Financial Crisis, Monetary Base Expansion and Risk

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Abstract

This paper examines the post-2008 European Central Bank's liquidity enhancing policies and provides evidence of risk-taking incentives of monetary policy. I build and estimate a dynamic, general equilibrium model that incorporates financial frictions in both the supply and demand for credit and allows banks to receive liquidity and hold reserves. When the central bank supplies liquidity during turbulent times, banks grant loans to riskier firms. This increases the firms' default probability on new credit and worsens the performance of the economy. Additionally, I find that borrower's risk increase can explain the recent reserve accumulation by the banking system. Lastly, I evaluate the effects of negative interest rates on excess reserves and assess the welfare implications of the liquidity provision policies.

JEL classification: D81, G01, G21, G33, E44, E52, E58

Keywords: Risk-taking channel; excess reserves; LTRO; unconventional monetary policy; negative interest rates; DSGE models

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1. Introduction

Since the onset of the Great Recession, central banks in the US and the Euro area have employed a number of non-standard monetary policy tools, most of them not previously analysed in the macroeconomic policy literature. The extension of existing reverse operations under longer maturities and the asset purchase programs were the most popular among those tools. Although the key scope of these direct funding programs was the stabilization of economic activity through a credit expansion, especially in the Eurozone, credit and output levels are still below pre-crisis levels. Additionally, on both continents there has been a rapid increase in banks' reserves holdings. This led commentators, analysts and policy makers to criticize banks for hoarding reserves out of emergency funds instead of lending them to the real sector.[1] At least in part to counter such hoarding, the ECB decided to penalize reserve holdings by charging negative interest rates on its reserves accounts. This paper studies these recent macroeconomic developments and their consequences for the Euro Area macroeconomy. The paper's main finding is that the ECB's liquidity provision, namely the Long Term Refinancing Operations (LTROs), was beneficial for the banking system but not for the macroeconomy due to the risk-taking channel of monetary policy (this channel is returned to momentarily).

This paper's main economic insight can be summarized as follows: Consider an increase in entrepreneurial risk which reduces firms' net worth, raises their probability of default and sets off a recession through a Bernanke, Gertler, and Gilchrist (1999) financial accelerator mechanism.[2] To counter such a recession, a central bank might provide liquidity directly to the banking sector as in the the case of ECB's LTROs. While such a policy would potentially halt the economic downturn, it simultaneously makes banks supply fresh credit to -now- riskier firms giving rise to a risk-taking channel. Reinforced by the lower cost of borrowing, firms leverage up their net worth (which in and of itself increases their likelihood of default). Higher default rates lead to higher bankruptcy costs and less available capital for production. A threshold exists where the capital gains from liquidity injections are equal to the capital losses due to bankruptcy costs. In the estimated model I find that the capital losses dominate the central bank's capital injections and lead to lower investment and output.

[1] Pisani-Ferry and Wolff (2012), The truth about all those excess reserves (*The Economist*), Central Bank reserve creation in the era of negative money multipliers (*Voxeu*), Draghi Unveils Historic Measures Against Deflation Threat (*Bloomberg*), ECB Doing Whatever It Takes Can't Make Euro-Area Banks Lend (*Bloomberg*) and many others. Philadelphia Fed President Charles Plosser expressed concern about what would occur "were all those excess reserves to start flowing out into the economy in the form of loans or purchases of other assets"

[2] The definition of entrepreneurial risk follows Christiano, Motto, and Rostagno (2014) risk shock, i.e., an increase in idiosyncratic production risk.

Additional results show that an increase of riskiness in the credit demand side is the reason behind the banks' excess reserves accumulation. Our results show that the central bank is able to induce banks to lower their reserves holdings and extend credit only when interest rates on reserves become significantly negative. Lastly, the adverse effects of an in-crisis liquidity mechanism are confirmed by a negative impact on consumers' welfare.

This study introduces agency problems associated with financial intermediation in an otherwise standard business cycles model and estimates the model for the Euro Area. A modelling framework is presented where banks are able to receive and store emergency liquidity funds from the central bank into their reserve accounts. By combining Gertler and Kiyotaki (2010) with Bernanke et al. (1999) (henceforth GK and BGG respectively) a setting is developed where increased risk (in the sense of risk shock by Christiano et al. (2014)) reduces firms' net worth, increases their likelihood of default and makes banks reduce credit.[3]

The main result of this study is in line with findings from the empirical literature on the risk-taking channel of monetary policy (see Jiménez, Ongena, Peydró, and Saurina (2014)). The risk-taking channel describes the notion that monetary policy affects the quality and not just the quantity of bank credit. Empirical studies show that expansionary monetary policy induces banks to grant loans to more risky firms which increases the borrowers likelihood of default. In the general equilibrium setting that I employ, this leads to negative effects to the macroeconomy.[4] There is need to emphasize that this paper examines only a channel and a specific instrument the ECB used, namely the LTROs. No general conclusions on the sign of the full impact of the ECB's accommodative monetary policy can be derived. To do that we must take into account all the other instruments used which is not the scope of this paper.

The ECB proceeded in measures aiming to support banks' liquidity funding and therefore encouraging banks to provide credit.[5] The main tool used, the LTRO, is a type of open

[3]This study does not claim that a risk shock was the sole source of the Euro Area Crisis. Following Christiano et al. (2014) showing that risk shocks account for a 60% of GDP fluctuations, they are used as the primary source of disturbance.

[4]Jiménez et al. (2014) using information on borrower quality from credit registry databases for Spain have identified that a monetary expansion induces risk-shifting. Dell'Ariccia, Laeven, and Suarez (2017) using a measure of ex-ante risk taking based on the banks assessment of risk at the time the loan was made find qualitatively similar results for the U.S. See also Allen and Gale (2000), Diamond and Rajan (2012), Ioannidou, Ongena, and Peydró (2014),Delis, Hasan, and Mylonidis (2017), Buch, Eickmeier, and Prieto (2014), Altunbas, Gambacorta, and Marques-Ibanez (2010), Maddaloni and Peydró (2011) and Lown and Morgan (2006) among other and the literature review in the end of this section.

[5]ECB's response was in two phases with the use of non-standard monetary policies labelled as "enhanced credit support". Firstly at the onset of financial crisis and later when the Euro sovereign crisis took place. These included the maturity extension of Long Term Refinancing Operations, the creation the Targeted Long Term Refinancing Operations (TLTROs), the reduction in banks' reserve requirements from 2% to 1%, an asset purchase program and numerous other non-standard measures described in detail by Cour-Thimann and Winkler (2012).

market operation that takes place as reverse transaction and is the main liquidity provision tool of the ECB. Starting from October 2008 the ECB steadily increased the maturities of the LTRO from 3 months to 36 months.[6]Therefore, financial intermediaries could have unlimited access to short term funding. At the same time a significant increase of the banks excess reserves took place.[7] LTRO funding and the banks' accumulation of excess liquidity are depicted in Figure 1.

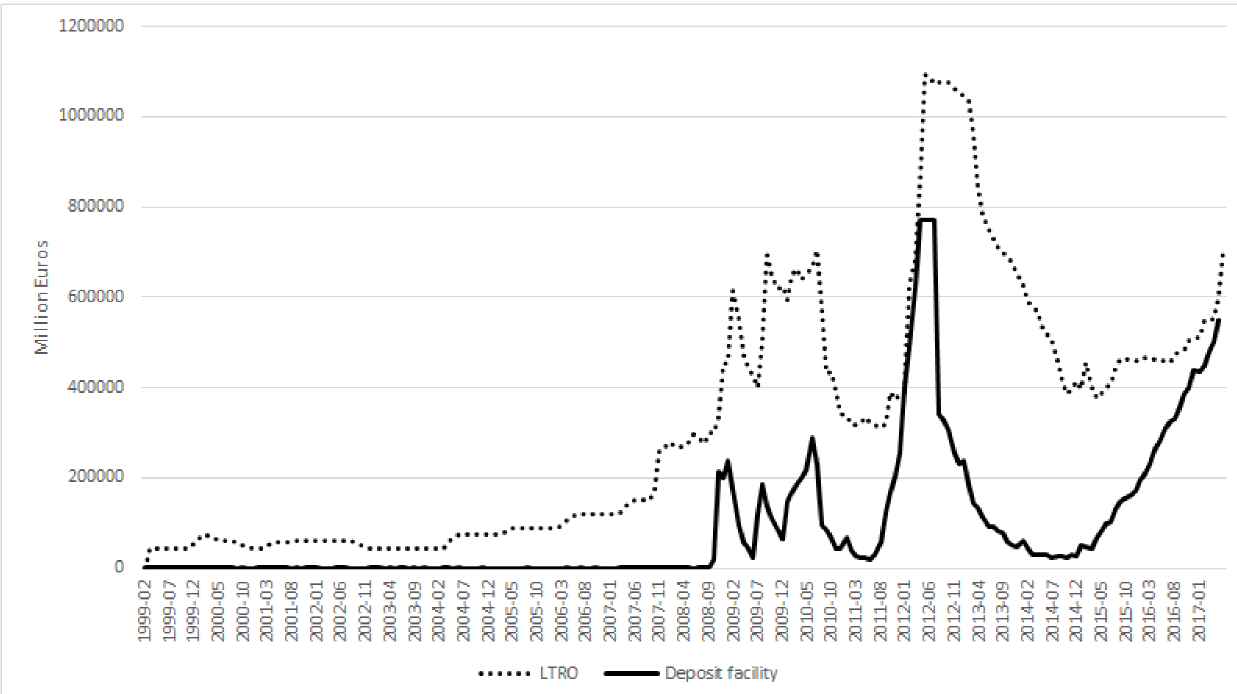

Fig. 1. LTRO and excess reserves in the Eurosystem. Data source: ECB

Despite the fact that the ECB has more than doubled its balance sheet, creating a remarkable expansion of the Eurosystem's monetary base, bank lending has not shown any signs of expansion yet as Figure 2 shows. Monetary base expansion, although unprecedented in its size, has not worked as intended. Banks' credit growth remains low in the Eurozone and hence investment.

[6]Only for it's second intervention, the ECB supplied to the banks 1 trillion Euro via the LTRO the scheme.

[7]In the Eurosystem framework, banks either hold their reserves as excess reserves where they get a zero remuneration or in the deposit facility, the account where banks make deposits with the central bank and earn an interest. Before 2008 both assets' level was insignificant and were only used for banking micro-management. Since I am not interested in the micro-management allocation of banks between the deposit facility and the current accounts, in the model I use the deposit facility account as the representative reserve account. The model can be extended easily to include also the current accounts (reserves outside of the deposit facility) as an asset that pays no interest.

