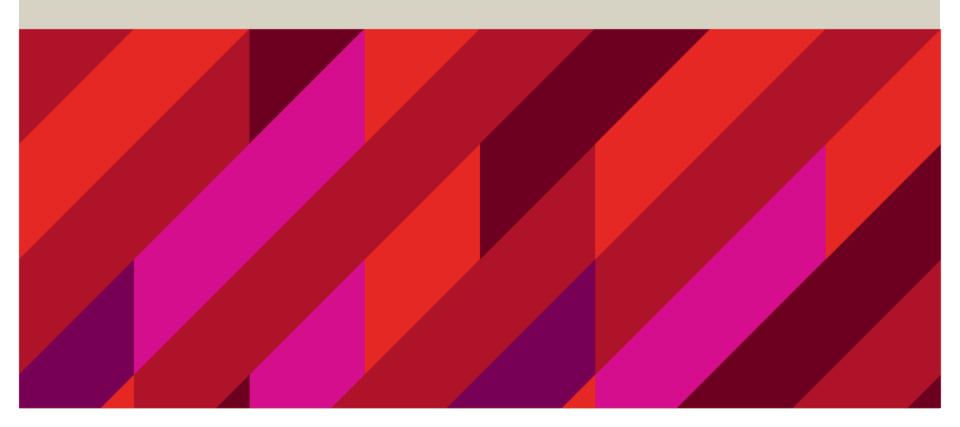


BIOL3110 Conservation & Ecological Genetics

LECTURE 4: CHARACTERIZING GENETIC VARIATION





AS A "BAROMETER" OF POPULATION GENETIC HEALTH

Useful comparisons:

(1) Among (sub)populations



(2) Over time (the same pop/species) as an index of change, i.e.:



$\frac{H_0}{H_T}$

Where:

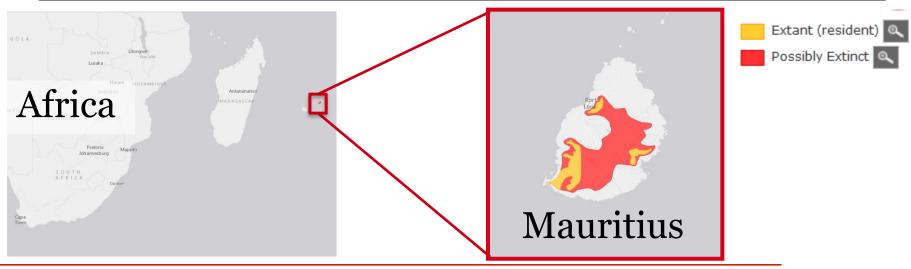
 $H_o = H$ at the time of observation

 $H_T = H$ as known for the past or predicted for the future...





MAURITIUS KESTREL (Falco punctatus)



Home > Falco punctatus (Mauritius Kestrel)



Driven down to 4 wild birds – 1 breeding pair in 1974!!

MAURITIUS KESTREL (Falco punctatus)



Population size:

1974: single mating pair bottleneck N=2

1997: N=500

Calculations using microsatellite markers extracted from museum material:

$$\frac{H_O}{H_T} = \frac{H_{1997}}{H_{1974}} = \frac{0.10}{0.23} = \mathbf{0.43}$$



Loss of more than 50% of *H* since 1974 ...even despite N increasing from 2 to ~500?





H-W equilibrium provides the "null expectation" for allele/genotype frequencies in a pop. Genotypes will accord to HWE unless:

- Inbreeding (>>homozygotes)
- Selection (not exp at neutral loci)
- Non-random mating (e.g. MHC)
- Small populations (Drift)
- Mutation takes a long time!







