BIOL334 Conservation & Ecological Genetics

PROBLEM SET #4

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OVERVIEW:

1. Effective population size (N_e)
2. Demography & N_e
   - Fluctuations in population size
   - Variation in family size
   - Departures of sex ratio

Effective Population Size (N_e)

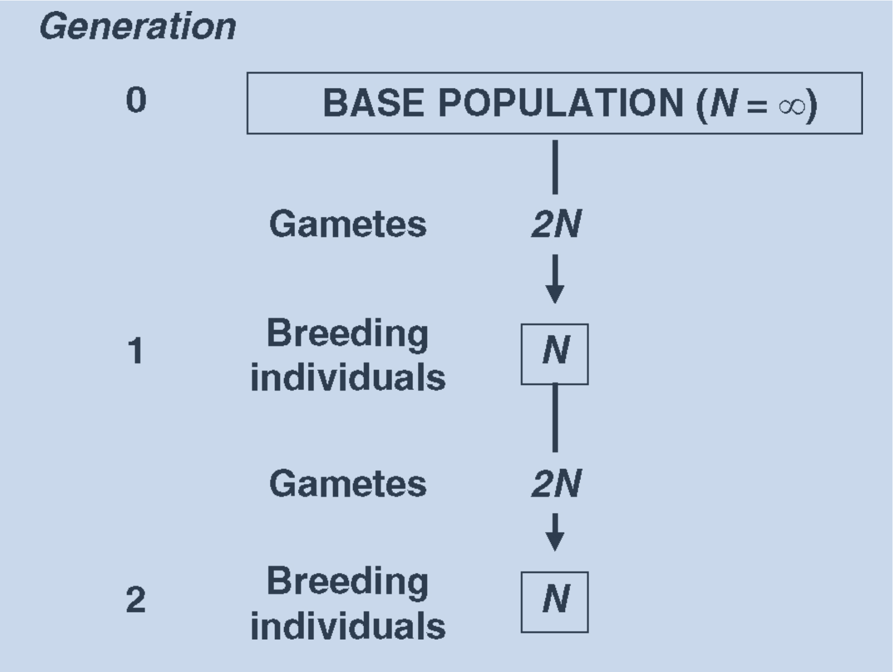
Recall from Lecture 13:

- N_e expresses N as a function of genetic variation
- Is key to assessing genetic robustness
- Often of interest is: N_e / N
- This is generally ~ 0.1 (or 10 %) in the long term

Theoretical concept of N_e

Idealized population as a "benchmark":

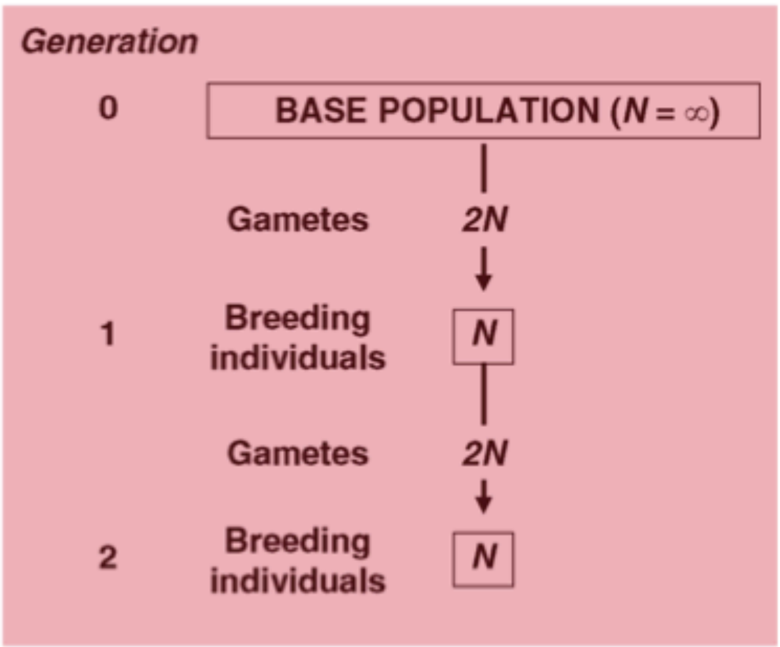
- Constant-size
- Closed population
- Random mating (hermaphrodites)
- Non-overlapping generations
- Negligible selection or mutation