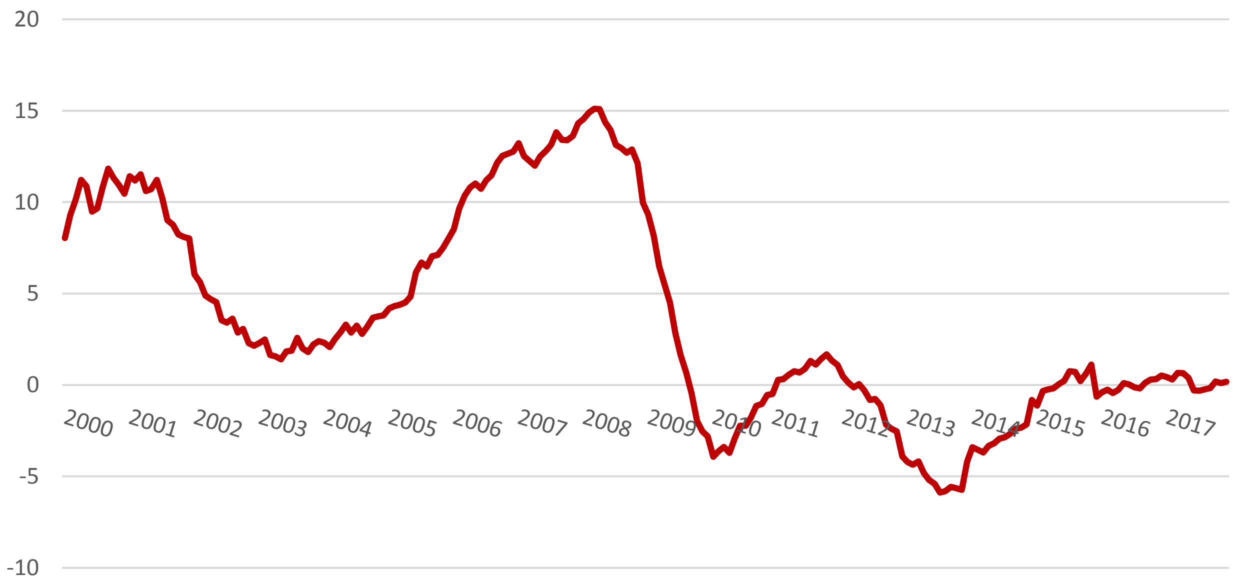

Fig. 2. Loans from Monetary Financial Institutions to Non-Financial Corporations in the Euro Area (Year on Year % Growth). Data source: ECB

This paper also analyses the effects of the newly introduced the negative interest rates or reserves practiced by the ECB and other central banks.[8] Interest rates in the present setting are constrained by the lower bound which is set by the model economy's riskless rate. Being unable to have negative rates, I employ a penalty function for accumulating reserves which turns positive when reserves exceed a threshold value, similar to a tax on reserves. When banks accumulate reserves below a specific threshold they have some gains (e.g. efficient and liquidity gains). When the level of reserves surpass the threshold banks pay a cost to the central bank. After an increase of the reserve penalty, a reduction of the banks' reserve position and an increase in credit follows which lead to an overall economic upturn. Lastly, using consumption equivalence measure based on conditional welfare as in Schmitt-Grohé and Uribe (2007), I find that the recent ECB's policies had a small but negative impact on welfare.

The modelling structure allows credit frictions to operate simultaneously originating from both the demand and the supply side of credit, an approach that has not yet been discussed in the literature. On the supply side, an agency problem between the depositors and the banks is introduced. The financial intermediaries can divert at any time a fraction of their assets and return it back to their families as in Hart and Moore (1998). This implies an

[8]Apart from the ECB, negative interest rates have been implemented also by Denmarks Nationalbank, Bank of Japan, Swiss National Bank and the Sveriges Riksbank.

endogenous constraint on the bank's ability to obtain funds that assures depositors' funds safety. A wedge between the interest rate on loans and the deposit interest rate is generated when the constraint is binding. As for the demand side friction, a costly state verification (CSV) problem as initially proposed by Townsend (1979) is introduced. Banks in order to observe the defaulting entrepreneurs payoff, must pay a monitoring cost. These monitoring costs can be interpreted as a cost of bankruptcy as in Bernanke (1981). A premium emerges between the interest rate on capital and the discount rate, the equivalent of the deposit rate in the model. An endogenously determined remain and exit probability of the entrepreneurs is introduced in this new framework. Entrepreneurs decide whether they exit taking as given the loan interest rate. They stay in life as long as the level of their leverage satisfies the minimum banks' profitability.

Related Literature. Macroeconomic models with financial frictions have populated a substantial fraction of the macro literature after the Great Recession following the seminal papers of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) (see Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2014) among many others).[9] Prior to the financial frictions models, most of the existing modern macroeconomic models do not take into account that monetary policy is implemented through the banking system, as it occurs in practice. Instead, most assume that central banks directly control interest rates or monetary aggregates and abstract from how the transmission of monetary policy may depend on the conditions of banks. Interactions between reserves, open market operations, banking and the macroeconomy introduced in this paper, aim to build a closer approach to the real world monetary policy implementation.

The risk-shifting channel of monetary policy, the main result of this paper, has regained attention after the Great Recession which has been characterised from substantial monetary easing from the central banks. Allen and Gale (2000), Diamond and Rajan (2012) were among the first to identify the risk-shifting channel of monetary policy. In an empirical framework Jiménez et al. (2014) and Ioannidou et al. (2014) find that monetary expansion induces banks to grant loans to more risky firms which increases the likelihood of default. Dell'Ariccia et al. (2017) find similar results for the U.S. [10] Adrian and Shin (2010) build a theoretical model and show that expansionary monetary policy increases the risk taking of the banking sector by relaxing the bank capital constraint due to moral hazard problems. In my knowledge the present paper is the first study that introduces the channel of risk-shifting

[9] Also Eggertsson and Woodford (2003), Curdia and Woodford (2011), Gertler and Karadi (2011). For a comprehensive literature review on the developments of models with financial factors see Gertler and Gilchrist (2018).

[10] For more studies that identify the risk-taking channels see: Delis et al. (2017), Buch et al. (2014), Altunbas et al. (2010), Maddaloni and Peydró (2011) and Lown and Morgan (2006) among others.

in lending after liquidity operations in a quantitative framework.

There are many studies on the ECB's LTROs which are close to the subject of this paper: Cahn, Matheron, and Sahuc (2017), Joyce, Miles, Scott, and Vayanos (2012), Bocola (2016), van der Kwaak (2017) to name a few. These, assume a direct relationship between the non-standard credit measures and the bank lending. Specifically, they omit the reserves that are being created from these operations. Thus, in these models it is assumed that all the emergency funding from the central bank transforms directly to credit, which is a strong assumption.

Finally, the last strand of literature that this paper relates to are the studies on banks' excess reserves. After the recent reserve accumulation by the banking sector there is a growing literature on the subject which goes back to Frost (1971). Bianchi and Bigio (2014) develop a new framework to study the implementation of monetary policy through the banking system. They find that the unprecedented increase in reserves is due to a substantial and persistent contraction in loan demand since the benefits of holding reserves relative to loans are increased. Their results are in line with this paper's findings. Primus (2017) designs a DSGE model where banks hold reserves but mainly focuses on the effects that reserve requirements can have in the middle-income countries.

Layout. The paper is organized as follows. Section 2 presents the model and section 3 describes the important economic mechanisms. Section 4 explains the data used and the estimation of the model. Finally, section 5 presents the quantitatively analysis and section 6 concludes.

2. The Model

The model is built on and extends two leading approaches in the credit market frictions literature: The seminal work of Bernanke et al. (1999) that introduced the "financial accelerator" in a general equilibrium setting and Gertler and Kiyotaki (2010). Due to the model length, the model is divided in two parts: The standard part of the model and the financial frictions. Section 2.1 describes the standard part of the model, employed in the most Real Business Cycles literature. Section 2.2 describes the financial frictions components. Finally, section 2.3 closes the model by providing the monetary and fiscal rules.

All variables are in real terms abstracting from the notion of money. There are five types of agents. Households, financial intermediaries, entrepreneurs, capital goods producers and retailers, and a government that conducts both fiscal and monetary policy. To enhance intuition on the model mechanism, the flows between agents are summarized in figure 3.

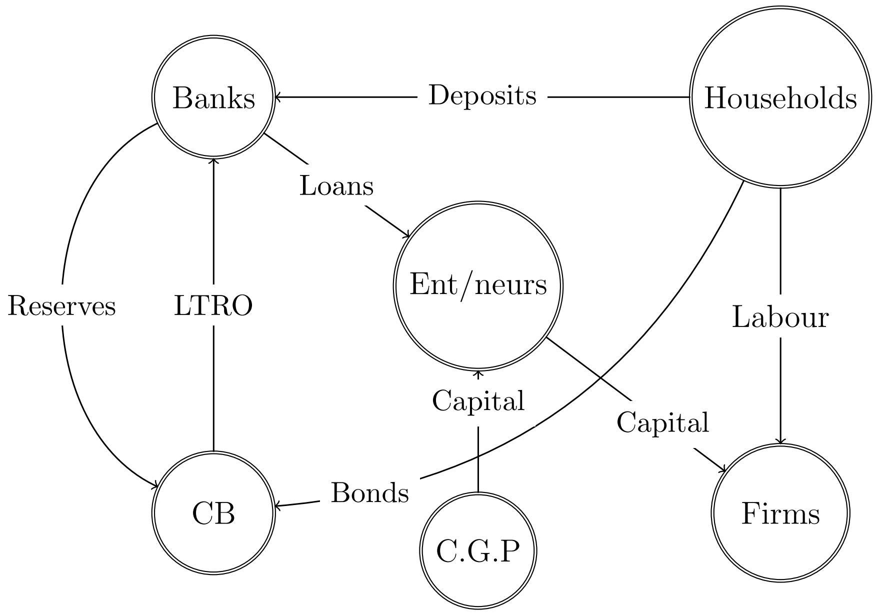

Fig. 3. Model Summary. CGP are the capital goods producers, CB is the central bank

2.1. *Standard Part of the Model*

Households.— There is a continuum of households with identical preferences. Within each household there are three different member types: ϖ workers, ς bankers and $(1-\varpi-\varsigma)$ entrepreneurs. Household members differ in the way they obtain earnings. Workers supply labour, bankers manage the financial intermediaries and entrepreneurs manage the non-financial firms. All return their earnings back to their families.[11] Within the family there is perfect consumption insurance.

The preferences of the representative household take the following form:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \zeta_{c,t} [\ln(C_{t+i}) - \frac{\chi}{1+\epsilon} N_{t+i}^{1+\epsilon}], \tag{1}$$

C_t denotes the per capita consumption of the household members and N_t the supply of labour. $\beta \in [0, 1]$ is the discount factor, ϵ is the inverse Frisch elasticity of labour supply, $\chi > 0$ is the relative utility weight of labour and $t + i$ is the time subscript. Finally, $\zeta_{c,t}$ is preference shock that follows an AR(1) process. Because of the stochastic setting, households make expectations for the future based on what they know in time t and $\mathbb{E}_t$ is the expectation operator at time t.

[11] This approach follows GK and allows for within-household heterogeneity but also sticks to the representative approach representation. Abstracting from consumption for the bankers and entrepreneurs makes the model presentation simpler.

The budget constraint of the representative household is

$$C_t + T_t + D_{h,t+1} = W_t N_t + \Pi_t + R_t D_{h,t}, \tag{2}$$

where

$$D_{h,t+1} = D_{t+1} + D_{g,t+1}. \tag{3}$$

Household allocates funds to consumption, taxes T_t and two types of savings: lending deposits D_{t+1} to banks and one period government bonds $D_{g,t+1}$. Both assets have no risk and are perfect substitutes of each other. R_t is the gross return for the bonds and the deposit holdings respectively (the interest factor) in period t. The household's financial resources are from labour income, W_t is the real wage, bond and deposits returns and the net payouts to the household from ownership of both non-financial firms and financial intermediaries Π_t.

The problem of the representative household is to choose $C_t, N_t, D_t, D_{h,t}$ in order to maximize its expected utility (1) subject to the budget constraint (2) at every period. Solution of the household's problem is shown in Appendix A. There is a turnover between workers, bankers and the entrepreneurs which ensures that bankers and entrepreneurs will never accumulate enough own funds to finance their activities. This will be explained in detail in the next section.[12]

Capital and Consumption Goods Production.— The non-financial firms are separated into two types: goods producers and capital producers. Capital evolves according to the law of motion of capital

$$K_{t+1} = k^q_{t+1}[I_t + (1-\delta)K^f_t]. \tag{4}$$

The variable K^f_t denotes the amount of capital available for time t production. [13] This is different than the amount of capital at the end of the previous period as some is lost because of monitoring costs. k^q_t denotes a capital quality shock and follows a first order autoregressive process. This is a simple way to introduce an exogenous source of variation in the value of capital.[14]

Goods Producers.— Goods producers are owned by the entrepreneurs. They combine capital received from the entrepreneurs at no cost, and labour to produce goods under a constant returns to scale production function. Production is also subject to a total factor

[12]This follows Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012).

[13]This follows the setting by Carlstrom, Fuerst, and Paustian (2016).

