PORTFOLIO CONSTRUCTION

FOR DIFFERENT COMMUNITIES

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Introduction & Motivation

- **Objective:**
- Help different types of people (e.g. students, working professionals, seniors, etc.) find optimal portfolios that fit them the best.
- Examine different investment strategies to create best plans.

- **Procedures**:
- Step 1: Specify expected level of risk and return in terms of financial background.
- Step 2: Assets - 50 Stocks from S&P 500 and 1 T-bills
- Step 3: Asset Allocation using Mean Variance Analysis and Risk Parity
- Step 4: Best Mix of Risky & Risk-free Assets
- Step 5: Evaluate the Portfolio (Risk Management)

Dataset

- **High Risk Asset**

  S&P 500 Index

  (4/23/2013 -- 4/20/2018 daily adjusted close price, from *Yahoo finance*)

  why S&P 500?

  -one of the most commonly followed equity indices. Many consider it one of the best representations of the U.S. stock market, and a bellwether for the U.S. economy.

- **Low Risk Asset**

  T-bill

  -balance a portfolio and negate stock market volatility

  -no credit risk & short maturities (no interest risk)

Stock Selection

Stock selection:

Select 50 stocks with highest Sharpe Ratios out of 500 companies.

$$\text{Sharpe ratio} = \frac{E(R) - \mu_f}{\sigma_R}$$

Also, we delete those stocks which have very high P/E (price-earnings) ratio (larger than 100).

ASSET ALLOCATION Methods

Mean Variance Analysis

- Assembling a portfolio of assets such that the expected return is maximized for a given level of risk.

- Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return.

- Under the model:

-- Portfolio return is the proportion-weighted combination of the constituent assets' returns.

-- Portfolio volatility is a function of the correlations ρ_{ij} of the component assets, for all asset pairs (i, j).

Risk Parity

- Focuses on allocation of risk, usually defined as volatility, rather than allocation of capital.

- It is similar to creating a minimum-variance portfolio subject to the constraint that each asset contributes equally to the portfolio overall volatility

- Equally-weighted risk contributions is not about "having the same volatility", it is about having each asset contributing in the same way to the portfolio overall volatility.

- Interest in the risk parity approach has increased since the late 2000s financial crisis as the risk parity approach fared better than traditionally constructed portfolios, as well as many hedge funds.

PROS and CONS

Mean Variance Analysis

PROS:

- Identifies portfolio with highest return at each risk level.
- Does not rely on negative weights.
- Computation costs are cheap and programs are readily available.

CONS:

- Number and nature of estimates can be overwhelming as number of asset classes increase
- Can result in undiversified portfolios
- Very sensitive to inputs

Risk Parity

PROS:

- Risk parity solutions are not sensitive to expected return, but Pays attention to risk
- More diversified portfolios
- Lower drawdowns

CONS:

- Weights sensitive to risk estimates
- Ignores returns
- Sample-dependent performance

Mathematical Model

Mean Variance Analysis

Expected Return:

$$\mathrm{E}(R_p) = \sum_i w_i \, \mathrm{E}(R_i)$$

Portfolio Return Variance:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$$

We created 200 portfolios from there and selected the one with the highest Sharpe ratios.

Risk Parity

Convex Optimization Problem

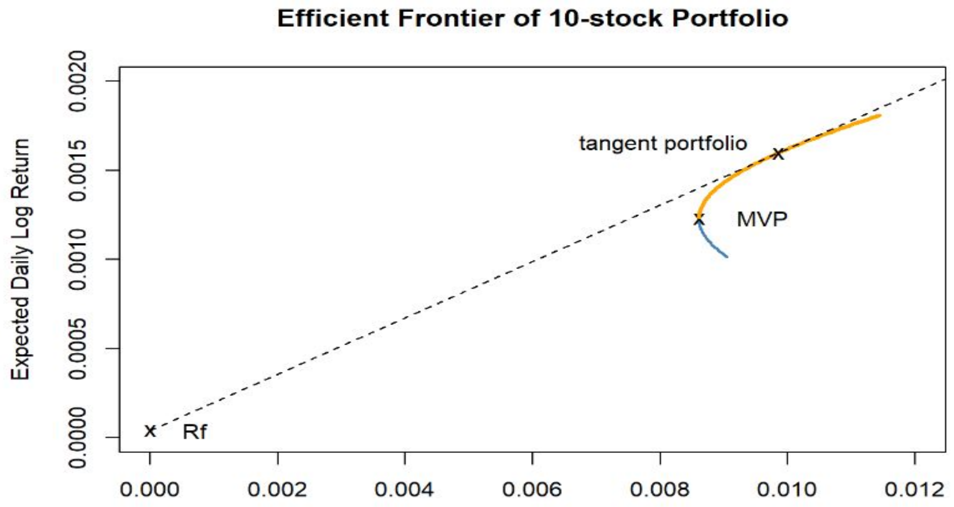
Marginal Risk Contribution of asset i:

