbitrage between the two assets and all assets' interest rates are equalized. On the other hand, when marginal transaction costs go to infinity, households' asset demands go to their respective fric-

⁶Note that under this setting, optimizing households could be also thought as financially constrained due to adjustment costs, similar to the wealthy hand to mouth consumers in [Kaplan, Violante, and Weidner \(2014\)](#). Also, another interpretation following [Kaplan et al. \(2018\)](#) is that bonds and stocks are illiquid assets and deposits are liquid assets.

tionless capacity values.

I consider two labour market specifications. Under the first setting labour market is competitive and each household chooses the quantity of hours supplied given the market wage W_t . In the second case wages are set by a labour union. Hours are demand driven by firms taking the wages as given by the union, households are ready to supply as many hours as required by the firms given the wage. Both wage specifications are analysed in section 2.2.

Rule of Thumb. Rule of thumb households account for a λ measure of households. Their participation in financial markets is restricted. They cannot smooth consumption either by trading securities or by acquiring bank deposits. They consume their net income at every period which is their labour income net of taxes. Their budget constraint is:

$$P_t C_t^r = P_t W_t L_t^r + P_t T_t^r. \quad (6)$$

C_t^r, L_t^r, T_t^r denote, respectively, consumption, hours worked and taxes (or transfers).

Rule of thumb agents maximize their utility subject to their budget constraint. Accordingly, the level of consumption will equate labour income specified by (6).

Rule of thumb agents' taxation is the only fiscal variable that matters for the model's fiscal allocation as is shown in Proposition 2. Optimizing agents internalize the government budget constraint through their government bond holdings. On the other hand, a change in the tax rate (or transfer) of the rule of thumb consumers implies a change in their taxes today or in the future.⁷ I study two transfer schemes for the rule of thumb consumers: a no-redistribution scheme where transfers to rule of thumb agents are zero and a fiscal rule that taxes the profits of the optimizing households and rebates them to hand to mouth consumers.

2.2. Wage Setting

Here I develop the two wage setting schemes of the model: perfectly competitive labour markets and wage-setting by unions.

2.2.1. Perfectly Competitive Labour Markets

In the case of perfect competition in labour markets, households choose optimally their labour supply taking wages as given. The optimality condition with respect to hours worked for a household of type s is

$$u_{c,t}^s W_t = \chi (L_t^s)^\epsilon. \quad (7)$$

In the case of the rule-of-thumb consumers, due to the very form of the logarithmic utility function, combining (6) and (7) we find an analytical expression of hours that

⁷Similar results are obtained for the TANK model in Bilbiie et al. (2013).

the rule of thumb agents optimally supply:

$$L_t^r = \left(\frac{1 - \frac{T_t^r}{C_t^r}}{\chi} \right)^{\left(\frac{1}{1+\epsilon} \right)}. \quad (8)$$

2.2.2. Wage Setting by Unions

In the second case it is assumed that wage decisions are delegated to a continuum of labour unions. Hours are determined by firms taking the wages set by unions as given.⁸ Households supply the hours required by the firms given the wage set by unions. Firms are also indifferent on the type of household they employ. Therefore, all households types supply the same working hours $L_t^o = L_t^r = L_t$.

Labour supply L_t is a composite of heterogeneous labour services

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dh \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \quad (9)$$

where $L_{h,t}$ is the supply of labour service h and ϵ_w is the elasticity of substitution between labour and consumption across household types.

At each period there is a probability $1 - \xi_w$ that the wage for each particular labour service $W_{h,t}$ is set optimally. The union buys homogeneous labour at nominal price $W_{h,t}$, repackages it by adding a mark-up and chooses the optimal wage W_t^* to maximize the objective function where labour income of the two types is weighed by their marginal utilities of consumption.

$$\lambda \left[u_{c,t}^r W_{h,t} L_{h,t} - \frac{\chi}{1 + \epsilon} L_t^{1+\epsilon} \right] + (1 - \lambda) \left[u_{c,t}^o W_{h,t} L_{h,t} - \frac{\chi}{1 + \epsilon} L_t^{1+\epsilon} \right] \quad (10)$$

Aggregation. Aggregate variables are given by the population weighted average of the corresponding variables of each household type.

$$C_t \equiv (1 - \lambda)C_t^o + \lambda C_t^r \quad (11)$$

$$L_t \equiv (1 - \lambda)L_t^o + \lambda L_t^r \quad (12)$$

$$T_t \equiv (1 - \lambda)T_t^o + \lambda T_t^r \quad (13)$$

The H superscript denotes the total asset holdings of households.

$$S_t^H \equiv (1 - \lambda)S_t^o$$

$$B_t^H \equiv (1 - \lambda)B_t^o$$

$$D_t^H \equiv (1 - \lambda)D_t^o$$

⁸For a detailed exposition on wage setting see Appendix A.

2.3. Financial Frictions

Banks. Banks are funded with deposits, receive reserves from the central bank during the QE, extend credit to non-financial firms and buy bonds from the government. Each bank j allocates its funds to buying a quantity $s_{j,t}$ of financial claims on non-financial firms at price Q_t and government bonds $b_{j,t+1}^B$ at price q_t . Banks' liabilities are made up from households' deposits $d_{j,t+1}^B$. When the central bank proceeds in securities' purchases ($Q_t S_t$ or $q_t B_t$) it pays back the bank with an equivalent value of reserves $m_{j,t}$.⁹ Finally, $n_{j,t+1}$ is the capital equity accumulated. Formally, the bank's balance sheet is:

$$Q_t s_{j,t}^B + q_t b_{j,t}^B + m_{j,t}^B = n_{j,t} + d_{j,t}^B. \quad (14)$$

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities.

$$n_{j,t+1} = R_{k,t} Q_{t-1} s_{j,t-1}^B + R_{b,t} q_{t-1} b_{j,t-1}^B + R_{m,t} m_{j,t}^B - R_t d_{j,t}^B.$$

Let Z_t be the net period income flow to the bank from a loan that is financing to a firm and δ the depreciation rate of capital being financed. Then the rate of return to the bank on the loan, $R_{k,t+1}$, is given by:

$$R_{k,t+1} = \frac{Z_t + (1 - \delta)Q_{t+1}}{Q_t}. \quad (15)$$

Long-term bond is a perpetuity that pays one euro per period indefinitely. The real rate of return on the bond $R_{b,t+1}$ is given by:

$$R_{b,t+1} = \frac{1/P_t + q_{t+1}}{q_t}.$$

Central bank reserves bear a zero weight in the banks' constraint and, as it will be shown momentarily, have a gross return $R_{m,t}$ equal to the risk-free rate R_t . It follows that banks have no incentive to hold reserves in equilibrium.

The bankers' objective at the end of period t , is to maximize the expected present value of future dividends. Since the banks are owned by the optimizing households, their stochastic discount factor Λ_{t+1} is used as the discounting measure.

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1 - \sigma_B) \sigma_B^{j-1} \Lambda_{t+1} n_{j,t+1}. \quad (16)$$

To motivate a limit on the banks ability to obtain deposits, I introduce a moral hazard problem in the same fashion as in [Gertler and Kiyotaki \(2010\)](#). A banker can abscond

⁹We can think $m_{j,t}^B$ as the sum of reserves a bank receives from the purchases not only of its own securities but also from the ones the households listed to the bank hold. The bank will transfer the exact same amount to the household's deposit account (see [McLeay, Radia, and Thomas \(2014\)](#)), keeping the balance sheet constraint intact.

a fraction of the assets it holds and transfer it back to their household members. In the case it does it, depositors can force the bank into bankruptcy and get the remaining fraction of assets. It is assumed that the banker can divert loans easier than it can divert bonds and reserves.

The depositors continue providing funds to the bank as long as the following incentive constraint is not violated:

$$V_{j,t} \geq \theta [Q_t s_{j,t}^B + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B]. \quad (17)$$

where θ is the fraction of assets that the banker may divert and $\Delta \in (0, 1)$ and $\omega \in (0, 1)$ are the ratios of how much bonds and reserves the banker can divert. On the left of (17) is the franchise value of the banker which is what the banker would lose from diverting while on the right are the banker's gains from diverting which is a fraction θ of its assets.

The value of the bank at the end of period $t - 1$ must satisfy the Bellman equation:

$$\begin{aligned} V_{j,t-1}(s_{j,t-1}^B, b_{j,t-1}^B, m_{j,t}^B, d_{j,t}^B) = & E_{t-1} \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1 - \sigma_B) n_{j,t} \\ & + \sigma_B \max_{d_{j,t}} [\max_{s_{j,t}^B, b_{j,t}^B, m_{j,t}^B} V_t(s_{j,t}^B, b_{j,t}^B, m_{j,t}^B, d_{j,t}^B)]\}. \end{aligned} \quad (18)$$

Banker's problem is to maximize (16) subject to the balance sheet (14) and their constraint (17).

Proposition 1. A solution to the banker's dynamic program is

$$V_{j,t}(s_{j,t}^B, b_{j,t}^B, d_{j,t}^B, m_{j,t}^B) = A_{j,t}^B.$$

The marginal value of the banker's net worth A^B is then:

$$A^B = \mu_t^s \phi_t + \nu_{d,j,t}.$$

μ_t^s is the stochastic spread between the loan and the deposit rates, ϕ_t is the maximum leverage and $\nu_{d,j,t}$ is the marginal loss from deposits.

Proof. See appendix B.

The proposition clarifies the role of the bank's net worth in the model. We can rewrite the incentive constraint using the linearity of the value function as

$$\frac{A^B}{\theta} \geq \frac{[Q_t s_{j,t}^B + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B]}{n_{j,t}^B}. \quad (19)$$

The adjusted leverage of a banker cannot be greater than A^B/θ . The right hand side shows that as the net worth of the banker decreases the constraint is more likely to bind. Proposition 1 also implies that even there is heterogeneity in the bankers' holdings and net worth, this does not affect aggregate dynamics. Hence, the transition from

the individual to aggregate variables takes place in the same way as in the previous section.

The maximum adjusted leverage ratio of the bank is defined as

$$\phi_{j,t} = \frac{\nu_{d,j,t}}{\theta - \mu_t^s}. \quad (20)$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits $\nu_{d,j,t}$ and reserves and on the excess value of bank assets μ_t^s . As the credit spread increases, banks franchise value V_t increases and the probability of a bank to divert its funds declines. From the other hand as the proportion of assets that a bank can divert, θ increases, the constraint binds more.

Aggregation. Let S_t^B be the total quantity of loans that banks intermediate, B_t^B the total number of government bonds they hold, M_t^B the total quantity of reserves and N_t their total net worth. Furthermore, by definition, total deposits acquired by the households D_t^H are equal with the total deposits of the banking sector. Using capital letters for the aggregate variables, the banks' aggregate balance sheet becomes

$$Q_t S_t^B + q_t B_t^B + M_t^B = N_t + D_t^H. \quad (21)$$

Since the leverage ratio (20) does not depend on factors associated with individual banks' characteristics we can sum up across banks and get the aggregate bank constraint in terms of the total net worth in the economy:

$$Q_t S_t^B + \Delta q_t B_t^B + \omega M_t^B = \phi_t N_t. \quad (22)$$

The above equation gives the overall demand for loans $Q_t S_t$. When the incentive constraint is binding, the demand for assets is constrained by the net worth of the bank adjusted by the leverage. We can get some intuition here for what changes in the bank's constraint during the QE. No matter the security the central bank purchases, since their weights are higher than the weight of reserves ($1 > \Delta > \omega$), the exchange of securities with reserves relaxes the constraint and stimulates lending to the non-financial sector.

Aggregate net worth is the sum of the new bankers' and the existing bankers' equity: $N_{t+1} = N_{y,t+1} + N_{o,t+1}$. Young bankers' net worth is the earnings from loans multiplied by ξ_B which is the fraction of asset gains that being transferred from households to the new bankers

$$N_{y,t+1} = \xi [R_{k,t} Q_{t-1} S_{t-1}^B + R_{b,t} q_{t-1} B_{t-1}^B + R_{m,t} M_{t-1}^B]$$

and the net worth of the old is the probability of survival for an existing banker multiplied by the net earnings from assets and liabilities

$$N_{o,t+1} = \sigma [R_{k,t} Q_{t-1} S_{t-1}^B + R_{b,t} q_{t-1} B_{t-1}^B + R_{m,t} M_{t-1}^B - R_t D_t^H].$$

2.4. Central Bank, Asset Purchases and the Treasury

Central Bank. Central bank uses two policy tools. Firstly, it adjusts the policy rate according to the Taylor rule specified momentarily. Secondly, it can engage in risky asset purchases from households and banks. When balance sheet constraints are tight, excess returns rise. Central bank purchases relax the incentive constraint of the banks and increase aggregate demand thus driving up asset prices.¹⁰

Under a QE operation, the central bank buys securities from banks and households. These can be either private assets S_t^G or bonds B_t^G . It does that by paying the assets purchased by their respective price Q_t and q_t . To finance those purchases it creates electronically reserves M_t that pay back purchases from households and banks:

$$Q_t S_t^G + q_t B_t^G = M_t.$$

It is assumed that the central bank turns over any profits to the treasury and receives transfers to cover any losses. The central bank's budget constraint is:

$$T_t^{CB} + R_t M_{t-1} + Q_t S_t^G + q_t B_t^G = R_{b,t} q_{t-1} B_{t-1}^G + R_{s,t} Q_{t-1} S_{t-1}^G + M_t \quad (23)$$

where T_t^{CB} are transfers of the central bank to the treasury.

