Quantitative Easing, Debt Stability and Inflation

Gleb Kurovskiy, Luisa Lambertini, Stylianos Tsiaras
Ecole Polytechnique Fédérale de Lausanne

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VERY PRELIMINARY DRAFT

Abstract

Since the COVID-19 pandemic there is an unprecedented increase in social insurance transfers both in the EA and the US. In this paper, we explore different fiscal and monetary strategies aiming in times of large debt accumulation and identify QE as a novel mechanism for debt stability. We build a New Keynesian DSGE model with household heterogeneity, financial frictions, nominal rigidities, and an unconstrained central bank that can purchase bonds in exchange of reserves. Profits earned from the bonds-reserves spread can be remitted from the central bank to the treasury and can be a substantial fiscal revenue. Together with QE's general equilibrium effects both can achieve debt stability after a transfer shock. We analyse and compare QE as a debt stabilization tool versus taxation changes under an conventional active and passive monetary policy framework.

Keywords: Fiscal Policy; Monetary Policy; Quantitative Easing ; DSGE;

1. Introduction

Since our world was hit by the COVID-19 pandemic, there was unprecedented increase in social insurance transfers both in the EA and the US. In the US the total legislation actions due to the pandemic event had a budgetary cost of more than \$5 trillion.[1] As Romer (2021) points out, this is about four times the amount spent on the 2009 American Recovery and Reinvestment Act aimed in helping the US economy to recover from the financial crisis. There were extraordinary in level explicit or implicit transfers to households and businesses that dramatically increased the US debt to GDP and posed questions on its stability. At the same time the US monetary policy reduced its rates to zero and started an extensive Quantitative Easing program that has accounted for asset purchases of more than \$4 trillion.

In this paper we ask how this new debt is going to be repaid and we study whether QE can provide sufficient fiscal revenues and/ or create general equilibrium effects for debt stability. Debt purchases impact the economy through two channels: reduce banking capital constraints and generate extra fiscal revenues for the Treasury. The fiscal revenues from QE are generated by the profits the central bank generates on the spread on the purchased bonds' return and the interest rate on reserves paid back to the financial institutions. This, given back to the Treasury as remittances can be a substantial fiscal revenue. Furthermore, general equilibrium effects through the relaxation of the banks' constraints can help in debt stability. Along QE, we study two other possible avenues for debt stabilization: classical fiscal adjustments, that is, taxation that respond inversely to changes in the debt-to-GDP ratio and a passive monetary policy that accommodates higher inflation to inflate out the new debt.

To shed more light on the fiscal revenues generated from QE, Figure 1 shows the FED remittances to the treasury and the debt service-to-gdp ratio. Any income received by the Federal Reserve in excess of the amount needed to pay expenses and dividends and to maintain surplus at the level of \$6.825 billion is, by law, remitted to Treasury and does not affect the capital position or value of the Federal Reserve (Bonis, Fiesthumel, and Noonan (2018)). As can be seen, during the years of extensive QE, the remittances to Treasury could account for over a third of the total fiscal revenues needed for the interest payments on the debt. QE profits are, and can potentially be even more, a substantial fiscal revenue for debt stability. Importantly, QE revenues are high in times of crisis, when the interest rate on reserves is low thus widening the spread between bond returns and reserve payments.

To study those questions we develop a Two Agents New Keynesian (TANK) DSGE model calibrated to the US with nominal rigidities, financial intermediaries, a rich fiscal sector and a central bank that can purchase assets from the banks by issuing reserves. Furthermore, to accommodate a passive monetary policy framework we have partially unfunded debt similarly to Bianchi, Faccini, and Melosi (2020). The QE framework follows Gertler and Karadi (2013) but a formal representation of reserves and the asset

[1] Committee for a Responsible Federal Budget (CRFB), "COVID Money Tracker," https://www.covidmoneytracker.org/.

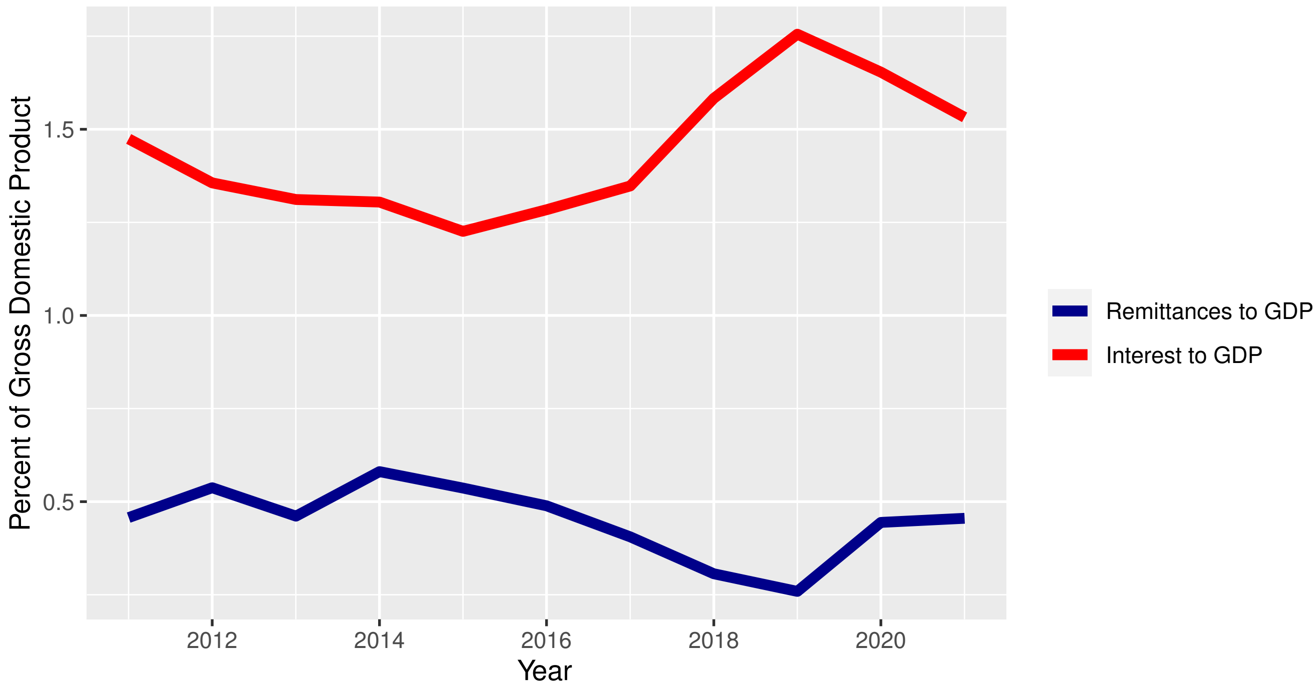


Fig. 1. Central Bank to Treasury Remittances and Federal Interest Expenses. Source: Board of Governors of the Federal Reserve System (US) and U.S. Bureau of Economic Analysis

swap mechanism induced by the QE is developed. The real economy part of the model follows closely Bianchi et al. (2020) and banks are modelled similarly to Gertler and Kiyotaki (2010): a costly enforcement problem creates a leverage constraint on the intermediaries. QE induces more lending through the relaxation of the leverage constraints of the financial intermediaries, similarly to Sims and Wu (2021), by the exchange of banks' government bonds with reserves and thus stimulates aggregate demand.

A setting is developed where central bank purchases of government bonds financed by reserves create profits for the central bank due to the positive spread. These profits are remitted to the treasury and can be used as fiscal revenue for debt repayment together with other fiscal tools, as in Reis (2017). QE in the model also works as a credit-stimulating mechanism to the real economy. It relaxes the leverage constraint of the banks and thus induces more lending to the real economy accelerating GDP and thus debt repayments.

For the first two cases of our analysis, namely debt stabilization through QE revenues and fiscal adjustments via an increase in revenues, monetary policy satisfies the Taylor principle and the fiscal authority is in control of debt stabilization. This is in words of Leeper (1991) active conventional monetary policy, which pays no attention to the state of government debt and is free to set its control variable as it sees fit, and a passive fiscal policy which responds to government debt shocks. The third case for debt stabilization, a passive monetary policy framework is implemented by the introduction of unfunded transfer shocks, similarly to Bianchi et al. (2020). These are transfers that are not backed by future fiscal adjustments, making a share of the overall government debt unfunded. The central bank accommodates the increase in inflation necessary to stabilize the unfunded amount of debt. As a result, these shocks trigger persistent movements in inflation and a decline in real interest rates, leading to a fiscal theory of trend inflation.

We show that after a large transfer to the households, the use of QE does not only reduce spreads and stimulate the economy but also provides debt stabilization even without any use of the classic fiscal adjustment tools or a passive monetary policy. QE as a debt stabilization tool, works by the revenue generated from the bond-reserves spread receivable by the central bank and also through the relaxation of the leverage constraint of the banks. It does not distort the economy by increasing consumption or labour taxation or by decreasing aggregate demand as in the case of a government spending reduction.

Passive monetary policy, which accommodates the inflation needed for debt stabilization, is the most efficient way for debt stability in our exercises. By sacrificing an increase in inflation, a reduction of the debt-to-gdp ratio leads to ample fiscal space for tax reduction and an increase to government spending and transfers to households that increase aggregate demand. Nevertheless, due to institutional reasons related with a high inflation target, we think of this scenario as possibly unrealistic.

Our theory regarding the debt stability is related to a recent small literature on the central bank remittances and price stability. Benigno (2020) develops a theory in which the central bank can control the price level without fiscal backing. He shows that control of the price level can be achieved by the central bank through an appropriate specification of the remittances policy. Park (2015) provides a framework that illustrates the relevance of the central bank's balance sheet and remittance policy for inflation.

There is a small but growing literature on remittances from the central bank to the treasury. Bernanke (2020) referrers to the asset purchase programs as "hugely profitable, with net profits resulting in remittances to the Treasury totaling about \$800 billion between 2009 and 2018, about triple pre-crisis rates". Cavallo, Del Negro, Frame, Grasing, Malin, and Rosa (2019) study the implication of quantitative tapering on central bank remittances for the US combining two widely used models maintained by the Federal Reserve Board. Similarly, Carpenter, Ihrig, Klee, Quinn, and Boote (2018) study monetary policy normalization scenarios with a focus on central bank remittances. Bassetto, Caracciolo et al. (2021), in the context of the Eurozone, study the transition mechanism of the ECB and national central banks' revenues to the treasuries of 19 Eurozone countries.

The outline of the paper is as follows. Section 2 presents some stylized facts that relate remittances from the Federal Reserve to the Treasury with revenue from QE operations. Section 3 presents the TANK model with banking that we use. The following sections present the main results of the paper and the last section concludes.

2. Empirical evidence

This section addresses possible avenues which can lead to debt stabilization by the use of QE. [2] In the first part, we demonstrate that QE can be viewed as an additional source of funding for the Treasury. In the second part of the section, we test empirically whether asset purchases response to new government debt issuance.

2.1. QE revenues

During the 2007-2008 financial crisis, the Federal Reserve initiated QE. The QE program has since expanded to include the purchase of mortgage-backed securities and long-term bonds. These assets are typically purchased from banks in exchange for newly issued reserves McLeay, Radia, and Thomas (2014). Banks had modest excess reserves prior to the Global Financial Crisis, but they have grown substantially since the start of QE programs. Figure 2.a shows that by 2022, the Federal Reserve's net bond purchases of the US Treasury were almost $4 trillion[3], which was in line with bank reserves.