Effective Population Size (N_e)

THE IDEALIZED POPULATION

As N departs from ∞ in the idealized population:



1. Loss of V_G due to drift
2. Increased chance of inbreeding ($F>0$)
3. $H_{obs} < H_{exp}$ (ie. H-W expected H)

N_e defined as the size of an idealized population that would **lose V_G at the rate observed** for the study population

N_e Example:

- For a study population of **N = 500**
- We measure change in V_G over time and find that V_G is lost at the same rate as an **idealized population** of **N = 50**
- Then we define $\mathbf{N_e = 50}$

Effectively (for purposes of V_G) dealing with N of 50 rather than 500

- Generally, $\mathbf{N_e}$ ~ 11 % (0.11) of **N** for many large populations

E.g. Spawning run in Chinook Salmon:
N = 2000 but $\mathbf{N_e}$ = 85 (4%)

Effective Population Size (N_e)

Can be estimated by:

(a) Genetic factors (if/when known)

(b) Demographic factors:

N_e reduced by (in order of significance):

(1) Fluctuations in population size

(2) Variation in family size

(3) Variation in sex ratio

Fluctuations in pop size over time

$$N_e = \frac{t}{\Sigma(\frac{1}{N})}$$

Where:

t = number of censuses (or generations)

N = N at each census

(or ideally, N_e if we knew it)

Fluctuations in population size over time

$N_e = t / \Sigma(1/N_i)$

Example:

t = **3** censuses (or generations)

$N_{(G1)}$ = 175,000 $N_{(G2)}$ = 20 $N_{(G3)}$ = 175,000

Question 4.1

FOR EACH POPULATION BELOW

(a) Calculate mean *N* over the 8 gens

(b) Estimate N_e for each population

$$N_e = t / \Sigma(1/N_i)$$

Generation	N
G1	220
G2	282
G3	121
G4	84
G5	32
G6	105
G7	170
G8 (Present)	**360**

Generation	N
G1	140
G2	120
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G4	120
G5	140
G6	120
G7	111
G8 (Present)	**130**

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Generation	N	1/N
G1	220	0.005
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G5	32	0.031
G6	105	0.010
G7	170	0.006
G8	360	0.003
Sum (Σ):	**1374**	**0.078**

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G1	140	0.007
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G4	120	0.008
G5	140	0.007
G6	120	0.008
G7	111	0.009
G8	130	0.008
Sum (Σ):	**1002**	**0.064**

(a) Calculate mean *N* over the 8 gens:

1374 / 8
= 171.75

1002 / 8
= 125.25

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Sum (Σ):	**1002**	**0.064**

(b) Estimate N_e for each population

8 / 0.078
= 102.56

8 / 0.064
= 125

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FOR EACH POPULATION BELOW

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(b) Estimate N_e for each population

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Sum (Σ):	**1374**	**0.078**
Mean N		**171.75**
N_e		**102.56**

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Sum (Σ):	**1002**	**0.064**
Mean N		**125.25**
N_e		**125**

Variation in family size

- N_e reduces with increasing variation in the sizes of contributing families
- Why?