Monetary policy is also characterised by a simple Taylor rule. It sets the nominal interest rate i_t such as to respond to deviations of inflation and output from its flexible price equilibrium level Y^* :

$$i_t = i + \kappa_\pi \pi + \kappa_y (Y - Y^*) + \epsilon_{m,t},$$

where i is the steady state level of the nominal interest rate and $\epsilon_{m,t}$ an exogenous monetary policy shock. The relation between nominal and real interest rates is given by the Fisher equation:

$$1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t}.$$

With the addition of the central bank in the model, three agents can hold assets or bonds: Optimizing households, banks and the central bank. The total quantity of loans therefore is decomposed as:

$$S_t = S_t^B + S_t^H + S_t^G \quad (24)$$

and for the bonds:

$$B_t = B_t^B + B_t^H + B_t^G. \quad (25)$$

If we combine these identities and insert them to the balance sheet constraint of the banks we have:

$$Q_t S_t \leq \phi N_t + Q_t S_t^H + Q_t S_t^G + \Delta(q_t B_t^G + q_t B_t^H - q_t B_t) \quad (26)$$

The above constraint implies that when government purchases either loans or bonds it

¹⁰See [Araújo, Schommer, and Woodford \(2015\)](#) for a same intuition under a different setting.

relaxes the balance sheet constraint of the banking sector. This can, in financial stress periods, reduce the excess returns and stimulate the economy. When this constraint does not bind and the inequality holds, asset or bond purchases by the government are neutral. This happens due to frictionless arbitrage that characterizes the economy when the banks has no binding constraint. Wallace (1981) in his seminal paper has make use of that assumption for the neutrality theorem of the open market operations.

Equation (26) gives another insight on the asset purchase mechanism. Buying loans or bonds does not have the same impact to the loosening of the banks' balance sheet constraint. In fact, since loans have an absconding fraction of 100%, purchases of loans by the central bank relaxes the constraint more than the purchase of bonds with a coefficient $\Delta < 100\%$. Intuitively, the central bank acquiring government bonds frees up less bank capital than does the acquisition of a similar amount of private loans.

It is now easier to understand when the irrelevance theorem holds. Since the government creates as many reserves as the value of the assets purchased ($M_t = q_t B_t^G + Q_t S_t^G$), then in the case of frictionless arbitrage between the existing assets ($R_{s,t} = R_{b,t} = R_t$), the market operations are indeed irrelevant. But since the financial frictions included in the model disrupt the frictionless arbitrage, asset purchases have effect on the real economy.

The share of the total assets that is purchased by the government follows a second order stochastic process.¹¹ Specifically,

$$S_t^G = \phi_{s,t} S_t,$$

$$B_t^G = \phi_{b,t} B_t.$$

Treasury. The treasury collects lump sum taxes $T_t = \lambda T_t^r + (1 - \lambda) T_t^o$ to finance its public expenditures which are fixed relative to output, $\bar{G} = \gamma^G Y^{ss}$. It also targets a constant real level of long-term debt, denoted by \bar{B} . It collects taxes at rate t_{pr} from non-financial firms' profits and redistribute them back to the hand to mouth households, $T_t^r = t_{pr} Profit_t$.

Treasury's budget constraint is:

$$\bar{G} + q_{t-1} R_{b,t} \bar{B} = q_t \bar{B} + T_t + T_t^{CB}. \quad (27)$$

Proposition 2. Fiscal policy matters only through the impact of taxes (transfers) on hand to mouth agents. Therefore, the only fiscal variables needs to be defined is the Hand to Mouth transfers (or taxes).

Proof. I make use of the optimizers budget constraint (2), the bank's -owned by optimizing agents- balance sheet (14), the taxes aggregator and the treasury and central bank's budget constraints (23), (27). Substituting the latter four equations in the optimizers'

¹¹As is shown in the calibration section, an AR(2) is the best way to simulate the ECB's Asset Purchase Program schedule.

budget constraint and using the financial variables aggregator, the aggregate resource constraint yields:

$$C_t^R + \frac{\bar{G}}{1 - \lambda} - \frac{\lambda}{1 - \lambda} T_t^K + adj\{B, S\} = W_t L_t^R. \quad (28)$$

Where $adj\{B, S\}$ are the adjustment costs for bonds and shares that households have to pay, defined in (2).

Taxes on optimizers and any short of government bond decision do not matter for the allocation. \square

2.5. Non-Financial Firms and Nominal Price Rigidities

The non-financial firms are separated into three types: intermediate, final goods firms (retailers) and capital goods producers. To allow for nominal price rigidities, I assume that the differentiated intermediate goods i produced by a continuum of monopolistically competitive intermediate goods firms are subject to Calvo price stickiness.

The final output composite is a CES composite of all indeterminate goods i : $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}$ where ζ denotes the elasticity of substitution across intermediate goods. Each period there is a fixed probability $1 - \gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p . The goods are then sold and used as inputs by a perfectly competitive firm producing the final good. Finally, the capital goods producers create new capital under investment adjustment costs and sell it to goods producers at a price Q_t . The non-financial sector problem is described in detail in Appendix C.

Capital stock evolves according to the law of motion of capital

$$K_{t+1} = I_t + (1 - \delta)K_t. \quad (29)$$

The intermediate good $i \in [0, 1]$ is produced by a monopolist who uses a constant returns to scale production function combining capital and labour:

$$Y_t(i) = A_t K_t(i)^\alpha L_t(i)^{1-\alpha}. \quad (30)$$

A_t is the total factor productivity. It finances its capital needs each period by obtaining funds from banks and households. To acquire the funds to buy capital, the firm issues $S_t(i)$ claims equal to the number of units of capital acquired $K_{t+1}(i)$ and prices each claim at the price of a unit of capital $Q_t(i)$. Then by arbitrage: $Q_t(i)S_t(i) = Q_t(i)K_{t+1}(i)$. The funds acquisition between goods firms and its lenders is under no friction. Firm's lenders can perfectly monitor the firms and there is perfect information.

Resource Constraint. Final output may be either transformed into consumption

good, invested or used by the government for government spending:

$$Y_t = C_t + I_t[1 + \tilde{f}\left(\frac{I_t}{I_{t-1}}\right)] + G.$$

3. Quantitative Analysis

In this section I present the model's calibration and the first set of results of the paper: the impact of the quantitative easing on inequality.

3.1. Calibration

The model's calibration is performed in order to match Euro Area stylized facts and is divided in conventional and banking parameters. It follows broadly the calibration of the updated version of the New Area-Wide Model (NAWM), ([Christoffel, Coenen, and Warne \(2008\)](#), [Coenen, Karadi, Schmidt, and Warne \(2018\)](#)), the DSGE model of the ECB. Parameters in the NAWM are estimated by the use of Bayesian methods in the time span of 1985Q1-2014Q4 using times series for 18 macroeconomic variables which feature prominently in the ECB/Eurosystem staff projections. One period in the model is one quarter. All the calibrated values are presented in Table 1.

Financial parameter values are chosen in order to match specific Euro Area banking characteristics namely the banks' average leverage, lending spread and planning horizon. There are three parameters that characterise the behaviour of the financial sector in the model. This is the absconding rate θ , the fraction of entering bankers initial capital fund ξ_B , and the steady-state value of the survival rate, σ_B . I calibrate these parameters to match certain steady-state moments following the moments reported in [Coenen et al. \(2018\)](#). The steady-state leverage of the banks is set equal to 6, which corresponds to the average asset-over-equity ratio of monetary and other financial institutions as well as non-financial corporations, with weights equal to their share of assets in total assets between 1999Q1 and 2014Q4 according to the euro area sectoral accounts. Second, the steady-state spread of the lending rate over the risk-free rate, $R_t^k - R_t$ is set to 2.17 percentage points at the steady state, which is the average spread between the long-term cost of private-sector borrowing and the EONIA rate from 2003Q1 to 2014Q4. The banks planning horizon is set equal to 5 years. These parameters are also in line with the related studies in the literature. Finally I set the fraction of bonds that can be absconded Δ to 50% targeting a steady state bond spread half to the lending spread. The absconding rate of reserves ω is set to zero. Since reserves are in essence central bank money it is assumed that the central bank has full control on them.

Regarding the bond market, the long term target of the real bonds supply by the treasury equals 70% of GDP. The fraction of long-term bonds held by banks is 25% which is consistent with the sovereign debt holdings of the banking sector according to EA data. This leaves the rest 75% of the bond holdings to the optimizers' portfolio. The fraction of shares held by optimizing households is 50%.

Parameters	Value	Definition
Households		
β	0.998	Discount rate
χ	4.152	Relative utility weight of labour
λ	0.20	Share of rule of thumb agents
ϵ	2	Inverse Frisch elasticity of labour supply
\bar{S}^R/S	0.500	Proportion of shares of the optimizers
\bar{B}^R/B	0.750	Proportion of bond holdings of the optimizers
κ	1	Portfolio adjustment cost parameter
Banks		
θ	0.20	Absconding rate
Δ	0.5	Absconding fraction for bonds
ω	0	Absconding fraction for reserves
ξ_B	0.0014	Entering bankers initial capital
σ_B	0.950	Bankers' survival rate
Intermediate and Capital Goods Firms		
δ	0.025	Depreciation of capital
α	0.36	Capital share
η	5.77	Inverse elasticity of net investment to the price of capital
Wage and Price Setting		
ζ	4.340	Elasticity of labour substitution
ξ_w	0.890	Probability of keeping the price constant
γ_w	0.417	Wage Indexation parameter
ζ	2.540	Elasticity of substitution between goods
γ	0.720	Probability of keeping the wages constant
γ_p	0.480	Indexation parameter
Treasury Policy		
γ^G	0.20	Steady state fraction of government expenditures to output
t_{pr}	0%-40%	Optimizers' profit tax rate
Monetary Policy		
κ_π	1.860	Inflation coefficient in the Taylor rule
κ_y	0.147	Output gap coefficient in the Taylor rule
ρ_m	0.860	Interest-rate smoothing
ρ_1	1.700	First AR coefficient of the bond purchase shock
ρ_2	-0.730	Second AR coefficient of the bond purchase shock
ψ	0.015	Initial asset purchase shock

Table 1: Parameter Values

The values for the share of capital α and the depreciation rate δ are chosen to 0.36 and 0.025 respectively following the estimation results of [Christoffel et al. \(2008\)](#). Similarly, the value of β is assigned to 0.998, chosen to be consistent with an annualised equilibrium real interest rate of 2%. The relative utility weight of labour χ is chosen to ensure a level of labour close to 1/3 in steady state, a fairly common benchmark in the literature (see [Corsetti, Kuester, Meier, and Müller \(2014\)](#)). The parameter of the inverse Frisch elasticity of labour supply ϵ is one difficult to identify. In the NAWM, this parameter is not estimated and calibrated ad-hoc to 2 which is the one I employ here as well. ϵ has a crucial role on the IADL results of the paper. I provide additional robustness checks in the Appendix G for a range of ϵ starting from 0.5 to 2. Results of the paper hold for all these values.

The elasticity of substitution between goods ζ and the capital adjustment costs also follow the NAWM and set to 2.54 and 5.77 respectively. The same holds for the wage setting parameters. The government spending as a fraction of the GDP is set to 20% also following other studies for the Euro Area. Retail firms parameters: the elasticity of substitution between goods, the Calvo probability and the price indexation parameter are set to the value estimated in the NAWM. The same holds for the monetary policy parameters: the inflation and output gap coefficients in the Taylor rule and the interest rate smoothing parameter.

The share of rule of thumb consumers is chosen to be $\lambda = 0.20$. Using the data from the Eurosystem Household Finance and Consumption Survey, as explained in Section 1, almost the bottom 20% of the Euro Area households hold essentially no net worth at all. This is also in line with the estimates of [Slacalek et al. \(2020\)](#). The same value is also used by a similar study for the EA with LAMP [Hohberger, Priftis, and Vogel \(2019b\)](#). The profits' tax rate used in the IADL results of the paper takes values from 0% to 40% in the exercises performed.¹²

The bond purchase shock is modelled as an AR(2) process.¹³ The AR(2) process in contrast with an AR(1) captures the expectation of the further expansion of central bank purchases in the future, which is the case in the ECB's APP started in 2015Q1. The history of APP net asset purchases is shown in Appendix E. Purchases for the first year are constant to 60 billion euro, then in 2016 increase to 80 billion for four quarters to eventually go back to 60 billion and fade out. Relative to 2015 GDP purchases increase from a 2% to almost 4% at their peak. To illustrate this pattern, the first AR coefficient is chosen to 1.700 and the second being -0.730 while the initial shock is chosen to 0.015. The choice of the monetary policy shock initial increase is chosen such that the GDP by the a magnitude similar to the one of the QE shock.

¹²Results remain qualitatively similar under any reasonable tax rate.

¹³This follows similar studies that conclude that the ECB's QE program is characterised by a AR(2) process (see [Andrade, Breckenfelder, De Fiore, Karadi, and Tristani \(2016\)](#), [Hohberger et al. \(2019b\)](#), [Carlstrom, Fuerst, and Paustian \(2017\)](#)).

3.2. Impulse Response Analysis

I proceed with a quantitative exercise on identifying i) what was the impact of the ECB's APP programme on the macroeconomy, ii) its impact on consumption and income inequality and iii) what's the difference with an accommodative monetary policy shock, assuming that the economy is not at the effective lower bound. I present the results of the model with sticky wages. For high levels of asset markets participation, as it occurs in the calibrated model, the two specifications offer qualitatively similar results. The model is solved non-linearly following [Lindé and Trabandt \(2019\)](#).

Central Bank Bond Purchases and Conventional Monetary Policy. How a bond purchasing programme similar to the APP and an expansionary monetary policy shock affect main macro variables? This is shown in Figure 3. The bond purchase shock is following the APP programme of the ECB. The monetary policy shock is set such that to produce a same increase in output of about 2.9%. In the case of the QE shock I set the nominal interest rate to remain constant for the first four quarters.¹⁴ In bold lines, the responses of a bond shock reflect the responses of a conventional interest rate reduction.