The Federal Reserve earns coupons for holding bonds and pays bank reserves interest. Because the latter is usually higher than the former, the Federal Reserve receives income that is given back to the Treasury. QE revenues can be summarized in (1).

$$QERevenue_t = \sum_{i=1}^{N} BondsParValue_{i,t} \times (Coupon_{i,t} - ReservesRate_t), \tag{1}$$

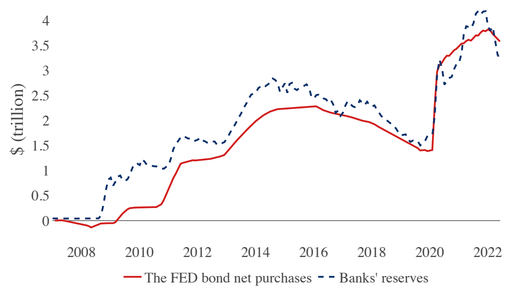
for period t and bond type i. More data collection details are presented in Appendix D.

Figure 2.b shows the total revenue from the ownership of bonds as a percentage of remittances to the Treasury and total government debt interest payments. Revenue hit 90% of Federal Reserve transfers to the Treasury at its peak in 2021. The majority of QE income is generated by the purchase of long-term interest bonds. They carry risk premia and yields are higher than the reserves rate in the period of expansionary monetary policy. Short interest rates react quickly to changes in policy rates 2.c and do not provide much additional income. However, QE income declined for all yields throughout the 2019 contractionary monetary policy. Long-term yields have remained stable, while other rates have caught up.

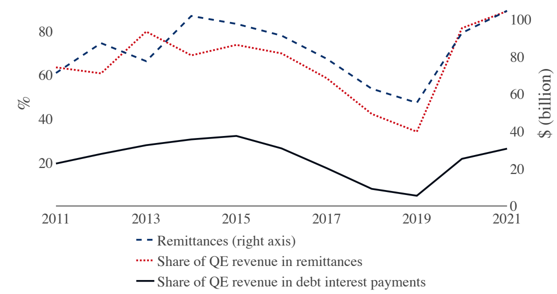
We conduct an empirical analysis to understand how the Fed funds rate affects yields at different maturities. The Fed funds rate, as shown in table 1, is the best at explaining shorter rather than longer yields. It makes conventional monetary policy instruments for lowering long-term interest rates insufficient. Figure 2.c shows that the coupon rate received by the Federal Reserve is closer to the 30-year yield than the 3-month yield. As a result, QE revenues are higher during expansionary monetary policy and lower during the contractionary monetary policy.

[2] From now on, under QE, we assume government bond purchases by the FED.

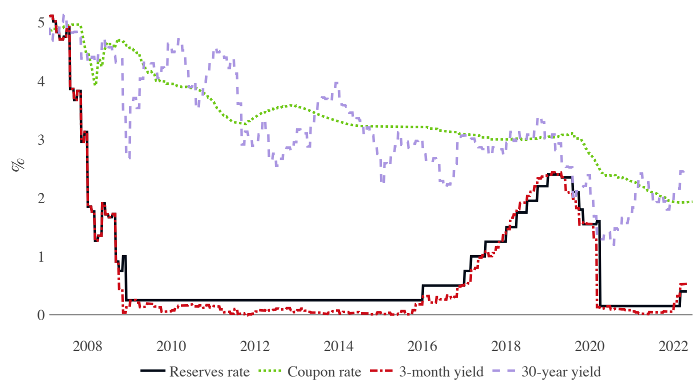
[3] Starting from January 2007



((a)) Banks' reserves and the Fed bond net purchases



((b)) The Fed QE revenues



((c)) Yields at different maturities

The panel (a) shows data on bank reserves in the United States as well as the Federal Reserve's net bond purchases at market prices. We use FRED data (WRESBAL) for bank reserves, and SOMA holdings of domestic securities for FED net purchases. Panel (b) depicts QE revenue as a percentage of interest payments and total FED remittances to the Treasury. QE revenues are calculated using the formula in eq (1). We use Fred's data for debt repayments (A091RC1Q027SBEA) and total remittances (RESPPLLOPNWW). Panel (c) displays the Investing.com 3-month and 10-year yields, the Fred reserves rate (IOER, IORB), and the coupon rate. The latter is calculated using SOMA data as the weighted average coupon rate across the Fed's bond holdings.

Fig. 2.

Another crucial element for remittances is that buying bonds by themselves lowers yields. Table 1 demonstrates that QE purchases reduce the yields on both the two- and ten-year bonds. We collect data on the Federal Reserve's bond purchases split by maturity, data on government yields, and episodes of various QE programs to verify this conclusion. Results indicate that bond purchases have a negative influence on bond yields, which is consistent with Bernanke (2020), Ihrig, Klee, Li, Wei, and Kachovec (2018). Figure 2.c shows that during the active QE phase from 2009 to 2016, short rates were at the zero lower bound while long-term yields were continuously declining.

	Dependent variable:	
	10-year yield	2-year yield
Fed funds rate	0.515***	0.912***
	(0.027)	(0.014)
Purchase of 5-10 year bonds	−0.006*	
	(0.004)	
Purchase of 6 month-2 year bonds		−0.008***
		(0.003)
Time effects	YES	YES
Observations	267	267
Adjusted R^2	0.739	0.946
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 1: Conventional and unconventional monetary policy effects on bond yields

2.2. QE rule

Motivated by a simultaneous sharp increase in Treasury bond issuance and bond purchases by the Federal reserve during COVID-19, we study whether the Federal Reserve was indirectly following a QE rule on how much debt to purchase in response to bond issuance. Debt purchases have two effects on the economy: they reduce banking capital constraints and generate additional fiscal revenues for the Treasury. Fiscal revenue reduces the debt burden by partially covering the costs of government debt interest payments. As a result, the Federal Reserve may purchase bonds to help stabilize the debt. However, during the COVID-19 meltdown, the Treasury and the Federal Reserve may coordinate debt issuance and purchases. It raises concerns about the endogeneity problem. To investigate causal relationship, we conduct a Granger causality test:

$$\begin{cases} \tilde{B}_t^{CB} = \sum_{k=1}^{j} \alpha_k \tilde{B}_{t-k}^{CB} + \beta_k \frac{\tilde{B}}{GDP}_{t-k} \\ \frac{\tilde{B}}{GDP}_t = \sum_{k=1}^{j} \gamma_k \tilde{B}_{t-k}^{CB} + \kappa_k \frac{\tilde{B}}{GDP}_{t-k} \end{cases} \tag{2}$$

where $\tilde{()}$ defines growth rates of B_t^{CB} central bank bond holdings at period t and $\frac{B}{GDP}_t$ government debt-to-GDP at period t. Both $\beta_1 = ... = \beta_k = 0$ and $\gamma_1 = ... = \gamma_k = 0$ are rejected, hence, we cannot exclude mutual relation between the variables. To avoid endogeneity problem, we further consider impact of debt-to-GDP with a lag. We estimate the following QE rule where central bank bond purchases react to debt-to-GDP:

$$\tilde{B}_t^{CB} = \rho_b^{CB} \tilde{B}_{t-1}^{CB} + (1-\rho_b^{CB})\gamma_{QE} \frac{\tilde{B}}{GDP}_{t-1,j} + \beta_{COVID} D_{COVID} \frac{\tilde{B}}{GDP}_{t,j} + QEshock_t \tag{3}$$

where j indicates maturity, D_{COVID} is a dummy variable equal to 1 in the second quarter 2020, the periods of most active debt issuance response by the Treasury. All data has been seasonality adjusted using X13-ARIMA and is calculated as a deviation from the Hodrick-Prescott trend. We divided the QE rule into maturities to determine which type of issued bond central bank is the most responsive. For Treasury bonds, we chose three maturity date cut-offs: two years, five years, and ten years. Considering only bonds with maturities of less than two years, for example, demonstrates how the central bank responds to short-term debt increases.

Our findings are summarized in Table 2. We divided the Federal Reserve purchases into maturities that corresponded exactly to the debt maturities. We show that a higher debt-to-GDP ratio predicts an increase in central bank bond purchases. However, the effect is heterogeneous across the maturity of the debt. If we consider long-term bonds, then the QE response will get stronger. Long-term yields are often higher and less volatile than short-term yields. The long-term bonds purchase sensitivity parameter γ^{QE} ranges from 3 before COVID-19 to 10 during the debt spike. We also show that, compared to other periods, the central bank significantly increased its debt purchases at the start of the COVID-19 pandemic.

	Dependent variable:				
	$\tilde{B}^{CB}$				
Maturity	All	$< 2years$	$> 2, < 5years$	$> 5years$	$> 10years$
$\frac{\tilde{B}}{GDP}\ (t-1)$	0.720***	−0.217	0.174	1.810***	1.259***
	(0.180)	(0.614)	(0.445)	(0.249)	(0.181)
$\tilde{B}^{CB}\ (t-1)$	0.724***	0.562**	0.860***	0.405***	0.591***
	(0.060)	(0.221)	(0.158)	(0.033)	(0.034)
$D_{COVID}\frac{\tilde{B}}{GDP}\ t$	2.558***	0.920	6.457***	3.156***	2.541***
	(0.116)	(1.613)	(0.593)	(0.158)	(0.131)
Constant	−0.016**	−0.168	0.012	−0.014*	−0.016***
	(0.007)	(0.111)	(0.047)	(0.007)	(0.006)
Observations	61	61	61	61	61
Adjusted R^2	0.897	0.302	0.726	0.955	0.957
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 2: Response of the Federal Reserve debt purchases to the debt issuance from January 1, 2007 to July 1, 2022. For each of type of bond we collect and match data by bond CUSIP for both issuance by the US Treasury and holdings of the Federal Reserve. This allow us to split data by maturities of 2, 5 and 10 years length. Data source: Federal Reserve, US Treasury.

3. The Model

The economy is populated by two types of households: savers and hand to mouth households that differ in their ability to participate in the assets market. A continuum of firms and financial intermediaries owned by the savers, labour wide unions that set the wages, capital goods producers and retailers, a monetary authority and the treasury complete the model economy.

There is a moral hazard problem between the savers and the banks. Banks can steal a fraction of their funds and return them to their families. This problem introduces an incentive constraint to the model to be followed by the banks. Furthermore, the central bank performs its (active and/or passive) conventional monetary policy under a Taylor rule, but can also engage in asset purchases and pay the banks that own the assets back the same value in newly created reserves. Lastly, there is a rich fiscal sector that aims for debt stabilization.

3.1. Households

All households are assumed to have identical preferences, given by

$$U^j = \log\left(C_t^j - \chi C_{t-1}^j\right) - \psi \frac{(L_t^j)^{1+\eta}}{1+\eta}. \tag{4}$$

C_t^j denotes the per capita consumption of the household members and L_t^j the supply of labour. The super-index $j \in [S, N]$ specifies the household type (S for "savers" or N for "hand to mouth"). $\beta \in [0,1]$ is the discount factor. Households derive utility from consumption following an internal habit in consumption where χ is the habit parameter. Due to the stochastic setting, households make expectations for the future based on what they know in time t and $\mathbb{E}_t$ is the expectation operator at time t. Finally, η is the inverse Frisch elasticity of labour supply and ψ is the relative utility weight of labour.

Savers. This category of households accounts to a measure $(1-\mu)$ of the economy's population. They allocate their funds in consumption C_t^S and consumption taxes τ_t^C, and short-term deposits D_t^S which are remunerated at the risk-free rate of the economy R_t. Savers' income is made of deposit returns, lump sum transfers Z_t^S, after tax labour income and the profits from firms and banks that they own, Π_t.

Savers' budget constraint then is:

$$\begin{aligned} P_t(1-\tau_t^C)C_t^S + R_t^{-1}D_t \\ = D_{t-1} + (1-\tau_t^L)W_t L_t^S + P_t Z_t^S + \Pi_t \end{aligned} \tag{5}$$

where W_t denotes the wage rate that applies to all household members, and τ_t^C, τ_t^L denote the tax rates on consumption and labour income respectively.

