ΚΑΝΟΝΙΚΗ 13

Θέμα 1ο

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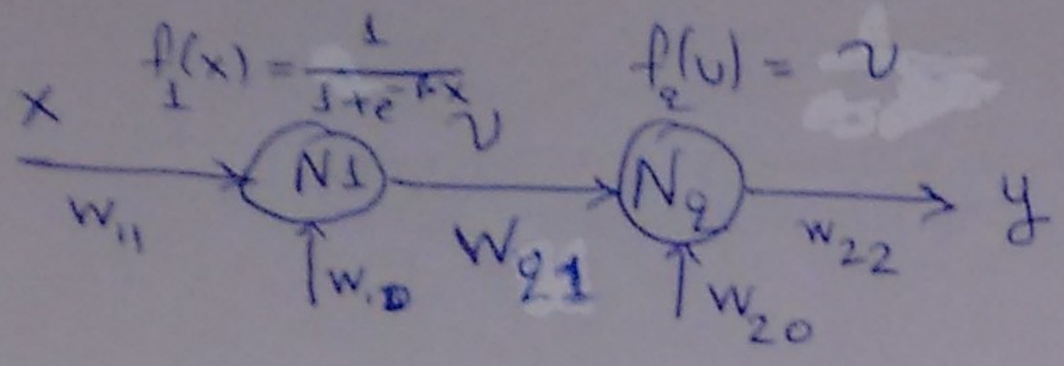
Θεσ. 130-134

a)



$$v_1 = w_{11} x + w_{10} \qquad E = \frac{1}{2} e_2^2$$

$$y_1 = f_1(v_1) = \frac{1}{1+e^{-\beta v_1}} \qquad y_2 = f_2(v_2) = v_2$$

$$v_2 = w_{21} y_1 + w_{20} \qquad e_2 = d_2 - y_2$$

$$\frac{\partial E}{\partial w_{20}} = \frac{\partial E}{\partial e_2} \cdot \frac{\partial e_2}{\partial y_2} \cdot \frac{\partial y_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial w_{20}} = e_2 \cdot (-1) \cdot 1 \cdot 1 = -e_2$$

$$\frac{\partial E}{\partial w_{21}} = \frac{\partial E}{\partial e_2} \cdot \frac{\partial e_2}{\partial y_2} \cdot \frac{\partial y_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial w_{21}} = e_2 \cdot (-1) \cdot 1 \cdot y_1 = -e_2 y_1$$

$$\frac{\partial E}{\partial \beta} = \frac{\partial E}{\partial e_2} \cdot \frac{\partial e_2}{\partial y_2} \cdot \frac{\partial y_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial y_1} \cdot \frac{\partial y_1}{\partial \beta} =$$

$$= e_2 \cdot (-1) \cdot 1 \cdot w_{21} \cdot \left(-\frac{1}{(1+e^{-\beta v_1})^2} \right) (-v_1)\, e^{-\beta v_1} =$$

$$= -e_2 \cdot w_{21} v_1 \cdot e^{-\beta v_1} \cdot \frac{1}{(1+e^{-\beta v_1})^2}$$

$$\Delta\beta = -\eta \cdot \frac{\partial E}{\partial \beta} \Rightarrow \beta(n+1) = \beta(n) + e_2 w_{21} v_1 \cdot e^{-\beta v_1} \cdot \frac{1}{(1+e^{-\beta v_1})^2}$$

$$w_{21}(n+1) = w_{21}(n) + \eta e_2 y_1 \,, \quad w_{20}(n+1) = w_{20}(n) + \eta e_2$$

$$\frac{\partial E}{\partial w_{10}} = \frac{\partial E}{\partial y_1}$$

$$w_{10}(n+1) = w_{10}(n) + \eta \delta_1 y_1 =$$

$$= w_{10}(n) - \eta \frac{\partial E}{\partial y_1} \varphi_1'(v_1) =$$

$$= \frac{\partial E}{\partial y_1} \varphi_1'(v_1) = \sum_k e_k(n) \varphi_k'(v_k) w_{k1} =$$

$$= (e_1 \varphi_1'(v_1) w_{11} +$$

$$\Delta w_{10} = \eta \delta_1 y_0$$

$$\Delta w_{11} = \eta \cdot \delta_1 \cdot y_1$$

, $y_0 = +1$

$$\delta_1 = f_1'(v_1) \sum_k e_k f_k'(v_k) w_{k1} =$$

$$= -\frac{1}{(1+e^{-\beta v_1})^2} \cdot e^{-\beta v_1}(-\beta) \cdot e_1 \cdot \left(-\frac{1}{(1+e^{-\beta v_2})^2}\right) e^{-\beta v_1}(-\beta) w_{11}$$

β) $E^{(i)} = \frac{1}{2}(d-y)^2 = \frac{1}{2}(d - w_1 p_1 - w_{12} p_{12} p_2 - w_2 p_2 - b)^2$

$$\frac{\partial E}{\partial w_1} = e(-p_1) \quad , \quad \frac{\partial E}{\partial w_2} = e \cdot (-p_2) \quad , \quad \frac{\partial E}{\partial w_{12}} = e(-p_1 p_2) \quad , \quad \frac{\partial E}{\partial b} = e(-1$$

$w_{12} =$

(ii) $(0,0)$: $y = b \rightarrow b = 0$, $(1,1)$: $w_1 + w_2 + w_{12} = 0$, $(0,1)$: $w_2 = 1$, $(1,0)$: $w_1 = 1$,