Bond purchases stimulate the economy and increase output as Figure 3 shows. The current calibration of the rule of thumb agents' measure to the EA average ($\lambda = 0.20$) leads to the case that both MP and QE shock increase aggregate demand. The main mechanism works through the loosening of the banks' constraint. Central bank intermediation increases asset prices Q_t and this leads to an increase in banks' valuation (net worth). Standard financial accelerator effects lead to a further increase of capital price and an economic upturn. An increase in the bonds' prices drives banks to buy more assets which leads to an increase in assets' prices. Excess returns reduce for both securities. The economic upturn is also affecting the real economy due to the higher demand for employment and wage increases. The responses due to the QE shock and those of the MP shock are at least qualitatively identical. For a policy rate reduction to produce the same effect of the APP programme, a 30 basis points reduction is needed assuming that the interest rate is not in its effective lower bound.

Income and Consumption Inequality. I move to the decomposition of income and consumption responses between the two agents in the economy. Figure 4 shows the responses of those variables after the same two shocks defined above. Both agents consumption increases. Rule of thumb agents' consumption strictly follows the real wage path which after both shocks goes up due to more demand for labour. Notice that were the nominal interest not constant for the first four periods, consumption of the optimizing agents would have been decreasing. This is because after a stimulating bond shock, the Taylor rule dictates the interest rate to raise. Through the standard intertemporal substitution mechanism the optimizing agents would have lowered their consumption. Consumption inequality defined as C_t^R/C_t decreases. This is in line with the well established fact that hand to mouth consumers have a higher marginal propensity to consume than the financially unconstrained agents ([Auclert \(2017\)](#), [Kaplan et al.](#)

¹⁴ECB after the initiation of its APP programme in 2015Q1 kept its main refinancing operations interest rate constant for a year.

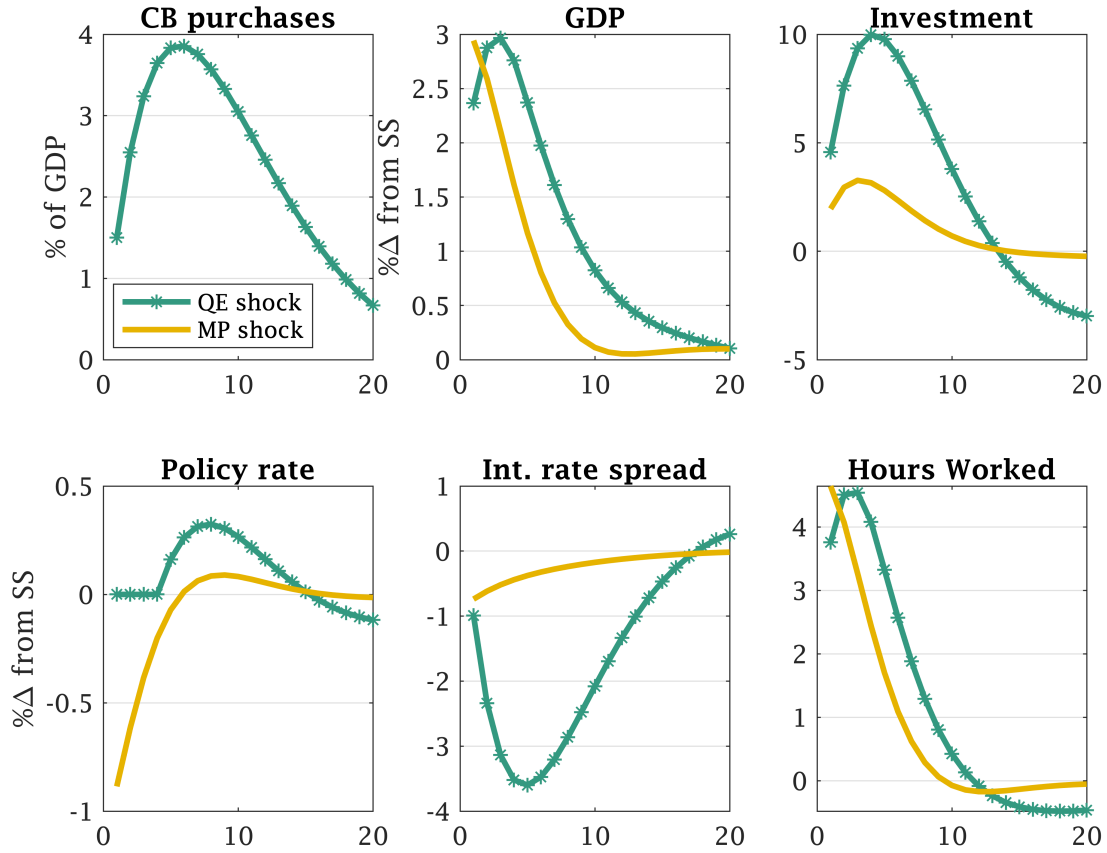


Fig. 3. Government Asset Purchase Shock and Conventional Monetary Policy Shock

(2018) among others).

Turning to the income responses of the two agents, depicted at the second row of Figure 4, optimizers' income decline. After a QE shock, optimizers reduce their bond holdings as their demand function for bonds shows (5). Optimizers hold a positive frictionless value of bonds and reduce their holdings as long as excess returns drop. This has a negative impact on their balance sheet since they lose from the interest rate differential and also from the risk free rate reduction after both shocks. Due to the exchange of bonds to reserves, the income reduction is much more amplified during the QE shock. Rule of thumb agents' income follows their labour wage which grows after both shocks. Consequently, income inequality drops for both accommodative policies but is much more amplified in the QE case due to the returns loss.

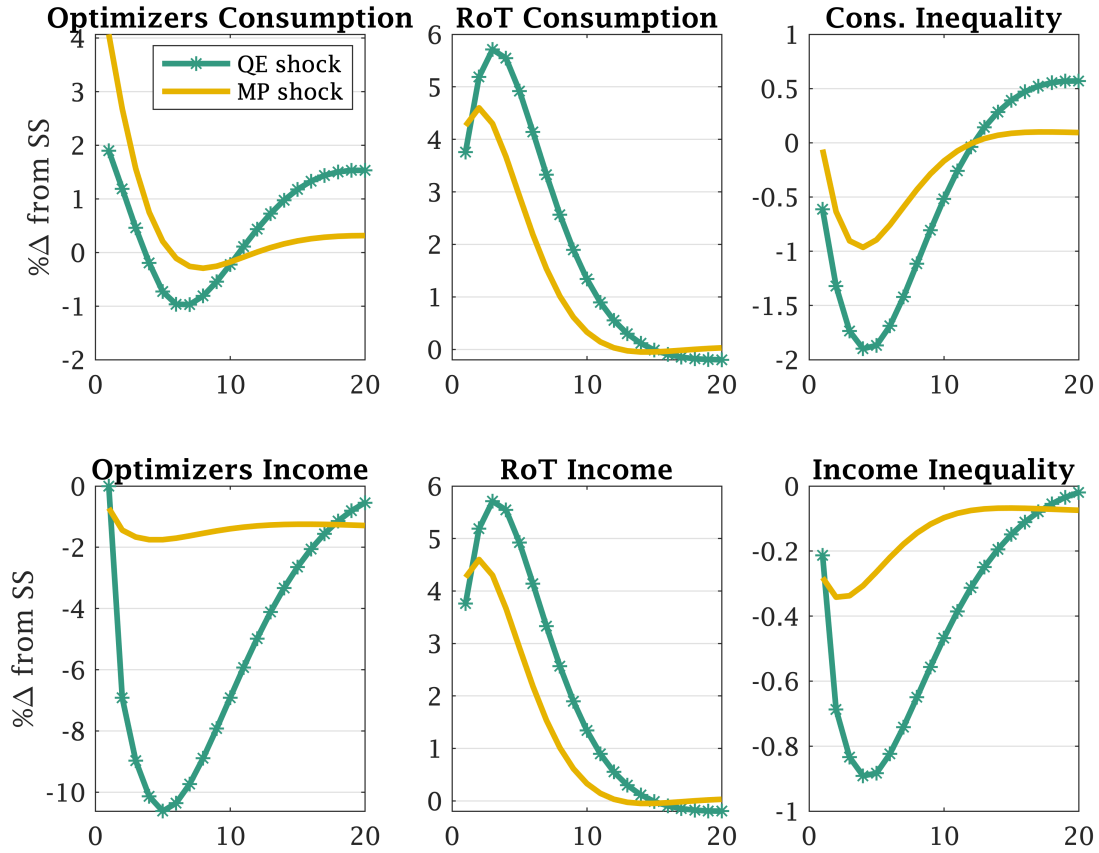


Fig. 4. Government Asset Purchase Shock and Conventional Monetary Policy Shock: Inequality

4. Unconventional Monetary Policy and Assets Market Participation

In the present section, I examine analytically and quantitatively the existence of the Inverted Aggregate Demand Logic (IADL) for the case of i) a conventional accommodative monetary policy shock and ii) a quantitative easing shock. IADL¹⁵ is the region where the accommodative monetary policy, when limited asset markets participation is low, can have contractionary effects instead of stimulating aggregate demand.¹⁶ I perform this exercise for the case of perfect labour market and also when wages are sticky. This is done under two different taxation schemes. Firstly, under a no redistribution scheme: transfers to rule of thumb agents are zero; secondly under a redistribu-

¹⁵Borrowing the term from Bilbiie (2008).

¹⁶A key depart from Bilbiie's work is that the present model includes capital.

tive scheme: rule of thumb agents get a proportion of the firms' profits as a lump-sum transfer.

When wages are flexible, QE can be contractionary for low levels of asset market participation, while when wages are sticky this result is muted. The contractionary effects can be avoided by fiscal redistribution of a portion of profits from the firm owners to the hand to mouth consumers. I provide analytical and numerical solutions for the first part of the analysis without transfers while for the case where transfers are on I show only the quantitative results since the analysis becomes substantially more complex.

4.1. No-Redistribution Scheme

For the first part of the analysis, I provide analytical expressions that show the *direct effect* of interest rate reduction and quantitative easing on output. Then, I show the fraction of constrained agents that pushes the model into the IADL area in both cases, that is making the total effect of the two policies contractionary. To pursue this, due to the high dimensionality of the model, I solve the model numerically.

In order to derive analytical results I make the following, not distorting, assumptions: Consumption and hours worked are equal among all the members in steady state.¹⁷ Therefore in steady state: $L = L^r = L^o$ and $C = C^r = C^o$. The first assumption can be implemented by a particular choice of χ , whereas the second by introducing a tax level that make optimizers' consumption equal to rule of thumb agents'. Furthermore, due to no-redistribution, I assume that rule of thumb agents taxation is zero: $T_t^r = 0$. Under these assumptions, we can express the consumption and labour aggregators (11), (12) as $l_t = \lambda l_t^r + (1 - \lambda)l_t^o$ and $c_t = \lambda c_t^r + (1 - \lambda)c_t^o$ respectively, where lower case letters denote log deviations from the non-stochastic steady state.

The optimality condition (8) without including any tax (or transfer) rule dictates that the labour supply of the rule of thumb agents in levels is always constant, therefore $l_t^r = 0$. The labour consumption optimality conditions are in log-linear terms: $c_t^r = w_t + l_t^r$ and $c_t^o = w_t - \epsilon l_t^o$. Using the aggregate consumption, labour consumption optimal choices, and the hours worked aggregator we get:¹⁸

$$w_t = c_t + \epsilon l_t. \quad (31)$$

Note that the above relation holds for both labour market settings, given that both agents have equal consumption and work the same hours in steady state. Substituting (31) in the labour optimality condition of the optimizing agents:

$$c_t^o = c_t - \epsilon \left(\frac{\lambda}{1 - \lambda} \right) l_t. \quad (32)$$

Trivially with no hand to mouth consumers $\lambda = 0$, c_t^o follow the aggregate consump-

¹⁷The latter holds without any further arrangement for the centralised wage setting market where firms choose uniformly the labour required given the wage set by the unions.

¹⁸The derivations of the main equations of this chapter are presented in Appendix F

tion schedule. Introducing limited asset market participation in the model makes optimizers' consumption reacting negatively to an increase of the *aggregate* employment. This is due to wage being the rule of thumb agents' only source of income.

Doing the same exercise for the rule of thumb agents:

$$c_t^r = c_t + \epsilon l_t.$$

Rule of thumb agents' consumption schedule reacts positively in changes of aggregate consumption and employment with elasticity ϵ . Having the above relations in hand I proceed with the derivation of the aggregate Euler equation.

The log-linearised versions of the production function and resource constrain are $y_t = \alpha k_t + (1 - \alpha)l_t$ and $y_t = c_t s_c + i_t s_i + s_g$ respectively. Inserting both equations in the optimizing agents' consumption function (32) and substituting the result to the optimizers' Euler equation $c_t^o = \mathbb{E}_t\{c_{t+1}^o\} + [\mathbb{E}_t\{\pi_{t+1}\} - r_t]$ we arrive to the aggregate Euler equation or IS curve:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\delta}[r_t - \mathbb{E}_t\{\pi_{t+1}\}] - \frac{1}{\delta} \frac{s_i}{s_c} \Delta i_{t+1} + \frac{1}{\delta} \frac{\epsilon \lambda}{(1 - \lambda)(1 - \alpha)} [\alpha \Delta k_{t+1}]. \quad (33)$$

where

$$\delta = \frac{1}{s_c} - \epsilon \frac{\lambda}{(1 - \lambda)(1 - \alpha)}$$

and $s_c = C^{ss}/Y^{ss}$, $s_i = I^{ss}/Y^{ss}$, $s_g = G^{ss}/Y^{ss}$.