[14]Many recent papers make use of the exogenous disturbance in the capital. See Gertler and Karadi (2011), Brunnermeier and Sannikov (2014) among others.

productivity shock A_t that follows a first order autoregressive process.[15]

$$Y_t = A_t(K_t^f)^\alpha N_t^{1-\alpha}.$$

The decision problem of the goods producers is to choose K_t^f and N_t in order to maximize their profits. Profit maximization implies standard input demands for labour and capital:

$$W_t = (1-\alpha)\big(\frac{K_t^f}{N_t}\big)^\alpha$$

$$Z_t = \alpha\big(\frac{N_t}{K_t^f}\big)^{1-\alpha}.$$

Capital Goods Producers.— Capital goods producers produce new capital and sell it to entrepreneurs at a price Q_t. Investment on capital (I_t) is subject to adjustment costs. Their objective is to choose $\{I_t\}_{t=0}^\infty$ to solve:

$$\max_{I_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Big\{ Q_t I_t - [1 + \tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) I_\tau] \Big\}.$$

where the adjustment cost function $\tilde{f}$ captures the cost of investors to increase their capital stock:

$$\tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) = \frac{\eta}{2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2 I_\tau.$$

η is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors yields the competitive price of capital:

$$Q_t = 1 + \Big(\eta \frac{I_\tau}{I_{\tau-1}}\Big(\frac{I_\tau}{I_{\tau-1}} - 1\Big) + \frac{\eta}{2}\Big(\frac{I_\tau}{I_{\tau-1}} - 1\Big)^2 - \eta\Lambda_{t,\tau}\frac{I_{\tau+1}^2}{I_\tau^2}\Big(\frac{I_\tau}{I_{\tau-1}} - 1\Big)\Big).$$

2.2. Financial Frictions

Entrepreneurs.— Each entrepreneur i purchases raw capital $k_{i,t+1}$ from the capital goods producers at price Q_t in a competitive market and fund this purchase with their equity $n_{i,t+1}^E$ and credit $l_{i,t+1}$ obtained from the financial institutions. The entrepreneur's balance sheet is:

$$Q_t k_{i,t+1} = l_{i,t+1} + n_{i,t+1}^E. \tag{5}$$

[15] It might be argued that since the idiosyncratic shock to entrepreneurs' capital is used, an aggregate productivity shock is redundant. Nevertheless I stick to the original BGG formulation and include the productivity shock.

The entrepreneur transfers the purchased capital to the retail firm in order to produce goods. Capital yields its marginal product Z_{t+1}. At the end of the period, she sells the undepreciated capital back to the capital goods producer at price Q_{t+1}. Therefore, the average return per nominal unit invested in period t is:

$$R_{k,t+1} = k^q_{t+1} \frac{[Z_{t+1} + (1-\delta)Q_{t+1}]}{Q_t}, \tag{6}$$

In every period t an idiosyncratic shock ψ_i transforms the newly purchased $k_{i,t+1}$ raw units of capital into $\psi_i k_{i,t+1}$ effective units of capital. It is assumed that ψ follows a unit-mean log normal distribution. The idiosyncratic shock is drawn from a density $f(\psi_t)$ and the probability of default is then given by:

$$p(\bar{\psi}) = \int_0^{\bar{\psi}} f(\psi) d\psi. \tag{7}$$

Following Christiano et al. (2014) I call the standard deviation of $\log(\psi)$ denoted by σ_t, the *risk shock*. It is the cross sectional dispersion in ψ and it is allowed to vary stochastically over time. Intuitively, is an increase in the volatility of the entrepreneurs distribution of good and bad signals. As it will be shown in the simulations, a positive risk shock will lead to an increase in the standard deviation σ_ψ of the idiosyncratic shock ψ that the entrepreneurs receive.

A threshold value of ψ_i called $\bar{\psi}_{t+1}$ divides the entrepreneurs that cannot pay back the loan and interest from those who can repay. It is defined by

$$R_{l,t+1} l_{i,t+1} = \bar{\psi}_{t+1} R_{k,t+1} Q_t k_{i,t+1}. \tag{8}$$

$R_{l,t+1}$ is the rate to be decided in the debt contract between the entrepreneur and the banker. When $\psi_i \geqslant \bar{\psi}_{t+1}$ the entrepreneur repays the bank the amount $R_{l,t+1} l_{i,t+1}$ keeps the profits equal to $\bar{\psi}_{t+1} R_{k,t+1} Q_t k_{i,t+1} - R_{l,t+1} l_{i,t+1}$ and continues production. If $\psi_i < \bar{\psi}_{t+1}$ the entrepreneur has negative net worth resulting in bankruptcy and default. When an entrepreneur defaults, is then being monitored by a bank which acquires her assets. The expected net worth of the entrepreneurs is

$$\mathbb{E}_t[(1 - \Gamma_t(\bar{\psi}_{t+1}))\bar{\psi}_{t+1} R_{k,t+1} Q_t k_{i,t+1}], \tag{9}$$

where

$$\Gamma_t(\bar{\psi}_{t+1}) = \int_0^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + \bar{\psi}_{t+1}(1 - p(\bar{\psi}_{t+1})).$$

and $1 - \Gamma_t(\bar{\psi}_{t+1})$ represents the average weight of the entrepreneurs' gains.

If there was no cost for the banker to observe the idiosyncratic shock $\psi_{i,t}$, then there would have been state-contingent contracts that would perfectly insure the banker. Instead, in order to make entrepreneurs' default costly for the banking sector, ψ_i is costlessly observed by the entrepreneur, but it is not observed by the lender unless he pays a fraction of their ex-post revenues. Specifically, the financial intermediary must pay a "monitoring cost" to observe the borrower's realized return on capital. This follows the "costly state verification" illustration proposed by Townsend (1979). Monitoring costs can be interpreted as legal costs that the banks pay in the case of borrowers' default. This cost destroys part of the capital produced by the project and equals a proportion μ of the gross payoff of the firms capital, i.e. $\mu\psi_{i,t+1}R_{k,t+1}Q_t k_{i,t+1}$.

The optimal contract maximizes the expected profits of the entrepreneur under the condition that the expected return on lending is no less that the opportunity cost of lending. In other words, for the financial intermediary to continue extending credit to entrepreneurs, their expected return from credit must be always greater or equal to the opportunity cost of its funds. The opportunity cost is the riskless rate R_t. The loan contract must satisfy:

$$(1-\mu)R_{k,t+1}Q_t k_{i,t+1}\int_0^{\bar{\psi}_{t+1}} \psi f(\psi)d\psi + (1 - p(\bar{\psi}_{t+1}))R_{l,t+1}l_{i,t+1} \geqslant R_t l_{i,t+1}. \tag{10}$$

The left hand side shows the expected gross return that the financial intermediary receives over all realizations of the shock and the right hand side the opportunity cost of lending that the intermediary has.

Using (7) the zero profit condition (10) becomes :

$$R_{k,t+1}Q_t k_{i,t+1}[\Gamma_t(\bar{\psi}_{t+1}) - \mu G_t(\bar{\psi}_{t+1})] \geqslant R_t(Q_t k_{i,t+1} - n^E_{i,t+1}), \tag{11}$$

where $G_t(\bar{\psi}_{t+1})$ are the expected monitoring costs:

$$G_t(\bar{\psi}_{t+1}) = \int_0^{\bar{\psi}_{t+1}} \psi f(\psi)d\psi$$

respectively. The optimal contract for the entrepreneur solves the entrepreneur's expected net worth (9) subject to the zero profit condition (11). The solution is presented in Appendix B. Combining the first order conditions leads to the external finance premium between the interest gain on capital and the riskless rate:

$$\mathbb{E}_t\, R_{k,t+1} = \mathbb{E}_t\, \rho(\bar{\psi}_{t+1})R_{t+1}, \tag{12}$$

$$\rho(\bar{\psi}_{t+1}) = \frac{\Gamma_t'(\bar{\psi}_{t+1})}{[(\Gamma_t(\bar{\psi}_{t+1}) - \mu G_t(\bar{\psi}_{t+1}))\Gamma_t'(\bar{\psi}_{t+1}) + (1 - \Gamma_t(\bar{\psi}_{t+1})(\Gamma_t'(\bar{\psi}_{t+1}) - \mu G_t'(\bar{\psi}_{t+1}))]}.$$

Aggregation.— At the end of the period a fraction $\sigma_{E,t}$ of entrepreneurs decides to remain and the rest disappear and replaced by an equal number of workers. This assumption ensures that entrepreneurs will not fund all investments from their own accumulated capital. The probability of remaining is not constant, in contrast with the BGG, and it is adjusted taking as given the loan interest rate that they have to pay to the banks. Specifically, it adjusts at every time t such that the level of leverage satisfies the zero profit condition (10). Exit doesn't necessarily mean default.Thus, $\sigma_{E,t}$ is a time varying probability. The probability of default and the remaining probability are characterized by a negative relationship.[16]

The new entrants receive a start up fund transferred from the old entrepreneurs which is equal to a proportion ξ_E of their wealth. By the law of large numbers the aggregate net worth for every entrepreneurs i at the end of the period t is $(1 - \Gamma_{t-1})\bar{\psi}_t R_{k,t} Q_{t-1} k_{i,t}$. Integrating over all entrepreneurs we get the aggregate net worth at the end of period t where capital letters denote aggregate variables.

$$N_{t+1}^E = (\sigma_{E,t} + \xi_E)([1 - \Gamma_{t-1}(\bar{\psi}_t)]\bar{\psi}_t R_{k,t} Q_{t-1} K_t).$$

***Banks.*—** Each bank j allocates its funds to credit $l_{j,t+1}$ and reserves $x_{j,t+1}$. It funds its operations by receiving deposit from households $d_{j,t+1}$, emergency funding from the central bank $m_{j,t}$ and also by raising equity $n_{j,t+1}^B$. From the above specification, it follows that the bank's balance sheet is:

$$l_{j,t+1} + x_{j,t+1} = n_{j,t}^B + d_{j,t+1} + m_{j,t+1}. \tag{13}$$

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities net the cost of holding excess reserves.

$$n_{j,t+1}^B = R_{l,t} l_{j,t}(1 - p(\bar{\psi}_t)) + R_{k,t} k_{j,t} Q_{t-1}(1-\mu) G_t(\bar{\psi}_t) + R_{x,t} x_{j,t} - R_t d_{j,t} - R_{m,t} m_{j,t} - \Phi(x_t). \tag{14}$$

$R_{x,t}$ is the interest rate of the deposit facility and $R_{m,t}$ the interest rate of the emergency funding (LTRO). Banks get repaid the principal plus the interest of the loans from the entrepreneurs with a probability of $(1 - p(\bar{\psi}))$. The first two terms in the right hand side of the equation is the expected return to the bank from the contract averaged over all realizations of the idiosyncratic shock.

Banks have to pay reserve accumulation costs $\Phi(x_t)$ when their reserve holdings exceed

[16]See Appendix C for further details.

a specific level. This captures the recent ECB's negative interest rate policy applied when banks' over-accumulated excess reserves. At first sight, introducing a penalty for accumulating reserves might seem odd. The reasoning behind that is that interest rates in the present setting are constrained by the lower bound which is set by the riskless rate R_t. Therefore, introducing negative interest rates is not a straightforward process making the reserve accumulation penalty a working alternative. To capture this I follow Glocker and Towbin (2012) and the penalty as a fraction of net worth depends on the size of the total excess reserves:

$$\Phi(x_t)n_t^B = \left(\frac{\kappa}{2}x_t^2 + \epsilon x_t\right)\zeta_{x,t},$$

or $\Phi(x_t) = \left(\frac{\kappa}{2}\Upsilon_t^2 n_t^B + \epsilon\Upsilon_t\right)\zeta_{x,t}$ and $\Upsilon_t = x_t/n_t^B$. In the parametrization I set ϵ to be negative. When excess reserves holdings are below the threshold $-\frac{2\epsilon}{\kappa}$ banks have efficiency gains from holding liquidity. As the excess reserves increase and overpass the threshold banks have costs due to the increased reserve penalty. $\zeta_{x,t}$ is a transitory reserve penalty shock. An unanticipated increase in the reserves' penalty will make banks reduces their reserves holdings and will induce credit to the real economy.