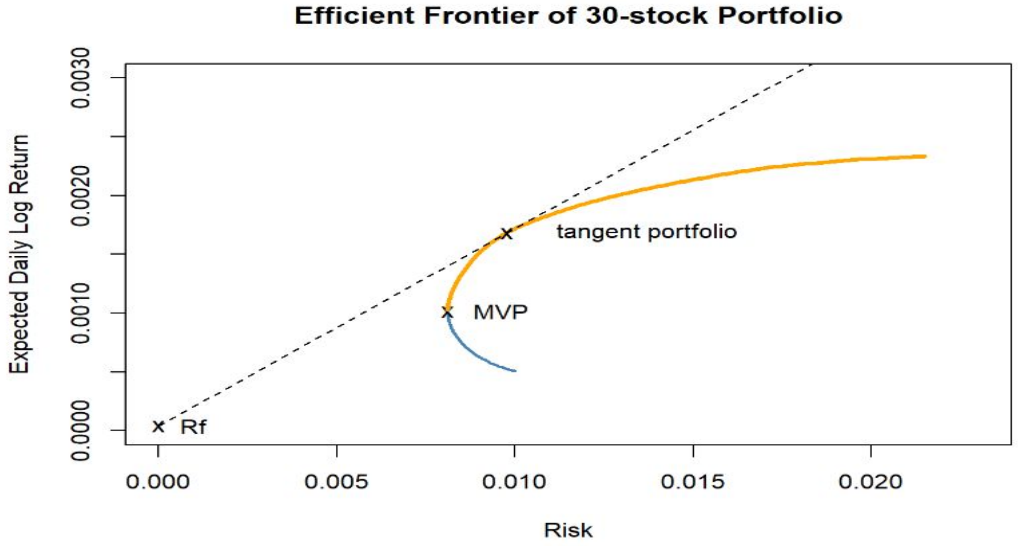
$$\sigma_i(w) = w_i \times \partial_{w_i} \sigma(w) = \frac{w_i (\Sigma w)_i}{\sqrt{w' \Sigma w}}$$

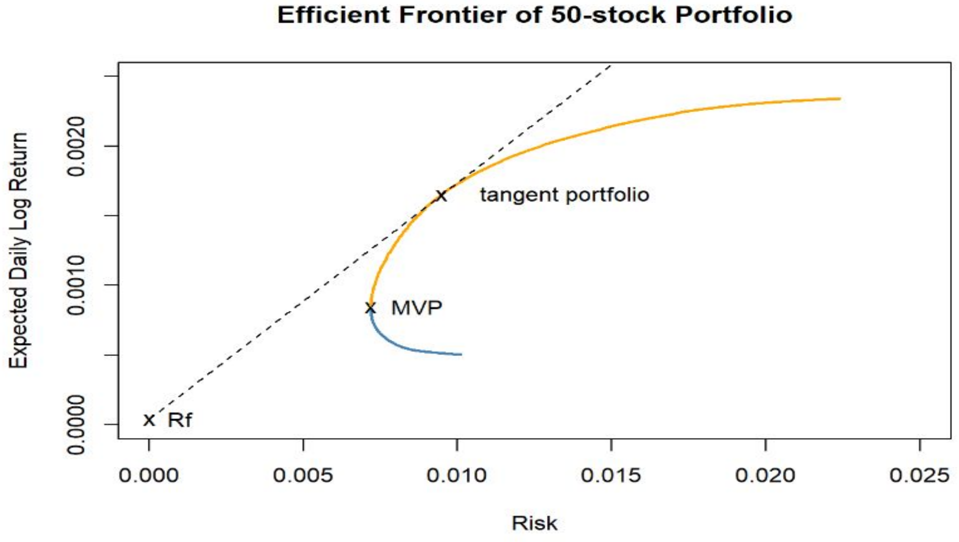
The weights can be found by solving optimization problem:

$$\arg\min_w \sum_{i=1}^{N} \left[w_i - \frac{\sigma(w)^2}{(\Sigma w)_i N} \right]^2$$

Influence of Portfolio's Size Mean-Variance







	Expected Return	Volatility
10 stocks	0.426	0.156
30 stocks	0.424	0.155
50 stocks	0.415	0.150

Influence of Portfolio's Size Risk Parity

Under both methods:

- the portfolio with 10 stocks has the maximized annual expected return;
- the portfolio with 50 stocks has the minimized risk.

why 10-stock portfolio has max expected return?

High correlation in stocks' upstanding industries: 3 military industrial enterprises, 3 technology companies, and 4 are from manufacturing industry.

why 50-stock portfolio has min risk?

Diversification decreases the risk.

	Expected Return	Volatility
10 stocks	0.3429	0.1459
30 stocks	0.3283	0.1483
50 stocks	0.2955	0.1386

Risky Portfolio with Maximized Sharpe Ratio

	10 stocks	30 stocks	50 stocks
Mean-Variance	2.7236	2.7344	2.7642
Risk Parity	2.2816	2.1454	2.0586

Under the 2 methods and 6 portfolios, we choose the one with the largest sharpe ratio, which is the portfolio with 50 stocks obtained from mean-variance analysis.

Assessment of the Portfolio

- **Sharpe Ratio**

- Symbolizes how well the return of an asset compensates the investor for the risk taken.
- A ratio higher than 2 is rated as very good.

- **Treynor Ratio**

$$Treynor\ Ratio = \frac{Rp - Rf}{\beta p}$$

- Rp = Expected Portfolio Return
- Rf – Risk Free Rate
- Beta(p) = Portfolio Beta

- Uses a portfolio's "beta" as its risk. Beta measures the volatility of an investment relative to the stock market.
- When the value of the Treynor ratio is high, it is a sign that an investor has generated high returns on each of the market risks he has assumed.

Assessment of the Portfolio

- **Jensen's Alpha**

$$Jensen's\ \alpha = Rp - Rf - \beta p(Rm - Rf)$$

Rp = Expected Portfolio Return

Rf – Risk Free Rate

Beta(p) = Portfolio Beta

Rm = Market Return

- Used to determine the abnormal return of a security or portfolio of securities over the theoretical expected return.