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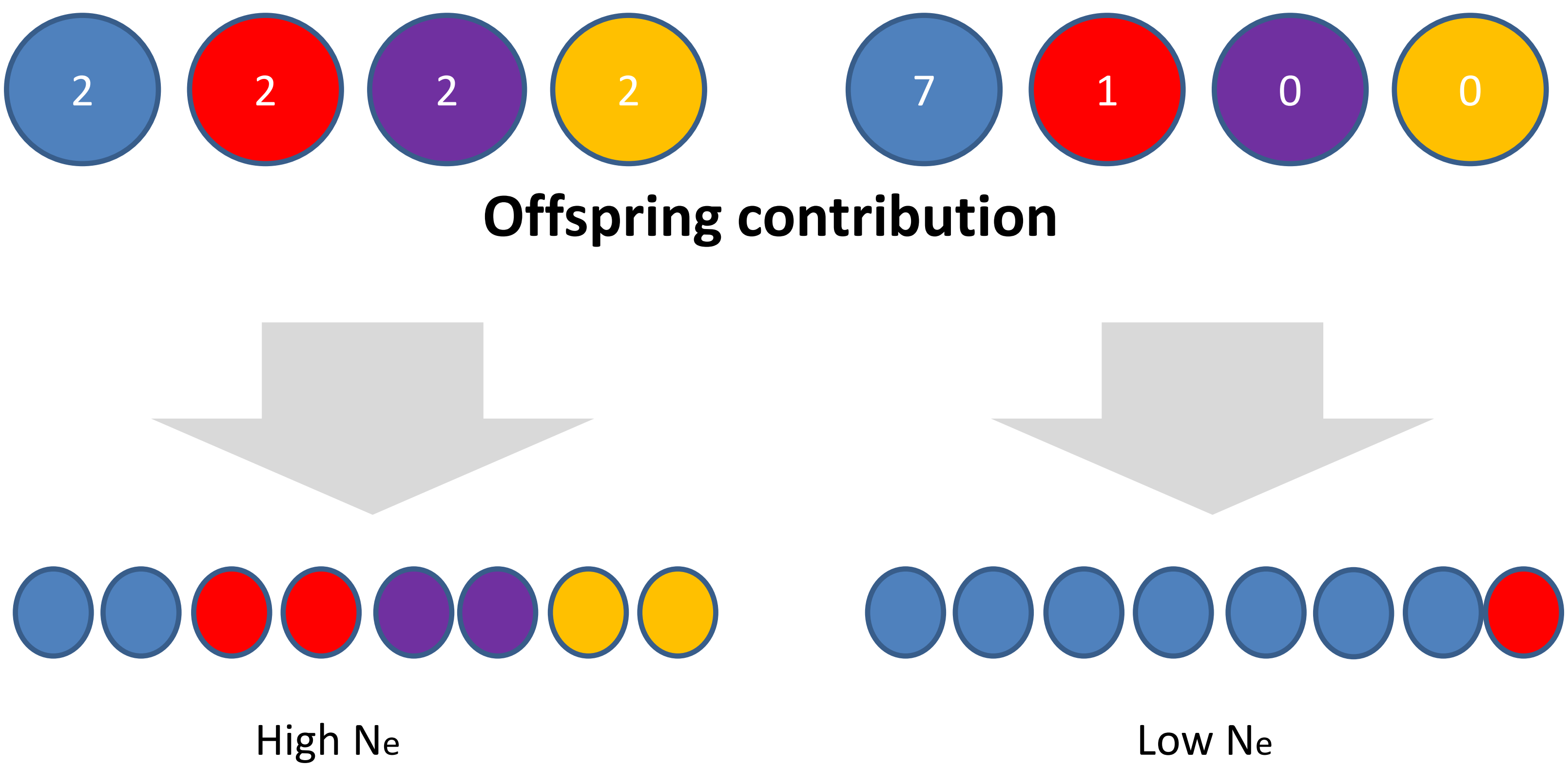


More generally:

$$N_e \sim \frac{4N}{V_k + 2}$$

Where:

N = observed **parental** population size

$\boldsymbol{V_k}$ = variance in family size **(where family size = number of offspring only)**

$\boldsymbol{N_e}$ = Effective population of **parental generation**

Idealized pop has $\boldsymbol{V_k = 2}$

(1 for each male & 1 for each female)