THE HARDY-WEINBERG (H-W) EQUILIBRIUM

For 2 alleles $(A_1 \text{ and } A_2)$ segregating at a locus, if we have:

- A large, closed population (no migration)
- Random mating (i.e. no selective mate choice)
- Equal fitness of genotypes
- Negligible mutation

Then we expect alleles to assort among genotypes as:

Genotypes:	A_1A_1	A_1A_2	A_2A_2
Frequencies:	p^2	2pq	q^2

Where p = frequency of A_1 and q = frequency of A_2



THE HARDY-WEINBERG (H-W) EQUILIBRIUM

Where $p = \text{frequency of } A_1 \text{ and } q = \text{frequency of } A_2$

Genotypes: A_1A_2 2pq

Frequencies:

For a locus with 2 alleles:

$$H_{exp} = 2pq$$



ALLELE COUNTING

Absolute				
frequencies	A_1A_1	$\mathbf{A_1}\mathbf{A_2}$	A_2A_2	Total
Individuals:	38	60	2	100
Alleles:	2 copies of A ₁	1 copy of A ₁	2 copies of A ₂	200
		1 copy of A ₂		



ALLELE COUNTING

Absolute				
frequencies	A_1A_1	$\mathbf{A_1}\mathbf{A_2}$	A_2A_2	Total
Individuals:	38	60	2	100
Alleles:	2 copies of A ₁	1 copy of A ₁ 1 copy of A ₂	2 copies of A ₂	200
A ₁ :	76	60	0	136
A ₂ :	0	60	4	64



ALLELE COUNTING

Absolute				
frequencies	A_1A_1	$\mathbf{A_1}\mathbf{A_2}$	A_2A_2	Total
Individuals:	38	60	2	100
Alleles:	2 copies of A ₁	1 copy of A ₁ 1 copy of A ₂	2 copies of A ₂	200
A ₁ :	76	60	0	136
A ₂ :	0	60	4	64

Relative frequencies (~%) of each allele:

$$A_1 = p = 136/200 = 0.68$$

 $A_2 = q = 64/200 = 0.32$ (...or simply 1-p)



USE p & q TO CALCULATE H-W EXPECTATION

$$p(A_1) = 0.68, q(A_2) = 0.32$$

	Genotypes			
	A_1A_1	Total		
H-W exp frequencies:	p^2	2pq	q^2	



USE p & q TO CALCULATE H-W EXPECTATION

$$p(A_1) = 0.68, q(A_2) = 0.32$$

	Genotypes				
	A_1A_1	$\begin{array}{c cccc} A_1A_1 & A_1A_2 & A_2A_2 \end{array}$			
H-W exp frequencies:	p^2	2 pq	q^2		
For p=0.68 & q =0.32:	0.682	2 x 0.68 x 0.32	0.32^{2}	1.0	
=	0.462	0.435	0.102	1.0	



USE p & q TO CALCULATE H-W EXPECTATION

$$p(A_1) = 0.68, q(A_2) = 0.32$$

	Genotypes				
	A_1A_1	$\begin{array}{c ccccc} A_1A_1 & A_1A_2 & A_2A_2 \end{array}$			
H-W exp frequencies:	p^2	2 pq	q^2		
For p=0.68 & q =0.32:	0.68^{2}	2 x 0.68 x 0.32	0.32^{2}	1.0	
=	0.462	0.435	0.102	1.0	

Expected genotypes given a sample of N=100		43.5	10.2	100
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USE p & q TO CALCULATE H-W EXPECTATION

$$p(A_1) = 0.68, q(A_2) = 0.32$$

	Genotypes				
	A_1A_1	$\begin{array}{c ccccc} A_1A_1 & A_1A_2 & A_2A_2 \end{array}$			
H-W exp frequencies:	p^2	2 pq	q^2		
For p=0.68 & q =0.32:	0.68^{2}	2 x 0.68 x 0.32	0.32^{2}	1.0	
=	0.462	0.435	0.102	1.0	

Expected genotypes given a sample of N=100	46.2	43.5	10.2	100
What we observed :	38	60	2	100

DO THESE DISTRIBUTIONS DIFFER?



THE "CHI-SQUARED" TEST FOR SIGNIFICANCE

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

Does your observed distribution of genotypes **differ** from the H-W expected distribution?