Their optimality conditions are:

$$\begin{aligned} \text{Euler Deposits } : 1 \quad &= \quad R_t \mathbb{E}_t\left[\Lambda_{t,t+1}^S\right] \\ \text{Stochastic Discount Factor} : \Lambda_{t,t+1}^S \quad &\equiv \quad \beta \frac{\lambda_{t+1}^S}{\lambda_t^S} \\ \text{where} : \lambda_t^S \quad &= \quad \frac{\frac{1}{C_t^S - \chi C_{t-1}^S} - \beta\chi \frac{1}{C_{t+1}^S - \chi C_t^S}}{1-\tau_t^C} \\ \text{Labour Supply} : -\frac{U_{L,t}^S}{\lambda_t^S(1-\tau_t^L)} \quad &= \quad -W_t \end{aligned}$$

Hand to Mouth. Hand-to-mouth households consume all of their disposable, after-tax income every period. Their income is made of their labour income and lump-sum government transfers. Profits are earned only by the savers that own the firms. It is assumed that the hand-to-mouth households supply differentiated labour services, and set their wage to be equal to the average wage that is optimally chosen by the savers, therefore there is a common wage level W_t for both groups, Their budget constraint is as follows:

$$P_t(1+\tau_t^C)C_t^N = (1-\tau_t^L)W_t L_t^N + P_t Z_t^N. \tag{6}$$

Their optimality conditions are:

$$\begin{aligned} \text{Labour Supply} : -\frac{U_{L,t}^N}{\lambda_t^N(1-\tau_t^L)} &= -W_t \\ \text{where} : \lambda_t^N &= \frac{\frac{1}{C_t^N-\chi C_{t-1}^N} - \beta\chi\frac{1}{C_{t+1}^N-\chi C_t^N}}{1-\tau_t^C} \end{aligned}$$

3.2. Banks

Banks are funded with deposits from the saver households, extend credit to non-financial firms and buy long-term bonds from the government. Each bank, allocates its funds to buying a quantity s_t of financial claims on non-financial firms at price Q_t and long term government bonds b_{t+1}^B at price P_t^B. We denote the long term government bond holdings of the banks with the superscript B, because in the model the central bank is allowed to hold government bonds as well. Banks' liabilities are made up from households' deposits d_{t+1}. When the central bank proceeds in bond purchases it pays back the bank with an equivalent value of reserves m_t. Finally, n_{t+1} is the capital equity accumulated. Formally, the bank's balance sheet is:

$$Q_t s_t + P_t^B b_t^B + m_t = n_t + d_t. \tag{7}$$

To limit bankers' ability to save and eventually overcome their financial constraint by using own funds, following Gertler and Kiyotaki (2010) we assume the following: Each period, a fraction $1-\sigma_B$ of bankers, exit and give retain earnings to owners, namely saver households. An equal number of new bankers enter at the same time. They begin with a start up fund of ξ given to them by saver households.

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities.

$$n_{t+1} = R_{k,t+1}Q_t s_t + R_{b,t+1}P_t^B b_t^B + R_{m,t+1}m_t - R_{t+1}d_t.$$

Following Woodford (2001) long-term government bond B_t with a maturity decaying at a constant rate $\rho \in [0,1]$ and duration $(1-\beta\rho)^{-1}$ can be purchased at price P_t^B.

Therefore, the realized return that the bank earns from the maturity bond is:

$$R_{b,t+1} = \frac{1 + \rho_B P^B_{t+1}}{P^B_t}$$

Let Z_t be the marginal product of capital of the firm and δ the depreciation rate of capital being financed. Then the state-contingent rate of return to the bank on the loan, $R_{k,t+1}$, is given by:

$$R_{k,t+1} = \frac{Z_t + (1-\delta)Q_{t+1}}{Q_t}. \tag{8}$$

The bankers' objective at the end of period t, is to maximize the expected present value of future dividends. Since the banks are owned by the optimizing households, their stochastic discount factor Λ_{t+1} is used as the discounting measure.

$$V_t = \mathbb{E}_t \sum_{j=1}^{\infty} (1-\sigma_B)\sigma_B^{j-1} \Lambda_{t,t+j} n_{t+j}. \tag{9}$$

To motivate a limit on the banks' ability to obtain deposits, a costly enforcement constraint is introduced in the same fashion as in Gertler and Kiyotaki (2010). A banker can abscond a fraction of her assets and transfer them back to her household members; since depositors anticipate the bank's incentive, they will not lend resources to the bank beyond the level at which the bank's value exceeds the value of the resources that can be diverted. More precisely, depositors impose an incentive compatibility constraint of the following form.

The depositors continue providing funds to the bank as long as the following incentive constraint is not violated:

$$V_t \geqslant \theta[Q_t s_t + \Delta P^B_t b^B_t + \omega m_t]. \tag{10}$$

where $\theta \in (0,1)$ is the fraction of assets that the banker may divert and $\Delta \in (0,1)$ and $\omega \in (0,1)$ are the fraction of bonds and reserves respectively the banker can divert. On the left of (10) is the franchise value of the banker.

The bank's incentive to invest in reserves versus long-term bond is driven by two considerations: the differential rate of return and the effect on the incentive compatibility constraint. Since $R_{m,t} < R_{b,t}$, reserves are dominated in terms of returns. The effect on the incentive compatibility constraint depends on Δ versus ω; we assume $\omega = 0$ because reserves are difficult to divert due to perfect central bank monitoring and, as a result, reserves are truly dominated as an asset, this is to say that banks would not invest in reserves.

The maximum leverage ratio of the bank ϕ_t (see Appendix) yields:

$$\phi_t = \frac{\mathbb{E}_t \Lambda_{t,t+1}\Omega_{t,t+1}R_{t+1}}{\theta - \mathbb{E}_t \Lambda_{t,t+1}\Omega_{t,t+1}(R_{k,t+1} - R_{t+1})}. \tag{11}$$

The maximum leverage falls as θ goes up, i.e. the bank's incentive to divert assets; it goes up with an increase in the discounted excess return on assets or the discounted safe rate.

Importantly, the maximum adjusted leverage ratio does not depend on any individual bank characteristics, therefore the heterogeneity in the bankers' holdings and net worth, does not affect aggregate dynamics. Hence, it is straightforward to express individual financial sector variables in aggregate form.

Aggregation. Let S_t be the total quantity of loans that banks intermediate, B_t^B the total number of government bonds they hold, M_t the total quantity of reserves and N_t their total net worth. Furthermore, by definition, total deposits acquired by the households D_t are equal with the total deposits of the banking sector. Using capital letters for the aggregate variables, the banks' aggregate balance sheet becomes

$$Q_t S_t + P_t^B B_t^B + M_t = N_t + D_t. \tag{12}$$

Since the leverage ratio (11) does not depend on factors associated with an individual bank's characteristics we can sum up across banks and get the aggregate bank constraint in terms of the total net worth in the economy:

$$Q_t S_t + \Delta P_t^B B_t^B + \omega M_t \leqslant \phi_t N_t. \tag{13}$$

The above equation gives the overall demand for loans $Q_t S_t$. When the incentive constraint is binding, the demand for assets is constrained by the net worth of the bank according to the maximum leverage. We can get some intuition here for what changes in the bank's constraint during the QE. Since the weight on bonds is higher than the weight of reserves ($\Delta > \omega$), the exchange of securities with reserves relaxes the constraint and stimulates lending to the non-financial sector and to a lesser extent to new bond purchases.

Aggregate net worth is the sum of the new bankers' and the existing bankers' equity: $N_{t+1} = N_{y,t+1} + N_{o,t+1}$. Young bankers' net worth is the earnings from loans multiplied by ξ_B which is the fraction of asset gains that is transferred from households to the new bankers:

$$N_{y,t+1} = \xi^B [R_{k,t} Q_{t-1} S_{t-1} + R_{b,t} P_{t-1}^B B_{t-1}^B + R_{m,t} M_{t-1}],$$

and the net worth of the old is the net worth of surviving banks, namely the fraction σ^B of total banks' net worth

$$N_{o,t+1} = \sigma^B [R_{k,t} Q_{t-1} S_{t-1} + R_{b,t} P_{t-1}^B B_{t-1}^B + R_{m,t} M_{t-1} - R_t D_t].$$

3.3. Conventional Monetary Policy and Asset Purchases

The central bank uses two policy tools. Firstly, it adjusts the policy rate according to the Taylor rule specified below. Secondly, it engages in asset purchases with the banks.

Asset Purchases. Under a QE operation, note that QE and asset purchases will be used interchangeably, the central bank buys bonds B_t^{CB} from banks at the market price

P_t^B. To finance those purchases it creates electronic reserves M_t that are used to pay banks. The central bank's balance sheet then is:

$$P_t^B B_t^{CB} = M_t. \tag{14}$$

It is assumed that the central bank turns over any profits to the treasury and receives transfers to cover any losses. The central bank's budget constraint is:

$$T_t^{CB} + R_t M_{t-1} + P_t^B B_t = R_{b,t} P_{t-1}^B B_{t-1}^{CB} + M_t$$

where T_t^{CB} are transfers of the central bank to the treasury.

Given (14) we can write the total remittances from the central bank to the treasury yielding from bond purchases as:

$$T_t^{CB} = (R_{b,t} - R_t) P_t^B B_t^{CB}. \tag{15}$$

The central bank remittances stem from the spread between the bonds purchases and the interest on reserves that is paid to the banks for the same value as the bond purchases as seen by (14). As long the spread is positive and the value of bonds purchased non zero, central bank will earn profits from QE and transfer them to the treasury.

The total quantity of bonds in the economy can be decomposed as:

$$B_t = B_t^B + B_t^{CB}. \tag{16}$$

If we combine these identities and insert them into the balance sheet constraint of the banks we have:

$$Q_t S_t \leqslant \phi N_t + \Delta(P_t^B B_t^{CB} - P_t^B B_t) \tag{17}$$

The above constraint implies that when government purchase bonds it relaxes the balance sheet constraint of the banking sector. This can, in financial stress periods, reduce the returns and stimulate the economy. When this constraint does not bind and the inequality holds, bond purchases made by the government are neutral. This happens due to friction-less arbitrage that characterizes the economy when the banks has no binding constraint. Wallace (1981) in his seminal paper has made use of that assumption for the neutrality theorem of the open market operations.

The share of the total assets that is purchased by the central bank follows a rule that responds to debt-to-gdp deviations similarly to the fiscal rules presented momentarily. This setting makes QE a fiscal stabilization tool. When debt is high, the central bank buys more bonds thus generating extra fiscal revenue for the treasury. The QE rule reads as follows:

$$\hat{b}_t^{CB} = \rho_{b^{CB}} \hat{b}_{t-1}^{CB} + (1 - \rho_{b^{CB}}) \gamma_{QE} \hat{b}_{t-1} + \zeta_{b^G,t} \tag{18}$$

where $b_t = \frac{P_t^B B_t}{P_t Y_t}$ is the debt-to-gdp ratio and b_t^{CB} the central bank's bond holdings. The variables in $\hat{x}_t$ denote the percentage deviation from their own steady state. γ_{QE} is the magnitude of the asset purchases relative to the debt deviations. A higher (lower) value for $\gamma_{b^{CB}}$ provides debt stability (instability). $\zeta_{b^{CB},t}$ is a QE shock following second order stochastic process.

