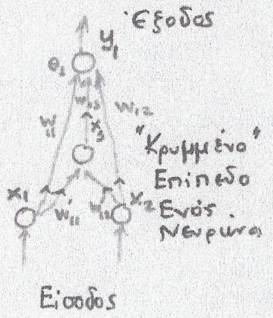
Ο Αλγόριθμος BACKPROPAGATION

ΜΑΘΗΣΗ "ΜΕ ΔΑΣΚΑΛΟ"

Κριτήριο Ελαχιστοποίησης

$$E_p = \frac{1}{2}\sum_j (d_{pj} - y_{pj})^2, \quad j=1$$

$$E = \sum_p E_p$$



p : training samples

j : αριθμός εξόδων

d : επιθυμητή έξοδος

y : πραγματική έξοδος

Άγνωστες Παράμετροι: $[w_{11}, w_{12}, w_{13}, \theta_1]$, $[w'_{11}, w'_{12}, \theta'_1]$.

Λειτουργία νευρώνων εξόδου - παρόμοια των "κρυμμένων Επιπέδων"

$$y_{pj} = \frac{1}{1+e^{-(\sum_i w_{ji}x_{pi}+\theta_j)}} \triangleq \frac{1}{1+e^{-z_{pj}}}$$

Τότε:

$$\Delta_p w_{ji} \approx -\eta \frac{\partial E_p}{\partial w_{ji}}$$

Αλγόριθμος Αναδρομικός από Νευρώνες Εξόδου → Εισόδου

$$\Delta_p w_{ji} = \begin{cases} \eta\, \delta_{pj}\, y_{pi} & \text{(Εξόδου)} \\ \eta\, \delta'_{pj}\, x_{pi} & \text{(Κρυμμένο Επίπεδο)} \end{cases}$$

$$\frac{\partial E_p}{\partial w_{ji}} = \underbrace{\frac{\partial E_p}{\partial z_{pj}}}_{-\delta_{pj}} \frac{\partial z_{pj}}{\partial w_{ji}} = -\delta_{pj} \cdot x_{pi} \Rightarrow$$

$$-\frac{\partial E_p}{\partial w_{ji}} = \delta_{pj}\, x_{pi} \Rightarrow \Delta_p w_{ji} = \eta\, \delta_{pj}\, x_{pi} \qquad (1)$$

$$\delta_{pj} = -\frac{\partial E_p}{\partial z_{pj}} = -\frac{\partial E_p}{\partial y_{pj}} \cdot \frac{\partial y_{pj}}{\partial z_{pj}} \qquad (2)$$

Όμως:

$$\frac{\partial y_{pj}}{\partial z_{pj}} = \frac{1}{(1+e^{-z_{pj}})^2} \cdot e^{-z_{pj}} = y_{pj}(1-y_{pj}) \qquad (3)$$

$(2) \wedge (3) \Rightarrow$

$$\delta_{pj} = -y_{pj}(1-y_{pj}) \frac{\partial E_p}{\partial y_{pj}} \qquad (4)$$

Υπολογισμός του $\frac{\partial E_p}{\partial y_{pj}}$
- (a) Νευρώνες Εξόδου
- (b) "Κρυμμένων Επιπέδων" $y_{pj} \equiv x_{pj}$

(a) $$\frac{\partial E_p}{\partial y_{pj}} = -(d_{pj} - y_{pj}) \qquad (5)$$

Άρα (1)∧(5)∧(4)

$$\Delta_p w_{ji} = n\,\delta_{pj}\,x_{pi} \quad \text{, όπου}$$

$$\delta_{pj} = y_{pj}(1-y_{pj})(d_{pj}-y_{pj}) \qquad (A)$$

(b) $\frac{\partial E_p}{\partial x_{pj}}$: Υπολογίζεται με κανόνα Μερικών Παραγώγων

$$\frac{\partial E_p}{\partial x_{pj}} = \sum_k \frac{\partial E_p}{\partial Z_{pk}} \cdot \frac{\partial Z_{pk}}{\partial x_{pj}} = \sum_k \frac{\partial E_p}{\partial Z_{pk}} \cdot \frac{\partial Z_{pk}}{\partial x_{pj}} = \sum_k \underbrace{\frac{\partial E_p}{\partial Z_{pk}}}_{-\delta_{pk}} w_{kj}$$

$$\Rightarrow \frac{\partial E_p}{\partial x_{pj}} = -\sum_k \delta_{pk} w_{kj}$$

Άρα

$$\Delta_p w_{ji} = n\,\delta'_{pj}\,x_{pi} \quad \text{όπου}$$

$$\delta'_{pj} = x_{pj}(1-x_{pj}) \sum_k \delta_{pk} w_{kj} \qquad (B)$$

Απόδειξη σύγκλισης Αλγόριθμου Perceptron

Έστω $\underline{w}^*$ η ζητούμενη λύση. Τότε για $0<\delta<1$

$(\underline{w}^*)^T \cdot \underline{x} > +\delta > 0$ για patterns x κλάσης 1

$(w^*)^T \underline{x} < -\delta < 0$ " " x " 2

Έστω $\varphi(w) = \dfrac{(w^*)^T \cdot w}{\|w^*\| \|w\|} \leq 1$, για κάποιο w

Είναι όμως:

Updating, όταν $(\underline{w}^{(k)})^T \underline{x} < 0$ αν $x \in C_1$

ή $(w^{(k)})^T \underline{x} > 0$ αν $x \in C_2$

$w^{(k+1)} = w^{(k)} + n x$ για $x \in C_1$

$w^{(k+1)} = w^{(k)} - n x$ για $x \in C_2$

$$\Longrightarrow (w^*)^T w^{(k+1)} = \begin{cases} (\underline{w}^*)^T \underline{w}^{(k)} + n(\underline{w}^*)^T \underline{x} & \text{αν } x \in C_1 \\ (\underline{w}^*)^T \underline{w}^{k} - n(w^*)^T \underline{x} & \text{αν } x \in C_2 \end{cases} \quad (1)$$

$$\|(w^{(k+1)}\|^2 = \begin{cases} (w^{(k)} + nx)^T (w^k + nx) & \text{αν } x \in C_1 \\ (w^{(k)} - nx)^T (w^k - nx) & \text{αν } x \in C_2 \end{cases} \quad (2)$$

∴

$$\left.\begin{aligned} (w^*)^T w^{(k+1)} &> (w^*)^T w^{(k)} + n\cdot\delta \\ \|(w^{(k+1)})\|^2 &< \|w^{(k)}\|^2 + n^2\|x\|^2 \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} (w^*)^T w^{(k_0+1)} &> n k_0 \delta \\ \|(w^{(k_0+1)})\|^2 &< n^2 k_0 \|x\|^2_{min} \end{aligned}\right\} \Rightarrow$$

$$\phi\left(w^{(k_0+1)}\right) = \frac{(w^*)^T w^{(k_0+1)}}{\|w^*\| \cancel{\phi} \|w^{(k_0+1)}\|} > \frac{k_0 \delta}{\sqrt{k_0}\,\|x\|_m \|w^*\|} = \sqrt{k_0}\,\delta'$$

πρέπει

< 1

$\Rightarrow k_0 < \infty$