- Alpha<0: the investment has earned too little for its risk.
- Alpha=0: the investment has earned a return adequate for the risk taken.
- Alpha>0: the investment has a return in excess of the reward for the assumed risk.

Performance of the Selected Portfolio

Besides Sharpe Ratio, we assessed the selected portfolio by Treynor Ratio and Jensen's Alpha.

Risk ratio	Value
Sharpe Ratio	2.764172
Treynor Ratio	0.3519954
Jensen's Alpha	0.2452098

Sharpe Ratio: higher than 2 is rated as very good.

Treynor Ratio>0: the portfolio has adequately compensated its investors for the risk it has subjected them to.

Jensen's Alpha>0: the investment has a return in excess of the reward for the assumed risk.

→ According to the three criterions, the performance of the selected portfolio is good.

Mix of the Stock portfolio & Treasury bill (Risk-free Asset)

Find best mix of risky and risk-free asset for an investor with a given risk aversion (A)

$$w^* = \frac{E(r_1) - r_f}{A \times \sigma_1^2}$$

E(r1) : the expected return of stock portfolio

rf : risk-free rate (expected return of T-bill)

A: risk aversion,which is the behavior of consumers and investors, when exposed to uncertainty, in attempting to lower that uncertainty

variance 1 : variance of stock portfolio

The Choices of A (risk aversion coefficient)

Investors differ in the amount of risk they are willing to take for a given return.

Investors who are risk averse require a greater return for a given amount of risk than a risk lover.

Increase of risk aversion of the investor → Decrease of weight on the risky asset (w*) of the portfolio

Compare The Result

A	w*	E(Rp)	sigma(Rp)
5.00	3.588045	1.465143	0.5394521
17.50	1.025156	0.42583	0.15413
25.46	0.7046435	0.296	0.105
40.02	0.4485056	0.19198	0.06743
62.26	0.2881501	0.1269	0.0433

How does the weight on the risky asset (w*) change as:

1. Expected return of the risky asset? Increases
2. Volatility of the risky asset? Decreases
3. Risk aversion of the investor? Decreases

A for Aggressive Investors

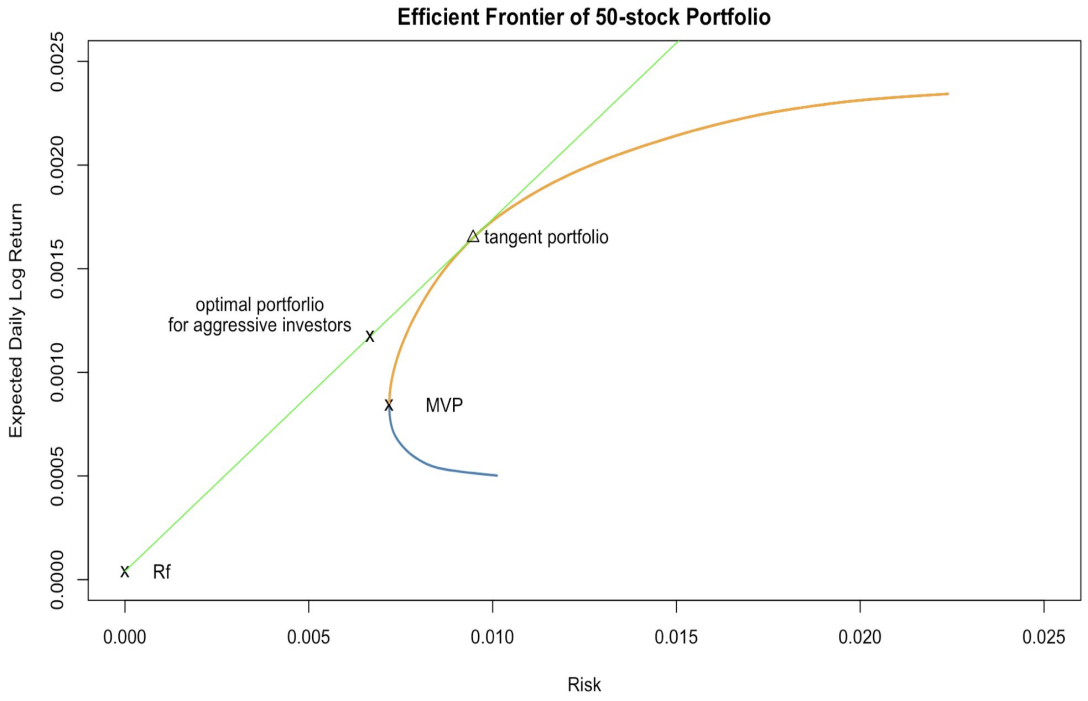
A= 25.46

$$w^* = \frac{(0.4156253 - 0.0101)}{25.46 \times 0.1503471^2}$$

$$w^* = 0.7046435$$

$$E[r_p] = r_f + w^* \times E[r_1 - r_f] = 0.0101 + 0.704 \times (0.4156 - 0.0101) = 0.296$$

$$\sigma_p = |w^*| \times \sigma_1 = 0.105$$



The result indicates that we invest **70.4%** in risky asset and **29.6%** in T-bills.