At the end of the period an exogenously determined constant fraction of bankers σ_B remains and the rest disappear and are replaced by an equal number of workers.

The banker's objective at the end of period t, is the expected present value of future dividends:

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty}(1-\sigma_B)\sigma_B^{j-1}\Lambda_{t+1}n_{j,t+1}^B. \tag{15}$$

In order to set a limit to the bankers borrowing from either the depositors or the central bank, I introduce an endogenous constraint on the banks ability to borrow in the same fashion as in GK and others. A banker j after collecting deposits from households and liquidity from the central bank may decide to divert a fraction of these funds. This occurs when the bank's value from diverting is higher than its franchise value. It is assumed that the bank can steal a fraction $\theta \in [0,1]$ of the expected non-defaulting loans net a fraction $\theta\omega \in [0,1] < \theta$ of the central bank liquidity. The cost of stealing for the banker is that the creditors can force the intermediary into bankruptcy at the beginning of the next period. This sets a limit to the bankers borrowing from either the depositors or the central bank. In order for the banks creditors to continue providing funds to the bank, the following incentive constraint must always hold:

$$V_{j,t} \geqslant \theta[(1-p(\bar{\psi}_t))l_{j,t} - \omega m_{j,t}]. \tag{16}$$

Bank's value must be greater or at least equal with the value of its divertable assets. When this constraint holds bankers have no incentive to steal from their creditors. In the case where the constraint binds a spread between the risky and the riskless interest rate emerges. As I will show below this will be the case in times of a negative shock. A reduction of the banker's net worth will make the constraint to bind and a spread increase occurs.

The value of the bank at the end of period $t-1$ must satisfy the Bellman equation:

$$V_{j,t-1}(l_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) = E_{t-1}\Lambda_{t-1,t}\sum_{i=1}^{\infty}\{(1-\sigma_B)n_{j,t}^B + \sigma_B \max_{d_{j,t}}[\max_{l_{j,t},x_{j,t},m_{j,t}} V_t(l_{j,t}, x_{j,t}, d_{j,t}, m_{j,t})]\}. \tag{17}$$

Banker's problem is to maximize (15) subject to the balance sheet (13) and liquidity constraint (16).

Proposition 1. A solution to the banker's dynamic program is

$$V_{j,t}(l_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}) = A^B n_{j,t}^B.$$

The marginal value of the banker's net worth A^B is then:

$$A^B = \mu_t \phi_t + \nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2.$$

μ_t is the spread, ϕ_t is the maximum leverage and $\nu_{d,j,t}$ is the marginal loss from deposits.

Proof. See appendix D.

The proposition clarifies the role of the bank's net worth in the model. We can rewrite the incentive constraint using the linearity of the value function as

$$\frac{A^B}{\theta} \geqslant \frac{[(1-p(\bar{\psi}_t))l_{j,t} - \omega m_{j,t}]}{n_{j,t}^B}.$$

The adjusted leverage of a banker cannot be greater than A^B/θ. The right hand side shows that as the net worth of the banker decreases the constraint is more likely to bind. Proposition 1 also implies that even there is heterogeneity in the bankers' holdings and net worth, this does not affect aggregate dynamics. Hence, the transition from the individual to aggregate variables takes place in the same way as in the previous section.

The maximum adjusted leverage ratio of the bank is defined as

$$\phi_{j,t} = \frac{\nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2}{(1-p(\bar{\psi}_t))\theta - \mu_t}. \tag{18}$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits $\nu_{d,j,t}$ and reserves and on the excess value of bank assets μ_t. As the credit spread increases, banks franchise value V_t increases and the probability of a bank to divert its funds declines. From the other hand as the proportion of assets that a bank can divert, θ increases, the constraint binds more.

Aggregation.— Aggregate net worth is the sum of the new bankers' and the existing bankers' equity: $N^B_{t+1} = N^B_{y,t+1} + N^B_{o,t+1}$. Young bankers' net worth is the earnings from loans multiplied by ξ_B which is the fraction of asset gains that being transferred from households to the new bankers

$$N^B_{y,t+1} = \xi_B[R_{l,t}L_t]$$

and the net worth of the old is the probability of survival for an existing banker multiplied by the net earnings from assets and liabilities

$$N^B_{o,t+1} = \sigma_B[R_{l,t}L_t + R_{x,t}X_t - R_m M_t - R_t D_t - \Phi_t(X_t)].$$

2.3. Fiscal, Monetary Policy and Resource Constraint

Government acts as both fiscal and monetary authority. Its fiscal role is limited on collecting lump sum taxes T_t to finance its public expenditures G_t. I assume that the level of the government expenditures is at a fixed level relative to output (γ^G) and subject to a transitory shock g_t that follows an AR(1) process. Hence, $G_t = (\gamma^G Y_t)g_t$. As a monetary authority, it supports the banking liquidity by providing M_t funds at interest rate $R_{m,t}$, it accommodates banks' excess reserves X_t at an interest rate $R_{x,t}$ and issues bonds to finance its expenses $D_{g,t}$, bought by households at an interest rate R_t. The government budget constraint thus is:

$$G + M_t - D_{g,t} - X_t = T_t + R_{m,t}M_{t-1} - R_t D_{g,t-1} - R_{x,t}X_{t-1}. \tag{19}$$

The monetary authority's liquidity policy follows the policy rule introduced by Gertler and Karadi (2011). At every crisis episode, loosely defined as a period when the credit spread increases, the central bank increases the liquidity provision to the banking sector according to the following rule:

$$\chi_{m,t} = \chi_m + \kappa_m \mathbb{E}_t[(R_{l,t+1} - R_{t+1}) - (R^{ss}_l - R^{ss})], \tag{20}$$

where $\chi_{m,t} = \frac{M_t}{L_t+X_t}$ is the fraction of the total bank assets financed through LTRO and χ_m is its steady state value. $(R^{ss}_l - R^{ss})$ is the steady state premium. The intensity of the liquidity intervention depends on the liquidity feedback parameter κ_m which is always positive. According to this rule, liquidity to the banking sector increases as the spread increases relative to its steady state level.

Finally, the resource constraint of the economy is:

$$Y_t = C_t + I_t[1 + \tilde{f}\Big(\frac{I_t}{I_{t-1}}\Big)] + G_t + \Phi(X_t) + \mu\psi_t R_{k,t} Q_t K_t.$$

Final output may be either transformed into consumption good, invested, used by the government for its spending or used up in monitoring costs and reserve costs. Lastly, the amount of capital available for production is given by $K_t^f = (1 - \mu G_t)K_t$. Available capital equals the initial capital net of the capital destroyed due to the expected monitoring costs.

3. Bankers' Optimal Asset Allocation and the Risk-Taking Channel

This section presents in detail the main mechanisms of the model and is divided in two parts. In the first, I show the optimal allocation decisions of the bankers along with how the risk-taking incentives affect the allocation of capital. In the second, the economic mechanism that drives the adverse effects of liquidity injections in the presence of risk and bankruptcy costs is explained. To enhance clarity, the explanation is accompanied by a graph that captures the main ingredients of the mechanism in a static framework.

3.1. Bankers' Optimal Allocation

The following relations describe how the bankers allocate their funds between reserves and loans and how the risk-taking channel emerges from the optimal decisions. These yield from the solution of the bankers problem[17]. Next, I describe how the interest rates are determined endogenously in the model.

At optimum, the demand for excess reserves for the bank is such that the marginal benefit for investing in one unit of reserves, $\nu_{x,j,t}$, equals the marginal cost from using on unit of short term debt $\nu_{d,j,t}$ and the marginal cost of raising one unit of reserves[18].

$$\nu_{x,j,t} = \nu_{d,j,t} + \Phi'(x_{j,t}).$$

[17] The full solution is presented in detail in Appendix D.

[18] This relation yields directly from the first order condition of the banker's problem with respect to excess reserves x_t.

The bank's credit supply to non-financial firms is:[19].

$$
l_{j,t} = \phi_{j,t} n^B_{j,t} + \underbrace{\frac{1}{1-p(\bar{\psi}_t)}(\omega m_{j,t})}_{\text{risk-taking}}. \tag{21}
$$

Available credit depends on two components: the banks' own funds and the liquidity received by the central bank. When the liquidity policy is absent ($m_{j,t} = 0$), then the bank adjust its loan supply according to the product between leverage $\phi_{j,t}$ and net worth. At turbulent times, when the central bank injects liquidity into the system ($m_{j,t} > 0$) banks that receive LTRO funds will increase their lending compared to the no liquidity case but they engage in risky lending. Banks search for yield and increase the lending to the non-financial firms which during crises have a higher likelihood of default. I denote this as the *risk-taking* component. Risky lending occurs using the central bank funds and this captures the risk-shifting channel of monetary policy.

The bank's demand for loans is determined from the expected lending rate.

$$
\mathbb{E}_t\, R_{l,t+1} = \underbrace{\frac{\lambda_t}{(1+\lambda_t)} \frac{\theta}{\mathbb{E}_t\, \Lambda_{t,t+1}\Omega_{t+1}}}_{\text{liquidity component}} + \underbrace{\mathbb{E}_t\, R_{t+1} \frac{1}{1-p(\psi_t)}}_{\text{risk component}}.
$$

Two components determine the expected lending rate. The first, is due to the binding funding constraints for the bankers. When the constraint binds, bankers cannot get new funding to explore new profitable activities. Hence they adjust the loan rate. This will be referred as the *liquidity component*. The second one reflects the compensation that bankers demand when the firms' probability of default increases. This is the *risk component*.[20]

The interest rate of the LTRO funding is endogenously determined as follows:

$$
R_{m,t} = \omega R_{l,t} + \left(1 - \omega \frac{1}{1-p(\bar{\psi}_t)}\right) R_t.
$$

The liquidity funding interest rate is a weighted average of the loan rate and the deposit rate. I calibrate the parameter values in order to have a liquidity funding interest rate below the loan rate but slightly above the riskless rate. Lastly, the interest rate on reserves is defined as a function of the riskless rate $R_{x,t} = \tau R_t$.

[19] The optimal lending decision of the banker yields from the compatibility constraint in conjunction with the FOC for l_t and m_t under the assumption that the constraint is always binding.

[20] Bocola (2016) using another source of uncertainty (an increase of future sovereign default) instead of the firms' default, shows the existence of the same two sources of frictions between the loan and the risk free rates.

3.2. The Adverse Effects of Liquidity Injections

The main result of this study is the negative consequences of the liquidity injections when borrowers' default is an equilibrium outcome. The economic mechanism is the following. Consider an increase of the entrepreneur's risk. This reduces firms' net worth, raises their probability of default due to standard financial accelerator effects and reduces banks' net worth. As equation (21) shows, when liquidity policy is absent, and the shock hits, banks cut on lending. The striking result comes up when a central bank in order to halt the recession provides liquidity to the banking sector.

Newly injected liquidity m_t relaxes the constraint of the banks as (16) shows. This, following equation (21), makes banks supply fresh credit to - now - riskier firms thus increasing the risk exposure of banks. As a result, firms probability of default will increase more and more monitoring -bankruptcy- costs have to be paid by the bankers thus more capital has to be destroyed. A threshold exists when the loss from the destruction of capital is equal to the gain from the new capital injections. When the capital destruction dominates the capital injections, as it is the case presented by the estimated model, less capital is available for production. Reinforced with financial accelerator effects, this makes the recession more severe.

Figure 4 gives a static example of the mechanism in the case of the marginal entrepreneur with zero net worth. In that case the balance sheet of the entrepreneur is $Q_t K_t = L_t$. All the loans from the banks are transformed into capital purchased from the capital goods producers. The initial mass of entrepreneurs is F and the initial capital is K_t^A. When a risk shock hits, the dispersion of the idiosyncratic shock of the entrepreneurs' increases and this leads to a higher number of defaults. The total mass of firms reduces to F^A. Due to the monitoring costs, the bank has to pay a fraction of the capital of the defaulting entrepreneurs. The capital that is destroyed by this operation is the horizontal line area on the top right of the graph. The available capital for production after the shock in the no liquidity case $K_t^{A,f}$ is shown by the doted area.