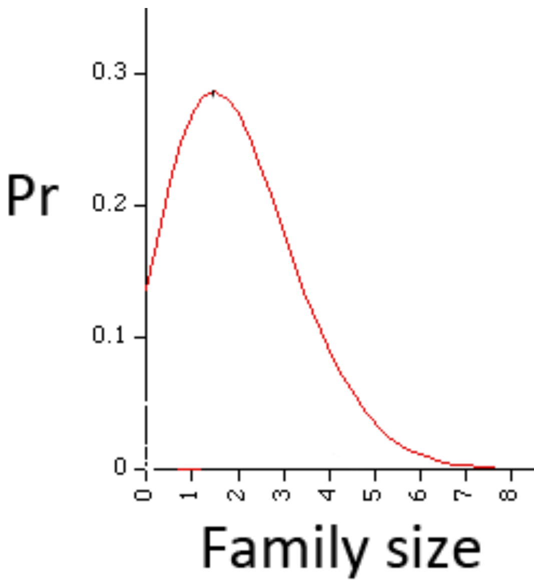
V_k of 2 is expected under the Poisson probability distribution, which describes random mating in a large population:

Idealized expectation for V_k

= what would happen in the "idealized population":

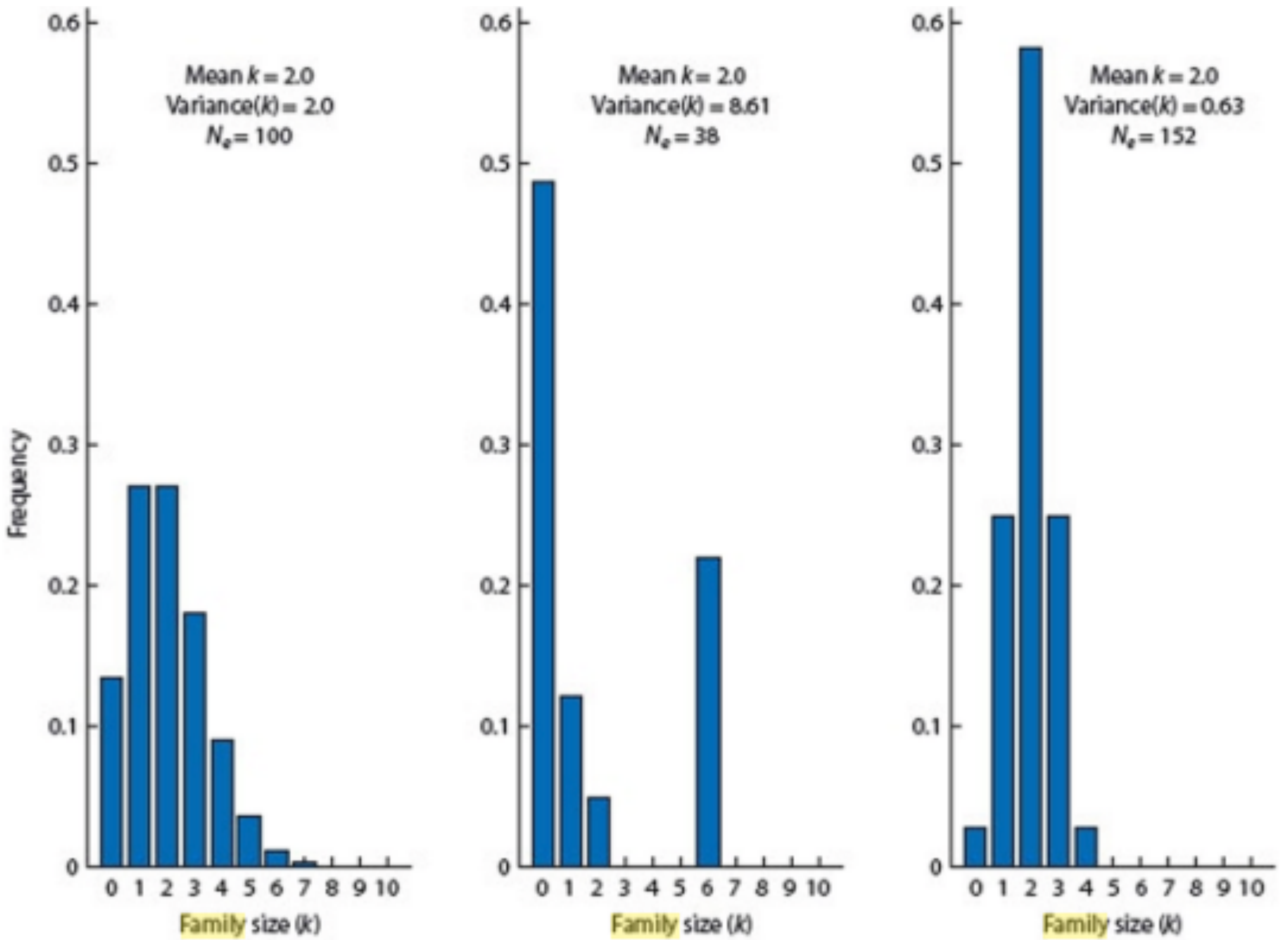
- Predicted by Poisson distribution
- It doesn't mean zero variation!!!