A for Neutral Investors

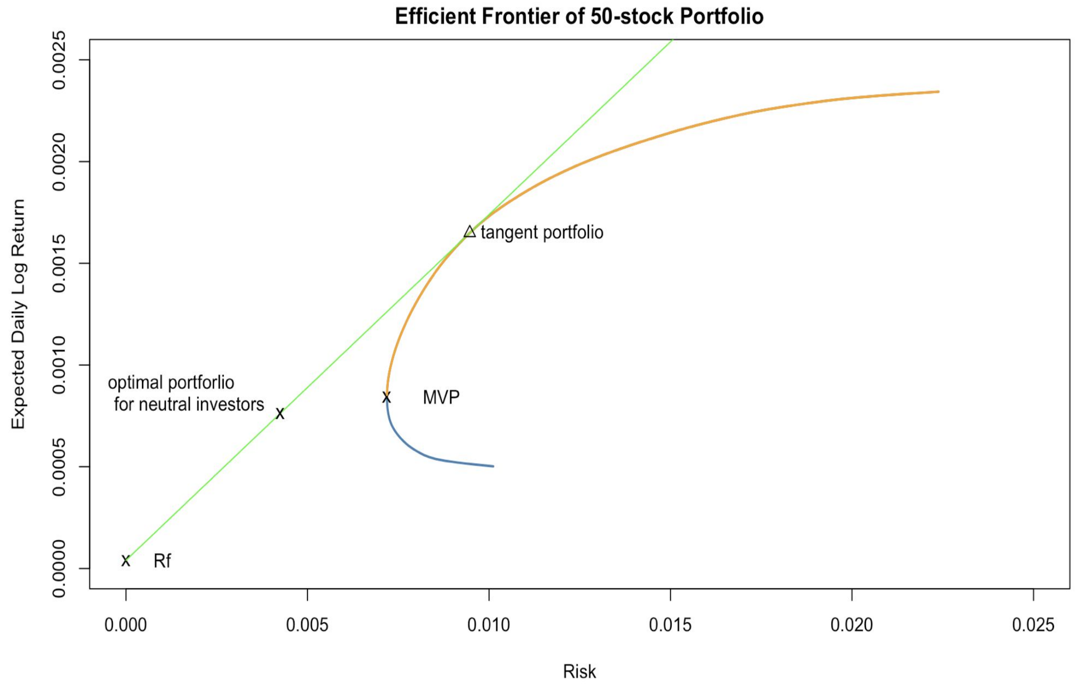
A= 40.02

$$w^* = \frac{(0.4156253 - 0.0101)}{40.02 \times 0.1503471^2}$$

$$w^* = 0.4485056$$

$$\mathrm{E}[r_p] = r_f + w^* \times \mathrm{E}[r_1 - r_f] = 0.0101 + 0.449 \times (0.4156 - 0.0101) = 0.19198$$

$$\sigma_p = |w^*| \times \sigma_1 = 0.06743$$



The result indicates that we invest **44.9%** in risky asset and **54.1%** in T-bills.

A for Conservative Investors

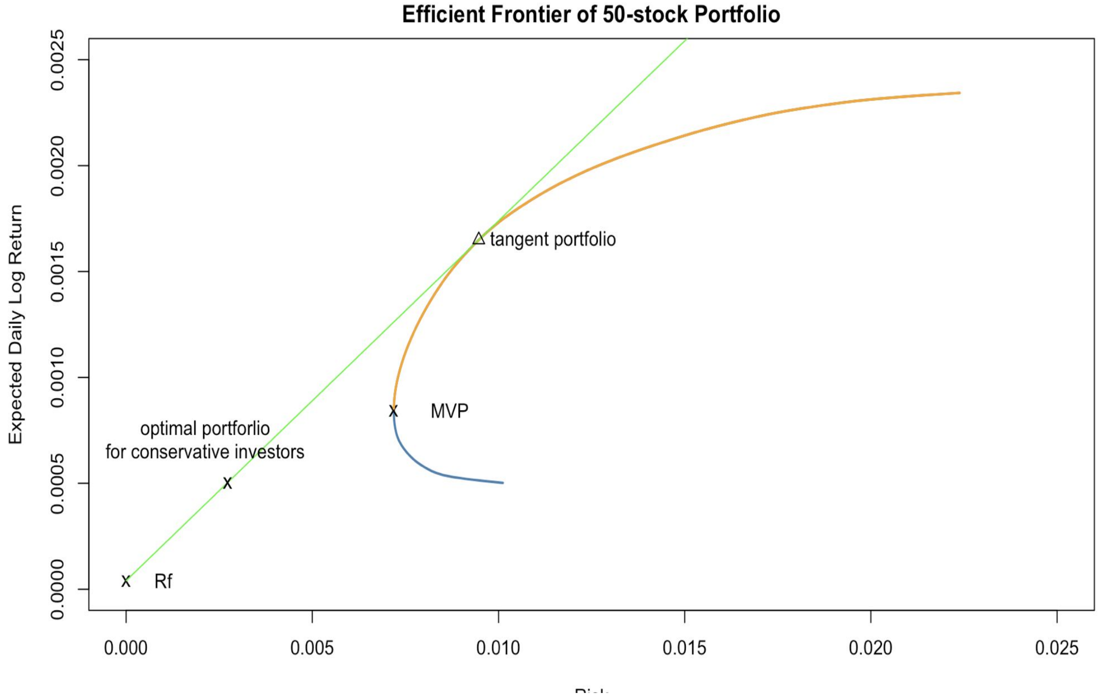
A= 62.26

$$w^* = \frac{(0.4156253 - 0.0101)}{62.26 \times 0.1503471^2}$$

$$w^* = 0.2881501$$

$$E[r_p] = r_f + w^* \times E[r_1 - r_f] = 0.0101 + 0.288 \times (0.4156 - 0.0101) = 0.1269$$

$$\sigma_p = |w^*| \times \sigma_1 = 0.0433$$



The result indicates that we invest **29%** in risky asset and **71%** in T-bills.

