Asset Purchases, Limited Asset Markets Participation and Inequality

Stylianos Tsiaras*

European University Institute

August 2020

Abstract

This paper examines the impact of quantitative easing (QE) on aggregate demand and inequality in a restricted financial participation economy. It shows that when labour markets are imperfect and the level of asset markets participation high, QE stimulates aggregate demand. It also provides redistribution by reducing income and consumption inequality. To study these phenomena, I develop and calibrate a Neo-Keynesian dynamic, general equilibrium model with sticky prices and wages for the Euro Area that incorporates limited assets market participation, financial frictions and allows central bank purchases from banks and households. Bond purchases increase aggregate demand and benefit financially restricted households more due to the dominance of QE's indirect effects, reducing income and consumption inequality. This is conditional on the level of wage stickiness and thus the cyclicality of profits. When wages are flexible and thus profits countercyclical, low financial participation levels invert QE's stimulating effects. The threshold value of participation that makes the policy contractionary depends on the fiscal policy redistribution. Using an external instrument SVAR, I find that profits move pro-cyclically supporting the sticky wage specification of the model and introducing wage rigidities, the negative effects of are muted.

JEL classification: E12; E44; E52; E58; E61

Keywords: Quantitative Easing; Inequality; Financial Participation; DSGE; SVAR; External Instrument

*Corresponding email: stylianos.tsiaras@eui.eu. I would like to thank Martin Kaae Jensen, Paul Levine, Ramon Marimon, Ricardo Nunes and Morten Ravn for their valuable feedback and discussions and also participants at conferences and workshops this paper has been presented.

1. Introduction

Asset purchase programmes following the Great Recession aimed to hold down long-term interest rates and stimulate aggregate demand. Although empirical literature has shown that the goal of the programmes has been achieved (see Krishnamurthy and Vissing-Jorgensen (2012) among others), a question lately posed by policy makers (Yellen, 2016; Bernanke, 2015; Draghi, 2016) and gained the media attention is whether and in what extend asset purchase programmes have contributed to the increase of inequality.[1] In this paper I answer two questions: firstly whether QE increases aggregate demand and secondly how it affect inequality.

I show that a QE policy does not only reduce the structural wedges in the Euro Area (EA) economy but also provides redistribution. I develop a DSGE model with banks, financially constrained and unconstrained households and a central bank that can purchase assets from banks and households. Firstly, I extend the result of Bilbiie (2008) and show that the sing of the QE impact depends on the assets market participation and the cyclicality of profits, a by-product of the wage stickiness. The key determinant of this behaviour is the profits and their impact on aggregate demand. In an economy with sticky wages, marginal costs do not increase as much as in the competitive labour market case, and profits are much less countercyclical. Therefore, the QE's contractionary effects are muted. To investigate the behaviour of profits in the EA, I use an external instrument SVAR approach and I show that profits move pro-cyclically after a QE shock supporting the recent evidence for the accelerating role of the QE policy. In a counterfactual exercise, employing the standard flexible wages NK specification and thus countercyclical profits, QE can have negative impact on aggregate demand conditional on the level of assets market participation; I show that this can be alleviated by fiscal redistribution. In detail I introduce a fiscal rule that redistributes share of the profits as transfers to hand to mouth consumers. The higher the redistribution, the less likely for QE to be contractionary for a low level of financial participation. This highlights the importance of monetary and fiscal policy coordination.

Turning to the inequality impact of the QE I show that the indirect (i.e general equilibrium) effects outweigh the direct effects leading to a reduction in income and consumption inequality. The economic intuition of the QE impact on inequality is as follows. Consider an increase in the bond holdings of the central bank in an economy with two types of consumers, asset holders and hand to mouth consumers. The outcome of this operation will have direct and indirect effects. The direct effects, namely the reduction of the interest rates and the asset price increases will harm and benefit the bond holders respectively. On the other hand, the indirect or general equilibrium effects such as the employment level and real wage increases will benefit hand to mouth consumers. To demonstrate this, I firstly develop a basic two-period model with a cash in advance constraint which shows analytically the direct effects of a quantitative easing

[1] Does Quantitative Easing Mainly Help the Rich? (CNBC), Debate rages on quantitative easing's effect on inequality (Financial Times), Quantitative easing helped vulnerable more than rich, says ECB (Financial Times).

in households' consumption and income. Then, using the full calibrated DSGE model I show quantitatively that QE is redistributive.

This study introduces limited asset markets participation (LAMP), agency problems associated with financial intermediation and a QE framework in an otherwise standard business cycles model with sticky prices and wages. By combining Galí, López-Salido, and Vallés (2007) (GLS hereafter), Bilbiie (2008) and Gertler and Karadi (2013) a setting is developed where central bank purchases of government bonds or private assets and the exchange of those with reserves, create direct and indirect effects on the real economy affecting differently those with and without access to financial markets. I evaluate inequality between the two groups in terms of consumption and income inequality following Krueger and Perri (2006).

Financial frictions play a prominent role in the analysis. Banks, are subject to a minimum capital requirement (MCR) constraint similar to Basel III. which implies that banks must have capital value greater than a proportion of their risk-weighted assets. Central bank bond purchases relax the bank's constraint and stimulate the demand for loans. QE in the model works as a credit stimulating mechanism to the real economy.

To evaluate the behaviour of profits after a QE shock in the Euro Area I employ an external instrument SVAR approach. This is based on the work of Mertens and Ravn (2013) and the high frequency identification approach of Gertler and Karadi (2015). To identify QE policy surprises I make use of the Euro Area Monetary Policy Event Study Database by Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019), I develop and use the QE factor as external instrument. Results show that profits move pro-cyclically supporting the specification of sticky wages in the model. The existence of the regulatory constraint together with households' transaction costs eliminate the perfect substitutability of assets and break the neutrality result of open market operations first shown by Wallace (1981) and more recently by Curdia and Woodford (2010).[2] The different risk weights for each asset held by bank lead to different returns thus making the assets imperfect substitutes.

Financial Inclusion in the Euro Area. I use household-level data, the Eurosystem Household Finance and Consumption Survey (HFCS), and document the fraction of the Euro Area households that are hand to mouth consumers. I restrict attention to the first wave of the HFCS data conducted mainly in 2009 and 2010. This is done in order to eliminate as possible the effects of the 2008 financial crisis. Also the survey was performed well before the start of ECB's QE in March 2015. The data has been collected from 15 Euro Area member states for a sample of more than 62,000 households.

Figure 1 reports the distribution of financial and real asset holdings of the Euro Area residents.[3] As the Figure shows, 20-30% of the Euro Area households hold a total

[2] Wallace (1981) was the first to show that open market operations are not effective under the assumption of the same return between money and assets purchased and a fixed fiscal policy stance. The result remains the same in future studies built on more relaxed assumptions. See also Sargent and Smith (1987); Chamley and Polemarchakis (1984); Eggertsson and Woodford (2003); Curdia and Woodford (2010).

[3] Financial assets include deposits (sight and saving accounts), mutual funds, bonds, shares, money owed to the households, value of voluntary pension plans and whole life insurance policies of household members and other financial assets item - which includes private non-self-employment businesses, assets

value of financial assets that is close to zero (blue, dotted column). In comparison all percentiles of Euro Area households holding real assets hold substantial values of real assets (orange column).

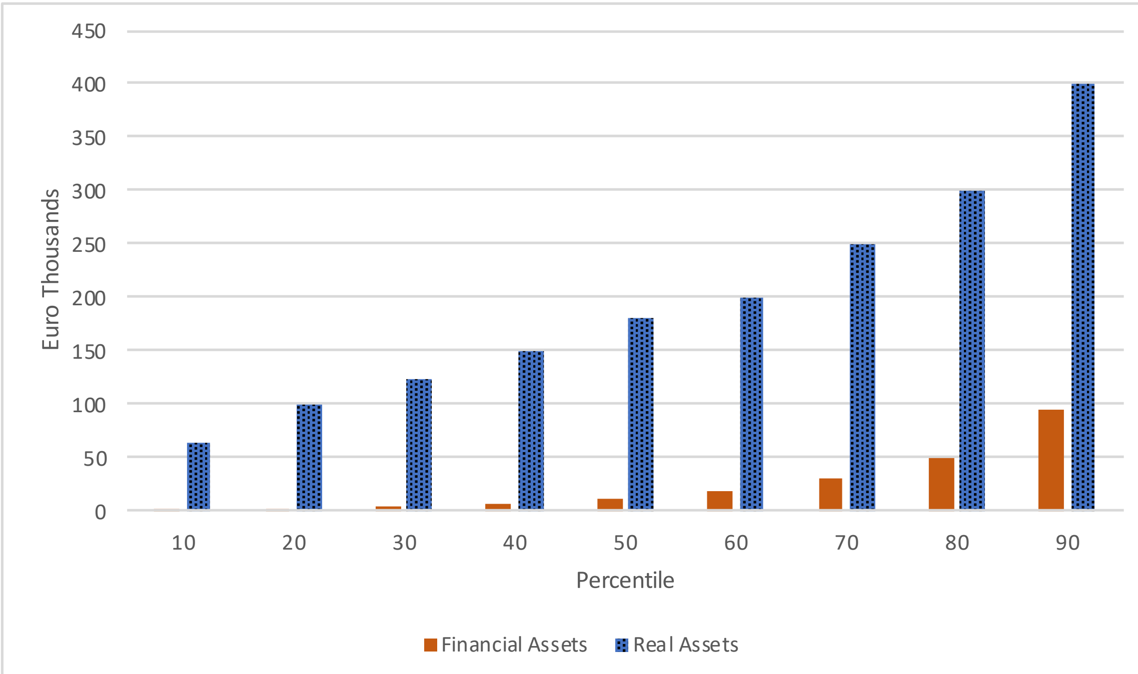

Fig. 1. Total Financial and Real Assets among EA Households

Heterogeneity in asset holdings is also present between the Euro Area member countries. Using the same dataset, I show the volume of financial assets held by households for each country of the Euro Area in Figure 2. Countries of the core of the EA such as Belgium, Luxembourg and Netherlands report financial assets holdings that are way above those in countries in the periphery such as Greece, Italy and Spain. I will show later that the same monetary policy can lead to different impact effects conditionally on a country's level of financial participation.

Existing Literature. This study relates to several strands of the macro-finance literature. Firstly, Two Agent Neo-Keynesian (TANK) models, that this study builds upon, is a compact tool to measure differences between two income groups. Galí et al. (2007) firstly introduced a TANK framework to study the effects of government spending consumption. Bilbiie (2008) using a version of GLS without capital accumulation shows that an expansionary monetary policy shock can have contractionary effects when asset markets participation is low, namely the existence of Inverted Aggregate Demand Logic (IADL). Building on Bilbiie's work, Colciago (2011) proves that the results no longer hold when wages are sticky and Broer, Harbo Hansen, Krusell, and Öberg (2020) identify as well the importance of wage rigidities. In this paper I use this modelling

in managed accounts and other types of financial assets. Real assets include the value of household's main residence.

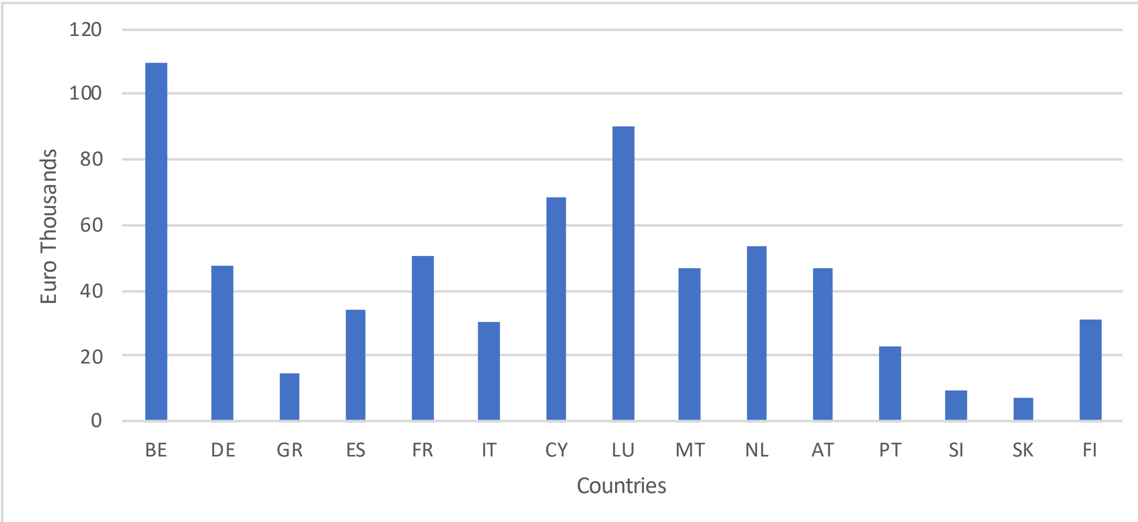

Fig. 2. Total Financial Assets among EA Households by Country

framework to show income and consumption inequality outcomes and I also extend the results of IADL in the case of QE, including sticky wages. [4]

Recently, Kaplan, Moll, and Violante (2018), McKay and Reis (2016) and Ravn and Sterk (2016) have developed an Aiyagari-type heterogeneous agent framework with New Keynesian nominal rigidities (HANK) making the characterization and study of the full income and wealth distribution feasible. As shown by Debortoli and Galí (2017), a two agents framework is able to identify differences in average consumption *between* the constrained and unconstrained agents but lacks on characterising consumption heterogeneity *within* the subset of unconstrained households. Since the main focus on this paper is on the interactions between the two types of agents, it suffices to use a less rich setting of heterogeneity.

The distributional effects of monetary policy, mainly focused on standard interest rate shocks, are well documented. To name a few Coibion, Gorodnichenko, Kueng, and Silvia (2017), Slacalek, Tristani, and Violante (2020) show that accommodative monetary policy reduces income and wealth inequality. Studies on the distributional effects of QE agree that its effects benefit mostly the lower end of income distribution in line with this paper's results. Lenza and Slacalek (2018) evaluate the impact of quantitative easing on income and wealth of individual euro area households. Empirical studies by Bunn, Pugh, Yeates et al. (2018) using Bank of England data and Bivens (2015) also concur on the relatively bigger effect to lower income households.[5]

Studies building on structural models are similarly mostly focused on conventional monetary policy. Dolado, Motyovszki, and Pappa (Forthcoming) focus on labour frictions and conclude that a monetary policy easing increases income inequality between

[4]Studies using a TANK framework also include Monacelli (2009) and Bilbiie, Monacelli, and Perotti (2013), Galí, López-Salido, and Vallés (2004).

[5]For a comprehensive literature review see Colciago, Samarina, and de Haan (2018).

skilled and unskilled workers. Gornemann, Kuester, and Nakajima (2016) use a heterogeneous agents framework, as developed by Kaplan et al. (2018), accompanied with matching frictions and propose an addition of unemployment stabilization to the dual mandate of the central banks to achieve higher welfare. Finally, Cui and Sterk (2019) build a heterogeneous agents model and show that QE is dominated by conventional policy in welfare terms.

Hohberger, Priftis, and Vogel (2019a) in parallel work conduct a similar study where they evaluate the effects of QE to consumption and income inequality. They employ a standard NK setting with two agents and show that consumption and income inequality fall after a QE policy, in line with this paper. In their analysis, the perfect substitutability of assets and the QE neutrality are eliminated through portfolio costs. In the present paper the Wallace neutrality is endogenously eliminated through the banks' MCR constraint. QE shifts the economy to its first best allocation. Furthermore, this study explores the effects of QE when asset market participation is low in two different labour market setting and provides evidence on the procyclicality of profits after a QE shock using a proxy SVAR approach. In my knowledge there is no other study employing a TANK model with financial frictions and an explicit framework for asset purchases by the central bank that measures changes in consumption and income inequality.

Lastly, related to this paper's empirical specification there is a vast literature on SVARs with different identification methods. A very comprehensive summary of this is in Ramey (2016). Given the dataset on the monetary policy surprises in the Euro Area by Altavilla et al. (2019) I employ the proxy - or external instruments - SVAR approach. This approach has been pioneered by Stock and Watson (2012) and Mertens and Ravn (2013) where hey compute the impact of tax changes on GDP and other macro variables using as external instrument narratively identified tax changes. Gertler and Karadi (2015) use this together with a high frequency identification approach for monetary policy changes in the US which is essentially the method I follow but using the Euro Area Monetary Policy Event Study Database to obtain the QE factors and use them as external instruments. Jarociński and Karadi (2020) develop this approach further to identify shocks and to decompose them to monetary policy and communication shocks.

2. A Simple Model with QE and a Banking Sector

To fix ideas regarding the impact of QE in the household's and bank's balance sheets and consequently in consumption and income inequality it is useful to consider a basic model with a cash in advance constraint. The analysis focuses on the direct effects from QE and abstracts from general equilibrium effects. These assumptions are relaxed in the DSGE model of the next section.

In the first part, I show how the the effect central bank purchases have on household's consumption and income when there is no banking sector, while in a second step I show the same in a household-bank-central bank model. Finally, I show that QE is effective in reducing the spread between assets' and the risk- free rate.