Profits.— Profits play a crucial role in the analysis. As it will be shown below is the primary reason of the IADL existence. Profits form non-financial corporations are given by $Prof_t = Y_t - W_t L_t - Z_t K_t$. Log-linearising it around the steady state (with $d_t = \ln((Prof_t - Prof)/Y)$) we get:

$$d_t = y_t - (w_t + l_t) - (z_t + k_t). \quad (34)$$

Profits move countercyclically in response to demand shocks, a standard feature of the NK models.¹⁹

4.1.1. Conventional Monetary Policy

The aggregate IS curve derived above, shows that the elasticity of aggregate demand to interest rates depends on whether we assume a representative agent specification or a LAMP setting. Specifically, the elasticity is s_c in the case of a representative agent model ($\lambda = 0$), and becomes $-1/\delta$ when LAMP is assumed. Solving for $\delta = 0$ we can find the threshold fraction of the rule of thumb agents λ^* that make the impact of the *direct effect* of an interest rate reduction ineffective:

$$\lambda^* = \frac{1 - \alpha}{1 - \alpha + \epsilon s_c}. \quad (35)$$

¹⁹This is also shown by Bilbiie (2019) in a model without capital and government sector.

Beyond this threshold level, a further reduction of the interest rate will have contractionary effects and this will be the region where the parameter δ changes sign.

For a low λ below the threshold value or equivalently when financial participation is high, output reacts inversely to real interest rate changes. As we move to higher values of λ this effect is becoming stronger. When $\lambda > \lambda^*$, and the fraction of hand to mouth consumers is big enough, δ becomes negative and distorts the well known stimulating effect of accommodative monetary policy using the policy rate. In that region lower interest rates restrain aggregate demand and we enter the Inverse Aggregate Demand Logic region. Finally as λ reaches its upper bound of 1 where no agent hold assets, $1/\delta$ decreases towards zero; the interest rate as a monetary policy tool becomes irrelevant.

Feeding the model with the parameter values from the model's calibration shown in Section 3.1, I show the *total* impact effect of a conventional interest rate reduction to main macro variables as a function of rule of thumb agents, where $\lambda \in [0, 0.9]$. The top chart of Figure 5 shows the *total* impact effect on aggregate output to a conventional accommodative monetary policy shock conditional on different fractions of rule of thumb agents. The bottom part of Figure 5 shows the *total* impact on profits. I show this for two cases: perfect labour market and imperfect labour markets (the sticky wages case). This distinction is important, as I will explain momentarily, the wage stickiness neutralises the countercyclical behaviour of profits which is the main factor that drags down aggregate demand.

Competitive labour markets. As the rule of thumb fraction increases this shifts the value of output upwards. This continues up to a point where aggregate demand reaches its maximum. When λ is over the threshold of $\lambda^* = 0.57$, then the reduction of the nominal interest rate has the opposite effect on the aggregate variables; expansionary monetary policy generates contractionary effects. As λ reaches its upper limit, and agents cannot have intertemporal decisions, monetary policy becomes ineffective. Under the baseline calibration, the *direct effect* of the interest rate reduction presented analytically in equation (35) yields a threshold value of λ^* of 0.52 which is fairly close to the *total effect* threshold shown by solving the model numerically.

To understand the reasoning behind the IADL it is useful to first focus on the region where there is restricted limited participation: $\lambda < \lambda^*$. A reduction in interest rates leads to an increase in aggregate demand. Wage increases from the intertemporal substitution of asset holders and this wage increase translates to a further increase in demand, since non-asset holders consume their wage income (assuming no transfers). This generates a shift in labour demand upwards. As Figure 5 shows this effect is not constant across the domain of λ values. To understand why this is the case is important to focus on the role of profits. Profits as shown above analytically and in the bottom panel of Figure 5 are countercyclical. Consequently, as the asset market participation lowers the less the negative consequences of the profits experienced by the majority of population, the non-asset holders. Therefore, as λ increases and until it reaches λ^* aggregate demand increases continuously. The countercyclicity of profits will induce aggregate demand to drop and there is a new equilibrium with lower output, consumption and wages. Finally, reaching the end of the λ domain, at $\lambda = 0.9$ almost no agent holds assets and the interest rate policy is ineffective.

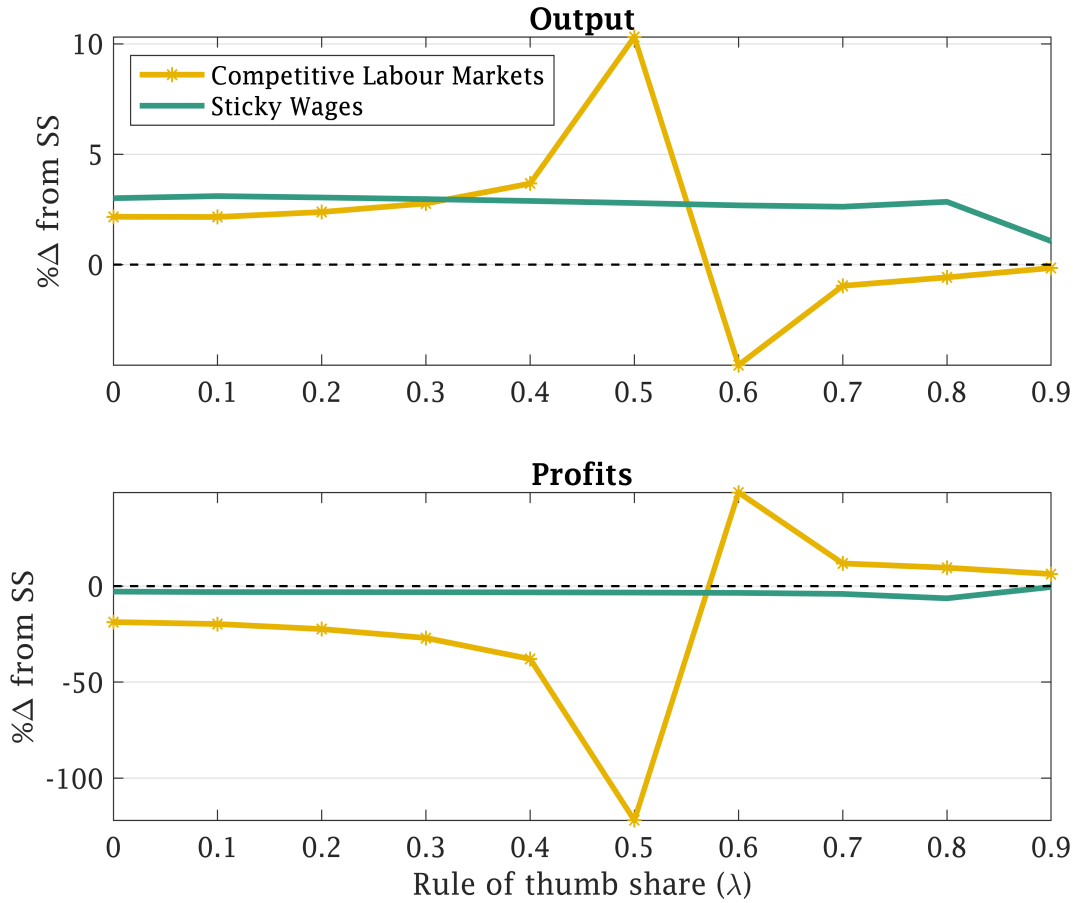


Fig. 5. Impact Effect Conditional on Asset Market Participation: Monetary Policy Shock

Sticky Wages. When we introduce labour unions that set the wages, results change. After an accommodative monetary policy shock of the same magnitude as before, we see that for all levels of asset market participation the impact effect of output does never turn negative. The introduction of sticky wages manages to keep marginal costs stable and therefore the impact effect of profits is still countercyclical but of a much smaller magnitude. Consequently, profits no longer drag aggregate demand down and output's response is always positive for the λ domain.

4.1.2. Quantitative Easing

In the same spirit with the contractionary effects of a conventional policy rate reduction, I show that a quantitative easing programme can have adverse effects in a LAMP setting. I look at this again for both labour market setting. The bond buying programme in the present setting is an one time increase in the government bond holdings and a simultaneous reduction of the holdings of banks and households. Finding the *direct*

effect of QE on output is a more tedious process than the one of the monetary policy interest rate change since QE is not present in the IS equation (33).

A way to introduce government bonds is through capital. From the capital market clearing (24) we have $K_t = K_t^B + K_t^H + K_t^G$. Log-linearising it around the steady-state yields:

$$k_t = s_k^H k_t^H + s_k^B k_t^B + s_k^G k_t^G, \quad (36)$$

where $s_k^H = K^H/K$, $s_k^B = K^B/K$, $s_k^G = K^G/K$. Log-linearising the aggregate incentive constraint of the bank around the steady state:

$$QS^B(\hat{Q}_t + k_t^B) + \Delta q B^B(\hat{q}_t + b_t^B) = \phi N(\hat{\phi} + n_t).$$

The small letters are the log-deviations of the variables from their steady state. \hat{Q}_t is the corresponding value for the price of capital and \hat{q}_t for the price of bonds. Solving for the bankers' capital holdings:

$$k_t^B = -\frac{\Delta q B^B}{QS^B} b_t^B - \frac{\Delta q B^B}{QS^B} \hat{q}_t + \frac{\phi N}{QS^B}(\hat{\phi} + n_t) - \hat{Q}_t. \quad (37)$$

Taking the log deviations of the capital market clearing (25) and solving for the banks' bond holdings:

$$b_t^B = -\frac{s_b^G}{s_b^B} b_t^G + \frac{b_t}{s_b^B} - \frac{s_b^H}{s_b^B} b_t^H, \quad (38)$$

where $s_b^H = B^H/B$, $s_b^B = B^B/B$, $s_b^G = B^G/B$.

Plugging (37),(38) into (36):

$$\begin{aligned} k_t = & s_k^H k_t^H + s_k^B \left[-\frac{\Delta q B^B}{QS^B} \left(-\frac{s_b^G}{s_b^B} b_t^G + \frac{b_t}{s_b^B} - \frac{s_b^H}{s_b^B} b_t^H \right) \right. \\ & \left. - \frac{\Delta q B^B}{QS^B} \hat{q}_t + \frac{\phi N}{QS^B}(\hat{\phi} + n_t) - \hat{Q}_t \right] + s_k^G k_t^G. \end{aligned} \quad (39)$$

Since we are interested on the direct effect of government bond purchases (assuming everything else remains constant) we are interested in

$$k_t = s_k^B \frac{\Delta q B^B}{QS^B} \frac{B^G}{B^B} b_t^G = \frac{\Delta B^G}{S} b_t^G. \quad (40)$$

The *direct effect* on output using the IS equation is:

$$-\frac{1}{\delta} \frac{\epsilon \lambda \alpha}{(1-\lambda)(1-\alpha)} \frac{\Delta B^G}{S} b_t^G. \quad (41)$$

Using the fact that $b_t^G = \frac{B_t^G - B^G}{B^G}$, $B^G = 0$, and after some algebra manipulation the above

equation becomes:

$$\frac{1}{\frac{(1-\lambda)(1-\alpha)K}{s_c \alpha \epsilon \lambda \Delta} - \frac{\epsilon \lambda K}{\alpha \epsilon \lambda \Delta}} B_t^G. \quad (42)$$

Setting the above expression equal to zero, we can find the threshold value λ^* that makes the direct effect of the quantitative easing policy ineffective. The result yields the same level of threshold with the conventional monetary policy case, $\lambda^* = 0.799$. Therefore, the value of λ that makes both the *direct* effect of quantitative easing and the interest rate reduction ineffective is equivalent.

In order to find the *total* impact effect of the QE, I proceed with the numerical solution of the model. The impact effects of the same macro variables shown in the previous exercise are presented in the top chart of Figure 6.

In the perfectly competitive labour market case, the total impact effect is positive and increasing as long as the asset market participation decreases. After the level of participation passes the threshold level λ^* , QE becomes contractionary. Nevertheless, the *total* impact effect of QE and MP shock is different and the threshold level of market participation λ^* that neutralizes the total effect of the two policies differs as well. The countercyclicality of profits, shown in the bottom chart of Figure 6, is also in this case the factor that produces the IADL.

Introducing sticky wages, as in the monetary policy shock case, neutralises the countercyclical role of profits. The impact effect of output is positive for most of the λ domain and it slightly turns negative when the asset market participation is too low, around 70%.

For robustness, I provide additional additional checks in the Appendix G for a range of ϵ starting from 0.5 to 2. Results of the paper hold for all these values.

4.2. Redistribution of Profits

We have seen that in the perfectly competitive labour market case, accommodative conventional and unconventional monetary policy can have negative effects. In this section I focus only in this labour market setting and provide results under the assumption that taxation is redistributive. That is, a percentage of the profits is allocated to the hand to mouth consumers who were entitled zero transfers under the baseline scenario examined before. What changes is as the rule of thumb consumers share of the profits increases, the IADL region shifts to the right. The negative effects of profits are shared between the two groups leading to a welfare increase.