Conclusion

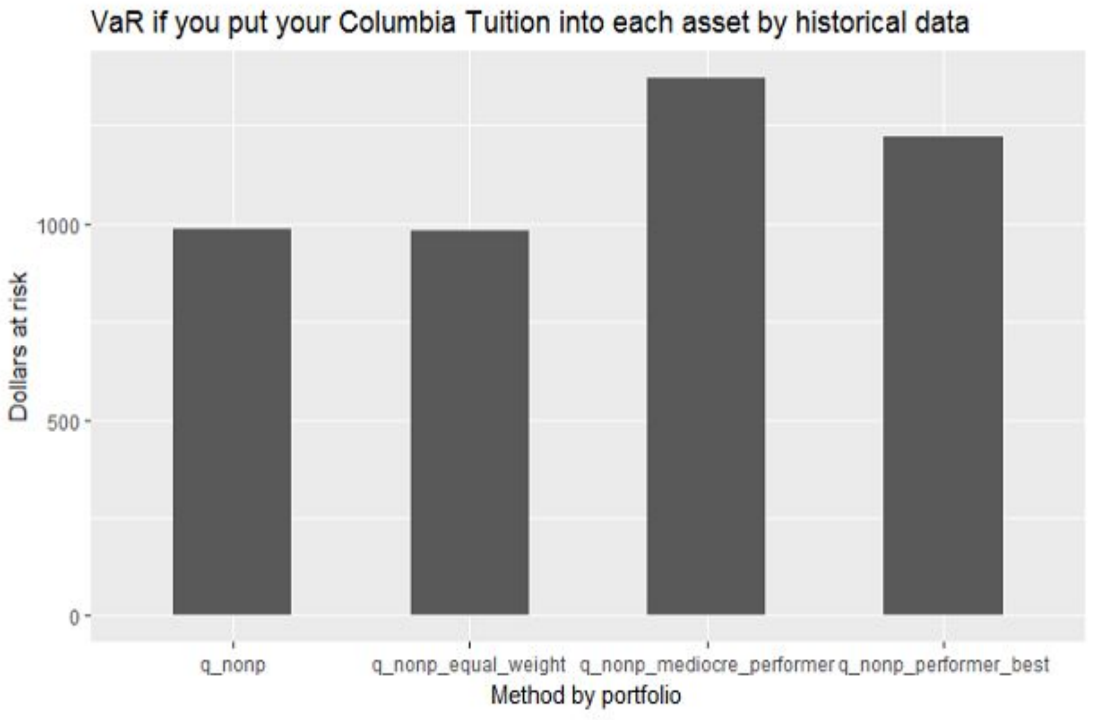
	A	w*	E(Rp)	sigma(Rp)
Aggressive investors	25.46	0.7046435	0.296	0.105
Neutral investors	40.02	0.4485056	0.19198	0.06743
Conservative investors	62.26	0.2881501	0.1269	0.0433

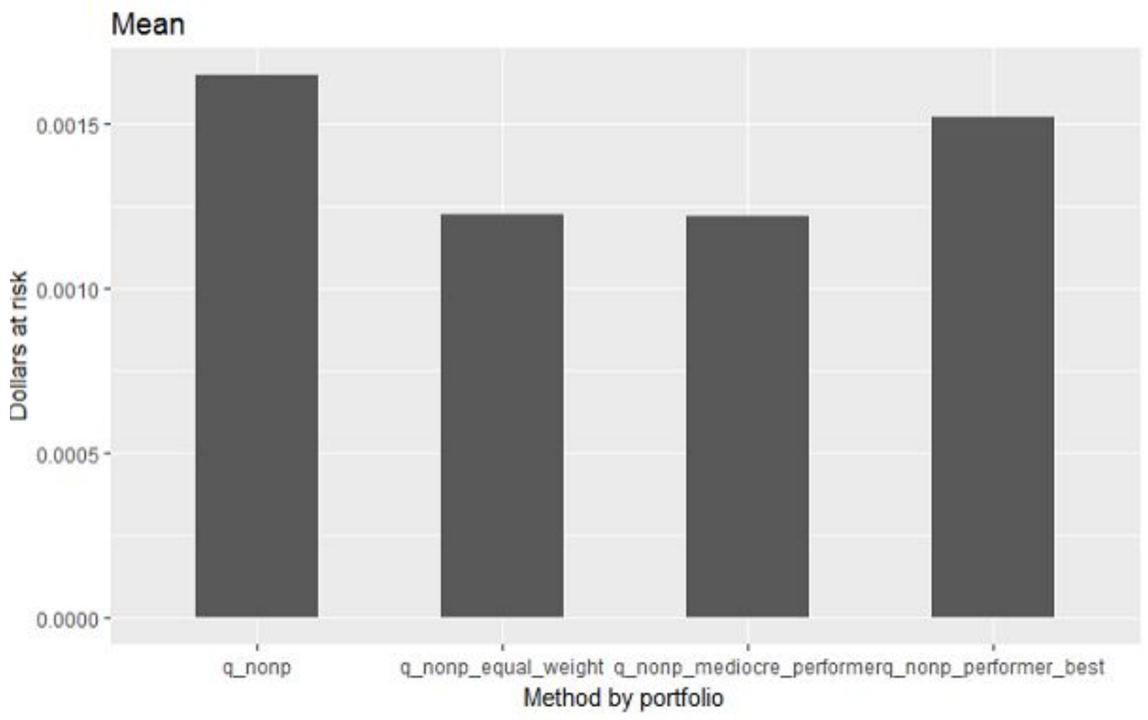
RISK MANAGEMENT

- **Why risk management matters?**
  Risk is the main cause of uncertainty in any organization. Thus, companies increasingly focus more on identifying risks and managing them before they even affect the business. The ability to manage risk will help companies act more confidently on future business decisions.