There is a continuum of a unit measure of identical households. Household members are bankers and workers and they share perfect insurance inside households. In period 1, workers are endowed with y goods. The representative household makes a deposit d in a bank and buys b^H bonds from the central bank. QE in this simple framework is characterised by a central bank buying b^{CB} from the household and giving back the same value in reserves m. The period 1 budget constraint is:

$$c + d + b^H + m \leqslant y, \tag{1}$$

while the period 2 budget constraint, with primes for the next period variables, is:

$$c' \leqslant Rd + R^b b^H + Rm + \pi. \tag{2}$$

On top of that households in the first period need to follow a cash in advance (CIA) constraint:

$$c_t \leqslant m_t. \tag{3}$$

Here we make the assumption that goods markets open before financial markets. The total amount of bonds is given by $b = b^H + b^{CB}$ where b is a constant and reserves equal central bank purchases, $m = b^{CB}$.

Both deposits d and reserves m are remunerated at R which is the risk-free rate of the economy. Bond holdings b are remunerated at the rate R^b. Finally, household earn the bank profits π. For the present analysis without banks, $\pi = 0$. For the moment, I assume that there is an exogenous wedge (think of i.e. portfolio adjustment costs) between the risk-free rate and the bond rate such that $R^b > R$. The intertemporal budget constraint yields:

$$c + \frac{c'}{R} + b - b^{CB} \leqslant y + \frac{\pi}{R} + \frac{R^B}{R}(b - b^{CB}). \tag{4}$$

Notice that since reserves are remunerated at the risk-free rate, they do not play any role in the household's decision.

Assume that households utility take the form of $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Households maximize

$$u(c) + \beta u(c'),$$

subject to the intertemporal budget constraint (4). Optimality conditions give the intertemporal Euler equation and the wedge between the risk free rate and the bond rate:

$$c' = \left(\frac{c^{-\gamma} - \mu}{\beta R}\right)^{-\frac{1}{\gamma}} \tag{5}$$

and

$$\frac{R^b}{R} = \frac{\lambda + \mu}{\lambda}. \tag{6}$$

where λ and μ are the Lagrange multipliers for the budget constraint and the cash in advance constraint respectively. The solution of the problem yields,

$$c + \frac{(c^{-\gamma} - \mu)^{-\frac{1}{\gamma}}}{\frac{(\beta R)^{-\frac{1}{\gamma}}}{R}} = y + \frac{\pi}{R} + \frac{R^b - R}{R}(b - b^{CB}) \tag{7}$$

For simplicity, given the non-linearity with respect to c, if we assume that the CIA constraint is not binding ($\mu = 0$) we have:

$$c = \frac{y + \frac{\pi}{R} + \frac{R^b - R}{R}(b - b^{CB})}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}} \tag{8}$$

and both period budget constraints are binding. Looking at the optimal consumption rule it's clear why the Wallace irrelevance proposition does not hold. When the CIA constraint is binding thus $R^b > R$, bond purchases have real effects on consumption.

Equations (7) and (8) show how consumption of household is affected by the government purchases when the CIA constraint is binding and when it's not. Since the total amount of bonds in the economy b is constant, an increase of central bank purchases b^{CB} has different outcome in each case. Firstly, when the CIA constraint is binding, $R^b > R$ and households reduce consumption (note the negative sign on b^{CB}). Therefore a QE policy translates directly to a downward shift in households' consumption. Secondly, in the case where the CIA constraint is loose, $R^b = R$ and we get back to the Wallace irrelevance proposition where government purchases have no real effect.

To enhance realism, I introduce a bank in the model and a friction between depositors and banks. Bankers are endowed with N units, they buy claims s from non-financial firms priced at the exogenous rate R^k, bonds b^B at rate R^b and receive deposits d from the households at the risk-free rate R while when the central bank buys some of their bonds, it pays back reserves m^B priced at the rate R. The economy's total bonds now are $b = b^H + b^B + b^{CB}$ and reserves are now divided in households' and banks' reserves: $m = m^B + m^H$. Bankers want to maximize their profits π:

$$\pi = \max[sR^k + b^B R^b + m^B R - dR] \tag{9}$$

and their balance sheet constraint is $s + b^B + m^B = N + d$. In absence of financial frictions and if the CIA constraint defined above is not binding, the above rates would be equal and we would have perfect asset substitution. Let's call the economy without financial frictions the *first best allocation*.To introduce financial frictions, similarly to the full model, we assume that the banker has an option to default. If the bank defaults, it takes a fraction θ of its claims, a fraction $\Delta\theta < \theta$ of its bonds and a fraction $\omega\theta < \theta$ of the reserves it gets from the central bank during a QE. The remaining assets are given to the depositors. Therefore, defaulting banks receive $\theta(sR^k + \Delta b^B R^b + \omega m R)$ and depositors $(1-\theta)sR^k + (1-\theta\Delta)b^B R^b + (1-\theta\omega)Rm$.

The banker chooses not to default if profits are higher than what the banker gets in case of default:

$$sR^k + b^B R^b + m^B R - dR \geqslant \theta(sR^k + \Delta b^B R^b + \omega m^B R). \tag{10}$$

Rearranging we see that the banker will choose not to default as long as depositors earn less in the no default regime.

$$(1-\theta)sR^k + (1-\theta\Delta)b^B R^b + (1-\omega\theta)m^B R \geqslant dR. \tag{11}$$

Firstly, I am interested in the effects on households consumption that stem from the banker's balance sheet. Given the introduced financial frictions, when the banker's constraint is binding, a wedge is created between R^k and R and R^b and R. Therefore we don't need the CIA assumption here to introduce interest rate wedges, the lack of which also helps in the analytical representation of the results.

Since households earn bankers profits in this family setting, in order to see how households' optimal consumption is affected by the QE, we need to expand (8) including the banks' profits. Substituting the profits (9) and the optimal decision for households deposits, the first period budget constraint, $d = y - c - m^H - b^H$, after some algebra we get:

$$c = \frac{R^k}{(\beta R)^{\frac{1}{\gamma}} + R^k}(N + y) + \frac{(b^h + b^B)(R^b - R^k) + (m^h + m^B)(R - R^k)}{(\beta R)^{\frac{1}{\gamma}} + R^k}. \tag{12}$$

Households' consumption increases with the endowment and the bank's net worth. On the other hand as long as $R^k > R^b > R$ households loose from QE in terms of consumption. In the calibrated full model the previous inequality always holds. They lose by $R - R^b$ when they get reserves for the bonds they or their banks own, and they also lose by $R^b - R^k$ when they hold bonds instead of firms' claims.

In the last part of this section I show why buying bonds from household and banks brings the economy to its first best. In the lines of Gertler and Karadi (2011), the central bank's *unconventional* monetary policy target is to reduce the spread increase between asset rates and the risk free rate. This is what happens also here when the QE is used. As I've shown above, when the central bank buys bonds consumption of households' falls. What happens in households' deposits is important for the analysis. Taking the first period budget constraint and interpreting the bond and reserve holdings in terms of bank holdings yields:

$$d \leqslant y - c - b + b^B + m^B$$

Since b is the total (constant) amount of bonds in the economy, and in a QE scenario m^B will increase by as much as b^B increase, deposits will increase. This is due to the inverse relationship between government purchases and private consumption. Rearranging (11) we get:

$$(1-\theta)sR^k \geqslant dR - (1-\theta\Delta)bR^b + (1-\theta\Delta)b^B R^b - (1-\omega\theta)m^B R. \tag{13}$$

In order for the QE to achieve its goal and reduce the spread, the central bank's actions should aim at relaxing the banker's incentive constraint. In the simple GK model, where no bond markets exist and no asset purchases occur, this occurs with a drop in deposits after an unconventional measure. Here, we established that deposits go up. What drives the relaxation of the constraint is the drop of bank bond holdings and the increase of bank reserves. Both reduce the right hand side of (13) and relax the constraint. How much, depends on the differential between the increase in bond purchases and the increase in deposits by households.

Let me summarize the results from this basic model. Firstly, both when a banking sector is included in the model or not, during a QE households' consumption falls. This is due to the interest rate differential between the bonds they sell and the reserves they receive. In the first case, the interest rate wedge is introduced through a CIA constraint and in the second through financial frictions between the banks and their depositors. Ii should be noted that in this simple model I only took into account the direct effects from the interest rates change since output is constant. In the case that output and wages go up after the central bank purchases, households that have not access to financial markets have a net gain from the QE. The net outcome for the ones that own bonds depends on the net difference in their income composition. Therefore, if the net income and consumption of the bond holders reduces, consumption and income inequality between the two groups will fall. Secondly, I showed that in a model with a constrained banking sector, QE relaxes the constraint and shifts the economy closer to its first best allocation, where all rates are equal to the risk-free rate.

3. The DSGE Model

The economy is populated by two types of households: Rule of thumb and optimising households that differ in their ability to participate in the assets market. A continuum of firms and financial intermediaries owned by the optimizers, labour wide unions that set the wages, capital goods producers and retailers, a monetary authority and the treasury complete the model economy. Banks are subject to a minimum capital requirements (MCR) constraint in the spirit of Basel III which restraints banks from having a low capital value. The central bank works under a conventional Taylor rule, but can also engage in asset purchases and pay the investors back the same value in newly created reserves.

3.1. *Households - The Two Agents Framework*

All households are assumed to have identical preferences, given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i [\ln(C^s_{t+i}) - \frac{\chi}{1+\epsilon} L^{1+\epsilon,s}_{t+i}], \tag{14}$$

C^s_{t+i} denotes the per capita consumption of the household members and N^s_{t+i} the supply of labour. The super-index $s \in [o, r]$ specifies the household type (o for "optimizers" or r for "rule of thumb"). $\beta \in [0, 1]$ is the discount factor. Due to the stochastic setting, households make expectations for the future based on what they know in time t and $\mathbb{E}_t$ is the expectation operator at time t. Finally, ϵ is the inverse Frisch elasticity of labour supply and χ is the relative utility weight of labour.

Optimizers.— Optimizers account to a measure of $(1 - \lambda)$ of the economy's population. Their portfolio includes one period government bonds B^o_t, bank deposits D^o_t and firm shares S^o_t. They can freely adjust their deposit holdings. However, they are not experts in trading bonds and shares. Transactions above or below a frictionless level $\bar{S}^o_t$ and $\bar{B}^o_t$ for shares and bonds respectively require broker expertise and this induces costs. Costs equal to $\frac{1}{2}\kappa(S^o_t - \bar{S}^o_t)^2$ for shares and $\frac{1}{2}\kappa(B^o_t - \bar{B}^o_t)^2$ for bonds deviating from their respective frictionless level.[6]

Optimizing households budget constraint then is

$$\begin{aligned} C^o_t + T^o_t + D^o_t + q_t[B^o_t + \frac{1}{2}\kappa(B^o_t - \bar{B}^o)^2] + Q_t[S^o_t + \frac{1}{2}\kappa(S^o_t - \bar{S}^o)^2] \\ = W_t L^o_t + \Pi_t + R_{d,t} D^o_{t-1} + R_{b,t} q_{t-1} B^o_{t-1} + R_{k,t} Q_{t-1} S^o_{t-1}, \end{aligned} \tag{15}$$

Total deposits D^o_t are the sum of households' private deposits and deposits created by the exchange of securities with reserves when the central bank purchases those during a QE. They are remunerated at the risk-free rate $R_{d,t}$. $R_{b,t}$ and $R_{k,t}$ are the gross returns for the bonds and shares respectively in period t. W_t is the real wage which both types of households take as given. T^o_t are taxes (or transfers if negative) that optimizing households pay every period. Finally, optimizers receive income Π_t from the ownership of both non-financial firms and financial intermediaries.

The problem of the optimizing household is to choose $C^o_t, L^o_t, D^o_t, B^o_t, S^o_t$ in order to maximize its expected utility (14) subject to the budget constraint (15) at every period. Let $u^o_{c,t}$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ denote the optimizing household's stochastic discount factor (the intertemporal marginal rate of substitution)

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{c^o,t+1}}{u_{c^o,t}}. \tag{16}$$

Maximizing optimizers' utility with respect to deposits yields their intertemporal optimality condition

$$\mathbb{E}_t \Lambda_{t,t+1} R_{d,t+1} = 1. \tag{17}$$

The choices for private securities and long-term government bonds are given by:

$$S^o_t = \bar{S}^o_t + \frac{\mathbb{E}_t \Lambda_{t,t+1}(R_{k,t+1} - R_{t+1})}{\kappa}$$

[6] Note that under this setting, optimizing households could be also thought as financially constrained due to adjustment costs, similar to the wealthy hand to mouth consumers in Kaplan, Violante, and Weidner (2014). Also, another interpretation following Kaplan et al. (2018) is that bonds and stocks are illiquid assets and deposits are liquid assets.

$$B_t^o = \bar{B}_t^o + \frac{\mathbb{E}_t \Lambda_{t,t+1}(R_{b,t+1} - R_{t+1})}{\kappa}$$

It follows that households hold always the frictionless amount of each asset. Their demand for extra units is increasing in the excess returns relative to the respective curvature parameter that governs the marginal transaction cost κ. As marginal transaction costs go to zero, excess returns disappear: There is frictionless arbitrage between the two assets and all assets' interest rates are equalized. On the other hand, when marginal transaction costs go to infinity, households' asset demands go to their respective frictionless capacity values.

I consider two labour market specifications. Under the first setting labour market is competitive and each household chooses the quantity of hours supplied given the market wage W_t. In the second case wages are set by a labour union. Hours are demand driven by firms taking the wages as given by the union, households are ready to supply as many hours as required by the firms given the wage. Both wage specifications are analysed in section 3.2.

Rule of Thumb.— Rule of thumb households account for a λ measure of households. Their participation in financial markets is restricted. They cannot smooth consumption either by trading securities or by acquiring bank deposits. They consume their net income at every period which is their labour income net of taxes. Their budget constraint is:

$$P_t C_t^r = P_t W_t L_t^r + P_t T_t^r. \tag{18}$$

C_t^r, L_t^r, T_t^r denote, respectively, consumption, hours worked and taxes (or transfers).

Rule of thumb agents maximize their utility subject to their budget constraint. Accordingly, the level of consumption will equate labour income specified by (18).

Rule of thumb agents' taxation is the only fiscal variable that matters for the model's fiscal allocation as is shown in Proposition 2. Optimizing agents internalize the government budget constraint through their government bond holdings. On the other hand, a change in the tax rate (or transfer) of the rule of thumb consumers implies a change in their taxes today or in the future.[7] I study two transfer schemes for the rule of thumb consumers: a no-redistribution scheme where transfers to rule of thumb agents are zero and a fiscal rule that taxes the profits of the optimizing households and rebates them to hand to mouth consumers.

3.2. *Wage Setting*

Here I develop the two wage setting schemes of the model: perfectly competitive labour markets and wage-setting by unions.

3.2.1. *Perfectly Competitive Labour Markets*

In the case of perfect competition in labour markets, households choose optimally their labour supply taking wages as given. The optimality condition with respect to

[7] Similar results are obtained for the TANK model in Bilbiie et al. (2013).

hours worked for a household of type $j \in \{o, r\}$ is

$$u^j_{c,t} W_t = \chi (L^j_t)^{\epsilon}. \tag{19}$$

In the case of the rule-of-thumb consumers, due to the very form of the logarithmic utility function, combining (18) and (19) we find an analytical expression of hours that the rule of thumb agents optimally supply:

$$L^r_t = \left(\frac{1 - \frac{T^r_t}{C^r_t}}{\chi} \right)^{\left(\frac{1}{1+\epsilon}\right)}. \tag{20}$$

Rule of thumb agents' taxation is the only fiscal variable that matters for the model's allocation as is shown in Proposition 2. It is this the case because optimizing agents internalize the government budget constraint through their government bond holdings. On the other hand, a change in the tax rate (or transfer) of the rule of thumb consumers implies a change in their taxes today or in the future.[8] I study two transfer schemes for the rule of thumb consumers: a no-redistribution scheme where transfers to rule of thumb agents are zero and a fiscal rule that taxes the profits of the optimizing households and rebates them to hand to mouth consumers.

3.2.2. Wage Setting by Unions

In the second case it is assumed that wage decisions are delegated to a continuum of labour unions. Hours are determined by firms taking the wages set by unions as given.[9] Households supply the hours required by the firms given the wage set by unions. Firms are also indifferent on the type of household they employ. Therefore, all households types supply the same working hours $L^o_t = L^r_t = L_t$.

Labour supply L_t is a composite of heterogeneous labour services

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dh \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{21}$$

where $L_{h,t}$ is the supply of labour service h and ϵ_w is the elasticity of substitution between labour and consumption across household types.

At each period there is a probability $1 - \xi_\omega$ that the wage for each particular labour service $W_{h,t}$ is set optimally. The union buys homogeneous labour at nominal price $W_{h,t}$, repackages it by adding a mark-up and chooses the optimal wage W^*_t to maximize the objective function where labour income of the two types is weighed by their marginal utilities of consumption.

$$\lambda \left[u^r_{c,t} W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] + (1 - \lambda) \left[u^o_{c,t} W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] \tag{22}$$

[8]Similar results are obtained for the TANK model in Bilbiie et al. (2013).

[9]For a detailed exposition on wage setting see Appendix A.

.