Taxation is following a simple fiscal rule of redistribution of profits to the hand to mouth consumers defined as:

$$T_t^r = t_{pr} Profit_t. \quad (43)$$

I assume three different taxation parameter values: 0% (baseline scenario), 20% and 40%. It's important to note that this is an ad-hoc choice for the profit tax parameter values. Since the purpose of this exercise is to identify the changes when transfers to rule of thumb agents are non-zero, the choice of a data driven parameter is not crucial. Due to the complexity of the model I abstract from the analytical solution of this case

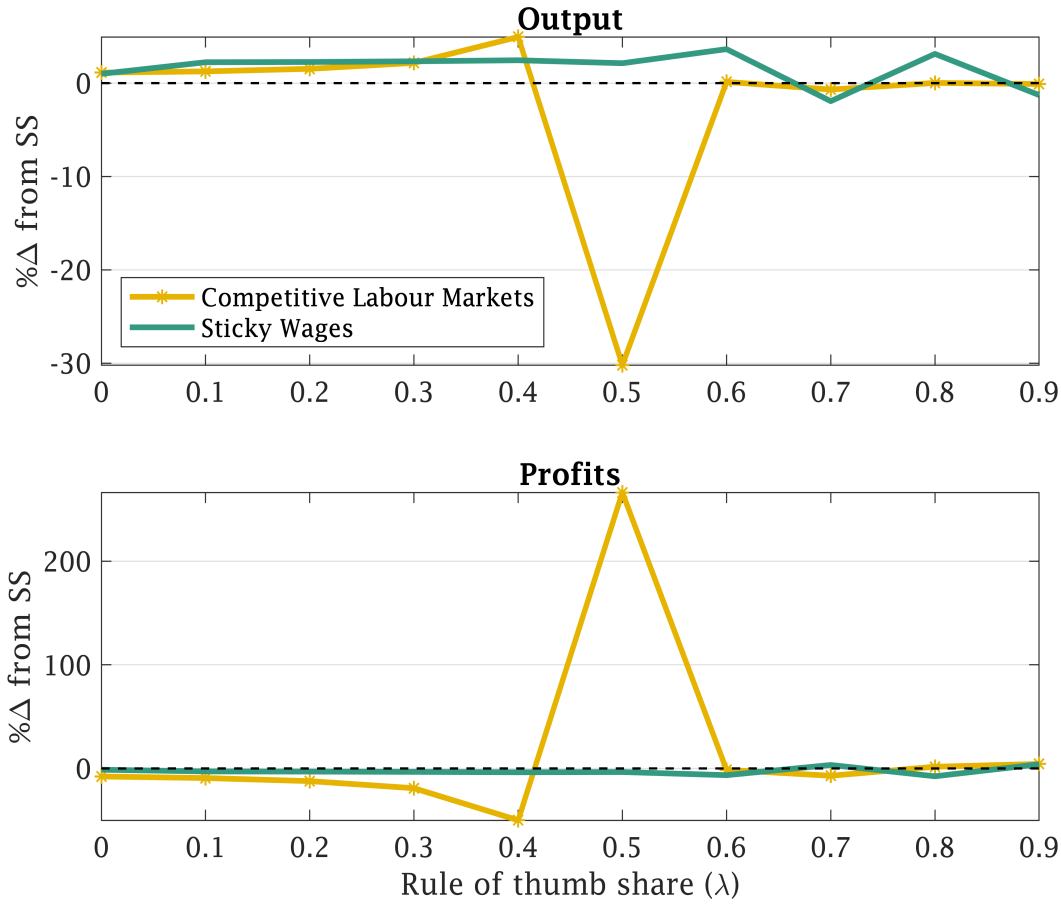


Fig. 6. Impact Effects Conditional on Asset Market Participation: QE Shock

and I show numerically what is the *total* impact effect of a monetary policy shock and a QE shock to output. To show the counter- or procyclicality of both policies under the taxation regime I focus on the impact effect of both policies on output.

Figure 7 shows the paths of the impact effect of output after both a conventional accommodative monetary policy shock (on the top panel) and a bond purchase shock (on the bottom panel). Both impact effects are plotted as a function of λ . Shocks follow the process specified in the calibration section. The yellow line corresponds to the baseline scenario of no redistribution, while the green line to a tax rate of 20% and the cyan line to a tax rate of 40%. What changes in comparison with the no redistribution case is that as the tax rate increases, the threshold of λ that makes both monetary policy tools contractionary shifts to the right. At the same time, the impact effect of output is milder for both cases of fiscal redistribution compared to the benchmark for reasonable values of λ (up to 0.7).²⁰

²⁰Note that the impact effect is plotted until $\lambda = 0.90$ since the analysis is restricted to the range of λ

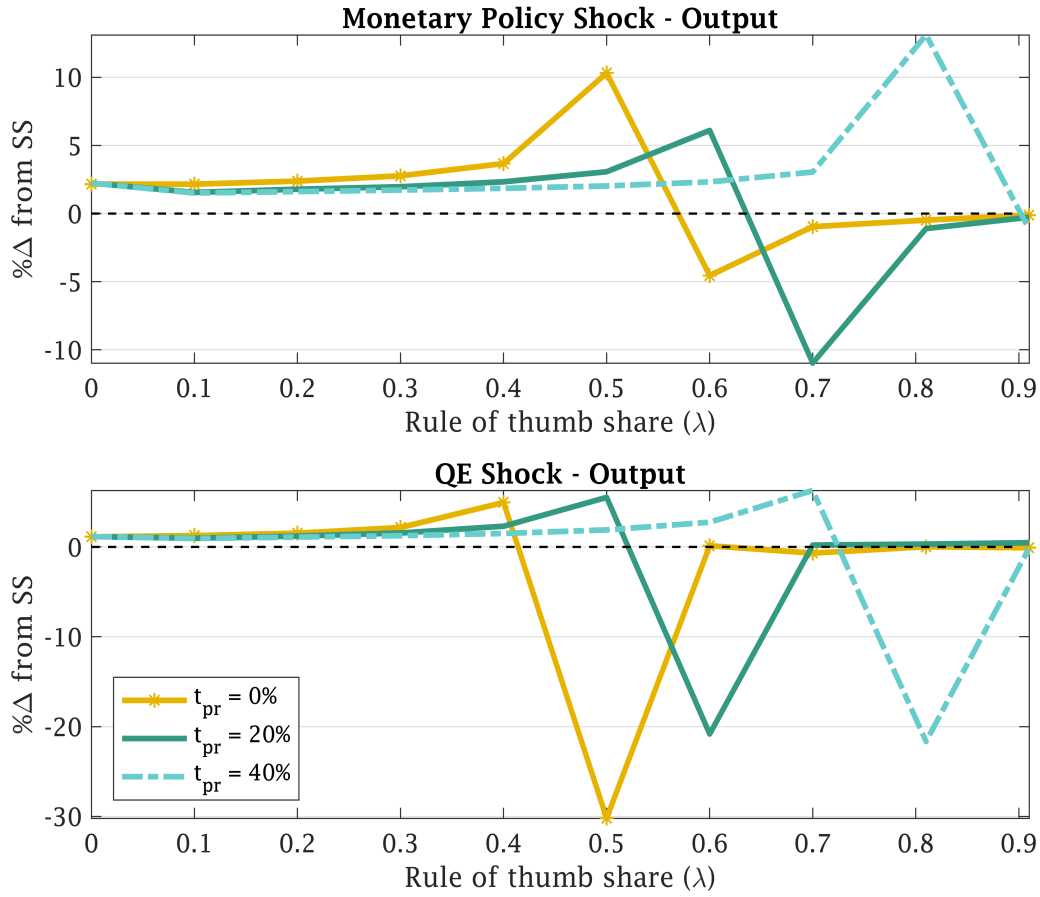


Fig. 7. Inverted Aggregate Demand Logic (Profit Redistribution)

Under fiscal redistribution, the rule of thumb agents share partially the negative effects of profits. As the financial participation level goes down, profits' role in output becomes limited. Opposed to the benchmark case, now rule of thumb agents internalize partially the adverse effects and thus aggregate demand does not increase as much as in the benchmark case. On the other hand, the impact effect of output remains positive for most of the domain λ especially in the high taxation case. This stops at a threshold level of λ where profits have been decreased by so much that induce a drop in aggregate demand. Redistributive fiscal policy preserves the procyclicality of accommodative monetary policy tools.

values consistent with a unique equilibrium.

5. VAR Evidence

In the present section I provide empirical evidence on how QE as in the case of Asset Purchase Programme of the ECB affected the main macro variables in the Euro Area. I show that QE is stimulative. Furthermore given the importance of the profits response in the model specification, I also show how profits move after a QE shock. Results imply that profits move procyclically. This leads to the fact that the correctly specified model for the EA should have sticky wages as also emphasised by [Broer et al. \(2020\)](#).

For the empirical exercise I employ the proxy-SVAR approach as introduced by [Stock and Watson \(2012\)](#) and [Mertens and Ravn \(2013\)](#). Due to the difficulty of identifying monetary shocks in the data as elaborated in [Ramey \(2016\)](#), this approach provides a novel way that makes use of external instruments for the structural shocks of interest. The method I use is most closely related to [Gertler and Karadi \(2015\)](#) high-frequency identification (HFI) approach. In order to identify external instruments for the QE shock I use the Euro Area Monetary Policy Event Study Database (EA-MPD) constructed in [Altavilla et al. \(2019\)](#) (ABGMR hereafter) together with their methodology to extract the factors. The novelty of this approach is that a QE factor can be extracted from the data and be used directly as an instrument.

Estimation Methodology. The VAR has the following general structural form:

$$\mathbf{A}\mathbf{V}_t = c + \sum_{i=1}^p \mathbf{C}_i \mathbf{V}_{t-i} + \epsilon_t,$$

where \mathbf{V}_t is a vector of the n economic and financial variables included in the estimation, \mathbf{C}_j for $j = 1 \dots p$ are $n \times n$ coefficient matrices and ϵ_t is a $n \times 1$ vector of structural white noise shocks. Multiplying each side by \mathbf{A}^{-1} we get the reduced form VAR:

$$\mathbf{V}_t = c + \sum_{i=1}^p \mathbf{B}_i \mathbf{V}_{t-i} + \mathbf{u}_t, \quad (44)$$

$u_t = \mathbf{S}\epsilon_t$ is the reduced form residuals, a function of the structural shocks ϵ_t . Also, $\mathbf{B}_j = \mathbf{A}^{-1}\mathbf{C}_j$ and $\mathbf{S} = \mathbf{A}^{-1}$.

We can partition the vector of structural shocks according to the structural shock of interest, in this case the QE shock, and the rest. That is

$$\epsilon_t = \begin{bmatrix} \epsilon_t^{QE} & \epsilon_t^R \end{bmatrix}$$

Let s denote the column matrix of \mathbf{S} which is associated with the impact of the reduced form residuals \mathbf{u}_t of the structural shock of interest ϵ_t^{QE} . To compute the impulse responses of the system to this shock we have to estimate:

$$\mathbf{V}_t = c + \sum_{i=1}^p \mathbf{B}_i \mathbf{V}_{t-i} + s\epsilon_t^{QE}$$

At this stage we could proceed by applying the widely used timing or coefficient restrictions as is common in the SVAR literature (see for example the coefficient restrictions in [Blanchard and Perotti \(2002\)](#) or sign restrictions in [Mountford and Uhlig \(2009\)](#)) in order to identify the elements in s . In a study with similar scope to the present, [Lenza and Slacalek \(2018\)](#) use a combination of zero and sign restrictions. They make the identifying assumption that an expansionary asset purchase shock decreases the term spread (defined as long-term minus short-term interest rate) and has a positive impact on the real economy of the four countries under analysis. As mentioned in [Gertler and Karadi \(2015\)](#) this is problematic for a VAR that includes financial variables, like the present, in which the policy indicator has contemporaneous effect on financial variables. Therefore, I follow the work of [Mertens and Ravn \(2013\)](#) and [Stock and Watson \(2012\)](#) and use the proxy-SVAR method to obtain covariance restrictions which allows for no direct, hard-wired assumptions on the elements of S .

Let Z_t be the instrument of interest that is correlated with the shock of interest, in this instance the QE shock ϵ_t^{QE} , but is also orthogonal to all the rest of the shocks ϵ_t^R , that is:

$$\mathbb{E}_t[Z_t \epsilon_t^{QE}] = \Phi \quad (45)$$

$$\mathbb{E}_t[Z_t \epsilon_t^R] = 0. \quad (46)$$

Condition (45) is the relevance condition that states that the correlation between the instrument and the structural policy shock must be different from zero where Φ is a scalar. Condition (46) is the exogeneity condition that implies the instrument is uncorrelated with any other structural shock. When these two conditions are met the instrument can be used as a proxy for the structural shock ϵ_t^{QE} . These two assumptions are the key identifying assumptions which add restrictions to the matrix S .

The estimation proceeds in the three following steps: First I estimate the reduced form VAR (44) with least squares to obtain estimates of the reduced form residuals vector u_t . We can partition the vector u_t into residual from the policy indicator equation and from the rest of the variables different from the policy indicator which yields $u_t = [u_t^{QE} u_t^R]$. In order to isolate the variation in u_t^{QE} that is due to the structural monetary policy shock ϵ_t^{QE} only, we regress the former on the vector of instrumental variables Z_t and a constant:

$$u_t^{QE} = \alpha + \beta Z_t + \psi_t.$$

The fitted value that yields from the regression $(u_t^{QE})^f$ can be used to estimate the ratio of s^R/s^{QE} :

$$u_t^R = \frac{s^R}{s^{QE}} (u_t^{QE})^f + \theta_t.$$

This yields an unbiased ratio of s^R/s^{QE} .

Identifying the QE surprises. I construct the instrument used for the QE shock using changes in the yields of risk-free rates at different maturities, spanning one-month to ten-years, around the EA policy meetings. Data is coming from the [Altavilla et al. \(2019\)](#) EA-MPD dataset that is continuously updated and covers the period 2002 to 2020. The

EA-MPD dataset reports median price changes around the time interval of past ECB monetary policy meetings for a broad class of assets and various maturities, including Overnight Index Swaps (OIS), sovereign yields, stock prices, and exchange rates. ECB monetary meetings have a distinct sequence, firstly there is the press release at 13.45 Central European Time where a policy decision is announced without further elaboration followed by the press conference at 14.30 where the monetary policy strategy and its details are explained more broadly.