When the central bank provides liquidity the incentive constraint of the bank relaxes and this leads to a credit extension. The new higher level of capital is K_t^B, above K_t^A. Due to the risk shock, the low price of capital and the low net worth now more firms default. When banks are willing to supply higher credit to risky firms this implies a higher probability of default, with larger expected monitoring costs for the lender. Therefore the total mass of firms reduces to F^B, which is lower than the mass of firms F^A in the no policy case. Now the total capital that is destroyed due to monitoring costs is the thick outlined square and the available capital for production $K_t^{B,f}$ is the graph area net of the capital destroyed due to monitoring.

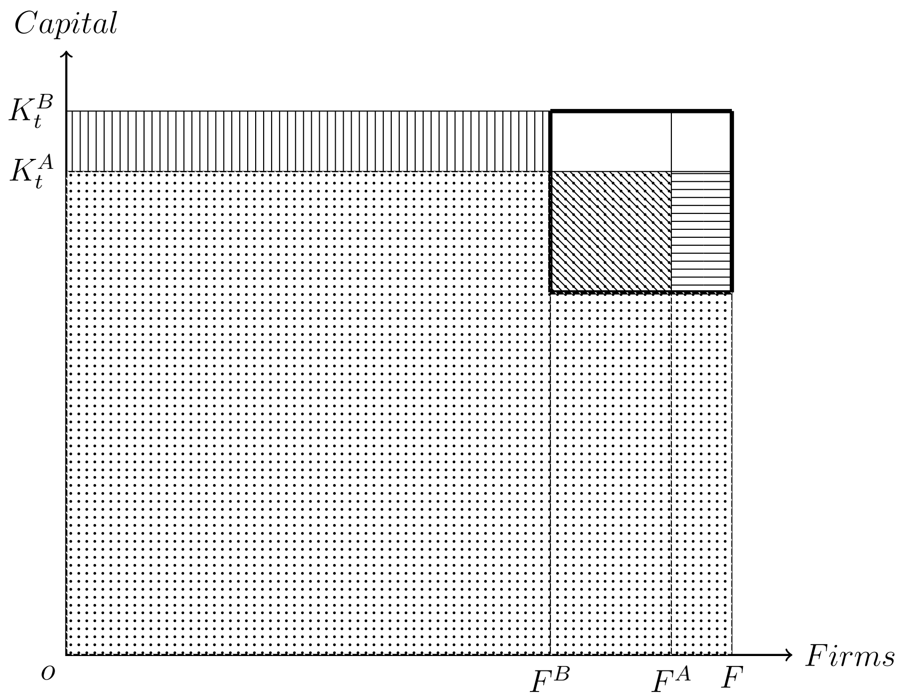

Fig. 4. Adverse Effects of Liquidity Injections

The following proposition presents the condition under which the gains from liquidity injections are smaller than the bankruptcy costs.

Proposition 2. If the gains from liquidity $(K_t^B - K_t^A)$ are smaller than the losses due to increased expected monitoring costs $(\mu G_t^B(\bar{\psi}_{t+1})K_t^B - \mu G_t^A(\bar{\psi}_{t+1})K_t^A)$, the available capital after the liquidity expansion $K_t^{B,f}$ will be lower than the available capital without the liquidity policy $K_t^{A,f}$.

Proof. The available capital after the destruction due to monitoring costs in the no policy case is: $K_t^{A,f} = (1 - \mu G_t^A)K_t^A$ and in the policy case: $K_t^{B,f} = (1 - \mu G_t^B)K_t^B$. The difference between the liquidity policy available capital and the no policy is:

$$K_t^{B,f} - K_t^{A,f} = (K_t^B - K_t^A) - (\mu G_t^B(\bar{\psi}_{t+1})K_t^B - \mu G_t^A(\bar{\psi}_{t+1})K_t^A)$$

Since $G_t(\bar{\psi}_{t+1})$ is increasing on the likelihood of default, for the above expression to be negative it must be that $(\mu G_t^B(\bar{\psi}_{t+1})K_t^B - \mu G_t^A(\bar{\psi}_{t+1})K_t^A) > (K_t^B - K_t^A)$. □

In the quantitative analysis following in the next section using the estimated model for the Euro Area, the gains from liquidity injections are proven to be smaller than the losses giving rise to the adverse effects of the liquidity policy.

4. Estimation and Model Inference

This section presents the model estimation, the priors and the posteriors for the analysis and descriptive statistics. Finally, I compare the model's moments with the Euro Area data moments at the prior values of the parameters.

4.1. Data

I use quarterly Eurozone data from Q1:2000 to Q1:2017. This includes four standard variables used in macroeconomics analyses: GDP, consumption, investment and the base interest rate of the ECB. Additionally, 4 financial variables are used: credit to non-financial corporations, credit spread between the lending rate and the short rate, bank reserves and non-financial firms net worth. The NFC net worth is obtained through the Dow Jones index for the Euro area. The rest of the variables are downloaded from the ECB Statistical Warehouse and the European Commission. Before the estimation all the variables apart from the credit spread and interest rate are transformed into real variables by dividing with the GDP delfator. Then they are expressed as per capita terms by dividing them with the active labour force.

Prior to the analysis following Christiano et al. (2014) I transform the data as follows: For GDP, consumption, investment, credit, reserves and net worth I take the logarithmic first difference and then remove the sample mean. I leave the interest rates in levels removing the sample mean.

4.2. Priors and Posteriors

The parameters in the model are divided into two categories. The first set of parameters is calibrated at the standard values in the business cycles literature and the second set is estimated. I fix the depreciation rate of capital δ at 0.025, the capital share α at 0.33 and the Inverse Frisch elasticity of labour supply ϵ at 0.33 as in Gerali, Neri, Sessa, and Signoretti (2010) and Gelain (2010) where both study the Euro Area economy. The relative utility of labour χ is calibrated at 5.584 such as to ensure a level of labour hours close to 1/3 in steady state. The ratio of government spending to GDP is fixed at 0.2, consistent with the Euro Area data (see for example Christoffel and Schabert (2015)) and the discount factor β at 0.9973 which is equivalent to a 4% annual interest rate, a value close to the historical time series of the interest rate and also in line with several estimations for the Euro Area.

I want to ensure that the model captures a bankers' leverage ratio of 4 and a bank capital to lending ratio of 0.25 close to the value suggested by Christoffel and Schabert

(2015). Therefore, σ_B and ξ_B are calibrated at 0.955 and 0.009 respectively. In order to capture the unconventional character of the LTRO policy, I choose a very low level for the steady state value of the LTRO operations χ_m equal to 0.1%. I set the rate on reserves equal to the rate of the riskless asset which is the case according to the pre-2009 Euro data and I define τ equal to one. Lastly, I target a marginally positive level of excess reserves of around 1% in steady state by calibrating ϵ to -0.2. In this way I allow for some liquidity management gains from holding reserves. The parameter values are presented in Table 1.

Parameters	Definition	Value
	Households	
β	Discount rate	0.99
χ	Relative utility weight of labor	5.584
ϵ^B	Inverse Frisch elasticity of labor supply	0.333
	Banks	
ω	Divertable fraction of LTRO	0.3
ξ_B	Entering bankers initial capital	0.009
σ_B	Bankers' survival rate	0.955
ϵ	Gains from reserves	-0.2
τ	Interest on reserves relative to the riskless rate	1
	Resource constraint and government policy	
δ	Depreciation of capital	0.025
α	Capital share	0.33
γ^G	Steady state fraction of government expenditures to output	0.2
χ_m	Steady state value of the LTRO	0.001

Table 1: Calibrated Parameter Values

The second set of parameters consists of the estimated parameters following the Bayesian techniques surveyed by An and Schorfheide (2007). There are two categories of the parameters, one related to the bankers', entrepreneurs' and investment parameters and the other set which are associated with the shocks in the model. Table 2 shows the prior distribution used for each of the parameter, its mean and standard deviation and also the mode of the posterior distribution.

The steady-state value of the risk shock has a mode of its posterior distribution of 0.3180 which is close to the findings of Queijo von Heideken (2009) for the Euro Area. The monitoring cost mode of the posterior distribution is 17.95%. It has been estimated by Queijo von

Parameters	Definition	Prior dist	Prior Mean	Std	Posterior Mode
	Economic Parameters				
κ	Costs of reserve holdings	beta	10	3.5	13.0122
μ	Monitoring costs	beta	0.15	0.073	0.1795
η	Inverse elasticity of net investment	norm	5	3	1.5074
ξ_E	Transfer to entering entrepreneurs	beta	0.005	0.002	0.0023
θ	Fraction of assets divertable	beta	0.15	0.07	0.1585
σ_ψ^{SS}	Steady-state idiosyncratic shock	beta	0.2	0.075	0.3180
	Shocks				
	Autocorrelations				
ρ_σ	Risk shock	beta	0.5	0.2	0.9796
ρ_ψ	Capital quality shock	beta	0.5	0.2	0.9936
ρ_A	Productivity shock	beta	0.5	0.2	0.8557
ρ_g	Gov. spending shock	beta	0.5	0.2	0.9318
ρ_ζ	Marginal efficiency of inv. shock	beta	0.5	0.2	0.9982
ρ_{ζ_c}	Consumption pref. shock	beta	0.5	0.2	0.8881
ρ_{ζ_x}	Excess reserve penalty shock	beta	0.5	0.2	0.9062
	Std, shock innovations				
σ	Risk	invg	0.0123	0.2	0.07169
ψ	Capital quality	invg	0.0123	0.2	0.04788
A	Productivity	invg	0.0123	0.2	0.04744
g	Gov. spending	invg	0.0123	0.2	0.02351
ζ	Marginal efficiency of investment	invg	0.0123	0.2	0.06162
ζ_c	Consumption pref.	invg	0.0123	0.2	0.02401
ζ_x	Excess reserves penalty	invg	0.0123	0.2	0.02301

Table 2: Estimated Parameter Values

Heideken (2009) that in the Euro area the monitoring costs are about 27% and it is close to the value suggested by Christiano et al. (2014) of 21.49%. The mode parameter for transfers to the new entrepreneurs ξ^E is 0.0023 and the steady-state idiosyncratic shock 0.3180, both close to the values shows in Christiano et al. (2014). The estimated diversion parameter θ yields a value close to the common found interval in the literature [0.15-0.30]. Lastly, the inverse elasticity of net investment to the price of capital η equal to 1.50 a value significantly lower than the estimated value from Gerali et al. (2010) for the Eurozone.

Table 3 reports the steady-state properties of the model when parameters are set to their mode under the prior distribution. The data values are calculated as the average of each variable relative to the average level of output. The model manages to deliver well the ratios of different variables. Consumption, investment, government spending and reserves follow closely the data moments. Credit to output is capturing the fact that is far above all the other statistics but the model overestimates it's value.

Variable	Model	Data
C/Y	0.592	0.561
I/Y	0.223	0.216
L/Y	3.22	1.68
G/Y	0.200	0.182
X/Y	0.013	0.011

Table 3: Steady State Properties at Priors vs. Euro Data

5. Quantitatively Analysis

This section illustrates the policy recommendations that the model can provide by performing two different sets of experiments. In what follows, I present the impulse response functions to a number of model's structural shocks and then I estimate the welfare gains (or costs) from a number of different policy actions. To solve the model I apply an approximation to the policy functions. The numerical strategy is based on perturbation methods as in Schmitt-Grohé and Uribe (2004) and is well-suited for the specific modelling framework, given the large number of state variables.

5.1. Impulse Response Functions

5.1.1. Risk-Taking Channel

The first objective is to simulate an economic downturn, similar to the one the Euro economy has experienced in the end of 2007, and see how the model economy responds. Then, I show the results of the same exercise when the central bank supplies liquidity following the feedback rule (20). I provide the impulse response functions to a 1% standard deviation increase in the risk shock for both cases. In the first exercise, the feedback parameter κ_m in the policy rule is set to 0 whereas in the second case to 100 following the value chosen by Gertler and Karadi (2011).