Aggregation.— Aggregate variables are given by the population weighted average of the corresponding variables of each household type.

$$C_t \equiv (1-\lambda)C_t^o + \lambda C_t^r \tag{23}$$
$$L_t \equiv (1-\lambda)L_t^o + \lambda L_t^r \tag{24}$$
$$T_t \equiv (1-\lambda)T_t^o + \lambda T_t^r \tag{25}$$

The H superscript denotes the total asset holdings of households.

$$S_t^H \equiv (1-\lambda)S_t^o$$
$$B_t^H \equiv (1-\lambda)B_t^o$$
$$D_t^H \equiv (1-\lambda)D_t^o$$

3.3. Financial Frictions

Banks.— Banks are funded with deposits, receive reserves from the central bank during the QE, extend credit to non-financial firms and buy bonds from the government. Each bank j allocates its funds to buying a quantity $s_{j,t}$ of financial claims on non-financial firms at price Q_t and government bonds $b^B_{j,t+1}$ at price q_t. Banks' liabilities are made up from households' deposits $d^B_{j,t+1}$. When the central bank proceeds in securities' purchases ($Q_t S_t$ or $q_t B_t$) it pays back the bank with an equivalent value of reserves $m_{j,t}$.[10] Finally, $n_{j,t+1}$ is the capital equity accumulated. Formally, the bank's balance sheet is:

$$Q_t s^B_{j,t} + q_t b^B_{j,t} + m^B_{j,t} = n_{j,t} + d^B_{j,t}. \tag{26}$$

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities.

$$n_{j,t+1} = R_{k,t} Q_{t-1} s^B_{j,t-1} + R_{b,t} q_{t-1} b^B_{j,t-1} + R_{m,t} m^B_{j,t} - R_t d^B_{j,t}.$$

Let Z_t be the net period income flow to the bank from a loan that is financing to a firm and δ the depreciation rate of capital being financed. Then the rate of return to the bank on the loan, $R_{k,t+1}$, is given by:

$$R_{k,t+1} = \frac{Z_t + (1-\delta)Q_{t+1}}{Q_t}. \tag{27}$$

Long-term bond is a perpetuity that pays one euro per period indefinitely. The real rate

[10]We can think $m^B_{j,t}$ as the sum of reserves a bank receives from the purchases not only of its own securities but also from the ones the households listed to the bank hold. The bank will transfer the exact same amount to the household's deposit account (see McLeay, Radia, and Thomas (2014)), keeping the balance sheet constraint intact.

of return on the bond $R_{b,t+1}$ is given by:

$$R_{b,t+1} = \frac{1/P_t + q_{t+1}}{q_t}.$$

Central bank reserves bear a zero risk weight in the banks' MCR constraint and, as it will be shown momentarily, have a gross return $R_{m,t}$ equal to the risk-free rate R_t. It follows that banks have no inventive to hold reserves in equilibrium.

The bankers' objective at the end of period t, is to maximize the expected present value of future dividends. Since the banks are owned by the optimizing households, their stochastic discount factor Λ_{t+1} is used as the discounting measure.

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1-\sigma_B)\sigma_B^{j-1} \Lambda_{t+1} n_{j,t+1}. \tag{28}$$

Banks are required to follow a minimum capital requirement constraint similar to the Basel III. The regulatory constraint specifies that the banks should have a franchise value always greater or at least equal with a fraction θ of a risk weighted measure of their assets. It is assumed that loans to firms have a risk-weighting of 1, sovereign bonds a risk coefficient of Δ and the central bank reserves' risk coefficient is ω. These are defined as $1 > \Delta > \omega$. In the calibration section I set these values follow the ones specified by the Basel III framework for BBB+ to BBB- graded bonds and firm shares. The regulatory constraint of the bank therefore is:

$$V_{j,t} \geqslant \theta[Q_t s_{j,t}^B + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B]. \tag{29}$$

The value of the bank at the end of period $t-1$ must satisfy the Bellman equation:

$$\begin{aligned} V_{j,t-1}(s_{j,t-1}^B, b_{j,t-1}^B, m_{j,t}^B, d_{j,t}^B) &= E_{t-1}\Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1-\sigma_B) n_{j,t} \\ &+ \sigma_B \max_{d_{j,t}} [\max_{s_{j,t}^B, b_{j,t}^B, m_{j,t}^B} V_t(s_{j,t}^B, b_{j,t}^B, m_{j,t}^B, d_{j,t}^B)]\}. \end{aligned} \tag{30}$$

Banker's problem is to maximize (28) subject to the balance sheet (26) and the minimum capital requirement constraint (29).

Proposition 1. A solution to the banker's dynamic program is

$$V_{j,t}(s_{j,t}^B, b_{j,t}^B, d_{j,t}^B, m_{j,t}^B) = A_{j,t}^B.$$

The marginal value of the banker's net worth A^B is then:

$$A^B = \mu_t^s \phi_t + \nu_{d,j,t}.$$

μ_t^s is the stochastic spread between the loan and the deposit rates, ϕ_t is the maximum leverage and $\nu_{d,j,t}$ is the marginal loss from deposits.

Proof. See appendix C.

The proposition clarifies the role of the bank's net worth in the model. We can rewrite the incentive constraint using the linearity of the value function as

$$\frac{A^B}{\theta} \geqslant \frac{[Q_t s^B_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t}]}{n^B_{j,t}}. \tag{31}$$

The adjusted leverage of a banker cannot be greater than A^B/θ. The right hand side shows that as the net worth of the banker decreases the constraint is more likely to bind. Proposition 1 also implies that even there is heterogeneity in the bankers' holdings and net worth, this does not affect aggregate dynamics. Hence, the transition from the individual to aggregate variables takes place in the same way as in the previous section.

The maximum adjusted leverage ratio of the bank is defined as

$$\phi_{j,t} = \frac{\nu_{d,j,t}}{\theta - \mu^s_t}. \tag{32}$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits $\nu_{d,j,t}$ and reserves and on the excess value of bank assets μ^s_t. As the credit spread increases, banks franchise value V_t increases and the probability of a bank to divert its funds declines. From the other hand as the proportion of assets that a bank can divert, θ increases, the constraint binds more.

Aggregation.—Let S^B_t be the total quantity of loans that banks intermediate, B^B_t the total number of government bonds they hold, M^B_t the total quantity of reserves and N_t their total net worth. Furthermore, by definition, total deposits acquired by the households D^H_t are equal with the total deposits of the banking sector. Using capital letters for the aggregate variables, the banks' aggregate balance sheet becomes

$$Q_t S^B_t + q_t B^B_t + M^B_t = N_t + D^H_t. \tag{33}$$

Since the leverage ratio (32) does not depend on factors associated with individual banks' characteristics we can sum up across banks and get the aggregate bank constraint in terms of the total net worth in the economy:

$$Q_t S^B_t + \Delta q_t B^B_t + \omega M^B_t = \phi_t N_t. \tag{34}$$

The above equation gives the overall demand for loans $Q_t S_t$. When the regulatory constraint is binding, the demand for assets is constrained by the net worth of the bank adjusted by the leverage. We can get some intuition here for what changes in the bank's constraint during the QE. No matter the security the central bank purchases, since their risk weights are higher than the weight of reserves ($1 > \Delta > \omega$), the exchange of securities with reserves relaxes the regulatory constraint.

Aggregate net worth is the sum of the new bankers' and the existing bankers' equity: $N_{t+1} = N_{y,t+1} + N_{o,t+1}$. Young bankers' net worth is the earnings from loans multiplied by ξ_B which is the fraction of asset gains that being transferred from households to the new bankers

$$N_{y,t+1} = \xi[R_{k,t}Q_{t-1}S^B_{t-1} + R_{b,t}q_{t-1}B^B_{t-1} + R_{m,t}M^B_{t-1}]$$

and the net worth of the old is the probability of survival for an existing banker multiplied by the net earnings from assets and liabilities

$$N_{o,t+1} = \sigma[R_{k,t}Q_{t-1}S^B_{t-1} + R_{b,t}q_{t-1}B^B_{t-1} + R_{m,t}M^B_{t-1} - R_tD^H_t].$$

3.4. *Central Bank, Asset Purchases and the Treasury*

Central Bank.— Central bank uses two policy tools. Firstly, it adjusts the policy rate according to the Taylor rule specified momentarily. Secondly, it can engage in risky asset purchases from households and banks. When balance sheet constrains are tight, excess returns rise. Central bank purchases relax the minimum capital requirement constraints of the banks and increase aggregate demand thus driving up asset prices.[11]

Under a QE operation, the central bank buys securities from banks and households. These can be either private assets S^G_t or bonds B^G_t. It does that by paying the assets purchased by their respective price Q_t and q_t. To finance those purchases it creates electronically reserves M_t that pay back purchases from households and banks:

$$Q_tS^G_t + q_tB^G_t = M_t.$$

Is is assumed that the central bank turns over any profits to the treasury and receives transfers to cover any losses. The central bank's budget constraint is:

$$T^{CB}_t + R_tM_{t-1} + Q_tS^G_t + q_tB^G_t = R_{b,t}q_{t-1}B^G_{t-1} + R_{s,t}Q_{t-1}S^G_{t-1} + M_t \tag{35}$$

where T^{CB}_t are transfers of the central bank to the treasury.

Monetary policy is also characterised by a simple Taylor rule. It sets the nominal interest rate i_t such as to respond to deviations of inflation and output from its flexible price equilibrium level Y^*:

$$i_t = i + \kappa_\pi\pi + \kappa_y(Y - Y^*) + \epsilon_{m,t},$$

where i is the steady state level of the nominal interest rate and $\epsilon_{m,t}$ an exogenous monetary policy shock. The relation between nominal and real interest rates is given by the Fisher equation:

$$1 + i_t = R_{t+1}\frac{P_{t+1}}{P_t}.$$

With the addition of the central bank in the model, three agents can holds assets or bonds: Optimizing households, banks and the central bank. The total quantity of loans therefore is decomposed as:

$$S_t = S^B_t + S^H_t + S^G_t \tag{36}$$

[11]See Araújo, Schommer, and Woodford (2015) for a same intuition under a different setting.

and for the bonds:

$$B_t = B_t^B + B_t^H + B_t^G. \tag{37}$$

If we combine these identities and insert them to the balance sheet constraint of the banks we have:

$$Q_t S_t \leqslant \phi N_t + Q_t S_t^H + Q_t S_t^G + \Delta(q_t B_t^G + q_t B_t^H - q_t B_t) \tag{38}$$

The above constraint implies that when government purchases either loans or bonds it relaxes the balance sheet constraint of the banking sector. This can, in financial stress periods, reduce the excess returns and stimulate the economy. When this constraint does not bind and the inequality holds, asset or bond purchases by the government are neutral. This happens due to frictionless arbitrage that characterizes the economy when the banks has no binding constraint. Wallace (1981) in his seminal paper has make use of that assumption to for the neutrality theorem of the open market operations.

Equation (38) gives another insight on the asset purchase mechanism. Buying loans or bonds does not have the same impact to the loosening of the banks' balance sheet constraint. In fact, since loans have a risk-weight ration of 100%, purchases of loans by the central bank relaxes the constraint more than the purchase of bonds with a risk-weight coefficient $\Delta < 100\%$. Intuitively, the central bank acquiring government bonds frees up less bank capital than does the acquisition of a similar amount of private loans.

It is now easier to understand when the irrelevance theorem holds. Since the government creates as many reserves as the value of the assets purchased ($M_t = q_t B_t^G + Q_t S_t^G$), then in the case of frictionless arbitrage between the existing assets ($R_{s,t} = R_{b,t} = R_t$), the market operations are indeed irrelevant. But since the financial frictions included in the model disrupt the frictionless arbitrage, asset purchases have effect on the real economy.

The share of the total assets that is purchased by the government follows a second order stochastic process.[12] Specifically,

$$S_t^G = \phi_{s,t} S_t,$$

$$B_t^G = \phi_{b,t} B_t.$$

Treasury.— The treasury collects lump sum taxes $T_t = \lambda T_t^r + (1-\lambda) T_t^o$ to finance its public expenditures which are fixed relative to output, $\bar{G} = \gamma^G Y^{ss}$. It also targets a constant real level of long-term debt, denoted by $\bar{B}$. It collects taxes at rate t_{pr} from non-financial firms' profits and redistribute them back to the hand to mouth households, $T_t^r = t_{pr} Prof_t$.

Treasury's budget constraint is:

$$\bar{G} + q_{t-1} R_{b,t} \bar{B} = q_t \bar{B} + T_t + T_t^{CB}. \tag{39}$$

[12] As is shown in the calibration section, an AR(2) is the best way to simulate the ECB's Asset Purchase Program schedule.

.

Proposition 2. Fiscal policy matters only through the impact of taxes (transfers) on hand to mouth agents. Therefore, the only fiscal variables needs to be defined is the Hand to Mouth transfers (or taxes).

Proof. I make use of the optimizers budget constraint (15), the bank's -owned by optimizing agents- balance sheet (26), the taxes aggregator and the treasury and central bank's budget constraints (35), (39). Substituting the latter four equations in the optimizers' budget constraint and using the financial variables aggregator, the aggregate resource constraint yields:

$$C_t^R + \frac{\bar{G}}{1-\lambda} - \frac{\lambda}{1-\lambda} T_t^K + adj\{B, S\} = W_t L_t^R. \tag{40}$$

Where $adj\{B, S\}$ are the adjustment costs for bonds and shares that households have to pay, defined in (15).

Taxes on optimizers and any short of government bond decision do not matter for the allocation. □

3.5. Non-Financial Firms and Nominal Price Rigidities

The non-financial firms are separated into three types: intermediate, final goods firms (retailers) and capital goods producers. To allow for nominal price rigidities, I assume that the differentiated intermediate goods i produced by a continuum of monopolistically competitive intermediate goods firms are subject to Calvo price stickiness.

The final output composite is a CES composite of all indeterminate goods i: $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}$ where ζ denotes the elasticity of substitution across intermediate goods. Each period there is a fixed probability $1-\gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p. The goods are then sold and used as inputs by a perfectly competitive firm producing the final good. Finally, the capital goods producers create new capital under investment adjustment costs and sell it to goods producers at a price Q_t. The non-financial sector problem is described in detail in Appendix B.

Capital stock evolves according to the law of motion of capital

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{41}$$

The intermediate good $i \in [0, 1]$ is produced by a monopolist who uses a constant returns to scale production function combining capital and labour:

$$Y_t(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha}. \tag{42}$$

A_t is the total factor productivity. It finances its capital needs each period by obtaining funds from banks and households. To acquire the funds to buy capital, the firm issues $S_t(i)$ claims equal to the number of units of capital acquired $K_{t+1}(i)$ and prices each claim at the price of a unit of capital $Q_t(i)$. Then by arbitrage: $Q_t(i)S_t(i) = Q_t(i)K_{t+1}(i)$. The funds acquisition between goods firms and its lenders is under no friction. Firm's lenders can perfectly monitor the firms and there is perfect information.

Resource Constraint. Final output may be either transformed into consumption good, invested or used by the government for government spending:

$$Y_t = C_t + I_t[1 + \tilde{f}\Big(\frac{I_t}{I_{t-1}}\Big)] + G.$$

4. Unconventional Monetary Policy and Assets Market Participation

In the present section, I exam analytically and quantitatively the existence of the Inverted Aggregate Demand Logic (IADL) for the case of i) a conventional accommodative monetary policy shock and ii) a quantitative easing shock. IADL[13] is the region where the accommodative monetary policy, when limited asset markets participation is low, can have contractionary effects instead of stimulating aggregate demand.[14]I perform this exercise for the case of perfect labour market and also when wages are sticky. This is done under two different taxation schemes. Firstly, under a no redistribution scheme: transfers to rule of thumb agents are zero; secondly under a redistributive scheme: rule of thumb agents get a proportion of the firms' profits as a lump-sum transfer.

When wages are flexible, QE can be contractionary for low levels of asset market participation, while when wages are sticky this result is muted. The contractionary effects can be avoided by fiscal redistribution of a portion of profits from the firm owners to the hand to mount consumers. I provide analytical and numerical solutions for the first part of the analysis without transfers while for the case where transfers are on I show only the quantitative results since the analysis becomes substantially more complex.

4.1. *No-Redistribution Scheme*

For the first part of the analysis, I provide analytical expressions that show the *direct effect* of interest rate reduction and quantitative easing on output. Then, I show the fraction of constrained agents that pushes the model into the IADL area in both cases, that is making the total effect of the two policies contractionary. To pursue this, due to the high dimensionality of the model, I solve the model numerically.

In order to derive analytical results I make the following, not distorting, assumptions: Consumption and hours worked are equal among all the members in steady

[13]Borrowing the term from Bilbiie (2008).

[14]A key depart from Bilbiie's work is that the present model includes capital.

state.[15] Therefore in steady state: $L = L^r = L^o$ and $C = C^r = C^o$. The first assumption can be implemented by a particular choice of χ, whereas the second by introducing a tax level that make optimizers' consumption equal to rule of thumb agents'. Furthermore, due to no-redistribution, I assume that rule of thumb agents taxation is zero: $T_t^r = 0$. Under these assumptions, we can express the consumption and labour aggregators (23), (24) as $l_t = \lambda l_t^r + (1-\lambda) l_t^o$ and $c_t = \lambda c_t^r + (1-\lambda) c_t^o$ respectively, were lower case letters denote log deviations from the non-stochastic steady state.