Using tick data, they document the price changes about 10 minutes before and after the meeting and they estimate by principal components the factors that yield from the monetary policy changes. To extract monetary policy surprises that are economically interpretable the factors are rotated as in [Gürkaynak \(2005\)](#). The rotation is made such that the QE factor has no impact in the 1 month OIS rates and also has no impact in the pre-crisis period of the dataset 2002-2008 (the factor is restricted to have the smallest variance in that period). Based on the risk free assets' maturity type those factors load, four factors are identified: the "Target" that loads only on the short rates, "Timing", "Forward Guidance" and the "QE" factor that loads only in the longer-term rates.

ABGMR have estimated the factors up to 2018. I proceed by updating the monetary policy factors until 2020 using the up to date EA-MPD dataset and following the work of [Gürkaynak, Sack, and Swanson \(2007\)](#) and the procedures of ABGMR described above I estimate and rotate the latent factors in the same fashion. Naturally, in my VAR exercise I use as an external instrument for QE surprises the QE factor. Given that the rest of the dataset is in quarterly frequency, I follow [Slacalek et al. \(2020\)](#) and sum all the intra-day surprises of the QE factor that occur in a quarter.

Data and Results. I analyse quarterly data from 1999Q1 to 2019Q4, starting at the birth year of the Euro Area and leaving out of my sample the current pandemic. The baseline VAR has nine variables, including two policy indicators, the 10 year Euro Area benchmark bond rate and the 3 month rate and seven economic and financial variables: the CPI, the real GDP, a EA stock prices index, the employment level, a measure for the wages, real consumption and real profits. This follows a similar specification by [Slacalek et al. \(2020\)](#). The VAR has two lags based on the AIC criterion. Data is coming - mainly - from the Area Wide Model dataset originally constructed by [Fagan, Henry, and Mestre \(2001\)](#). The updated AWM database starts in 1970Q1 (for most variables) and is available until 2017Q4. To update the data further, I make use of publicly updated data from Eurostat, ECB and the OECD.

Given that the analysis is focused on the Quantitative Easing, the instrument is used for the period 2014 to the end of the sample, which is the period QE took place in the Euro Area. I thus use the full sample 1999Q1 to 2019Q4 to estimate the lag coefficients and obtain the reduced form residuals in equation (44). Then I use the reduced form residuals for the period 2014 to 2019 to identify the impact of QE surprises (i.e the vector S).

Figure 8 shows the impulse responses after a QE shock. The shock is scaled such as it reduces the ten-year rate by 95 bps on impact. Given the recent analysis of [Eser, Lemke, Nyholm, Radde, and Vladu \(2019\)](#) on the APP's impact on the yield curve: "A 10 year term premium compression of around 50 bps was associated with the

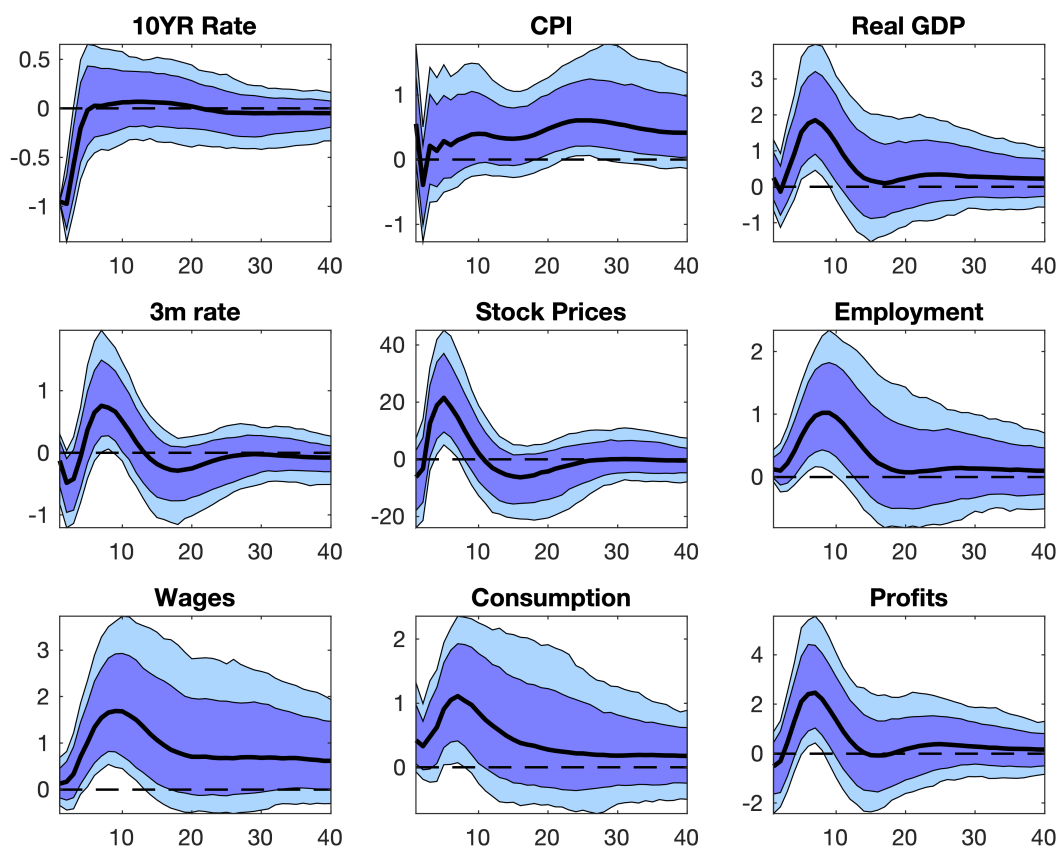


Fig. 8. Impulse Responses to a QE Shock. *Notes: The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. The X axis shows the quarters while the Y axis the percent change.*

initial APP announcement in January 2015. With the expansion of the programme the yield curve impact has become more marked and is estimated to be around 95 bps in June 2018.” The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. The shock after the first two quarters weakens but remains below the initial state for three years.

There is a significant increase in real output, employment, stock prices, profits and also wages. The peak in real GDP and in most other variables in the VAR comes in about two and a half years after the shock while wages and employment peak later. The three month rate also responds fast to the shock. The price level increases too but the changes are not statistically significant. Undoubtedly a QE shock stimulates the economy. Profits follow output’s response thus they have a procyclical behaviour. For robustness, the

same exercise using the Cholesky identification is presented in Appendix [H](#).

6. Conclusion

Quantitative easing have been substantially acknowledged to stimulate the economy through various monetary policy channels. In this paper I provide answers using both a theoretical and an empirical specification on how QE affects inequality and whether QE can be contractionary when asset markets participation is low.

I build and calibrate a DSGE model for the Euro Area economy with limited assets market participation, financially constraint banks and price and wage rigidities while I also employ an external instrument SVAR using a QE shock identified through a high frequency approach. Results from the DSGE model show that QE is stimulative and reduces income and consumption inequality when the assets market participation level is set to the Euro Area average. Furthermore, quantitative easing can be contractionary for low levels of financial participation when wages are fully flexible while sticky wages mute the contractionary effects. Using the SVAR I verify empirically the stimulative effects of the QE and I show that profits move procyclically motivating the sticky wage specification of the model.

Arguably, a substantial limitation of this model is the absence of housing which has been left out to reduce the model's complexity. [Slacalek et al. \(2020\)](#) provide a characterization of Euro Area households based on their holdings of liquid and illiquid assets. They can be summarized as optimizers, wealthy hand to mouth and poor hand to mouth. Differently to this model, optimizers and wealthy hand to mouth hold housing on top of their other assets which, importantly, are very similar in volume. Therefore, accommodative monetary policy would have had the same positive effect through house prices on the income of the wealthy hand to mouth and optimizers, leaving, at least qualitatively, the inequality results of this paper between the two groups intact.

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Appendix A Wage-Setting by Unions

The problem of the union is to maximize its objective function (in the main text).

$$\lambda \left[u_{c,t}^r W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] + (1-\lambda) \left[u_{c,t}^o W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right]$$

subject to

$$L_{h,t} = \left(\frac{W_{f,t}}{W_t^*} \right)^{-\epsilon_w} L_t$$

The first order condition yields:

$$\left(\frac{\lambda}{u_{c,t}^r u_{l,t}^r} + \frac{1-\lambda}{u_{c,t}^o u_{l,t}^o} \right) W_t = \mu^W$$

where $\mu^W = \frac{\epsilon_w}{\epsilon_w - 1}$ and $u_{c,t}^j u_{l,t}^j$ is the marginal rate of substitution of agent of type j .

Appendix B Bank's Problem

This appendix describes the method used for solving the banker's problem. I solve this, with the method of undetermined coefficient in the same fashion as in [Gertler and Kiyotaki \(2010\)](#). I conjecture that a value function has the following linear form:

$$V_t(s_{j,t}, d_{j,t}, b_{j,t}^B, m_{j,t}^B) = \nu_{l,j,t} s_{j,t} + \nu_{b,j,t} b_{j,t}^B + \nu_{m^B,j,t} m_{j,t}^B - \nu_{d,j,t} d_{j,t} \quad (\text{B.1})$$

where $\nu_{s,j,t}$ is the marginal value from credit for bank j , $\nu_{d,t}$ the marginal cost of deposits, $\nu_{m^B,j,t}$ the marginal value from the central bank reserves and $\nu_{b^B,j,t}$ the marginal value from purchasing one extra unit of sovereign bonds. The banker's decision problem is to choose $s_{j,t}, b_{j,t}^B, m_{j,t}^B, d_{j,t}$ to maximize $V_{j,t}$ subject to the incentive constraint (17) and the balance sheet constraint (14). Using (14) we can eliminate $d_{j,t}$ from the value function. This yields:

$$V_{j,t} = s_{j,t}(\nu_{s,t} - \nu_{d,t} Q_t) + b_{j,t}^B(\nu_{b,j,t} - \nu_{d,j,t} q_t) + m_{j,t}^B(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t} n_{j,t}^B.$$

Let \mathcal{L} be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t [V_t - \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)] = (1 + \lambda_t) V_t - \lambda_t \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B).$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta s_{j,t}} : (1 + \lambda_t) \left(\frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t} \right) = \lambda_t \theta \quad (\text{B.2})$$

$$\frac{\theta \mathcal{L}}{\theta b_{j,t}^B} : (1 + \lambda_t) \left(\frac{\nu_{b^B,j,t}}{q_t} - \nu_{d,j,t} \right) = \Delta \lambda_t \theta \quad (\text{B.3})$$

$$\frac{\theta \mathcal{L}}{\theta m_{j,t}^B} : (1 + \lambda_t) (\nu_{m^B,j,t} - \nu_{d,j,t}) = \omega \lambda_t \theta \quad (\text{B.4})$$

The Kuhn-Tucker condition yields:

$$\begin{aligned} KT : & \lambda_t [s_{j,t}(\nu_{s,j,t} - \nu_{d,j,t} Q_t) + b_{j,t}^B(\nu_{b^B,j,t} - \nu_{d,j,t} q_t) + m_{j,t}^B(\nu_{m^B,j,t} - \nu_{d,j,t}) \\ & + \nu_{d,j,t} n_{j,t}^B - \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)] = 0. \end{aligned} \quad (\text{B.5})$$

I define the excess value of bank's financial claim holdings as

$$\mu_t^s = \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}. \quad (\text{B.6})$$

The excess value of bank's bond holdings relative to deposits

$$\mu_t^b = \frac{\nu_{b^B,j,t}}{q_t} - \nu_{d,j,t},$$

and the excess value of bank's reserve holdings relative to deposits

$$\mu_t^m = \nu_{m^B,j,t} - \nu_{d,j,t}.$$

Then from the first order conditions we have:

$$\mu_t^b = \Delta \mu_t^s. \quad (\text{B.7})$$

Setting the fraction of the absconding rate for reserves ω to 0%, the reserves first order condition (B.4) implies that

$$\nu_{m^B,j,t} = \nu_{d,j,t}. \quad (\text{B.8})$$

This relationship implies that the gain from one extra unit of reserves is exactly the same with the cost of raising one extra unit of deposits. This helps us to show that when reserves is a strictly riskless asset, the bank is not taking them into account when the optimization problem is formulated. From (B.5) and (B.7) when the constraint is

binding ($\lambda_t > 0$) we get:

$$\begin{aligned}
s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b^B,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m,j,t}^B - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} &= \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B) \\
s_{j,t}(\mu_t^s Q_t) + b_{j,t}^B(\mu_t^b q_t) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t} &= \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B) \\
Q_t s_{j,t}(\mu_t^s - \theta) + q_t b_{j,t}^B(\Delta \mu_t^s - \Delta \theta) + m_{j,t}^B(\omega \mu_t^s - \omega \theta) + \nu_{d,t}n_{j,t} &= 0 \\
Q_t s_{j,t}(\mu_t^s - \theta) + \Delta q_t b_{j,t}^B(\mu_t^s - \theta) + \omega m_{j,t}^B(\mu_t^s - \theta) + \nu_{d,t}n_{j,t} &= 0
\end{aligned}$$

and by rearranging terms, we get equation the adjusted leverage constraint:

$$Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B = \frac{\nu_{d,t}n_{j,t}}{\theta - \mu_t^s} \quad (\text{B.9})$$

which gives the bank asset funding. It is given by the constraint at equality, where ϕ_t is the maximum leverage allowed for the bank. The constraint limits the portfolio size to the point where the bank's required capital is exactly balanced by the fraction of the weighted measure of its assets. Hence, in times of crisis, where a deterioration of banks' net worth takes place, supply for assets will decline.