The results are reported in Figure 5. The blue line (circles) shows the responses to an 1% standard deviation increase of the risk shock when the central bank does not provide liquidity. The economic mechanism here is as follows: as the riskiness of the entrepreneurial project increases banks charge higher interest rates to cover the costs, thus the spread increases. Entrepreneurs now are more likely to default as it's more difficult to repay back their loans.

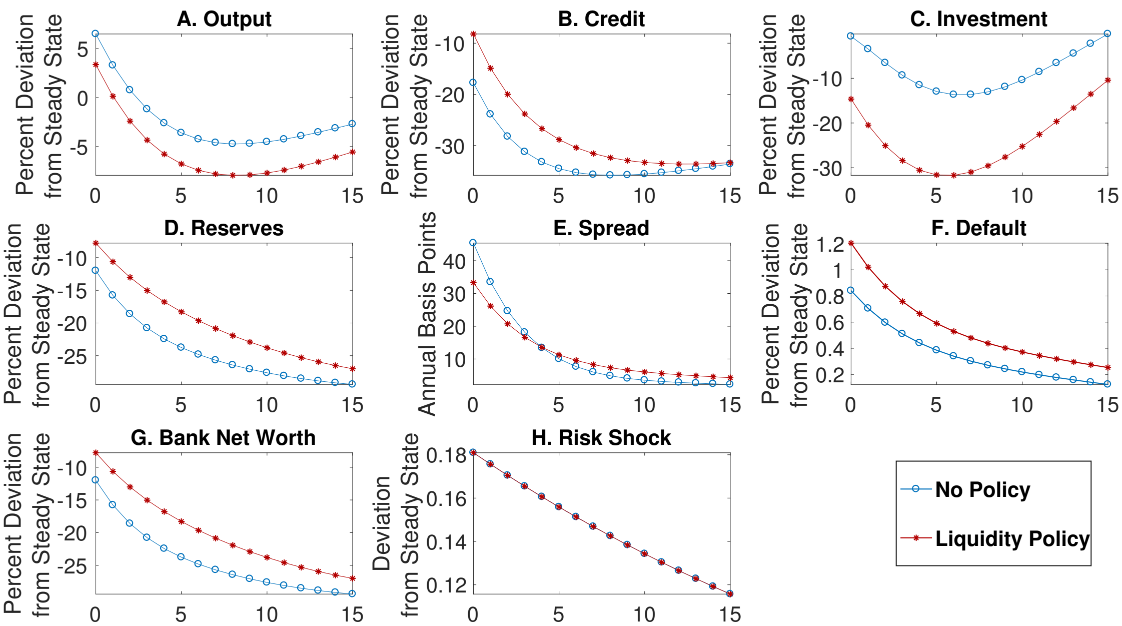

Fig. 5. Impulse responses to an increase of the risk shock

Banks lend less and credit drops. With fewer financial resources, entrepreneurs purchase less capital, which reduces investment. This leads to a fall in output and consumption. The fall in investment produces a fall in the price of capital, which reduces the net worth of entrepreneurs, and this magnifies the impact of the jump in risk through financial accelerator effects.

The red line (stars) displays the responses when the central bank follows the liquidity feedback rule. Extra liquidity provides extra funds for the banks, relaxes their constraint and allows them to reduce the lending interest rate and increase credit. They also increase their reserve holdings as they use a portion of the fresh liquidity to invest in the safe asset. The central bank policy improves the health of the financial institutions and that can be seen by the increase in their net worth. On the credit demand side, the higher level of credit increases the firms' likelihood of default as they leverage more due to the lower cost of credit. This occurs in conjunction with the low level of net worth and capital price. Since more defaults occur, monitoring costs for banks increase and more capital is being destroyed. Therefore, lower entrepreneurial net worth leads to less capital purchase and a higher drop in investment and output compared to the no policy scenario. According to this result and following Proposition 2, the capital losses from the bankruptcy costs dominate the capital gains from the liquidity injections.

The above mechanism describes the potential problem of the open market operations in turbulent times. Although banks spend the liquidity injected to new credit, this credit ends up to insolvent non-financial corporations. The liquidity provided by the central bank is driving excessive risk-taking from the banks as the riskiness of the firms has increased and banks face moral hazard problems.

5.1.2. Negative Interest Rates

I continue with an exercise trying to capture the effect of the negative rates on reserves. This is simulated by an increase in the penalty rate for holding reserves. In other words, banks have to pay more to accumulate excess reserves. It encapsulates the recent European Central Bank policy of charging fees to reserves. Figure 6 shows the response of a set of variables to an 1% standard deviation increase in the reserves' penalty level.

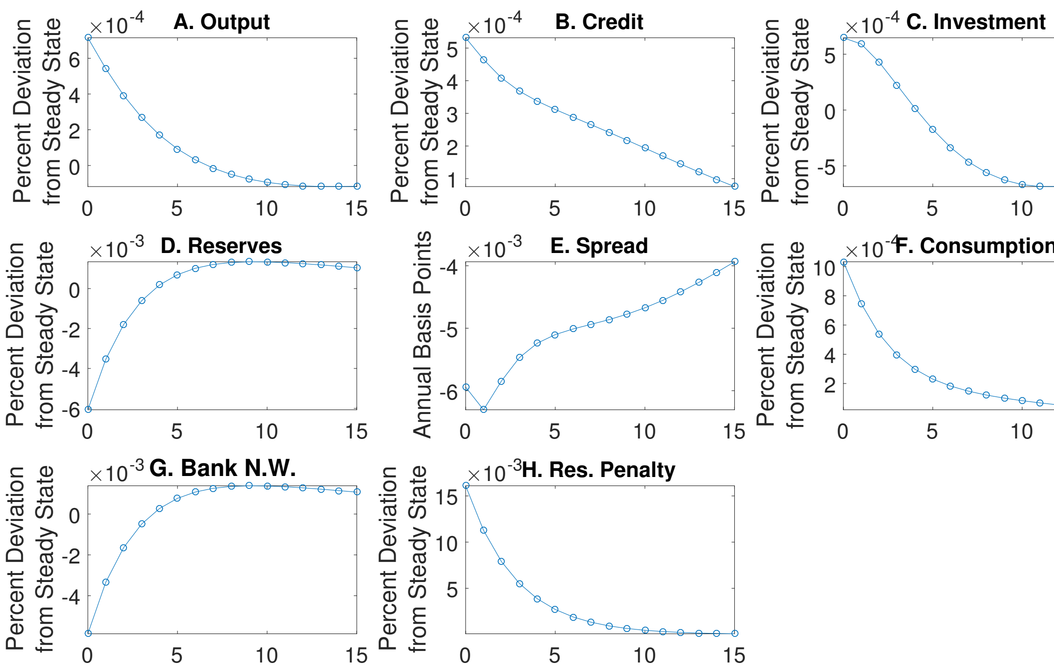

Fig. 6. Impulse responses to an increase of the reserve penalty

It can be seen that as the penalty for reserves increases, banks at least in the sort run give up their reserves position and extend their credit supply. That gives a push to the economy. Entrepreneurs borrow more and hence they invest more. This has an immediate consequence on output and consumption which both increase. Since the model cannot account for negative interest rates for the aforementioned reasons described in the model section, there is no estimate on what is the optimal level of interest rates that will stimulate lending. Nevertheless, the above exercise presents a general evidence that the recently announced policy of the European Central Bank to tax reserves can stimulate lending.

As a second exercise associated with the negative rates, I measure the stochastic steady state path of reserves and credit for different values of the penalty parameter rate that the central bank sets. Figure 7 shows the stochastic steady state path of reserves and credit for parameter values $\kappa_m \in [0, 100]$. As the penalty rate increases, banks hold less reserves and expand their credit to non-financial corporations thus increasing the welfare gains. This comes in line with the unprecedented policy of the ECB to charge the banks of the Euro Area for holding reserves. As the cost of reserves increases, banks will reduce their reserve holdings and increase credit. At the same time, in order to achieve the reserves reduction to a substantial level, the penalty parameter must increase to almost ten times the initial steady state value. Bringing the above results to the recent central bank unconventional measures, the general intake is that negative interest rates will make the banks adverse in increasing credit but only when the rates that are charged are negative enough.

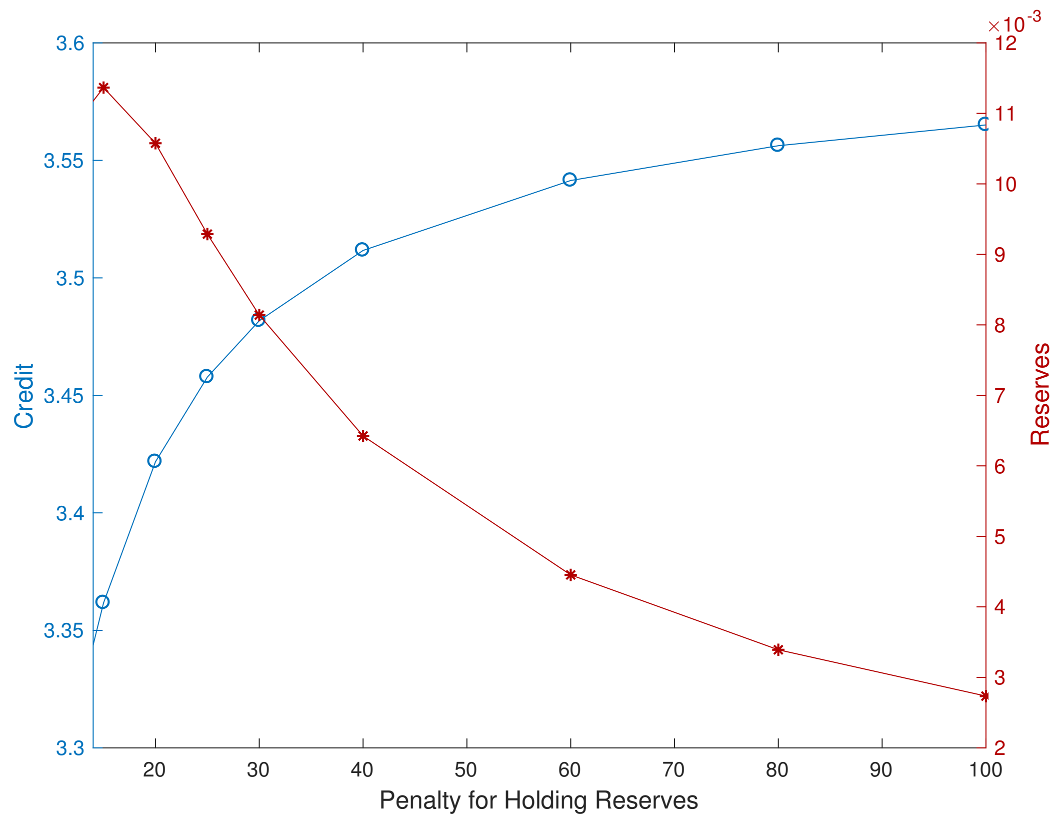

Fig. 7. Stochastic Steady State Path of Credit and Reserves

5.2. *Measuring Welfare Costs*

In order to conduct policy analysis, I will now present the welfare costs (or gains) in terms of consumption units between i) the adoption of aggressive liquidity supply scheme by the central bank and ii) the no policy rule.

Since the non-stochastic steady state for the two different regimes is different, the unconditional expectation of welfare leaves out the dynamics associated with the stochastic steady state. Therefore, following Schmitt-Grohé and Uribe (2007) I proceed with the welfare conditional on the initial state being the non-stochastic steady state. At time zero, the state vector is the same for both policies, in other words all state variables equal their steady states. This ensures that in both regimes we start from the same initial values. Given that in a first order approximation the welfare $\mathcal{W}_t$ equals to it's non-stochastic steady state I will proceed with a second order approximation to determine the effects of different regimes on lifetime utility. I define the welfare associated with the no policy scheme conditional on a particular state of the economy in period 0 as:

$$\mathcal{W}_0^n = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^n, N_t^n),$$

where the C_t^n, N_t^n denote the consumption units and labour hours spend under the no policy scheme. In a similar way I define the conditional welfare associated with the liquidity supply scheme as:

$$\mathcal{W}_0^l = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^l, N_t^l),$$

where C_t^l, N_t^l denote the consumption units and labour hours spend under the liquidity supply scheme.