The optimality condition (20) without including any tax (or transfer) rule dictates that the labour supply of the rule of thumb agents in levels is always constant, therefore $l_t^r = 0$. The labour consumption optimality conditions are in log-linear terms: $c_t^r = w_t + l_t^r$ and $c_t^o = w_t - \epsilon l_t^o$. Using the aggregate consumption, labour consumption optimal choices, and the hours worked aggregator we get:[16]

$$w_t = c_t + \epsilon l_t. \tag{43}$$

Note that the above relation holds for both labour market settings, given that both agents have equal consumption and work the same hours in steady state. Substituting (43) in the labour optimality condition of the optimizing agents:

$$c_t^o = c_t - \epsilon \left(\frac{\lambda}{1-\lambda} \right) l_t. \tag{44}$$

Trivially with no hand to mouth consumers $\lambda = 0$, c_t^o follow the aggregate consumption schedule. Introducing limited asset market participation in the model makes optimizers' consumption reacting negatively to an increase of the *aggregate* employment. This is due to wage being the rule of thumb agents' only source of income.

Doing the same exercise for the rule of thumb agents:

$$c_t^r = c_t + \epsilon l_t.$$

Rule of thumb agents' consumption schedule reacts positively in changes of aggregate consumption and employment with elasticity ϵ. Having the above relations in hand I proceed with the derivation of the aggregate Euler equation.

The log-linearised versions of the production function and resource constrain are $y_t = \alpha k_t + (1-\alpha) l_t$ and $y_t = c_t s_c + i_t s_i + s_g$ respectively. Inserting both equations in the optimizing agents' consumption function (44) and substituting the result to the optimizers' Euler equation $c_t^o = \mathbb{E}_t\{c_{t+1}^o\} + [\mathbb{E}_t\{\pi_{t+1}\} - r_t]$ we arrive to the aggregate Euler equation or IS curve:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\delta}[r_t - \mathbb{E}_t\{\pi_{t+1}\}] - \frac{1}{\delta}\frac{s_i}{s_c}\Delta i_{t+1} + \frac{1}{\delta}\frac{\epsilon\lambda}{(1-\lambda)(1-\alpha)}[\alpha \Delta k_{t+1}]. \tag{45}$$

[15] The latter holds without any further arrangement for the centralised wage setting market where firms choose uniformly the labour required given the wage set by the unions.

[16] The derivations of the main equations of this chapter are presented in Appendix D

where

$$\delta = \frac{1}{s_c} - \epsilon\frac{\lambda}{(1-\lambda)(1-\alpha)}$$

and $s_c = C^{ss}/Y^{ss}$, $s_i = I^{ss}/Y^{ss}$, $s_g = G^{ss}/Y^{ss}$.

Profits.— Profits play a crucial role in the analysis. As it will be shown below is the primary reason of the IADL existence. Profits form non-financial corporations are given by $Prof_t = Y_t - W_tL_t - Z_tK_t$. Log-linearising it around the steady state (with $d_t = ln((Prof_t - Prof)/Y))$ we get:

$$d_t = y_t - (w_t + l_t) - (z_t + k_t). \tag{46}$$

Profits move countercyclically in response to demand shocks, a standard feature of the NK models.[17]

4.1.1. Conventional Monetary Policy

The aggregate IS curve derived above, shows that the elasticity of aggregate demand to interest rates depends on whether we assume a representative agent specification or a LAMP setting. Specifically, the elasticity is s_c in the case of a representative agent model ($\lambda = 0$), and becomes $-1/\delta$ when LAMP is assumed. Solving for $\delta = 0$ we can find the threshold fraction of the rule of thumb agents λ^* that make the impact of the *direct effect* of an interest rate reduction ineffective:

$$\lambda^* = \frac{1-\alpha}{1-\alpha+\epsilon s_c}. \tag{47}$$

Beyond this threshold level, a further reduction of the interest rate will have contractionary effects and this will be the region where the parameter δ changes sign.

For a low λ below the threshold value or equivalently when financial participation is high, output reacts inversely to real interest rate changes. As we move to higher values of λ this effect is becoming stronger. When $\lambda > \lambda^*$, and the fraction of hand to mouth consumers is big enough, δ becomes negative and distorts the well known stimulating effect of accommodative monetary policy using the policy rate. In that region lower interest rates restrain aggregate demand and we enter the Inverse Aggregate Demand Logic region. Finally as λ reaches its upper bound of 1 where no agent hold assets, $1/\delta$ decreases towards zero; the interest rate as a monetary policy tool becomes irrelevant.

Feeding the model with the parameter values from the model's calibration shown in detail momentarily in Section 6.1, I show the *total* impact effect of a conventional interest rate reduction to main macro variables as a function of rule of thumb agents, where $\lambda \in [0, 0.9]$. The top chart of Figure 3 shows the *total* impact effect on aggregate output to a conventional accommodative monetary policy shock conditional on different fractions of rule of thumb agents. The bottom part of Figure 3 shows the *total* impact on profits. I show this for two cases: perfect labour market and imperfect labour markets

[17] This is also shown by Bilbiie (2019) in a model without capital and government sector.

(the sticky wages case). This distinction is important, as I will explain momentarily, the wage stickiness neutralises the countercyclical behaviour of profits which is the main factor that drags down aggregate demand.

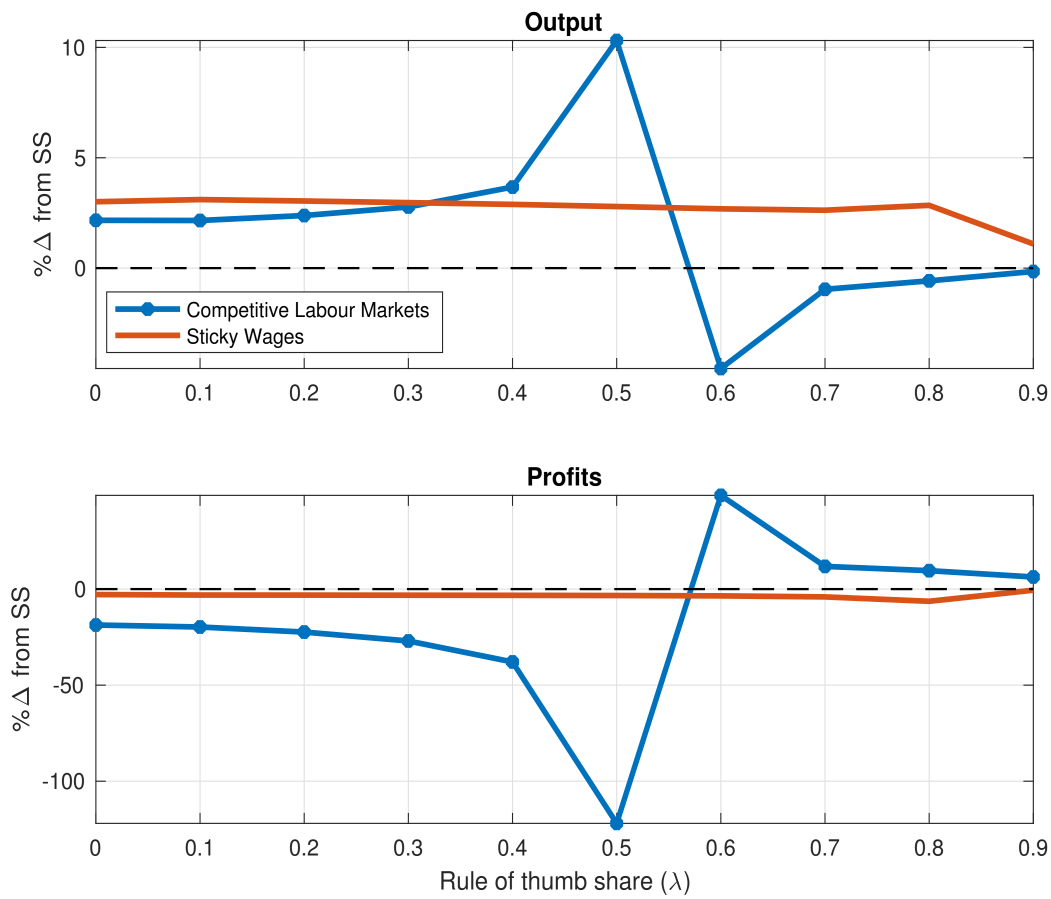

Fig. 3. Impact Effects Conditional on Asset Market Participation: Monetary Policy Shock

Competitive labour markets. As the rule of thumb fraction increases this shifts the value of output upwards. This continues up to a point where aggregate demand reaches its maximum. When λ is over the threshold of $\lambda^* = 0.57$, then the reduction of the nominal interest rate has the opposite effect on the aggregate variables; expansionary monetary policy generates contractionary effects. As λ reaches its upper limit, and agents cannot have intertemporal decisions, monetary policy becomes ineffective. Under the baseline calibration, the *direct effect* of the interest rate reduction presented analytically in equation (47) yields a threshold value of λ^* of 0.52 which is fairly close to the *total effect* threshold shown by solving the model numerically.

To understand the reasoning behind the IADL it is useful to first focus on the region where there is restricted limited participation:$\lambda < \lambda^*$. A reduction in interest rates leads to an increase in aggregate demand. Wage increases from the intertemporal substitution of asset holders and this wage increase translates to a further increase in demand, since non-asset holders consume their wage income (assuming no transfers). This generates a shift in labour demand upwards. As Figure 3 shows this effect is not constant across the domain of λ values. To understand why this is the case is important to focus on the role of profits. Profits as shown above analytically and in the bottom panel of Figure 3 are countercyclical. Consequently, as the asset market participation lowers the less the negative consequences of the profits experienced by the majority of population, the non-asset holders. Therefore, as λ increases and until it reaches λ^* aggregate demand increases continuously. The countercyclicality of profits will induce aggregate demand to drop and there is a new equilibrium with lower output, consumption and wages. Finally, reaching the end of the λ domain, at $\lambda = 0.9$ almost no agent holds assets and the interest rate policy is ineffective.

Sticky Wages. When we introduce labour unions that set the wages, results change. After an accommodative monetary policy shock of the same magnitude as before, we see that for all levels of asset market participation the impact effect of output does never turn negative. The introduction of sticky wages manages to keep the impact effect of profits still countercyclical but of a much smaller magnitude. Consequently, profits no longer drag aggregate demand down and output's response is always positive for the λ domain.

4.1.2. Quantitative Easing

In the same spirit with the contractionary effects of a conventional policy rate reduction, I show that a quantitative easing programme can have adverse effects in a LAMP setting. I look at this again for both labour market setting. The bond buying programme in the present setting is an one time increase in the government bond holdings and a simultaneous reduction of the holdings of banks and households. Finding the *direct effect* of QE on output is a more tedious process than the one of the monetary policy interest rate change since QE is not present in the IS equation (45).

A way to introduce government bonds is through capital. From the capital market clearing (36) we have $K_t = K_t^B + K_t^H + K_t^G$. Log-linearising it around the steady-state yields:

$$k_t = s_k^H k_t^H + s_k^B k_t^B + s_k^G k_t^G, \tag{48}$$

where $s_k^H = K^H/K, s_k^B = K^B/K, s_k^G = K^G/K$. Log-linearising the aggregate incentive constraint of the bank around the steady state:

$$QS^B(\hat{Q}_t + k_t^B) + \Delta q B^B(\hat{q}_t + b_t^B) = \phi N(\hat{\phi} + n_t).$$

The small letters are the log-deviations of the variables from their steady state. $\hat{Q}_t$ is the corresponding value for the price of capital and $\hat{q}_t$ for the price of bonds. Solving for the bankers' capital holdings:

$$k_t^B = -\frac{\Delta q B^B}{QS^B} b_t^B - \frac{\Delta q B^B}{QS^B}\hat{q}_t + \frac{\phi N}{QS^B}(\hat{\phi} + n_t) - \hat{Q}_t. \tag{49}$$

Taking the log deviations of the capital market clearing (37) and solving for the banks' bond holdings:

$$b_t^B = -\frac{s_b^G}{s_b^B} b_t^G + \frac{b_t}{s_b^B} - \frac{s_b^H}{s_b^B} b_t^H, \tag{50}$$

where $s_b^H = B^H/B, s_b^B = B^B/B, s_b^G = B^G/B$.

Plugging (49),(50) into (48):

$$\begin{aligned} k_t = s_k^H k_t^H + s_k^B[-\frac{\Delta q B^B}{QS^B}\left(-\frac{s_b^G}{s_b^B} b_t^G + \frac{b_t}{s_b^B} - \frac{s_b^H}{s_b^B} b_t^H\right) \\ -\frac{\Delta q B^B}{QS^B}\hat{q}_t + \frac{\phi N}{QS^B}(\hat{\phi} + n_t) - \hat{Q}_t] + s_k^G k_t^G. \end{aligned} \tag{51}$$

Since we are interested on the direct effect of government bond purchases (assuming everything else remains constant) we are interested in

$$k_t = s_k^B \frac{\Delta q B^B}{QS^B}\frac{B^G}{B^B} b_t^G = \frac{\Delta B^G}{S} b_t^G. \tag{52}$$

The *direct effect* on output using the IS equation is:

$$-\frac{1}{\delta}\frac{\epsilon\lambda\alpha}{(1-\lambda)(1-\alpha)}\frac{\Delta B^G}{S} b_t^G. \tag{53}$$

Using the fact that $b_t^G = \frac{B_t^G - B^G}{B^G}$, $B^G = 0$, and after some algebra manipulation the above equation becomes:

$$\frac{1}{\frac{(1-\lambda)(1-\alpha)K}{s_c\alpha\epsilon\lambda\Delta} - \frac{\epsilon\lambda K}{\alpha\epsilon\lambda\Delta}} B_t^G. \tag{54}$$

Setting the above expression equal to zero, we can find the threshold value λ^* that makes the direct effect of the quantitative easing policy ineffective. The result yields the same level of threshold with the conventional monetary policy case, $\lambda^* = 0.799$. Therefore, the value of λ that makes both the *direct* effect of quantitative easing and the interest rate reduction ineffective is equivalent.

In order to find the *total* impact effect of the QE, I proceed with the numerical solution of the model. The impact effects of the same macro variables shown in the previous exercise are presented in the top chart of Figure 4.

In the perfectly competitive labour market case, the total impact effect is positive and increasing as long as the asset market participation decreases. After the level of participation passes the threshold level λ^*, QE becomes contractionary. Nevertheless, the *total* impact effect of QE and MP shock is different and the threshold level of market participation λ^* that neutralizes the total effect of the two policies differs as well. The countercyclicality of profits, shown in the bottom chart of Figure 4, is also in this case the factor that produces the IADL.

Introducing sticky wages, as in the monetary policy shock case, neutralises the countercyclical role of profits. The impact effect of output is positive for most of the λ domain and it slightly turns negative when the asset market participation is too low, around 70%.

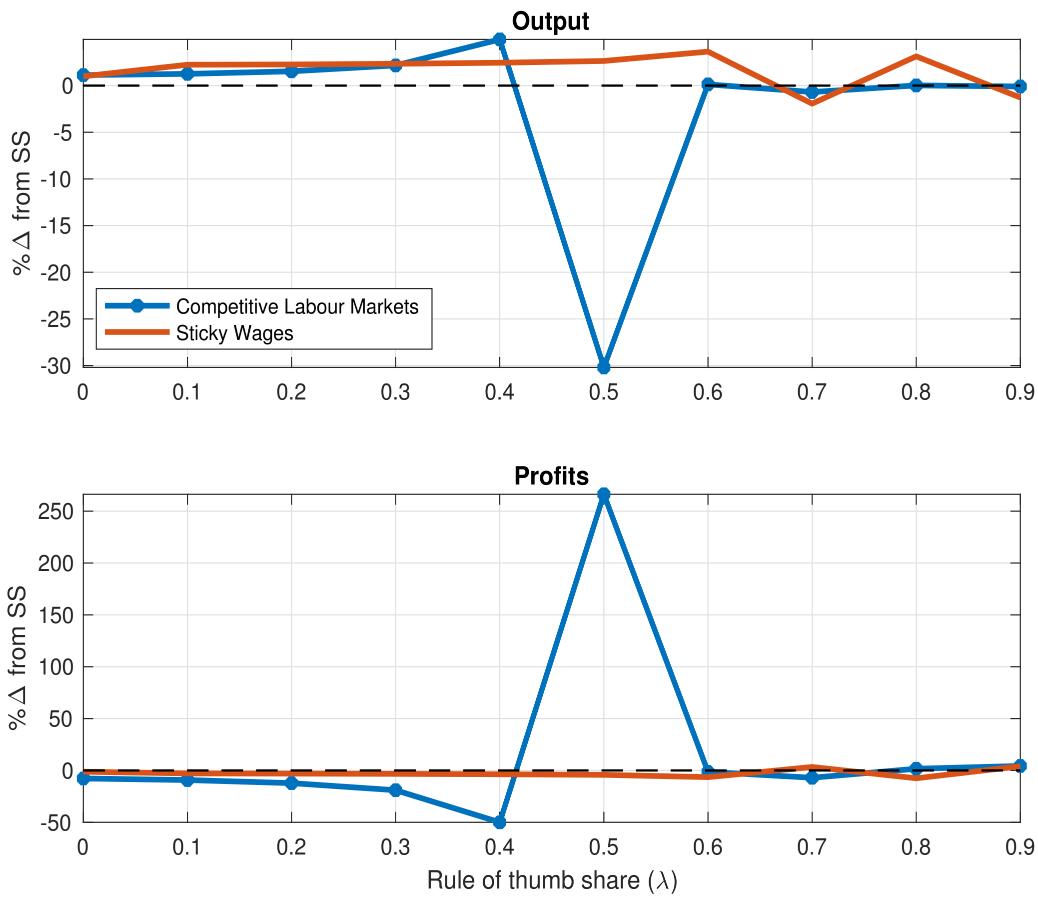

Fig. 4. Impact Effects Conditional on Asset Market Participation: QE Shock

4.2. *Redistribution of Profits*

We have seen that in the perfectly competitive labour market case, accommodative conventional and unconventional monetary policy can have negative effects. In this section I focus only in this labour market setting and provide results under the assumption that taxation is redistributive. That is, a percentage of the profits is allocated to the hand to mouth consumers who were entitled zero transfers under the baseline scenario examined before. What changes is as the rule of thumb consumers share of the profits increases, the IADL region shifts to the right. The negative effects of profits are shared between the two groups leading to a welfare increase.