Conventional Monetary Policy. The central bank also sets the nominal interest rate in response to movements in inflation originating by funded fiscal shocks but also it accommodates the inflation necessary to stabilize the unfunded portion of debt. Following Bianchi et al. (2020) the latter can be captured by a rule in which the central bank reacts to deviations of inflation from the level needed to stabilize the unfunded share of debt. Conventional monetary policy is active when it responds to deviations of inflation, $\hat{\pi}_t$, from the inflation needed to stabilize the unfunded share of debt, $\hat{\pi}_t{}^F$.

The Taylor rule is as follows:

$$\hat{R}_{n,t} = \rho_r \hat{R}_{n,t-1} + (1-\rho_r)[\phi_\pi(\hat{\pi}_t - \hat{\pi}_t^F) + \phi_y \hat{y}_t)] + \epsilon_{MPS}$$

where ϵ_{MPS} is a monetary policy shock.

Utilising this, it provides a tractable way to have active and passive conventional monetary policy in the same framework. The central bank responds differently depending on whether the increase in debt is funded (back by fiscal tools or QE) or unfunded.

3.4. *Fiscal Policy*

The government budget constraint is:

$$P_t^B B_t + \tau_t^L W_t L_t + \tau_t^C P_t C_t + P_t T_t^{CB} = (1+\rho P_t^B) B_{t-1} + P_t G_t + P_t Z_t \qquad (19)$$

where $C_t = \mu_t C_t^N + (1-\mu_t) C_t^S$ denotes aggregate consumption and $Z_t = Z_t^N = Z_t^S$ means that every household receives the same government transfer regardless of its type. The government finances government expenditures and transfers and the pays interest on the long-term debt by raising taxes and issuing new long term debt.

The budget constraint can also be written in terms of debt-to-gdp ratio as follows. Let $b_t = \frac{P_t^B B_t}{P_t Y_t}$ be the debt-to-GDP ratio.

$$b_t = \frac{b_{t-1} R_{b,t-1}}{\frac{Y_t}{Y_{t-1}}} - (\tau_t^C C_t + \tau_t^L W_t L_t)/Y_t - T_t^{CB}/Y_t + G_t/Y_t + Z_t/Y_t + \epsilon_t^b \qquad (20)$$

where ϵ_t^b is a debt shock.

The fiscal authority adjusts government spending G_t, transfers Z_t, and tax rates on labour income, and consumption τ^J, where $J \in C, L$ as a function of the debt-to-gdp deviations from its steady state value. Note that government consumption and transfers move pro-cyclically with the debt-to-gdp deviations, while taxes move counter-cyclically.

$$
\begin{aligned}
\hat{g}_t &= \rho_G \hat{g}_{t-1} - (1-\rho_G)\gamma_G \hat{b}^M_{t-1} + \zeta_{g,t} && (21)\\
\hat{z}_t &= \rho_Z \hat{z}_{t-1} - (1-\rho_Z)\gamma_Z \hat{b}^M_{t-1} + \zeta^F_{Z,t} + \zeta^M_{Z,t} && (22)\\
\hat{\tau}_t^{\,J} &= \rho_J \hat{\tau}^J_{t-1} + (1-\rho_J)\gamma_J \hat{b}^M_{t-1}. && (23)
\end{aligned}
$$

$\hat{b}^M_{t-1}$ is the portion of the debt to gdp ratio that the government is committed to stabilize with fiscal adjustments. This, is possible given that the elasticities of the government rules $\gamma_G, \gamma_J, \gamma_Z > 0$ are large enough to guarantee that $\hat{b}^M_{t-1}$ remains on the stable path. The fiscal authority does not make fiscal adjustments in response to the remaining, unfunded, portion of debt $\hat{b}^F_{t-1}$. The unfunded portion is the responsibility of a passive monetary policy.

The variables $\zeta^F_{Z,t}$and $\zeta^M_{Z,t}$ denote shocks to transfers that are respectively unfunded and funded. The shocks $\zeta^F_{Z,t}$, $\zeta^M_{Z,t}$ and $\zeta_{g,t}$ follow AR(1) Gaussian stochastic processes.

4. Quantitative Analysis

4.1. *Model Parametrization*

We calibrate the model to the US data. For the real economy we follow Bianchi et al. (2020). Given that the two models are very similar we benefit from their model estimation for the US economy. We take these parameters as given. For the financial sector, we take the values from Gertler and Karadi (2013) who they calibrate their model to the US banking sector.

Given the fact that our real model is very close to Bianchi et al. (2020) we take advantage of their estimation and we set our parameter values according to their paper. Importantly we follow them for the parametrization of the elasticities of the fiscal rules. The government spending elasticity, γ_G is set to 0.38. The transfer policy elasticity, γ_Z, set to 0.0017. Regarding the elasticities of the different taxes we slightly abstract from BFM. They set consumption taxes to zero, while estimating capital and labour taxes. Here, we use consumption and labour taxes. Capital taxes would have been needed to enter the bankers' problem, making it unrealistic. We set $\gamma_L = 0.0163$ similar to BFM and and $\gamma_C = 0.043$. These are the baseline values for our taxation parameters. When we consider debt stability achieved only by the tax rules, given the above values there is no stable equilibrium. This is clear in figure 5 in the next subsection which shows the stability regions depending on the rule parameter values. Therefore, to compare the different fiscal and QE rules for debt stability we increase the tax elasticities at the smallest value that can offer determinacy, around 0.35. Regarding the QE elasticity to government debt changes, γ_{QE}, there is no counterpart to this in the data and we set it in the baseline scenario to 15. As can be seen in figure 5 this value provides debt stability when no other rule is active.

Financial parameter values are chosen in order to match specific US banking characteristics namely the banks' average leverage, lending spread and the expected horizon of the banker. There are three parameters that characterise the behaviour of the financial sector in the model: the absconding rate θ, the fraction of entering bankers initial capital fund ξ_B , and the value of the survival rate, σ_B. We calibrate these parameters to match certain steady-state moments following the moments reported in Gertler and Karadi (2013) and Sims and Wu (2021). The steady-state leverage of the banks is set equal to 6, which corresponds to the average asset-over-equity ratio of monetary and other financial institutions as well as non-financial corporations between 1999Q1 and 2014Q4. Second, the steady-state spread of the lending rate over the risk-free rate, $R^k - R$ is set to 100 basis points points in annual frequency at the steady state. We set the fraction of bonds that can be absconded Δ to 50% targeting a steady state bond spread of half the lending spread, at 50 basis points. Following Gertler and Karadi (2013) we set the steady-state target for the excess return on bonds on estimates of the term premium by Ludvigson and Ng (2009). For private securities they use information on the pre-2007 spreads between mortgage rates and government bonds and between BAA corporate versus government bonds, in conjunction with the evidence on the term premium. The absconding rate of reserves ω is set to zero; similarly to Sims and Wu (2021) we assume that reserves are fully recoverable by depositors in the event of bankruptcy. Since these are deposits are the central bank, we believe they cannot be diverted without knowledge by the central bank. Finally the σ_B parameter is set such the expected horizon for bankers is ten years.

4.2. *Rule-Specific Determinate Equilibria*

We explore the determinacy regions for different values of the feedback parameters of the three policy rules we are interested in: the QE rule, the monetary Taylor rule and the tax rule. Table 3 summarizes the three policy rules, the tools that they use and when they can be characterised as active or passive.[4]

Figure 5 shows the determinacy regions for the taxation and the QE rules. Both rules respond to debt-to-gdp deviations from its steady state level as shown in (18) and (21) While searching for the stability region of every rule, the other rule coefficients are set to zero and the Taylor monetary rule follows an active monetary policy.

Policy	Tool	Active	Passive
QE	Bond Purchases	Set to zero	Debt-to-gdp
Conventional MP	Nominal Interest Rate	Inflation & Output	Set to zero
Fiscal Policy	Taxes	Set to zero	Debt-to-gdp

Table 3: Parameter Values

The top part of figure 5 shows the QE coefficient behaviour where the blue shaded

[4]Following Leeper (1991): a policy is "active" or "passive" depending on its responsiveness to government debt shocks. In this context, we can then call these policies "passive" when they respond to debt-to-gdp changes.

regions show the unique saddle-path equilibrium area and the white the instability area. When γ^{QE} is too low, between zero and four, there is no saddle path stable solution. The debt dynamics are explosive. A low response of QE to debt changes cannot create the fiscal revenues and the stimulating general equilibrium effects to provide stability.[5] The same follows for high values of QE where QE over-reacts to debt changes. The lower part of the figure shows the two tax rates stability regions. Similarly, for low rates on consumption and labour taxes no stable equilibrium can be achieved due to the explosive debt dynamics. Above a certain value, tax rates can stabilize the debt and therefore for all values of $\gamma^{\tau_j} > 0.25$ where $j = \{c, l\}$ there is a unique saddle-path equilibrium.

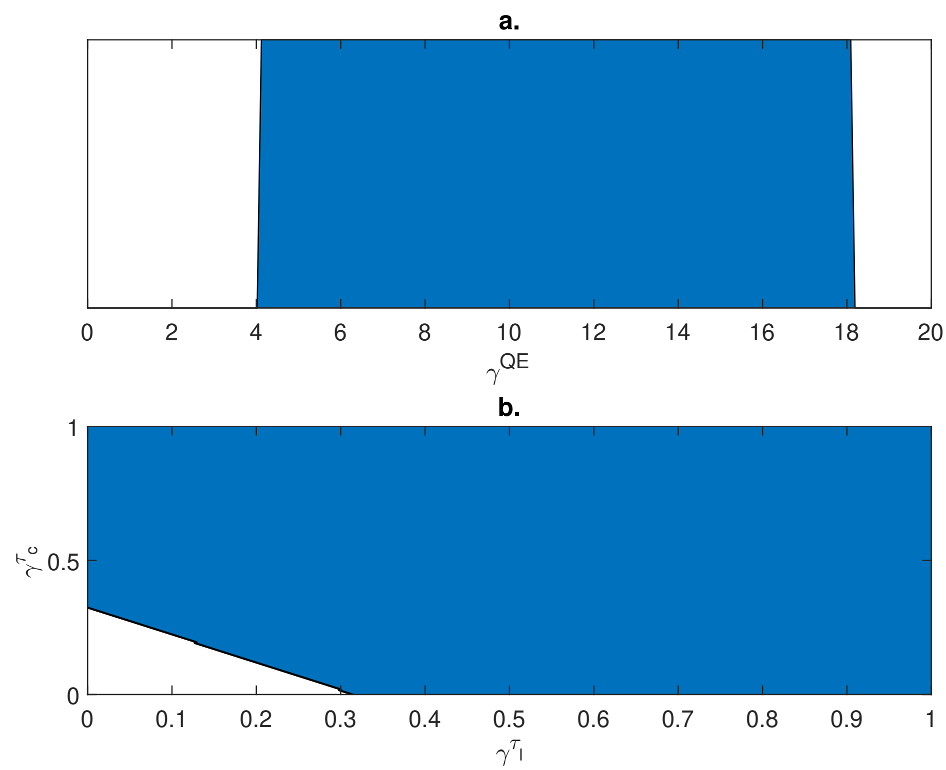


Fig. 3. Stability Regions for Policy Rules. a. QE rule, b. Tax Rule.
Notes. Blue area: determinacy, white area: no stable solution. Results are from simulations for 200 grid points in the parameter space.

We pursue to study the equilibrium determinacy of a passive QE policy along with a passive conventional monetary policy. Here, we focus on the cases where the QE rule and the monetary Taylor rule are the only rules responsible for providing debt stability; without any aid from fiscal policy. To do so we have set the tax coefficients to zero. We analyse the stability regions for a wide range of parameters for both rules. Specifically,

[5] We can say that the QE rule is not in its passive policy region.

we consider values of the Taylor rule response to inflation from 0 to 1 (passive monetary policy) and 1 to 2 (active monetary policy). The monetary rule coefficient to output changes (ϕ_y) is set to zero, which is close to its value in the calibrated model.