Let λ^c be the conditional welfare cost (or gain) for the consumer of adopting a liquidity policy rather than a no action policy by the central bank. In other words λ^c is the fraction of consumption that the household would need each period in the liquidity supply regime to yield the same welfare as would be achieved in the no policy regime. Formally λ^c is chosen to solve

$$\mathcal{W}_0^l = E_0 \sum_{t=0}^{\infty} \beta^t U((1+\lambda^c)C_t^n, N_t^n).$$

A positive value for λ^c means that the household prefers the liquidity policy regime - i.e. it would need extra consumption when the liquidity regime is on to be indifferent between the two regimes. In contrast, a negative value of λ^c means that the household prefers the no policy regime. Substituting the utility function given in equation (1) we can rewrite the above expression as:

$$\begin{aligned}\mathcal{W}_0^l &= E_t \sum_{i=0}^{\infty} \beta^i [\ln((C_{t+i} - \gamma C_{t+i-1})(1+\lambda^c)) - \frac{\chi}{1+\epsilon} N_{t+i}^{1+\epsilon}] \\ &= \frac{\ln(1-\lambda^c)}{1-\beta} + \mathcal{W}_0^n.\end{aligned}$$

Solving for λ^c we have:

$$\lambda^c = \exp\{(\mathcal{W}_0^l - \mathcal{W}_0^n)(1-\beta)\} - 1. \tag{22}$$

Table 4 shows the welfare analysis results. It presents the total value of conditional welfare in the scenario with liquidity policy and the no policy scenario and also the consumption equivalent metric that yields from the transition between the two policies. The consumption equivalence is measured in percentage terms. This metric is an indication of how much consumption units in percent are lost or gained from the transition to the new policy. The conditional welfare as is reported in Table 4 decreases as we move from the no policy regime to the liquidity policy regime. The loss is about -0.075 % of consumption units. Hence, the liquidity policy is not considered to be welfare improving.

Additional to the conditional welfare comparisons, I present the second moments of selected variables for the two different policy regimes. As expected, consumption volatility increases after the liquidity policy, from 0.73 to 0.77. Output and credit volatility behave in a similar manner and also the discount rate and the credit spread as the liquidity policy stabilizes and reduces the spread.

	No Policy	Liquidity Policy
Welfare		
Conditional Welfare Cost	0	-0.07487
Standard Deviation		
Output	0.73164	0.76671
Consumption	0.74892	0.77856
Investment	1.27563	1.39319
Credit	0.39258	0.49376
Spread	0.14932	0.29376
Discount Rate	0.57421	0.59422

Table 4: Welfare Costs and Second Moments

6. Conclusion

Since 2008, the ECB has massively increased its balance sheet in order to provide liquidity to financial institutions. Nevertheless, the macroeconomic environment seems still fragile. Banks have increased their reserves holdings while credit growth is in low levels. In this paper I assess the effectiveness of the main liquidity mechanism employed by the ECB, the LTROs, using an estimated DSGE model with financial frictions on the demand and the supply side of credit. I find that LTROs improved the banks' health but the macroeconomy would have been better off should the liquidity policy hasn't taken place. This result follows from the risk-shifting channel of monetary policy.

The main economic intuition is as follows. Consider an increase in entrepreneurial risk which reduces firms' net worth, raises their probability of default and sets off a recession through a Bernanke et al. (1999) financial accelerator mechanism. To counter such a recession, a central bank might provide liquidity directly to the banking sector as in the the case of ECB's LTROs. While such a policy would potentially halt the economic downturn, it simultaneously makes banks supply fresh credit to - now - riskier firms giving rise to a risk-taking channel. Reinforced by the lower cost of borrowing, firms leverage up their net worth (which in and of itself increases their likelihood of default). Higher default rates lead to higher bankruptcy costs and less available capital for production. A threshold exists where the capital gains from liquidity injections are equal to the capital losses due to bankruptcy costs. In the estimated model I find that capital losses dominate the capital injections and lead to lower investment and output.

Measuring the welfare costs of the liquidity provision against the no liquidity scenario confirms the above result. Specifically, I show that there is a welfare loss of -0.075% in consumption equivalent metric, constituting this policy not welfare improving.

Finally, I assess the effectiveness of negative interest rates. Given the impossibility of interest rates in the negative territory in the model due to the lower bound constraint, I employ a penalty function for accumulating reserves. When banks accumulate reserves below the threshold they have some gains (e.g. efficient and liquidity gains). When the level of reserves surpass the threshold banks pay a cost to the central bank similar to a tax on reserves. I show that an increase in the reserve penalty will reduce banks' reserve position and increase credit supply.

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Appendix A Household's Problem

Let $u_{c,t}$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ denote the household's stochastic discount factor (the intertemporal marginal rate of substitution):

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{c,t+1}}{u_{c,t}}, \tag{A.1}$$

$$u_{c,t} = (C_t - \gamma C_{t-1})^{-1} - \beta \, \mathbb{E}_t \, \gamma (C_{t+1} - \gamma C_t)^{-1}.$$

Let λ be the Lagrange multiplier associated with the household problem, the Lagrangian is

$$\mathcal{L} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \big\{ \ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} N_{t+i}^{1+\epsilon} + \lambda_t [W_t N_t + \Pi_t + R_t D_{h,t} - (C_t + T_t + D_{h,t+1})] \big\}.$$

The first order conditions yield:

$$\frac{\theta \mathcal{L}}{\theta C_t} : u_{c,t} - \lambda_t = 0 \tag{A.2}$$

$$\frac{\theta \mathcal{L}}{\theta D_{h,t+1}} : -\lambda_t + \beta \lambda_{t+1} (R_{t+1}) = 0 \tag{A.3}$$

$$\frac{\theta \mathcal{L}}{\theta N_t} : -\chi N_t^{\epsilon} + \lambda_t W_t = 0 \tag{A.4}$$

Combining (A.2) and (A.3) we get the Euler equation

$$\mathbb{E}_t \, \Lambda_{t,t+1} R_{t+1} = 1$$

and by combining (A.2) and (A.4) we get the optimality condition for labour supply

$$u_{c,t} W_t = \chi N_t^{\epsilon}$$

Appendix B Entrepreneur's Problem

Let $\mathcal{L}$ be the Lagrangian of the maximization problem and λ_t^e the Lagrange multiplier associated with the zero profit condition.

$$\mathcal{L} = [1 - \Gamma(\overline{\psi_{t+1}}) R_{k,t+1} Q_t K_{t+1}] + \lambda_t^e [R_{k,t} Q_t K_t [\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})] - R_{t+1}(Q_t K_t - N_t^e)].$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta\mathcal{L}}{\theta K_t} : 1 - \Gamma(\overline{\psi_{t+1}})R_{k,t+1} + \lambda_t^e[\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})R_{k,t+1} - R_{t+1}] = 0 \tag{B.1}$$

$$\frac{\theta\mathcal{L}}{\theta\overline{\psi_{t+1}}} : -\Gamma'(\overline{\psi_{t+1}}) + \lambda_t^e[\Gamma'(\overline{\psi_{t+1}}) - \mu G'(\overline{\psi_{t+1}})] = 0 \tag{B.2}$$

From equation B.2 we get

$$\lambda_t = \frac{\Gamma'(\overline{\psi_{t+1}})}{\Gamma'(\overline{\psi_{t+1}}) - \mu G'(\overline{\psi_{t+1}})}. \tag{B.3}$$

Inserting B.3 to B.1 we get:

$$R_{k,t} = \frac{\Gamma'(\overline{\psi_{t+1}})}{(\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}}))\Gamma'(\overline{\psi_{t+1}}) + (1 - \Gamma(\overline{\psi_{t+1}}))(\Gamma'(\overline{\psi_{t+1}}) - \mu G'(\overline{\psi_{t+1}})}R_t,$$

which gives the external finance premium as shown in the BGG:

$$\mathbb{E}_t\, R_{k,t+1} = \mathbb{E}_t\, \rho(\overline{\psi_{t+1}})R_{t+1}$$

where $\rho(\overline{\psi_{t+1}})$ is given by

$$\rho(\overline{\psi_{t+1}}) = \frac{\Gamma'(\overline{\psi_{t+1}})}{[(\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}}))\Gamma'(\overline{\psi_{t+1}}) + (1 - \Gamma(\overline{\psi_{t+1}})(\Gamma'(\overline{\psi_{t+1}}) - \mu G'(\overline{\psi_{t+1}}))]}.$$

Appendix C Entrepreneur's choice of remain

Proof. The zero profit condition is

$$R_{k,t}Q_tK_t[\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})] \geqslant R_{t+1}(Q_tK_t - N_t^e)$$

and divided by N_t^e becomes

$$R_{k,t}\frac{Q_tK_t}{N_t^e}[\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})] \geqslant R_{t+1}(\frac{Q_tK_t}{N_t^e} - 1).$$

Substituting the definition of N_t^e

$$R_{k,t}\frac{Q_tK_t}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t))R_{k,t}Q_{t-1}K_{t-1}}[\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})] \geqslant R_{t+1}(\frac{Q_tK_t}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t))R_{k,t}Q_{t-1}K_{t-1}} - 1)$$

we have

$$\frac{[\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})]}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t))} \geqslant R_{t+1}(\frac{1}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t))R_{k,t}} - 1)$$

and we get the equation for σ_t^e

$$\sigma_t^e = \frac{1}{R_k(1 - \Gamma(\bar{\psi}_t))} - \frac{\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})}{R_t(1 - \Gamma(\bar{\psi}_t))} - \xi$$

and the derivative with respect to $\bar{\psi}$

$$\frac{\partial \sigma_t^e}{\partial \bar{\psi}} = \frac{\Gamma'(\bar{\psi}_t))R_k}{[R_k(1 - \Gamma'(\bar{\psi}_t))]^2} - \frac{\Gamma'(\overline{\psi_{t+1}}) - \mu G'(\overline{\psi_{t+1}})}{R_t(1 - \Gamma'(\bar{\psi}_t)))} - \frac{\Gamma(\overline{\psi_{t+1}}) - \mu G(\overline{\psi_{t+1}})R_t\Gamma'(\overline{\psi_{t+1}})}{[R_t((1 - \Gamma'(\bar{\psi}_t))]^2}.$$

The $\sigma_{E,t}$ the values of $[0, 1]$ (so it is actually a probability measure), when $\bar{\psi} \in [0.49, 0.65]$, everything else remain constant. In the calibration there should be a restriction in the values of $\bar{\psi}$. That is in the variance of $\bar{\psi}$, σ_ψ.

For those values of $\bar{\psi}$ as ψ increases, $\sigma_{E,t}$ decreases. Hence the derivative is negative for those values. The path of $\sigma_{E,t}$ for the values of $\bar{\psi}$ is shown in Figure 8.