Taxation is following a simple fiscal rule of redistribution of profits to the hand to mouth consumers defined as:

$$T_t^r = t_{pr} Prof_t. \tag{55}$$

I assume three different taxation parameter values: 0% (baseline scenario), 20% and 40%. It's important to note that this is an ad-hoc choice for the profit tax parameter values. Since the purpose of this exercise is to identify the changes when transfers to rule of thumb agents are non-zero, the choice of a data driven parameter is not crucial. Due to the complexity of the model I abstract from the analytical solution of this case and I show numerically what is the *total* impact effect of a monetary policy shock and a QE shock to output. To show the counter- or procyclicality of both policies under the taxation regime I focus on the impact effect of both policies on output.

Figure 5 shows the paths of the impact effect of output after both a conventional accommodative monetary policy shock (on the top panel) and a bond purchase shock (on the bottom panel). Both impact effects are plotted as a function of λ. Shocks follow the process specified in the calibration section. The blue solid line corresponds to the baseline scenario of no redistribution, while the red dashed line to a tax rate of 20% and the yellow dotted line to a tax rate of 40%. What changes in comparison with the no redistribution case is that as the tax rate increases, the threshold of λ that makes both monetary policy tools contractionary shifts to the right. At the same time, the impact effect of output is milder for both cases of fiscal redistribution compared to the benchmark for reasonable values of λ (up to 0.7).[18]

Under fiscal redistribution, the rule of thumb agents share partially the negative effects of profits. As the financial participation level goes down, profits' role in output becomes limited. Opposed to the benchmark case, now rule of thumb agents internalize partially the adverse effects and thus aggregate demand does not increase as much as in the benchmark case. On the other hand, the impact effect of output remains positive for most of the domain λ especially in the high taxation case. This stops at a threshold level of λ where profits have been decreased by so much that induce a drop in aggregate demand. Redistributive fiscal policy preserves the procyclicality of accommodative monetary policy tools.

5. VAR Evidence on the Cyclicality of Profits

I employ a VAR empirical specification to understand what the data says about role of profits and income inequality after a QE shock. To do this I use the proxy-SVAR

[18]Note that the impact effect is plotted until $\lambda = 0.90$ since the analysis is restricted to the range of λ values consistent with a unique equilibrium.

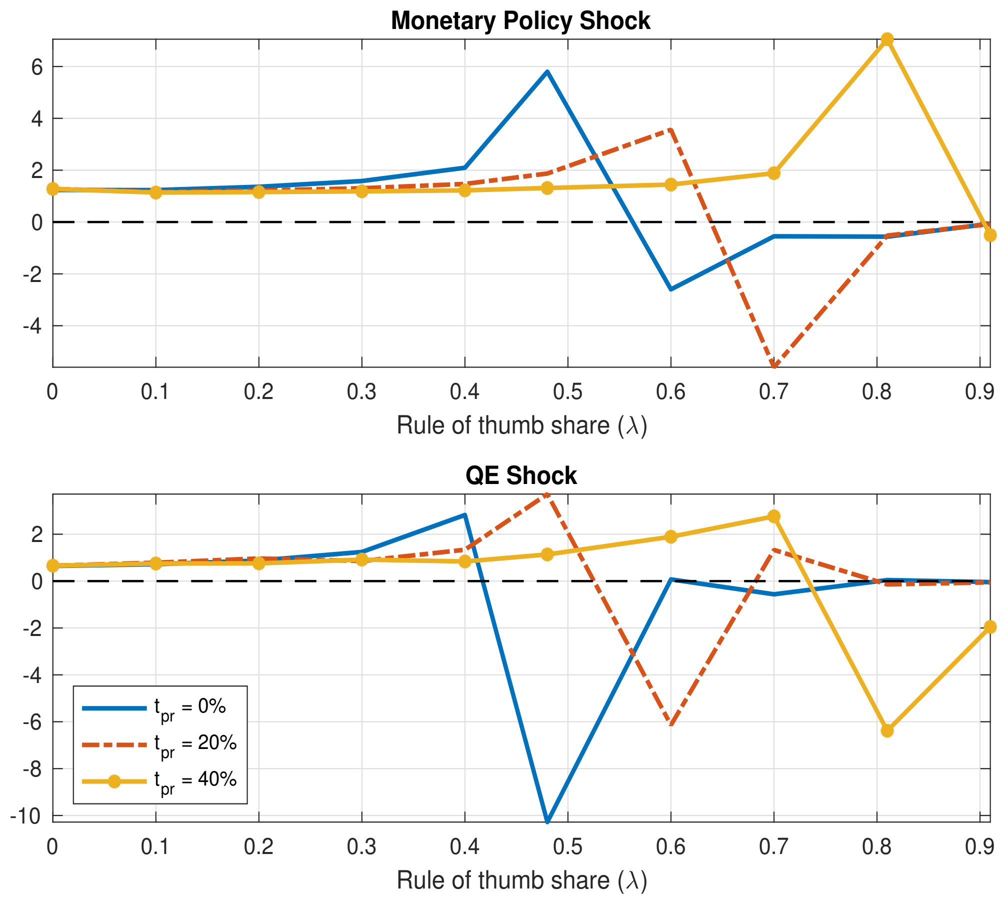

Fig. 5. Inverted Aggregate Demand Logic (Profit Redistribution)

approach firstly introduced by Stock and Watson (2012) and Mertens and Ravn (2013). Due to the difficulty of identifying monetary shocks in the data as elaborated in Ramey (2016), this approach provides a novel way that makes use of external instruments for the structural shocks of interest. Concretely, I employ the Gertler and Karadi (2015) high-frequency identification (HFI) approach. In order to identify external instruments for the QE I use the Euro Area Monetary Policy Event Study Database (EA-MPD) constructed in Altavilla et al. (2019) (ABGMR hereafter) together with their methodology to extract the factors. The novelty of this approach is that a QE factor is identified and can be used directly as an instrument.

The EA-MPD dataset reports median price changes around the time interval of past ECB monetary policy meetings for a broad class of assets and various maturities, including Overnight Index Swaps (OIS), sovereign yields, stock prices, and exchange rates. ECB monetary meetings have a distinct sequence, firstly there is the press release at at 13.45 Central European Time where a policy decision in announced without further elaboration followed by the press conference at 14.30 where the monetary policy strategy and its details are explained more broadly. Using tick data, Altavilla et al. (2019) document the asset prices changes about 10 minutes before and after the meeting. After documenting the changes, they proceed on estimating by principal components analysis and rotating different factors that yield from the monetary policy changes. This is done by firstly identifying the factors according to their statistical significance on the timing of the monetary policy event i.e. whether they are significant in the press release or press conference window. Secondly, the factors are rotated according to the orthogonality conditions assumed so that they are economically interpretable. Based and on which maturity of the risk free assets those factors load, four factors are identified: "Target", "Timing", "Forward Guidance" and "QE".

ABGMR have estimated the factors up to 2018. I proceed by updating the monetary policy factors until 2020 using the (continuously) up to date EA-MPD dataset that spans the time period of 2002 to 2020. In particular, following the work of Gürkaynak, Sack, and Swanson (2007) and the procedures of ABGMR described above I estimate latent factors from yield changes of risk-free rates at different maturities and then rotate these factors in order to identify the four different interpretable factors. Naturally, in my VAR exercise I use as an external instrument for QE surprises the QE factor. For the quarterly VAR, I follow Slacalek et al. (2020) and sum all the intra-day surprises of the QE factor that occur in a quarter.

The VAR has the following reduced form specification:

$$V_t = c + \sum_{i=1}^{p} B_j V_{t-1} + S\epsilon_t$$

where V_t is a vector of nine economic and financial variables and ϵ_t a vector of structural white noise shocks.

The variables include the 10 year Euro Area benchmark bond rate, CPI, the real GDP, the 3 month rate, a stock prices index, the employment level, a measure for the wages, real consumption and real profits. The VAR specification has two lags based on the AIC criterion. I use quarterly data from 1987Q1 to 2019Q4, leaving out of my sample the current pandemic. The data is coming -mainly- from the Area Wide Model dataset originally constructed by Fagan, Henry, and Mestre (2001). The updated AWM database starts in 1970Q1 (for most variables) and is available until 2017Q4. To update the data further, I make use of publicly updated data from Eurostat, ECB and the OECD. Finally, due to the stock market data (Stoxx50) available data starting in 1987Q1 I disregard the first 17 years of the data.

Following Mertens and Ravn (2013) and Stock and Watson (2012) the instrument Z_t must be correlated with the policy shock of interest, in this case the QE shock ϵ_t^{QE} and orthogonal with all the rest of the shocks ϵ_t^{R}:

$$\begin{aligned} \mathbb{E}_t[Z_t \epsilon_t^{QE}] &= \Phi \\ \mathbb{E}_t[Z_t \epsilon_t^{R}] &= 0. \end{aligned}$$

As elaborated above, the instrument used is the QE factor identified using the ABGMR methodology building on the EA-MPD dataset. Given that the big phase of the Asset Purchases Programme in the Euro Area started at 2014, the instrument is used for the period 2014Q1-2019Q4.

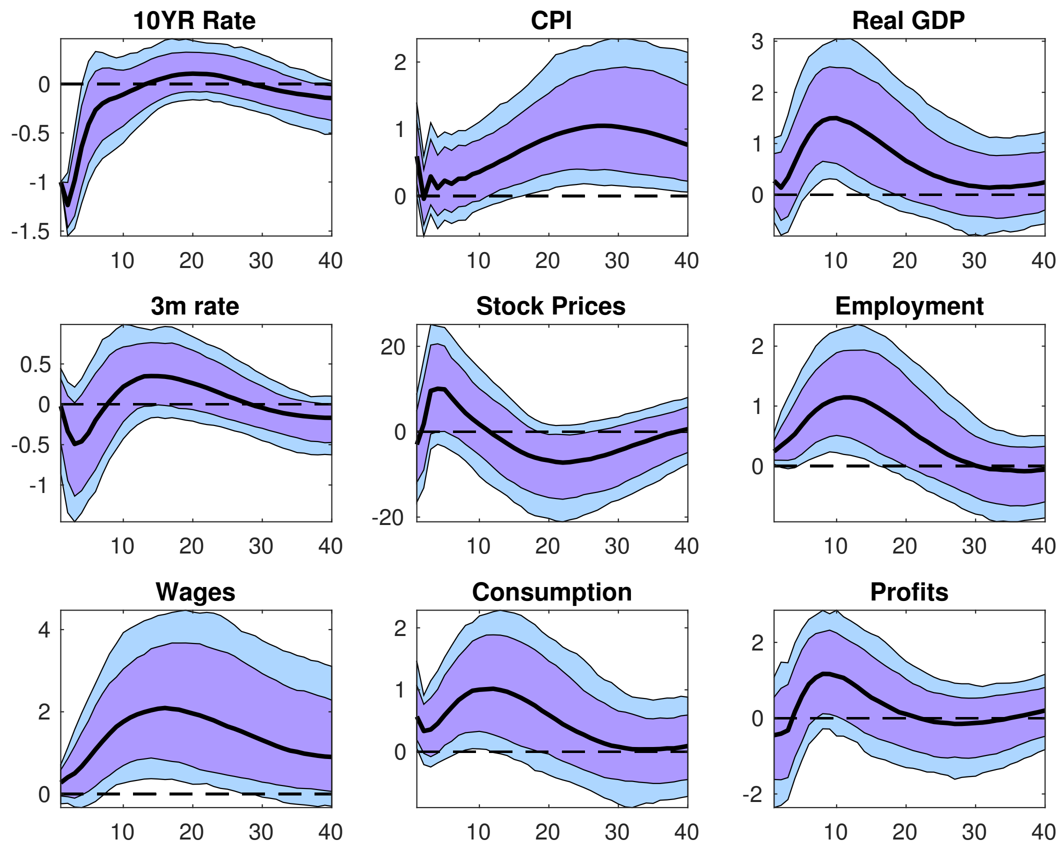

Fig. 6. Ten-Year Rate Shock

Figure 6 shows the impulse responses after a accommodative monetary policy shock that reduces the 10 year rate by 100 bp. The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. As expected from the conventional theory, a shock of this nature stimulates output, consumption and employment and also pushes wages up and - statistically insignificantly- the price level. Most notably, given our current question on the procyclicality of the profits, profits seem to move procyclical with output. They are not moving one to one with the real GDP but although statistically insignificant, they follow GDP response.

6. Quantitative Analysis

In this section I present the model's calibration and the second set of results of the paper: the impact of the quantitative easing and interest rate reduction in inequality.

6.1. *Calibration*

The model's calibration is performed in order to match Euro Area stylized facts and is divided in conventional and banking parameters. It follows broadly the updated version of the New Area-Wide Model (NAWM), (Christoffel, Coenen, and Warne (2008), Coenen, Karadi, Schmidt, and Warne (2018)), the DSGE model of the ECB. Exception to this is the calibration of the banking sector which is done in accordance to the Basel III framework and also in order to meet specific Euro Area banking targets. Parameters in the NAWM are estimated by the use of Bayesian methods in the time span of 1985Q1-2011Q4 using times series for 18 macroeconomic variables which feature prominently in the ECB/Eurosystem staff projections. One period in the model is one quarter. All the calibrated values are presented in Table 1.

Financial parameter values are chosen in order to match specific EA characteristics and to be in line with the Basel III framework. Calculating the Minimum Capital Requirement ratio or Capital Adequacy Ratio (CAR) is a non-trivial exercise. Basel III framework instructs that the total capital ratio must be no lower than 8%. Nevertheless, this is not the actual ratio that banks follow since this ratio depends on different buffers and pillars. Specifically, sine 2010 banks must hold a minimum Common Equity Tier 1 of 4.5% risk-weighed assets. Additionally, there is a mandatory "capital conservation buffer", equivalent to 2.5% of risk-weighted assets and a "discretionary counter-cyclical buffer" that varies from 0% to 2.5% and depends on the national bank regulators. Lastly, on the systemic important institutions (SII), three more buffers are added to the previous three: The Global systemically important institutions (G-SII) buffer and, subject to national discretion, other systemically important institutions buffer (O-SII). Also the "systemic risk buffer" varying from 0% to 3% has to be added to the previous ratios.[19] As a representative example, Deutsche Bank's Total Capital Requirements for 2018 are 14.5%.[20] In the calibration I set the total capital buffer θ to 20% of the risk weighted assets. In this sense, I consider the representative bank as a big systemic bank with the maximum values for each specific buffer.

The risk weights for each asset are parametrised as follows: The risk-weight for the bonds (Δ) is 50%. This corresponds to bonds with BBB+ to BBB- grade (following the Standard & Poor's notation) which is the median in the bond grade ranking. The risk-weight for the claims on BBB+ to BBB- corporates is 100%. Lastly, the risk-weight for the banks' reserves (ω) is 0%.[21]

Regarding the bond market, the long term target of the real bonds supply by the

[19] More information from the European Systemic Risk Board

[20] Minimum capital requirements and additional capital buffers

[21] BIS - Minimum Capital Requirements

Parameters	Value	Definition
		Households
β	0.998	Discount rate
χ	4.152	Relative utility weight of labour
λ	0.20	Share of rule of thumb agents
ϵ	2	Inverse Frisch elasticity of labour supply
$\bar{S}^R/S$	0.500	Proportion of shares of the optimizers
$\bar{B}^R/B$	0.750	Proportion of bond holdings of the optimizers
κ	1	Portfolio adjustment cost parameter
		Banks
θ	0.20	Minimum Capital Requirements (MCR)
Δ	0.5	Risk-weighting coefficient on bonds
ω	0	Risk-weighting coefficient on reserves
ξ_B	0.0014	Entering bankers initial capital
σ_B	0.950	Bankers' survival rate
		Intermediate and Capital Goods Firms
δ	0.025	Depreciation of capital
α	0.36	Capital share
η	5.77	Inverse elasticity of net investment to the price of capital
		Wage and Price Setting
ζ	4.340	Elasticity of labour substitution
ξ_w	0.890	Probability of keeping the price constant
γ_w	0.417	Wage Indexation parameter
ζ	2.540	Elasticity of substitution between goods
γ	0.720	Probability of keeping the wages constant
γ_p	0.480	Indexation parameter
		Treasury Policy
γ^G	0.20	Steady state fraction of government expenditures to output
t_{pr}	0%-40%	Optimizers' profit tax rate
		Monetary Policy
κ_π	1.860	Inflation coefficient in the Taylor rule
κ_y	0.147	Output gap coefficient in the Taylor rule
ρ_m	0.860	Interest-rate smoothing
ρ_1	1.700	First AR coefficient of the bond purchase shock
ρ_2	-0.730	Second AR coefficient of the bond purchase shock
ψ	0.015	Initial asset purchase shock

Table 1: Parameter Values

treasury equals 70% of GDP. The fraction of long-term bonds held by banks is 25% which is consistent with the sovereign debt holdings of the banking sector according to EA data. This leaves the rest 75% of the bond holdings to the optimizers' portfolio. The fraction of shares held by optimizing households is 50%.

