Θέμα 3ο

a) $x_1 = [0 \; 1]$

$x_2 = [1 \; 0]$

Ορίζουμε τον πυρήνα $k(x, x_i) = (1 + x^T x_i)^2$, $i = 1,2$

$\Rightarrow k(x, x_i) = 1 + x_1^2 x_{i_1}^2 + 2 x_1 x_2 x_{i_1} x_{i_2} + x_2^2 x_{i_2}^2 +$

$+ 2 x_1 x_{i_1} + 2 x_2 x_{i_2}$

Επειδή $k(x, x_i) = \phi^T(x_i)\phi(x)$, λαμβάνουμε:

$\phi(x) = [1, x_1^2, \sqrt{2} x_1 x_2, x_2^2, \sqrt{2} x_1, \sqrt{2} x_2]^T$

$\phi(x_i) = [1, x_{i_1}^2 + \sqrt{2} x_{i_1} x_{i_2}, x_{i_2}^2, \sqrt{2} x_{i_1}, \sqrt{2} x_{i_2}]^T$, $i = 1,2$

Υπολογίζουμε τον πίνακα Gram: $\phi(x_1) = [1, 0, 0, 1, 0, \sqrt{2}]^T$

$\phi(x_2) = [1, 1, 0, 0, \sqrt{2}, 0]^T$

$K = \{k(x_i, x_j)\}_{i,j=1}^{2}$ ⊛ $\longrightarrow K = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$

$k(x_1, x_1) = 1 + 0 \cdot 0 + 2 \cdot 0 \cdot 1 \cdot 0 \cdot 1 + 1 \cdot 1 + 2 \cdot 0 \cdot 0 + 2 \cdot 1 \cdot 1 =$

$= 1 + 1 + 2 = 4$

$k(x_1, x_2) = 1 + 0 \cdot 1 + 2 \cdot 0 \cdot 1 \cdot 1 \cdot 0 + 1 \cdot 0 + 2 \cdot 0 \cdot 1 + 2 \cdot 1 \cdot 0 =$

$= 1$

$k(x_2, x_1) = 1 + 1 \cdot 0 + 2 \cdot 1 \cdot 0 \cdot 0 \cdot 1 + 0 \cdot 1 + 2 \cdot 1 \cdot 0 + 2 \cdot 0 \cdot 1 =$

$= 1$

$k(x_2, x_2) = 1 + 1 \cdot 1 + 2 \cdot 1 \cdot 0 \cdot 0 \cdot 1 + 0 \cdot 0 + 2 \cdot 1 \cdot 1 + 2 \cdot 0 \cdot 0 =$

$= 1 + 1 + 2 = 4$

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Υποθέτουμε $d_1 = -1$, $d_2 = 1$.

Τότε, $Q(a) = \sum_{i=1}^{2} a_i - \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} a_i a_j d_i d_j k(x_i, x_j) =$

$$= a_1 + a_2 - \frac{1}{2}\left(4a_1^2 - a_1 a_2 - a_2 a_1 + 4a_2^2\right) =$$

$$= a_1 + a_2 - \frac{1}{2}\left(4a_1^2 - 2a_1 a_2 + 4a_2^2\right) =$$

$$= a_1 + a_2 - 2a_1^2 + a_1 a_2 - 2a_2^2$$

Η βελτιστοποίηση της $Q(a)$ ως προς τους δύο πολ/ες Lagrange είναι:

$$\left.\begin{aligned} \frac{\partial Q}{\partial a_1} = 0 &\Rightarrow 1 - 4a_1 + a_2 = 0 \Rightarrow a_2 = 4a_1 - 1 \\ \frac{\partial Q}{\partial a_2} = 0 &\Rightarrow 1 + a_1 - 4a_2 = 0 \Rightarrow a_1 = 4a_2 - 1 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow a_1 = 4(4a_1 - 1) - 1 \Rightarrow$$

$$\Rightarrow a_1 = 16a_1 - 4 - 1 \Rightarrow$$

$\cancel{b = 1 - w^T x_2 \ (d_2 = 1)}$

$$\left.\begin{aligned} &\Rightarrow 15a_1 = 5 \Rightarrow a_1 = \frac{1}{3} \\ &a_2 = \frac{4}{3} - 1 = \frac{1}{3} \end{aligned}\right\} a_1 = a_2 = \frac{1}{3}$$

$$w = \sum_{i=1}^{2} a_i d_i \varphi(x_i) = \frac{1}{3}\left(-\varphi(x_1) + \varphi(x_2)\right) = \frac{1}{3}[0, 1, 0, -1, \sqrt{2}, -$$

Το πρώτο στοιχείο του w υποδεικνύει ότι $b = 0$.

β) [-10, -2] : πολικό ψύχος

[-5, 3] : ψύχος

[2, 8] : πολύ κρύο

[5, 16] : κρύο

[15, 22] : δροσιά

[21, 27] : ζέστη

[25, 34] : ~~καύ~~ πολλή ζέστη

[33, 45] : καύσωνας.