Θέμα 2º

a) $w_i^{t+1} = w_i^t + \eta(t)[x - w_i^t]$

$\Rightarrow w_i^{t+1} - x = w_i^t + \eta(t)[x - w_i^t] - x \Rightarrow$

$\Rightarrow w_i^{t+1} - x = w_i^t + \eta(t)x - \eta(t)w_i^t - x \Rightarrow$

$\Rightarrow w_i^{t+1} - x = (1 - \eta(t))(w_i^t - x)$

$\Rightarrow |w_i^{t+1} - x| = |1 - \eta(t)| \cdot |w_i^t - x|$

Ομοίως, $|w_j^{t+1} - x| = |1 - \eta(t)| \cdot |w_j^t - x|$ $\quad \div \quad \Rightarrow$

~~Προφανώς~~

$$\Rightarrow \frac{|w_i^{t+1} - x|}{|w_j^{t+1} - x|} = \frac{|w_i^t - x|}{|w_j^t - x|}$$

Άρα αν $|w_i^t - x| < |w_j^t - x|$, θα ισχύει ς $|w_i^{t+1} - x| < |w_j^{t+1} - x|$

b) $w_1 = [7, 2]$

$w_2 = [1, 2]$

$w_3 = [-3, 0]$

$x = [4, 4]$

$a = 0.5$

(i) $S_1 = x \cdot w_1 = 36$ ← max, νικητής

$S_2 = x \cdot w_2 = 9$

$S_3 = x \cdot w_3 = -12$

$w_1(t+1) = w_1(t) + a(x - w_1(t)) =$

$= [5.5, 3]$

$w_2(t+1) = w_2(t) + a(x - w_2(t)) = [2.5, 3]$

$w_3(t+1) = w_3(t) + a(x - w_3(t)) = [0.5, 2]$

(ii) $d_1 = \|x-w_1\|^2 = 13$

$d_2 = \|x-w_2\|^2 = 13$ $\Big\}$ νικητές (min)

$d_3 = \|x-w_3\|^2 = 65$

Ομοίως για τα βάρη. Δε μπορούμε να επιλέξουμε νικητή-νευρώνα, κατάσταση "αναποφάσιστου".

) $W_{kj} = \dfrac{1}{N}\sum_m \xi_k^m \xi_j^\mu \quad \Big\} \quad \left|\sum_{j\neq k}\sum_{m\neq p} \xi_k^\mu \xi_j^m \xi_j^p\right| < M+N-1.$

$$y_k = f(u_k) = f\left(\sum_j w_{kj}\xi_j^p\right) = f\left(\frac{1}{N}\sum_j\sum_m \xi_k^\mu \xi_j^\mu \xi_j^p\right) =$$

$$= f\left(\frac{1}{N}\sum_m \xi_k^\mu \sum_j \xi_j^\mu \xi_j^p\right) =$$

$$= f\left(\frac{1}{N}\sum_m \xi_k^\mu \xi_k^\mu \xi_k^p + \frac{1}{N}\sum_m \xi_k^\mu \sum_{j\neq k}\xi_j^\mu \xi_j^p\right) =$$

$$= f\left(\frac{1}{N}\xi_k^p \cdot M + \frac{1}{N}\xi_k^p \sum_{j\neq k}\xi_j^p \xi_j^p + \frac{1}{N}\sum_{m\neq p}\xi_k^\mu \sum_{j\neq k}\xi_j^\mu \xi_j^p\right)$$

$$= f\left(\frac{1}{N}\xi_k^p \cdot M + \frac{1}{N}\xi_k^p (N-1) + \frac{1}{N}\sum_{j\neq k}\sum_{m\neq p}\xi_k^\mu \xi_j^\mu \xi_j^p\right)$$

$$= f\left(\frac{1}{N}\xi_k^p \left[M+N-1\right] + \frac{1}{N}\sum_{j\neq k}\sum_{m\neq p}\xi_k^\mu \xi_j^\mu \xi_j^p\right) =$$

$= \xi_k^p$ αν ο όρος $M+N-1$ υπερτερεί του $\left|\sum_{j\neq k}\sum_{m\neq p}\xi_k^\mu \xi_j^\mu \xi_j^p\right|$, δηλαδή αν $\left|\sum_{j\neq k}\sum_{\mu\neq p}\xi_k^\mu \xi_j^\mu \xi_j^p\right| < M+N-1$ ✓