For the remainder of the parameters of the banking sector, I set the values for parameters ξ, σ_B such that the model yields a steady state leverage (ϕ) equal to 6 for the banks and a bank capital to lending ratio of 0.25 close to the value suggested by Christoffel and Schabert (2015).

The values for the share of capital α and the depreciation rate δ are chosen to 0.36 and 0.025 respectively following the estimation results of Christoffel et al. (2008). Similarly, the value of β is assigned to 0.998, chosen to be consistent with an annualised equilibrium real interest rate of 2 %. The relative utility weight of labour χ is chosen to ensure a level of labour close to $1/3$ in steady state, a fairly common benchmark in the literature (see Corsetti, Kuester, Meier, and Müller (2014)). The parameter of the inverse Frisch elasticity of labour supply ϵ is one difficult to identify. In the NAWM, this parameter is not estimated and calibrated ad-hoc to 2 which is the one I employ here as well. ϵ has a crucial role on the IADL results of the paper. I provide additional robustness checks in the Appendix G for a range of ϵ starting from 0.5 to 2. Results of the paper hold for all these values.

The elasticity of substitution between goods ζ and the capital adjustment costs also follow the NAWM and set to 2.54 and 5.77 respectively. The same holds for the wage setting parameters. The government spending as a fraction of the GDP is set to 20% also following other studies for the Euro Area. Retail firms parameters: the elasticity of substitution between goods, the Calvo probability and the price indexation parameter are set to the value estimated in the NAWM. The same holds for the monetary policy parameters: the inflation and output gap coefficients in the Taylor rule and the interest rate smoothing parameter.

The share of rule of thumb consumers is chosen to be $\lambda = 0.20$. Using the data from the Eurosystem Household Finance and Consumption Survey, as explained in Section 1, almost the bottom 20% of the Euro Area households hold essentially no net worth at all. This is also in line with the estimates of Slacalek et al. (2020). The same value is also used by a similar study for the EA with LAMP Hohberger, Priftis, and Vogel (2019b). The profits' tax rate used in the IADL results of the paper takes values from 0% to 40% in the exercises performed.[22]

The bond purchase shock is modelled as an AR(2) process.[23] The AR(2) process in contrast with an AR(1) captures the expectation of the further expansion of central bank purchases in the future, which is the case in the ECB's APP started in 2015Q1. The history of APP net asset purchases is shown in Appendix F. Purchases for the first year are constant to 60 billion euro, then in 2016 increase to 80 billion for four quarters to eventually go back to 60 billion and fade out. Relative to 2015 GDP purchases increase

[22] Results remain qualitatively similar under any reasonable tax rate.

[23] This follows similar studies that conclude that the ECB's QE program is characterised by a AR(2) process (see Andrade, Breckenfelder, De Fiore, Karadi, and Tristani (2016), Hohberger et al. (2019b), Carlstrom, Fuerst, and Paustian (2017)).

from a 2% to almost 4% at their peak. To illustrate this pattern, the first AR coefficient is chosen to 1.700 and the second being -0.730 while the initial shock is chosen to 0.015. The choice of the monetary policy shock initial increase is chosen such that the GDP by the a magnitude similar to the one of the QE shock.

6.2. *Impulse Response Analysis*

I proceed with a quantitative exercise on identifying i) what was the impact of the ECB's APP programme on the macroeconomy, ii) its impact on consumption and income inequality and iii) what's the difference with an accommodative monetary policy shock, assuming that the economy is not at the effective lower bound. I present the results of the model with sticky wages. For high levels of asset markets participation, as it occurs in the calibrated model, the two specifications offer qualitatively similar results. The model is solved non-linearly following Lindé and Trabandt (2019).

Central Bank Bond Purchases and Conventional Monetary Policy.— How a bond purchasing programme similar to the APP and an expansionary monetary policy shock affect main macro variables? This is shown in Figure 7. The bond purchase shock is following the APP programme of the ECB. The monetary policy shock is set such that to produce a same increase in output of about 2.9%. In the case of the QE shock I set the nominal interest rate to remain constant for the first four quarters.[24] In bold lines, the responses of a bond shock reflect the responses of a conventional interest rate reduction.

Bond purchases stimulate the economy and increase output as Figure 7 shows. The current calibration of the rule of thumb agents' measure to the EA average ($\lambda = 0.20$) leads to the case that both MP and QE shock increase aggregate demand. The main mechanism works through the loosening of the banks minimum capital requirements constraint. Central bank intermediation increases asset prices Q_t and this leads to an increase in banks' valuation (net worth). Standard financial accelerator effects lead to a further increase of capital price and an economic upturn. An increase in the bonds' prices drives banks to buy more assets which leads to an increase in assets' prices. Excess returns reduce for both securities. The economic upturn is also affecting the real economy due to the higher demand for employment and wage increases. The responses due to the QE shock and those of the MP shock are at least qualitatively identical. For a policy rate reduction to produce the same effect of the APP programme, a 30 basis points reduction is needed assuming that the interest rate is not in its effective lower bound.

Income and Consumption Inequality.— I move to the decomposition of income and consumption responses between the two agents in the economy. Figure 8 shows the responses of those variables after the same two shocks defined above. Both agents consumption increases. Rule of thumb agents' consumption strictly follows the real wage path which after both shocks goes up due to more demand for labour. Notice

[24] ECB after the initiation of its APP programme in 2015Q1 kept its main refinancing operations interest rate constant for a year.

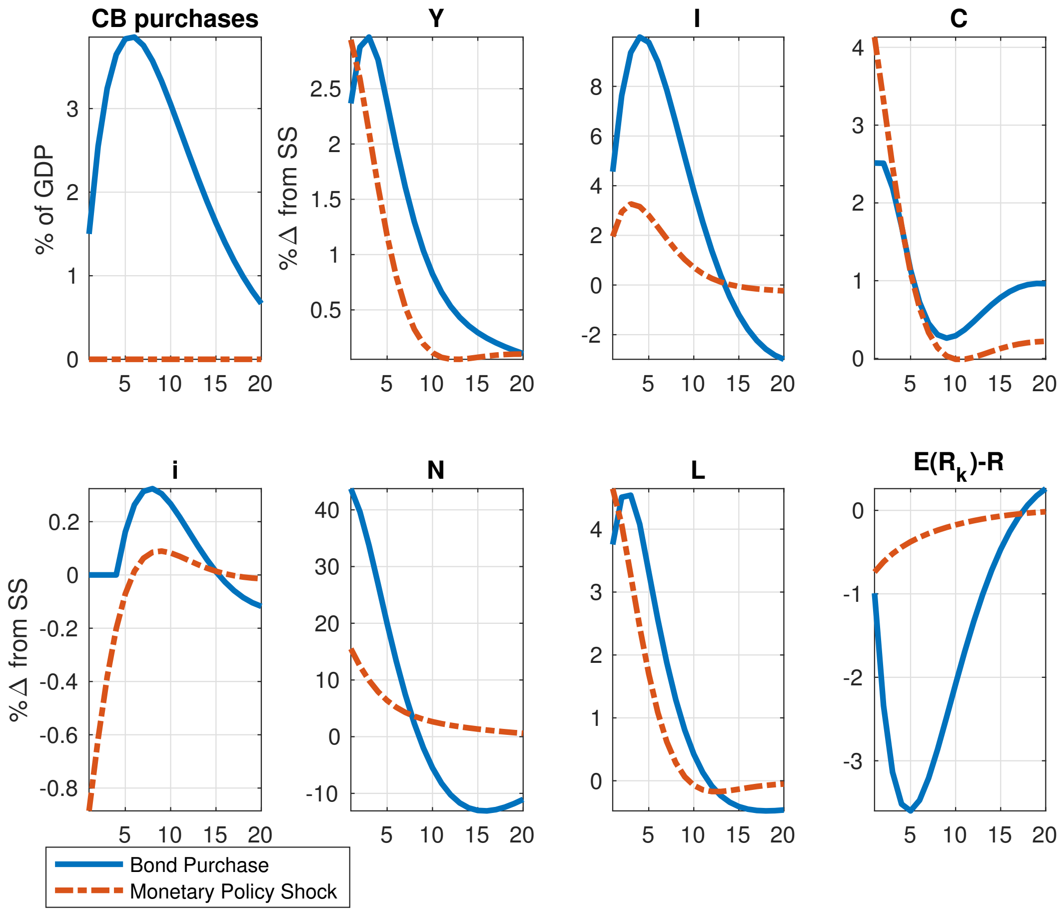

Fig. 7. Government Asset Purchase Shock

that were the nominal interest not constant for the first four periods, consumption of the optimizing agents would have been decreasing. This is because after a stimulating bond shock, the Taylor rule dictates the interest rate to raise. Through the standard intertemporal substitution mechanism the optimizing agents would have lowered their consumption. Consumption inequality defined as C_t^R/C_t decreases. This is in line with the well established fact that hand to mouth consumers have a higher marginal propensity to consume than the financially unconstrained agents (Auclert (2017), Kaplan et al. (2018) among others).

Turning to the income responses of the two agents, depicted at the second row of Figure 8, optimizers' income decline. After a QE shock, optimizers are forced to sell a fraction of their bond holdings to the central bank and exchange them with risk free interest bearing reserves. This has a negative impact on their balance sheet since they lose from the interest rate differential and also from the risk free rate reduction after both shocks. Furthermore due to the drop of excess returns, households they tend to hold less shares reducing further their income. Due to the exchange of bonds to reserves, the

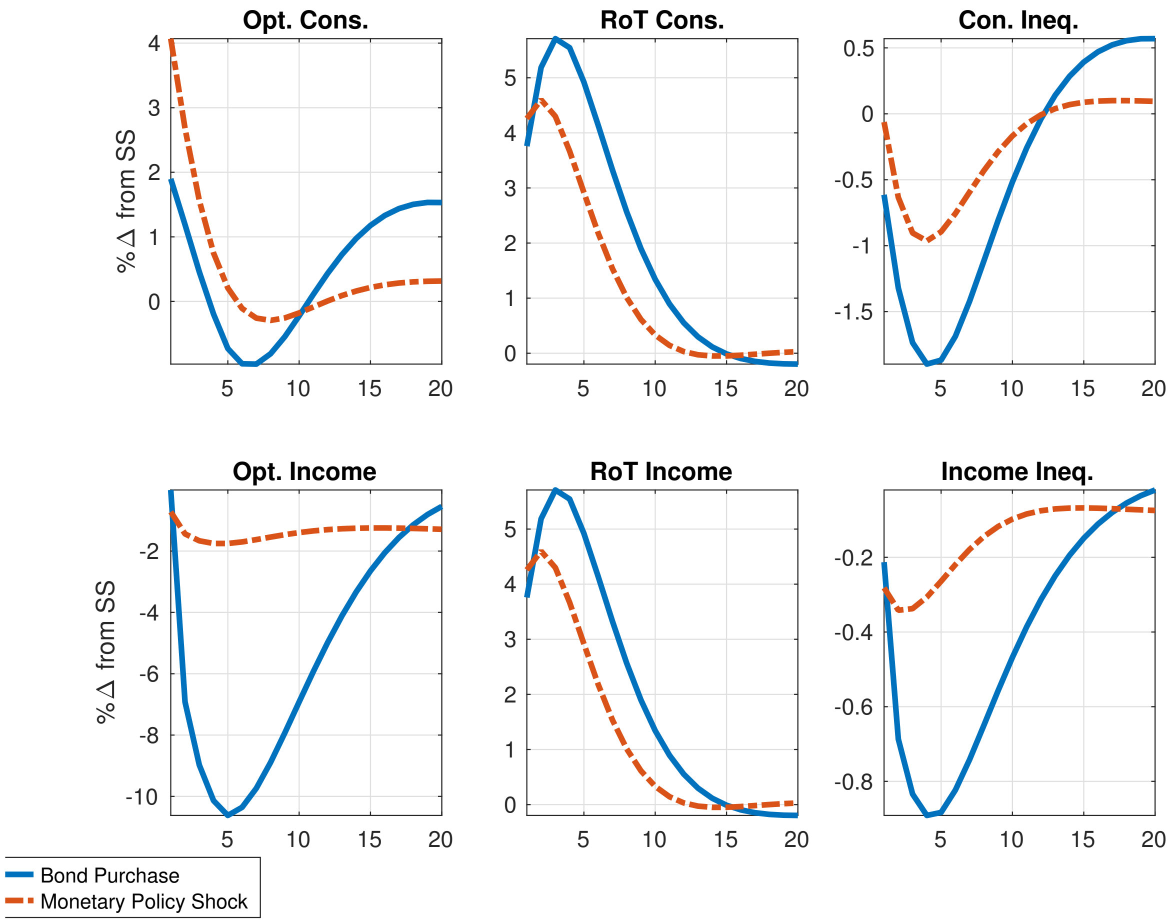

Fig. 8. Government Asset Purchase Shock: Inequality

income reduction is much more amplified during the QE shock. Rule of thumb agents' income follows their labour wage which grows after both shocks. Consequently, income inequality drops for both accommodative policies but is much more amplified in the QE case due to the returns loss.

7. Conclusion

In this paper I ask two questions regarding the recent QE programmes. Firstly, how do they affect consumption and income inequality and secondly whether they can be contractionary at a low level of financial participation. To fix ideas regarding QE's impact on inequality I set up a basic two period model that shows the adverse direct effects of QE on financially participating households' consumption and income. Then to understand the general equilibrium effects, I present and calibrate a model for the Euro Area economy with limited assets market participation, financially constraint banks and price and wage rigidities. Results show that QE reduces income and consumption inequality when the assets market participation level is set to the Euro Area average. Furthermore, I show that quantitative easing is contractionary for low levels of financial participation when wages are fully flexible. Sticky wages mute the contractionary effects.

Arguably, a substantial limitation of this model is the absence of housing which has been left out to reduce the model's complexity. Slacalek et al. (2020) provide a characterization of Euro Area households based on their holdings of liquid and illiquid assets. They can be summarized as optimizers, wealthy hand to mouth and poor hand to mouth. Differently to this model, optimizers and wealthy hand to mouth hold housing on top of their other assets which, importantly, are very similar in volume. Therefore, accommodative monetary policy would have had the same positive effect through house prices on the income of the wealthy hand to mouth and optimizers, leaving, at least qualitatively, the inequality results of this paper between the two groups intact.

References

ALTAVILLA, C., L. BRUGNOLINI, R. S. GÜRKAYNAK, R. MOTTO, AND G. RAGUSA (2019): "Measuring euro area monetary policy," *Journal of Monetary Economics*, 108, 162–179.

ANDRADE, P., J. H. BRECKENFELDER, F. DE FIORE, P. KARADI, AND O. TRISTANI (2016): "The ECB's asset purchase programme: an early assessment," *ECB Working Paper 1956*.

ARAÚJO, A., S. SCHOMMER, AND M. WOODFORD (2015): "Conventional and unconventional monetary policy with endogenous collateral constraints," *American Economic Journal: Macroeconomics*, 7, 1–43.

AUCLERT, A. (2017): "Monetary policy and the redistribution channel," Tech. rep., National Bureau of Economic Research.

BERNANKE, B. (2015): "Monetary policy and inequality," *Ben BernankeâĂŹs Blog at Brookings*.

BILBIIE, F. O. (2008): "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of Economic Theory*, 140, 162–196.

——— (2019): "The new keynesian cross," *Journal of Monetary Economics*.

BILBIIE, F. O., T. MONACELLI, AND R. PEROTTI (2013): "Public debt and redistribution with borrowing constraints," *The Economic Journal*, 123, F64–F98.

BIVENS, J. (2015): "Gauging the impact of the Fed on inequality during the Great Recession," *Hutchins Center Working Papers*.

BROER, T., N.-J. HARBO HANSEN, P. KRUSELL, AND E. ÖBERG (2020): "The New Keynesian transmission mechanism: A heterogeneous-agent perspective," *The Review of Economic Studies*, 87, 77–101.

BUNN, P., A. PUGH, C. YEATES, ET AL. (2018): "The distributional impact of monetary policy easing in the UK between 2008 and 2014," Working paper, Bank of England.

CARLSTROM, C. T., T. S. FUERST, AND M. PAUSTIAN (2017): "Targeting long rates in a model with segmented markets," *American Economic Journal: Macroeconomics*, 9, 205–42.

CHAMLEY, C. AND H. POLEMARCHAKIS (1984): "Assets, general equilibrium and the neutrality of money," *The Review of Economic Studies*, 51, 129–138.

CHRISTOFFEL, K. P., G. COENEN, AND A. WARNE (2008): "The new area-wide model of the euro area: a micro-founded open-economy model for forecasting and policy analysis," .