Figure 4 shows the results of this exercise. Similarly to the previous figure, the blue shaded regions show the determinacy areas, the white the no stability area and, new to this graph, the grey shaded area shows the indeterminacy region, or explosiveness. Given a low coefficient for the QE rule, that is an active QE rule that does not respond strongly to debt changes, determinacy is achieved only for values of $\phi_\pi < 1$, thus a passive monetary policy. In this case the central bank is ready to accommodate the level of inflation needed to stabilize the debt. On the contrary the monetary rule with $\phi_\pi > 1$ cannot stabilize the debt dynamics due to its strong response to inflation. At the same time the QE rule coefficient is too low for stability leading to no no saddle path stable solution.

An increase of the QE rule parameter above $\gamma^{QE} > 4$, making QE policy "passive" can lead to three different outcomes. This is conditional on the monetary policy parameter value. If monetary policy is active, that is $\phi_\pi > 1$, given that QE is in its passive regime, there is a saddle path stable equilibrium. If monetary policy is unresponsive to inflation, that is $\phi_\pi < 1$, then there is indeterminacy. Each policy authority acts passively in order to balance the government's budget. Without one rule being active this leads to a no solution outcome. Lastly, for any value of the nominal rate's response to inflation, if γ^{QE} is very high the model has no stable solution.

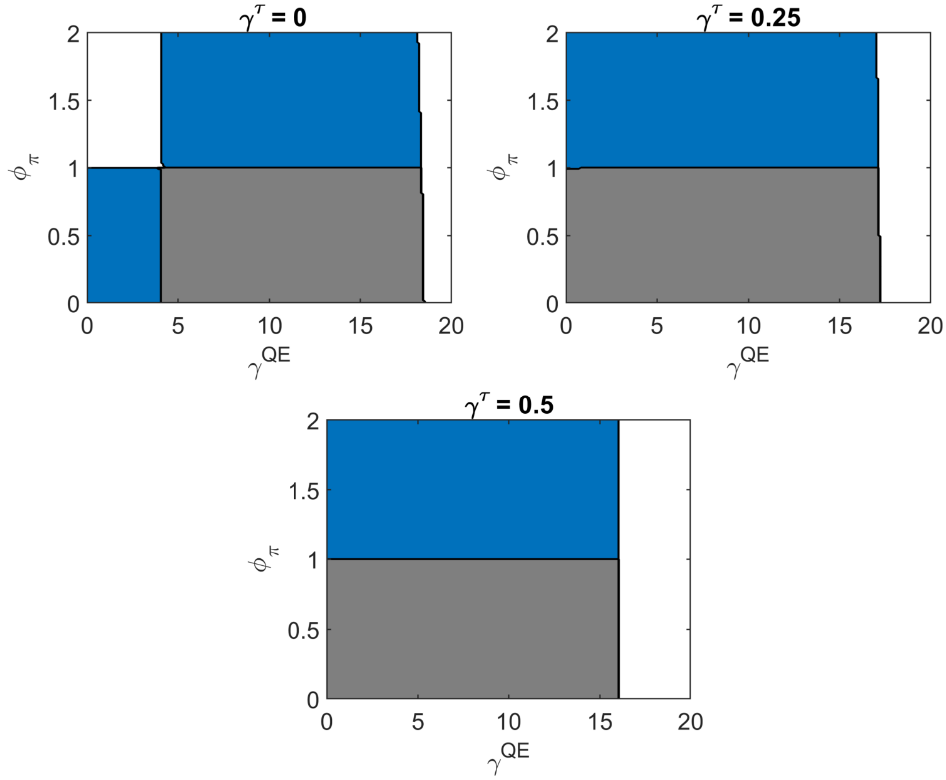


Fig. 4. Stability Regions for Policy Parameters. QE and Conventional Monetary Policy. Blue area: determinacy, white area: no stable solution, grey area: indeterminacy. Results are from simulations for 200 grid points in the parameter space.

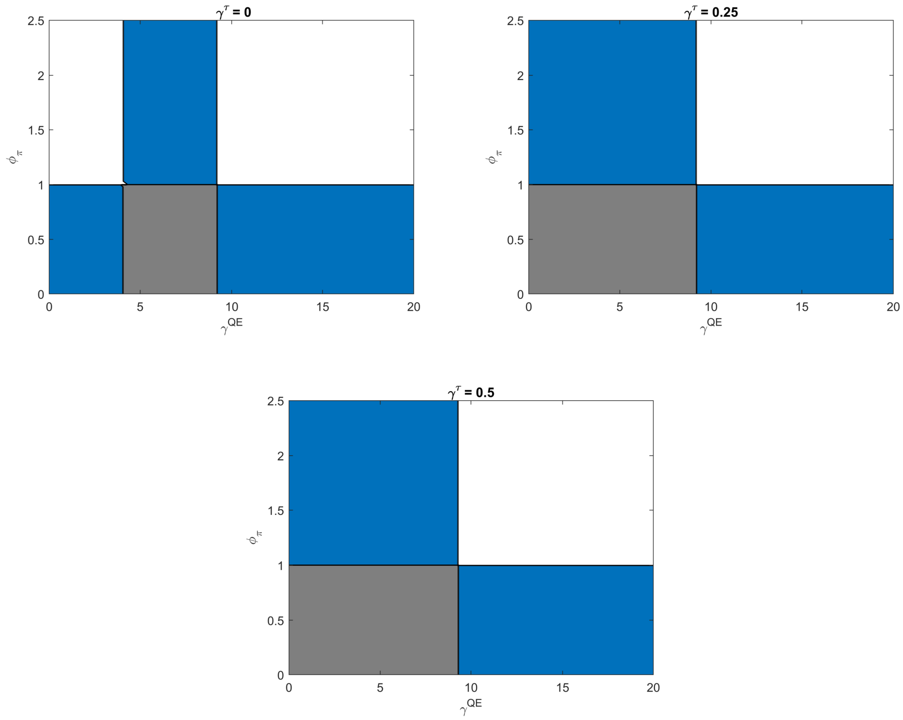


Fig. 5. Stability Regions for Policy Rule. QE responding to previous Debt. a. QE rule, b. Tax Rule.

Notes. Blue area: determinacy, white area: no stable solution. Results are from simulations for 200 grid points in the parameter space.

4.3. Transfer Shocks and Debt Stability

In this section we proceed by showing the responses of our model economy to an one time transfer to households. This captures in a crude way the Covid-19 transfers took place after 2019. The question we seek to answer is in what way the increase in debt could be stabilized conditional on minimizing any potential economic loss. We provide three different ways that debt stability can be achieved: first, by fiscal adjustments; an increase in taxes following the deviations of debt-to-gdp ratio from its initial equilibrium value. The second, an increase in the assets purchased by the central bank that follows the debt-to-gdp deviations (QE). This generates revenues for the treasury together with general equilibrium effects. In the case of QE we allow also for taxes to respond to debt changes.[6] The response parameters of the two rules are chosen given the stability regions such as they provide a saddle path stable equilibrium. In the third scenario, we assume that the central bank is ready to accommodate a level of inflation necessary to provide debt stability, therefore conducting a passive monetary policy and an active fiscal policy. In practical terms, the coefficients of the Taylor monetary rule

[6]Results remain similar even if we set the tax response parameters to zero.

and the tax response coefficient are all set to zero. Fiscal policy becomes active and monetary policy passive. To isolate the effects of government spending changes we have assumed that $\gamma_G = 0$ in this exercise.

Figure 6 shows the responses of some important macro variables to a transfer shock. The transfer shock is expansionary for both the cases of QE and passive monetary policy, while reduces output in the case of the tax fiscal adjustment. The fiscal adjustment case provides debt stability by increasing taxes and thus reducing the future income of the households. QE manages to provide debt stability by providing more fiscal space and by relaxing the banks' leverage constraint and reducing spreads therefore stimulating output and stabilizing debt. Taxation in the QE case does not need to increase as much as in the fiscal adjustment scenario. Furthermore, the increase in the debt-to-gdp also deteriorates much faster. Finally, a passive monetary policy provides the best response in terms of output. This is achieved by a fall in the real interest rate, as the fiscal and the monetary authorities coordinate to let inflation rise to stabilize the increase in transfers.

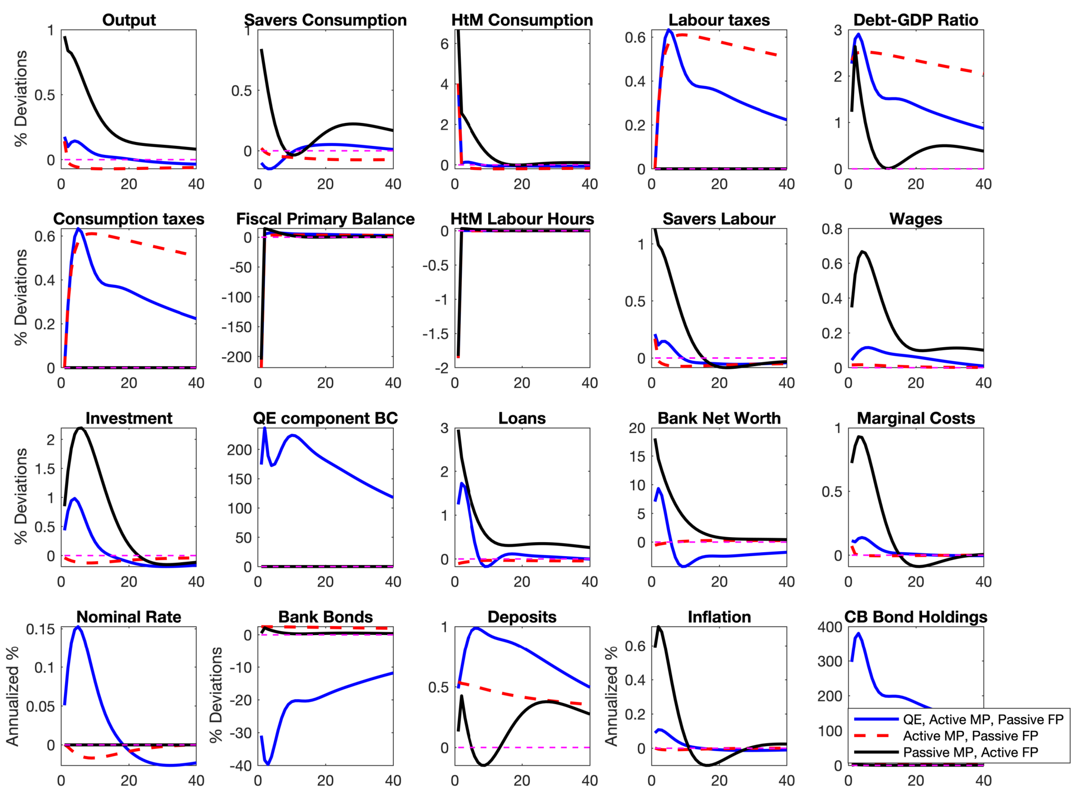


Fig. 6. A Transfer Shock under Different Debt Stabilizers

We need to stress that although a passive monetary policy provides the best response, this happens at a cost of leaving inflation to increase six times more than in the case of QE. Furthermore, the central bank is not in control of its policy rate given that it should stay be unresponsive to changes in inflation. On the contrary, in the QE scenario, the central bank is in full control of the policy rate which follows its Taylor rule. The transfer shock is still expansionary without letting inflation to deviate much from its steady state value. At the QE scenario, the central bank does not have to disobey its inflation targeting mandate. Having said that we will focus on the differences between the fiscal adjustment scenario and the QE scenario as they are the two more plausible scenarios.