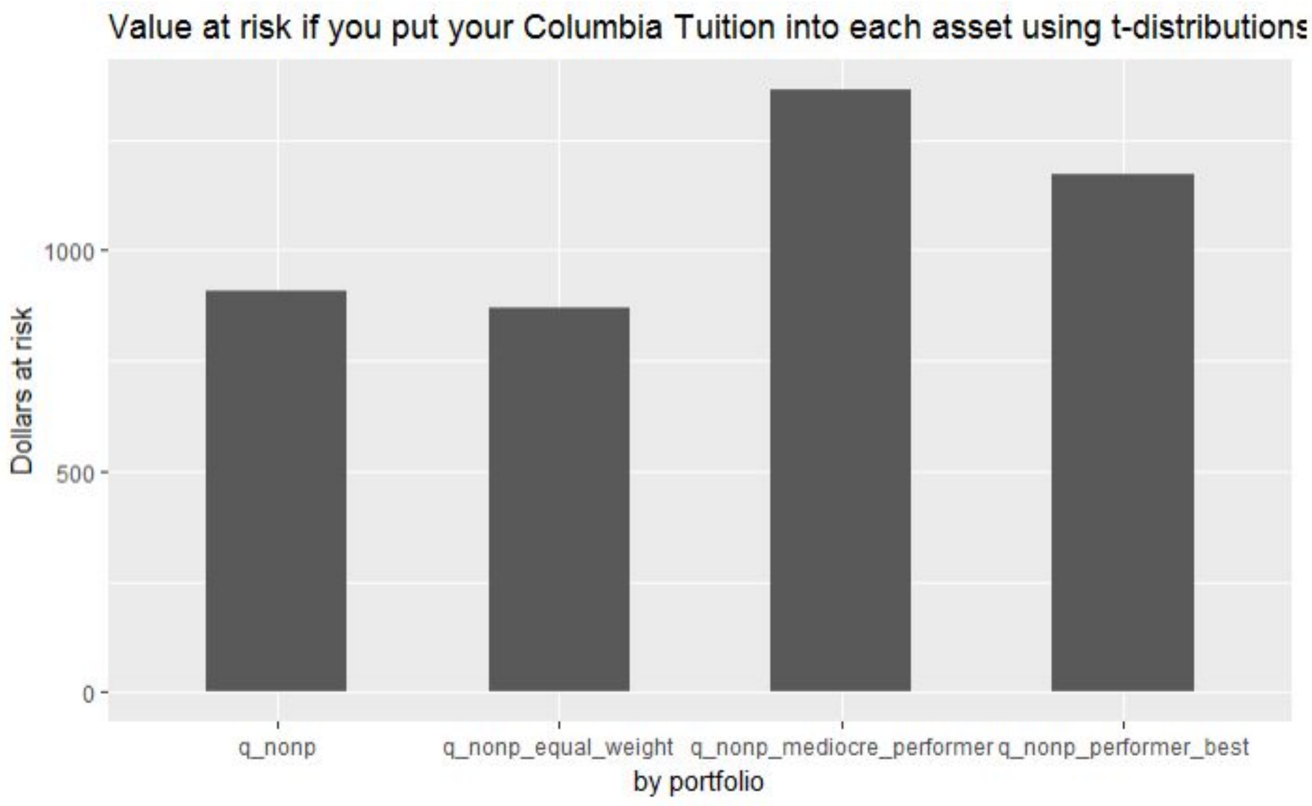
- **Method we use to calculate Value at Risk**
  Three way of calculating VaR:
    - **Empirical method**
    - **T-distribution**
    - **Time Series**