CHRISTOFFEL, K. P. AND A. SCHABERT (2015): "Interest rates, money, and banks in an estimated euro area model," *Working Paper No. 1791, European Central Bank (May 2015)*.

Coenen, G., P. Karadi, S. Schmidt, and A. Warne (2018): "The New Area-Wide Model II: an extended version of the ECB's micro-founded model for forecasting and policy analysis with a financial sector," *ECB Working Paper, No 2200*.

Coibion, O., Y. Gorodnichenko, L. Kueng, and J. Silvia (2017): "Innocent Bystanders? Monetary policy and Inequality," *Journal of Monetary Economics*, 88, 70–89.

Colciago, A. (2011): "Rule-of-Thumb Consumers Meet Sticky Wages," *Journal of money, credit and banking*, 43, 325–353.

Colciago, A., A. Samarina, and J. de Haan (2018): "Central bank policies and income and wealth inequality: A survey," *De Nederlandsche Bank Working Paper No. 594*.

Corsetti, G., K. Kuester, A. Meier, and G. J. Müller (2014): "Sovereign risk and belief-driven fluctuations in the euro area," *Journal of Monetary Economics*, 61, 53–73.

Cui, W. and V. Sterk (2019): "Quantitative easing," .

Curdia, V. and M. Woodford (2010): "The Central-Bank Balance Sheet as an Instrument of Monetary Policy," Working Paper 16208, National Bureau of Economic Research.

Debortoli, D. and J. Galí (2017): "Monetary policy with heterogeneous agents: Insights from TANK models," *Manuscript, September*.

Dolado, J. J., G. Motyovszki, and E. Pappa (Forthcoming): "Monetary Policy and Inequality under Labor Market Frictions and Capital-Skill Complementarity," *American Economic Journal: Macroeconomics*.

Draghi, M. (2016): "Stability, equity and monetary policy," *2nd DIW Europe Lecture*.

Eggertsson, G. B. and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 1, 139–233.

Fagan, G., J. Henry, and R. Mestre (2001): "An area-wide model (AWM) for the euro area," Tech. rep., European Central Bank.

Galí, J., J. D. López-Salido, and J. Vallés (2004): "Rule-of-Thumb Consumers and the Design of Interest Rate Rules," 36, 739–763.

——— (2007): "Understanding the effects of government spending on consumption," *Journal of the European Economic Association*, 5, 227–270.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58, 17–34.

——— (2013): "Qe 1 vs. 2 vs. 3...: A framework for analyzing large-scale asset purchases as a monetary policy tool," *International Journal of Central Banking*, 9, 5–53.

——— (2015): "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

Gertler, M. and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, 3, 547–599.

Gornemann, N., K. Kuester, and M. Nakajima (2016): "Doves for the rich, hawks for the poor? Distributional consequences of monetary policy," .

Gürkaynak, R. S., B. P. Sack, and E. T. Swanson (2007): "Market-based measures of monetary policy expectations," *Journal of Business & Economic Statistics*, 25, 201–212.

Hohberger, S., R. Priftis, and L. Vogel (2019a): "The Distributional Effects of Conventional Monetary Policy and Quantitative Easing: Evidence from an Estimated DSGE model," *Journal of Banking & Finance*.

——— (2019b): "The macroeconomic effects of quantitative easing in the Euro area: evidence from an estimated DSGE model," *Journal of Economic Dynamics and Control*, 108, 103756.

Jarociński, M. and P. Karadi (2020): "Deconstructing Monetary Policy Surprises-The Role of Information Shocks," *American Economic Journal: Macroeconomics*, 12, 1–43.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary policy according to HANK," *American Economic Review*, 108, 697–743.

Kaplan, G., G. L. Violante, and J. Weidner (2014): "The wealthy hand-to-mouth," Tech. rep., National Bureau of Economic Research.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The aggregate demand for treasury debt," *Journal of Political Economy*, 120, 233–267.

Krueger, D. and F. Perri (2006): "Does income inequality lead to consumption inequality? Evidence and theory," *The Review of Economic Studies*, 73, 163–193.

Lenza, M. and J. Slacalek (2018): "How does monetary policy affect income and wealth inequality? Evidence from quantitative easing in the euro area," .

Lindé, J. and M. Trabandt (2019): "Resolving the missing deflation puzzle," .

McKay, A. and R. Reis (2016): "The role of automatic stabilizers in the US business cycle," *Econometrica*, 84, 141–194.

McLeay, M., A. Radia, and R. Thomas (2014): "Money creation in the modern economy," *Bank of England, Quarterly Bulletin 2014 Q1*.

Mertens, K. and M. O. Ravn (2013): "The dynamic effects of personal and corporate income tax changes in the United States," *American economic review*, 103, 1212–47.

Monacelli, T. (2009): "New Keynesian models, durable goods, and collateral constraints," *Journal of Monetary Economics*, 56, 242–254.

Ramey, V. A. (2016): "Macroeconomic shocks and their propagation," in *Handbook of macroeconomics*, Elsevier, vol. 2, 71–162.

Ravn, M. O. and V. Sterk (2016): "Macroeconomic fluctuations with HANK & SAM: An analytical approach," *manuscript, UCL*.

Sargent, T. J. and B. D. Smith (1987): "Irrelevance of open market operations in some economies with government currency being dominated in rate of return," *The American Economic Review*, 78–92.

Slacalek, J., O. Tristani, and G. L. Violante (2020): "Household balance sheet channels of monetary policy: A back of the envelope calculation for the Euro Area," *Journal of Economic Dynamics and Control*.

Stock, J. H. and M. W. Watson (2012): "Disentangling the channels of the 2007-09 recession," *Brookings Papers on Economic Activity*, 81–135.

Wallace, N. (1981): "A Modigliani-Miller theorem for open-market operations," *The American Economic Review*, 71, 267–274.

Yellen, J. L. (2016): "Perspectives on inequality and opportunity from the Survey of Consumer Finances," *RSF*.

Appendix A Wage-Setting by Unions

The problem of the union is to maximize its objective function (in the main text).

$$\lambda\left[u^r_{c,t}W_{h,t}L_{h,t} - \frac{\chi}{1+\epsilon}L_t^{1+\epsilon}\right] + (1-\lambda)\left[u^o_{c,t}W_{h,t}L_{h,t} - \frac{\chi}{1+\epsilon}L_t^{1+\epsilon}\right]$$

.

subject to

$$L_{h,t} = \left(\frac{W_{f,t}}{W_t^*}\right)^{-\epsilon_w} L_t$$

The first order condition yields:

$$\left(\frac{\lambda}{u^r_{c,t}u^r_{l,t}} + \frac{1-\lambda}{u^o_{c,t}u^o_{l,t}}\right)W_t = \mu^W$$

where $\mu^W = \frac{\epsilon_w}{\epsilon_w - 1}$ and $u^j_{c,t}u^j_{l,t}$ is the marginal rate of substitution of agent of type j.

Appendix B Price Setting

Final-Good Firms.— The profit maximization problem of the retail firm is:

$$\max_{Y_t(j)} P_t\left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} - \int_0^1 P_t(i)Y_t(i)di.$$

The first order condition of the problem yields:

$$P_t\frac{\zeta}{\zeta-1}\left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}-1}\frac{\zeta-1}{\zeta}Y_t(i)^{\frac{\zeta-1}{\zeta}-1} = P_t(i).$$

Combining the previous FOC with the definition of the aggregate final good we get:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\zeta} Y_t.$$

Nominal output is the sum of prices times quantities across all retail firms i:

$$P_tY_t = \int_0^1 P_t(i)Y_t(i)di.$$

Using the demand for each retailer we get the aggregate price level:

$$P_t = \left(\int_0^1 P_t(i)^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}.$$

Intermediate-Good Firms.— Intermediate good firms are not freely able to change prices each period. Following the Calvo price updating specification each period there is a fixed probability $1-\gamma$ that a firm will be able to adjust its price.

The problem of the firm can be decomposed in two stages. Firstly, the firm hires labour and rents capital to minimize production costs subject to the technology constraint (42). Thus, it is optimal to minimize their costs which are the rental rate to capital and the wage rate for labour:

$$\min_{K_t(i), L_t(i)} P_t W_t l_t(i) + P_t Z_t K_t(i)$$

subject to

$$A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha} \geqslant \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

The problem's first order conditions are:

$$W_t = \frac{P_{m,t}^{nom}(i)}{P_t}(1-\alpha) A_t \frac{Y_t(i)}{L_t(i)}, \tag{B.1}$$

$$Z_t = \frac{P_{m,t}^{nom}(i)}{P_t} \alpha A_t \frac{Y_t(i)}{K_t(i)}. \tag{B.2}$$

$P_{m,t}^{nom}$ is the Lagrange multiplier of the minimization problem and the marginal cost of the firms with $P_{m,t} = \frac{P_{m,t}^{nom}(i)}{P_t}$ being the real marginal cost. Standard arguments lead to that marginal cost is equal across firms. Solving together the above equations we find an expression for the real marginal cost $P_{m,t}$ which is independent of each specific variety:

$$P_{m,t} = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} W_t^{1-\alpha} Z_t^{\alpha}.$$

In the second stage of the firm's problem, given nominal marginal costs, the firm chooses its price to maximize profits. Firms are not freely able to change prices each period. Each period there is a fixed probability $1-\gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p. They discount profits s periods in the future by the stochastic discount factor $\Lambda_{t,t+s}$ and the probability that a price price chosen at t will remain the same for some periods γ^s. The second stage of the updating firm at time t us to choose $P_t^*(i)$ to maximize discounted real profits:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} - P_{m,t+s} \right) Y_{t+s}(i)$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^{s} (1 + \pi_{\tau+\kappa-1})^{\gamma_p} \right)^{-\zeta} Y_{t+s}.$$

where π_t is the rate of inflation from $t-i$ to t. The first order condition of the problem is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^{s} (1 + \pi_{\tau+\kappa-1})^{\gamma_p} - P_{m,t+s} \frac{\zeta}{\zeta - 1} \right) Y_{t+s}(i) = 0.$$

Using the constraint and rearranging we get:

$$P_t^*(i) = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{m,t+s} P_{t+s}^{\zeta} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{t+s}^{\zeta-1} \prod_{\kappa=1}^{s} (1 + \pi_{\tau+\kappa-1})^{\gamma_p} Y_{t+s}}.$$

Since nothing on the right hand side depends on each firm i, all updating firms will update to the same reset price, P_t^*. By the law of large numbers the evolution of the price index is given by:

$$P_t = [(1-\gamma)(P_t^*)^{1-\zeta} + \gamma(\Pi_{t-1}^{\gamma_p} P_{t-1})^{1-\zeta}]^{\frac{1}{1-\zeta}}.$$

Capital Goods Producers.— Capital goods producers produce new capital and sell it to goods producers at a price Q_t. Investment on capital (I_t) is subject to adjustment costs. Their objective is to choose $\{I_t\}_{t=0}^{\infty}$ to solve:

$$\max_{I_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Bigg\{ Q_t I_t - [1 + \tilde{f}\left(\frac{I_\tau}{I_{\tau-1}} \right) I_\tau] \Bigg\}.$$

where the adjustment cost function $\tilde{f}$ captures the cost of investors to increase their capital stock:

$$\tilde{f}\left(\frac{I_\tau}{I_{\tau-1}} \right) = \frac{\eta}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 I_\tau.$$

η is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors yields the competitive price of capital:

$$Q_t = 1 + \Big(\eta \frac{I_\tau}{I_{\tau-1}} \Big(\frac{I_\tau}{I_{\tau-1}} - 1 \Big) + \frac{\eta}{2} \Big(\frac{I_\tau}{I_{\tau-1}} - 1 \Big)^2 - \eta \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2} \Big(\frac{I_\tau}{I_{\tau-1}} - 1 \Big) \Big).$$

Profits. Firms' nominal profits are: $Prof_t(i) = P_t(i) Y_t(i) - W_t P_t L_t(i) - Z_t P_t K_t(i)$. Using (B.1) and (B.2) we get $W_t P_t L_t(i) = P^{nom}_{m,t}(i)(1-\alpha)A_t Y_t(i)$ and $Z_t P_t K_t(i) = P^{nom}_{m,t}(i)\alpha A_t Y_t(i)$, We then can write real profits as: $\frac{Prof_t(i)}{P_t} = \frac{P_t(i)}{P_t}Y_t(i) - P_{m,t}Y_t(i)$.[25]

Aggregation. Total profits of non financial firms are equal to the sum of profits earned by intermediate good firms:

$$Prof_t = \int_0^1 Prof_t(i)di.$$

Under standard arguments and using that supply should equal demand in all markets: $\int_0^1 N_t(i)di = N_t$, $\int_0^1 K_t(i)di = K_t$, we get that total profits of the firms are:

$$Prof_t = Y_t - W_t L_t - Z_t K_t. \tag{B.3}$$

Appendix C Bank's Problem

This appendix describes the method used for solving the banker's problem. I solve this, with the method of undetermined coefficient in the same fashion as in Gertler and Kiyotaki (2010). I conjecture that a value function has the following linear form:

$$V_t(s_{j,t}, d_{j,t}, b^B_{j,t}, m^B_{j,t}) = \nu_{l,j,t}s_{j,t} + \nu_{b,j,t}b^B_{j,t} + \nu_{m^B,j,t}m^B_{j,t} - \nu_{d,j,t}d_{j,t} \tag{C.1}$$

where $\nu_{s,j,t}$ is the marginal value from credit for bank j, $\nu_{d,t}$ the marginal cost of deposits, $\nu_{m^B,j,t}$ the marginal value from the central bank reserves and $\nu_{b^B,j,t}$ the marginal value from purchasing one extra unit of sovereign bonds. The banker's decision problem is to choose $s_{j,t}, b^B_{j,t}, m^B_{j,t}, d_{j,t}$ to maximize $V_{j,t}$ subject to the minimum capital requirement constraint (29) and the balance sheet constraint (26). Using (26) we can eliminate $d_{j,t}$ from the value function. This yields:

$$V_{j,t} = s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b^B_{j,t}(\nu_{b,j,t} - \nu_{d,j,t}q_t) + m^B_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n^B_{j,t}.$$

Let $\mathcal{L}$ be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t[V_t - \theta(Q_t s_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t})] = (1+\lambda_t)V_t - \lambda_t\theta(Q_t s_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t}).$$

[25] In Gertler and Karadi (2011) firms derive revenues from selling their good and selling the undepreciated portion of the physical capital back to the capital producers. Therefore profits are $Prof_t = P_t(i)Y_t(i) + Q_t(i)(1-\delta)K_t(i) - W_t P_t L_t(i) - R_{k,t}Q_{t-1}(i)K_t(i)$. Substituting $R_{k,t}$ from (27) we get the same equation for aggregate real profits as in (B.3).

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta s_{j,t}} : (1+\lambda_t)(\frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}) = \lambda_t \theta \tag{C.2}$$

$$\frac{\theta \mathcal{L}}{\theta b^B_{j,t}} : (1+\lambda_t)(\frac{\nu_{b^B,t}}{q_t} - \nu_{d,j,t}) = \Delta \lambda_t \theta \tag{C.3}$$

$$\frac{\theta \mathcal{L}}{\theta m^B_{j,t}} : (1+\lambda_t)(\nu_{m^B,t} - \nu_{d,j,t}) = \omega \lambda_t \theta \tag{C.4}$$

The Kuhn-Tucker condition yields:

$$\begin{aligned} KT : \lambda_t[& s_{j,t}(\nu_{s,j,t} - \nu_{d,t} Q_t) + b^B_{j,t}(\nu_{b^B,j,t} - \nu_{d,j,t} q_t) + m^B_{j,t}(\nu_{m^B,j,t} - \nu_{d,j,t}) \\ & + \nu_{d,j,t} n^B_{j,t} - \theta(Q_t s_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t})] = 0. \end{aligned} \tag{C.5}$$

I define the excess value of bank's financial claim holdings as

$$\mu^s_t = \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}. \tag{C.6}$$

The excess value of bank's bond holdings relative to deposits

$$\mu^b_t = \frac{\nu_{b^B,t}}{q_t} - \nu_{d,j,t},$$

and the excess value of bank's reserve holdings relative to deposits

$$\mu^m_t = \nu_{m^B,j,t} - \nu_{d,j,t}.$$

Then from the first order conditions we have:

$$\mu^b_t = \Delta \mu^s_t. \tag{C.7}$$

If we implement the Basel III direction of attaching 0% of risk-weighting to reserves ($\omega = 0$), the reserves first order condition (C.4) implies that

$$\nu_{m^B,t} = \nu_{d,j,t}. \tag{C.8}$$

This relationship implies that the gain from one extra unit of reserves is exactly the same with the cost of raising one extra unit of deposits. This helps us to show that when reserves is a strictly riskless asset, the bank is not taking them into account when the optimization problem is formulated. From (C.5) and (C.7) when the constraint is binding ($\lambda_t > 0$) we get:

$$
\begin{aligned}
s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b^B_{j,t}(\nu_{b^B,j,t} - \nu_{d,j,t}q_t) + m^B_{j,t}(\nu^B_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} &= \theta(Q_t s_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t}) \\
s_{j,t}(\mu^s_t Q_t) + b^B_{j,t}(\mu^b_t q_t) + m^B_{j,t}(\mu^m_t) + \nu_{d,t}n_{j,t} &= \theta(Q_t s_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t}) \\
Q_t s_{j,t}(\mu^s_t - \theta) + q_t b^B_{j,t}(\Delta\mu^s_t - \Delta\theta) + m^B_{j,t}(\omega\mu^s_t - \omega\theta) + \nu_{d,t}n_{j,t} &= 0 \\
Q_t s_{j,t}(\mu^s_t - \theta) + \Delta q_t b^B_{j,t}(\mu^s_t - \theta) + \omega m^B_{j,t}(\mu^s_t - \theta) + \nu_{d,t}n_{j,t} &= 0
\end{aligned}
$$

and by rearranging terms, we get equation the adjusted leverage constraint:

$$
Q_t s_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t} = \frac{\nu_{d,t}n_{j,t}}{\theta - \mu^s_t} \tag{C.9}
$$

which gives the bank asset funding. It is given by the constraint at equality, where ϕ_t is the maximum leverage allowed for the bank. The constraint limits the portfolio size to the point where the bank's required capital is exactly balanced by the fraction of the risk-weighted measure of its assets. Hence, in times of crisis, where a deterioration of banks' net worth takes place, supply for assets will decline.