An one time increase in transfers increases debt for all three cases given the extra resources needed to pay for the transfer. It makes the taxes go up for the fiscal adjustment case for a prolonged period. In the QE case central bank bond purchases increase to about four times their steady state level and as a result bank bonds go down while bank reserves increase by the same amount. Taxes here also increase but much less and for a shorter period than in the case of no QE. Lastly, passive monetary policy keeps the nominal rate constant and thus increases inflation. By definition, taxes do not respond to debt changes and the debt stability comes from the big increase in inflation. Hand to mouth consumption increases in all three cases due to the transfers. Savers consumption reduces on impact in the QE and the fiscal adjustment cases due to the increase in taxes, but rebounds quickly in the QE scenario due to its stimulating effects.

QE manages to stimulate the banking variables in contrast with the fiscal adjustment regime. Banks' net worth and loans go up. This is due to the reduction of bond and credit spreads as can be seen in Figure 7. Given the less bonds and more reserves in the banks' balance sheet their constraint loosens and thus spreads drop. Bond and capital prices increase which lead to an increase in the net worth. In the fiscal adjustment scenario the opposite happens due to the contractionary effects of the policy: capital drops and therefore capital and bond (due to arbitrage) returns increase.

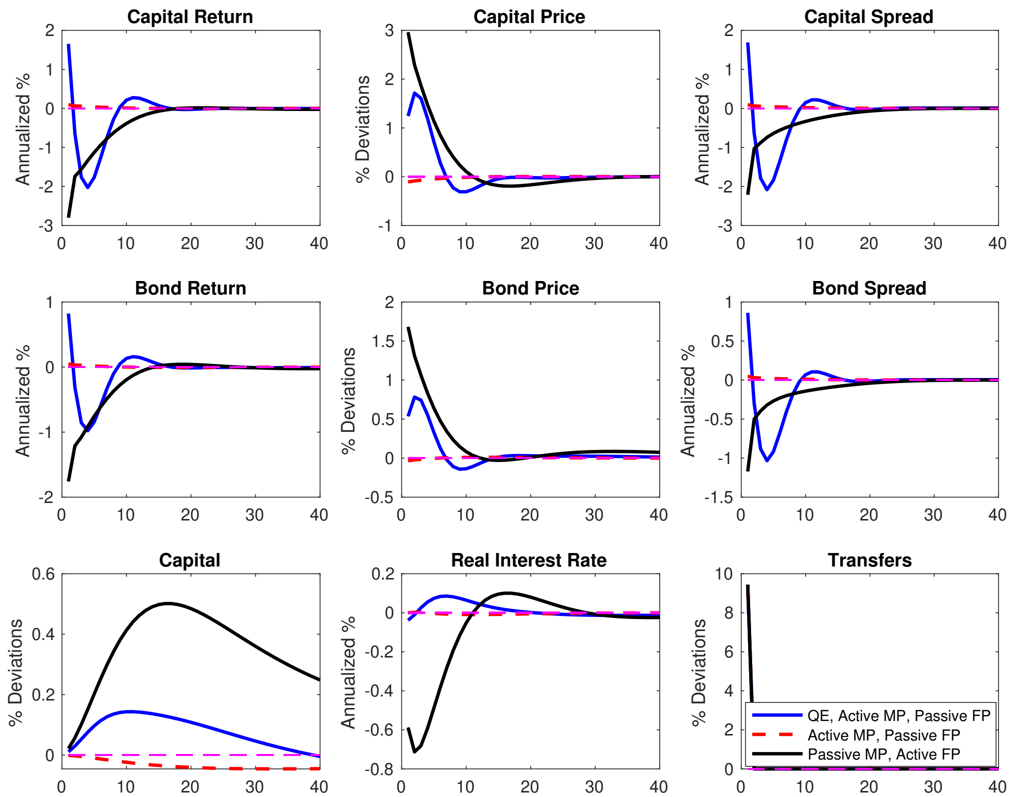


Fig. 7. A Transfer Shock under Different Debt Stabilizers

Given the large amounts of bonds purchases and the positive spread between the bonds and reserves, the central bank and therefore the treasury have an increase in the revenues as can be seen from the QE component of the consolidated government budget constraint. This helps, together with the general equilibrium effects coming from the banking sector, to pay the debt faster and thus to increase taxes less than in the fiscal adjustment case.

Comparing the passive monetary policy regime with the fiscal adjustments and the QE as a fiscal stabilizers yields two main points. Firstly, the passive monetary policy is the most efficient in terms of debt stability; providing even a reduction in debt after a transfer shock. This is achieved by an increase in inflation that is accommodated by the central bank. Secondly, the second best policy mix of those compared is fiscal adjustments with a parallel use of QE. This manages to keep taxes, government spending reduction and the recession at a much lower level than in the case where only the classical fiscal adjustment tools are employed.

4.4. *Risk Premium Shocks and QE*

QE rule can be more or less effective depending i) on the shock hitting the economy and ii) the variable(s) it responds to. In this section we want to simulate an exogenous increase in the risk premium. Recent experience shows that QE is used by central banks to reduce market distress and keeping interest rate spreads low. Here, we compare a QE policy that reacts to spreads versus a tax policy and a passive monetary policy, as defined above.

An example of this action comes from the recent UK experience. The UK with its recent "mini budget" plan by chancellor Kwasi Kwarteng The Bank of England intervened with a promise to buy up to £65bn of government bonds to save funds responsible for managing money on behalf of UK pensioners from collapse.

In the model a risk premium shock will increase both the bond and the capital interest spreads, reduce asset prices and increase the returns, as shown in figure 9

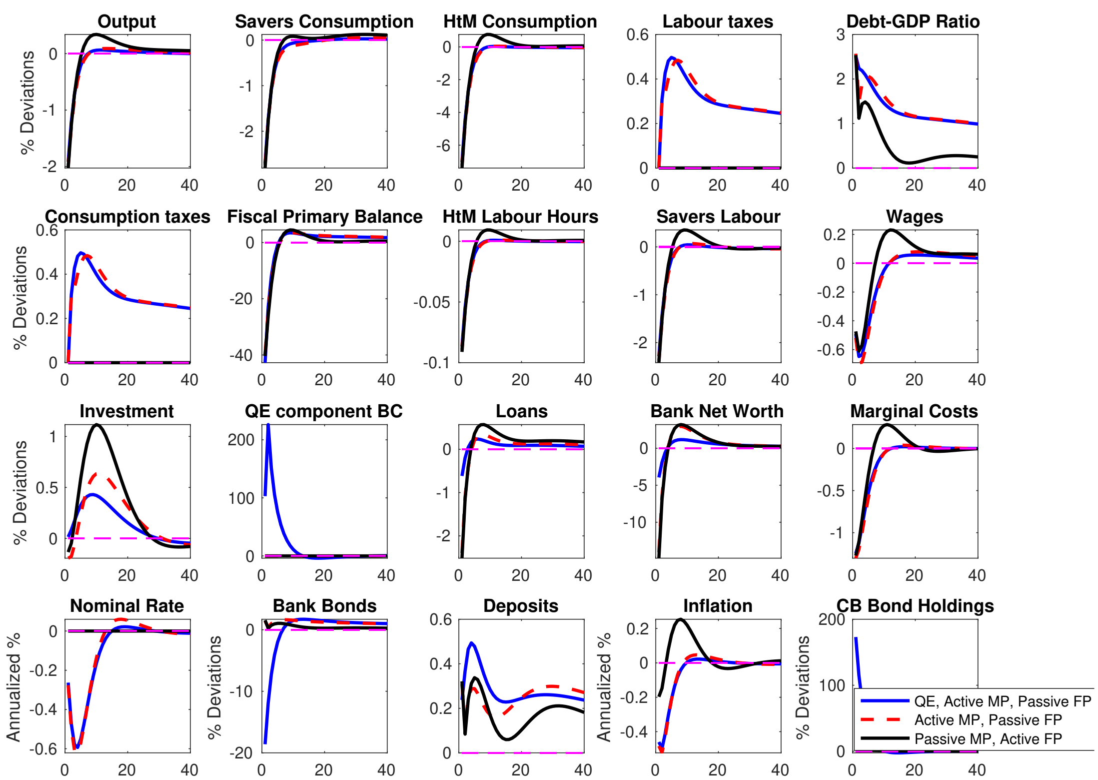


Fig. 8. A Risk Premium Shock

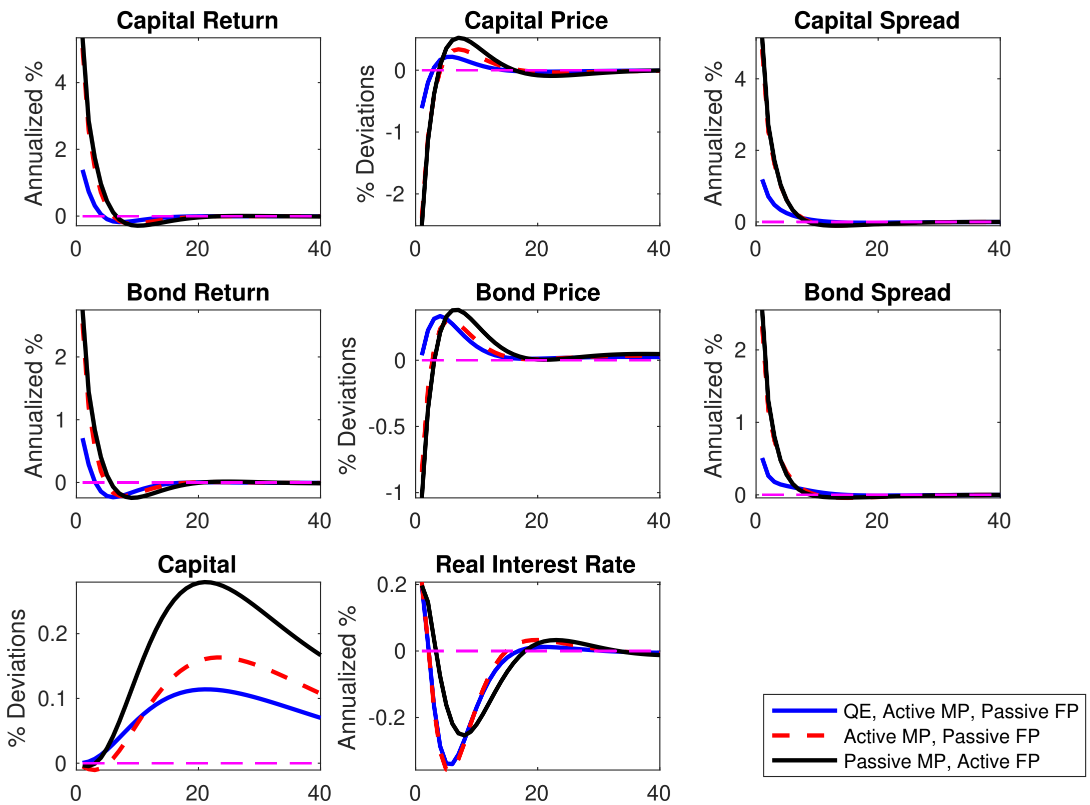


Fig. 9. A Risk Premium Shock

5. Conclusion

Since COVID-19 pandemic there is an unprecedented increase in social insurance transfers both in the EA and the US. In this paper, we explore different fiscal and monetary strategies in times of large debt accumulation. We build a New Keynesian DSGE model with household heterogeneity, financial frictions, nominal rigidities, and an unconstrained central bank that can purchase bonds in exchange of reserves. We identify QE as a potential tool for debt stabilization. Profits earned from the bonds-reserves spread can be remitted from the central bank to the treasury and can be a substantial fiscal revenue. We analyse and compare QE as a debt stabilization tool versus taxation changes under an active (conventional) monetary policy and a passive monetary policy framework.