As $\bar{\psi}$ increases the probability of default increase too. It is much more likely for $\psi \leqslant \bar{\psi}$. Therefore, as the probability of default increases, the remain probability decrease up to the point it becomes zero. □

Appendix D Bank's Problem

This appendix describes the method used for solving the banker's problem. I solve this, with the method of undetermined coefficient in the same fashion as in Gertler and Kiyotaki (2010). I conjecture that a value function has the following linear form:

$$V_t(l_{j,t}, d_{j,t}, x_{j,t}, m_{j,t}) = \nu_{l,j,t} l_{j,t}(1 - p) + \nu_{x,j,t} x_{j,t} - \nu_{d,j,t} d_{j,t} - \nu_{m,j,t} m_{j,t} - \Phi(x_t), \tag{D.1}$$

where $\nu_{s,j,t}$ is the marginal value from credit for bank j, $\nu_{d,t}$ the marginal cost of deposits, $\nu_{x,j,t}$ the marginal value from the deposit facility and $\nu_{m,j,t}$ the marginal cost of the emergency funding. The banker's decision problem is to choose $s_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}$ to maximize $V_{j,t}$ subject to the incentive constraint (16) and the balance sheet constraint (13). Using (13) we can eliminate $d_{j,t}$ from the value function. This yields:

$$V_{j,t} = l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{k,j,t} Q_t k_t + \nu_{d,t} n_{j,t}^B - \Phi(x_t).$$

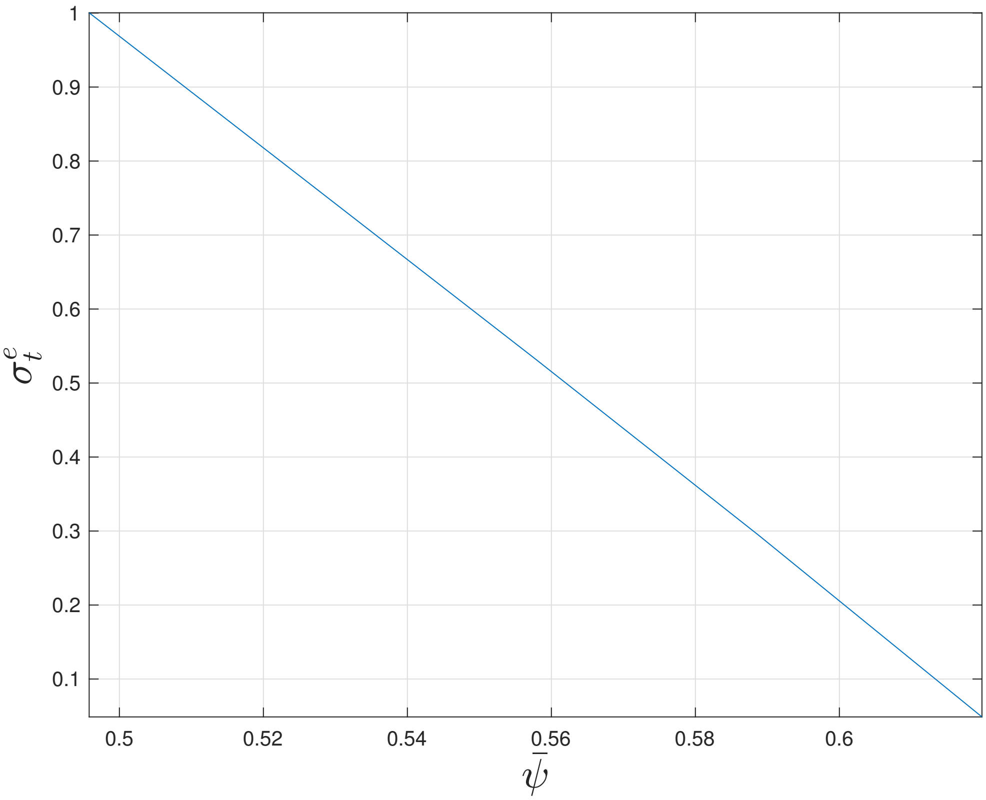

Fig. 8. Path of $\sigma_{E,t}$ for the values of $\bar{\psi}$

I define the ratio of excess liquidity to the net worth as

$$\Upsilon_t = \frac{x_t}{n_t^B}$$

and assume that the reserves penalty function has the following form:

$$\Phi(x_t) = \left(\frac{\kappa}{2}\Upsilon_t^2 n_t^B + \epsilon\Upsilon_t\right)\zeta_t.$$

Let $\mathcal{L}$ be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t[V_t - \theta((1-p)l_t - \omega m_t)] = (1+\lambda_t)V_t - \lambda_t\theta((1-p)l_t - \omega m_t).$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta\mathcal{L}}{\theta l_{j,t}} : (1+\lambda_t)(\nu_{l,j,t}(1-p) - \nu_{d,t}) = \lambda_t(1-p)\theta \tag{D.2}$$

$$\frac{\theta\mathcal{L}}{\theta \chi_{j,t}} : (1+\lambda_t)((\nu_{x,j,t} - \nu_{d,t})n_t - \kappa\Upsilon_t n_t) = 0 \tag{D.3}$$

$$\frac{\theta\mathcal{L}}{\theta m_{j,t}} : (1+\lambda_t)(\nu_{m,t} - \nu_{d,j,t}) = \omega\lambda_t\theta \tag{D.4}$$

$$\frac{\theta\mathcal{L}}{\theta k_{j,t}} : (1+\lambda_t)\nu_{k,j,t}Q_t = 0 \tag{D.5}$$

Equation (D.3) shows the optimal rule for the reserves' supply of the bank:

$$\nu_{x,j,t} - \nu_{d,j,t} = \kappa\Upsilon_t - \epsilon.$$

The Kuhn-Tucker condition yields:

$$\begin{aligned} KT : \lambda_t[&l_{j,t}(\nu_{l,j,t}(1-p) - \nu_{d,t}) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) \\ &+ \nu_{d,j,t}n^B_{j,t} - \Phi_t - \theta((1-p)l_{j,t} - \omega m_{j,t})] = 0. \end{aligned} \tag{D.6}$$

I define the excess value of bank's financial claim holdings as

$$\mu_t = \nu_{l,j,t}(1-p) - \nu_{d,j,t}. \tag{D.7}$$

The excess cost to a bank of LTRO credit relative to deposits

$$\mu^m_t = \nu_{m,j,t} - \nu_{d,j,t}.$$

Then from the first order conditions we have:

$$\mu^m_t = \omega\mu_t\frac{1}{1-p}. \tag{D.8}$$

From (D.6) and (D.8) when the constraint is binding ($\lambda_t > 0$) we get:

$$\begin{aligned}
l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} - \Phi_t &= \theta((1-p)l_t - \omega m_t) \\
l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) + \Upsilon_t n_t(\kappa\Upsilon_t) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} - \frac{\kappa}{2}\Upsilon_t^2 n_t &= \theta((1-p)l_t - \omega m_t) \\
l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t &= \theta((1-p)l_t - \omega m_t) \\
l_{j,t}(\theta(1-p) - \mu_t) - m_{j,t}(\omega\theta - \mu_t^m) &= \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t \\
l_{j,t}(\theta(1-p) - \mu_t) - m_{j,t}(\omega\theta - \omega\mu_t\frac{1}{1-p}) &= \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t \\
l_{j,t}(\theta(1-p) - \mu_t) - \frac{1}{1-p}\omega m_{j,t}(\theta(1-p) - \mu_t) &= \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t
\end{aligned}$$

and by rearranging terms, we get equation (21) on the main text :

$$l_{j,t} - \frac{1}{1-p}(\omega m_{j,t}) = \frac{(\nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2)n_t}{\theta(1-p) - \mu_t},$$

which gives the bank asset funding. It is given by the constraint at equality, where ϕ_t is the maximum leverage allowed for the bank. The constraint limits the portfolio size to the point where the bank's incentive to cheat is exactly balanced by the cost of losing the franchise value. Hence, in times of crisis, where a deterioration of banks' net worth takes place, supply for assets will decline.

Now, in order to find the unknown coefficients I return to the guessed value function

$$V_{j,t} = l_{j,t}(\mu_t) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\mu_t^m) + \nu_{d,t}n_{j,t}^B - \Phi_t. \tag{D.9}$$

Substituting (21) into the guessed value function yields:

$$\begin{aligned}
V_t &= (n_{j,t}\phi_t + \frac{1}{1-p}(\omega m_{j,t}))\mu_t + x_{j,t}\kappa\Upsilon_t - m_{j,t}\mu_t^m + \nu_{d,j,t}n_{j,t} - \Phi_t \Leftrightarrow \qquad \text{(D.10)} \\
V_t &= (n_{j,t}\phi_t + \frac{1}{1-p}(\omega m_{j,t}))\mu_t + \kappa\Upsilon_t^2 n_t - m_{j,t}\mu_t^m + \nu_{d,j,t}n_{j,t} - \frac{\kappa}{2}\Upsilon_t^2 n_t \Leftrightarrow \\
\Leftrightarrow V_t &= n_{j,t}(\phi_t\mu_t + \nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2) - m_{j,t}(\mu_t^m - \omega\mu_t\frac{1}{1-p})
\end{aligned}$$

and by (D.8) the guessed value function (D.10) becomes:

$$V_t = n_{j,t}^B(\phi_t\mu_t + \nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2).$$

Given the linearity of the value function we get that

$$A^B = \phi_t \mu_t + \nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2. \tag{D.11}$$

The Bellman equation (17) now is:

$$\begin{aligned} V_{j,t-1}(s_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) &= \mathbb{E}_{t-1} \, \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1-\sigma_B) n_{j,t}^B \\ &+ \sigma_B(\phi_t \mu_t + \nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2) n_{j,t}^B\}. \end{aligned} \tag{D.12}$$

By collecting terms with $n_{j,t}$ the common factor and defining the variable Ω_t as the marginal value of net worth:

$$\Omega_{t+1} = (1-\sigma_B) + \sigma_B(\mu_{t+1} \phi_{t+1} + \nu_{d,t+1} + \frac{\kappa}{2} \Upsilon_t^2). \tag{D.13}$$

The Bellman equation becomes:

$$\begin{aligned} V_{j,t}(s_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}) &= E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}^B = \\ &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t} l_{j,t-1}(1-p) + R_{x,t} x_{j,t} - R_t d_{j,t} - R_{m,t} m_{j,t} - \Phi_t]. \end{aligned} \tag{D.14}$$

The marginal value of net worth implies the following: Bankers who exit with probability $(1-\sigma_B)$ have a marginal net worth value of 1. Bankers who survive and continue with probability σ_B, by gaining one more unit of net worth, they can increase their assets by ϕ_t and have a net profit of μ_t per assets. By this action they acquire also the marginal cost of deposits $\nu_{d,t}$ which is saved by the extra amount of net worth instead of an additional unit of deposits and also the additional cost of reserves $\frac{\kappa}{2} \Upsilon_t^2$. Using the method of undetermined coefficients and comparing (D.1) with (D.14) we have the final solutions for the coefficients:

$$\begin{aligned} \nu_{l,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{l,t+1} \\ \nu_{x,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{x,t+1} \\ \nu_{m,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{m,t+1} \\ \nu_{d,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\ \mu_t &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{l,t+1}(1-p) - R_{t+1}] \\ \mu_t^x &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{x,t+1} - R_{t+1}] \qquad \text{(D.15)} \\ \mu_t^m &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{m,t+1} - R_{t+1}] \qquad \text{(D.16)} \end{aligned}$$

The first order condition (D.2) implies that when the incentive constraint is not binding ($\lambda_t = 0$), $\mu_t = 0$ the spread is zero, but in the case where constraint is binding ($\lambda_t > 0$) excess value of assets is positive $\mu_t > 0$. The same follows for μ_t^x and μ_t^m by equations (D.3) and (D.4) respectively. An important feature is that two effects take place to form the marginal value of the loans for the bank. The one is the case of the binding constraint and the other is the case of increased default probability. Taking equations (D.7) and the FOC (D.2) we have that

$$\nu_{l,j,t} = \frac{\lambda_t}{(1+\lambda_t)}\theta + \nu_{d,j,t}\frac{1}{1-p}.$$

The marginal value from extending a unit of loan is equal to the marginal cost from getting deposits which is increasing in default (as the banks' net worth is decreasing), plus the cost from the binding constraint.

From (D.9) we can get the following relationship between the expected loan rate, the riskless rate and the default probability.

$$E_t \Lambda_{t,t+1} \Omega_{t+1} R_{l,t+1} = \frac{\lambda_t}{(1+\lambda_t)}\theta + E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \frac{1}{1-p(\psi_t)} \tag{D.17}$$

.

This shows the two effects on the expected loan rate. The first, is due to the binding funding constraints for the bankers. This can be referred as the liquidity component. The second one reflects the compensation that bankers demand when the firms' probability of default increases. This can be called as risk component.