Now, in order to find the unknown coefficients I return to the guessed value function

$$
V_{j,t} = Q_t s_{j,t}(\mu^s_t) + q_t b^B_{j,t}(\mu^b_t) + m^B_{j,t}(\mu^m_t) + \nu_{d,t}n^B_{j,t}. \tag{C.10}
$$

Substituting (C.9) into the guessed value function yields:

$$
\begin{aligned}
V_t &= (n_{j,t}\phi_t - \Delta q_t b^B_{j,t} - \omega m^B_{j,t})\mu^s_t + q_t b^B_{j,t}(\mu^b_t) + m^B_{j,t}(\mu^m_t) + \nu_{d,t}n^B_{j,t} \Leftrightarrow \\
V_t &= (n_{j,t}\phi_t)\mu^s_t + q_t b^B_{j,t}(\mu^b_t - \Delta\mu^s_t) + m^B_{j,t}(\mu^m_t - \omega\mu^s_t) + \nu_{d,t}n^B_{j,t}
\end{aligned} \tag{C.11}
$$

and by (C.7) the guessed value function (C.11) becomes:

$$
V_t = (n_{j,t}\phi_t)\mu^s_t + \nu_{d,j,t}n_{j,t}
$$

Given the linearity of the value function we get that

$$
A^B = \phi_t\mu^s_t + \nu_{d,j,t}. \tag{C.12}
$$

The Bellman equation (30) now is:

$$
\begin{aligned}
V_{j,t-1}(s_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) = \mathbb{E}_{t-1}\,\Lambda_{t-1,t}\sum_{i=1}^{\infty}\{(1-\sigma_B)n^B_{j,t} \\
+ \sigma_B(\phi_t\mu^s_t + \nu_{d,j,t})n^B_{j,t}\}.
\end{aligned} \tag{C.13}
$$

By collecting terms with $n_{j,t}$ the common factor and defining the variable Ω_t as the marginal value of net worth:

$$
\Omega_{t+1} = (1-\sigma_B) + \sigma_B(\mu^s_{t+1}\phi_{t+1} + \nu_{d,t+1}). \tag{C.14}
$$

The Bellman equation becomes:

$$
\begin{aligned}
V_{j,t}(s_{j,t}, b^B_{j,t}, m^B_{j,t}, d_{j,t}) &= E_t \Lambda_{t,t+1} \Omega_{t+1} n^B_{t+1} = \\
&= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t} Q_{t-1} s_{j,t-1} + R_{b,t} q_{t-1} b^B_{j,t-1} + R_t m^B_{j,t} - R_t d_{j,t}]. \qquad \text{(C.15)}
\end{aligned}
$$

The marginal value of net worth implies the following: Bankers who exit with probability $(1 - \sigma_B)$ have a marginal net worth value of 1. Bankers who survive and continue with probability σ_B, by gaining one more unit of net worth, they can increase their assets by ϕ_t and have a net profit of μ_t per assets. By this action they acquire also the marginal cost of deposits $\nu_{d,t}$ which is saved by the extra amount of net worth instead of an additional unit of deposits and also the additional cost of reserves $\frac{\kappa}{2}\Upsilon^2_t$. Using the method of undetermined coefficients and comparing (C.1) with (C.15) we have the final solutions for the coefficients:

$$
\begin{aligned}
\nu_{s,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{k,t+1} Q_t \\
\nu_{b^B,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{b,t+1} q_t \\
\nu_{m^B,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\
\nu_{d,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\
\mu^s_t &= \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t+1} - R_{t+1}] \\
\mu^b_t &= \frac{\nu_{b,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{b,t+1} - R_{t+1}] \qquad \text{(C.16)} \\
\mu^m_t &= \nu_{m,j,t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{t+1} - R_{t+1}] = 0 \qquad \text{(C.17)}
\end{aligned}
$$

Appendix D Derivations for Section 4

Proof of (43) for the case of perfect labour market:

$$
\begin{aligned}
c_t &= \lambda c^r_t + (1-\lambda) c^o_t \\
c_t &= \lambda w_t + (1-\lambda)(w_t - \epsilon l^o_t) \\
c_t &= \lambda w_t + (1-\lambda)(w_t - \epsilon \frac{l_t}{1-\lambda})
\end{aligned}
$$

therefore

$$
w_t = c_t + \epsilon l_t.
$$

Proof of (43) for the case of wage setting by unions: The first order condition for the wage setting problem yields:

$$
\left(\frac{\lambda}{MRS^r_t} + \frac{1-\lambda}{MRS^o_t} \right) W_t = \mu^W
$$

where $MRS^o_t = u^o_{c,t} u^o_{l,t}$ and $MRS^r_t = u^r_{c,t} u^r_{l,t}$. Log-linearising this around the steady state yields:

$$w_t = \psi_r c_t^r + \psi_o c_t^o + \epsilon(\psi_r + \psi_o) l_t \tag{D.1}$$

where $\psi_r = \mu^W \frac{\lambda W}{MRS^r}$ and $\psi_o = \mu^W \frac{(1-\lambda)W}{MRS^o}$. Since both agents provide the same labour hours at any time and consumption in steady state is equalized between both agents, in steady state $MRS^o = MRS^r$. Therefore we can write (D.1) as:

$$w_t = c_t + \epsilon l_t.$$

Proof of IS equation (45): Assuming no TFP process in the production function, its log-linearised form is: $y_t = \alpha k_t + (1-\alpha)n_t$. Solving for n_t and substituting to (44) we get

$$c_t^o = c_t - \left(\frac{\lambda}{1-\lambda}\right)\left[\frac{y_t - \alpha k_t}{1-\alpha}\right] \tag{D.2}$$

Log-linearising the resource constraint we get $y_t = c_t s_c + i_t s_i + s_g$ since the proportion of government bond and shares are zero in the steady state. Solving for $c_t = \frac{y_t - i_t s_i - s_g}{s_c}$ and inserting the resource constraint we have:

$$c_t^o = y_t\left(\frac{1}{s_c} - \epsilon\frac{\lambda}{(1-\lambda)}\frac{1}{(1-\alpha)}\right) - i_t\frac{s_i}{s_c} - \frac{s_g}{s_c} + \epsilon\frac{\lambda}{(1-\lambda)}\frac{1}{(1-\alpha)}\alpha k_t. \tag{D.3}$$

Inserting the above into the optimizers Euler equation $c_t^o = \mathbb{E}_t\{c_{t+1}^o\} + [\mathbb{E}_t\{\pi_{t+1}\} - r_t]$, we get

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\delta}[r_t - \mathbb{E}_t\{\pi_{t+1}\}] - \frac{1}{\delta}\frac{s_i}{s_c}\Delta i_{t+1} + \frac{1}{\delta}\frac{\epsilon\lambda}{(1-\lambda)(1-\alpha)}[\alpha\Delta k_{t+1}]. \tag{D.4}$$

where

$$\delta = \frac{1}{s_c} - \epsilon\frac{\lambda}{(1-\lambda)(1-\alpha)} \tag{D.5}$$

and $s_c = C^{ss}/Y^{ss}$, $s_i = I^{ss}/Y^{ss}$ and $s_g = G^{ss}/Y^{ss}$.

Appendix E Steady State

As it is shown on the main text, the rule of thumb agents will always supply constant labour hours equal to and the first order condition for labour supply the rule of thumb agents:

$$L^r = \left(\frac{1}{\chi}\right)^{\left(\frac{1}{1+\epsilon}\right)}$$

From labour hours the aggregator (24) we get the labour hours supplied by the optimizing agents:

$$L^o = \frac{L - \lambda L^r}{1-\lambda}.$$

Rearranging the optimizing agents' first order condition for labour, utilizing the fact that $U_c^o = 1/C^o$, we can get an expression between consumption of the agents and labour supply:

$$C^o = \frac{W}{\chi(L^o)^\epsilon}.$$

Utilizing the above relation and the optimal consumption path of the rule of thumb agents, the consumption aggregator (23) becomes

$$C = \lambda W L^r + (1-\lambda)\frac{W}{\chi(L^o)^\epsilon}.$$

After some algebraic manipulation we end up to the total consumption coming from the demand side of the economy:

$$C = W\left[\lambda L^r + (1-\lambda)^{1+\epsilon}\frac{(L-\lambda L^r)^{-\epsilon}}{\chi}\right] \tag{E.1}$$

In addition, from the resource constraint we have:

$$C = Y - I - G - \tau_b B^G - \tau_s S^G,$$

where in a steady state $B^G = S^G = 0$. Therefore:

$$C = L\left[(1-\gamma)\left(\frac{K}{L}\right)^\alpha - \delta\frac{K}{N}\right] \tag{E.2}$$

To get an expression of K/L we make use of the marginal product of capital (B.2):

$$\frac{L}{K} = (\frac{Z}{A\alpha})^{\frac{1}{1-\alpha}},$$

yielding

$$\frac{K}{L} = \left(\frac{\alpha\left(\frac{\epsilon-1}{\epsilon}\right)}{R_k-(1-\delta)}\right)^{\frac{1}{1-\alpha}}. \tag{E.3}$$

Thus, combining the expressions (E.1), (E.2), (E.3) we obtain an equation depending only on parameters, calibrated values ($spread_{Rk}$) and L^k and determines steady state hours L. Having found L, using the labour hours aggregator (24) we can easily find the labour hours worked by the rule of thumb agents L^o. Thus, consumption of the optimizing agents can be pinned down. Notice that an equation between optimizers' consumption and aggregate consumption can be found by combining the first order condition for labour supply and the demand side aggregate consumption equation(E.1) and solving for W. Then:

$$C^o = \frac{C(1-\lambda)^\epsilon/\chi}{\lambda L^o(L-\lambda L^o)^\epsilon + (1-\lambda)^{1+\epsilon}/\chi} \tag{E.4}$$

Appendix F ECB's Asset Purchase Program

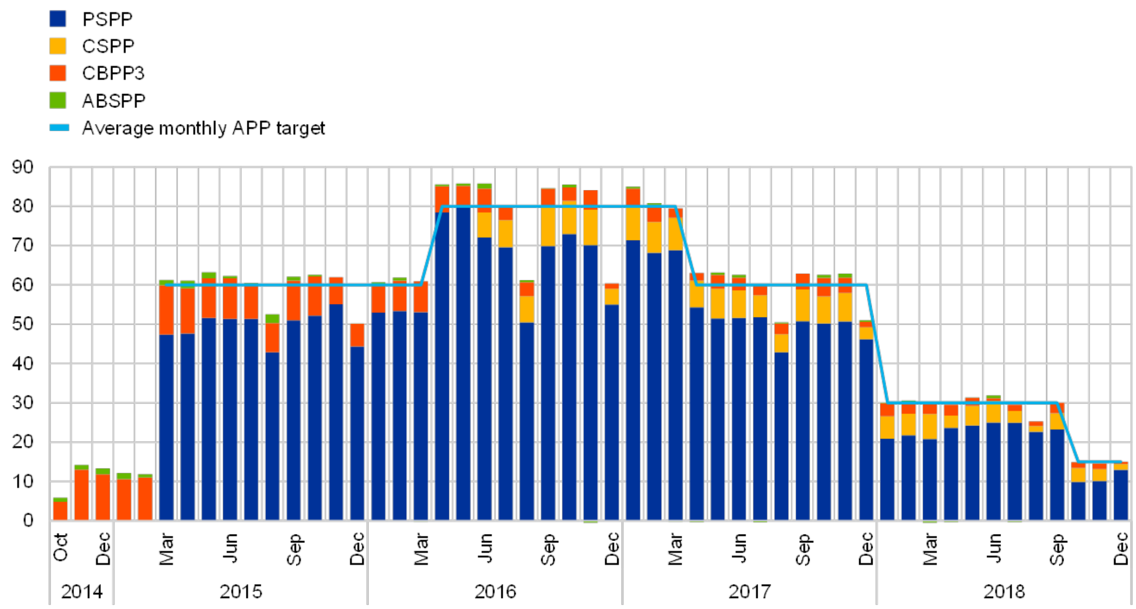

Fig. 9. Pace and composition of net APP purchases. Source: ECB

Appendix G Robustness

In this Appendix, I show that the IADL for the case of QE and a monetary policy shock holds for any reasonable parametrization of the inverse Frisch elasticity of labour supply. Figures 10 and 11 show the impact effect of output after a monetary policy and QE shock conditional on the degree of asset markets participation. This is repeated for four different values for the inverse Frisch elasticity: 0.5, 1 (baseline), 1.5 ans 2. What is clear, is that under every parametrization of the Frisch elasticity IADL remains valid. In all cases, impact effect on output grows as the asset markets participation is decreasing. This holds until a threshold value of λ, different for each case, which makes the impact effect negative until it reaches again a value close to zero.

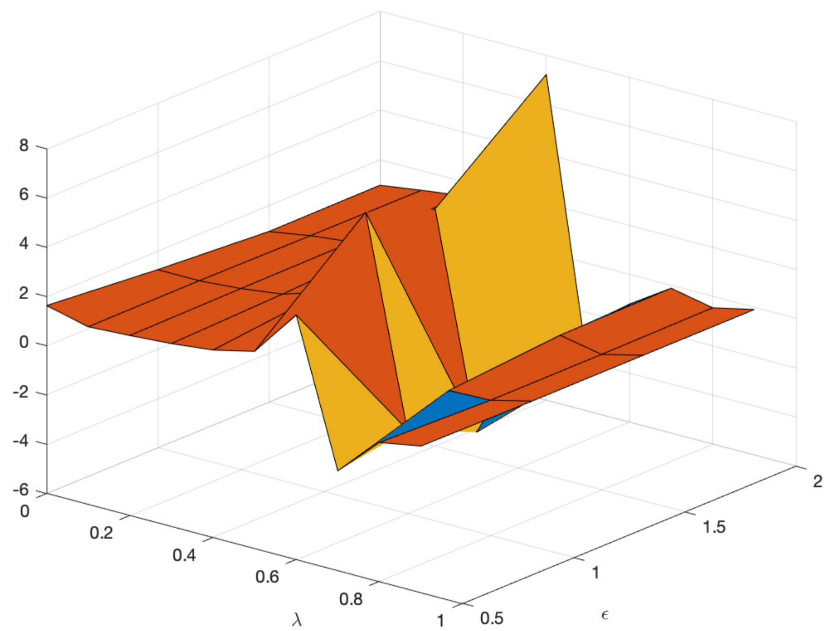

Fig. 10. Sensitivity to Inverse Frisch Elasticity Values: MP Shock

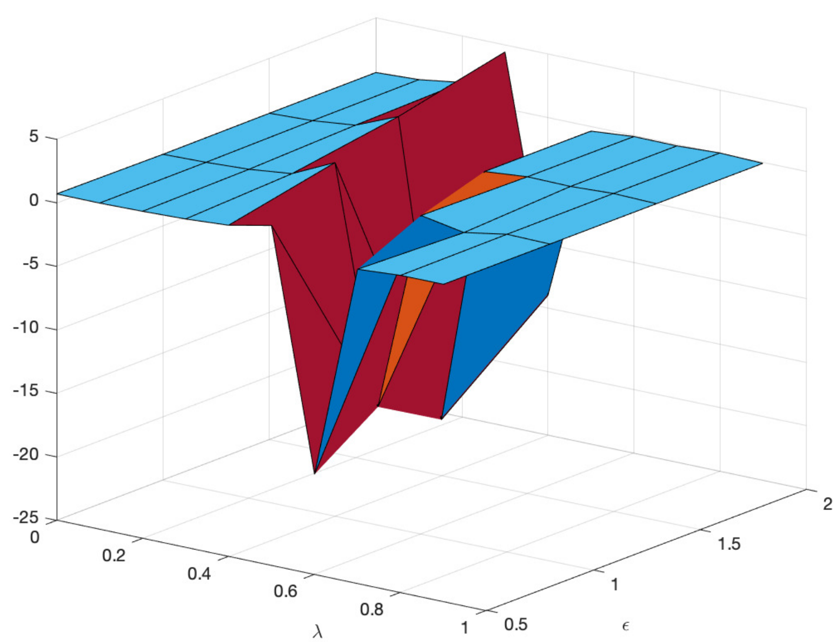

Fig. 11. Sensitivity to Inverse Frisch Elasticity Values: QE Shock