Now, in order to find the unknown coefficients I return to the guessed value function

$$V_{j,t} = Q_t s_{j,t}(\mu_t^s) + q_t b_{j,t}^B(\mu_t^b) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t}^B. \quad (\text{B.10})$$

Substituting (B.9) into the guessed value function yields:

$$\begin{aligned}
V_t &= (n_{j,t}\phi_t - \Delta q_t b_{j,t}^B - \omega m_{j,t}^B)\mu_t^s + q_t b_{j,t}^B(\mu_t^b) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t}^B \Leftrightarrow \\
V_t &= (n_{j,t}\phi_t)\mu_t^s + q_t b_{j,t}^B(\mu_t^b - \Delta \mu_t^s) + m_{j,t}^B(\mu_t^m - \omega \mu_t^s) + \nu_{d,t}n_{j,t}^B
\end{aligned} \quad (\text{B.11})$$

and by (B.7) the guessed value function (B.11) becomes:

$$V_t = (n_{j,t}\phi_t)\mu_t^s + \nu_{d,j,t}n_{j,t}$$

Given the linearity of the value function we get that

$$A^B = \phi_t \mu_t^s + \nu_{d,j,t}. \quad (\text{B.12})$$

The Bellman equation (18) now is:

$$\begin{aligned}
V_{j,t-1}(s_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) &= \mathbb{E}_{t-1} \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1 - \sigma_B)n_{j,t}^B \\
&+ \sigma_B(\phi_t \mu_t^s + \nu_{d,j,t})n_{j,t}^B\}.
\end{aligned} \quad (\text{B.13})$$

By collecting terms with $n_{j,t}$ the common factor and defining the variable Ω_t as the marginal value of net worth:

$$\Omega_{t+1} = (1 - \sigma_B) + \sigma_B(\mu_{t+1}^s \phi_{t+1} + \nu_{d,t+1}). \quad (\text{B.14})$$

The Bellman equation becomes:

$$\begin{aligned} V_{j,t}(s_{j,t}, b_{j,t}^B, m_{j,t}^B, d_{j,t}) &= E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}^B = \\ &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t} Q_{t-1} s_{j,t-1} + R_{b,t} q_{t-1} b_{j,t-1}^B + R_t m_{j,t}^B - R_t d_{j,t}]. \end{aligned} \quad (\text{B.15})$$

The marginal value of net worth implies the following: Bankers who exit with probability $(1 - \sigma_B)$ have a marginal net worth value of 1. Bankers who survive and continue with probability σ_B , by gaining one more unit of net worth, they can increase their assets by ϕ_t and have a net profit of μ_t per assets. By this action they acquire also the marginal cost of deposits $\nu_{d,t}$ which is saved by the extra amount of net worth instead of an additional unit of deposits and also the additional cost of reserves $\frac{\kappa}{2} \Upsilon_t^2$. Using the method of undetermined coefficients and comparing (B.1) with (B.15) we have the final solutions for the coefficients:

$$\begin{aligned} \nu_{s,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{k,t+1} Q_t \\ \nu_{b^B,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{b,t+1} q_t \\ \nu_{m^B,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\ \nu_{d,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\ \mu_t^s &= \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t+1} - R_{t+1}] \\ \mu_t^b &= \frac{\nu_{b,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{b,t+1} - R_{t+1}] \end{aligned} \quad (\text{B.16})$$

$$\mu_t^m = \nu_{m,j,t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{t+1} - R_{t+1}] = 0 \quad (\text{B.17})$$

Appendix C Price Setting

Final-Good Firms.— The profit maximization problem of the retail firm is:

$$\max_{Y_t(j)} P_t \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} di \right)^{\frac{\zeta}{\zeta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

The first order condition of the problem yields:

$$P_t \frac{\zeta}{\zeta-1} \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} di \right)^{\frac{\zeta}{\zeta-1}-1} \frac{\zeta-1}{\zeta} Y_t(i)^{\frac{\zeta-1}{\zeta}-1} = P_t(i).$$

Combining the previous FOC with the definition of the aggregate final good we get:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

Nominal output is the sum of prices times quantities across all retail firms i :

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di.$$

Using the demand for each retailer we get the aggregate price level:

$$P_t = \left(\int_0^1 P_t(i)^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}.$$

Intermediate-Good Firms.— Intermediate good firms are not freely able to change prices each period. Following the Calvo price updating specification each period there is a fixed probability $1 - \gamma$ that a firm will be able to adjust its price.

The problem of the firm can be decomposed in two stages. Firstly, the firm hires labour and rents capital to minimize production costs subject to the technology constraint (30). Thus, it is optimal to minimize their costs which are the rental rate to capital and the wage rate for labour:

$$\min_{K_t(i), L_t(i)} P_t W_t l_t(i) + P_t Z_t K_t(i)$$

subject to

$$A_t K_t(i)^\alpha L_t(i)^{1-\alpha} \geq \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

The problem's first order conditions are:

$$W_t = \frac{P_{m,t}^{nom}(i)}{P_t} (1 - \alpha) A_t \frac{Y_t(i)}{L_t(i)}, \quad (C.1)$$

$$Z_t = \frac{P_{m,t}^{nom}(i)}{P_t} \alpha A_t \frac{Y_t(i)}{K_t(i)}. \quad (C.2)$$

$P_{m,t}^{nom}$ is the Lagrange multiplier of the minimization problem and the marginal cost of the firms with $P_{m,t} = \frac{P_{m,t}^{nom}(i)}{P_t}$ being the real marginal cost. Standard arguments lead to that marginal cost is equal across firms. Solving together the above equations we find an expression for the real marginal cost $P_{m,t}$ which is independent of each specific variety:

$$P_{m,t} = \left(\frac{1}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha W_t^{1-\alpha} Z_t^\alpha.$$

In the second stage of the firm's problem, given nominal marginal costs, the firm chooses its price to maximize profits. Firms are not freely able to change prices each period. Each period there is a fixed probability $1 - \gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p . They discount profits s periods in the future by the stochastic discount

factor $\Lambda_{t,t+s}$ and the probability that a price chosen at t will remain the same for some periods γ^s . The second stage of the updating firm at time t us to choose $P_t^*(i)$ to maximize discounted real profits:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} - P_{m,t+s} \right) Y_{t+s}(i)$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^s (1 + \pi_{\tau+\kappa-1})^{\gamma_p} \right)^{-\zeta} Y_{t+s}.$$

where π_t is the rate of inflation from $t - i$ to t . The first order condition of the problem is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^s (1 + \pi_{\tau+\kappa-1})^{\gamma_p} - P_{m,t+s} \frac{\zeta}{\zeta - 1} \right) Y_{t+s}(i) = 0.$$

Using the constraint and rearranging we get:

$$P_t^*(i) = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{m,t+s} P_{t+s}^{\zeta} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{t+s}^{\zeta-1} \prod_{\kappa=1}^s (1 + \pi_{\tau+\kappa-1})^{\gamma_p} Y_{t+s}}.$$

Since nothing on the right hand side depends on each firm i , all updating firms will update to the same reset price, P_t^* . By the law of large numbers the evolution of the price index is given by:

$$P_t = [(1 - \gamma)(P_t^*)^{1-\zeta} + \gamma(\Pi_{t-1}^{\gamma_p} P_{t-1})^{1-\zeta}]^{\frac{1}{1-\zeta}}.$$

Capital Goods Producers.— Capital goods producers produce new capital and sell it to goods producers at a price Q_t . Investment on capital (I_t) is subject to adjustment costs. Their objective is to choose $\{I_t\}_{t=0}^{\infty}$ to solve:

$$\max_{I_t} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_{\tau} I_{\tau} - [1 + \tilde{f}\left(\frac{I_{\tau}}{I_{\tau-1}}\right)] I_{\tau} \right\}.$$

where the adjustment cost function \tilde{f} captures the cost of investors to increase their capital stock:

$$\tilde{f}\left(\frac{I_{\tau}}{I_{\tau-1}}\right) = \frac{\eta}{2} \left(\frac{I_{\tau}}{I_{\tau-1}} - 1 \right)^2 I_{\tau}.$$

η is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors yields the competitive price of capital:

$$Q_t = 1 + \left(\eta \frac{I_{\tau}}{I_{\tau-1}} \left(\frac{I_{\tau}}{I_{\tau-1}} - 1 \right) + \frac{\eta}{2} \left(\frac{I_{\tau}}{I_{\tau-1}} - 1 \right)^2 - \eta \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_{\tau}^2} \left(\frac{I_{\tau}}{I_{\tau-1}} - 1 \right) \right).$$

Profits. Firms' nominal profits are: $Prof_t(i) = P_t(i)Y_t(i) - W_tP_tL_t(i) - Z_tP_tK_t(i)$. Using (C.1) and (C.2) we get $W_tP_tL_t(i) = P_{m,t}^{nom}(i)(1-\alpha)A_tY_t(i)$ and $Z_tP_tK_t(i) = P_{m,t}^{nom}(i)\alpha A_tY_t(i)$. We then can write real profits as: $\frac{Prof_t(i)}{P_t} = \frac{P_t(i)}{P_t}Y_t(i) - P_{m,t}Y_t(i)$.²¹

Aggregation. Total profits of non financial firms are equal to the sum of profits earned by intermediate good firms:

$$Prof_t = \int_0^1 Prof_t(i)di.$$

Under standard arguments and using that supply should equal demand in all markets: $\int_0^1 N_t(i)di = N_t$, $\int_0^1 K_t(i)di = K_t$, we get that total profits of the firms are:

$$Prof_t = Y_t - W_tL_t - Z_tK_t. \quad (C.3)$$

Appendix D Steady State

As it is shown on the main text, the rule of thumb agents will always supply constant labour hours equal to and the first order condition for labour supply the rule of thumb agents:

$$L^r = \left(\frac{1}{\chi}\right)^{\left(\frac{1}{1+\epsilon}\right)}$$

From labour hours the aggregator (12) we get the labour hours supplied by the optimizing agents:

$$L^o = \frac{L - \lambda L^r}{1 - \lambda}.$$

Rearranging the optimizing agents' first order condition for labour, utilizing the fact that $U_c^o = 1/C^o$, we can get an expression between consumption of the agents and labour supply:

$$C^o = \frac{W}{\chi(L^o)^\epsilon}.$$

Utilizing the above relation and the optimal consumption path of the rule of thumb agents, the consumption aggregator (11) becomes

$$C = \lambda W L^r + (1 - \lambda) \frac{W}{\chi(L^o)^\epsilon}.$$

²¹In [Gertler and Karadi \(2011\)](#) firms derive revenues from selling their good and selling the undepreciated portion of the physical capital back to the capital producers. Therefore profits are $Prof_t = P_t(i)Y_t(i) + Q_t(i)(1 - \delta)K_t(i) - W_tP_tL_t(i) - R_{k,t}Q_{t-1}(i)K_t(i)$. Substituting $R_{k,t}$ from (15) we get the same equation for aggregate real profits as in (C.3).

After some algebraic manipulation we end up to the total consumption coming from the demand side of the economy:

$$C = W \left[\lambda L^r + (1 - \lambda)^{1+\epsilon} \frac{(L - \lambda L^r)^{-\epsilon}}{\chi} \right] \quad (\text{D.1})$$

In addition, from the resource constraint we have:

$$C = Y - I - G - \tau_b B^G - \tau_s S^G,$$

where in a steady state $B^G = S^G = 0$. Therefore:

$$C = L \left[(1 - \gamma) \left(\frac{K}{L} \right)^\alpha - \delta \frac{K}{N} \right] \quad (\text{D.2})$$

To get an expression of K/L we make use of the marginal product of capital (C.2):

$$\frac{L}{K} = \left(\frac{Z}{A\alpha} \right)^{\frac{1}{1-\alpha}},$$

yielding

$$\frac{K}{L} = \left(\frac{\alpha \left(\frac{\epsilon-1}{\epsilon} \right)}{R_k - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}. \quad (\text{D.3})$$

Thus, combining the expressions (D.1), (D.2), (D.3) we obtain an equation depending only on parameters, calibrated values ($spread_{Rk}$) and L^k and determines steady state hours L . Having found L , using the labour hours aggregator (12) we can easily find the labour hours worked by the rule of thumb agents L^o . Thus, consumption of the optimizing agents can be pinned down. Notice that an equation between optimizers' consumption and aggregate consumption can be found by combining the first order condition for labour supply and the demand side aggregate consumption equation (D.1) and solving for W . Then:

$$C^o = \frac{C(1 - \lambda)^\epsilon / \chi}{\lambda L^o (L - \lambda L^o)^\epsilon + (1 - \lambda)^{1+\epsilon} / \chi} \quad (\text{D.4})$$

Appendix E ECB's Asset Purchase Program

Figure E shows the path for the ECB's APP starting in March 2015. The path is reflected in the process of the QE shock in the model by using a first AR coefficient of 1.700 and a second coefficient of -0.730 while the initial shock is chosen to 0.015.