Family size (k)	"Idealized" Probability
0	0.135
1	0.271
2	0.271
3	0.180
4	0.090
5	0.036
> 5	0.017



Idealized expectation for V_k

= what would happen in the "idealized population":



Question 4.2

Find N_e for a pop of N = 360 and where:

(a) $V_k = 2$

(b) $V_k = 3.41$

(c) What if all families are equal size?

$$N_e \sim 4N / (V_k + 2)$$

Where:

N = observed population size

V_k = variance in family size

Question 4.2

$N_e \sim 4N / (V_k + 2)$

Find N_e for a pop of N = 360 and where:

(a) $V_k = 2$

(b) $V_k = 3.41$

(c) What if all families are equal size?

Question 4.2

(a) When Vk = 2, Ne = 4x360/2+2 = 360 (obviously – this is the idealized situation)

(b) When Vk = 3.41, Ne = 4x360/3.41+2 = 1440/5.41 = 266.17

(c) When Vk = 0, Ne = 4x360/2 = 720!!

Question 4.3

Given the following population,

Calculate:

(a) Total **parental** population size (*N*)
(b) Average family size (offspring ONLY)
(c) Family size variance (V_k)
(Use “Var.s” function in excel)

Family	Offspring
1	4
2	5
3	1
4	2
5	1
6	2
7	1
8	3

Then calculate N_e for this population

Question 4.3

Given the following population,

Calculate:

(a) Total population size (*N*)

(b) Average family size

(c) Family size variance (V_k)

Then calculate N_e

Family	Offspring
1	4
2	5
3	1
4	2
5	1
6	2
7	1
8	3

Question 4.3

Pop size is: **16 (Parental generation)**

Average family size is: 2.375

Variance is: 2.2678

$$N_e = (4N) / (V_K + 2)$$

$$= (4 \times 16) / (2.2678 + 2)$$

$$= 64 / 4.42678$$

$$= 14.457$$

Sex ratio & N_e

Point estimate:

$$N_e \sim \frac{4N_f \, x \, N_m}{N_f + N_m}$$

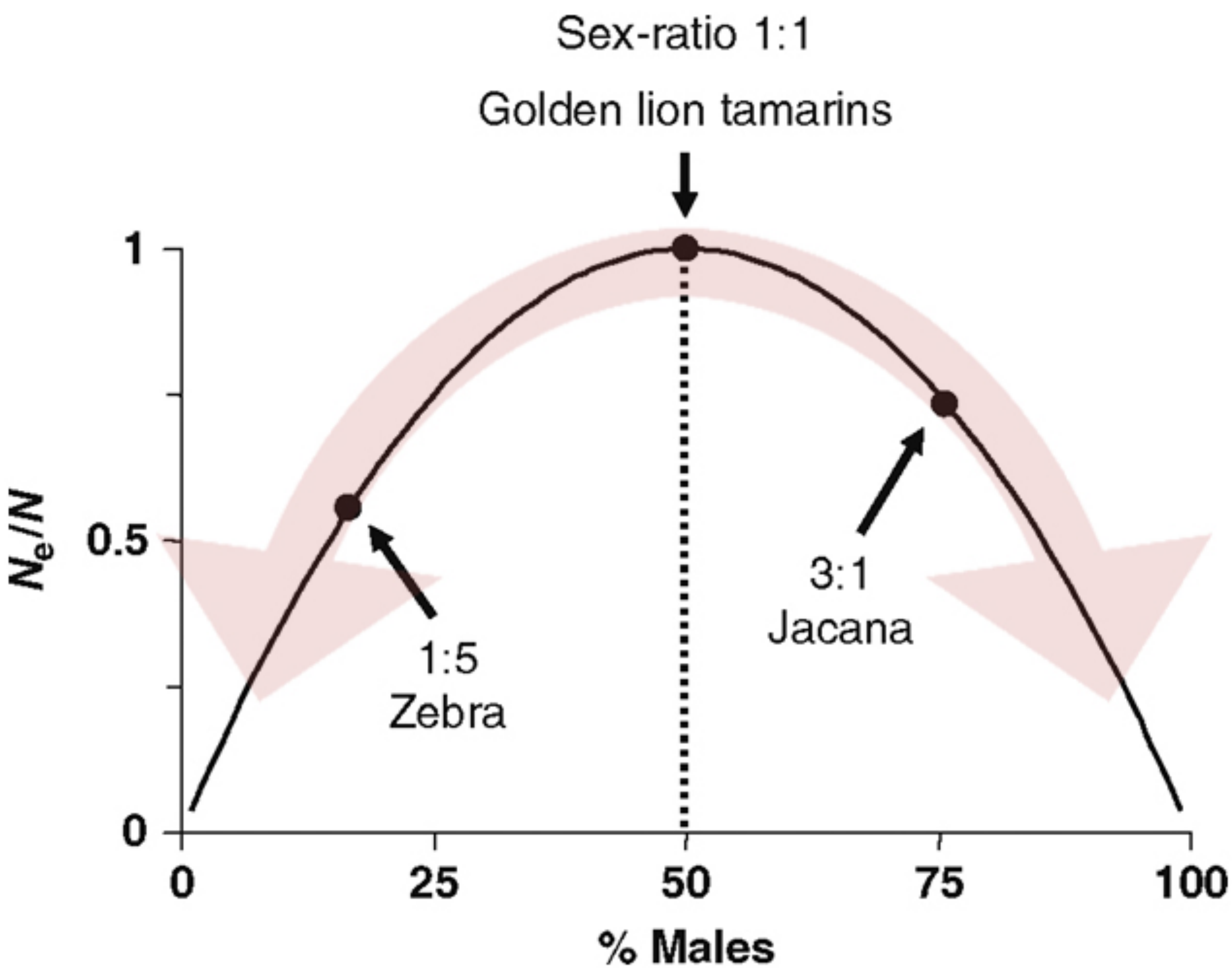
Where:

N_f = number of breeding females

N_m = number of males

As sex-ratio departs from 1:1 (in either direction), N_e declines from N…

Both genders are conduits for (and can leak) V_G!!



Frankham et al.
Fig 11.7

As sex-ratio departs from 1:1 (in either direction), N_e declines from N…

Both genders are conduits for (and can leak) V_G!!

- Rare males may be able to sustain the numbers in a population for longer than rare females (Due to their higher potential reproductive rate).

- However, the effect on the erosion of genetic variation is equivalent.

    - Therefore, the long-term genetic-consequence is equally as bad.

Question 4.4

In elephant-seals, most females breed, but males monopolize 'harems'.

$$N_e \sim \frac{4N_f \, x \, N_m}{N_f + N_m}$$

(a) Find N_e for a population of 360 breeders with $N_f = 300$ & $N_m = 60$

(b) Then, satisfy yourself that N_e would be higher if the sex-ratio of effective breeders was 50:50…

A happy harem-master

$$N_e \sim \frac{4N_f \; x \; N_m}{N_f + N_m}$$

Question 4.4

(a) Find N_e for a population of 360 breeders with N_f = 300 & N_m = 60

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Question 4.4

The calculations:

(a) Ne = (4x300x60)/360 = **200**

(b) Ne = (4x180x180)/360 = **360**

Discuss: Management implications for small & captive populations

N_e proportional to loss of heterozygosity – so, aim is to maximise N_e / N. How to best spread the genetic variance across individuals??

- Equalization of family sizes (EFS)… then $N_e \geq N$
- Equalisation of sex-ratios
- Reduction of reproductive skew (often means levelling the playing-field for males in the pop)

Are these measures likely to prove practical?