We show that QE can provide fiscal stability on top of its general equilibrium effects. This happens due to the revenues it generates for the central bank and can be remitted to the treasury to be used as fiscal revenues for debt repayments. After a transfer to the households, QE is the most efficient way of repayment relative to the classical fiscal adjustments. We also show that a passive monetary policy that increases inflation is also beneficial but hard to justify institutionally due to the inflation target mandate of the central banks.

Appendix A Bank's Problem

This appendix describes the method used for solving the banker's problem. We solve this, with the method of undetermined coefficient in the same fashion as in Gertler and Kiyotaki (2010). We conjecture that a value function has the following linear form:

$$V_t(s_t, d_t, b_t^B, m_t) = \nu_t s_t + \nu_t b_t^B + \nu_{m,t} m_t - \nu_{d,t} d_t \tag{A.1}$$

where $\nu_{s,t}$ is the marginal value from credit for bank j, $\nu_{d,t}$ the marginal cost of deposits, $\nu_{m^B,t}$ the marginal value from the central bank reserves and $\nu_{b^B,t}$ the marginal value from purchasing one extra unit of sovereign bonds. The banker's decision problem is to choose s_t, b_t^B, m_t, d_t to maximize V_t subject to the incentive constraint (10) and the balance sheet constraint (7). Using (7) we can eliminate d_t from the value function. This yields:

$$V_t = s_t(\nu_{s,t} - \nu_{d,t} Q_t) + b_t^B(\nu_{b,t} - \nu_{d,t} P_t^B) + m_t(\nu_{m,t} - \nu_{d,t}) + \nu_{d,t} n_t.$$

Let $\mathcal{L}$ be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t[V_t - \theta(Q_t s_t + \Delta P_t^B b_t^B + \omega m_t)] = (1 + \lambda_t)V_t - \lambda_t \theta(Q_t s_t + \Delta F b_t^B + \omega m_t).$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta s_t} : (1 + \lambda_t)(\frac{\nu_{s,t}}{Q_t} - \nu_{d,t}) = \lambda_t \theta \tag{A.2}$$

$$\frac{\theta \mathcal{L}}{\theta b_t^B} : (1 + \lambda_t)(\frac{\nu_{b^B,t}}{P_t^B} - \nu_{d,t}) = \Delta \lambda_t \theta \tag{A.3}$$

$$\frac{\theta \mathcal{L}}{\theta m_t} : (1 + \lambda_t)(\nu_{m^B,t} - \nu_{d,t}) = \omega \lambda_t \theta \tag{A.4}$$

The Kuhn-Tucker condition yields:

$$\begin{aligned} KT : \lambda_t[s_t(\nu_{s,t} - \nu_{d,t} Q_t) + b_t^B(\nu_{b^B,t} - \nu_{d,t} P_t^B) + m_t(\nu_{m^B,t} - \nu_{d,t}) \\ + \nu_{d,t} n_t - \theta(Q_t s_t + \Delta P_t^B b_t^B + \omega m_t)] = 0. \end{aligned} \tag{A.5}$$

We define the excess value of bank's financial claim holdings as

$$\mu_t^s = \frac{\nu_{s,t}}{Q_t} - \nu_{d,t}. \tag{A.6}$$

The excess value of bank's bond holdings relative to deposits

$$\mu_t^b = \frac{\nu_{b^B,t}}{P_t^B} - \nu_{d,t},$$

and the excess value of bank's reserve holdings relative to deposits

$$\mu_t^m = \nu_{m,t} - \nu_{d,t}.$$

Then from the first order conditions we have:

$$\mu_t^b = \Delta \mu_t^s. \tag{A.7}$$

Setting the fraction of the absconding rate for reserves ω to 0%, the reserves first order condition (A.4) implies that

$$\nu_{m,t} = \nu_{d,t}. \tag{A.8}$$

This relationship implies that the marginal impact of deposits and reserves is identical. From (A.5) and (A.7) when the constraint is binding ($\lambda_t > 0$) we get:

$$\begin{aligned} s_t(\nu_{s,t} - \nu_{d,t}Q_t) + b_t^B(\nu_{b^B,t} - \nu_{d,t}P_t^B) + m_t(\nu_{m,t}^B - \nu_{d,t}) + \nu_{d,t}n_t &= \theta(Q_t s_t + \Delta P_t^B b_t^B + \omega m_t) \\ s_t(\mu_t^s Q_t) + b_t^B(\mu_t^b P_t^B) + m_t(\mu_t^m) + \nu_{d,t}n_t &= \theta(Q_t s_t + \Delta P_t^B b_t^B + \omega m_t) \\ Q_t s_t(\mu_t^s - \theta) + P_t^B b_t^B(\Delta\mu_t^s - \Delta\theta) + m_t(\omega\mu_t^s - \omega\theta) + \nu_{d,t}n_t &= 0 \\ Q_t s_t(\mu_t^s - \theta) + \Delta P_t^B b_t^B(\mu_t^s - \theta) + \omega m_t(\mu_t^s - \theta) + \nu_{d,t}n_t &= 0 \end{aligned}$$

and by rearranging terms, we get equation the maximum leverage relative to divertible assets:

$$Q_t s_t + \Delta P_t^B b_t^B + \omega m_t = \phi_t n_t \tag{A.9}$$

this constraint shows the maximum leverage, ϕ_t, the bank can take relative to its measure of divertible assets, where the adjusted leverage is defined as:

$$\phi_t = \frac{\nu_{d,t}}{\theta - \mu_t^s}. \tag{A.10}$$

We follow Gertler and Karadi (2013) and call it adjusted leverage given that it is not the total assets over net worth. Instead, it is the measure of assets multiplied by their respective weights.

Now, in order to find the unknown coefficients we return to the guessed value function

$$V_t = Q_t s_t(\mu_t^s) + P_t^B b_t^B(\mu_t^b) + m_t(\mu_t^m) + \nu_{d,t}n_t. \tag{A.11}$$

Substituting (A.9) into the guessed value function yields:

$$\begin{aligned} V_t &= (n_t\phi_t - \Delta P_t^B b_t^B - \omega m_t)\mu_t^s + P_t^B b_t^B(\mu_t^b) + m_t(\mu_t^m) + \nu_{d,t}n_t \Leftrightarrow \\ V_t &= (n_t\phi_t)\mu_t^s + P_t^B b_t^B(\mu_t^b - \Delta\mu_t^s) + m_t(\mu_t^m - \omega\mu_t^s) + \nu_{d,t}n_t \end{aligned} \tag{A.12}$$

and by (A.7) the guessed value function (A.12) becomes:

$$V_t = \phi_t \mu_t^s n_t + \nu_{d,t}n_t$$

Given the linearity of the value function we get that

$$A_t^B = \phi_t \mu_t^s + \nu_{d,t}.$$

The Bellman equation now becomes:

$$\begin{aligned}
V_{t-1}(s_{t-1}, x_{t-1}, d_t, m_{t-1}) = \mathbb{E}_{t-1}\,\Lambda_{t-1,t}\sum_{i=1}^{\infty}\{(1-\sigma_B)n_t \\
+ \sigma_B(\phi_t\mu_t^s + \nu_{d,t})n_t\}. \qquad \text{(A.13)}
\end{aligned}$$

By collecting terms with n_t the common factor and defining the variable Ω_t as the marginal value of net worth:

$$\Omega_{t+1} = (1-\sigma_B) + \sigma_B(\mu_{t+1}^s\phi_{t+1} + \nu_{d,t+1}). \qquad \text{(A.14)}$$

The Bellman equation becomes:

$$\begin{aligned}
V_t(s_t, b_t^B, m_t, d_t) &= E_t\Lambda_{t,t+1}\Omega_{t+1}n_{t+1}^B = \\
&= E_t\Lambda_{t,t+1}\Omega_{t+1}[R_{k,t+1}Q_t s_t + R_{b,t+1}P_t^B b_t^B + R_{t+1}m_t - R_{t+1}d_t]. \qquad \text{(A.15)}
\end{aligned}$$

The marginal value of net worth implies the following: Bankers who exit with probability $(1-\sigma_B)$ have a marginal net worth value of 1. Bankers who survive and continue with probability σ_B, by gaining one more unit of net worth, they can increase their assets by ϕ_t and have a net profit of μ_t per assets. By this action they acquire also the marginal cost of deposits $\nu_{d,t}$ which is saved by the extra amount of net worth instead of an additional unit of deposits. Using the method of undetermined coefficients and comparing (A.1) with (A.15) we have the final solutions for the coefficients:

$$\begin{aligned}
\nu_{s,t} &= E_t\Lambda_{t,t+1}\Omega_{t+1}R_{k,t+1}Q_t \\
\nu_{b^B,t} &= E_t\Lambda_{t,t+1}\Omega_{t+1}R_{b,t+1}P_t^B \\
\nu_{m^B,t} &= E_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1} \\
\nu_{d,t} &= E_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1} \\
\mu_t^s &= \frac{\nu_{s,t}}{Q_t} - \nu_{d,t} = E_t\Lambda_{t,t+1}\Omega_{t+1}[R_{k,t+1} - R_{t+1}] \\
\mu_t^b &= \frac{\nu_{b,t}}{P_t^B} - \nu_{d,t} = E_t\Lambda_{t,t+1}\Omega_{t+1}[R_{b,t+1} - R_{t+1}] \qquad \text{(A.16)} \\
\mu_t^m &= \nu_{m,t} - \nu_{d,t} = E_t\Lambda_{t,t+1}\Omega_{t+1}[R_{t+1} - R_{t+1}] = 0 \qquad \text{(A.17)}
\end{aligned}$$

Appendix B Price Setting

Final-Good Firms.— The profit maximization problem of the retail firm is:

$$\max_{Y_t(j)} P_t \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

The first order condition of the problem yields:

$$P_t \frac{\zeta}{\zeta-1} \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}-1} \frac{\zeta-1}{\zeta} Y_t(i)^{\frac{\zeta-1}{\zeta}-1} = P_t(i).$$

Combining the previous FOC with the definition of the aggregate final good we get:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

Nominal output is the sum of prices times quantities across all retail firms i:

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di.$$

Using the demand for each retailer we get the aggregate price level:

$$P_t = \left(\int_0^1 P_t(i)^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}.$$

Intermediate-Good Firms.— Intermediate good firms are not freely able to change prices each period. Following the Calvo price updating specification each period there is a fixed probability $1-\gamma$ that a firm will be able to adjust its price.