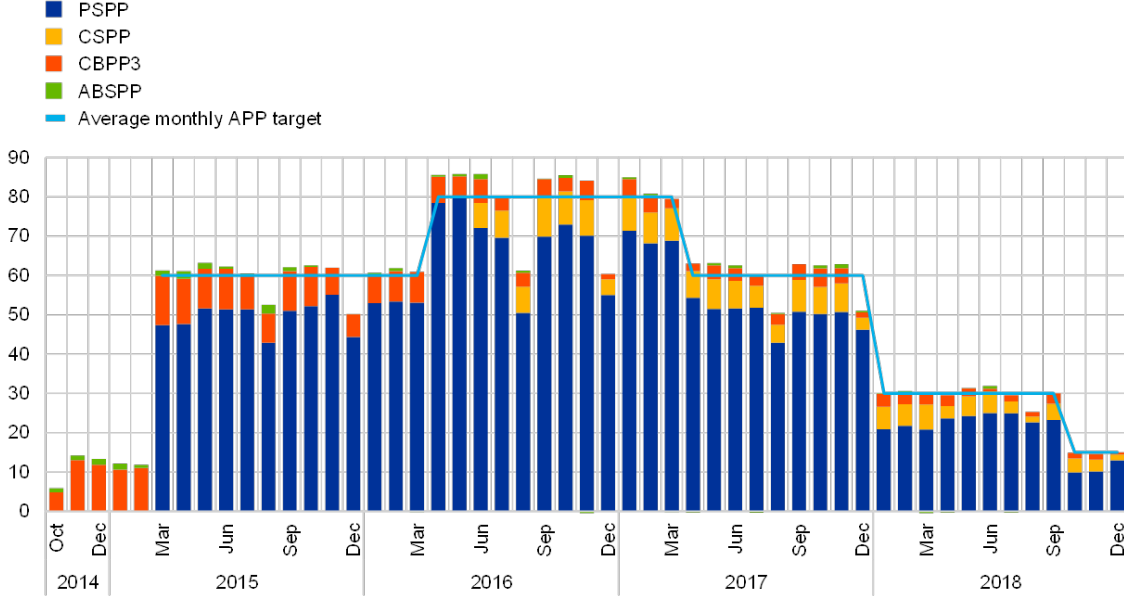


Fig. 9. Pace and composition of net APP purchases. Source: ECB

Appendix F Derivations for Section 4

Proof of (31) for the case of perfect labour market:

$$\begin{aligned}
 c_t &= \lambda c_t^r + (1 - \lambda) c_t^o \\
 c_t &= \lambda w_t + (1 - \lambda)(w_t - \epsilon l_t^o) \\
 c_t &= \lambda w_t + (1 - \lambda)(w_t - \epsilon \frac{l_t}{1 - \lambda})
 \end{aligned}$$

therefore

$$w_t = c_t + \epsilon l_t.$$

Proof of (31) for the case of wage setting by unions: The first order condition for the wage setting problem yields:

$$\left(\frac{\lambda}{MRS_t^r} + \frac{1 - \lambda}{MRS_t^o} \right) W_t = \mu^W$$

where $MRS_t^o = u_{c,t}^o u_{l,t}^o$ and $MRS_t^r = u_{c,t}^r u_{l,t}^r$. Log-linearising this around the steady state yields:

$$w_t = \psi_r c_t^r + \psi_o c_t^o + \epsilon(\psi_r + \psi_o) l_t \quad (\text{F.1})$$

where $\psi_r = \mu^W \frac{\lambda W}{MRS^r}$ and $\psi_o = \mu^W \frac{(1-\lambda)W}{MRS^o}$. Since both agents provide the same labour hours at any time and consumption in steady state is equalized between both agents,

in steady state $MRS^o = MRS^r$. Therefore we can write (F.1) as:

$$w_t = c_t + \epsilon l_t.$$

Proof of IS equation (33): Assuming no TFP process in the production function, its log-linearised form is: $y_t = \alpha k_t + (1 - \alpha)n_t$. Solving for n_t and substituting to (32) we get

$$c_t^o = c_t - \left(\frac{\lambda}{1 - \lambda} \right) \left[\frac{y_t - \alpha k_t}{1 - \alpha} \right] \quad (\text{F.2})$$

Log-linearising the resource constraint we get $y_t = c_t s_c + i_t s_i + s_g$ since the proportion of government bond and shares are zero in the steady state. Solving for $c_t = \frac{y_t - i_t s_i - s_g}{s_c}$ and inserting the resource constraint we have:

$$c_t^o = y_t \left(\frac{1}{s_c} - \epsilon \frac{\lambda}{(1 - \lambda)(1 - \alpha)} \right) - i_t \frac{s_i}{s_c} - \frac{s_g}{s_c} + \epsilon \frac{\lambda}{(1 - \lambda)(1 - \alpha)} \alpha k_t. \quad (\text{F.3})$$

Inserting the above into the optimizers Euler equation $c_t^o = \mathbb{E}_t\{c_{t+1}^o\} + [\mathbb{E}_t\{\pi_{t+1}\} - r_t]$, we get

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\delta} [r_t - \mathbb{E}_t\{\pi_{t+1}\}] - \frac{1}{\delta} \frac{s_i}{s_c} \Delta i_{t+1} + \frac{1}{\delta} \frac{\epsilon \lambda}{(1 - \lambda)(1 - \alpha)} [\alpha \Delta k_{t+1}]. \quad (\text{F.4})$$

where

$$\delta = \frac{1}{s_c} - \epsilon \frac{\lambda}{(1 - \lambda)(1 - \alpha)} \quad (\text{F.5})$$

and $s_c = C^{ss}/Y^{ss}$, $s_i = I^{ss}/Y^{ss}$ and $s_g = G^{ss}/Y^{ss}$.

Appendix G Robustness

In this Appendix, I show that the IADL for the case of QE and a monetary policy shock holds for any reasonable parametrization of the inverse Frisch elasticity of labour supply. Figures 10 and 11 show the impact effect of output after a monetary policy and QE shock conditional on the degree of asset markets participation. This is repeated for four different values for the inverse Frisch elasticity: 0.5, 1, 1.5 and 2. What is clear, is that under every parametrization of the Frisch elasticity IADL remains valid. In all cases, impact effect on output grows as the asset markets participation is decreasing. This holds until a threshold value of λ , different for each case, which makes the impact effect negative until it reaches again a value close to zero.

Appendix H VAR with Cholesky Identification

Figure H shows the impulse responses to a QE shock using the standard Cholesky identification. The shock is normalised such as to produce a 95 bps drop in the ten-

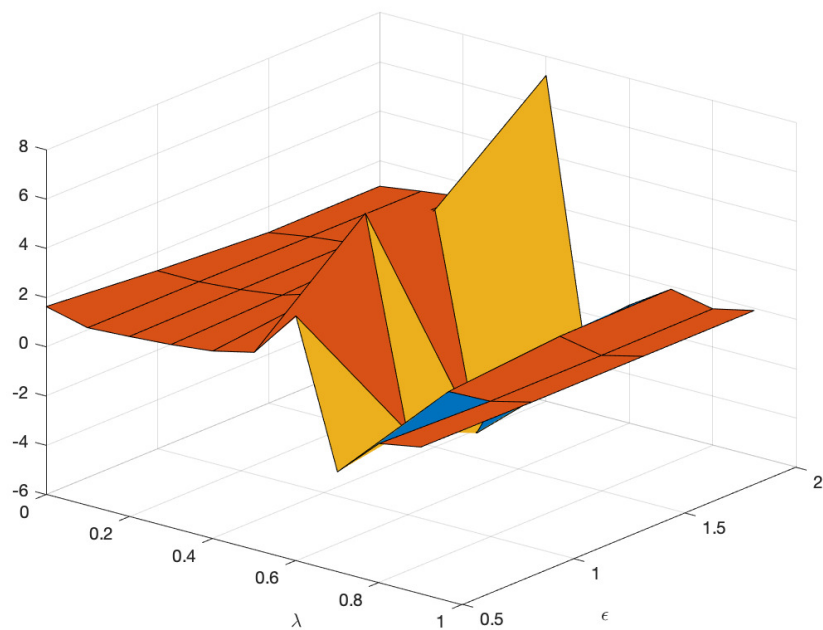


Fig. 10. Sensitivity to Inverse Frisch Elasticity Values: MP Shock

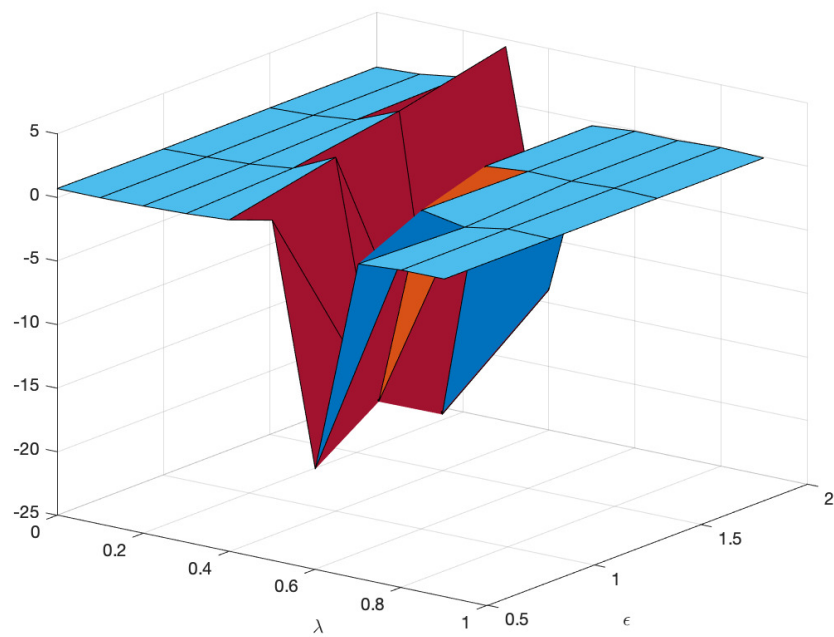


Fig. 11. Sensitivity to Inverse Frisch Elasticity Values: QE Shock

year rate. In comparison with the impulse responses using the external instrument approach, the two methods give similar results. All variables are responding as expected after an accommodative monetary shock with the exception of the price index. The CPI drops for the first 10 quarters and then increases though insignificantly.

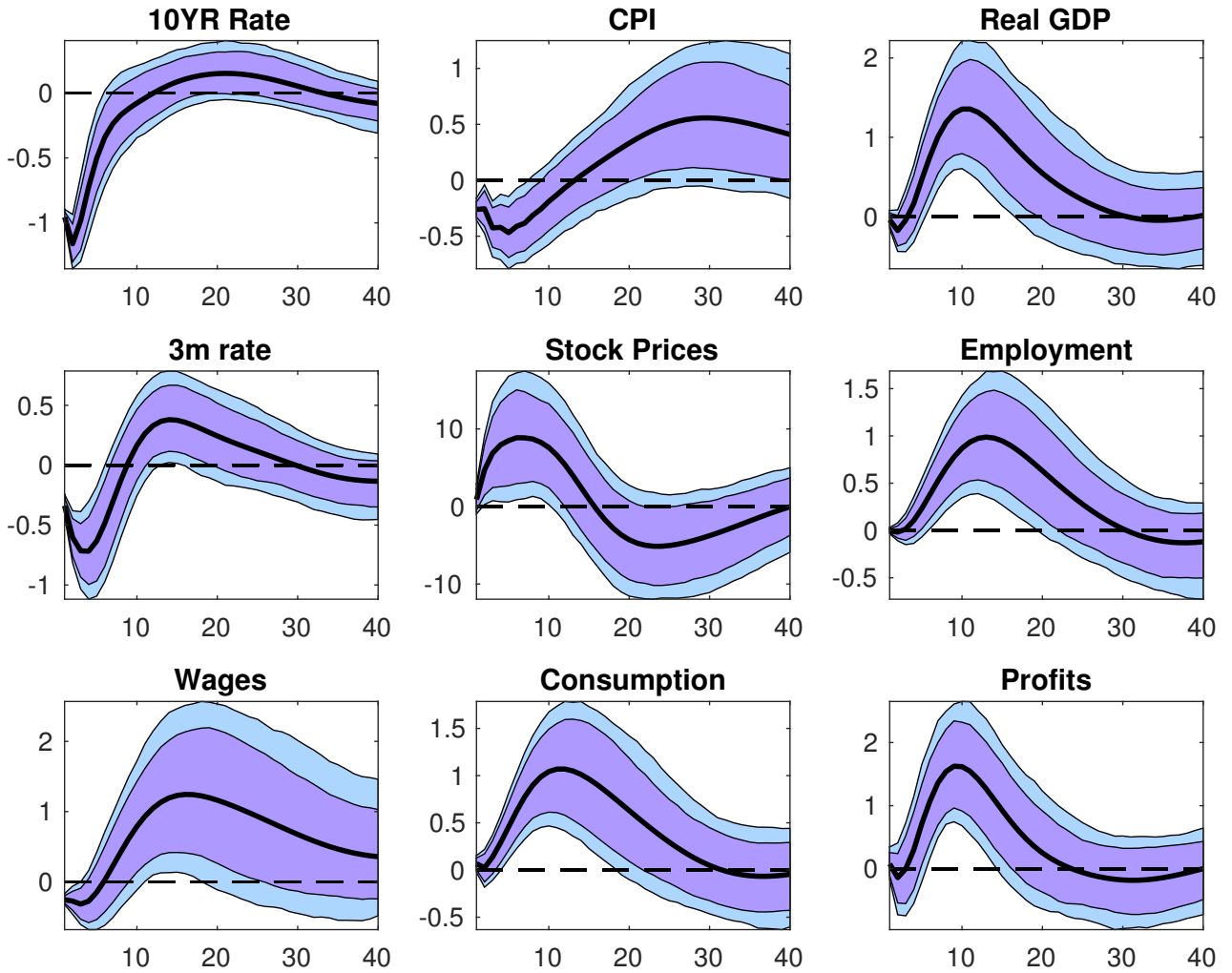


Fig. 12. Impulse Responses to a QE Shock with Cholesky Identification.

Notes: The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. The X axis shows the quarters while the Y axis the percent change.