Empirical Method

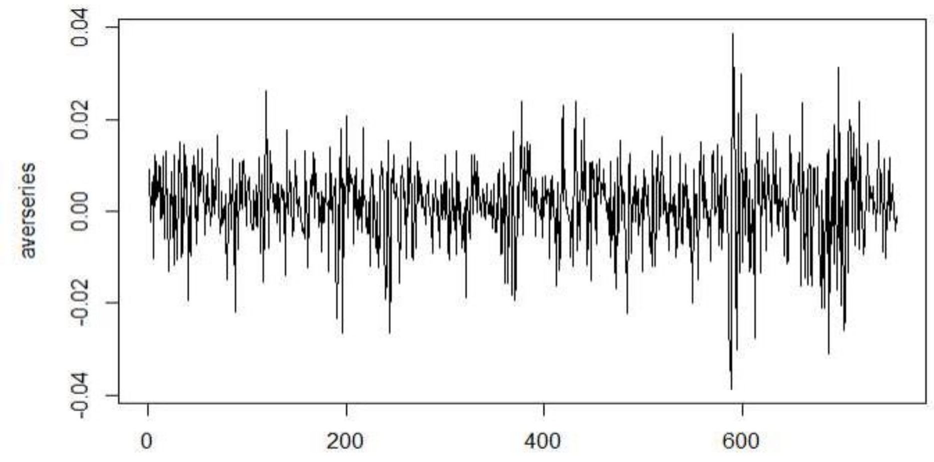


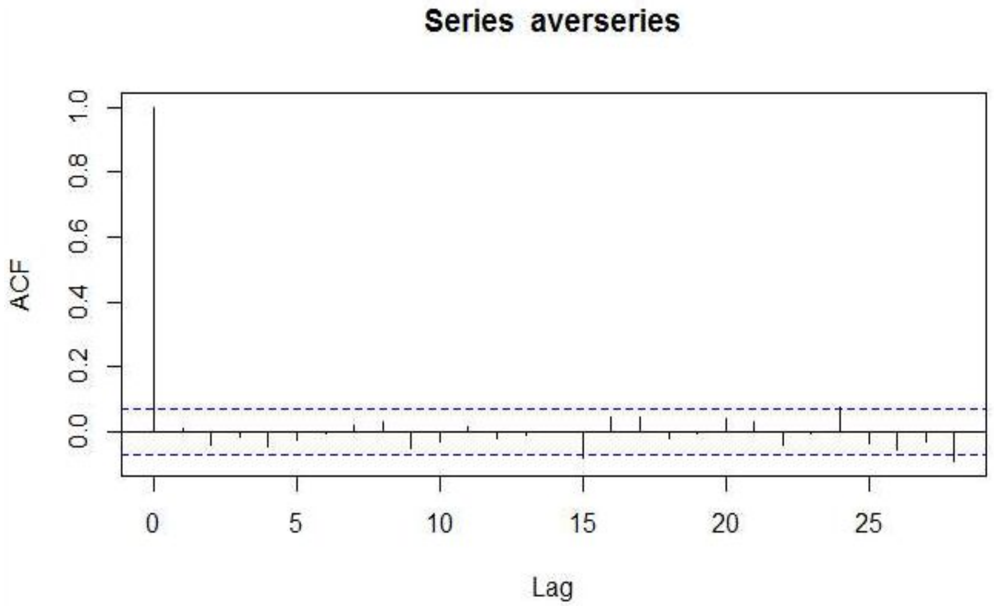


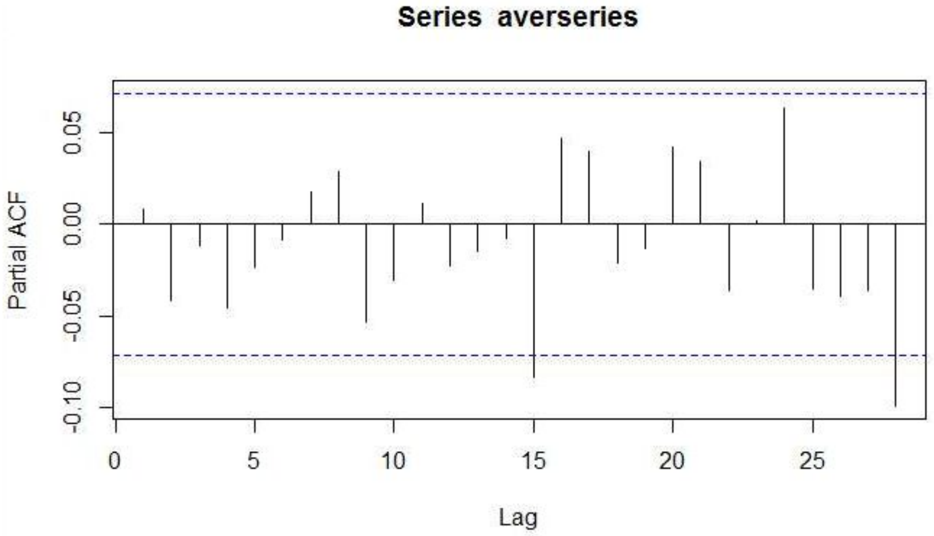
T Distribution Method



Time-series Analysis Method





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Series: portfolio
ARIMA(0,0,0) with non-zero mean