The problem of the firm can be decomposed in two stages. Firstly, the firm hires labour and rents capital to minimize production costs subject to the technology constraint. Thus, it is optimal to minimize their costs which are the rental rate to capital and the wage rate for labour:

$$\min_{K_t(i), L_t(i)} P_t W_t l_t(i) + P_t Z_t K_t(i)$$

subject to

$$A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha} \geqslant \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

The problem's first order conditions are:

$$W_t = \frac{P_{m,t}^{nom}(i)}{P_t} (1-\alpha) A_t \frac{Y_t(i)}{L_t(i)}, \tag{B.1}$$

$$Z_t = \frac{P_{m,t}^{nom}(i)}{P_t} \alpha A_t \frac{Y_t(i)}{K_t(i)}. \tag{B.2}$$

$P_{m,t}^{nom}$ is the Lagrange multiplier of the minimization problem and the marginal cost of the firms with $P_{m,t} = \frac{P_{m,t}^{nom}(i)}{P_t}$ being the real marginal cost. Standard arguments lead to that marginal cost is equal across firms. Solving together the above equations we find an expression for the real marginal cost $P_{m,t}$ which is independent of each specific variety:

$$P_{m,t} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} W_t^{1-\alpha} Z_t^{\alpha}.$$

In the second stage of the firm's problem, given nominal marginal costs, the firm chooses its price to maximize profits. Firms are not freely able to change prices each period. Each period there is a fixed probability $1-\gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p. They discount profits s periods in the future by the stochastic discount factor $\Lambda_{t,t+s}$ and the probability that a price price chosen at t will remain the same for some periods γ^s. The second stage of the updating firm at time t us to choose $P_t^*(i)$ to maximize discounted real profits:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} - P_{m,t+s} \right) Y_{t+s}(i)$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^{s} (1+\pi_{\tau+\kappa-1})^{\gamma_p} \right)^{-\zeta} Y_{t+s}.$$

where π_t is the rate of inflation from $t-i$ to t. The first order condition of the problem is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^{s} (1+\pi_{\tau+\kappa-1})^{\gamma_p} - P_{m,t+s} \frac{\zeta}{\zeta-1} \right) Y_{t+s}(i) = 0.$$

Using the constraint and rearranging we get:

$$P_t^*(i) = \frac{\zeta}{\zeta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{m,t+s} P_{t+s}^{\zeta} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{t+s}^{\zeta-1} \prod_{\kappa=1}^{s} (1+\pi_{\tau+\kappa-1})^{\gamma_p} Y_{t+s}}.$$

Since nothing on the right hand side depends on each firm i, all updating firms will update to the same reset price, P_t^*. By the law of large numbers the evolution of the price index is given by:

$$P_t = [(1-\gamma)(P_t^*)^{1-\zeta} + \gamma(\Pi_{t-1}^{\gamma_p} P_{t-1})^{1-\zeta}]^{\frac{1}{1-\zeta}}.$$

Capital Goods Producers.— Capital goods producers produce new capital and sell it to goods producers at a price Q_t. Investment on capital (I_t) is subject to adjustment costs. Their objective is to choose $\{I_t\}_{t=0}^{\infty}$ to solve:

$$\max_{I_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Big\{ Q_t I_t - [1 + \tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) I_\tau] \Big\}.$$

where the adjustment cost function $\tilde{f}$ captures the cost of investors to increase their capital stock:

$$\tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) = \frac{\eta}{2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2 I_\tau.$$

η is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors yields the competitive price of capital:

$$Q_t = 1 + \Big(\eta \frac{I_\tau}{I_{\tau-1}}\Big(\frac{I_\tau}{I_{\tau-1}} - 1\Big) + \frac{\eta}{2}\Big(\frac{I_\tau}{I_{\tau-1}} - 1\Big)^2 - \eta \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2}\Big(\frac{I_\tau}{I_{\tau-1}} - 1\Big)\Big).$$

Appendix C Banking Model and Risk Weights

The aggregate bank constraint in terms of the total net worth in the economy is:

$$\omega Q_t S_t^B + \Delta P_t^B B_t^B + \zeta M_t = \phi_t N_t. \tag{C.1}$$

QE, the purchase of bonds or shares by the central bank loosens the constraint leading to a reduction in spreads. In general, when the CB buys bonds, banks can leverage up and increase their loans S_t^B. The magnitude of the positive QE effects depends on the risk weights ω and Δ of the shares and bonds respectively. The higher the ω, the more risky the loans and the tighter the banking constraint therefore the smaller effects from a QE. On the other hand when the CB buys bonds reducing the amount of B_t, the smaller the parameter Δ is the smaller the effect on the banking constraint.

Figures 10 and 11 show the impulse responses after a QE shock with different weight on loans and bonds. Figure 10 shows the responses with $\omega = 1$ and $\Delta = 0.5$ versus the same shock with $\omega = 2$ and $\Delta = 0.5$ while figure 11 shows the responses with $\omega = 1$ and $\Delta = 0.5$ versus the same shock with $\omega = 1$ and $\Delta = 0.1$. In both cases, the QE shock is expansionary by general equilibrium effects. In 10 when the loan risk weight is 200% due to the tighter bank constraint the positive effects are scaled down. In 11 when the risk weight on bonds becomes 10%, the central bank operation has much less impact than the case of $\Delta = 0.5$.

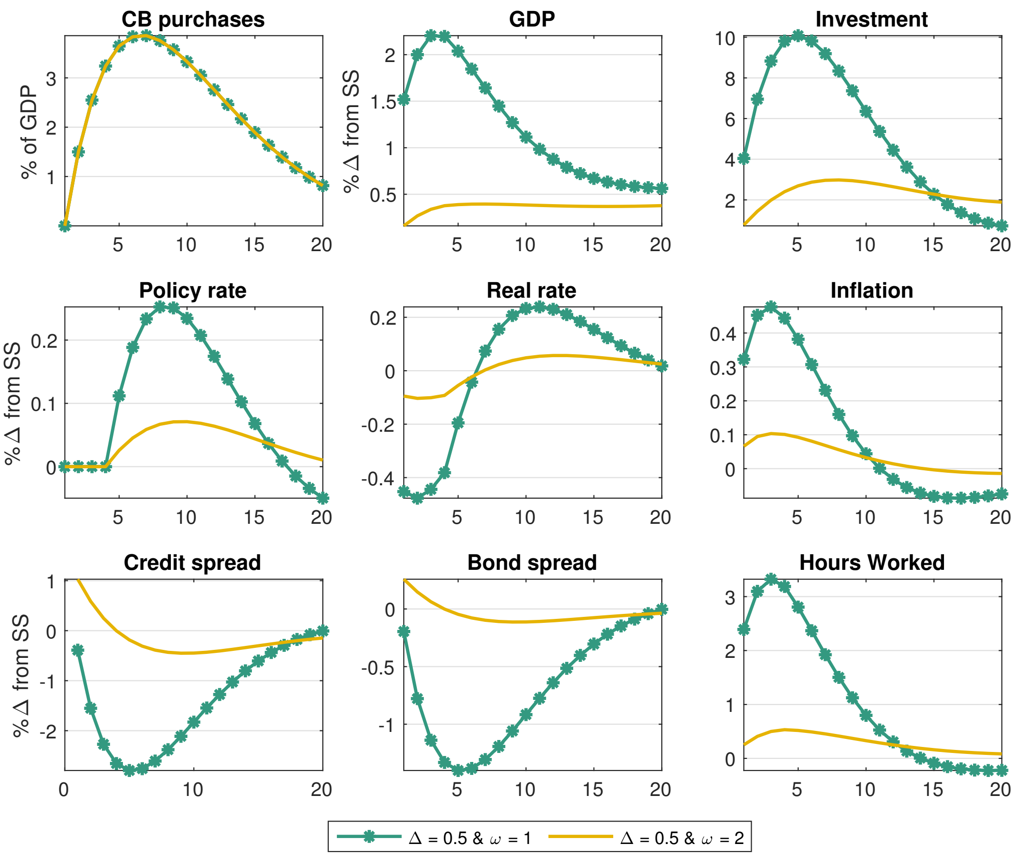


Fig. 10. QE under Different Risk Weights for Loans

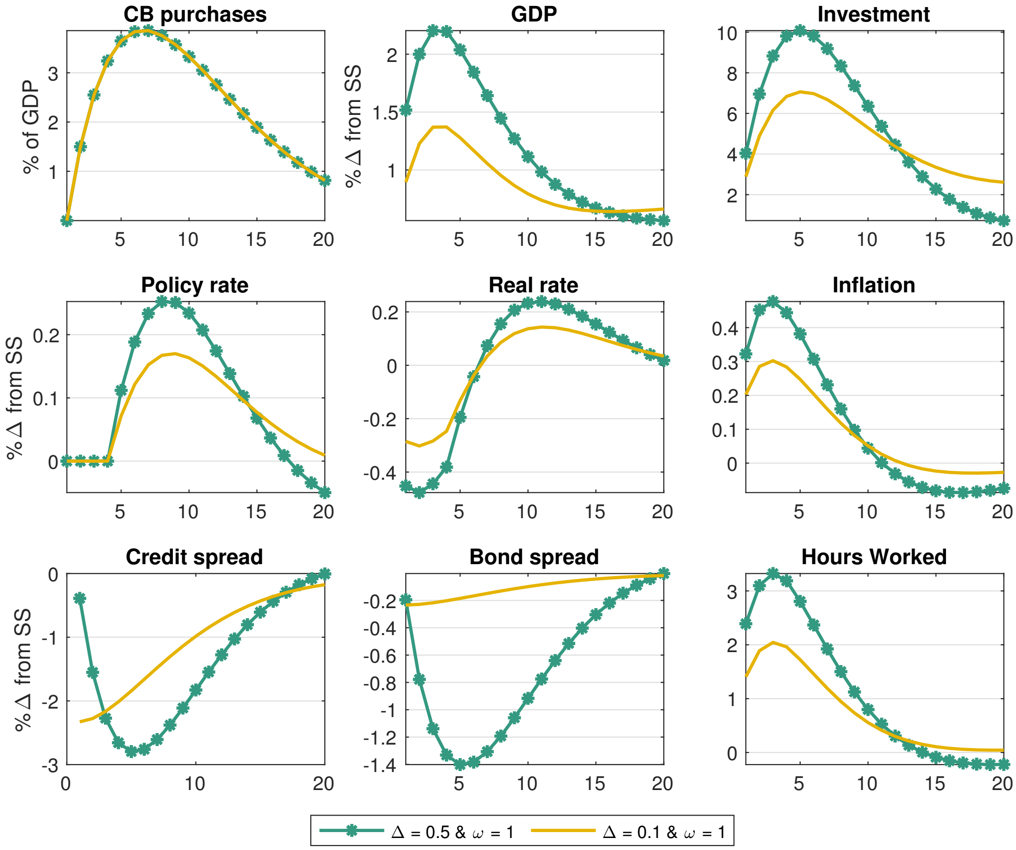


Fig. 11. QE under Different Risk Weights for Bonds

Appendix D QE revenue data

We use the data from the Federal Reserve Bank of New York to retrieve purchases and sales of the Treasury Securities (TSY). A detailed description of the API is provided on newyorkfed.org p24, p46-49. The data replication consists of two steps (i) get CUSIP (bonds' id) of the Fed purchases and (ii) use CUSID to download Treasury Holdings of the Federal Reserve at each given date.

Below we provide a detailed procedure of data construction.

1. Download data on bond IDs at weekly frequency. For ex. bond ID gives us all bond purchases in 2005. The data can only be downloaded for one year at a time.
2. The main table column is bond's ID: CUSIP.
3. We repeat the download for each year.
4. We select all unique CUSIPs from the sample.
5. For each CUSIP, we download historical holdings of the Federal Reserves at each week. For ex. Bond CUSIP 91282CED9

6. We repeat the download for each of 1407 different types of bonds and combine all the data into one table.
7. The columns of interest are: date of holding, maturity date, CUSIP, security type (tips, bills, notes and bonds), coupon, par value of the bond.
8. We sum up par value across all different CUSIP, which gives us Treasury holdings of the Federal Reserve at each given date.

   Further we describe the way to calculate revenues from QE
9. For every week of data, we know par value of bonds holdings of each bond type, the coupons paid on this bonds and reserves rate paid by the Fed to the banks.
10. We calculate weekly revenue as

$$\sum_{i=1}^{1407} \frac{BondsParValue_{i,t} \times (Coupon_{i,t} - ReservesRate_t)}{NumberOfWeeksInYear} \tag{D.1}$$

for week t and CUSIP i.

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