Calculate using **absolute genotype frequencies** (the numbers that we just calculated)

Use a significance table to look up a **P-value**

CHI-SQUARED" TEST FOR SIGNIFICANCE

~2 —	∇	$(\mathbf{Obs} - \mathbf{Exp})^2$
χ_ —		Exp

	A_1A_1	A_1A_2	A_2A_2	Sum
H-W Exp:	46.2	43.5	10.2	100
Obs:	38	60	2	100



CHI-SQUARED" TEST FOR SIGNIFICANCE

	A_1A_1	$\begin{array}{c c} \mathbf{A_1} \mathbf{A_2} & \mathbf{A_2} \mathbf{A_2} \end{array}$		Sum
H-W Exp:	46.2	43.5	10.2	100
Obs:	38	60	2	100
$\frac{(Obs - Exp)^2}{Exp}$	$\frac{(38 - 46.2)^2}{46.2}$	$\frac{(60 - 43.5)^2}{43.5}$	$\frac{(2-10.2)^2}{10.2}$	

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

CHI-SQUARED" TEST FOR SIGNIFICANCE

$$\chi^2 = \sum \frac{(\boldsymbol{Obs} - \boldsymbol{Exp})^2}{\boldsymbol{Exp}}$$

	A_1A_1	A_1A_2	A_2A_2	Sum
H-W Exp:	46.2	43.5	10.2	100
Obs:	38	60	2	100
$\frac{(Obs - Exp)^2}{Exp}$	$\frac{(38 - 46.2)^2}{46.2}$	$\frac{(60 - 43.5)^2}{43.5}$	$\frac{(2-10.2)^2}{10.2}$	
=	1.49	6.29	6.59	= 14.34

Chi Square statistic = **14.34**, **P** < **0.05** (from a table) Hence, conclude a significant deviation from HW...

CHI-SQUARE TABLE



Critical values of the Chi-square distribution with d degrees of freedom

	Probability of exceeding the critical value						
d	0.05	0.01	0.001	d	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312
9	16.919	21.666	27.877	19	30.144	36.191	43.820
10	18.307	23.209	29.588	20	31.410	37.566	45.315

Degrees of freedom = number of genotypes -1number of alleles -1

Or more simply – number of genotypes number of alleles

$$Df = 1$$





2 ALLELES:

A₁(p) A₂(q)

Genotypes				
A_1A_1	$A_1 A_2 A_2$			
p^2	2 pq	q^2		

3 ALLELES:

A₁(p)
A₂(q)
A₃(r)

	Genotypes					
A_1A_1 A_1A_2 A_1A_3 A_2A_2 A_2A_3 A_3A_3						A_3A_3
	p^2	2 pq	2 pr	q^2	2 qr	r ²





Loci on the **Sex Chromosomes**

Humans, primates, mammals & most insects:



Females XX, males XY

(females double-dose of X-linked alleles)

Birds & butterflies:

Females ZW, males ZZ



This changes the allele counting process...

see Frankham et al. Table 4.6 & Example 4.7



AS A "BAROMETER" OF POPULATION GENETIC HEALTH

Heterozygosity versus allelic diversity...

A: average allelic diversity

- Sensitive to sample size (N)
- Statistical control for unequal N

$$\mathbf{A} = \frac{\sum (N_A)}{N_T}$$

Where:

 N_A = number of different alleles across all loci N_T = total number of loci examined

n_e: effective number of alleles

- Less sensitive to sample size
- Less influenced by rare alleles
- Highest when more alleles present but at equal frequencies

$$n_e = \frac{1}{\sum P_i^2}$$

Where:

 P_i = frequency of each allele



LINKAGE DISEQUILIBRIUM

• Non-random associations between alleles at different loci is called **linkage disequilibrium** (D)

e.g.: 2 alleles at A & B loci :				
A_1B_1	A_1B_2	A_2B_1	A_2B_2	

- Can result from strong selection (e.g. mate preference & trait alleles in sexual selection)
- Can indicate bottlenecks, recent pop mixing, etc.

see Frankham et al. Ch. 4 (Box 4.3 & 4.4, Table 4.9)

Coming up:



Genetics from the phenotypic level