Coefficients:
        mean
      0.0016
s.e.  0.0003

sigma^2 estimated as 8.97e-05:  log likelihood=2450.38
AIC=-4896.76   AICc=-4896.75   BIC=-4887.51
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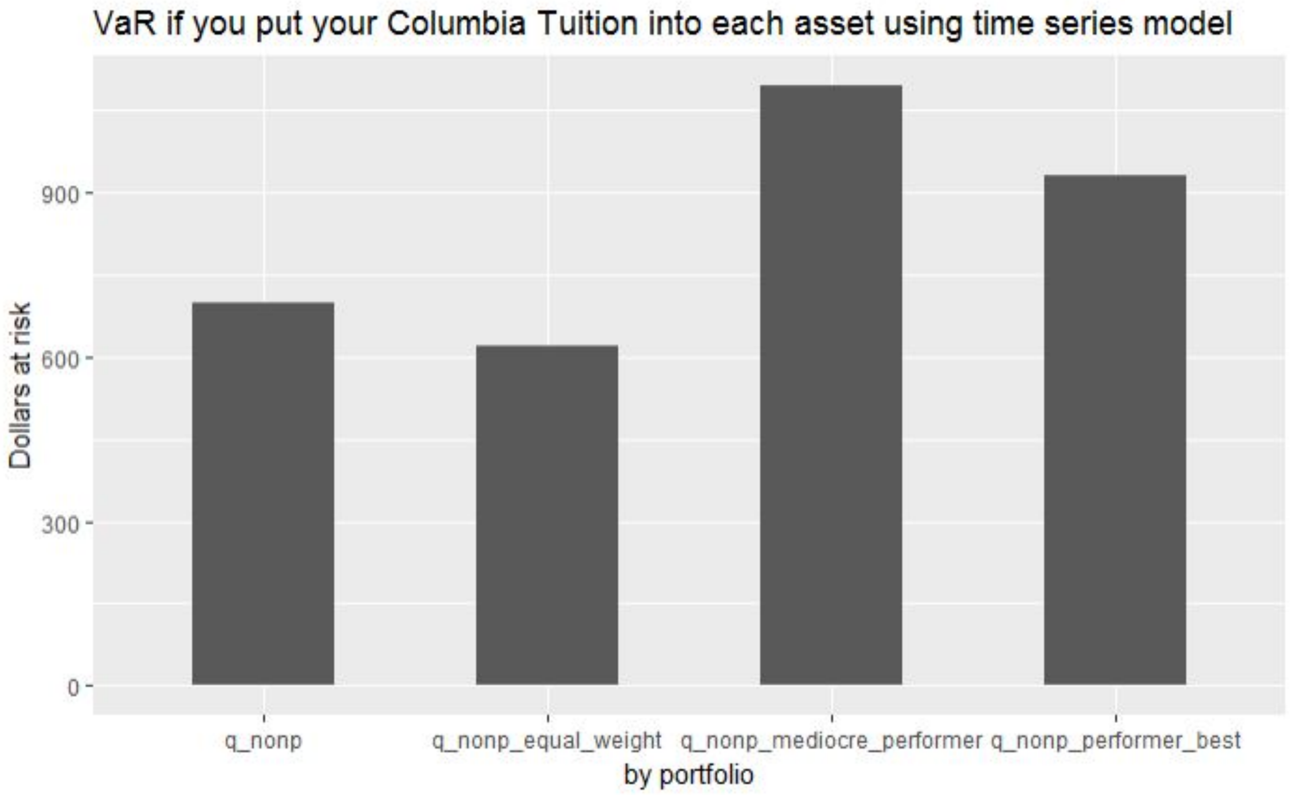
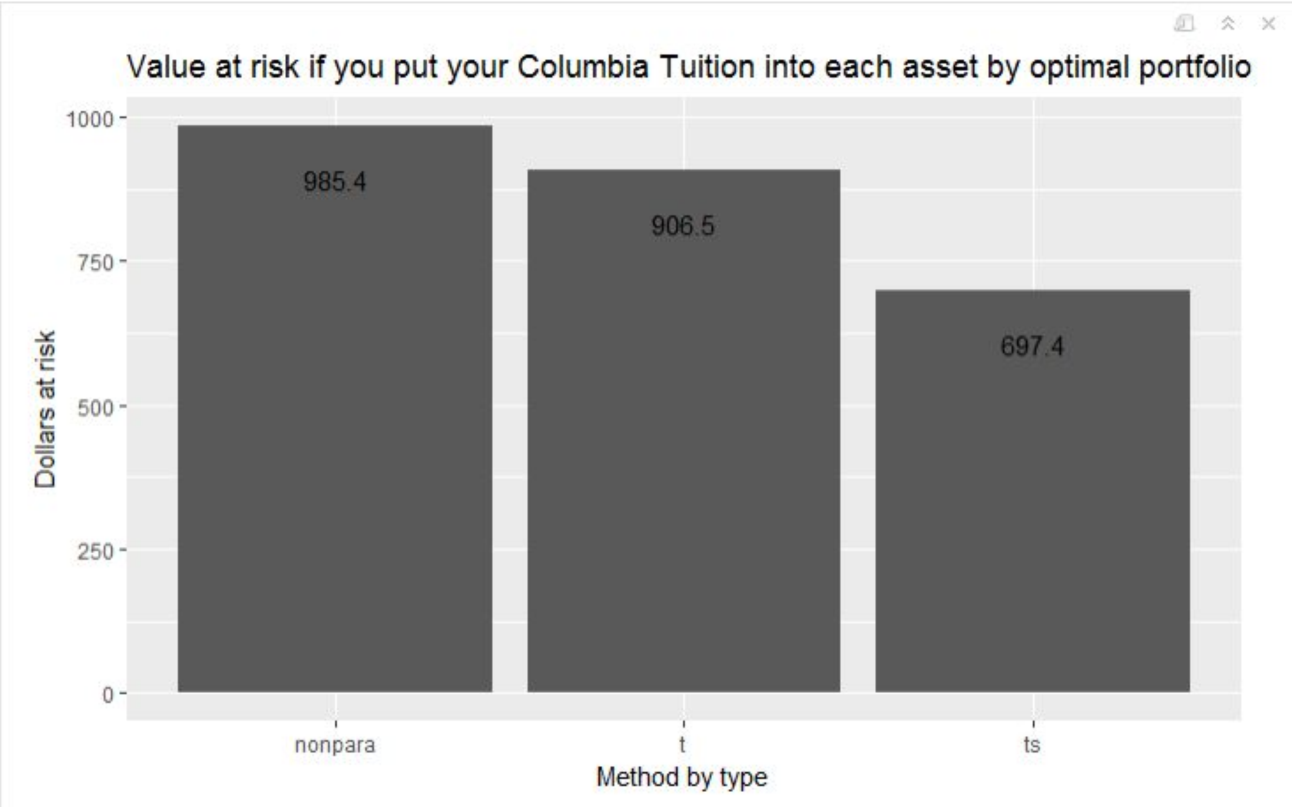
Time Series



VaR if you put your Columbia Tuition into each asset using time series model
Dollars at risk
0
300
600
900
q_nonp
q_nonp_equal_weight
q_nonp_mediocre_performer
q_nonp_performer_best
by portfolio

Conclusion



By comparing three ways of evaluating VaR, we find that time-series analysis gets a better result and the value of VaR of our portfolio is 697.4, which is desirable as we want.

THANKS!